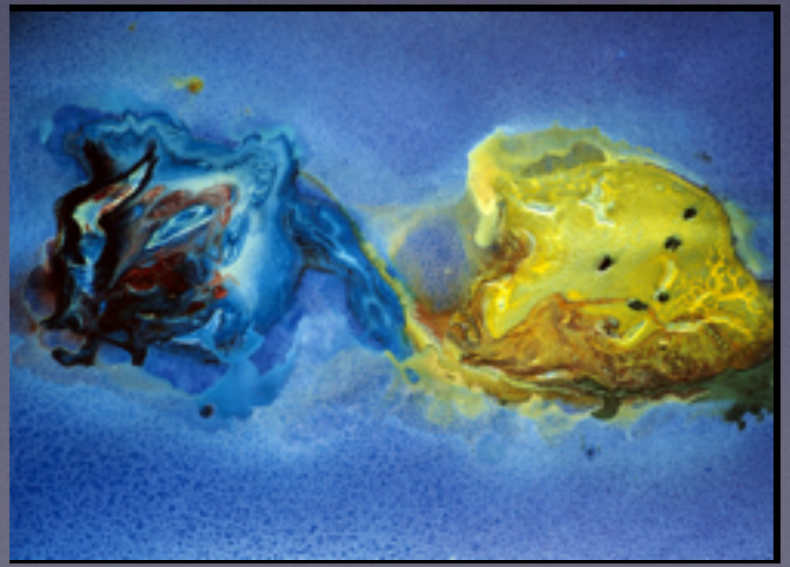
TECHNISCHE UNIVERSITÄT MÜNCHEN

EFTs for Nonrelativistic bound and near threshold states

NORA BRAMBILLA



PHYSIK DEPARTMENT TUM
T30F

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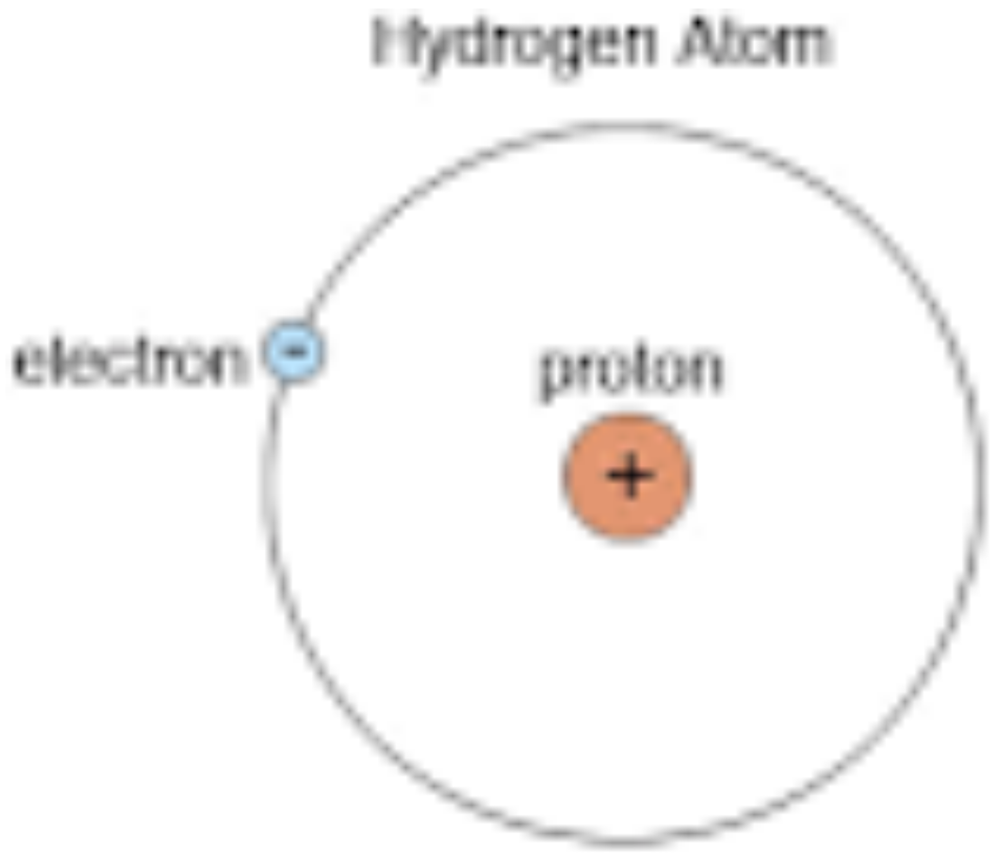
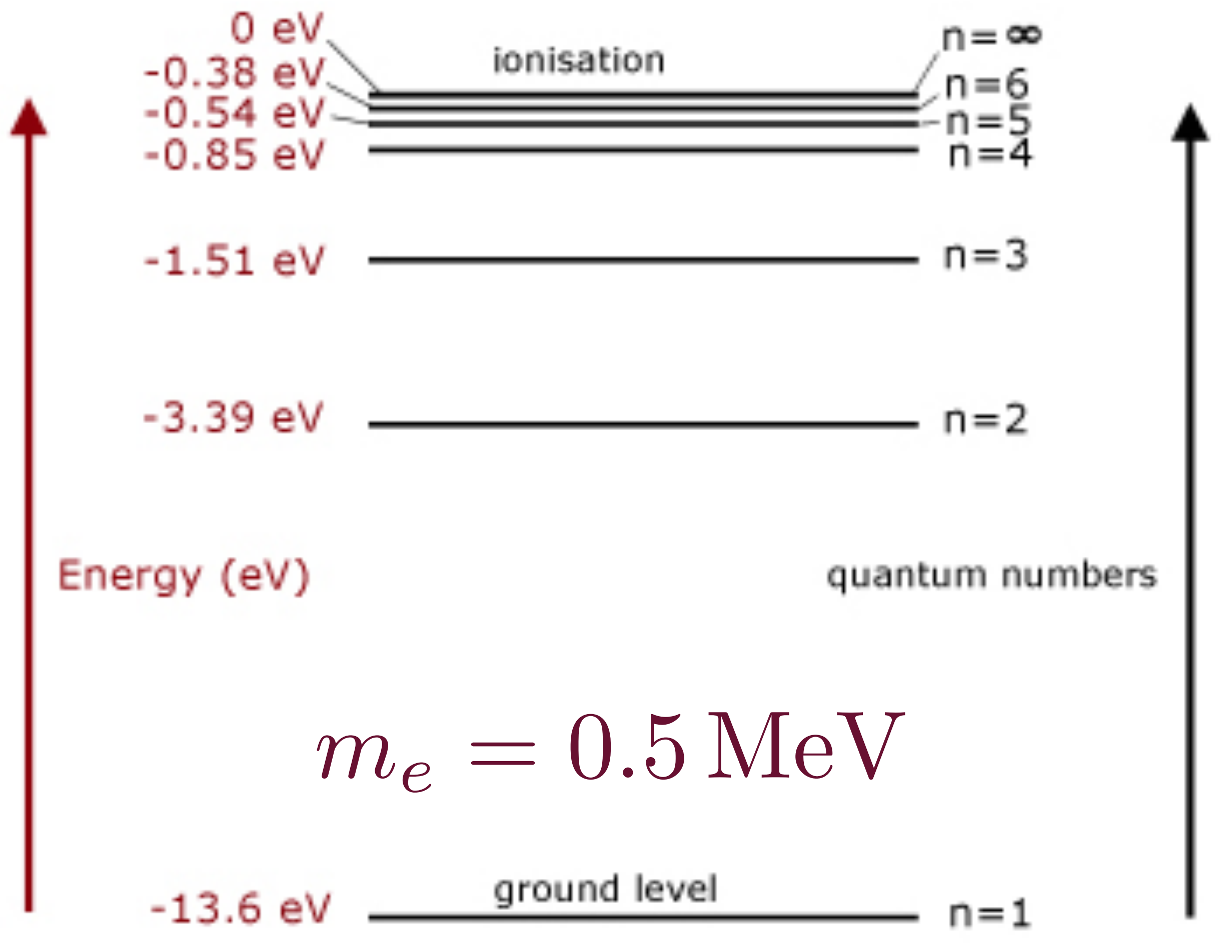
- Interplay of EFT and lattice and combination of different EFTs

Prototype of NR states: Hydrogen Atom





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for the hydrogen atom

$$v = \alpha = \frac{e^2}{4\pi}$$

fine structure constant

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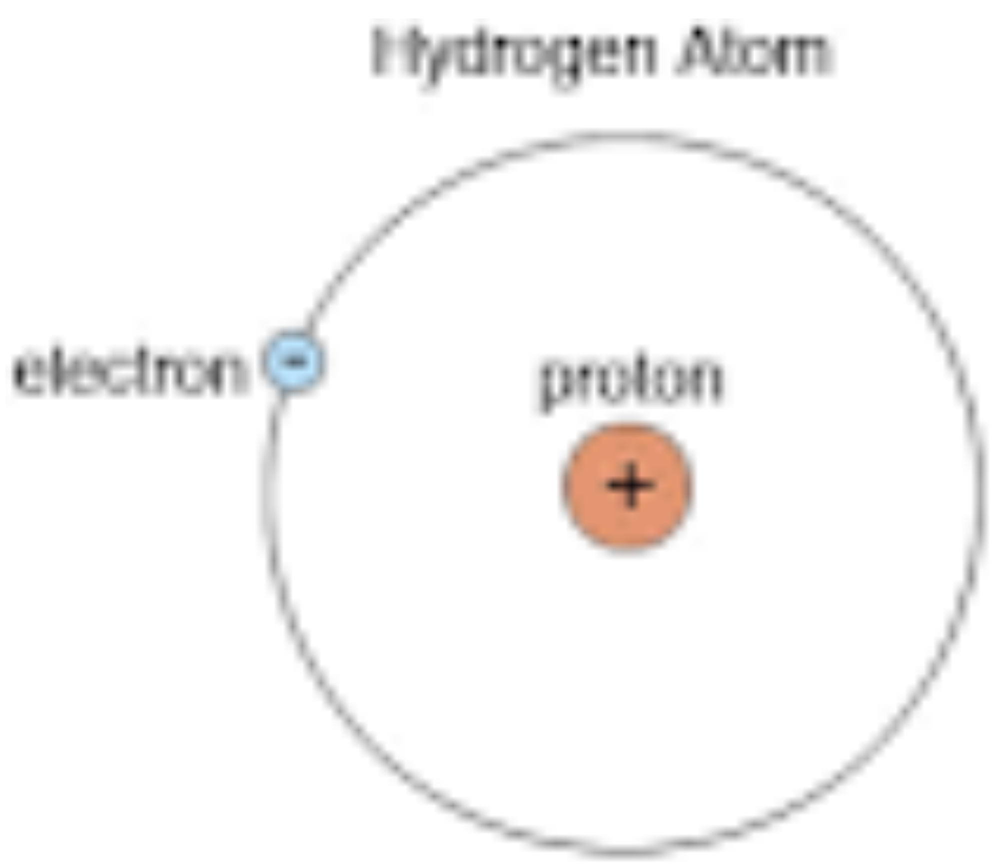

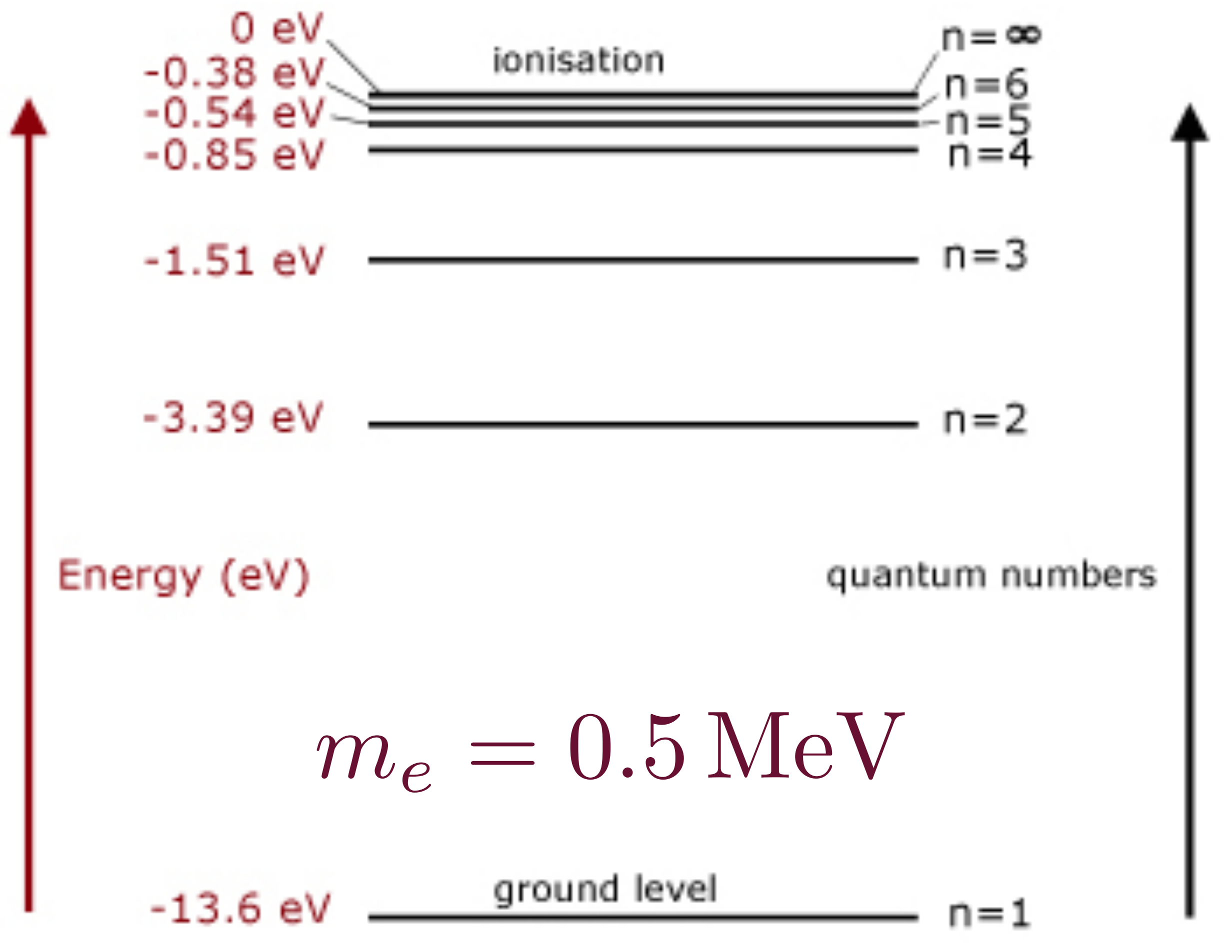

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The relevant scales of the non-relativistic bound state dynamics are

- $E \sim \frac{\mathbf{p}^2}{2m} \sim V \sim mv^2$,
- $p \sim 1/r \sim mv$;

a crucial observation: if v(elocity)$\ll 1$, then $m \gg mv \gg mv^2$.

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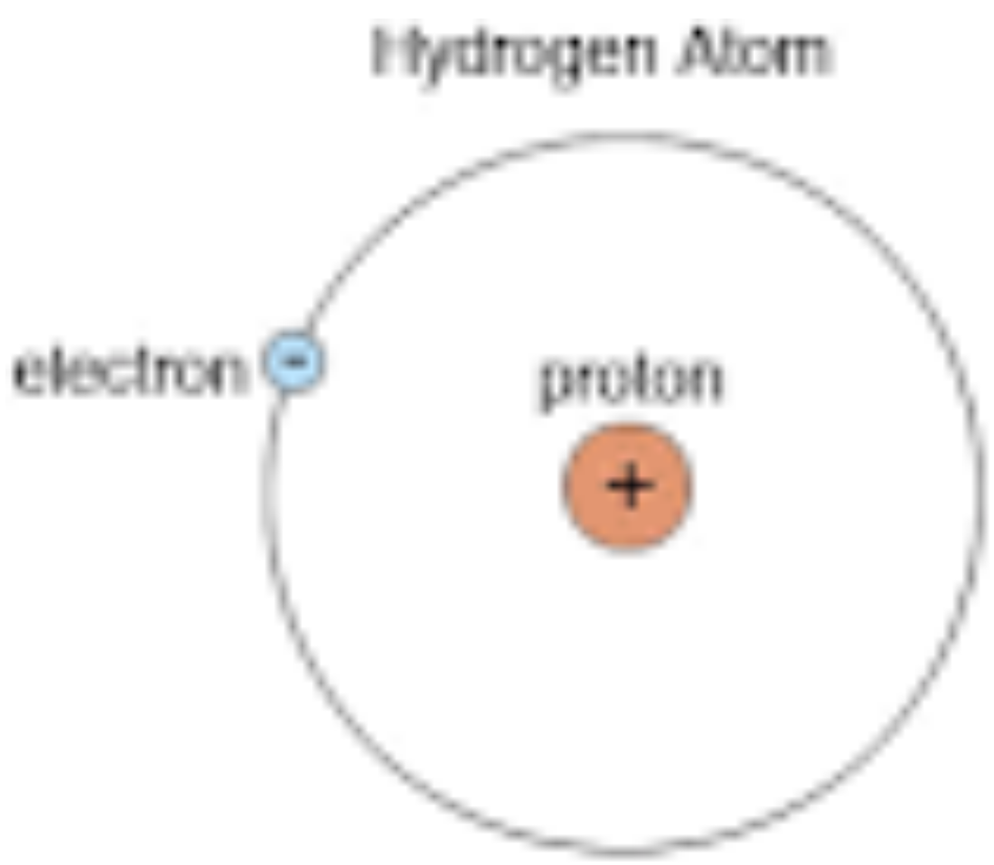

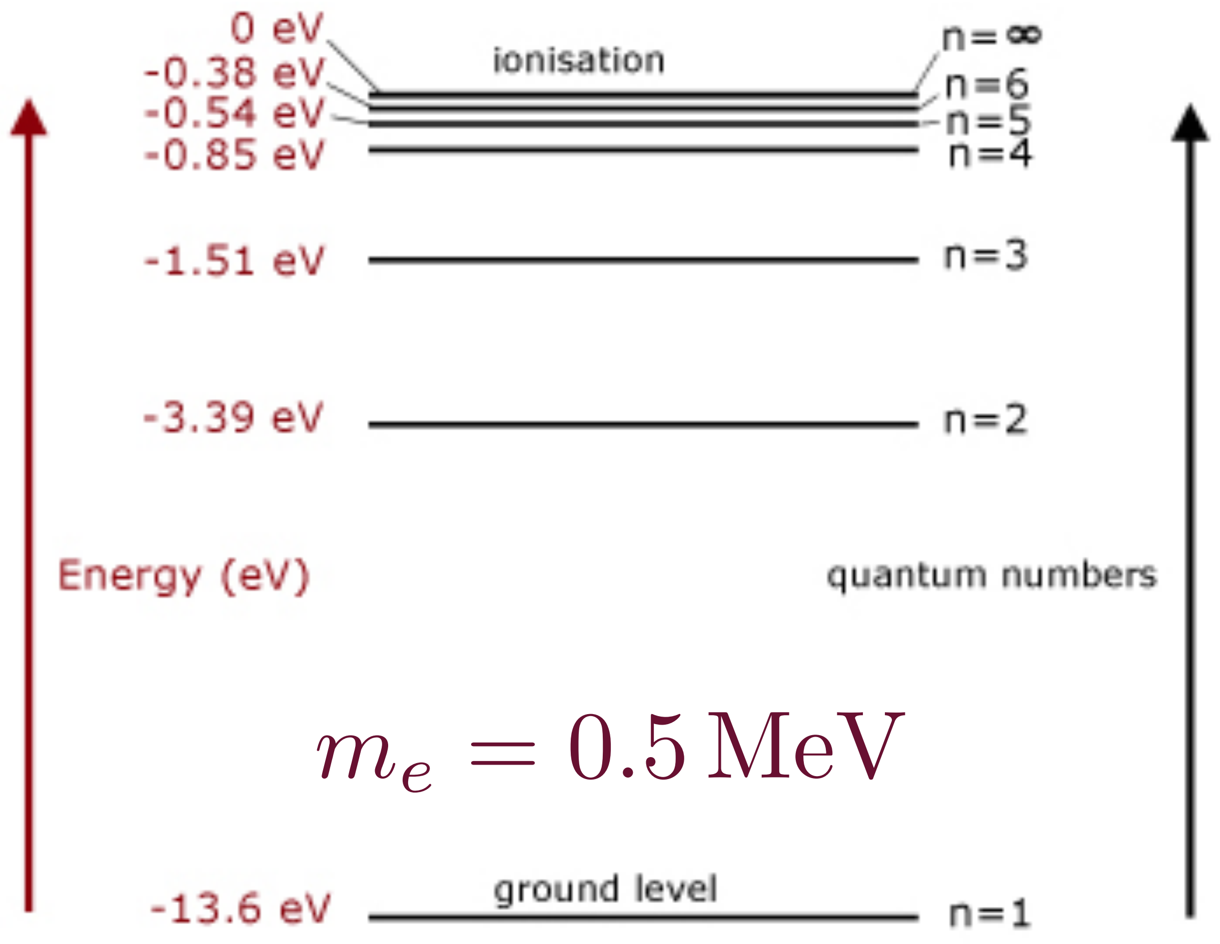

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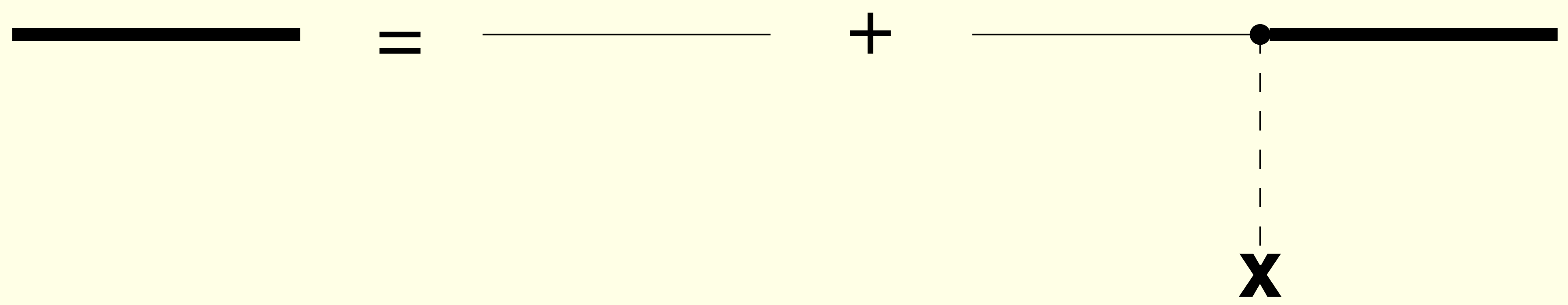
Non-relativistic (NR) bound states accompanied the history of the quantum theory from its inception to the establishing of the quantum theory of fields

Nonrelativistic Quantum Theory of bound states

- 1926 Schrödinger equation: $\left(\frac{\mathbf{p}^2}{2m} + V\right)\phi = E\phi$

$$\begin{cases} g = g_0 + g_0(-iV)g \\ g_0 = \frac{i}{E - \mathbf{p}^2/(2m)} \end{cases}$$



- 1927 Pauli equation: $\left(\frac{(\mathbf{p} - e\mathbf{A})^2}{2m} + V - \frac{\boldsymbol{\sigma} \cdot e\mathbf{B}}{2m}\right)\phi = E\phi$

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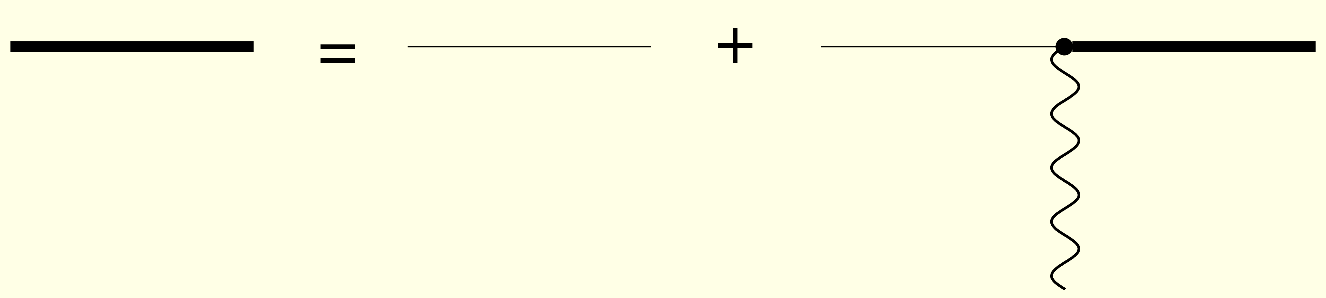

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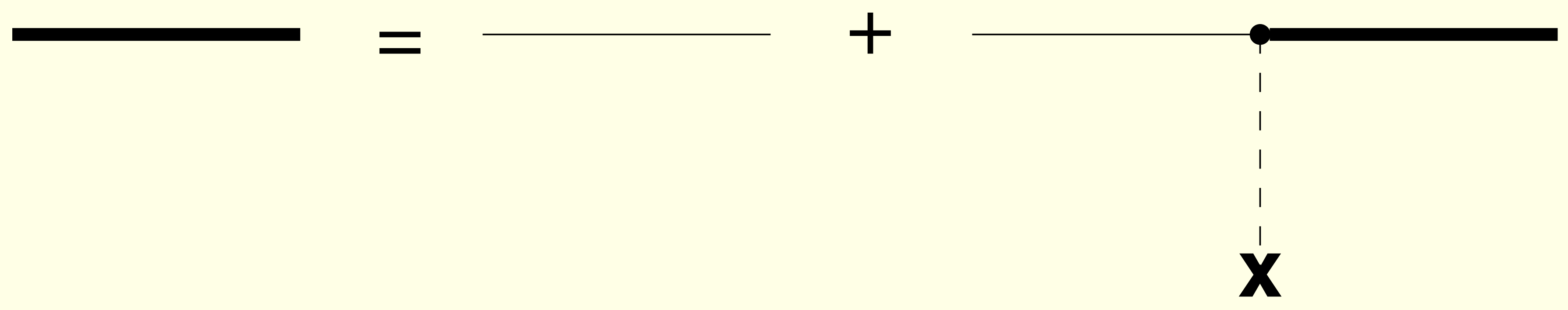

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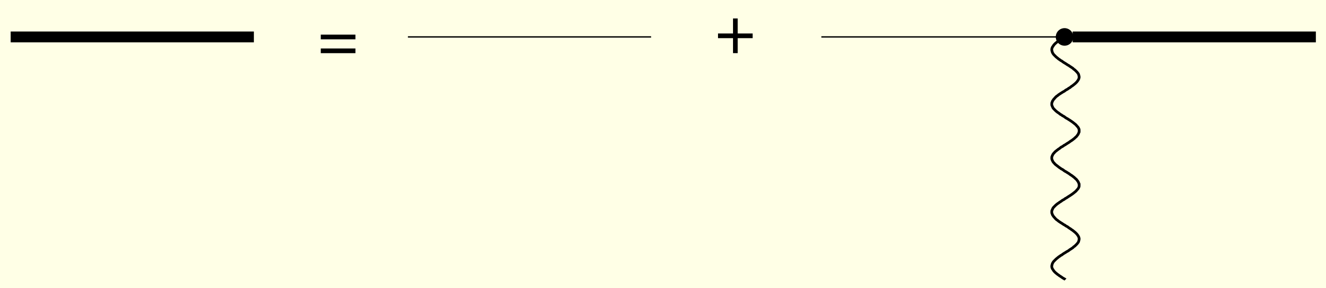

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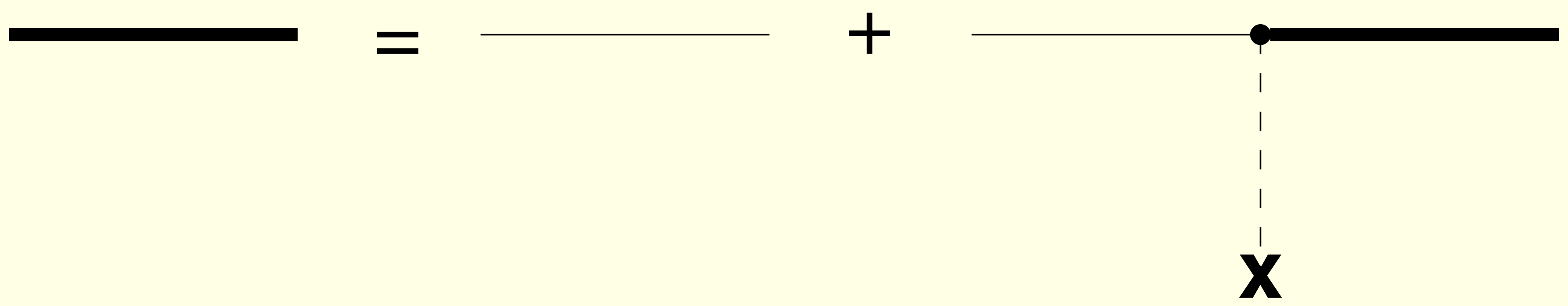

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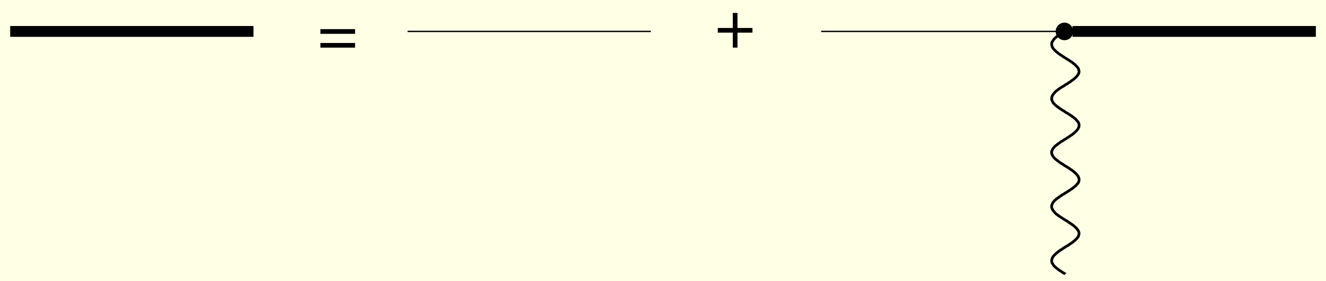

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it produces spin-corrections (spin-orbit), it does not give the Lamb shift (radiative corrections)

Relativistic Quantum Field Theory of bound states

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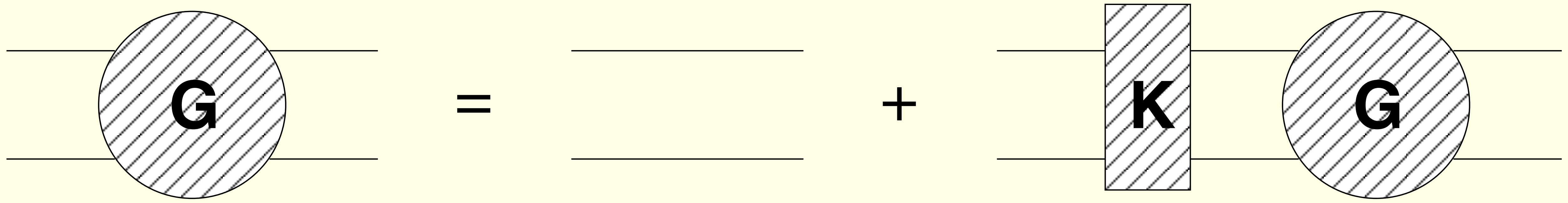

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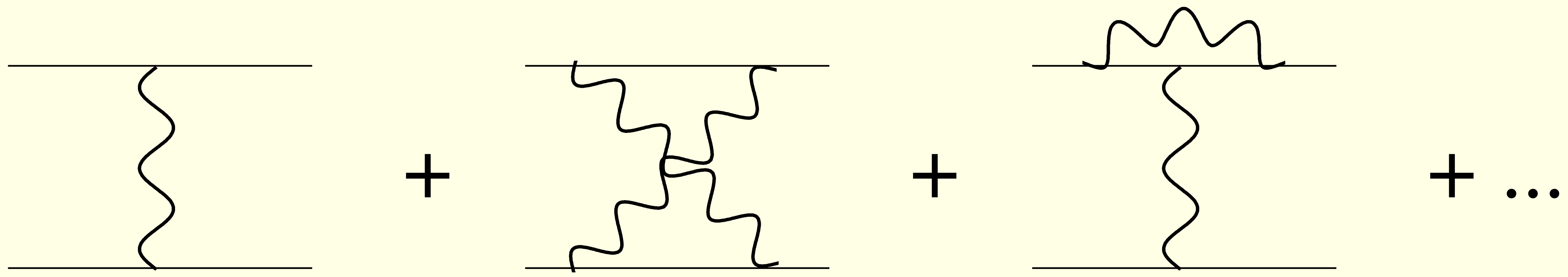

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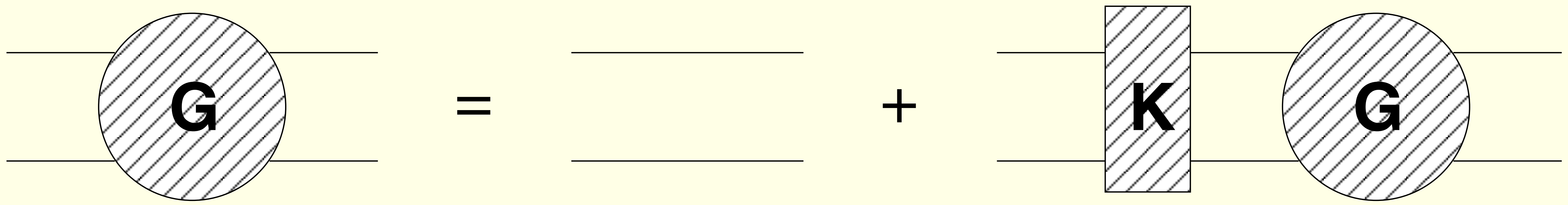

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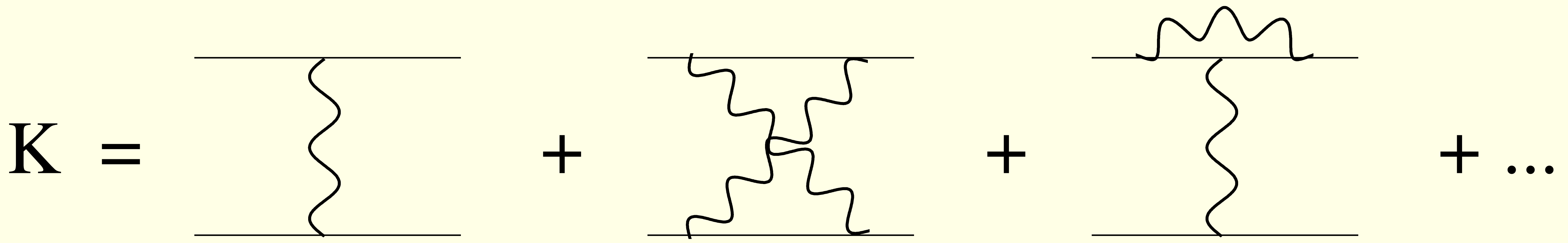

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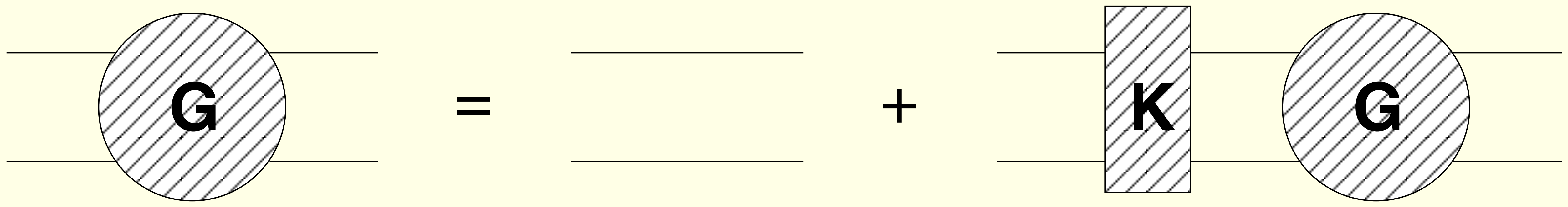

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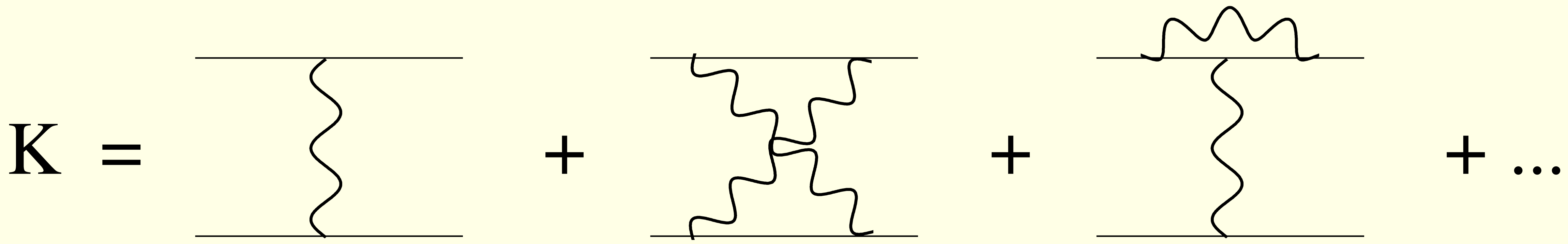

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<u>Ex.</u>

- It shows the difficulty of the approach the fact that going from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln\alpha$ term took twenty-five years!
  - `Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36 Bodwin Yennie PR 43(78)267`

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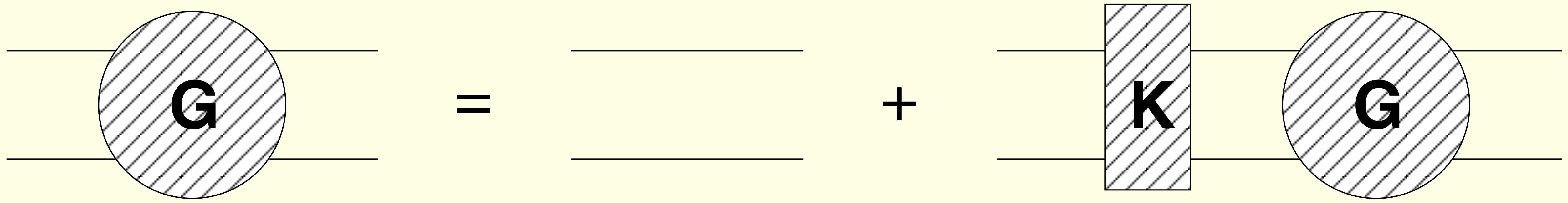

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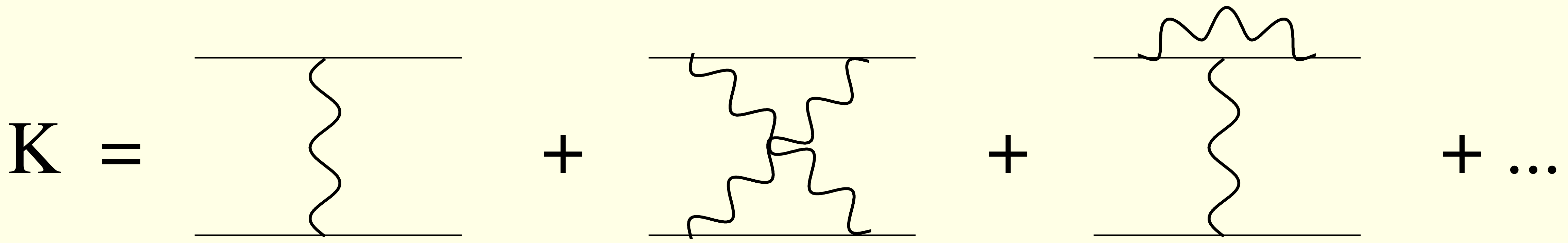

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B. It is very poorly suited to achieve factorization (especially important in QCD)

THE EFT APPROACH: PNREFT

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Relevant for: atomic physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), muonium ttbar threshold production; Dark matter annihilation and production close to threshold; SUSY particles annihilation and production; QQbar, QQq and QQQ with small radius; extraction of SM parameters

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II. In QCD (or in a strongly coupled theory) it factorizes automatically high energy contributions (perturbative) from low-energy (nonperturbative, thermal) ones

Relevant for: pionium and precision chiral dynamics; nucleon-nucleon systems; Quarkonium, Exotic X, Y, Z states, Quarkonium in hot QCD medium in heavy ion collisions; NR states in early universe; confinement and nonperturbative effects

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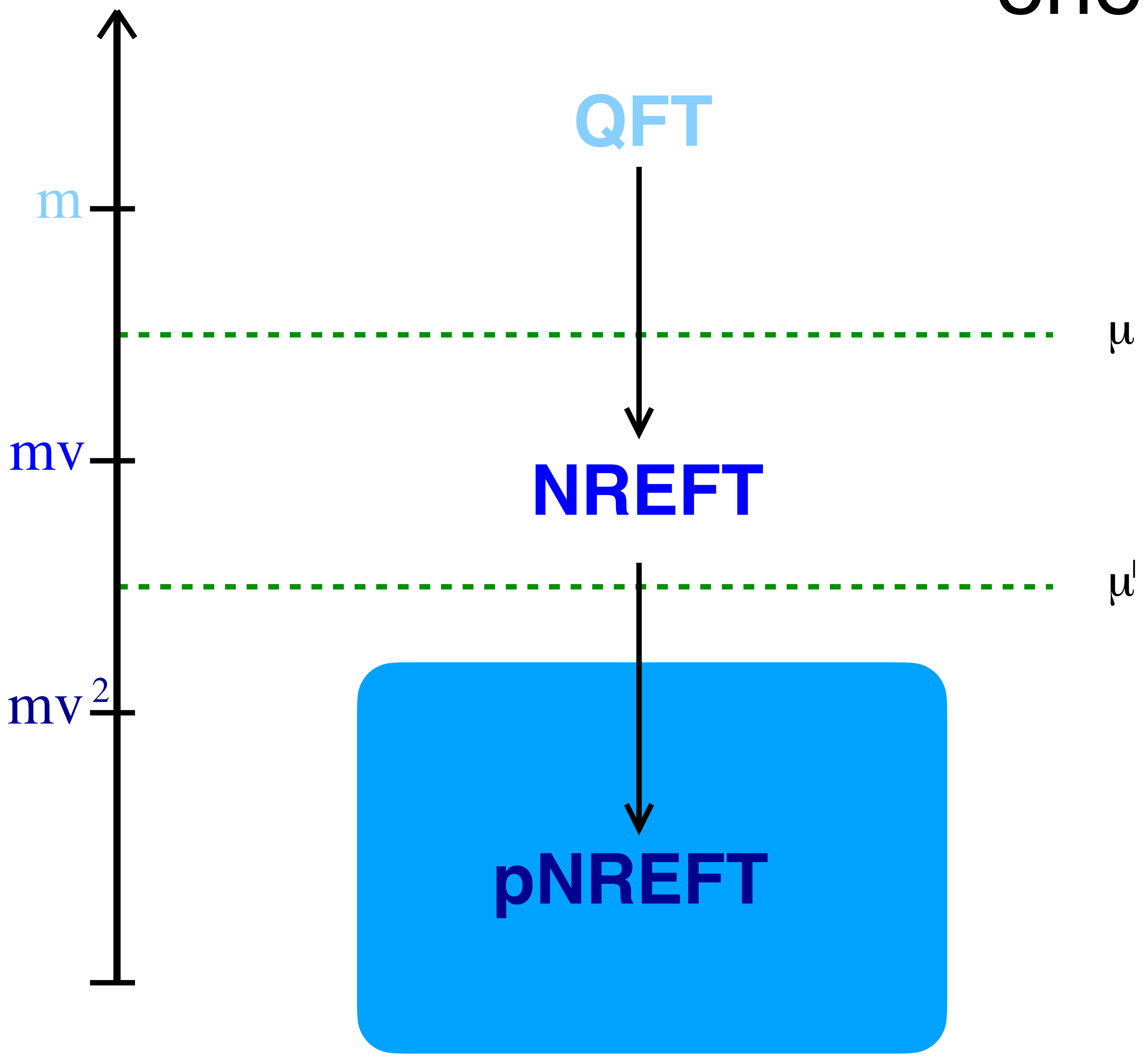
IV. It allows to define in QFT objects of great importance like potentials

More conceptually V. It provides a field theoretical foundation of the Schroedinger eq.:

the Lagrangian $$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

separates the Schroedinger dynamics of the two particle field ϕ from the low energy dynamics encoded in $\Delta\mathcal{L}$



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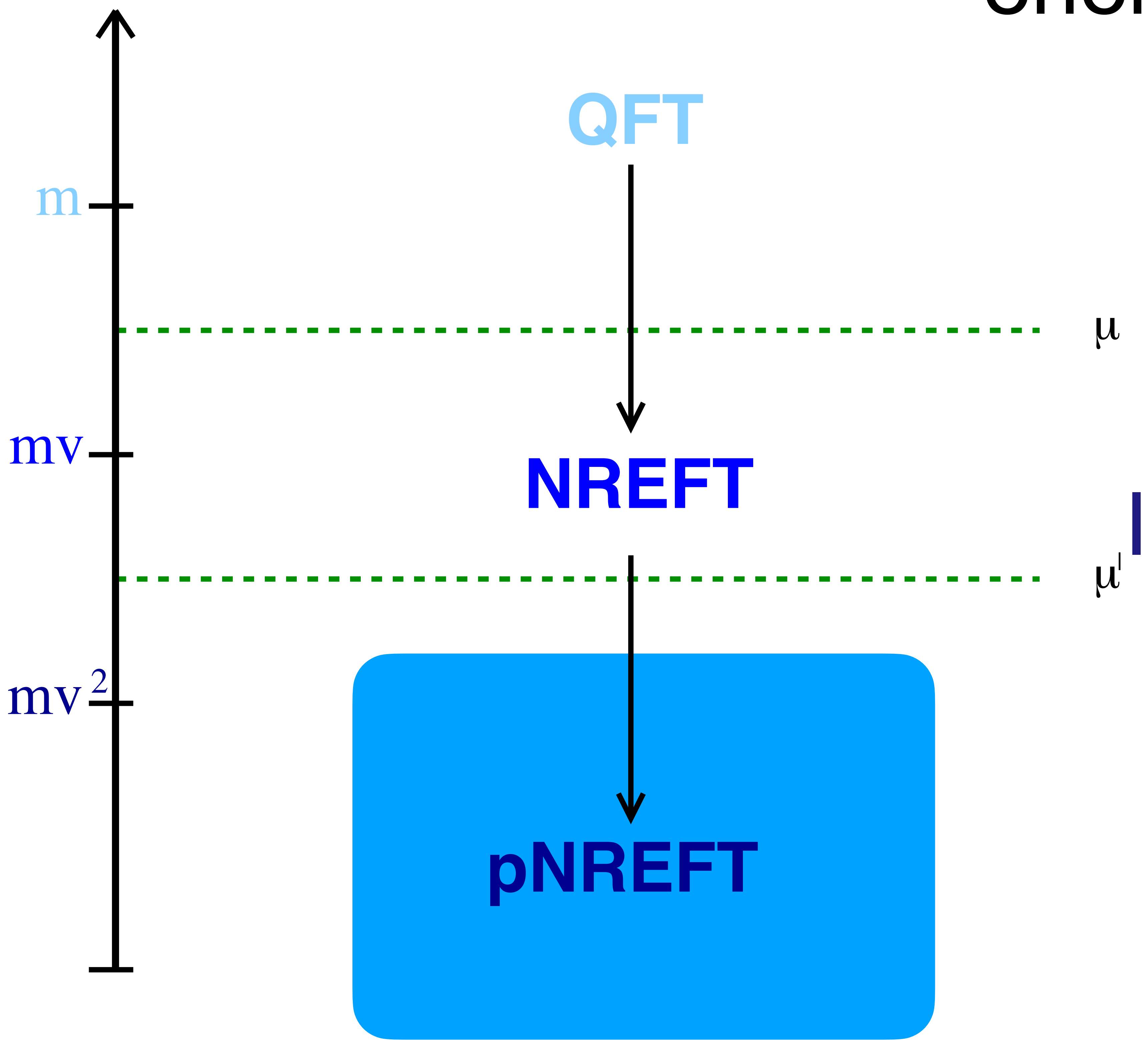

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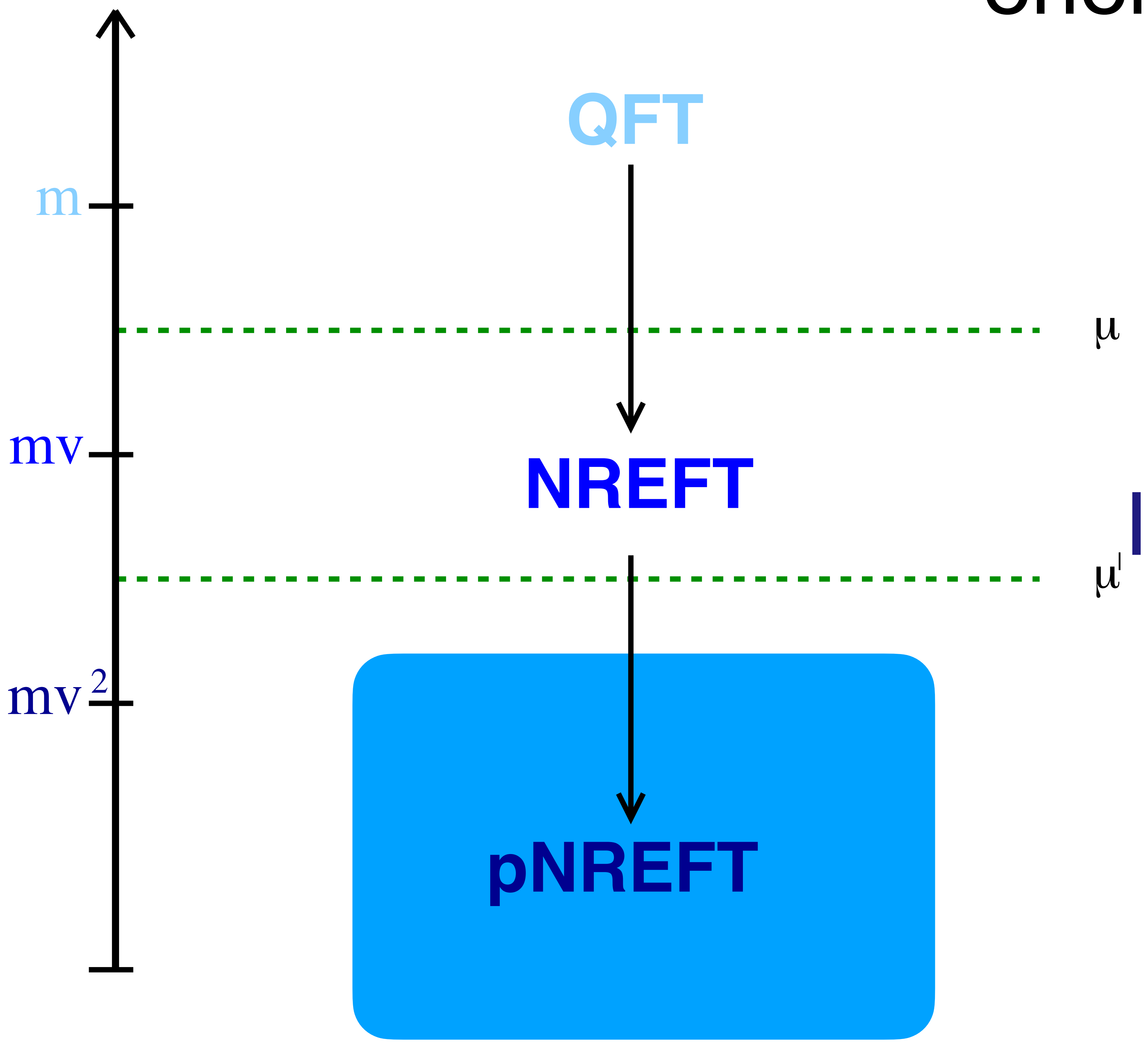

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if QFT = QED, pNRQED (Pineda,Soto 1998) gives a proper version of Quantum Mechanics

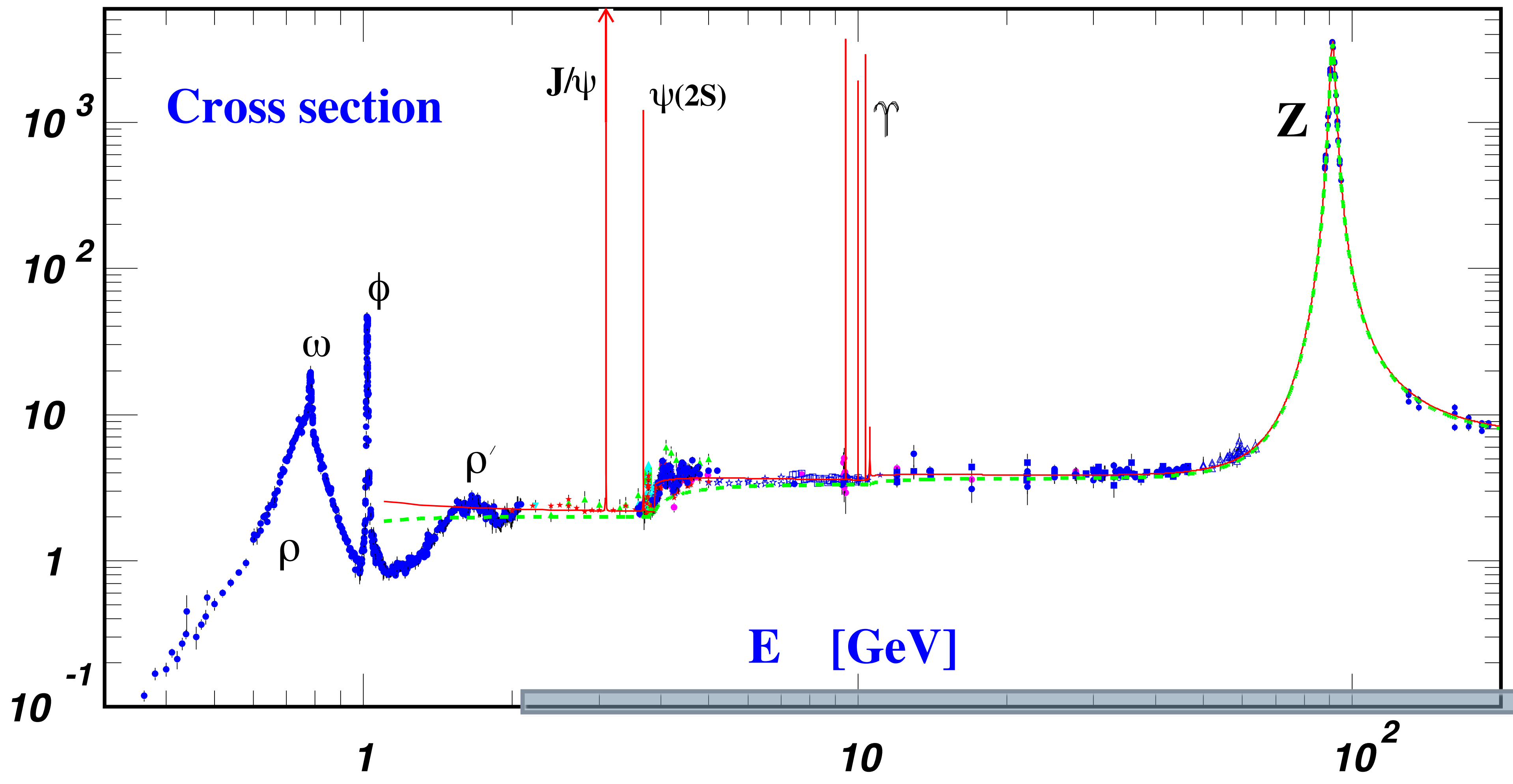
A rich physical example of NR state: quarkonium

in 1974 the J/psi discovery triggered the
November revolution:
charm discovery and confirmation of asymptotic freedom

Today from the physical point of view it is a golden probe of strong interactions

From the technical point of view it is a pretty interesting system, that add $\Lambda_{\rm QCD}$ to the physical scales and requires two different versions of pNREFT, weakly coupled and strongly coupled, for $mv > \Lambda_{\rm QCD}$ and $mv \sim \Lambda_{\rm QCD}$ respectively

NR bound states formed by heavy quarks offer a privileged access to strong interactions



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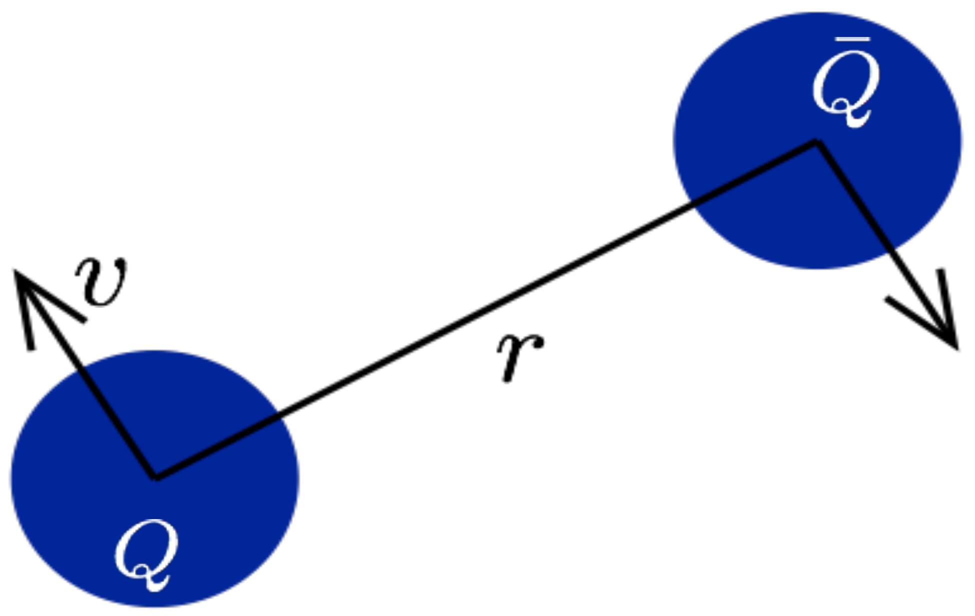

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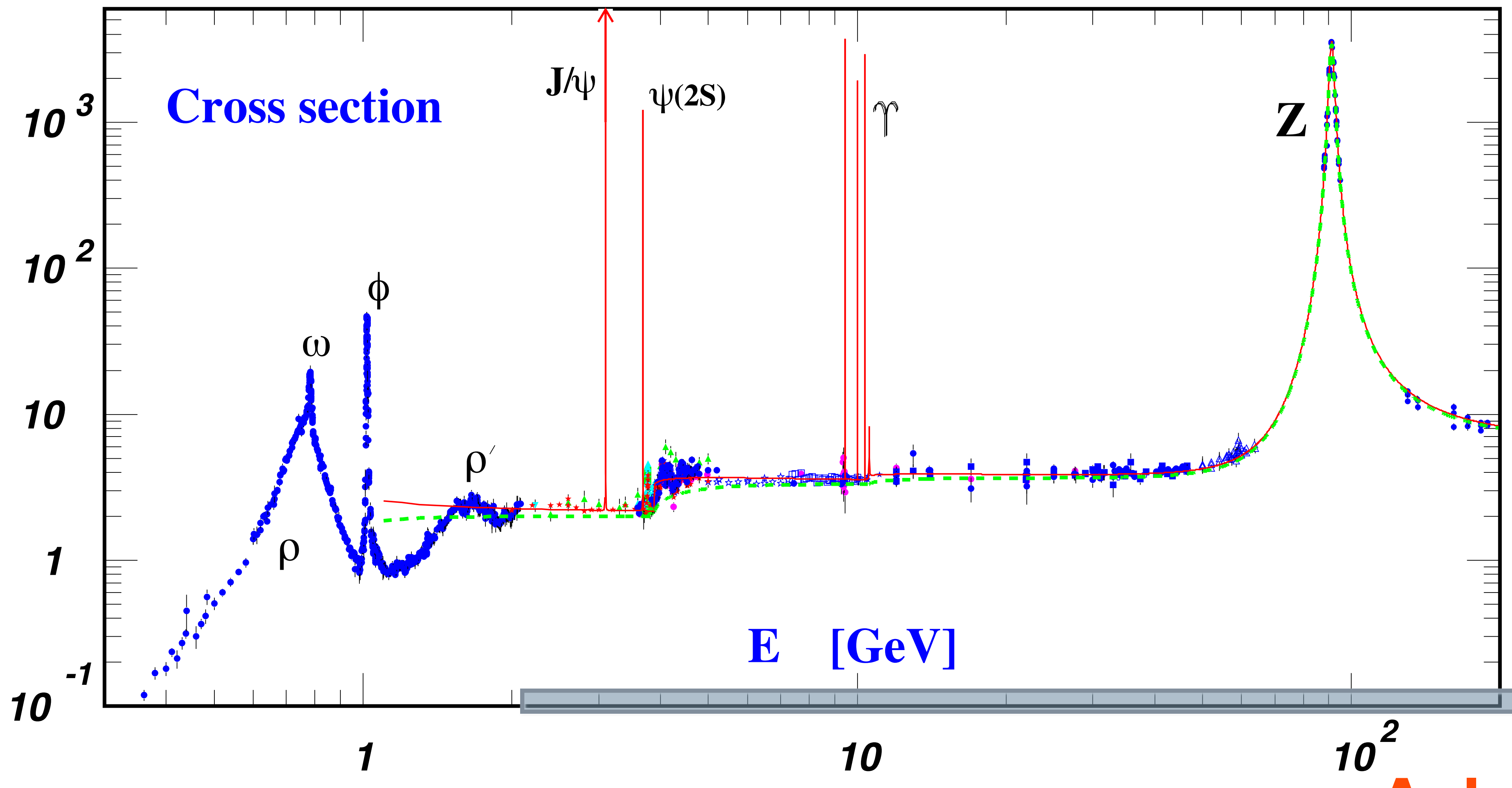

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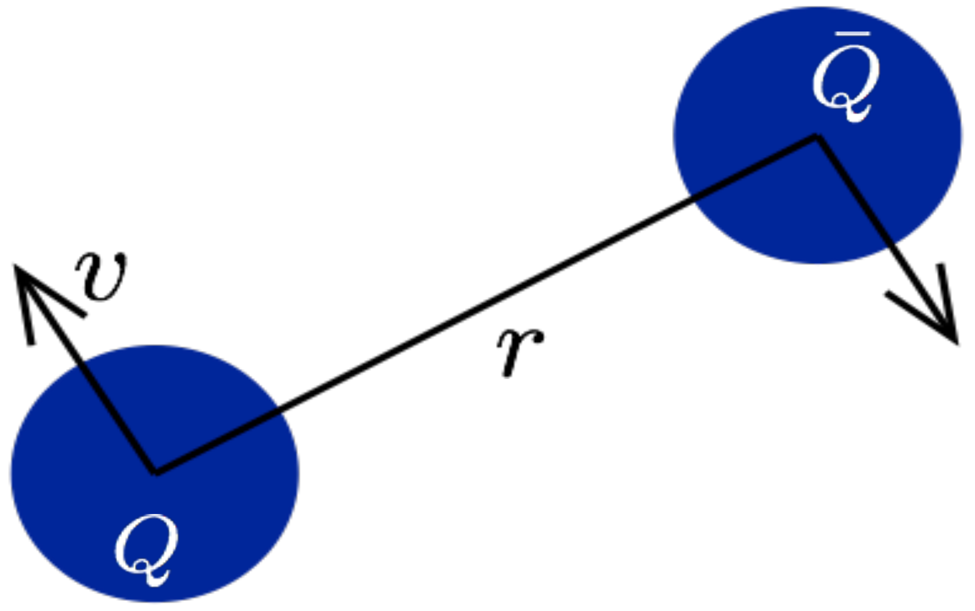

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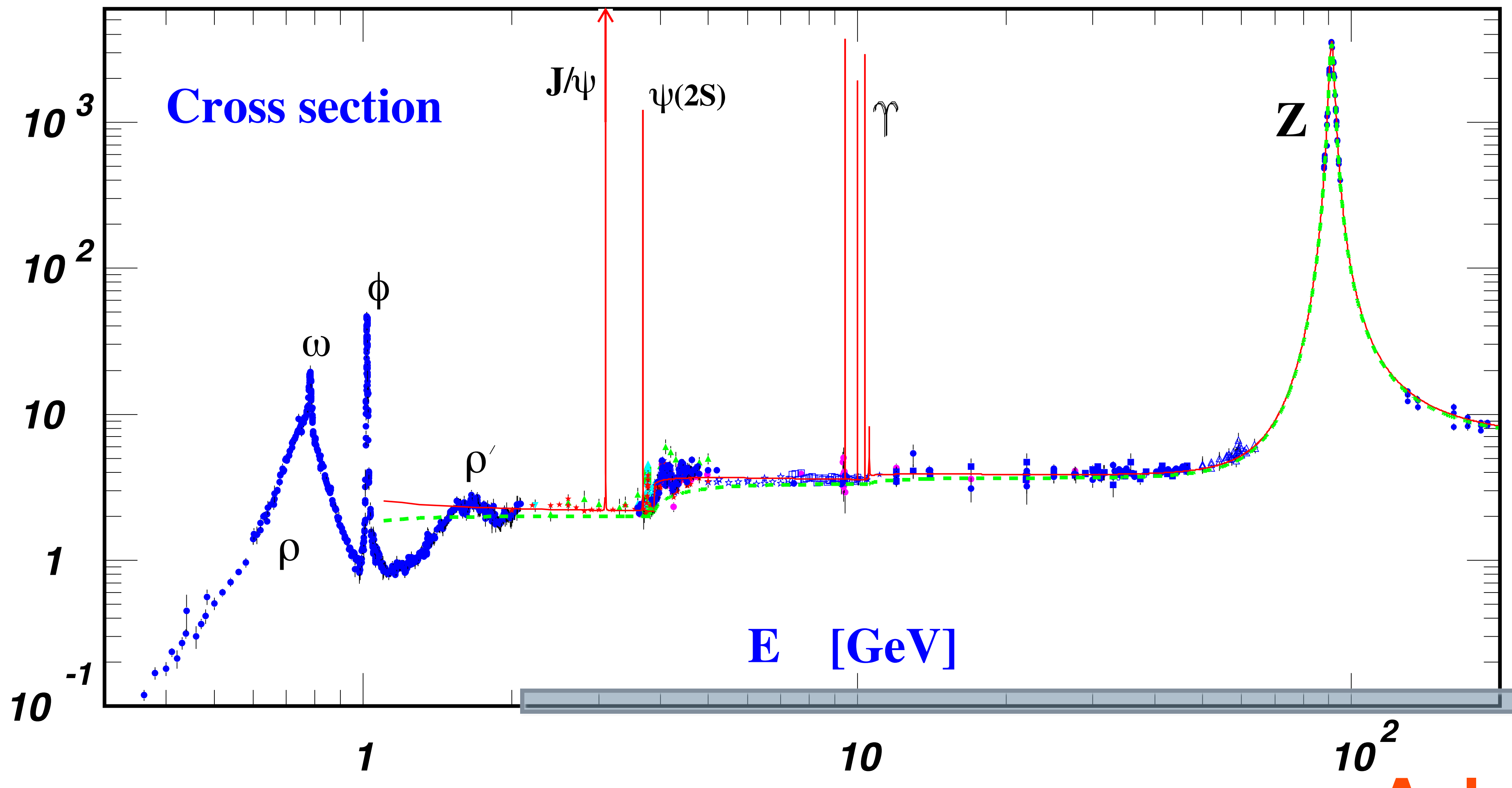

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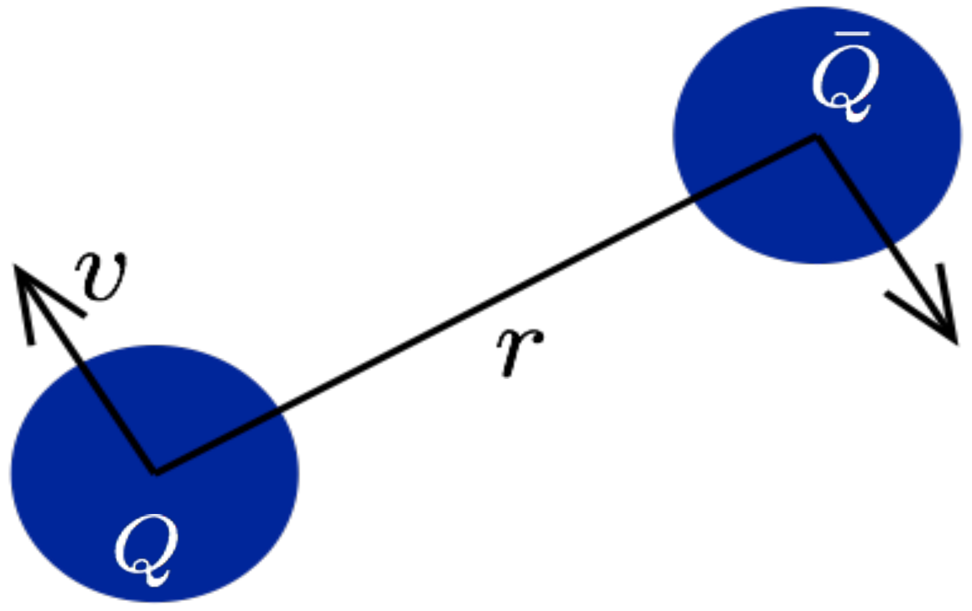

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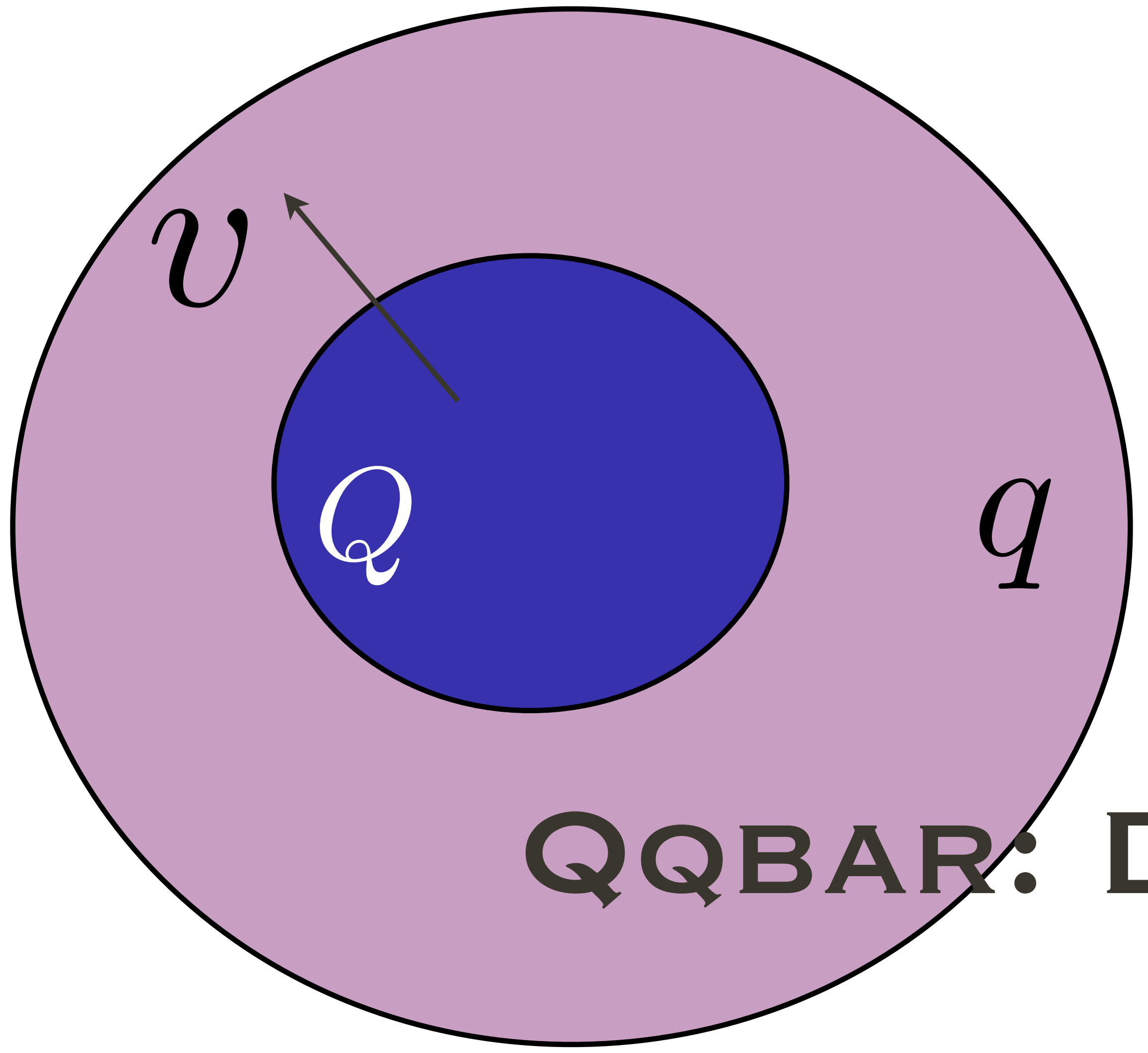
Heavy quarkonium is very different from heavy-light hadrons



different physics from the heavy light meson where only two scales exist m and Λ_{QCD}

Qqbar: D, B mesons

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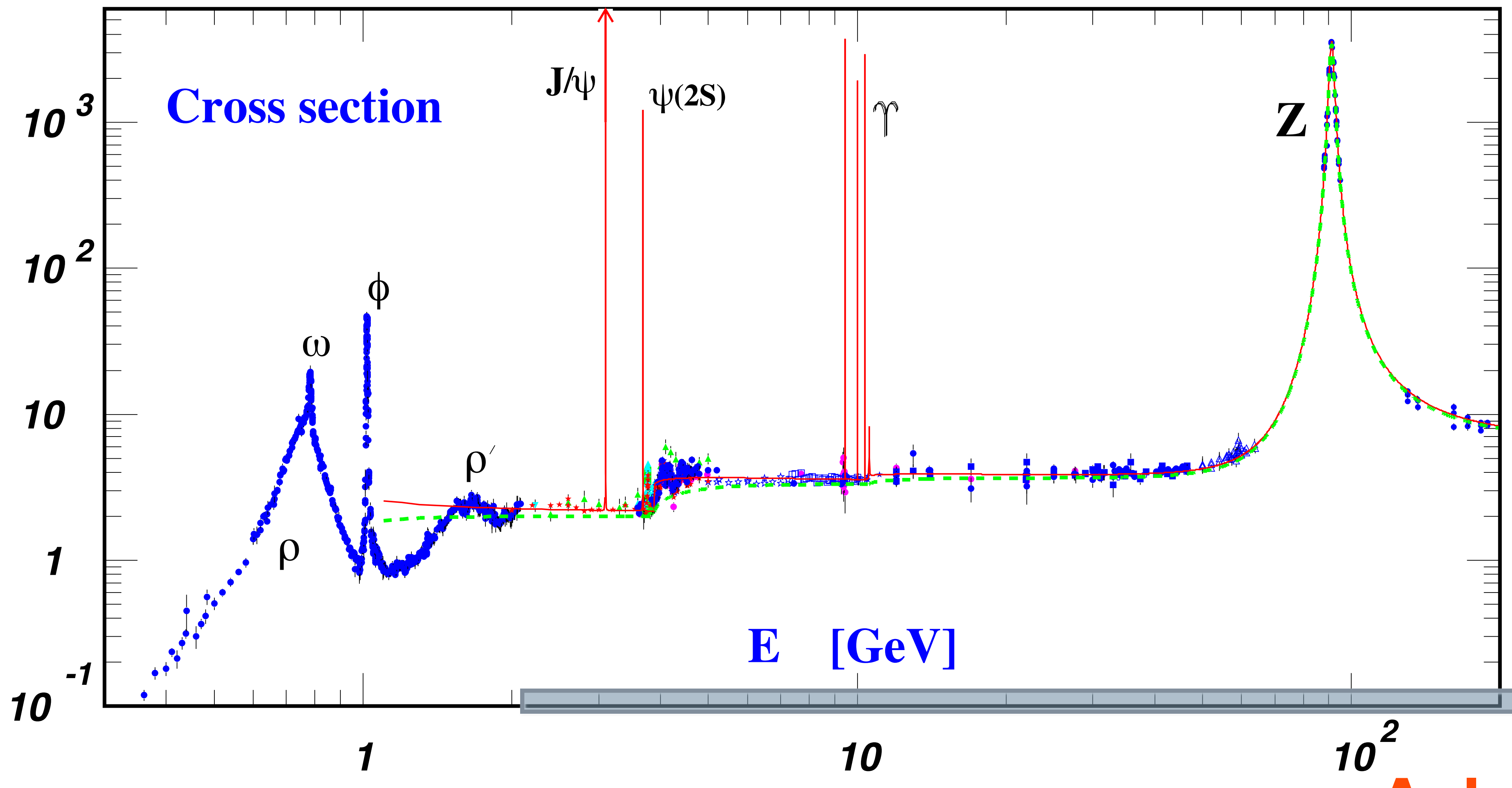

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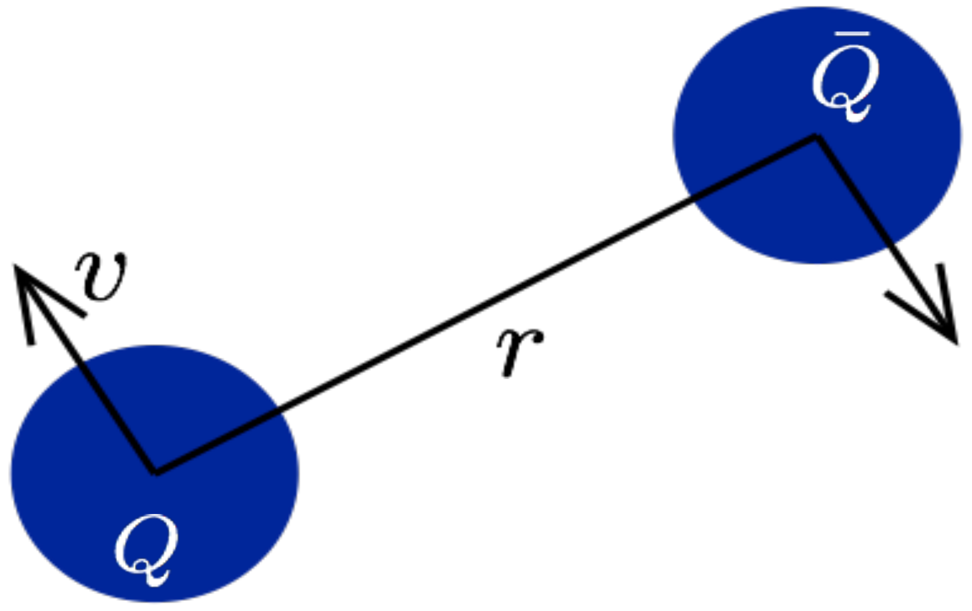

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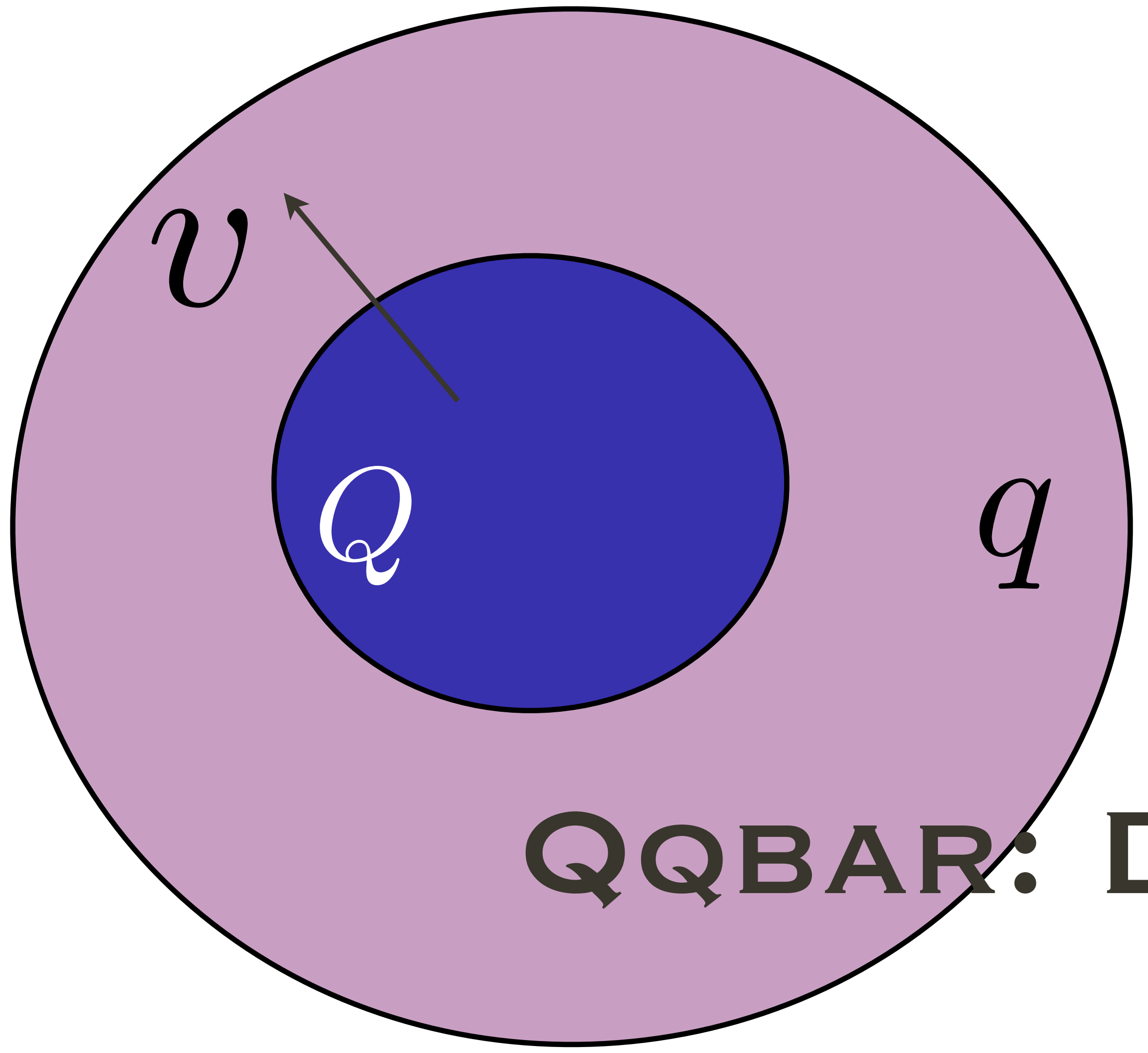

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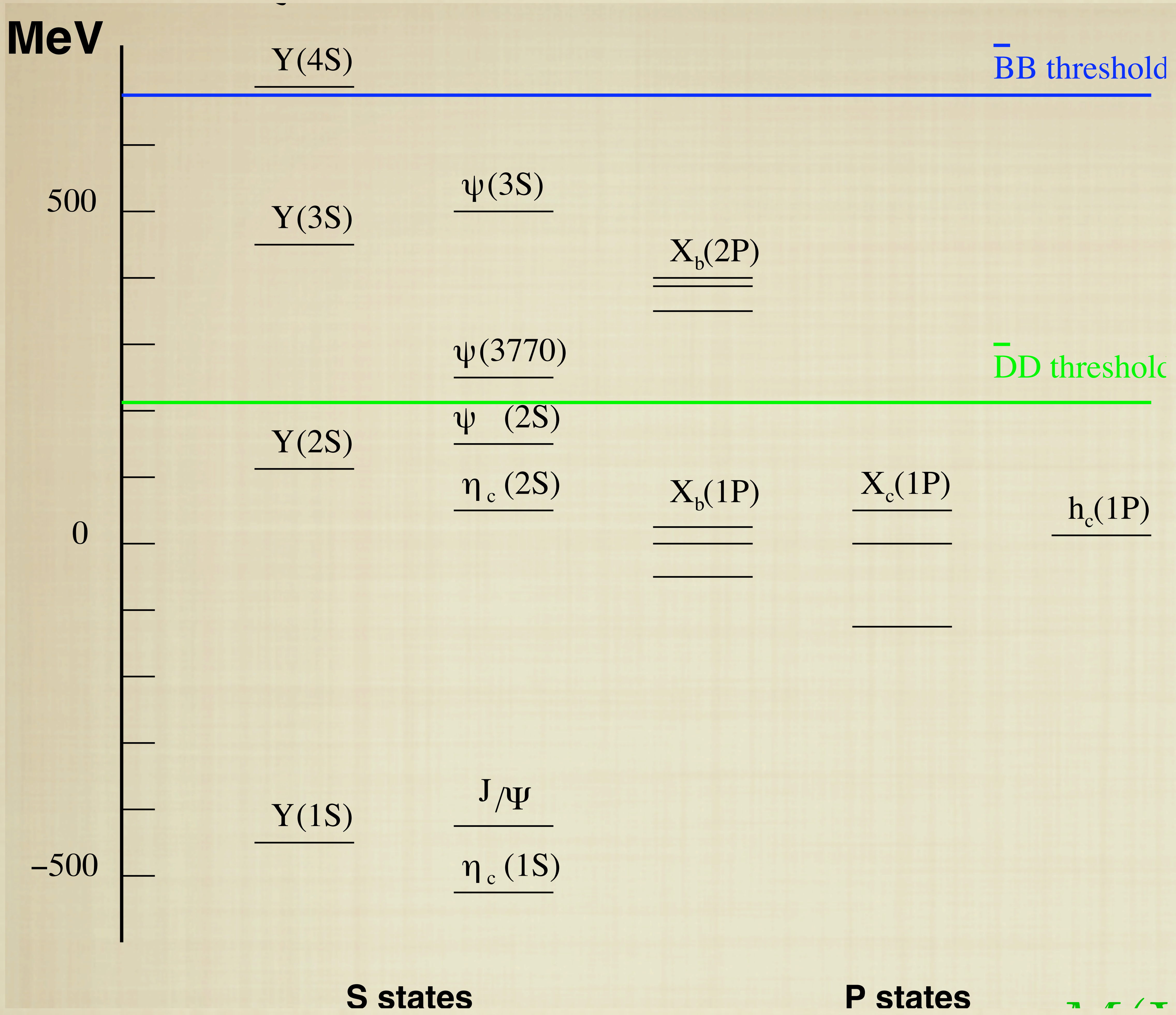
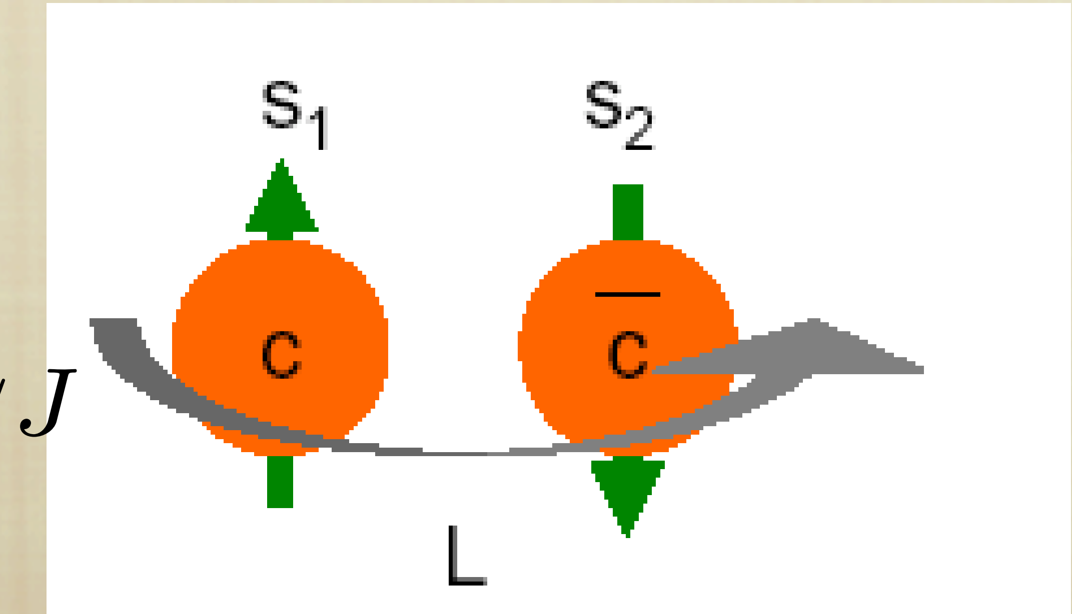
for heavy-light the EFT is HQET (Heavy Quark Effective Theory)

Quarkonium Scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$



$$M(\Upsilon(1S)) = 9460\,\mathrm{GeV}$$

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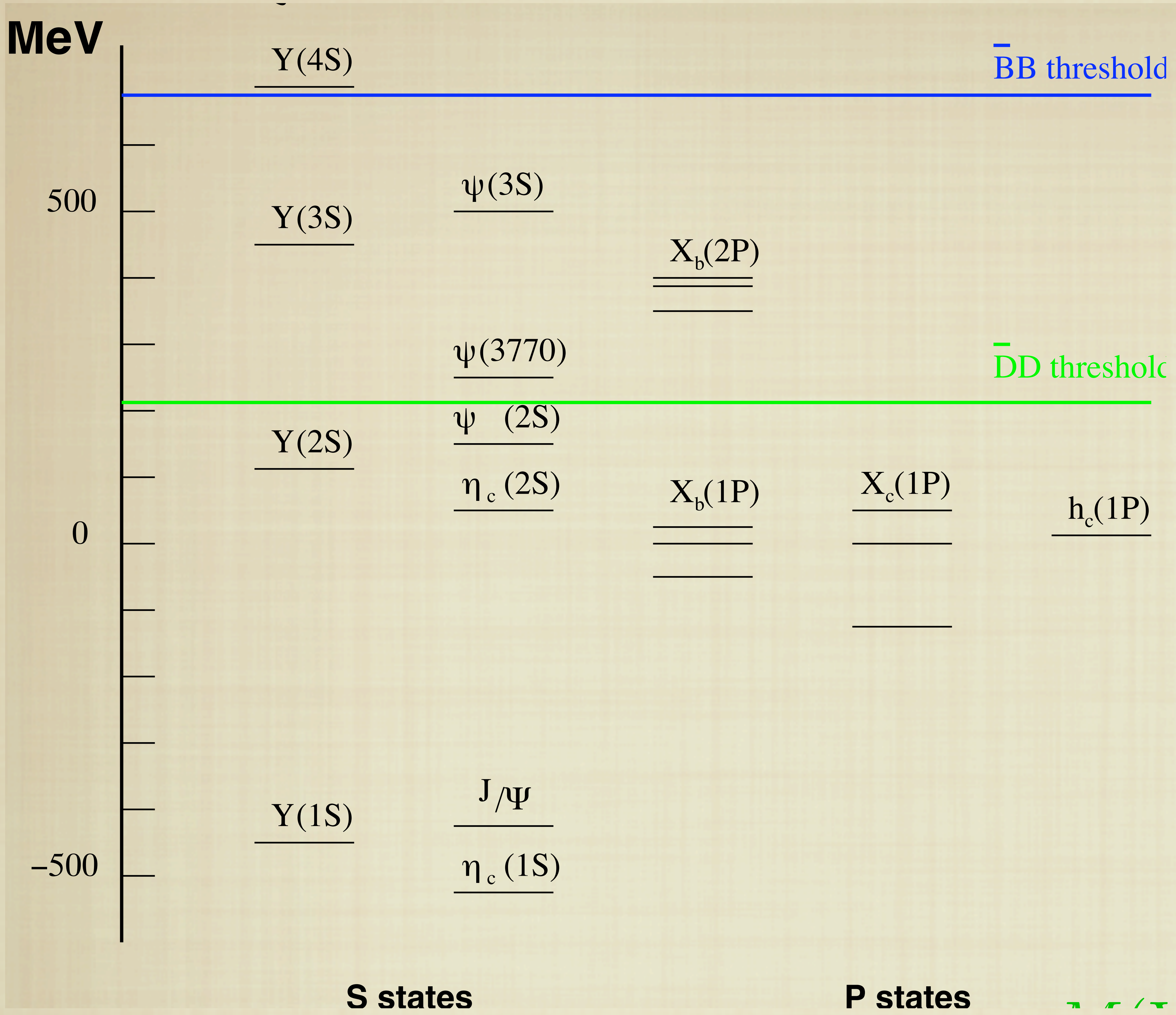

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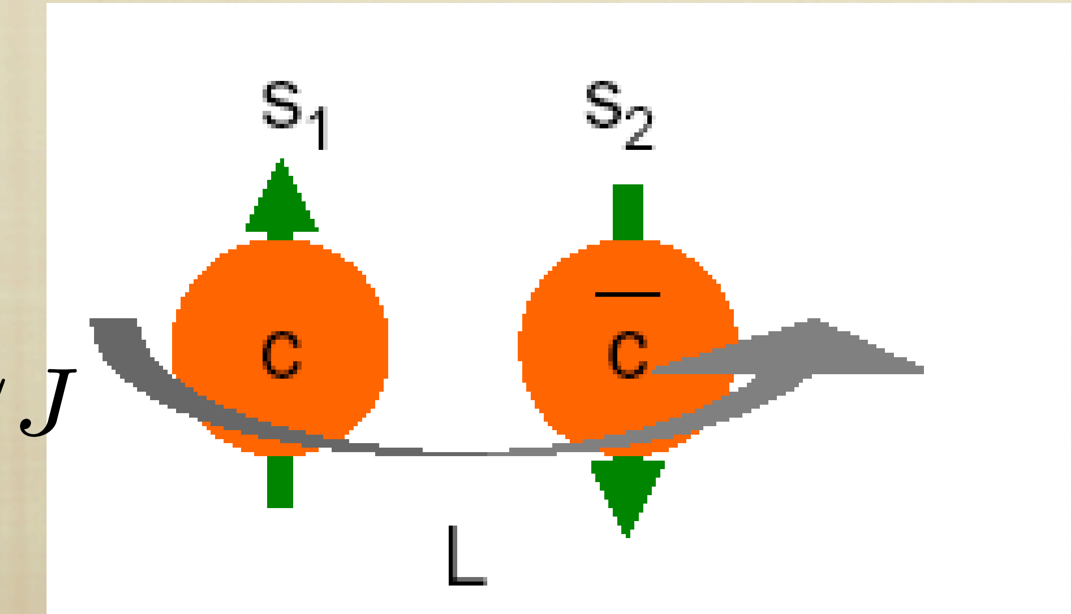

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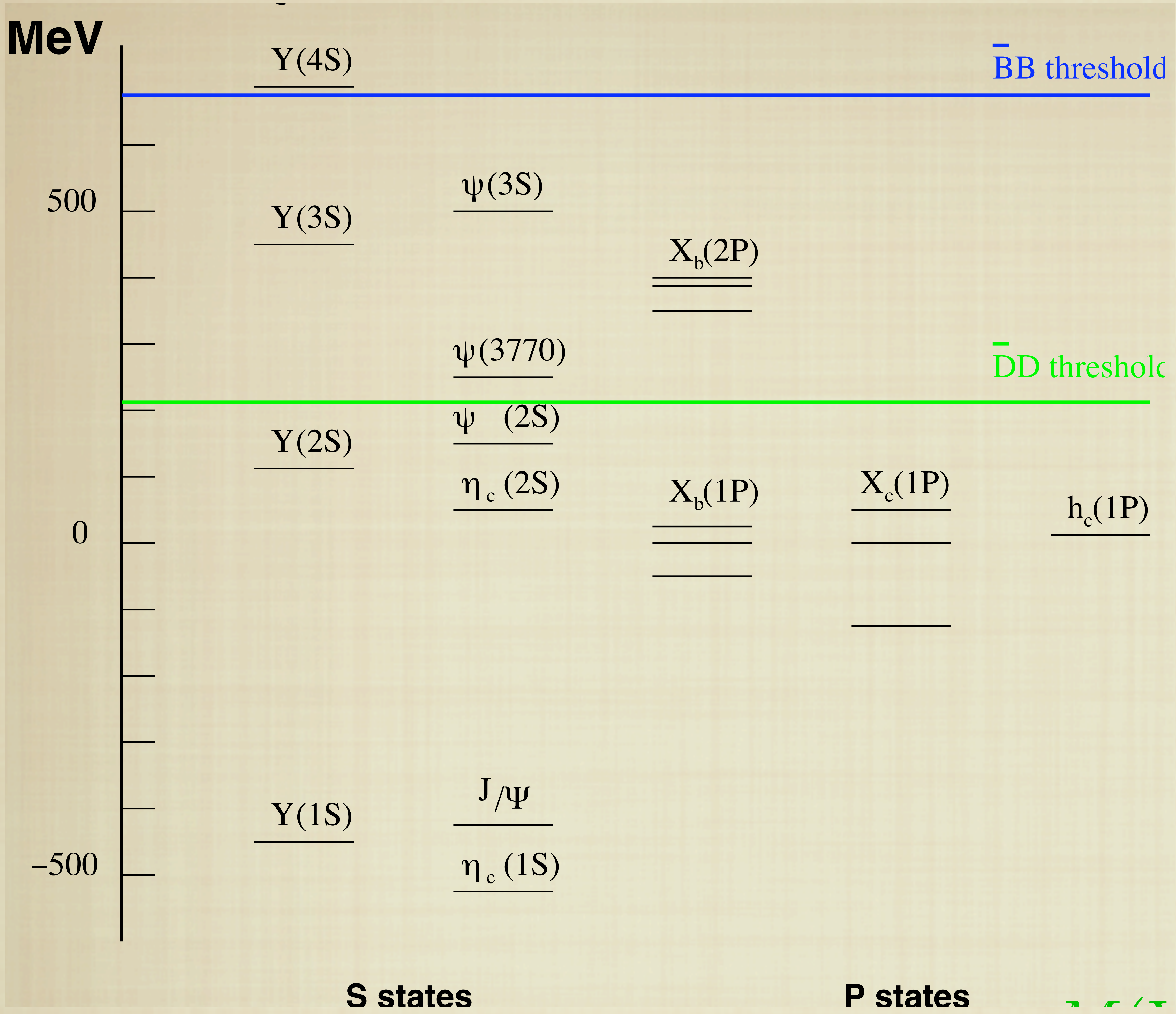

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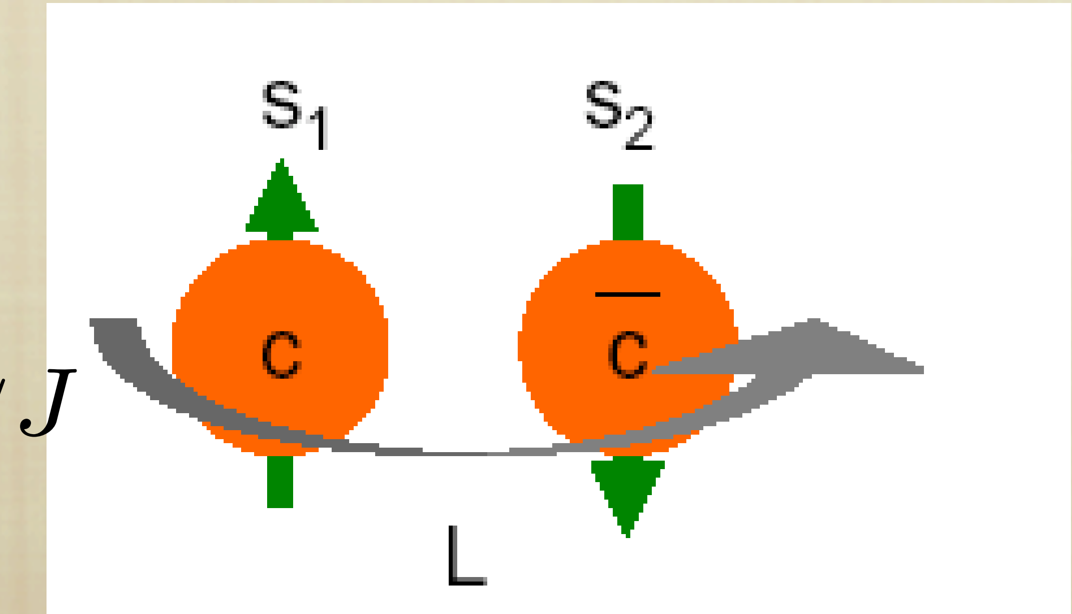

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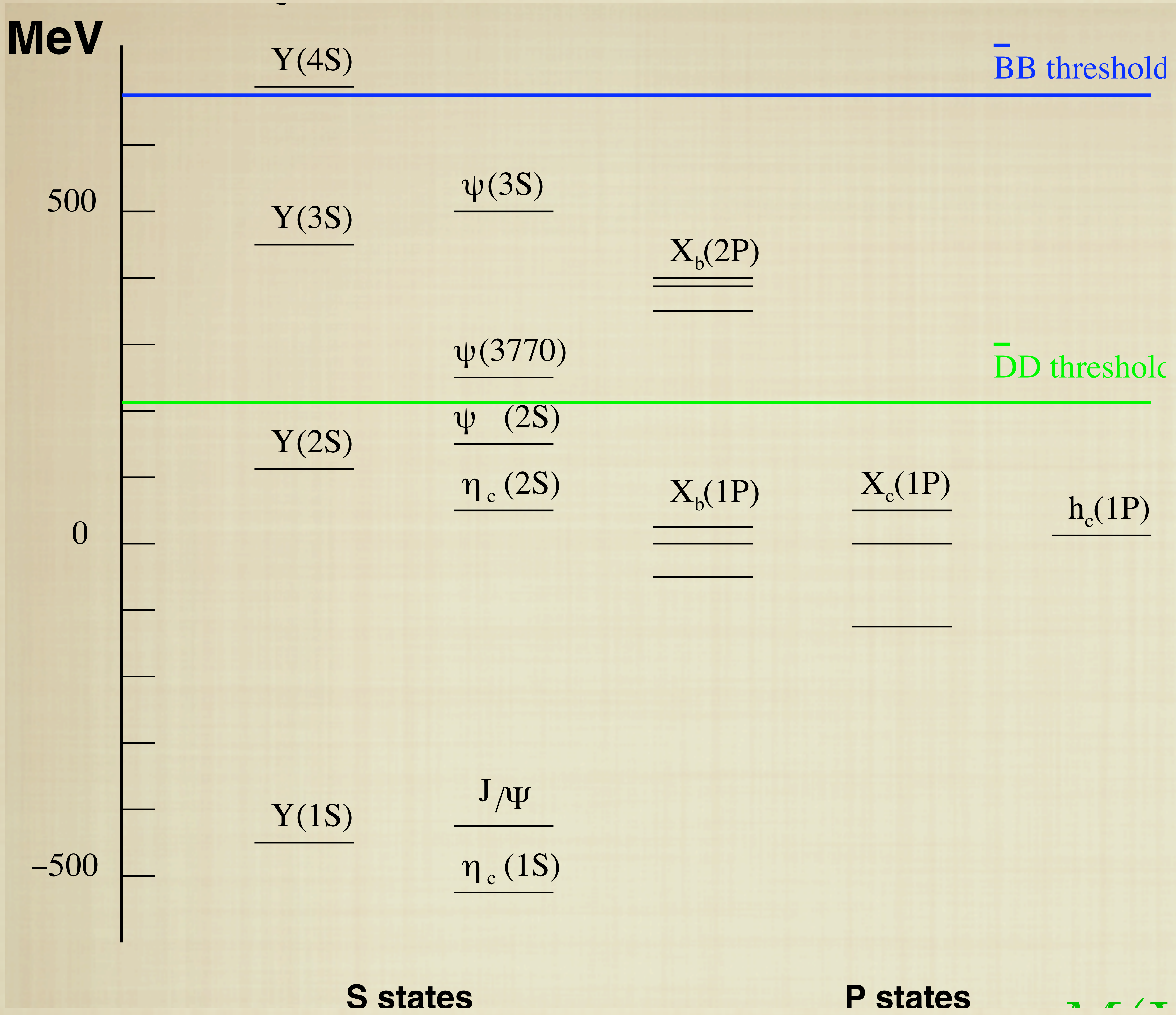

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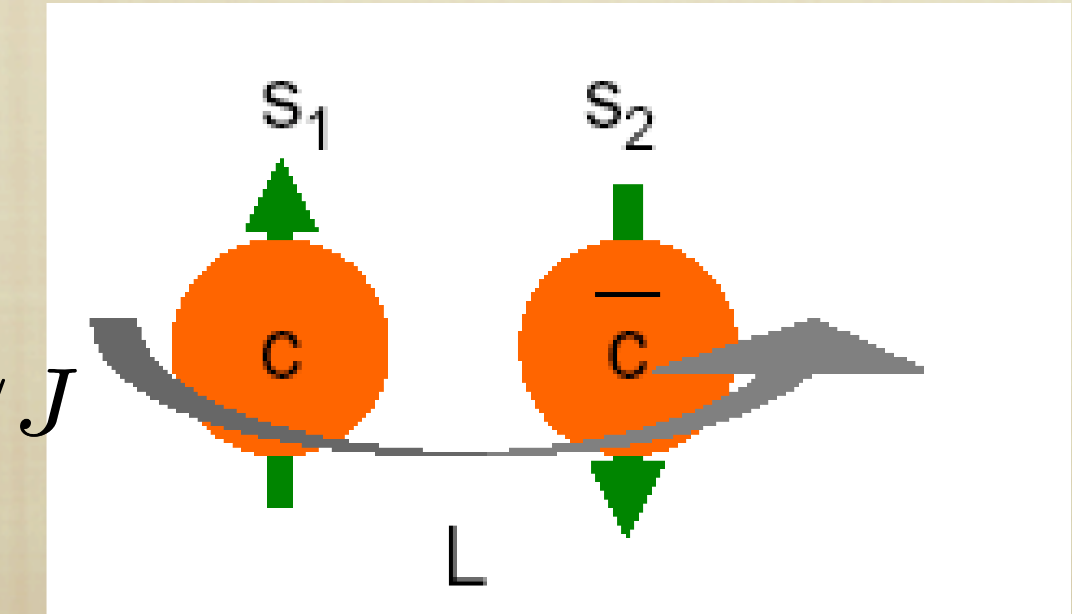

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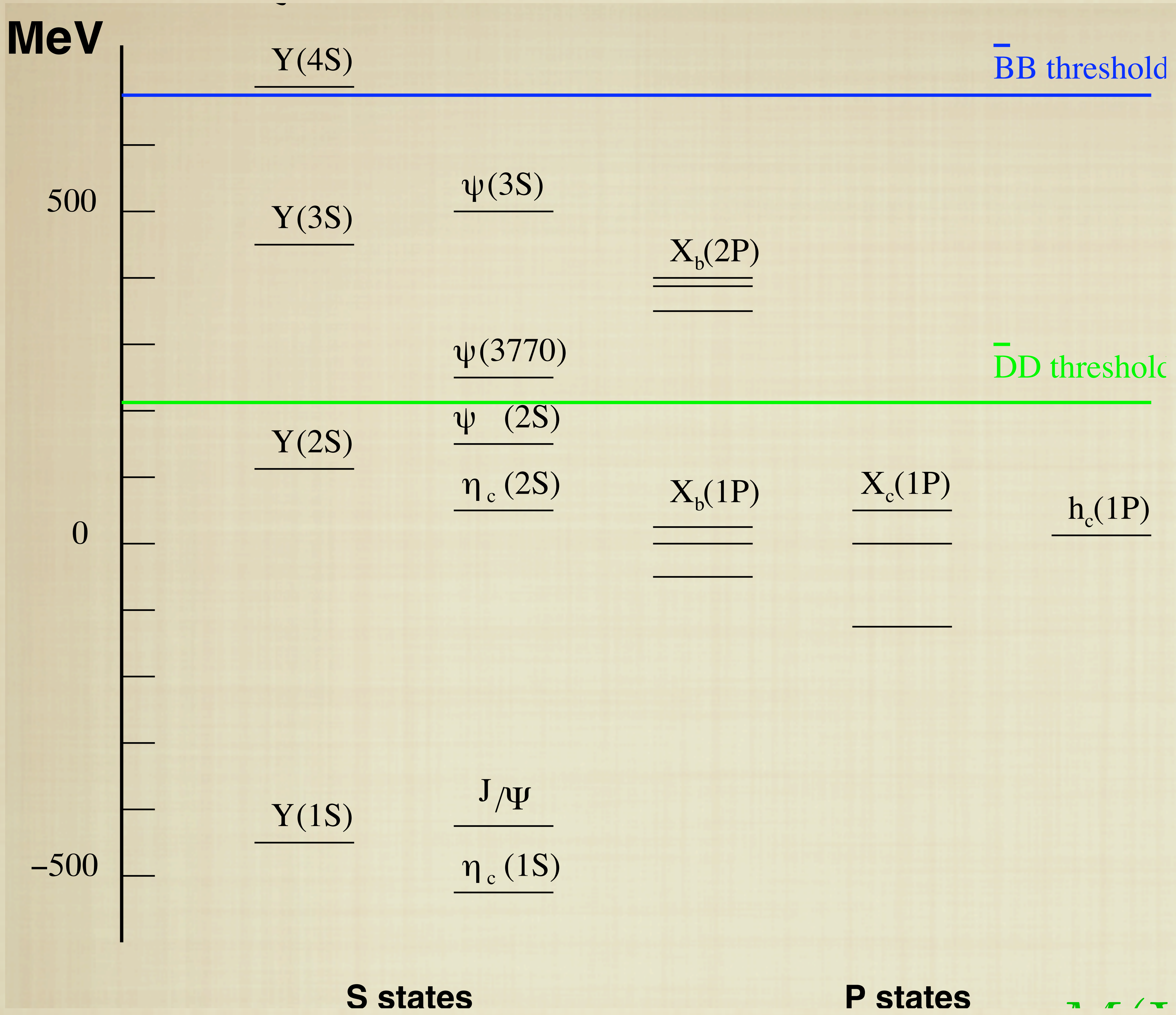

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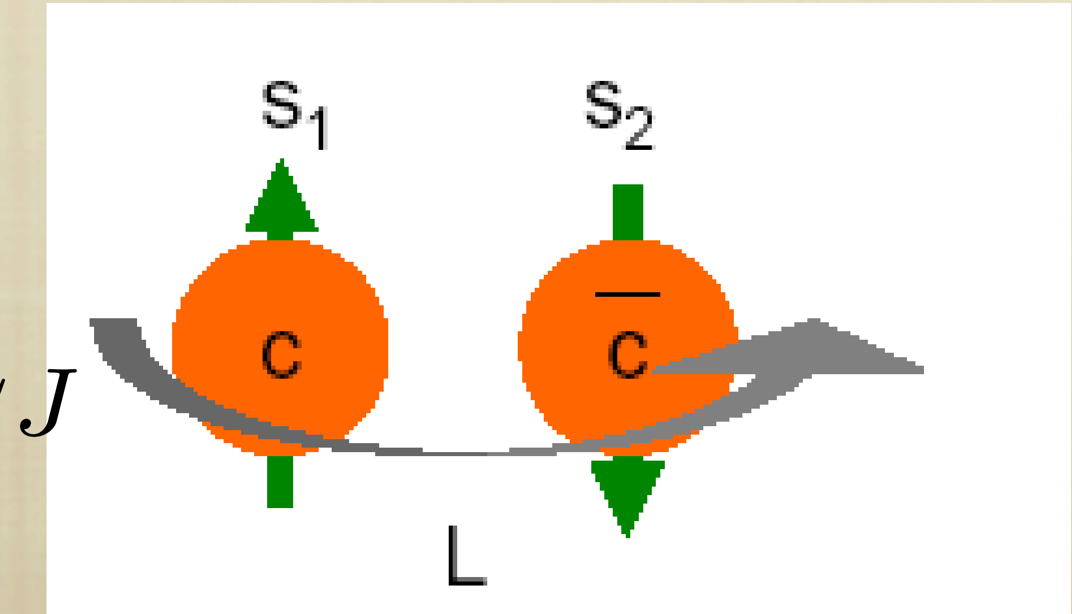

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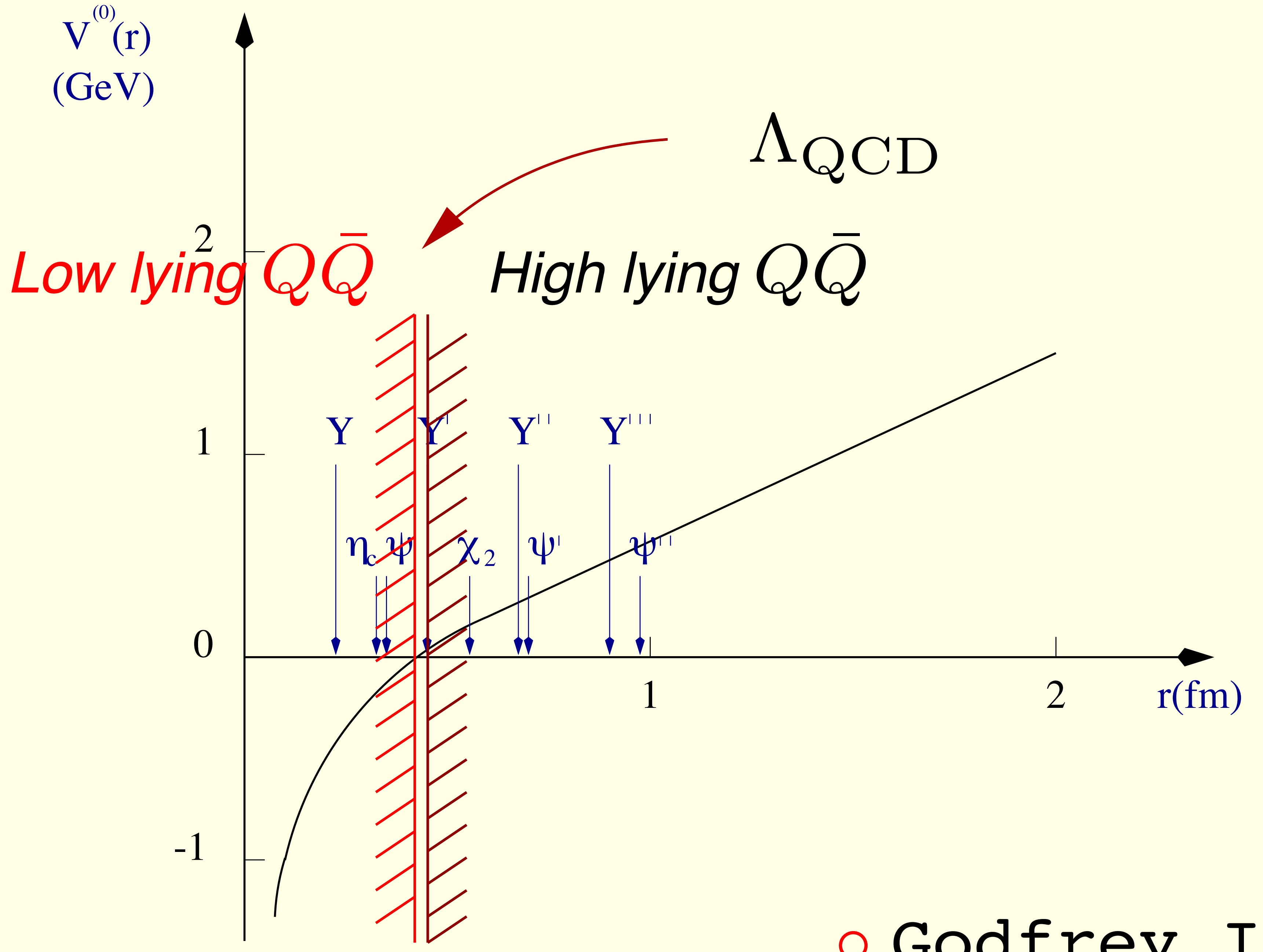
Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

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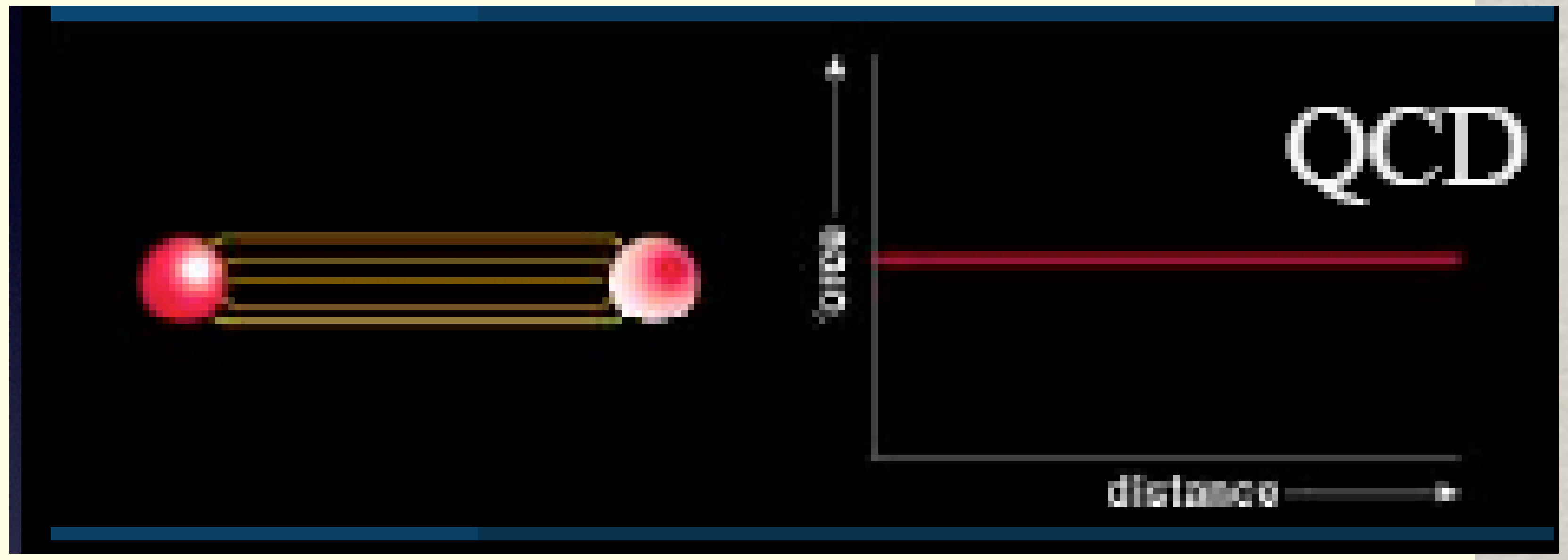
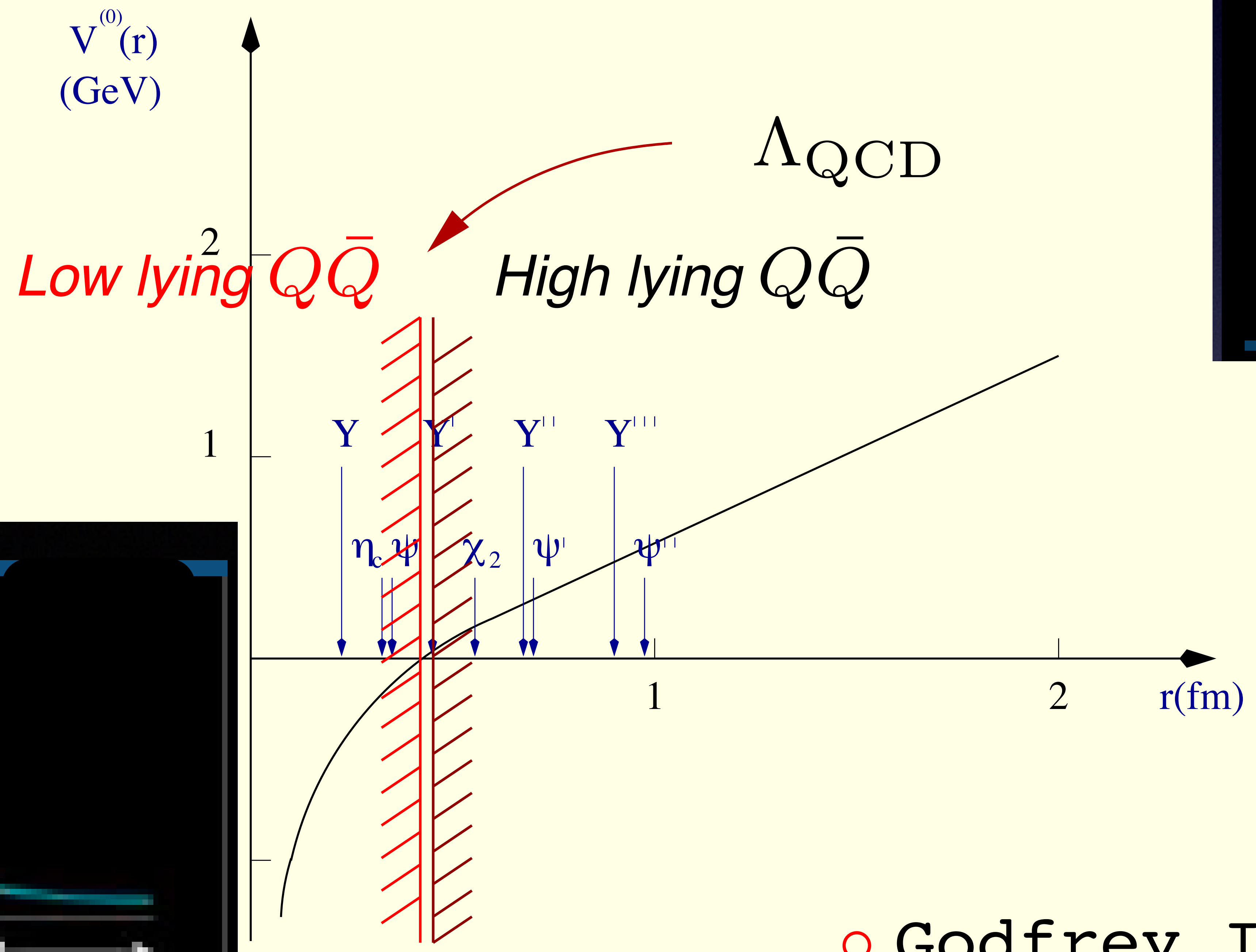
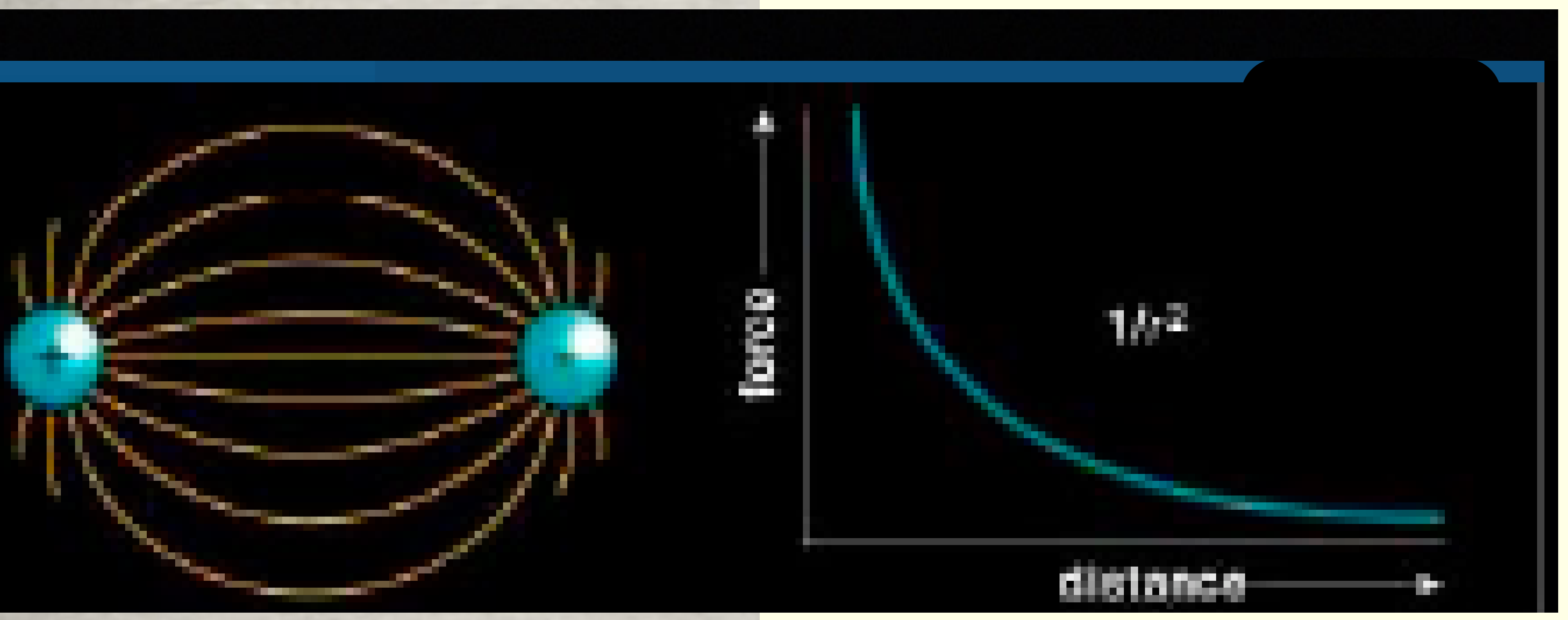




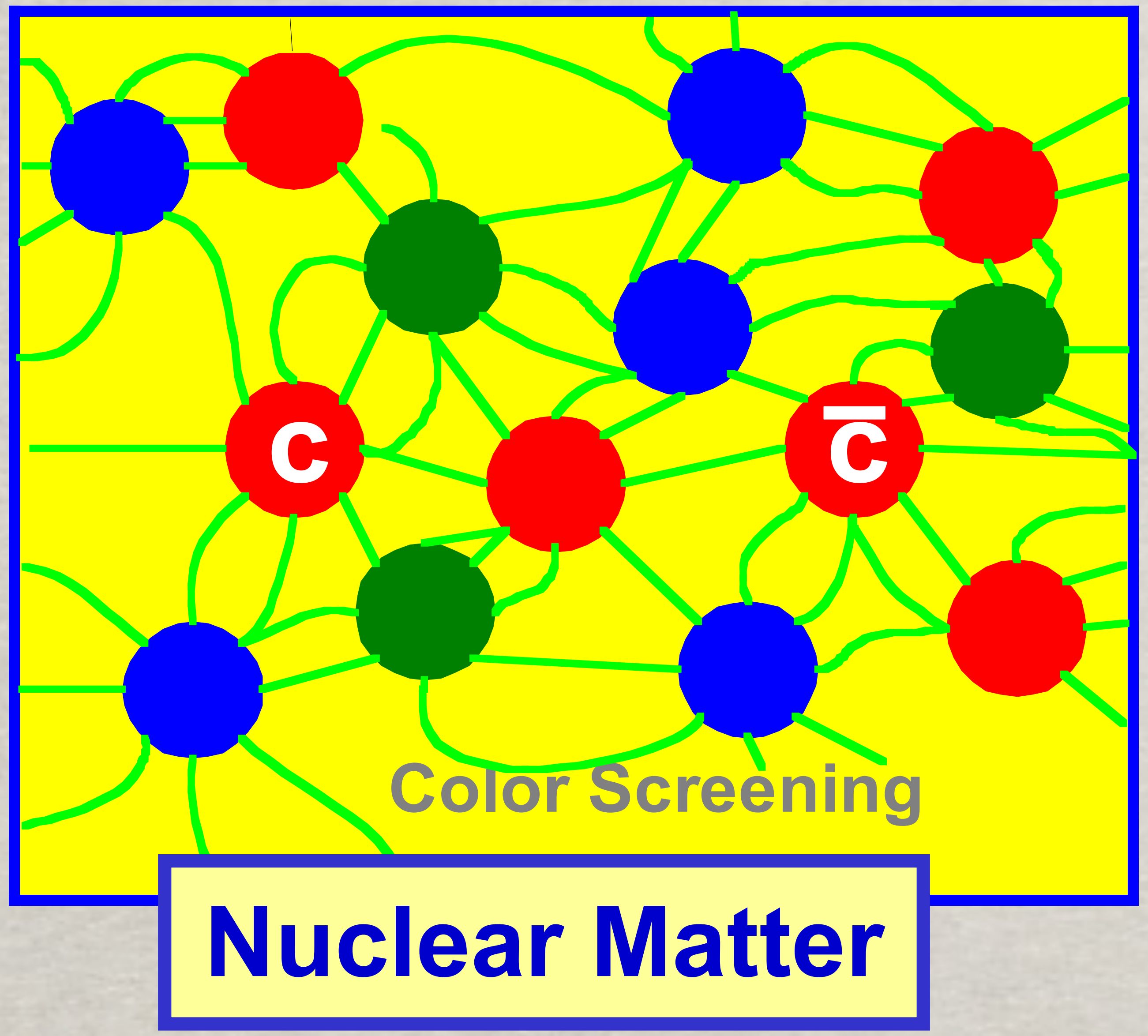


- Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r: they help us to understand the structure of the nonperturbative corrections in a strongly coupled theory

Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening $m_D \sim gT$

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

Matsui Satz 1986

A simulated collision of lead ions, courtesy the A
experiment at CERN

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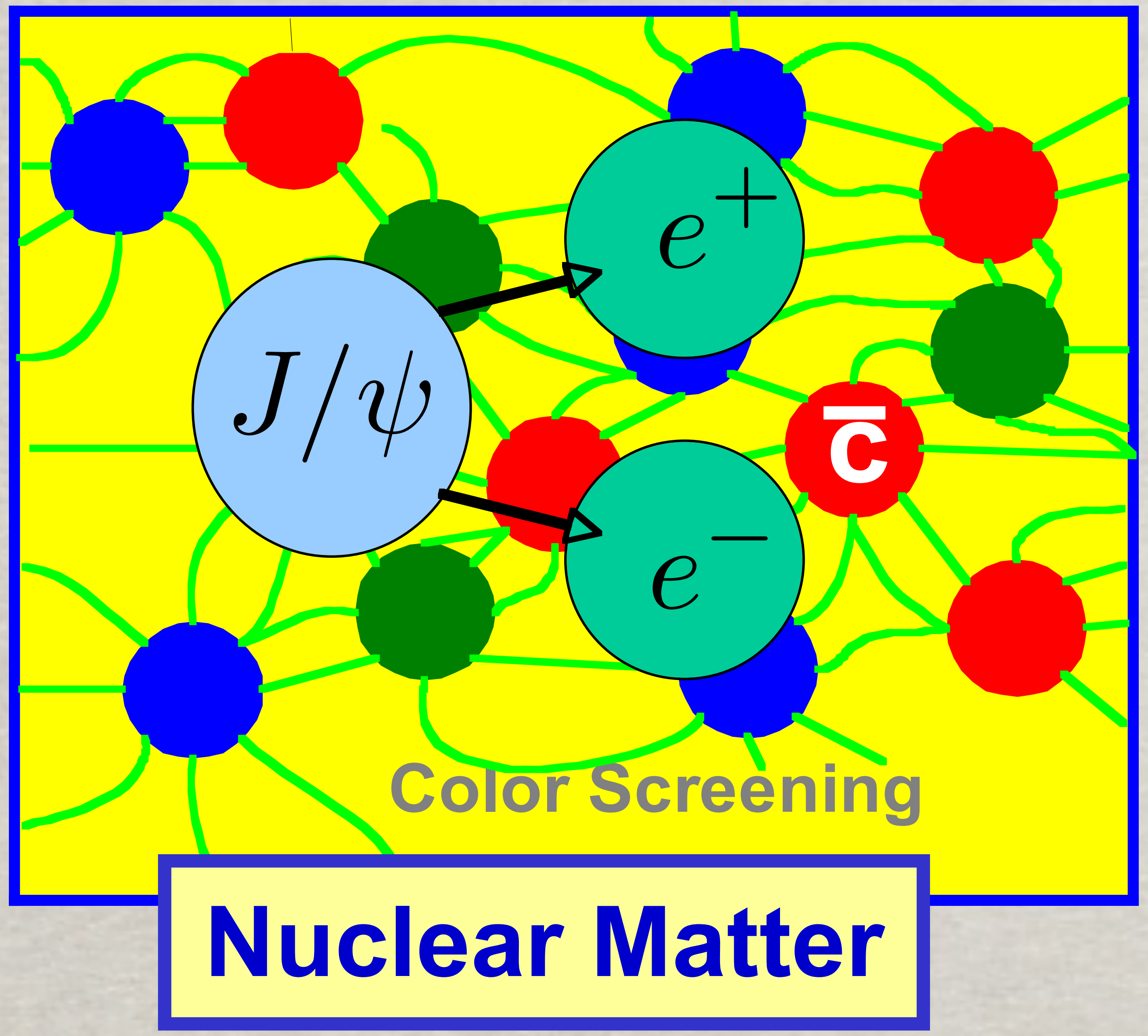

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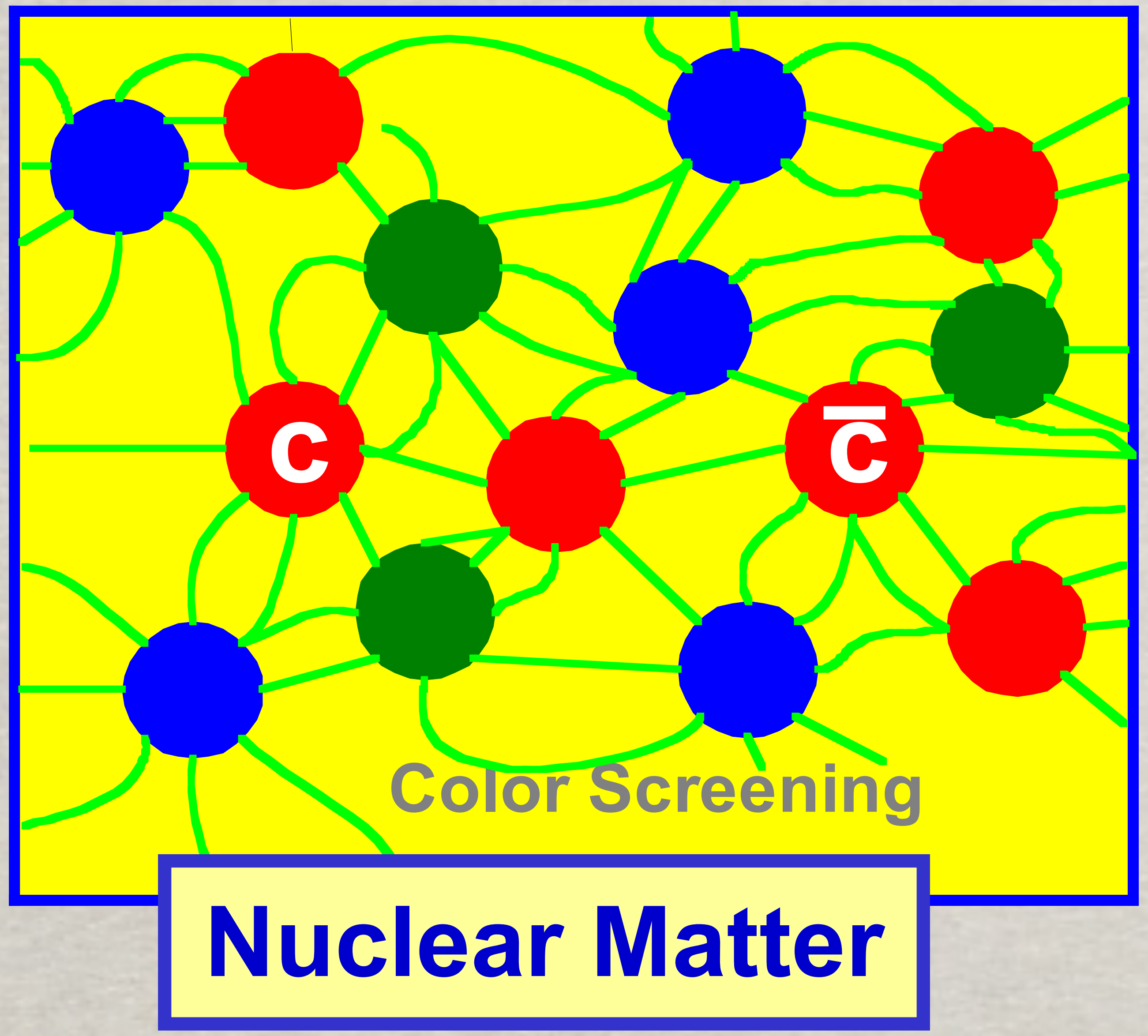

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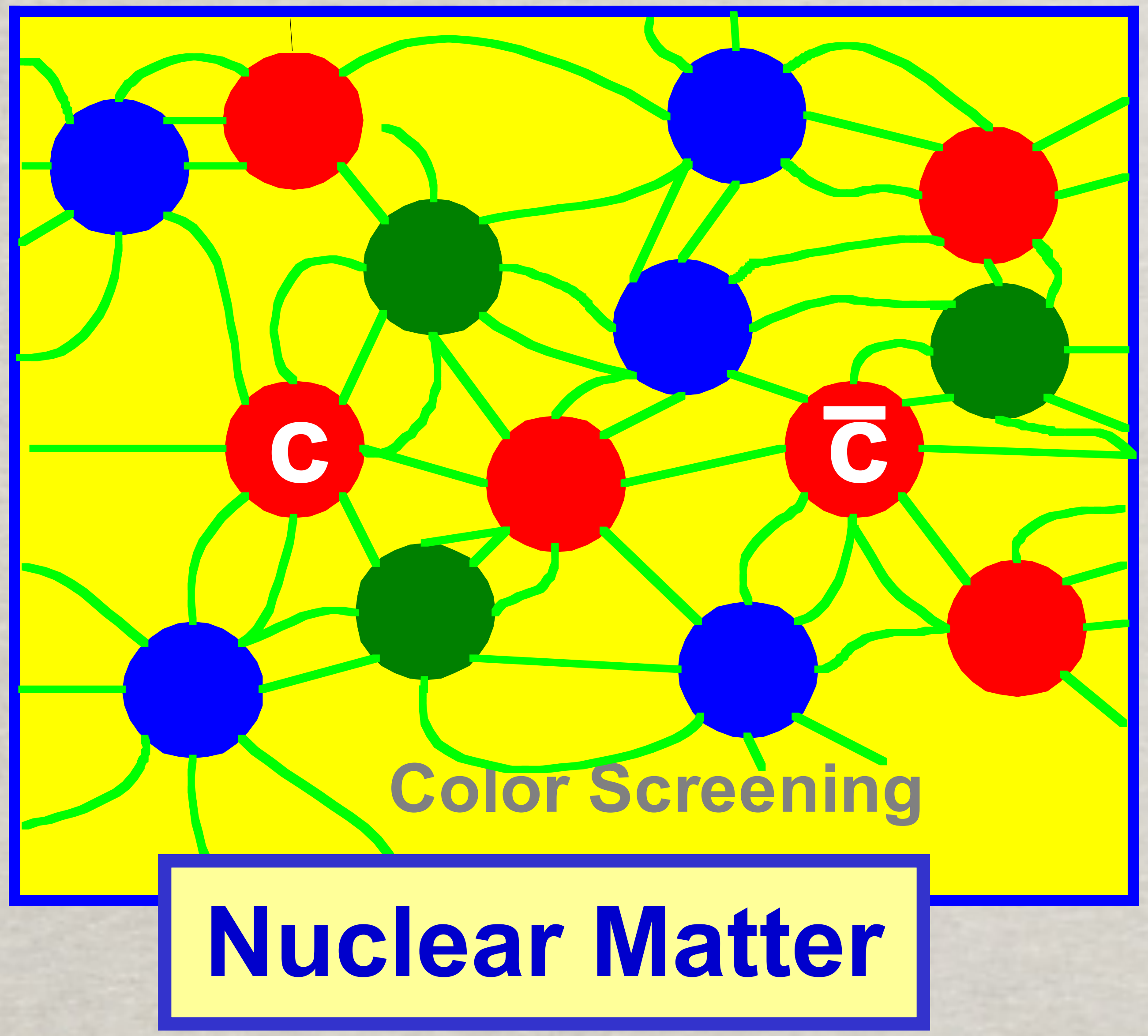

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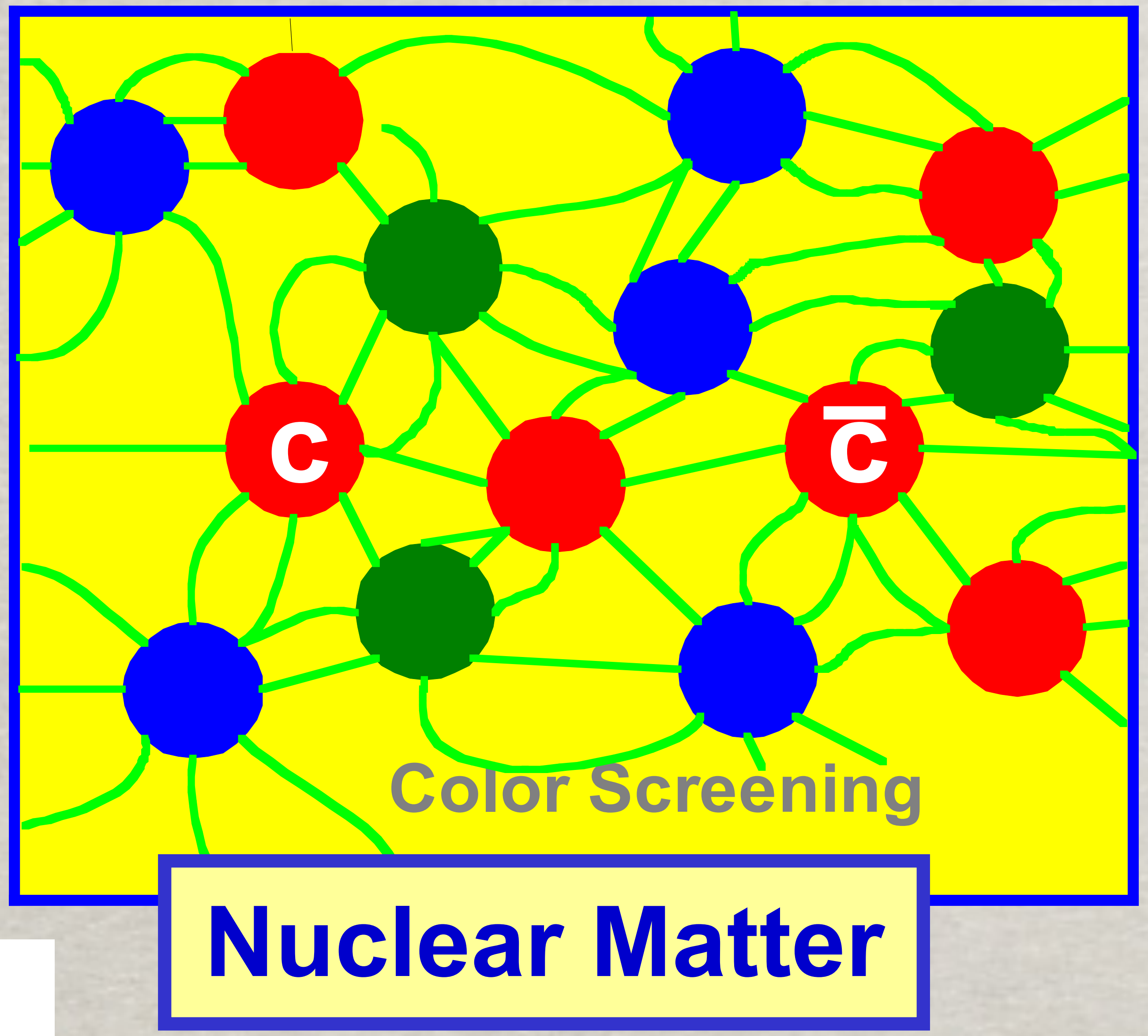

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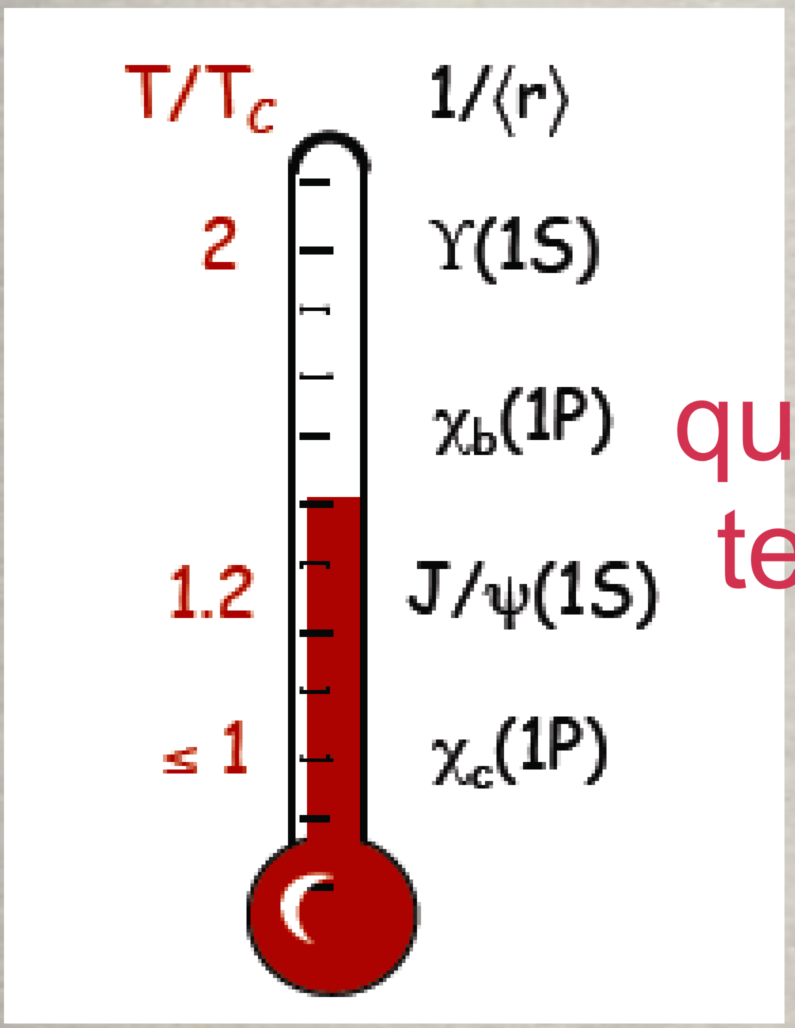

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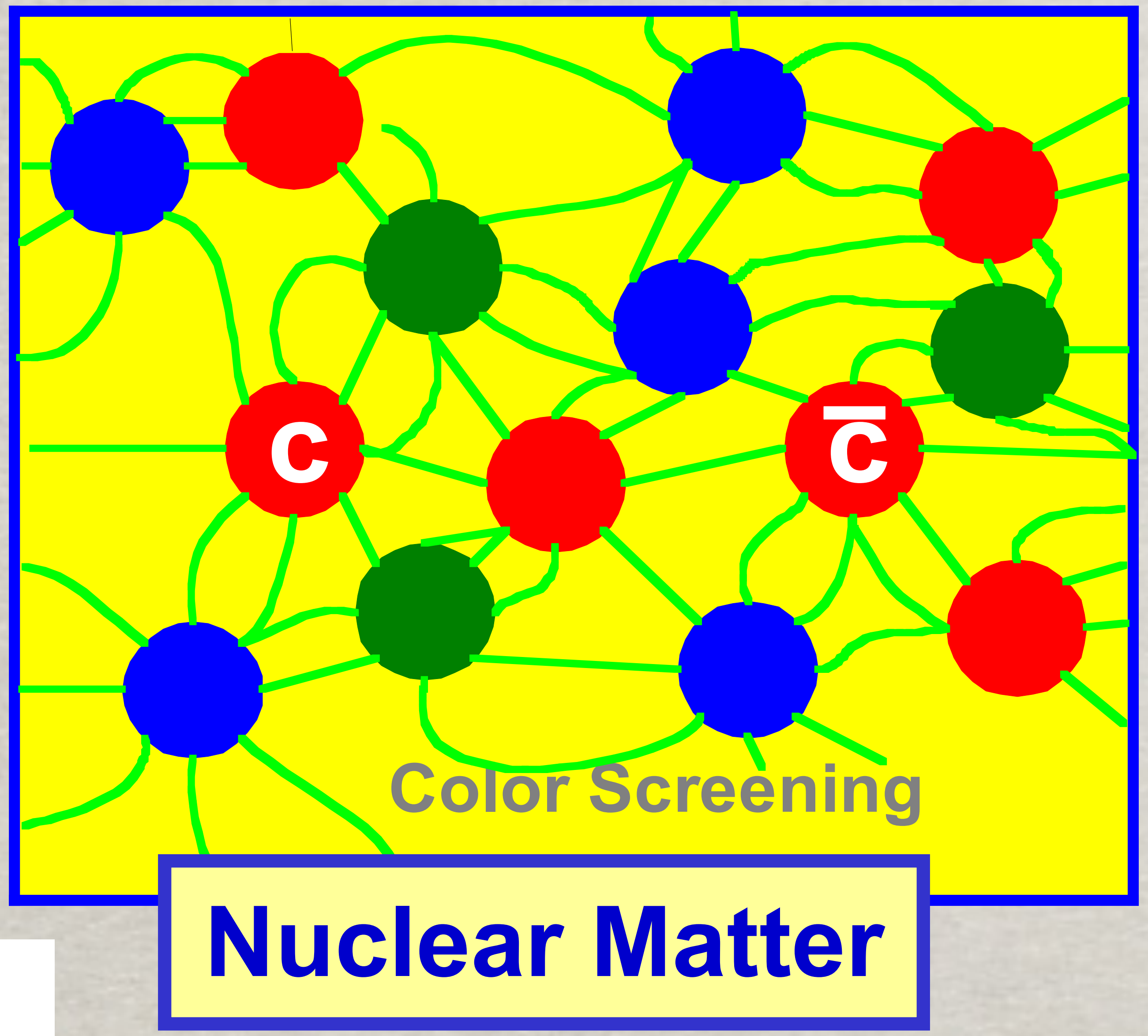

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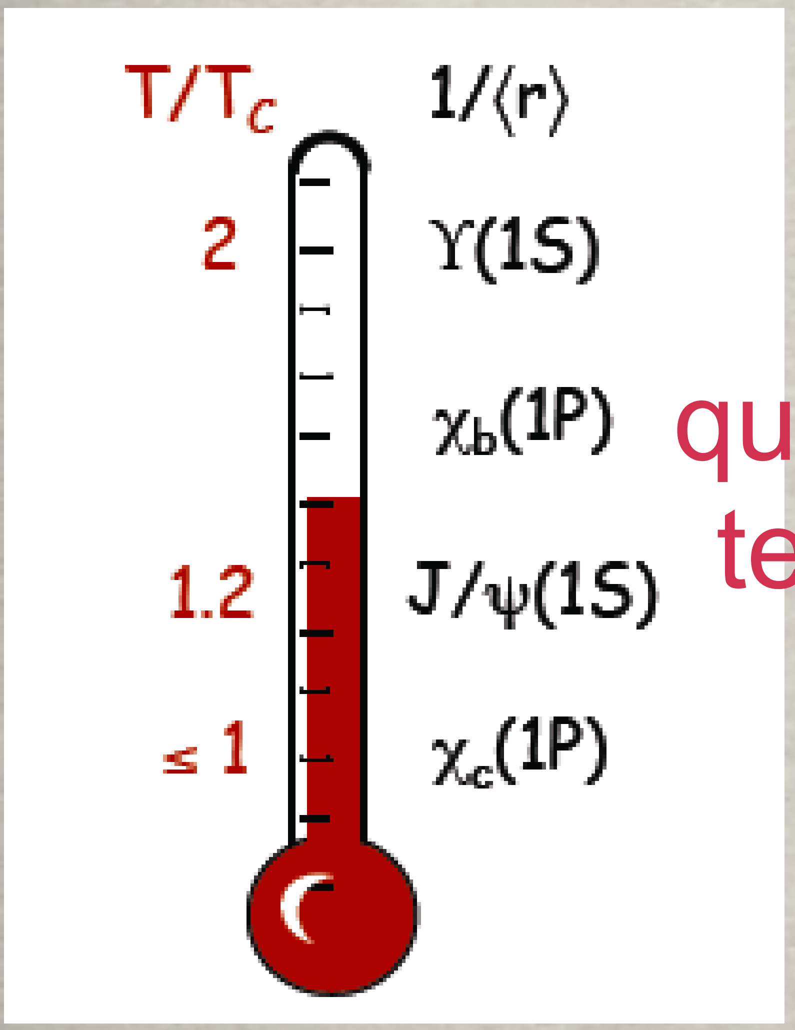
nuclear modificatio

$$R_{AA} = \frac{Yield^{q\bar{q}}_{AA}}{\langle N_{coll} \rangle \times Yield^{q\bar{q}}_{pp}}$$



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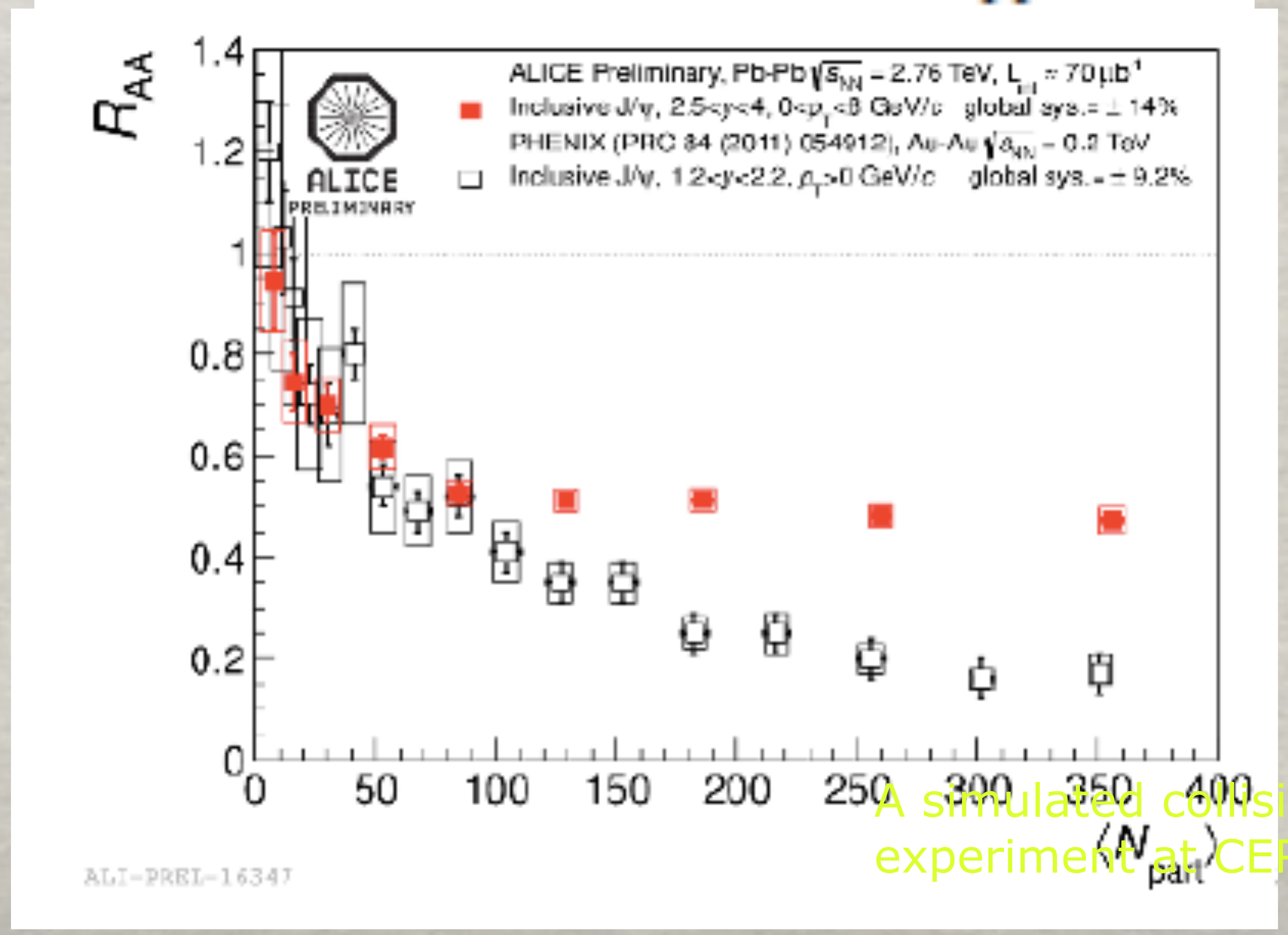

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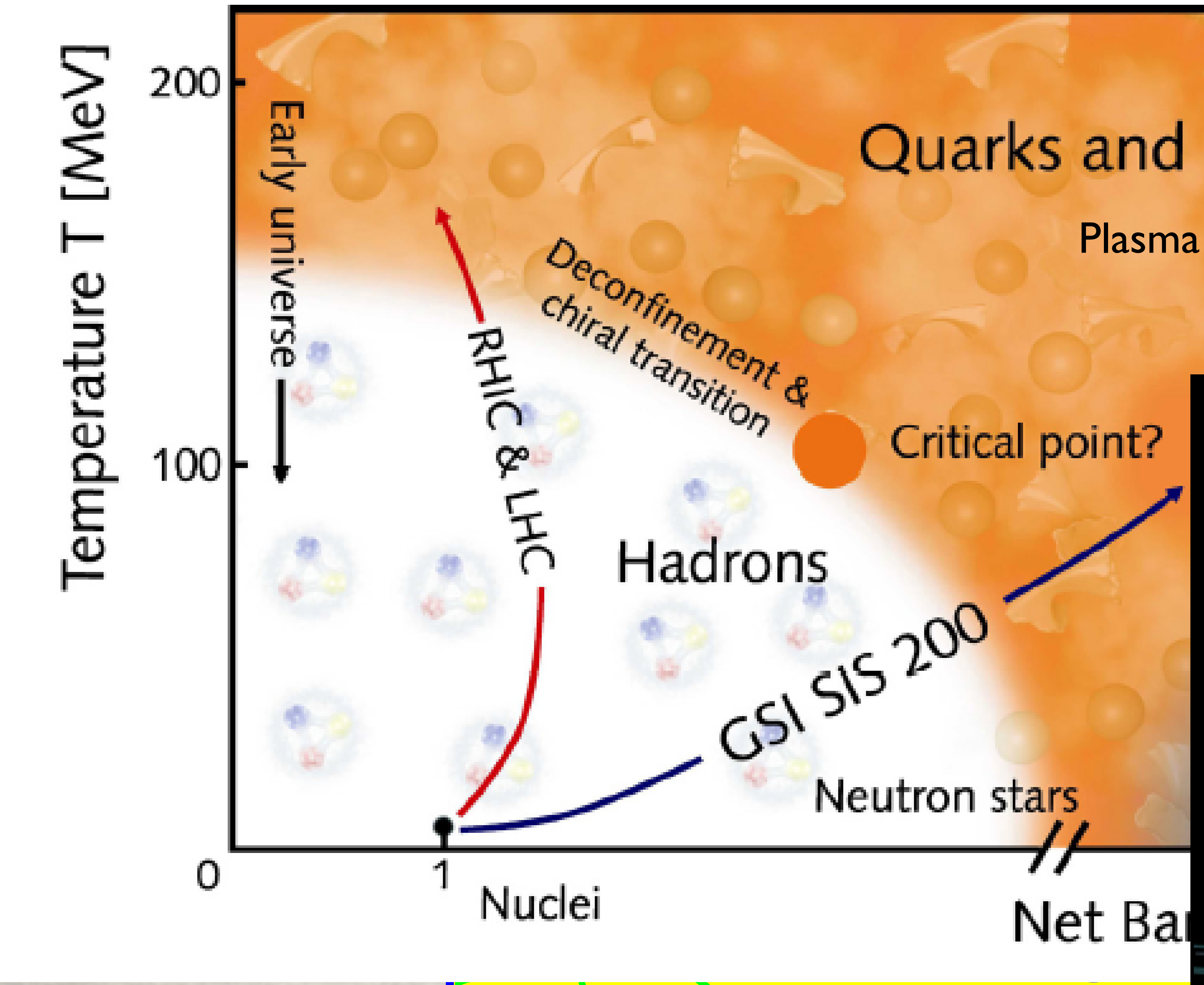
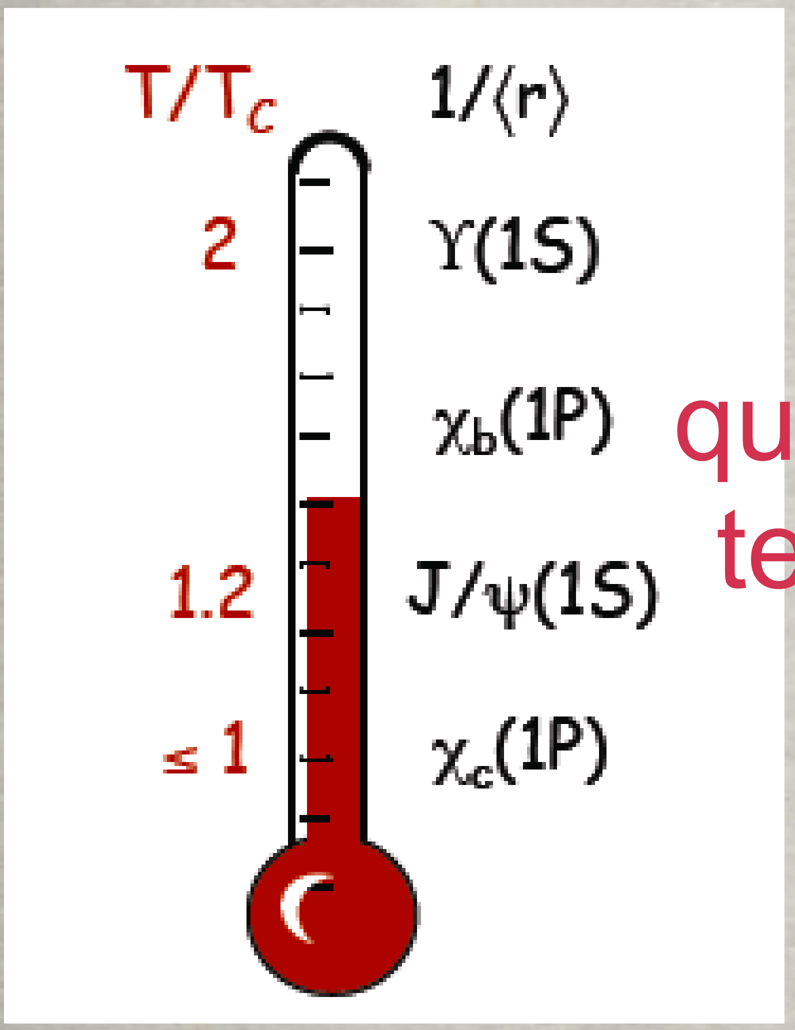


Nuclear Matter



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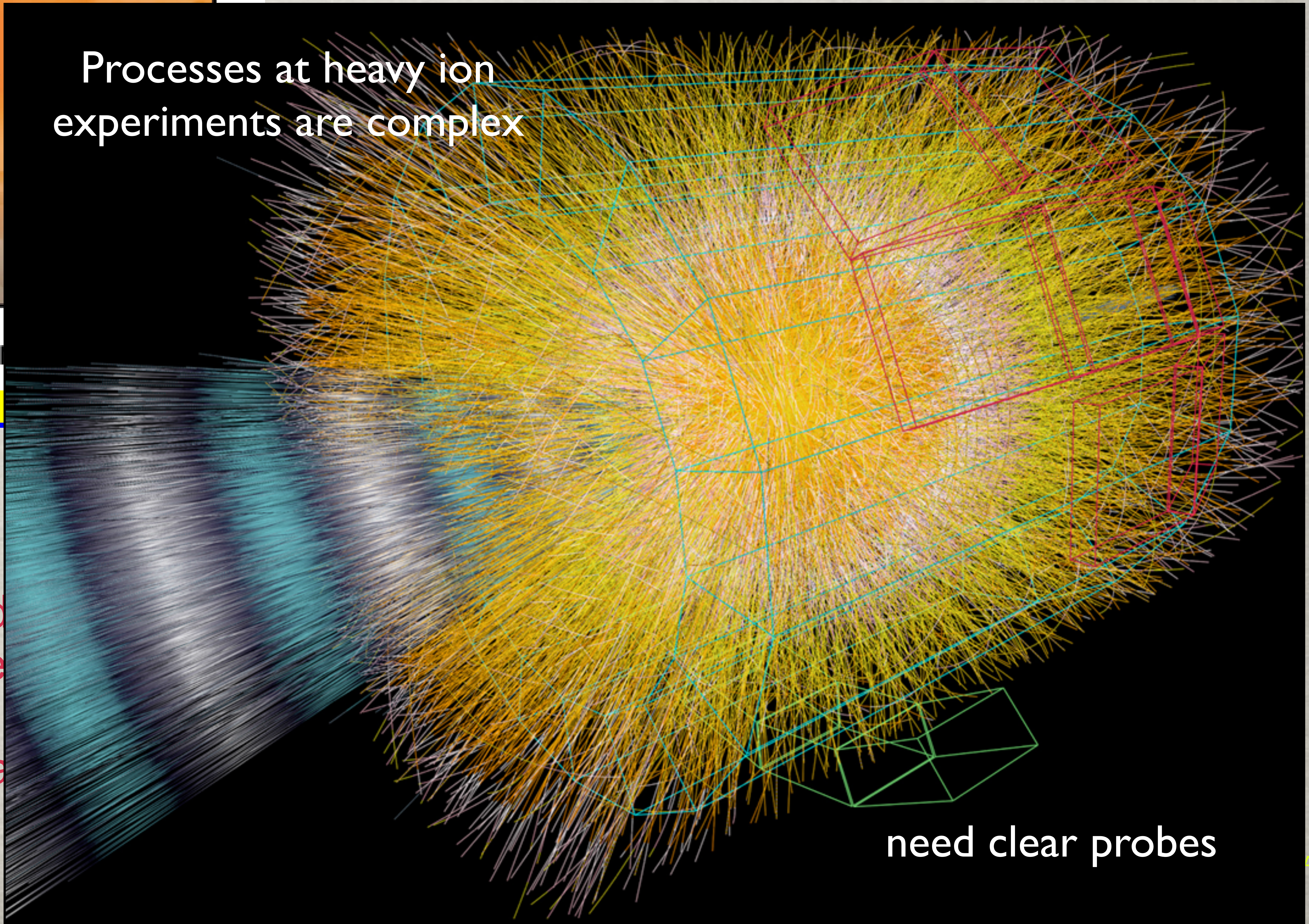

QCD theory of quarkonium: a very challenging problem

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Close to the bound state $\alpha_s \sim v$

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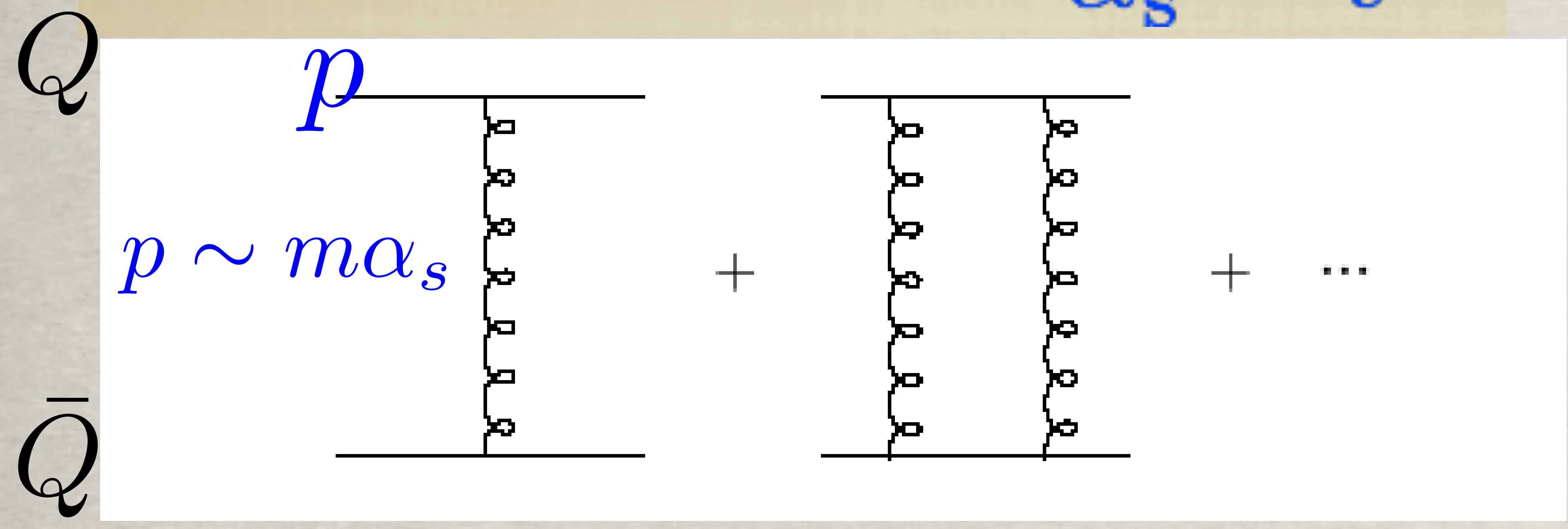

QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

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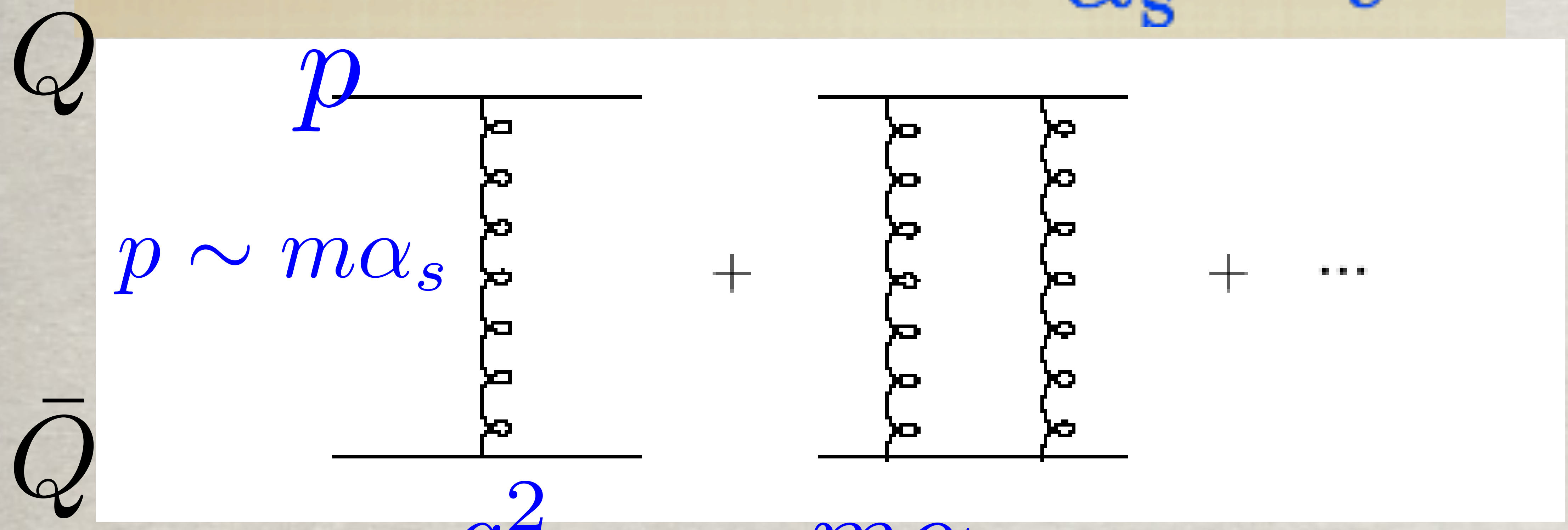


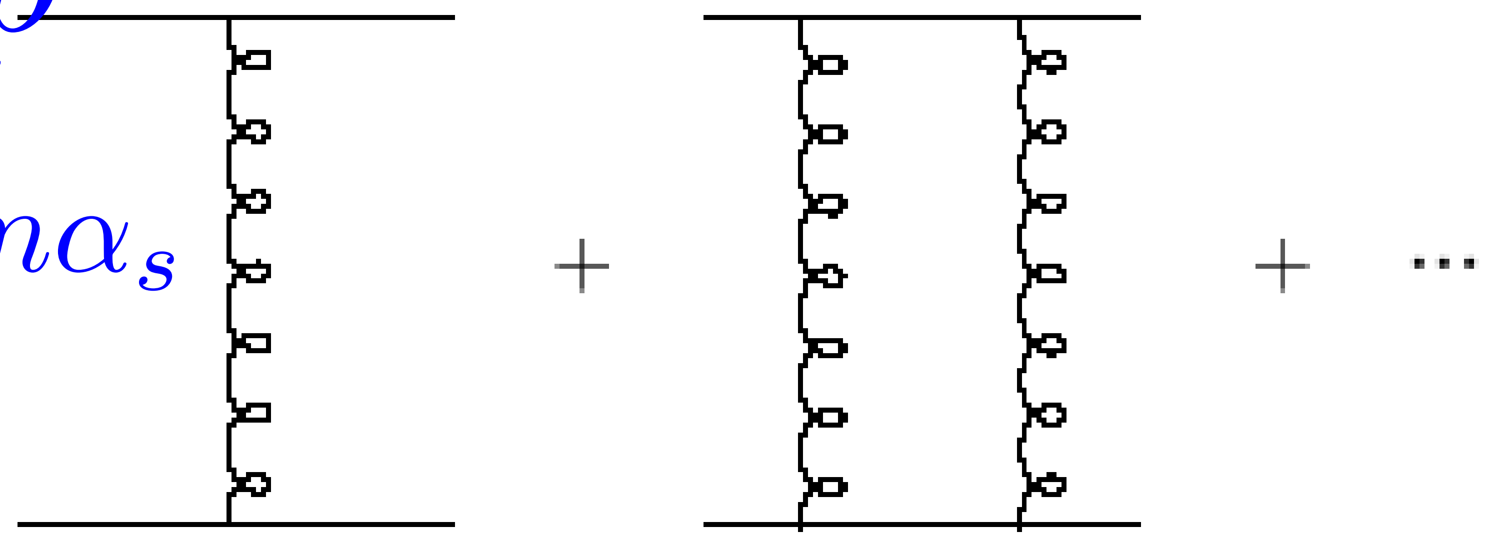
$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

Close to the bound state $\alpha_s \sim v$

Q p

$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

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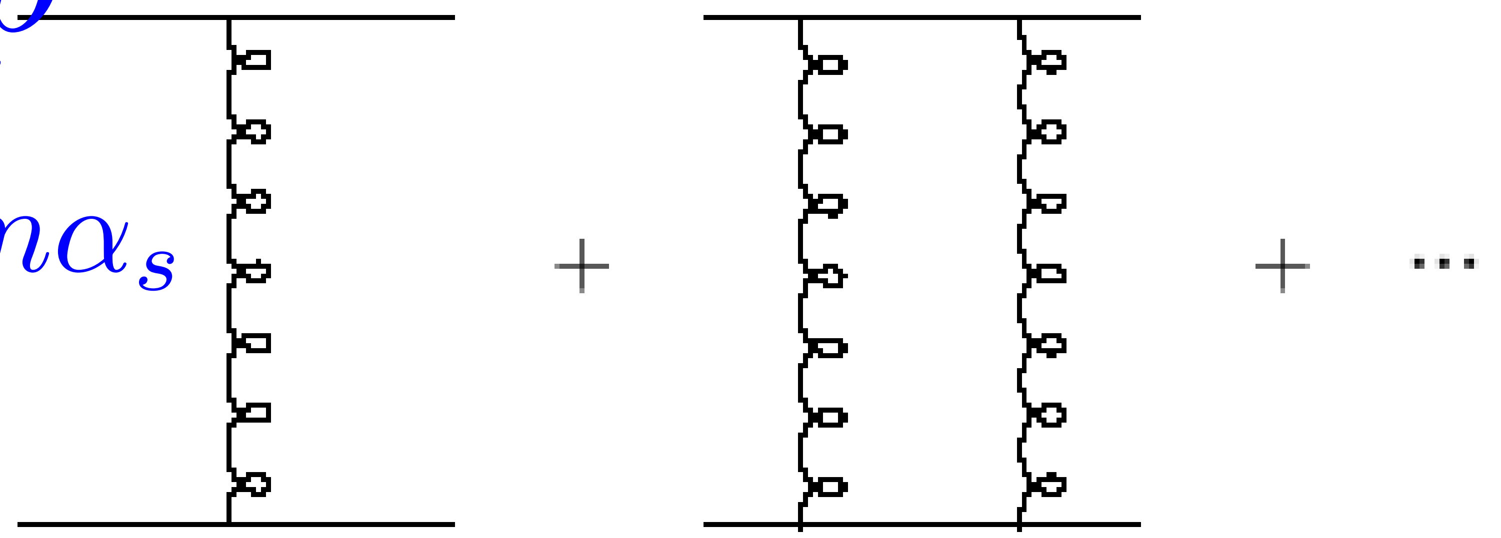

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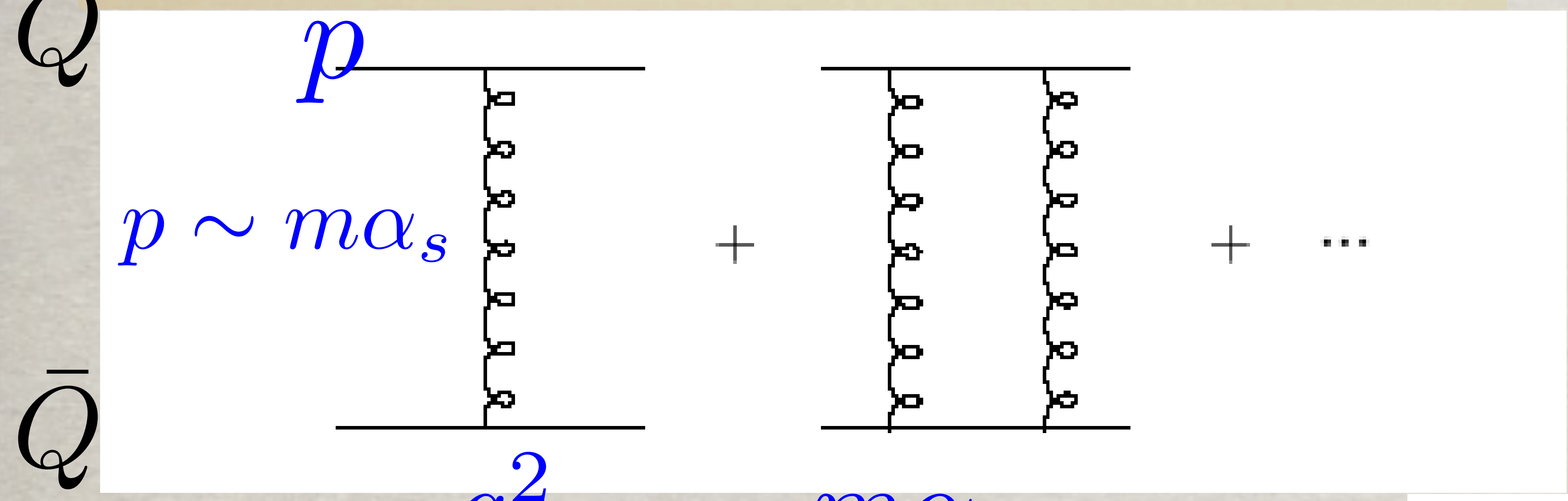

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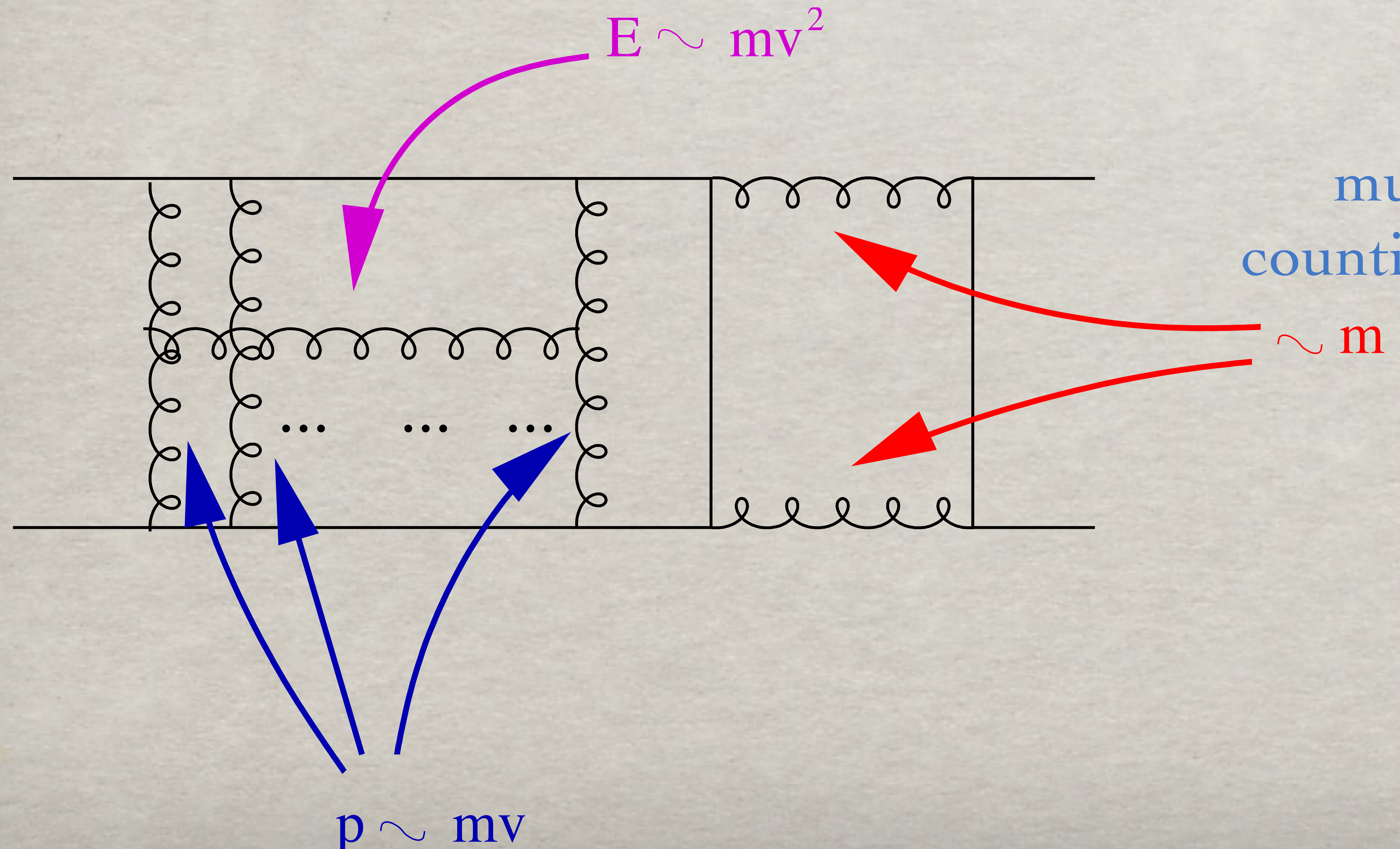
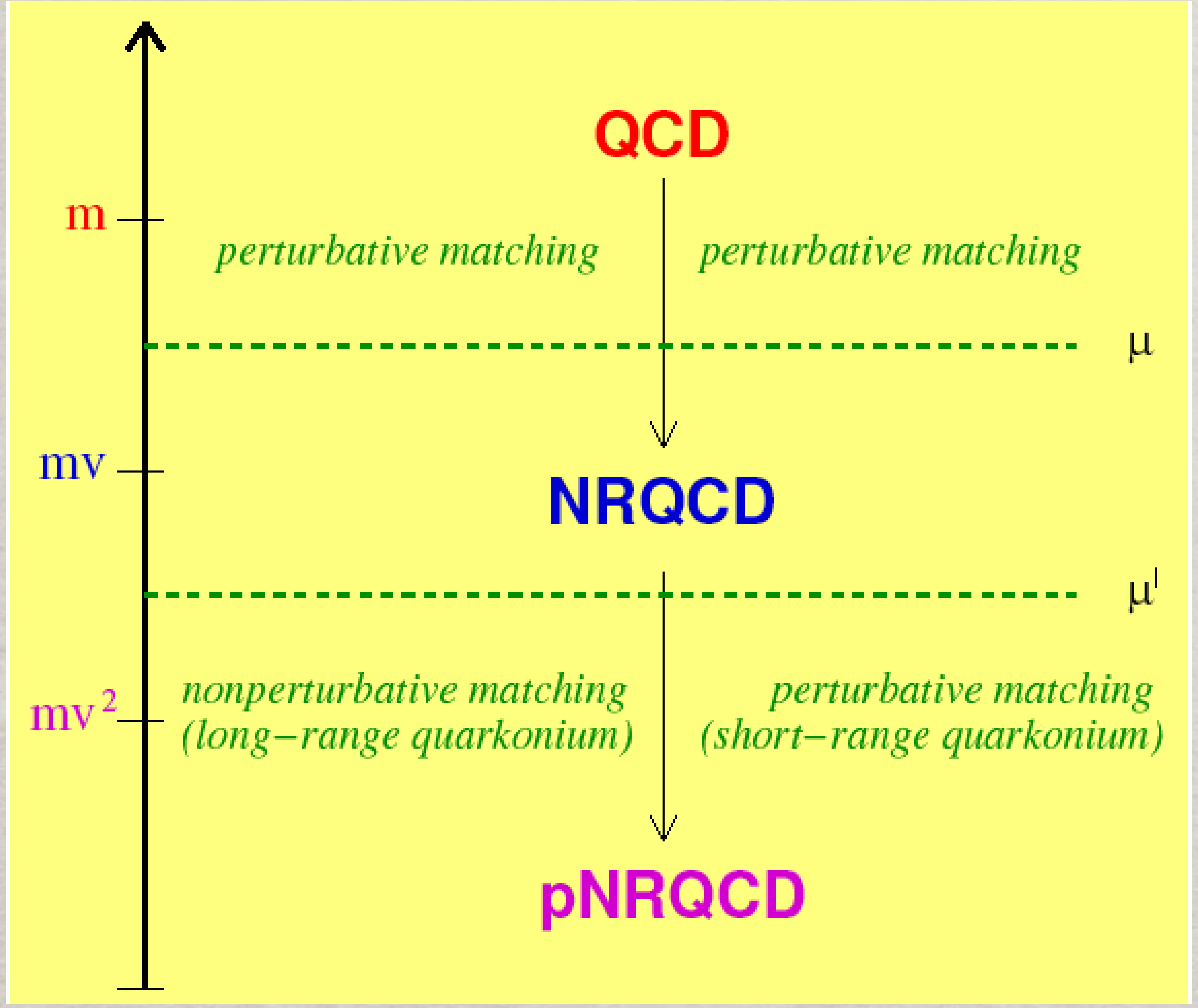


multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

NREFTs for quarkonium



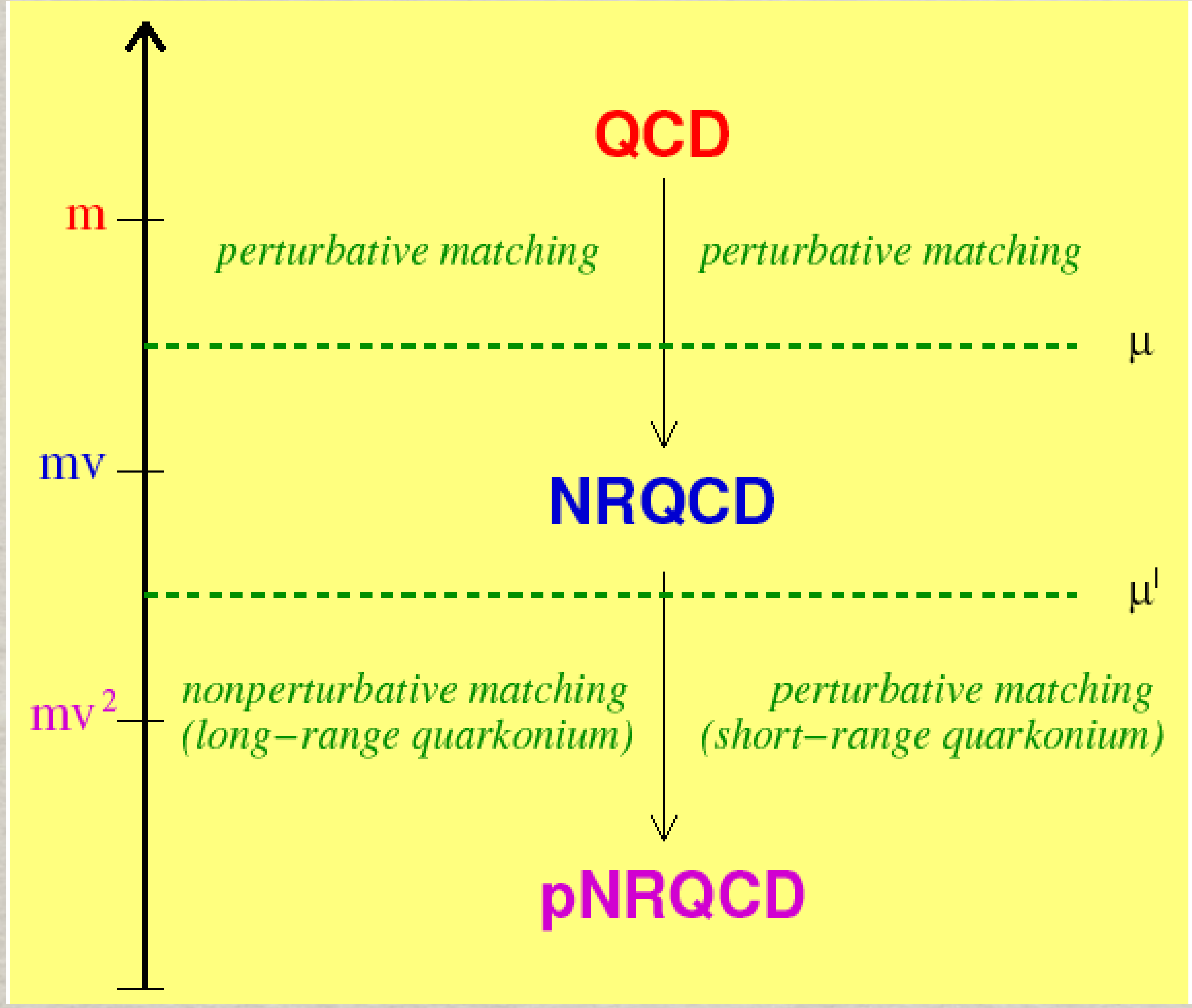
Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

NREFTs for quarkonium



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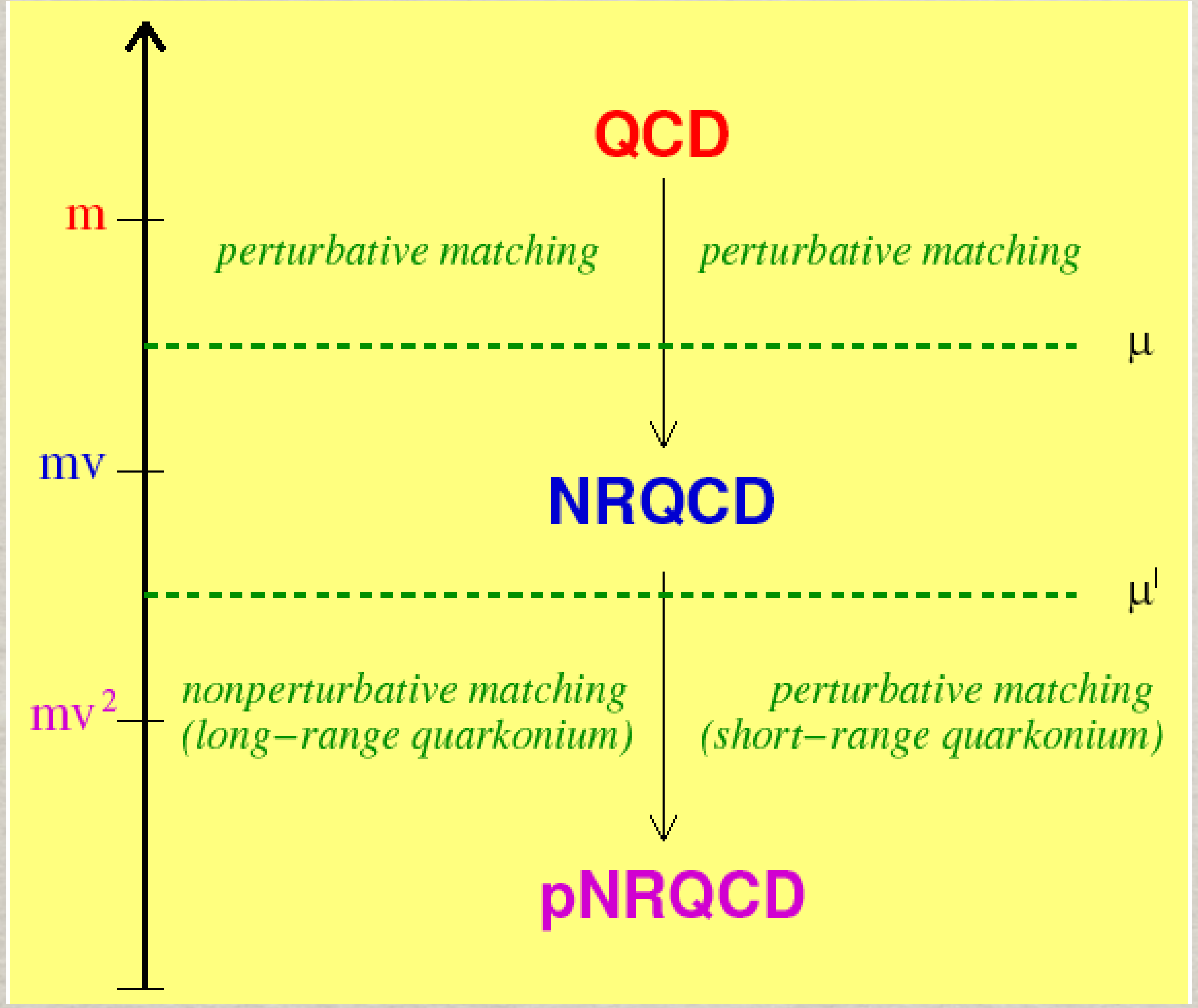

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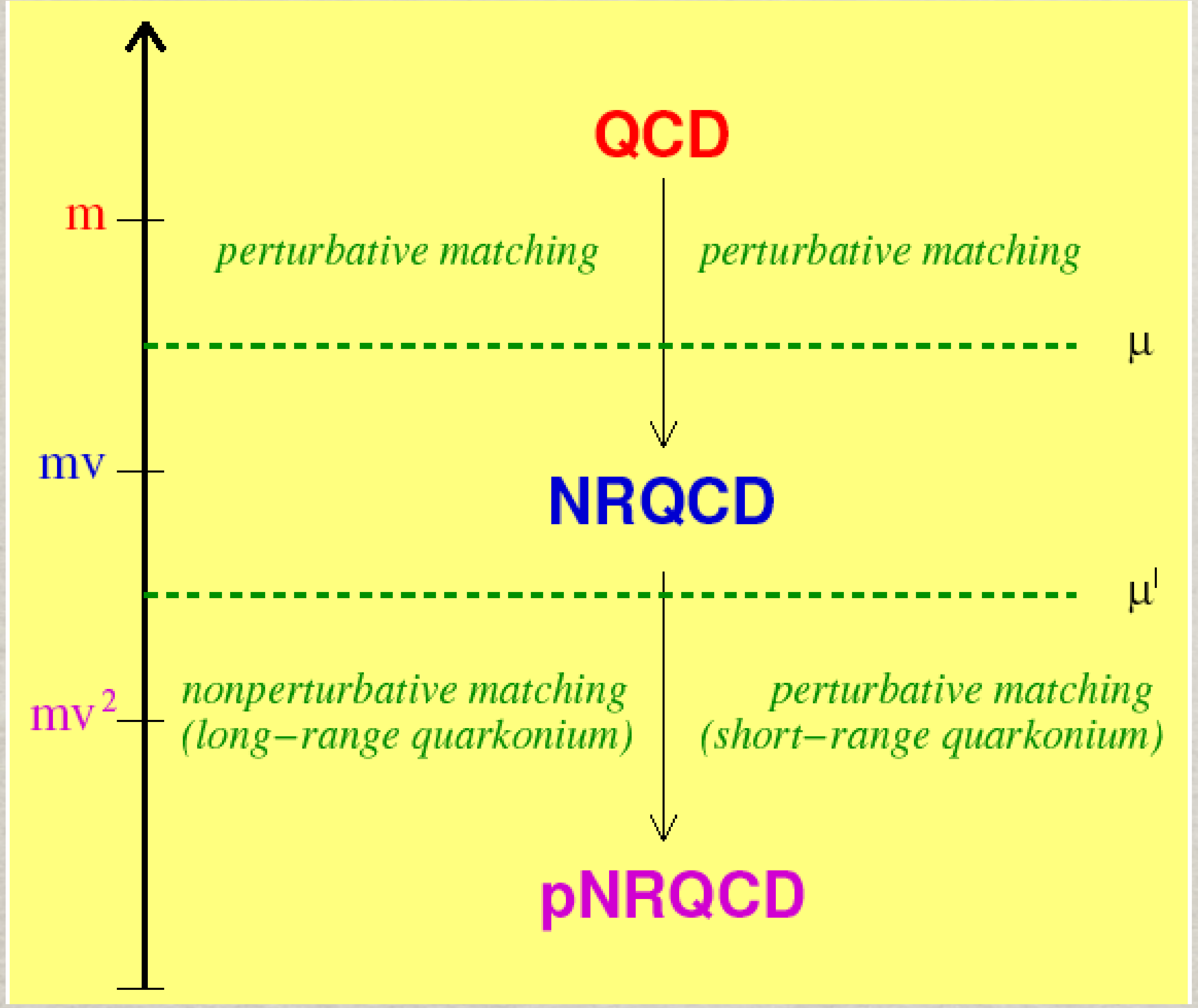

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$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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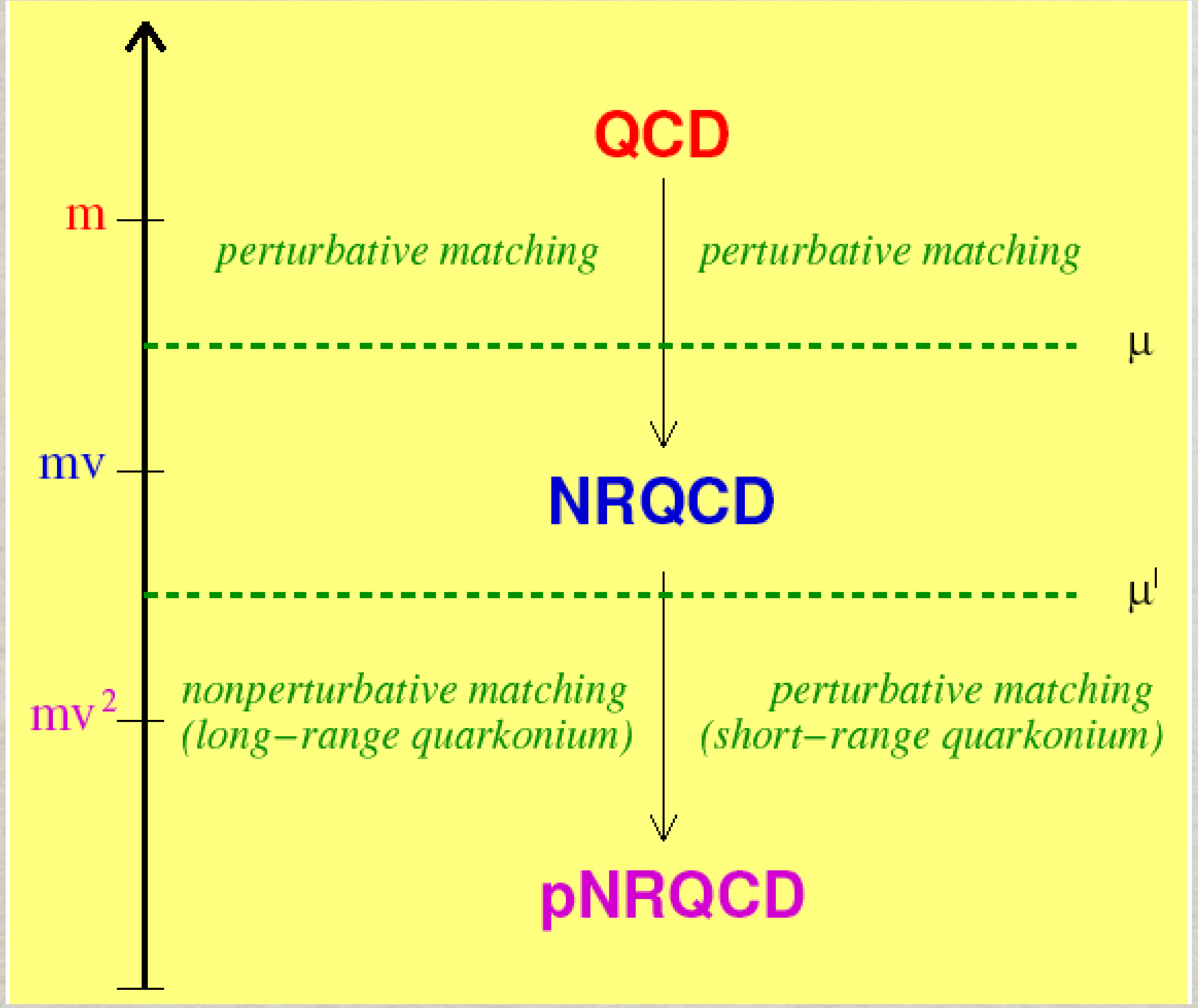

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Soft
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momentum)

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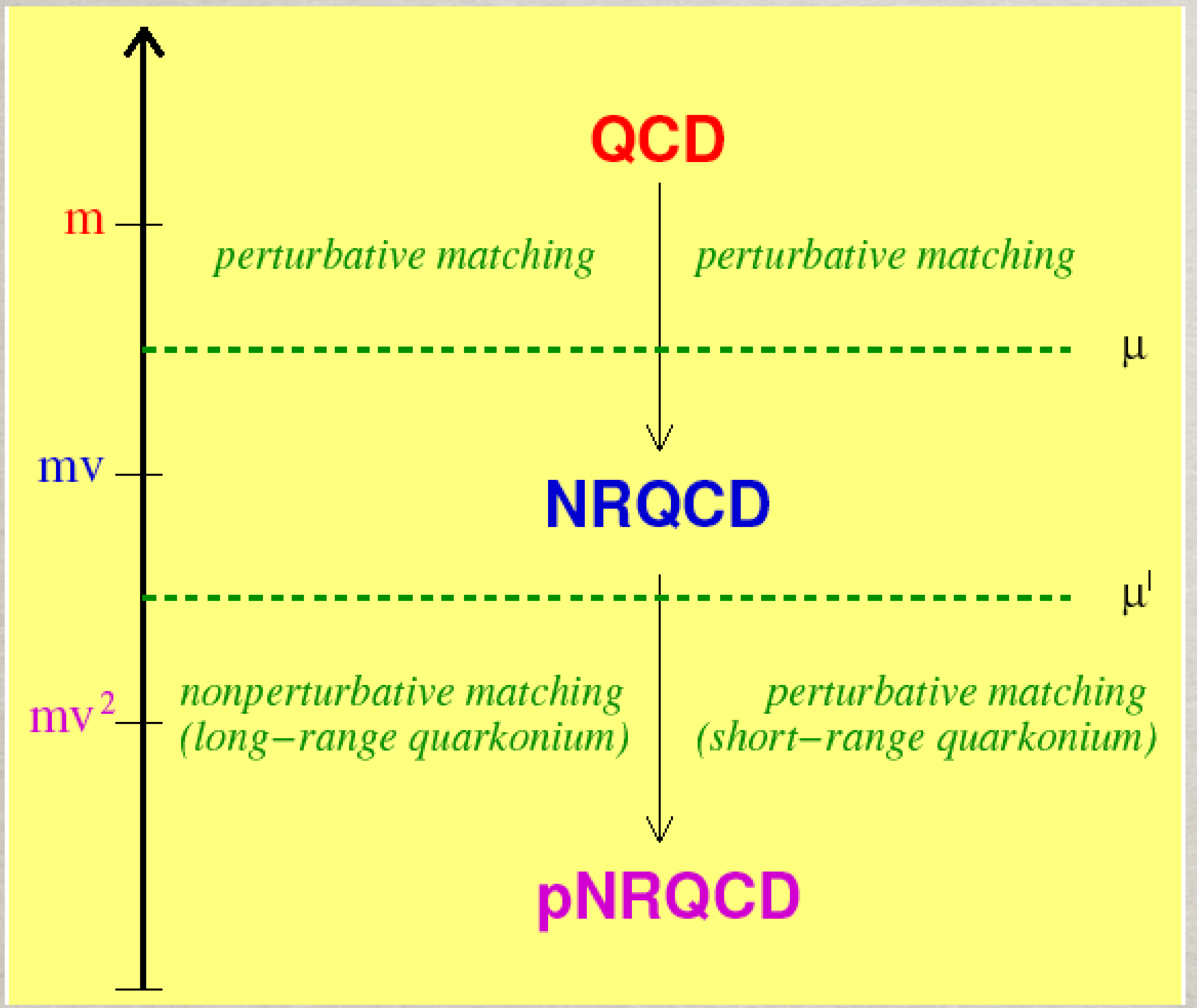
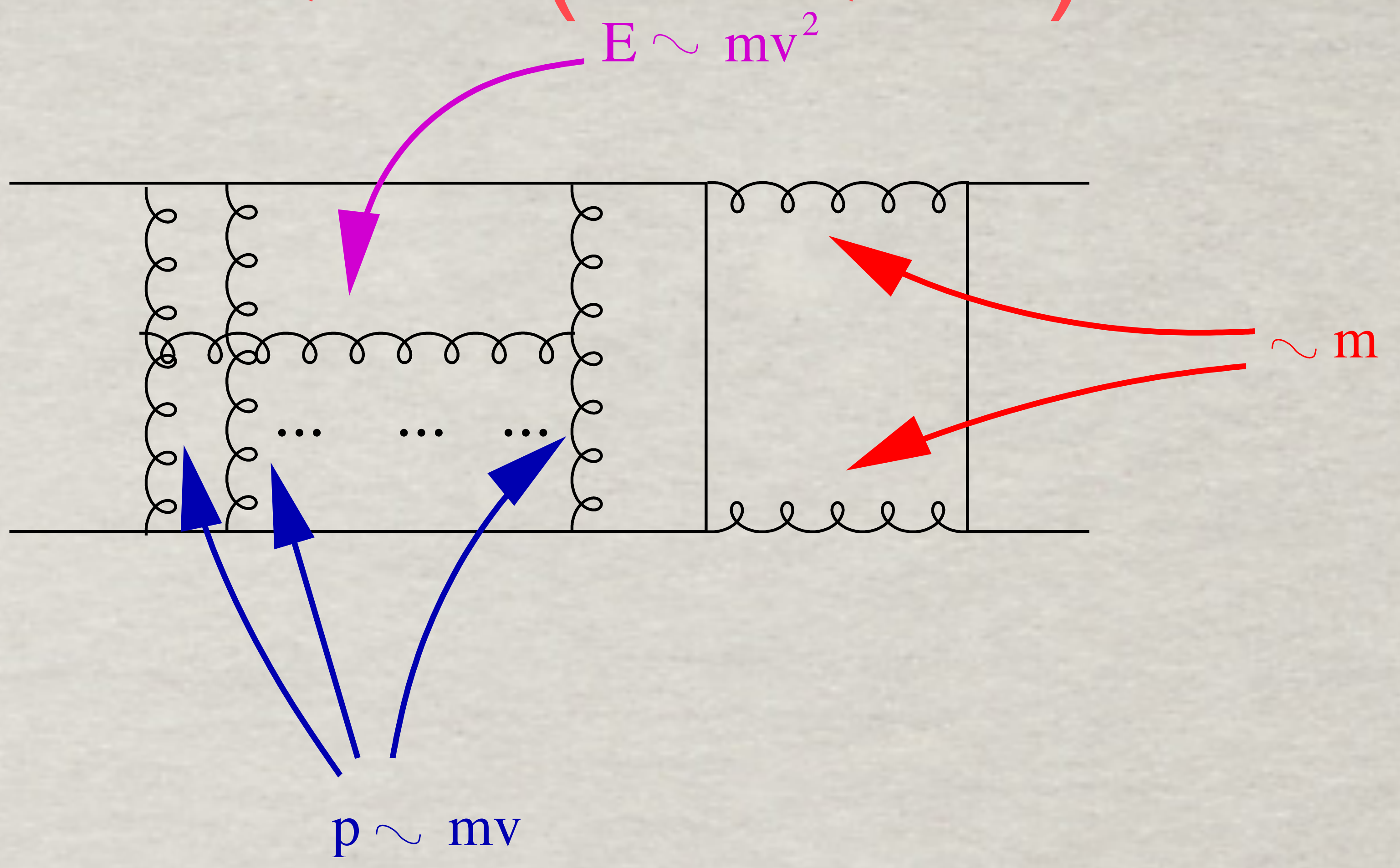
Quarkonium with NREFTs: Non Relativistic QCD (NRQCD)





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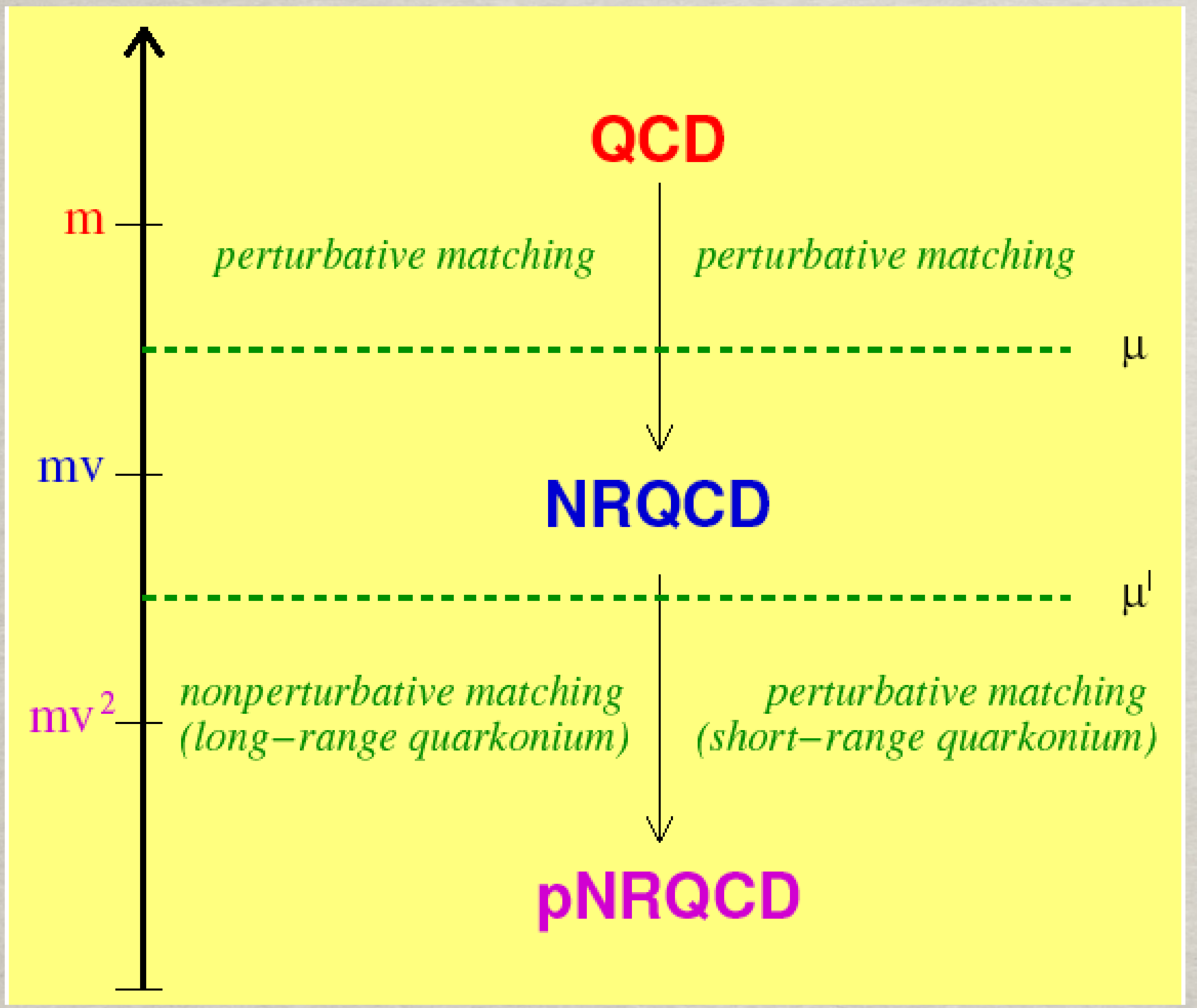

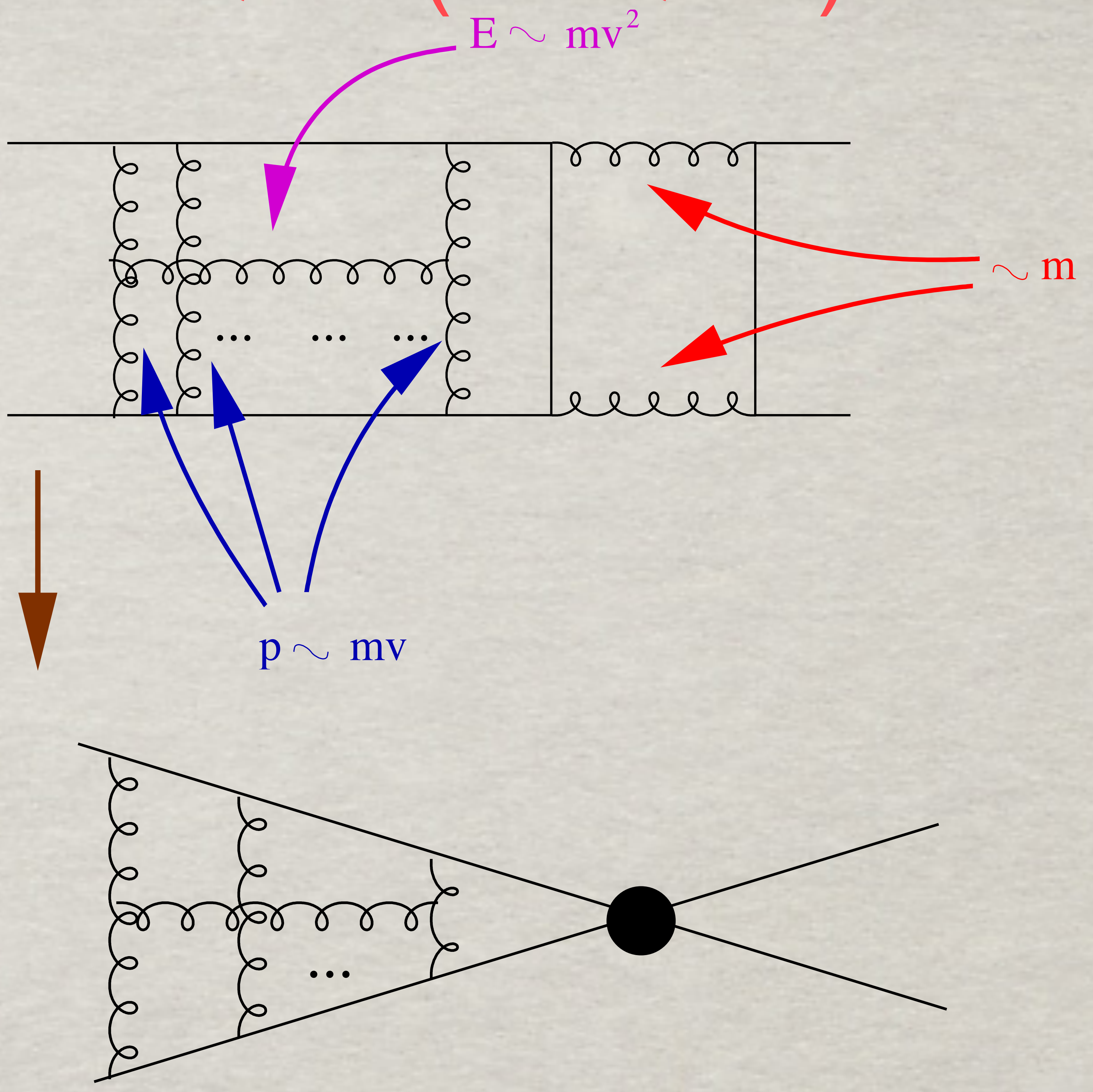

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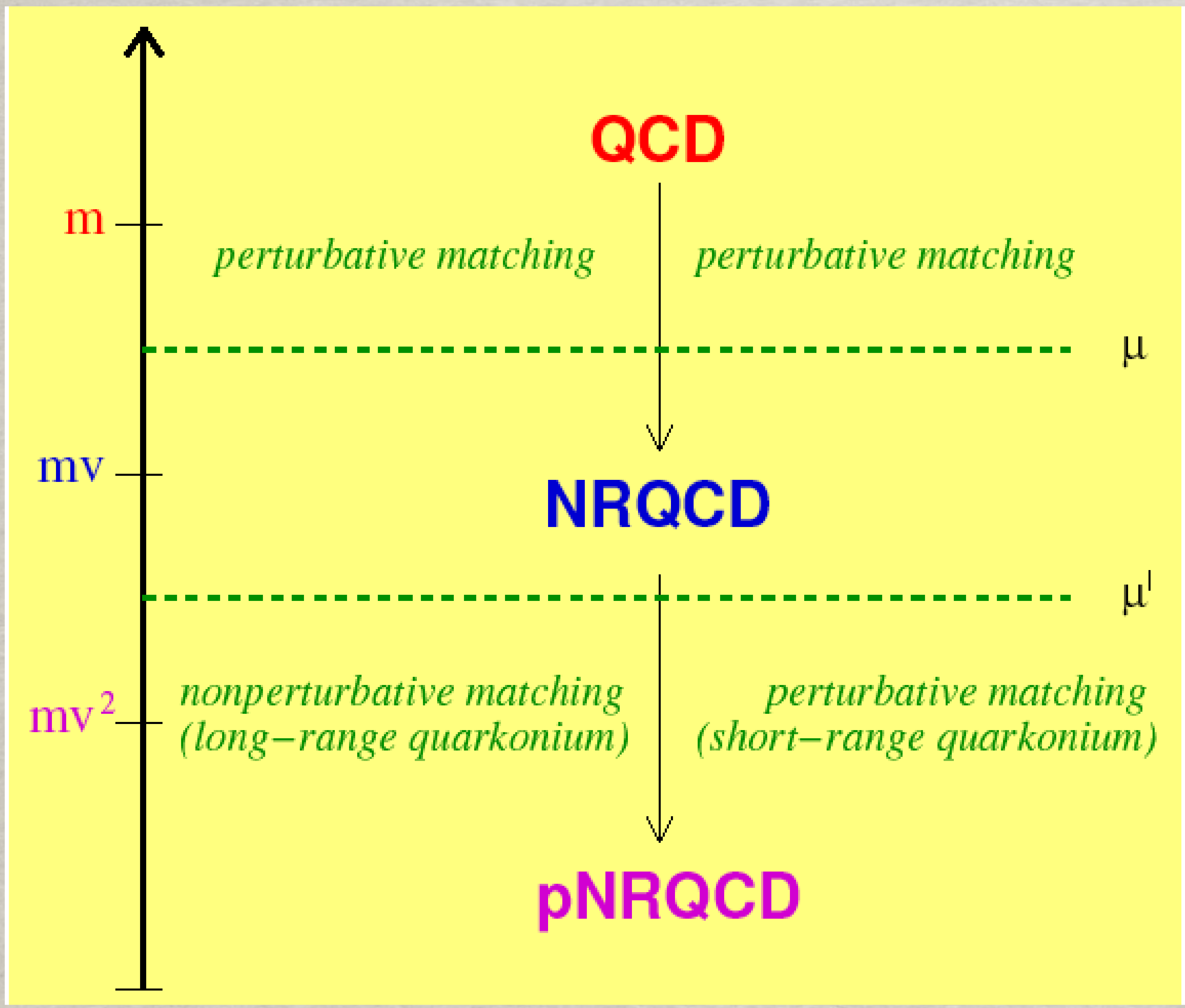
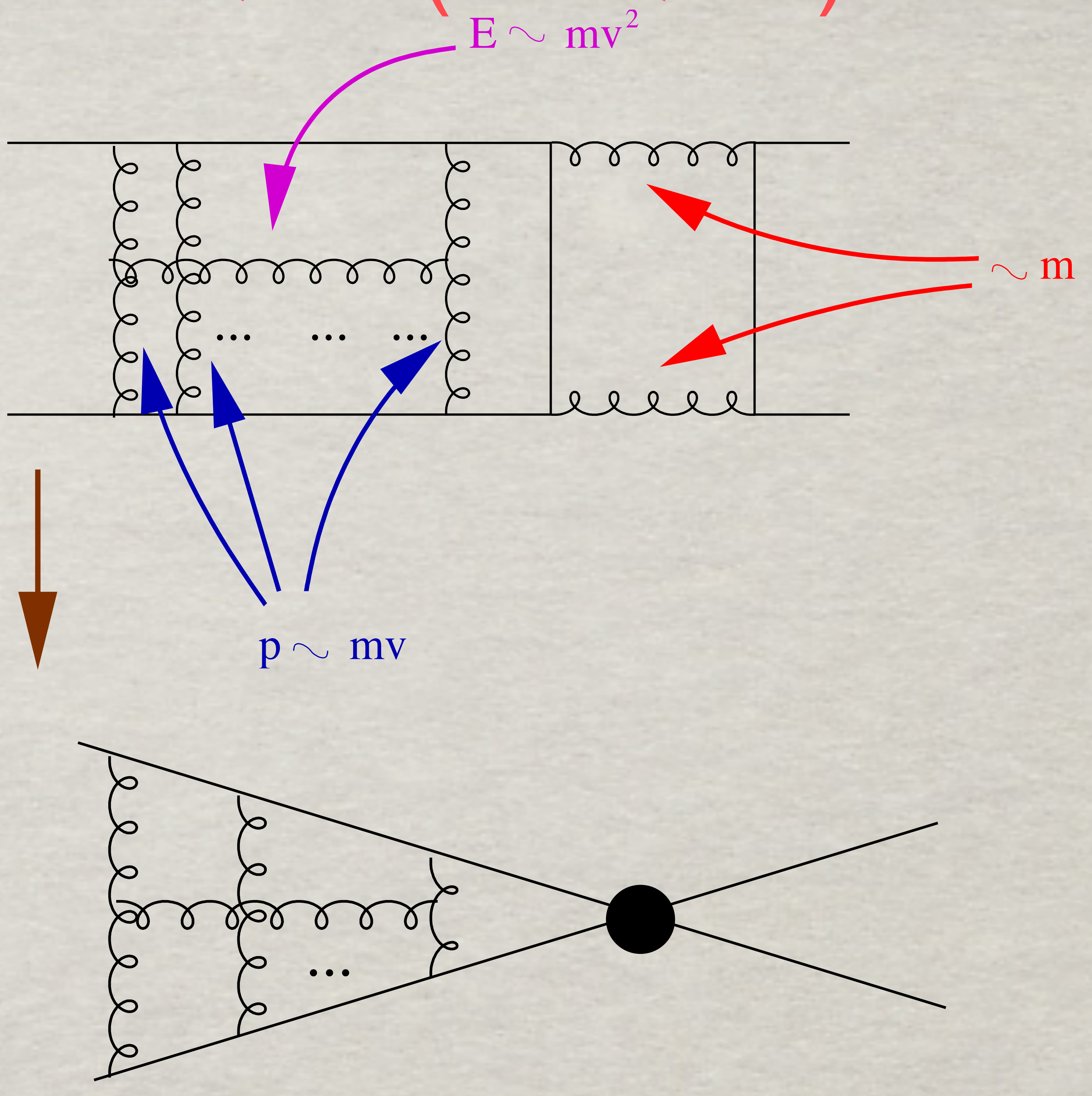
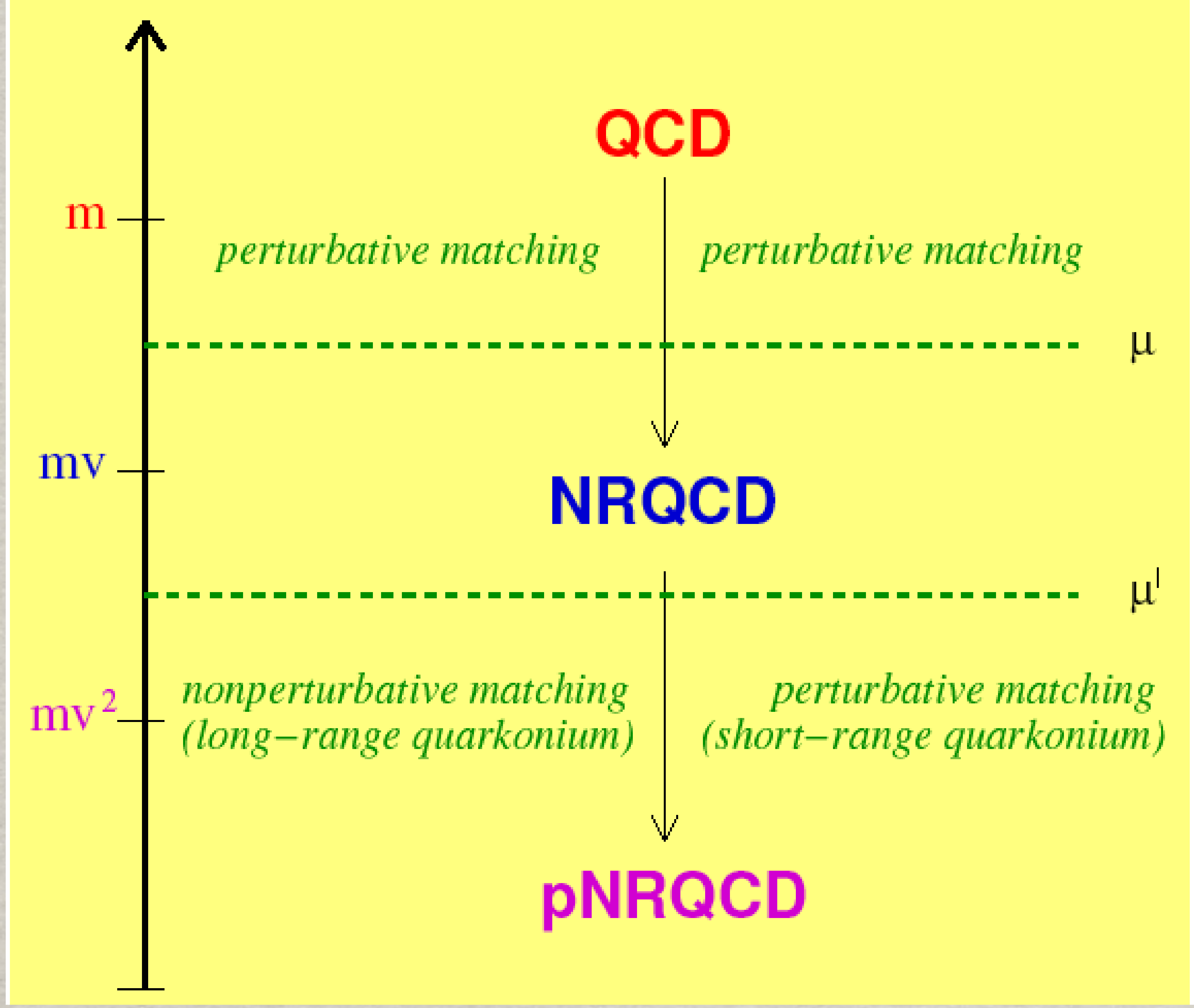
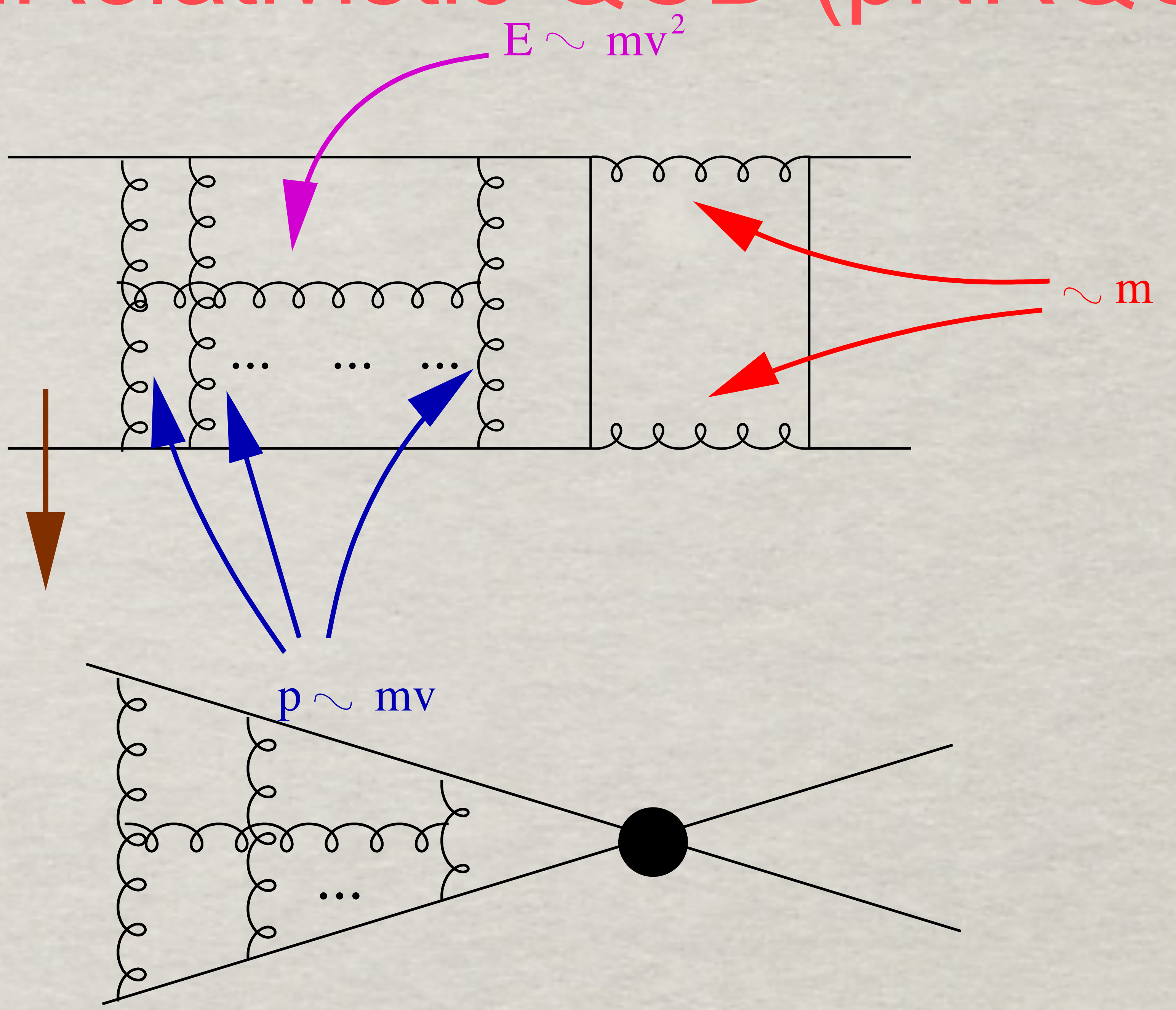




$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_{\mathrm{s}}(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





(with perturbative matching)

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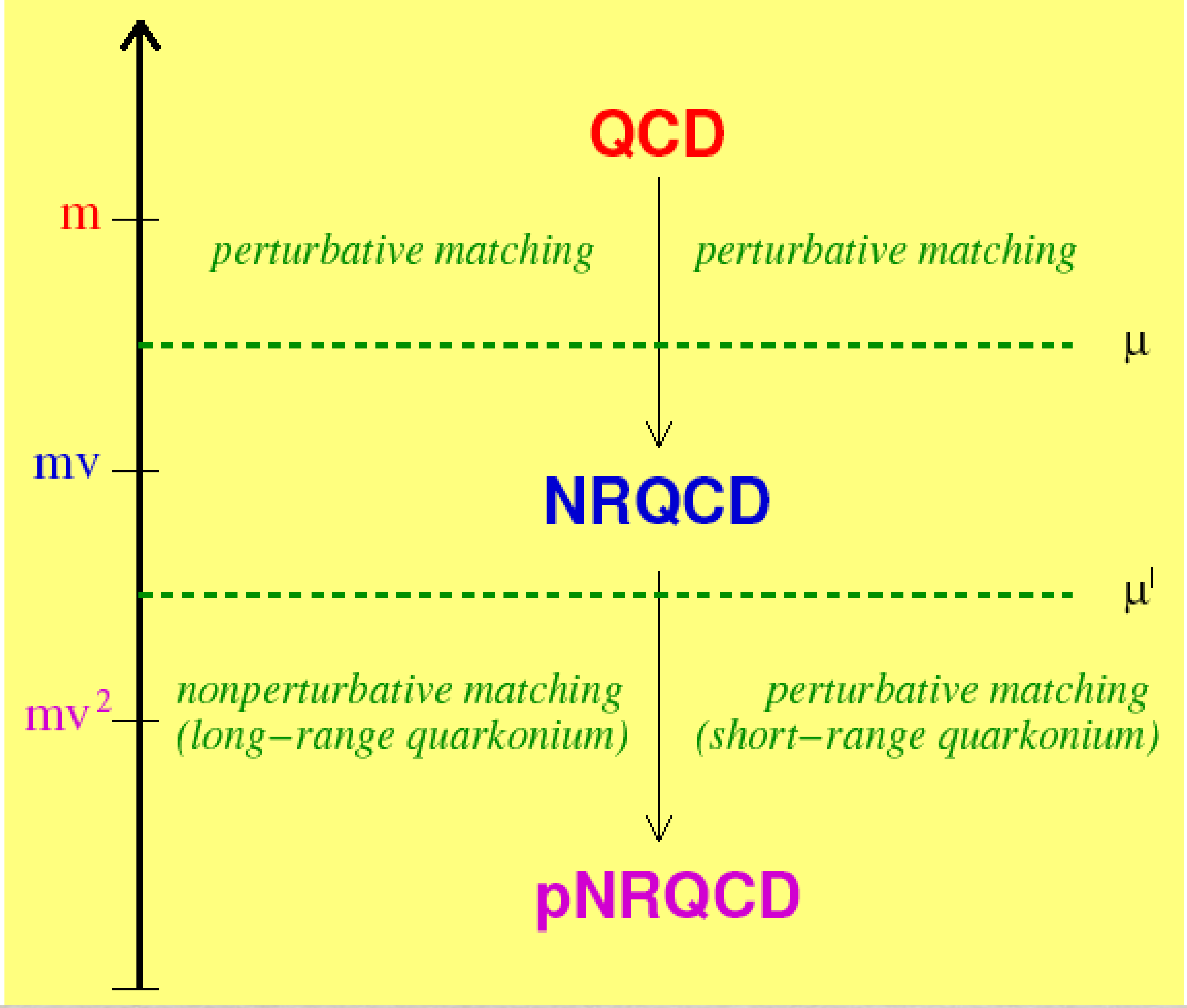
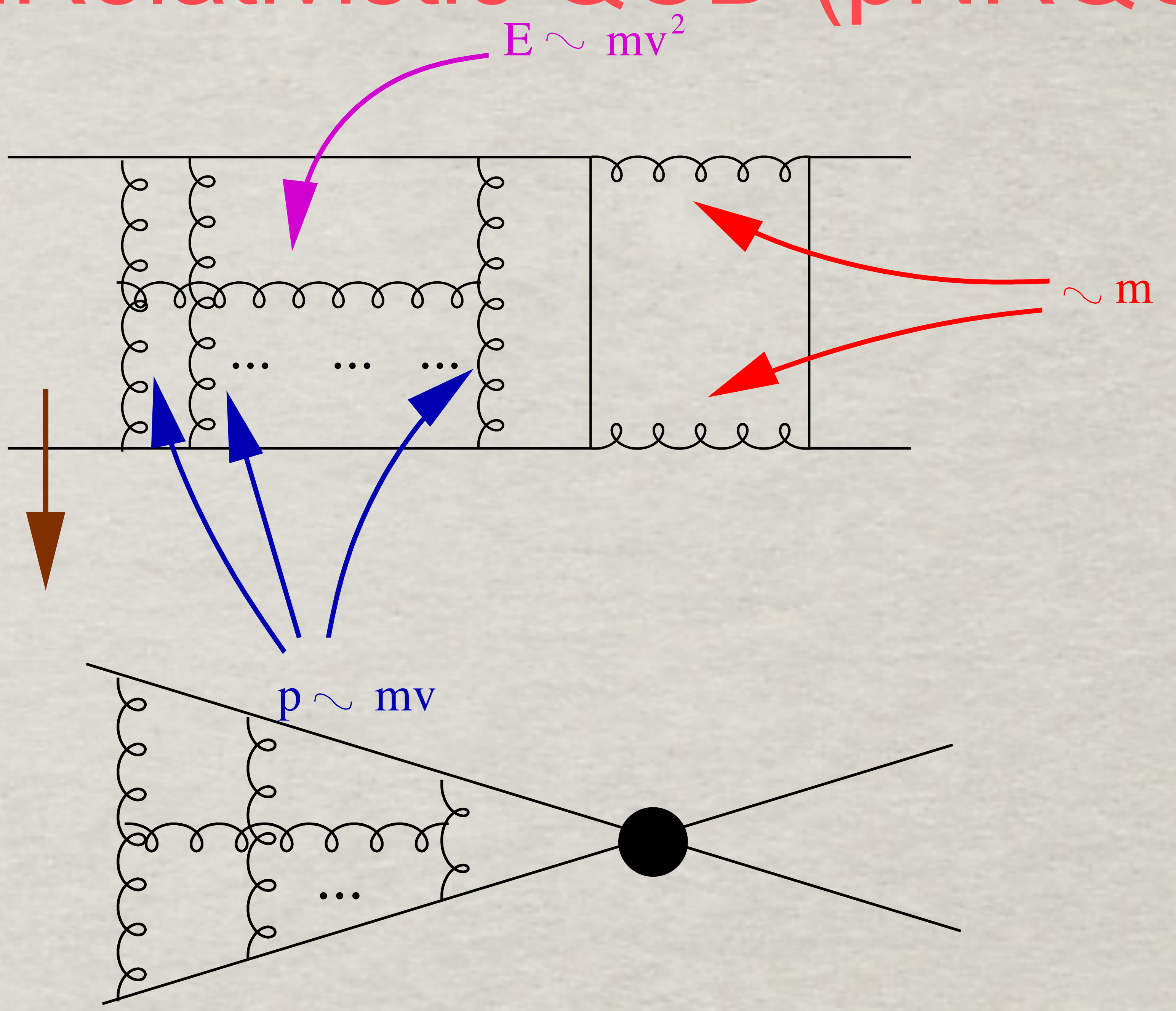
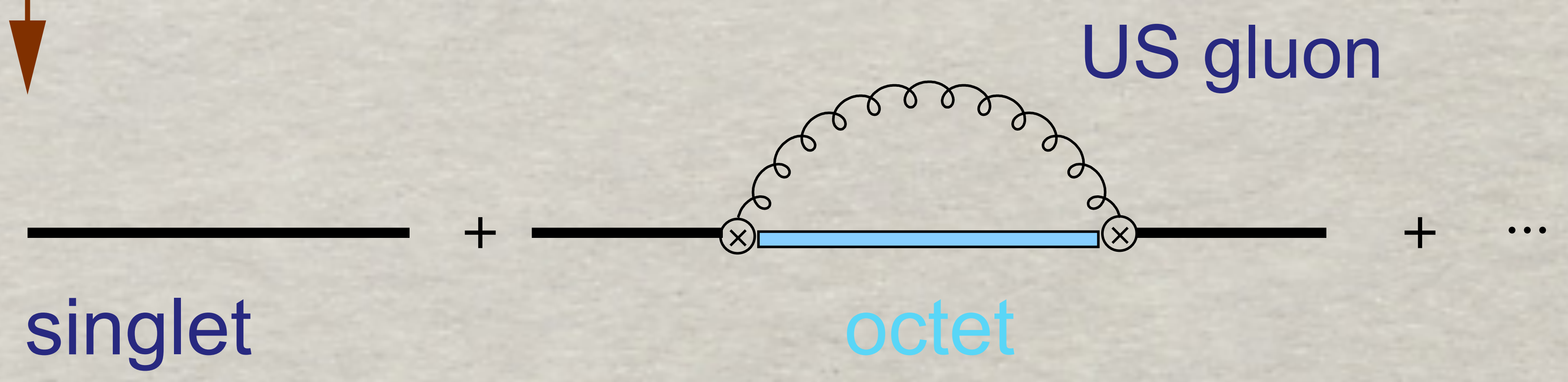
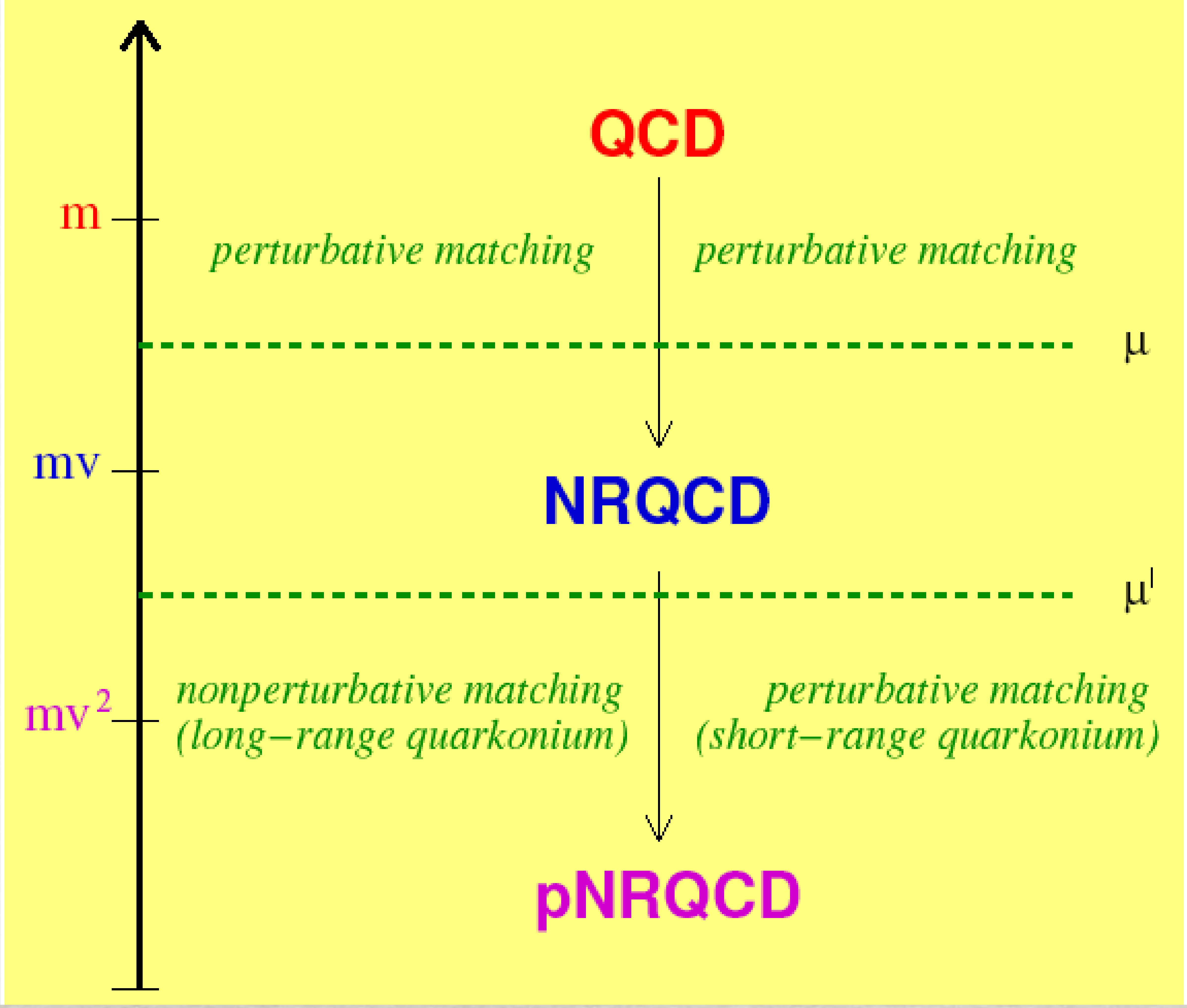


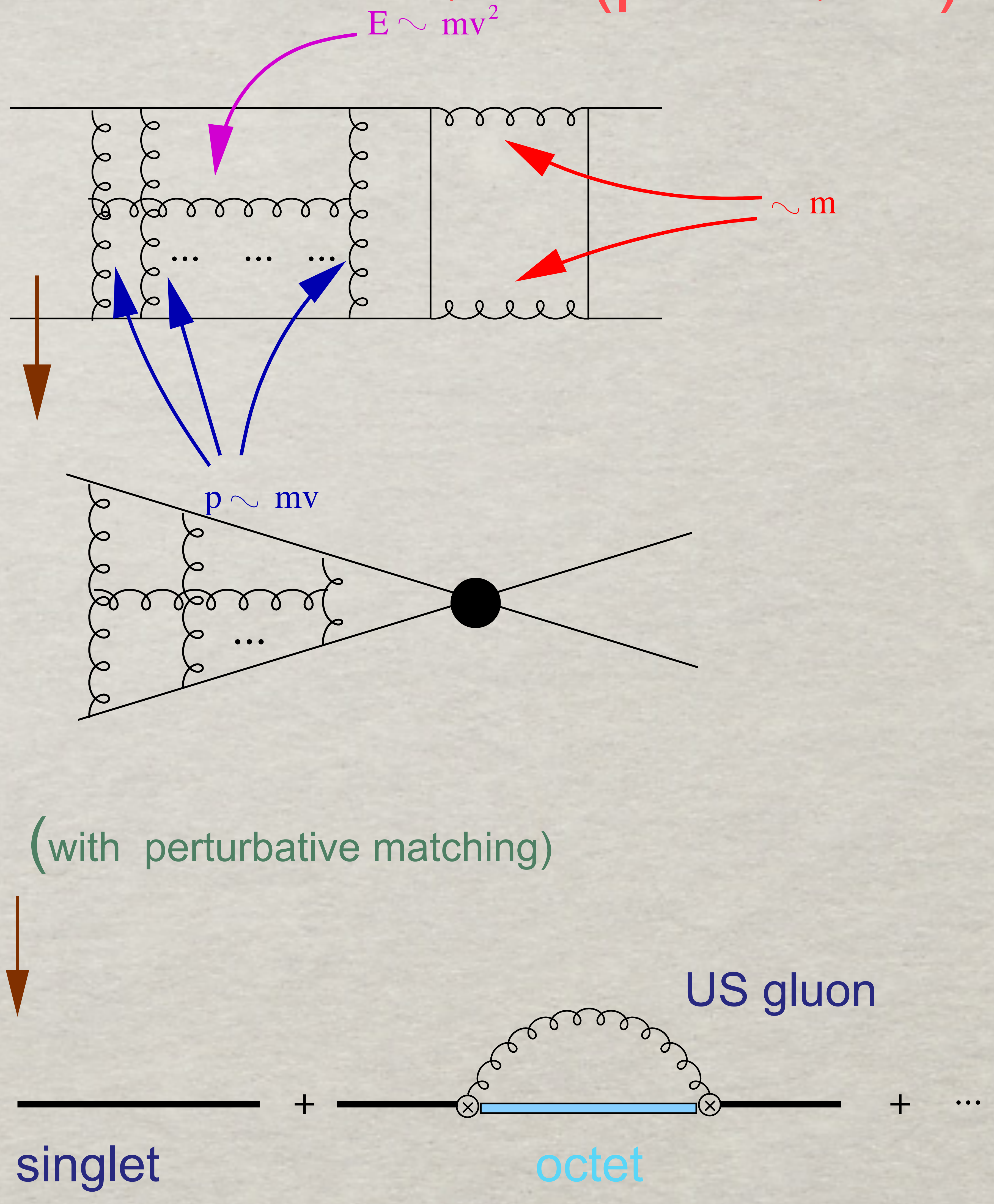


(with perturbative matching)



Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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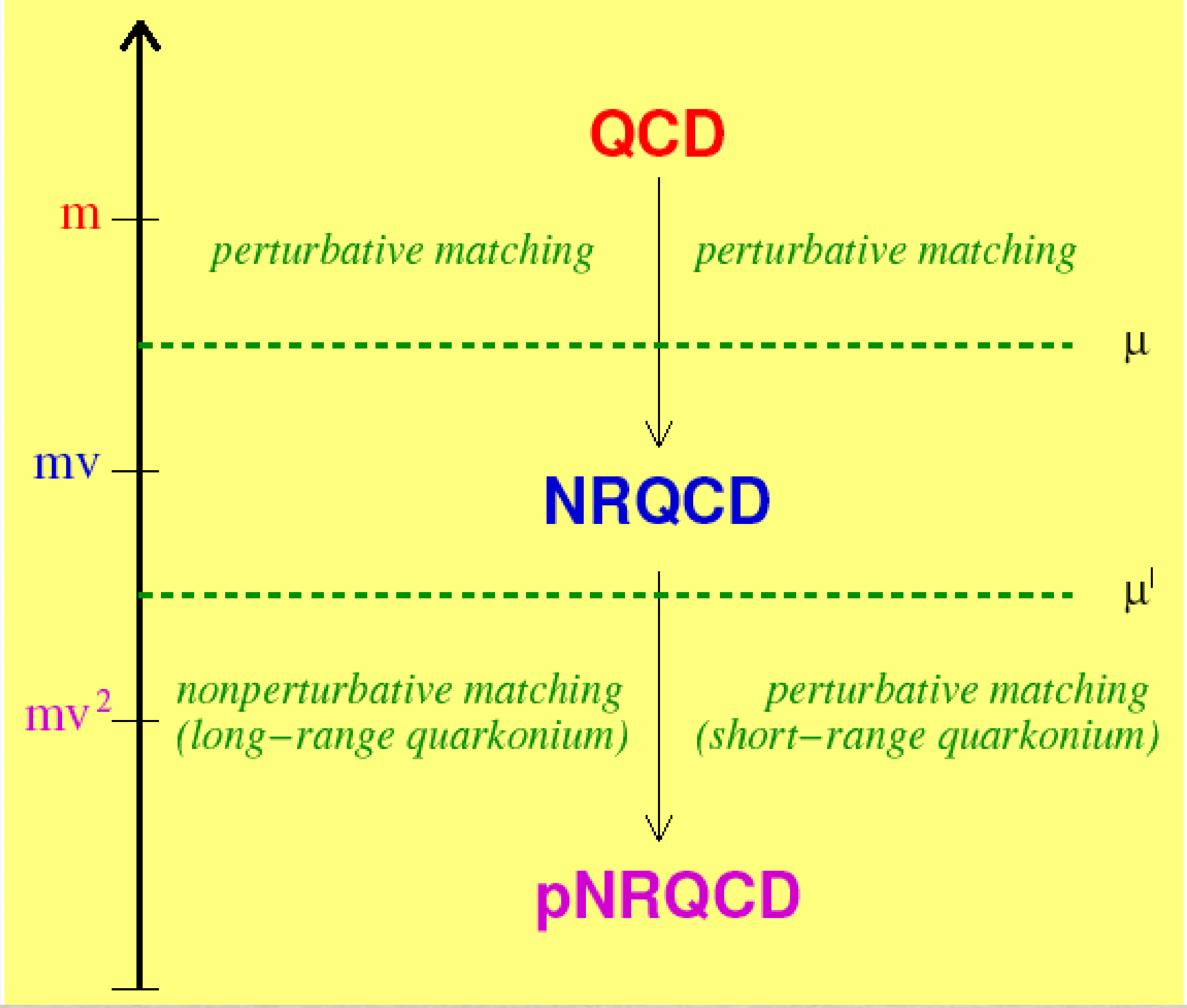

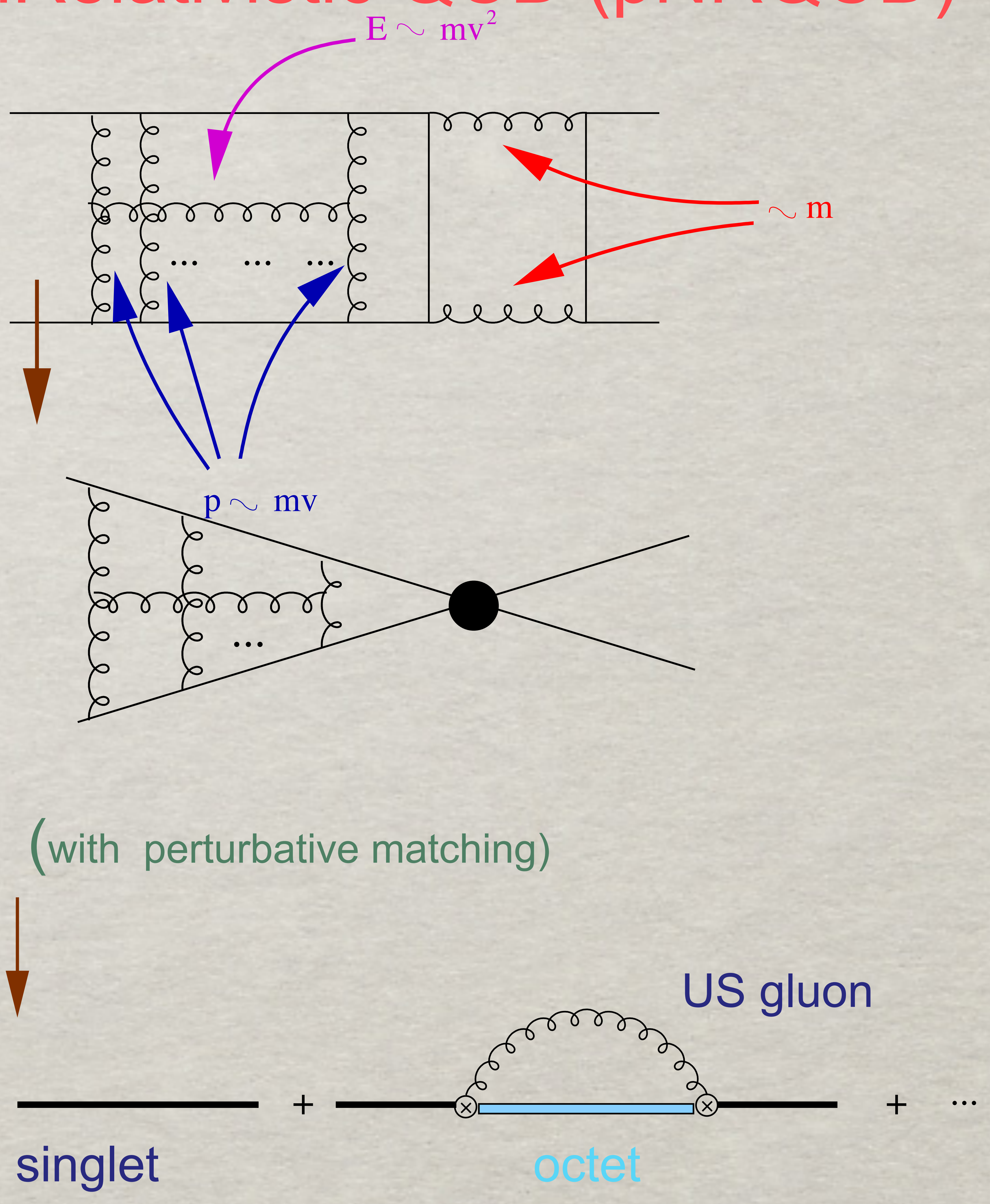

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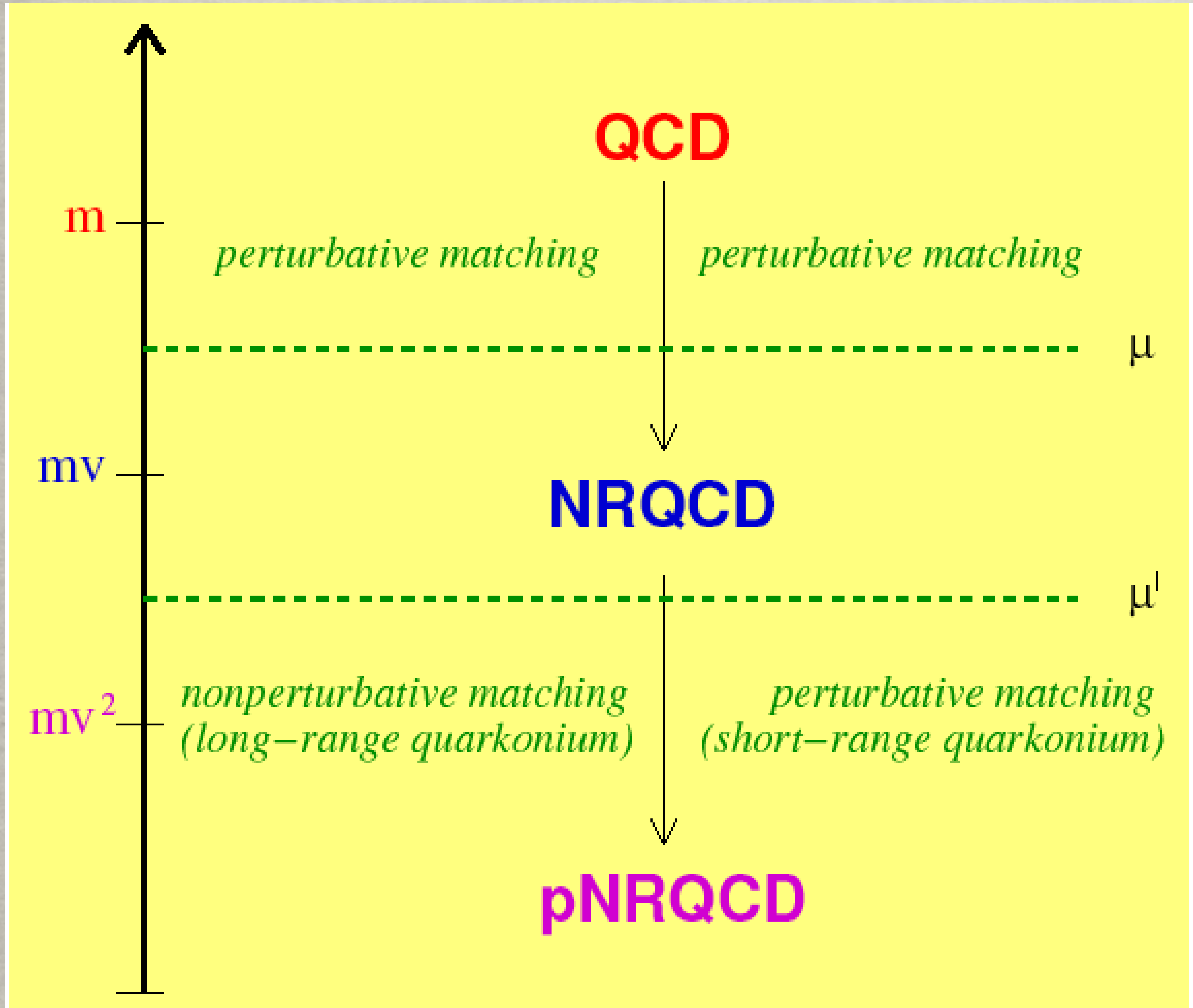

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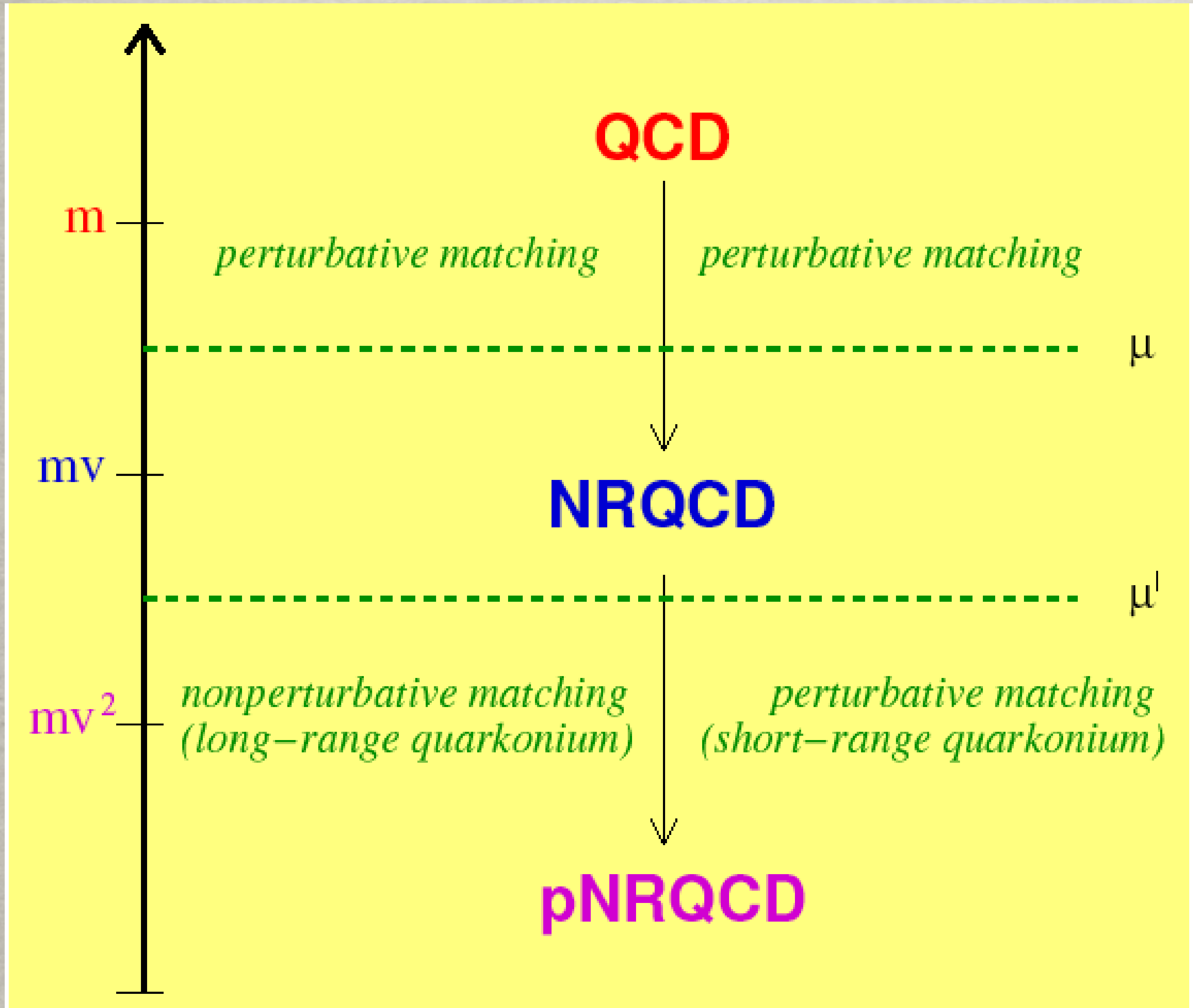

In QCD another scale is relevant $\Lambda_{\rm QCD}$

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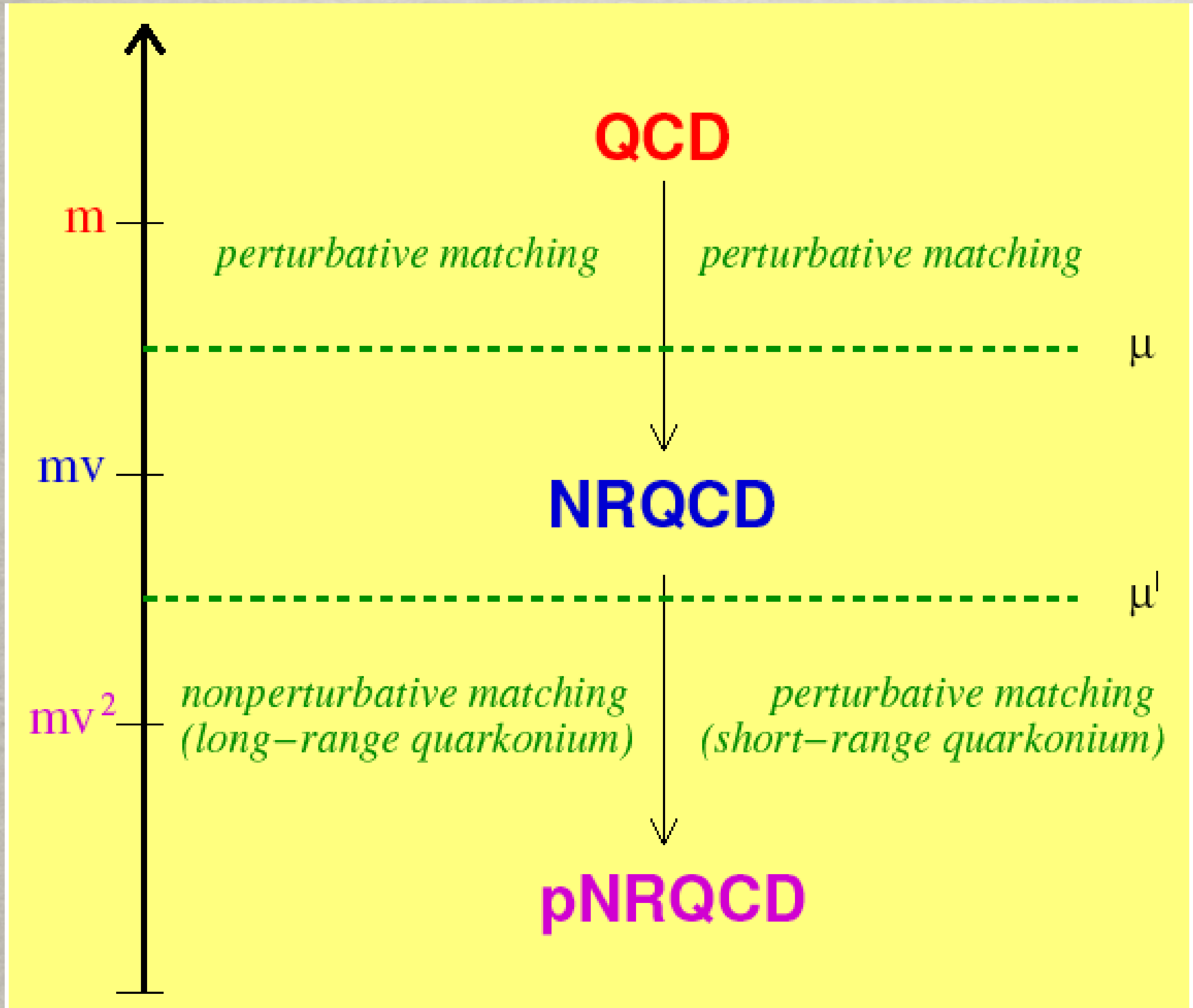

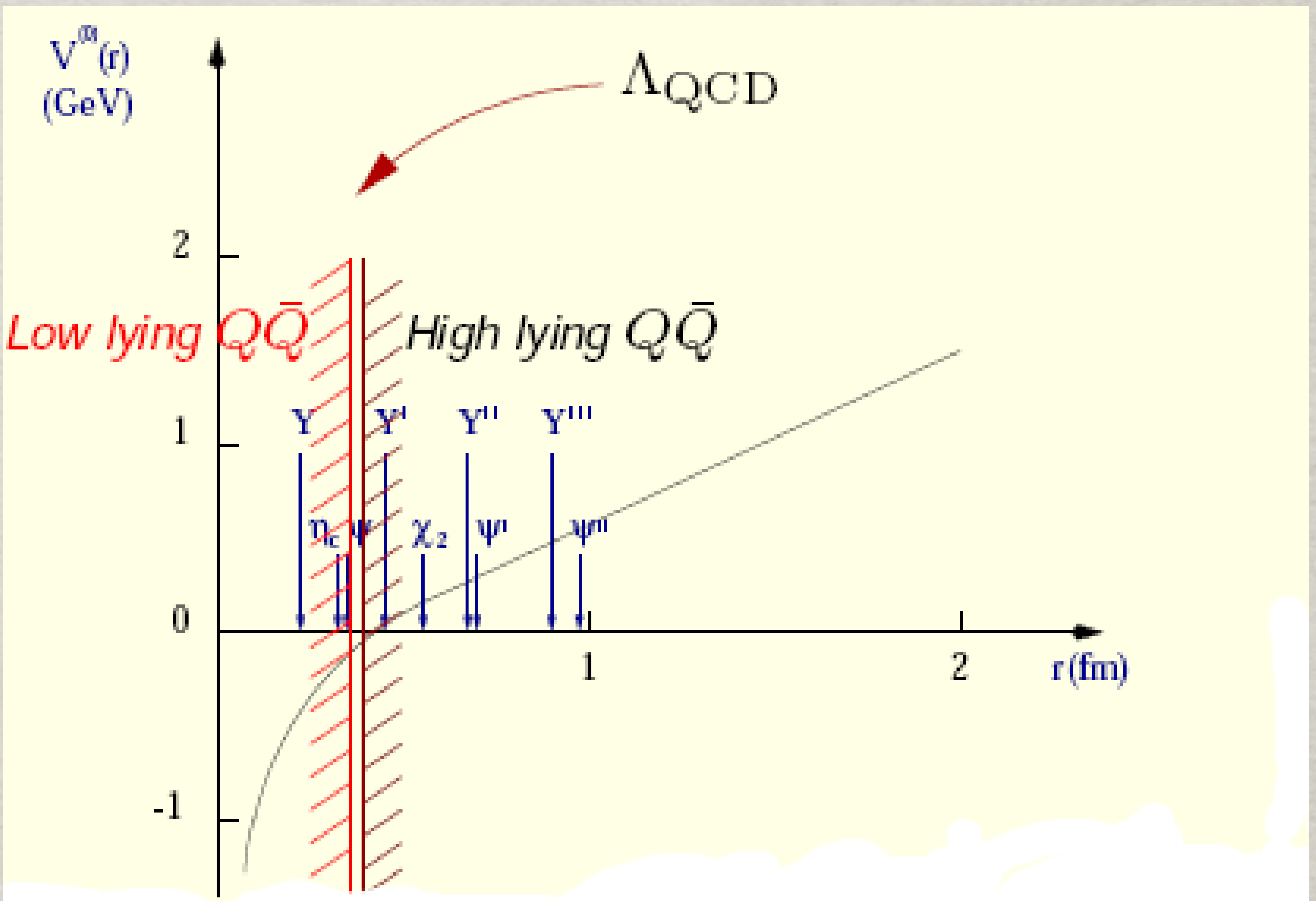

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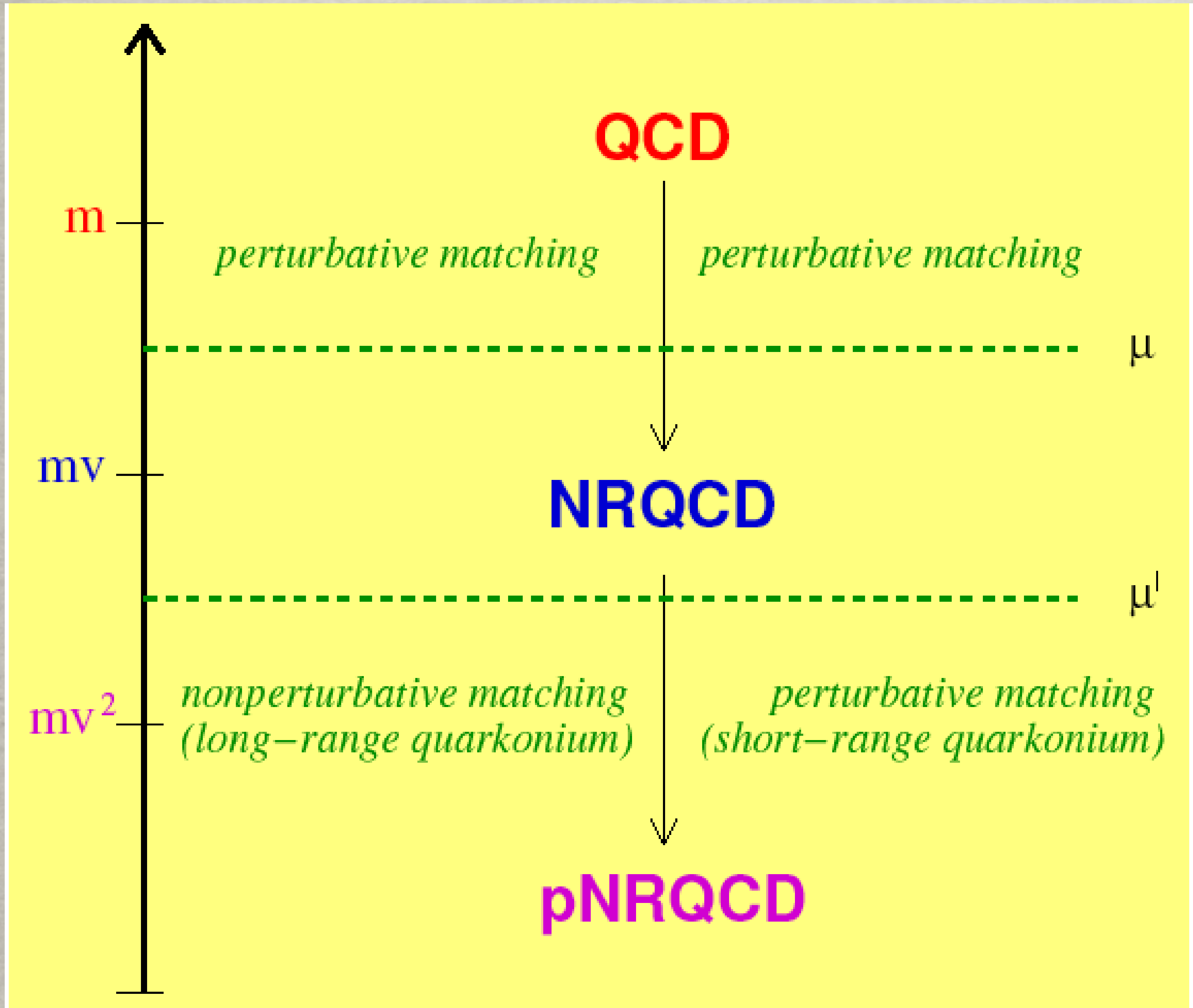

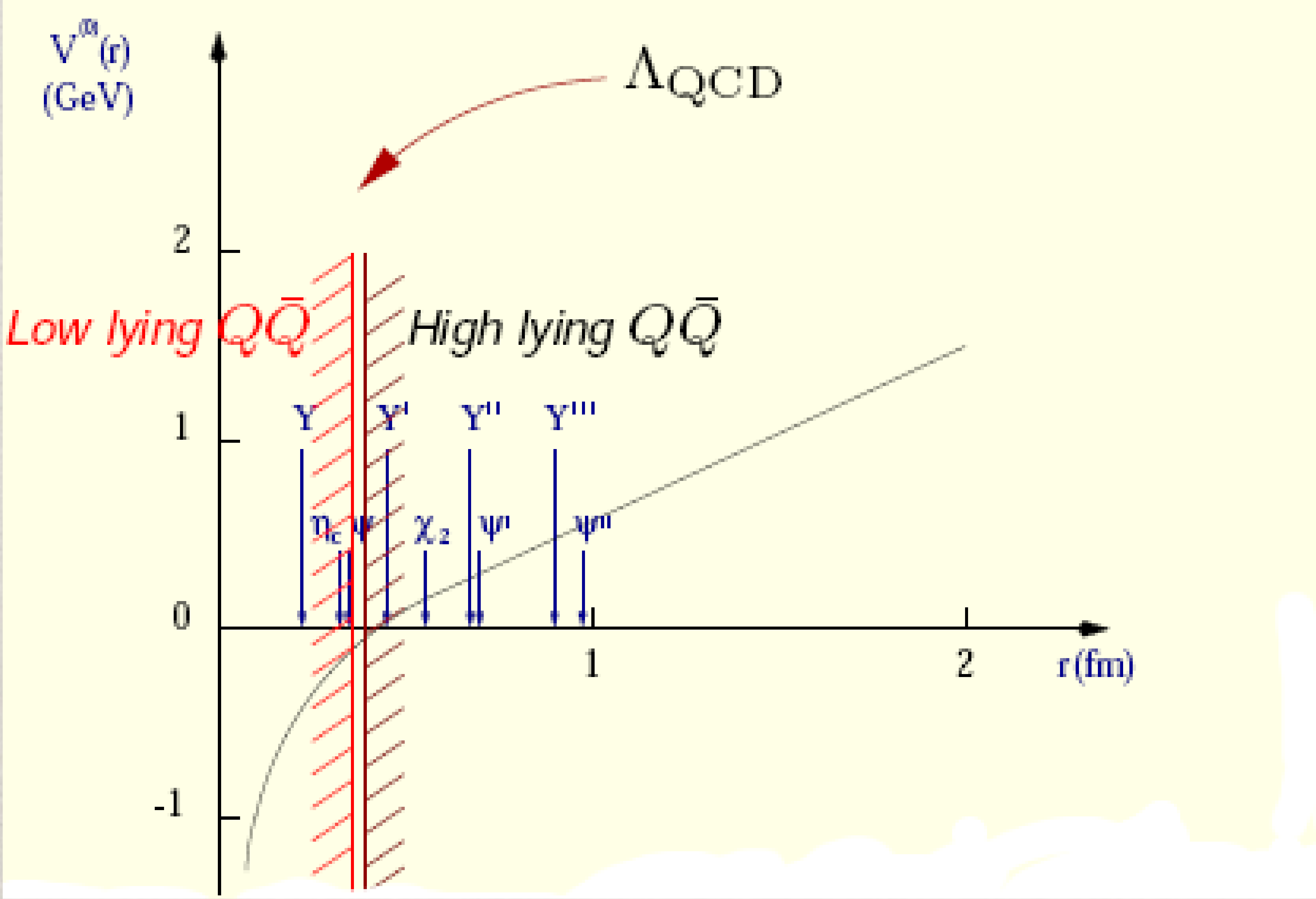

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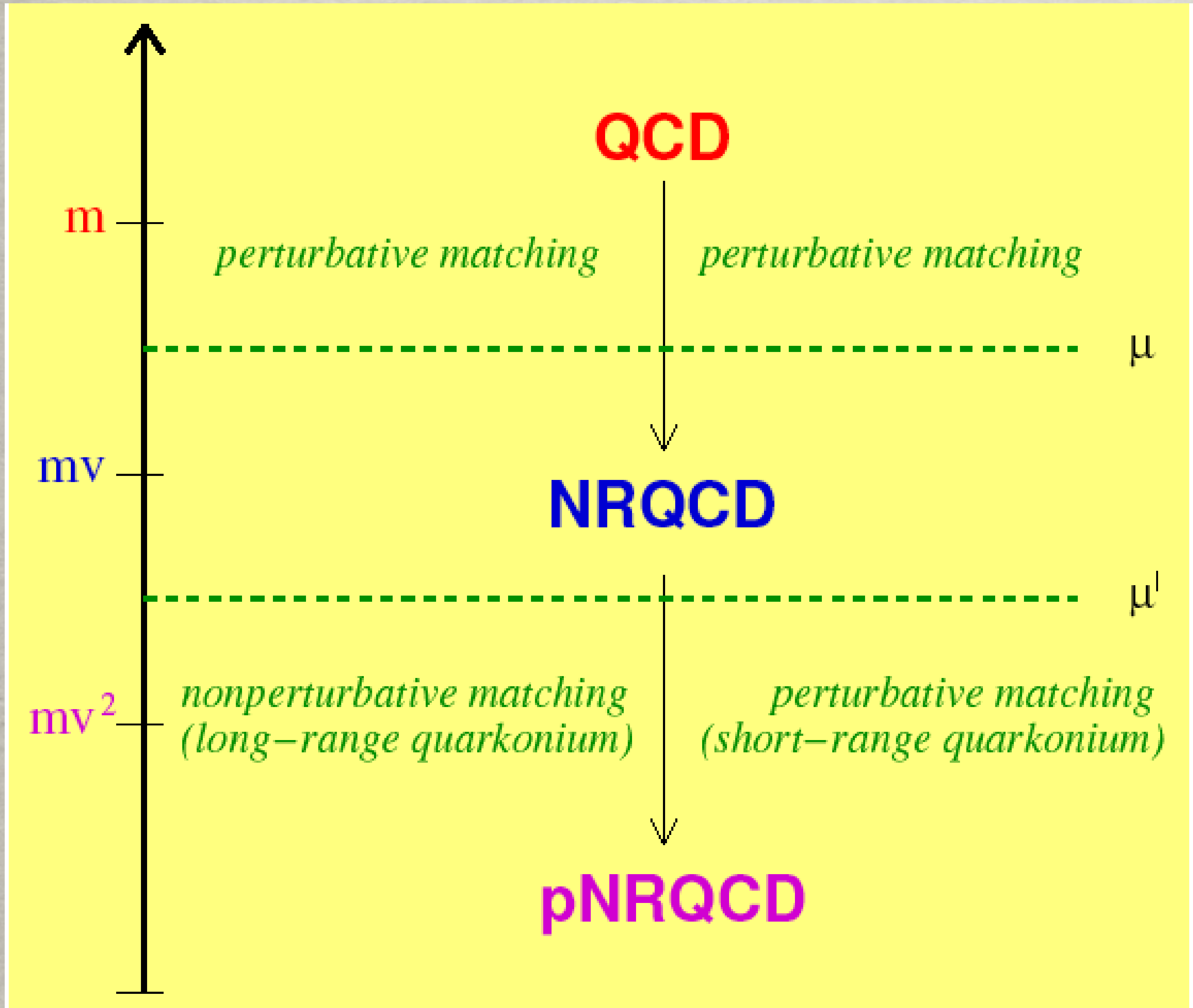

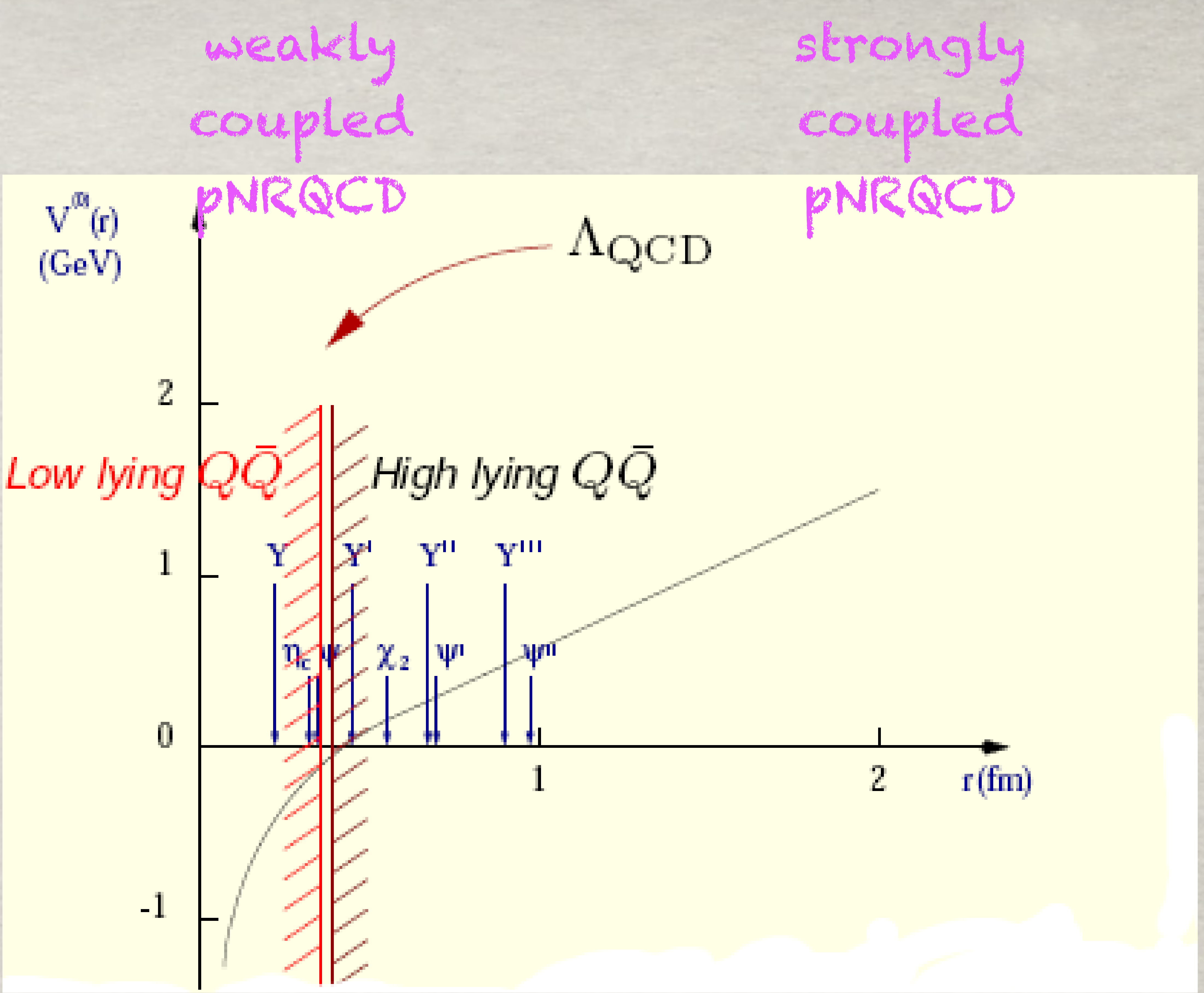

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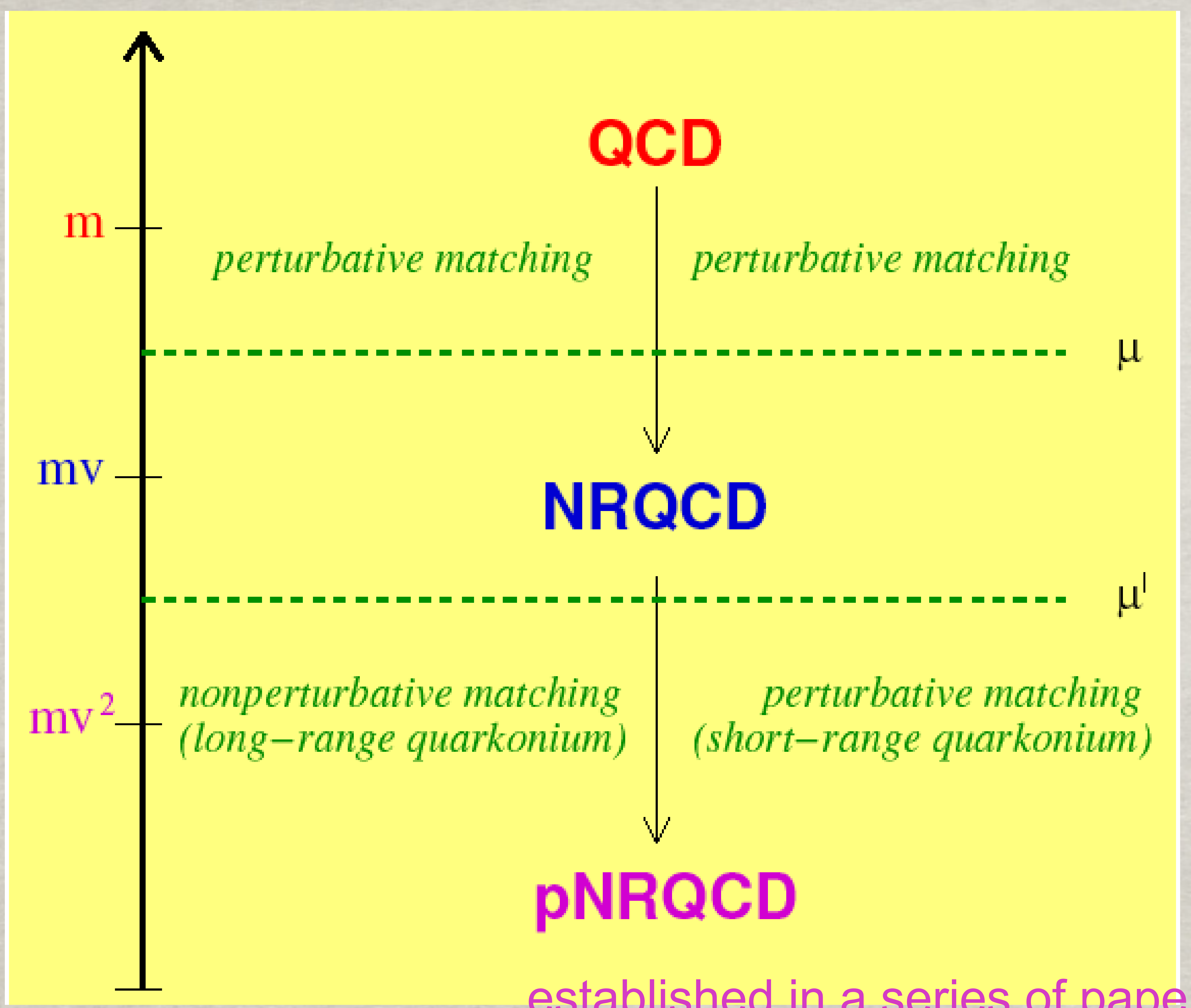
Quarkonium with NREFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, et al. 00–020
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

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The propagators of the NREFT have a noncovariant structure—> we have created a dedicated automatic symbolic calculation program to address these calculations (tree, 1 loop) see 'FeynOnium' 2006.15451

N.B., H. S. Chung, V. Shtabovenko, A. Vairo

NRQCD

(Caswell Lepage 86, Thacker, Lepage 88, 91, Bodwin Braaten Lepage 95)

$$\mathcal{L}^{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + ic_s \frac{\mathbf{S}\cdot[\mathbf{D}\times, g\mathbf{E}]}{4m^2} + \dots \right)\psi \quad + \chi^\dagger(\quad \dots \quad)\chi$$

$$+O(1/m^3)$$

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ψ (χ) is the field that annihilates (creates) the (anti)fermion

expansion in v and $\alpha_s(m)$

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Imaginary parts give the decay

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quarkonium state H

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NRQCD had a tremendous impact on spectrum lattice calculations, has given a theoretical framework for quarkonium production at colliders and for decays

pNRQCD

pNRQCD is the EFT for nonrelativistic quark-antiquark pairs ($Q\bar{Q}$) near threshold.

with respect to
$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- QFT = QCD
- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$
- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.
- The power counting is
  $\rightarrow p \sim 1/r \sim mv$ (soft scale),
  $\rightarrow E \sim \mathbf{p}^2/2m \sim V^{(0)} \sim \mathbf{P}_{\rm cm} \sim 1/\mathbf{R}_{\rm cm} \sim mv^2$ (ultrasoft scale),
  $\rightarrow$ operators in $\Delta\mathcal{L}$ scale like $(mv^2)^{\rm dimension}$.

◦ Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275
◦ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

pNRQCD

Matching the potential

$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- The potential is a matching coefficient of the EFT that may be computed from first principle by matching Green's functions in QCD with Green's function in pNRQCD, it is scheme and scale dependent, and undergoes renormalization. It may be organized as an expansion in $1/m$:

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

contains spin dependent terms

- The interaction terms contained in $\Delta\mathcal{L}$ provide corrections to the quantum mechanical picture.

○ Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

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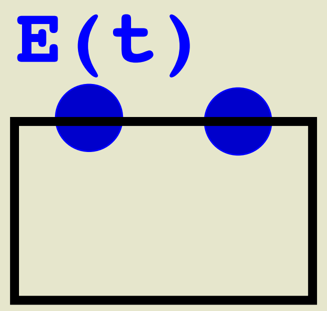
$$V^{(0)}(r) \; = \; \lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle - \Delta\mathcal{L}\,\text{effects}; \qquad \Box = \exp\left\{ ig \oint_{r\times T} dz^\mu A_\mu \right\}$$

Wilson loops (as matching Green's functions) guarantee gauge invariance.

- The $1/m$ potential:

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \; \langle \; \text{[Wilson loop with two } E(t) \text{ insertions]} \; \rangle \quad -\Delta\mathcal{L}\,\text{effects}$$



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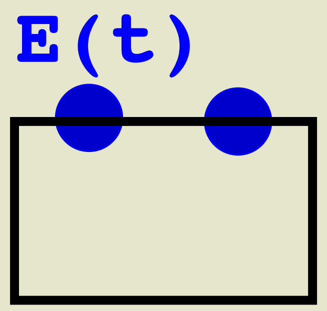

and so on..

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POINCARE' INVARIANCE IN NREFTS

Poincare’ Invariance in NREFTs

EFTs preserve all the invariances of the fundamental QFT.
Therefore NREFTs are constrained by the Poincaré invariance of the fundamental QFT, although Lorentz invariance is apparently broken by the nonrelativistic expansion.

It has been suggested, even before the establishing of EFTs, that Poincaré invariance provides non trivial constraints on the form of the potentials.

○ Dirac RMP 21 (1949) 302

Within NREFTs these constraints may be implemented in a rigorous setting. They allow to fix some of the matching coefficients/potentials of the NREFT to all orders and nonperturbatively without computing them. In QCD, these constraints can be tested against lattice determinations.

Poincare’ invariance gives the same constraints as reparameterization invariance (relations among the matching coefficients of the bilinear fermion terms in NRQCD) PLUS new relations, among the coefficients of the 4 fermions terms in NRQCD and among the potentials in pNRQCD

○ Brambilla Gromes Vairo PRD 64 (2001) 076010, PLB 576 (2003) 314
Berwein Brambilla Hwang Vairo PRD 99 (2019) 094008

Heinonen, Hill, Solon PRD86 (2012) 094020

Poincare' algebra

For any Poincare invariant theory the generators H, P , J and K of time translation, space translation, rotations and Lorentz boosts satisfy the Poincare algebra:

$$\begin{aligned}
[\mathbf{P}^i, \mathbf{P}^j] &= 0 \\
[\mathbf{P}^i, H] &= 0 \\
[\mathbf{J}^i, \mathbf{P}^j] &= i\epsilon_{ijk}\mathbf{P}^k \\
[\mathbf{J}^i, H] &= 0 \\
[\mathbf{J}^i, \mathbf{J}^j] &= i\epsilon_{ijk}\mathbf{J}^k \\
[\mathbf{P}^i, \mathbf{K}^j] &= -i\delta_{ij}H \\
[H, \mathbf{K}^i] &= -i\mathbf{P}^i \\
[\mathbf{J}^i, \mathbf{K}^j] &= i\epsilon_{ijk}\mathbf{K}^k \\
[\mathbf{K}^i, \mathbf{K}^j] &= -i\epsilon_{ijk}\mathbf{J}^k
\end{aligned}$$

Once P and J are written in terms of the EFTs fields, and H and K have been matched, the algebra constraints the matching coefficients of H, which include the potentials, and K.

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\end{aligned}
$$

—>The Poincare algebra imposes the following constraints on the potentials

$$V_{LS}^{(1)}(r) - V_{LS}^{(2)}(r) + \frac{1}{2r}V^{(0)\prime}(r) = 0$$

$$4V_{L^2}^{(1)}(r) - 2V_{L^2}^{(2)}(r) + rV^{(0)\prime}(r) = 0$$

$$4V_{p^2}^{(1)}(r) - 2V_{p^2}^{(2)}(r) + V^{(0)}(r) - rV^{(0)\prime}(r) = 0$$

The constraints are generic and do not depend on the dynamical content of the EFT. They are satisfied by any potential defined in an EFT and derived from a relativistic QFT.

Once P and J are written in terms of the EFTs fields, and H and K have been matched, the algebra constraints the matching coefficients of H, which include the potentials, and K.

WEAKLY COUPLED PNRQCD: $mv \gg \Lambda_{QCD}$
AND THE PERTURBATIVE MATCHING

Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \dots$

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\,\Big\{S^\dagger\Big(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots\Big)S + O^\dagger\Big(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots\Big)O +$$

LO in r

$$+V_A(S^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger \mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\Big\} + \dots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

Weakly coupled pNRQCD

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NLO in r

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The matching coefficients are the Coulomb potential

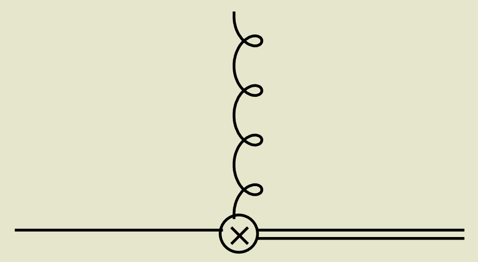
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \ldots,$$

$$V_A = 1 + \mathcal{O}(\alpha_s^2),\; V_B = 1 + \mathcal{O}(\alpha_s^2).$$

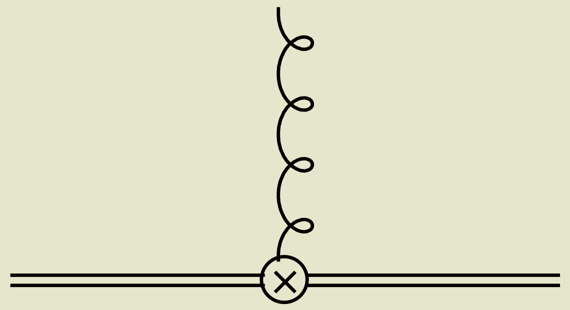
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$



$= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

pNRQCD

- pNRQCD provides a QM description from field theory: the Schroedinger equation and the potentials appear once all scales above the binding energy have been integrated out: this provides a clear definition of the potential

- The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture. Retardation effects are typically related to the nonperturbative physics

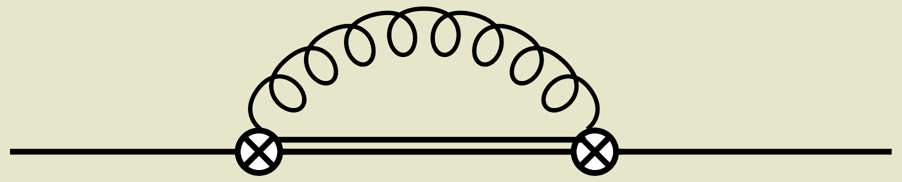
- The Quantum Mechanical divergences are cancelled by the NRQCD matching coefficients.

- Poincare' invariance is intact and is realized via exact relations among the matching coefficients (potentials)

Weakly coupled static potential

○ Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

$$V^{(0)}(r,\mu') = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle -$$



$$+\cdots$$

$$= E_0(r) + \frac{i}{N}\int_0^\infty dt\, e^{-it(V_o - V_S)} \langle \mathrm{Tr}\, \mathbf{r}\cdot g\mathbf{E}(t)\, \mathbf{r}\cdot g\mathbf{E}(0)\rangle(\mu') + \cdots$$

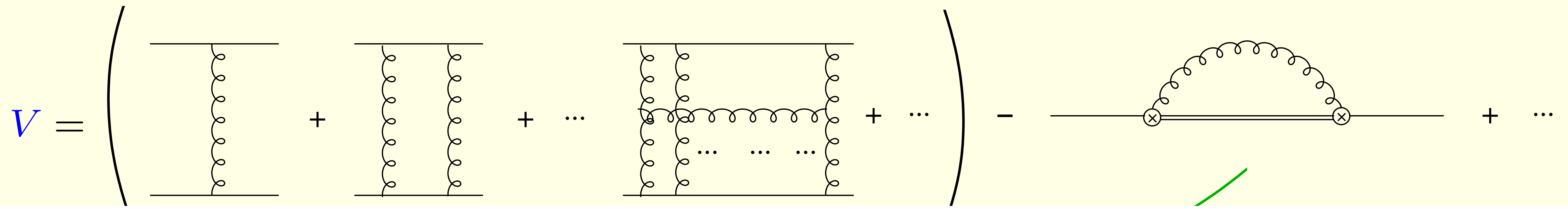
The static energy $E_0(r)$ is known at three loops:

$$E_0(r) = \Lambda_s - \frac{C_F \alpha_s}{r}\left(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln \alpha_s + \#\alpha_s^4 \ln^2 \alpha_s + \#\alpha_s^4 \ln \alpha_s + \ldots\right)$$

○ Anzai Kiyo Sumino PRL 104 (2010) 112003
A.Smirnov V.Smirnov Steinhauser PRL 104 (2010)

$\ln \alpha_s$ in E_0 signals the cancellation of contributions coming from soft and ultrasoft gluons

$$\ln \alpha_s = \ln \frac{\mu'}{1/r} + \ln \frac{\alpha_s/r}{\mu'}$$



Infrared logarithms in the potential may be computed in the EFT solving the ADM problem.

○ Appelquist Dine Muzinich PRD 17 (1978) 2074

QCD singlet static potential at N^4LO

$$V_s(r,\mu) = -C_F \frac{\alpha_s(1/r)}{r}\left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 \right.$$

$$+ \left(\frac{16\,\pi^2}{3} C_A^3 \ln r\mu + a_3\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3$$

$$\left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2 C_A^3 \beta_0(-5 + 6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\right]$$

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a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vair 3LOOPS REDUCES TO 1LOOP IN THE EFT

a_4^{L2}, a_4^L N.B., Garcia, Soto, V 4LOOPS REDUCES TO 2LOOPS IN THE EFT

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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$$V_s(r,\mu) = -C_F\frac{\alpha_s(1/r)}{r}\left[1 + a_1\frac{\alpha_s(1/r)}{4\pi} + a_2\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 + \left(\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + a_3\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 + \left(a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\right]$$

Two problems:

1)Bad convergence of the series due to renormalons

2) Large logs

QCD singlet static potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \quad = \quad & -C_F \frac{\alpha_s(1/r)}{r} \left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \right. \\
& + \left(\frac{16\,\pi^2}{3} C_A^3 \ln r\mu + a_3 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \\
& \left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0 (-5 + 6\ln 2) \right) \ln r\mu + a_4 \right) \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \right]
\end{aligned}
$$

Two problems:

1)Bad convergence of the series due to renormalons

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The eft cures both:

1) Renormalon subtracted scheme   Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

2) Renormalization group summation of the logs (RG with correlated scales Luke and Savage, Manohar and Stewart, Pineda and Soto)

up to N^3LL $(\alpha_s^{4+n} \ln^n \alpha_s)$ N. B Garcia, Soto Vairo 2007, 2009,

Resummation of logarithms

The potential satisfies renormalization group equations

$$\begin{cases} \mu' \dfrac{d}{d\mu'} V^{(0)} = -\dfrac{2}{3} C_F \dfrac{\alpha_s}{\pi} r^2 \left[V_o^{(0)} - V^{(0)}\right]^3 \left(1 + \dfrac{\alpha_s}{\pi} c\right) \\ \mu' \dfrac{d}{d\mu'} V_o^{(0)} = \dfrac{1}{N} \dfrac{\alpha_s}{\pi} r^2 \left[V_o^{(0)} - V^{(0)}\right]^3 \left(1 + \dfrac{\alpha_s}{\pi} c\right) \\ \mu' \dfrac{d}{d\mu'} \alpha_s = \alpha_s \beta(\alpha_s); \qquad c = \dfrac{-5n_f + C_A(6\pi^2 + 47)}{108} \end{cases}$$

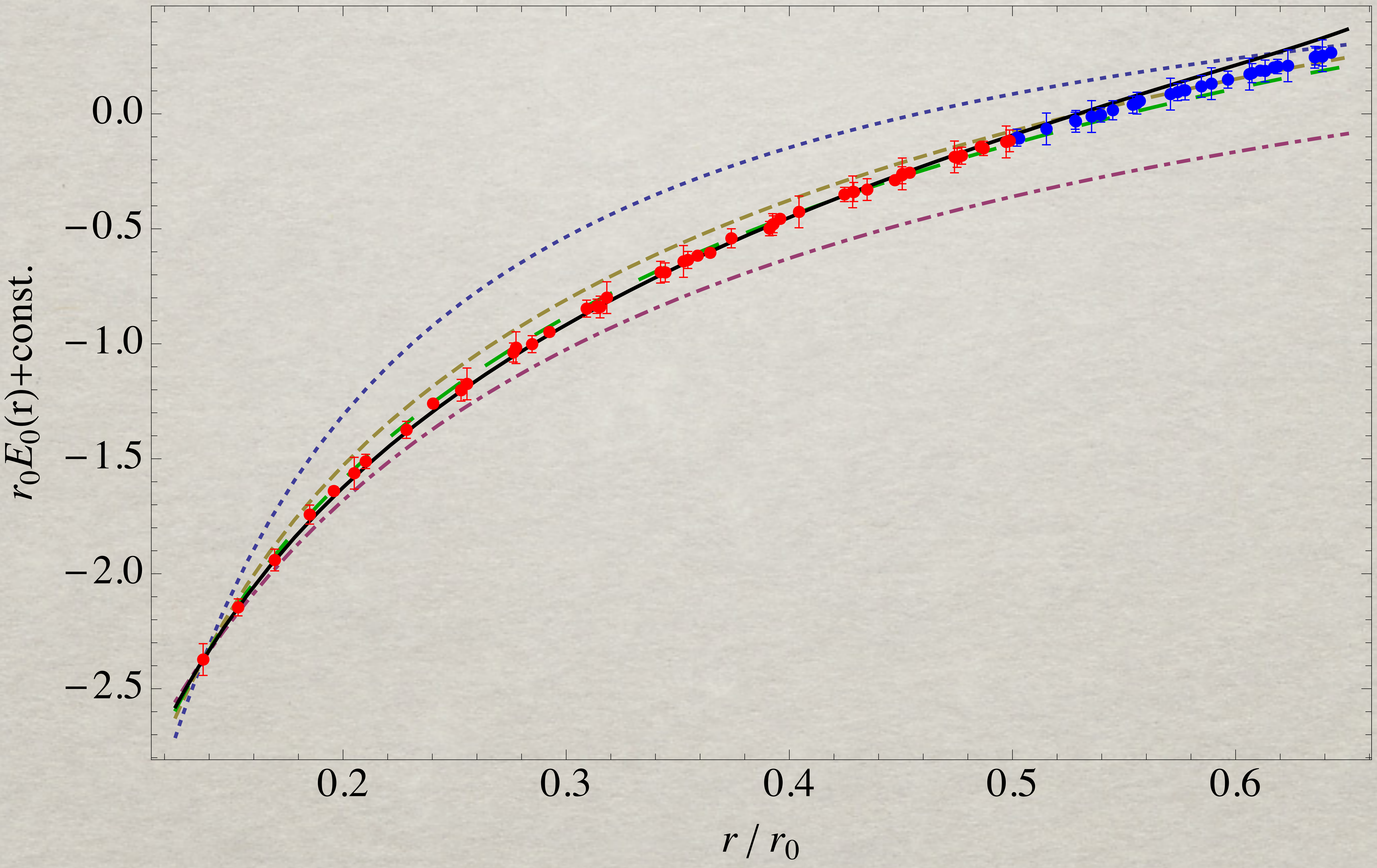
whose solution provides $V^{(0)}$ with N^{3}LL accuracy:

$$V^{(0)}(r, \mu') = V^{(0)}(r, 1/r) - \frac{C_F C_A^3}{6\beta_0} \frac{\alpha_s^3(1/r)}{r} \left\{ \left(1 + \frac{3}{4} \frac{\alpha_s(1/r)}{\pi} a_1\right) \ln \frac{\alpha_s(1/r)}{\alpha_s(\mu')} \right.$$
$$\left. \left(\frac{\beta_1}{4\beta_0} - 6c\right) \left[\frac{\alpha_s(\mu')}{\pi} - \frac{\alpha_s(1/r)}{\pi}\right] \right\}$$

- Pineda Soto PLB 495 (2000) 323
  Brambilla Garcia Soto Vairo PRD 80 (2009) 034016

QQbar singlet static energy at N^3LL in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019



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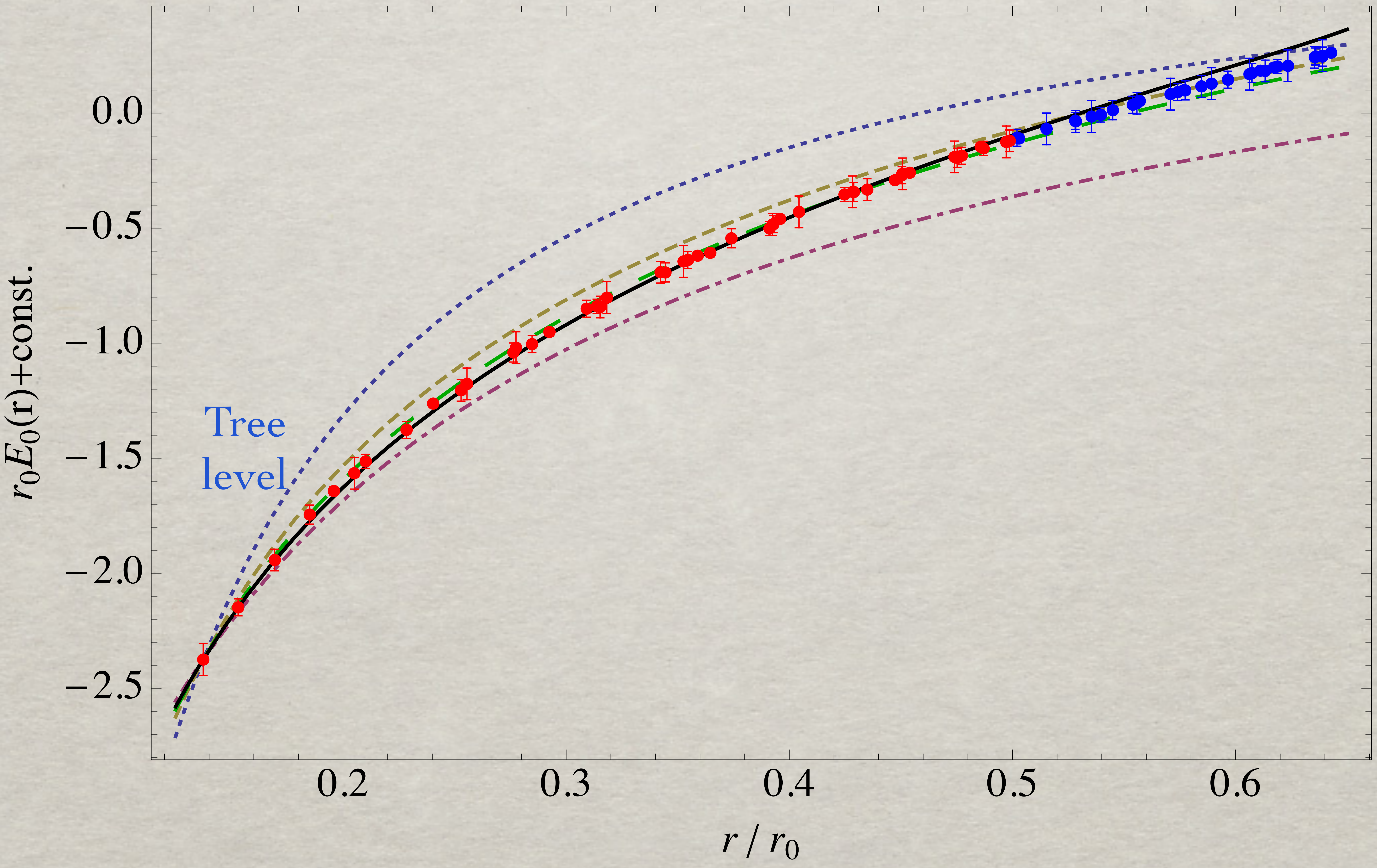

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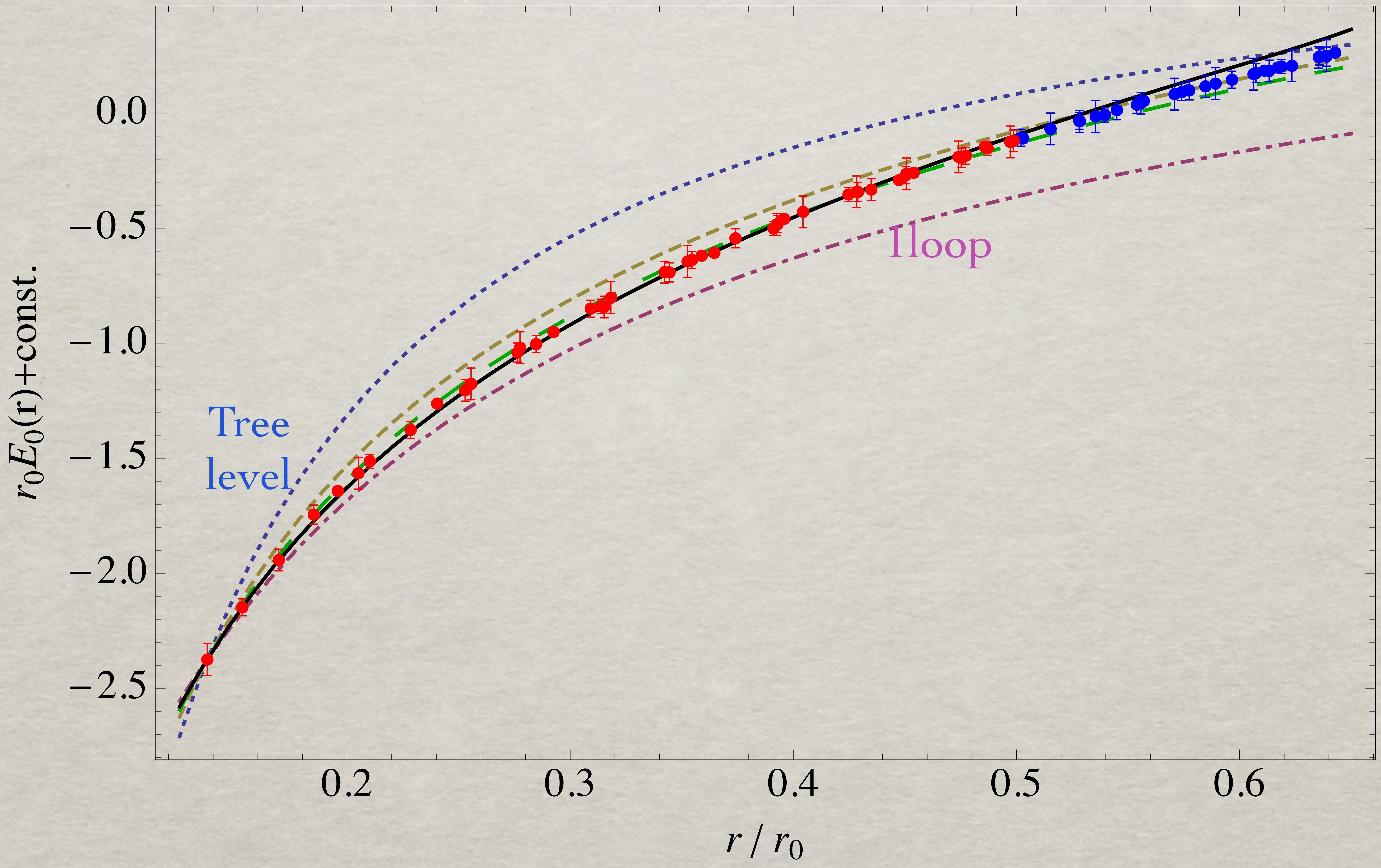

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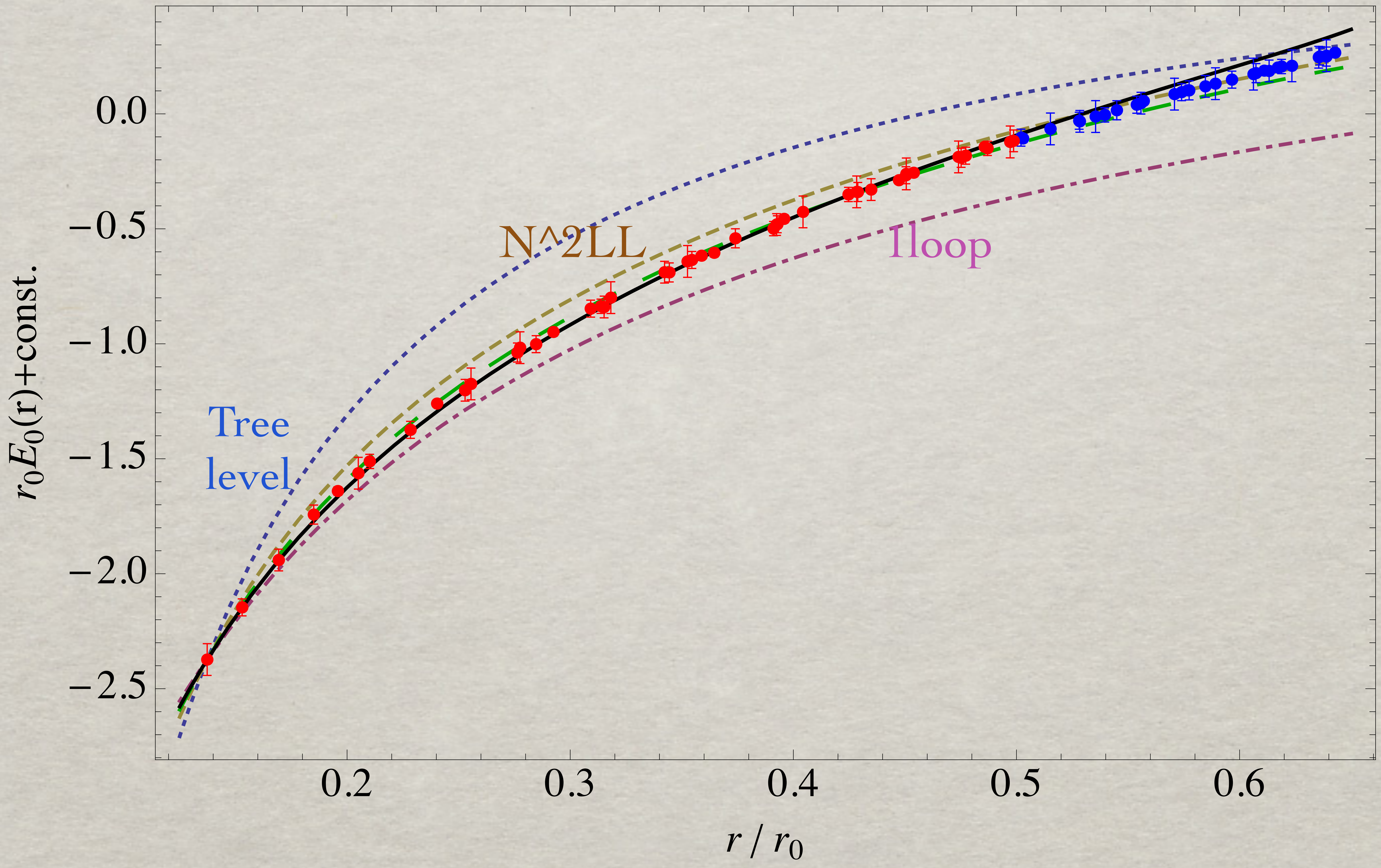

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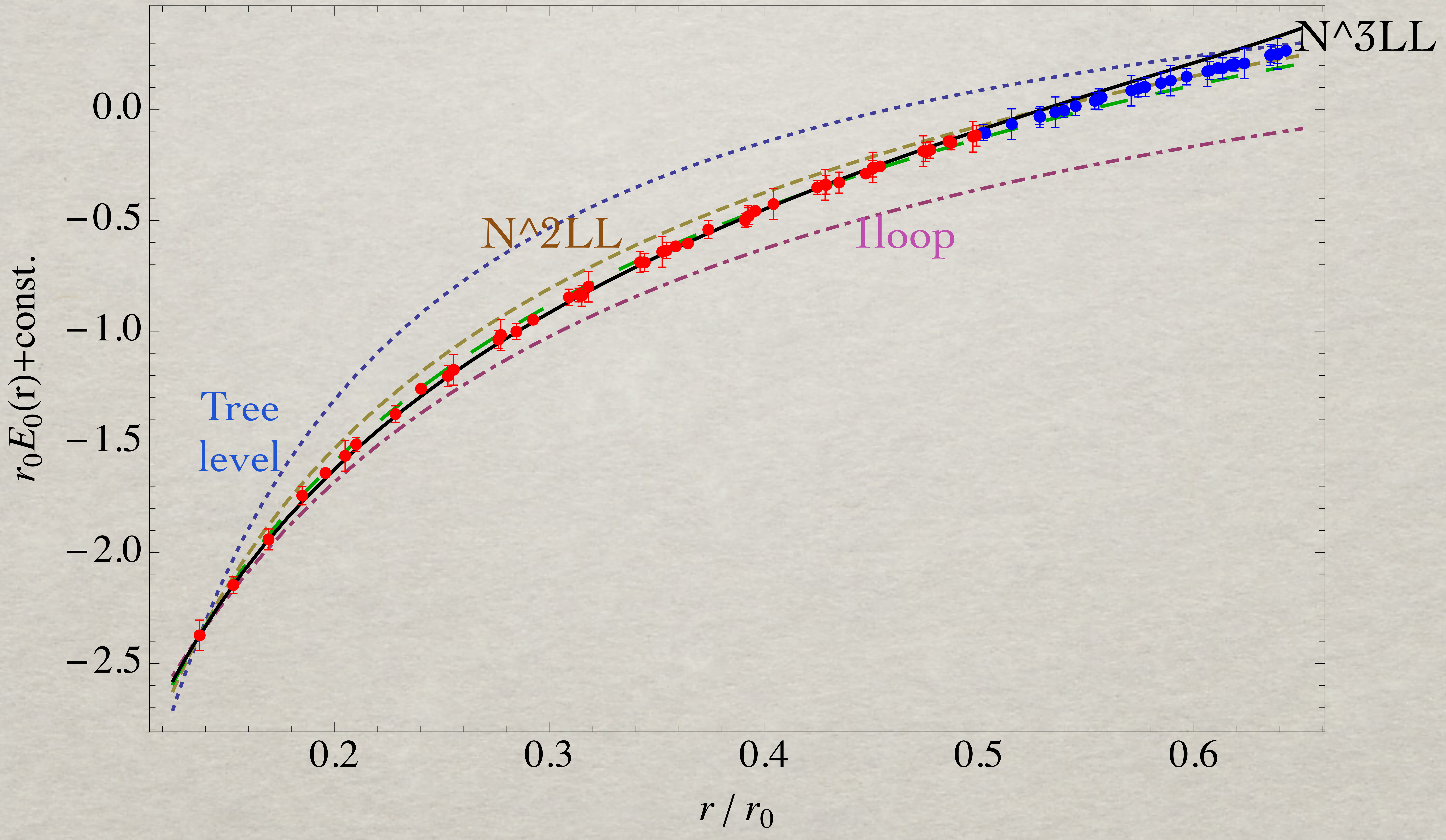

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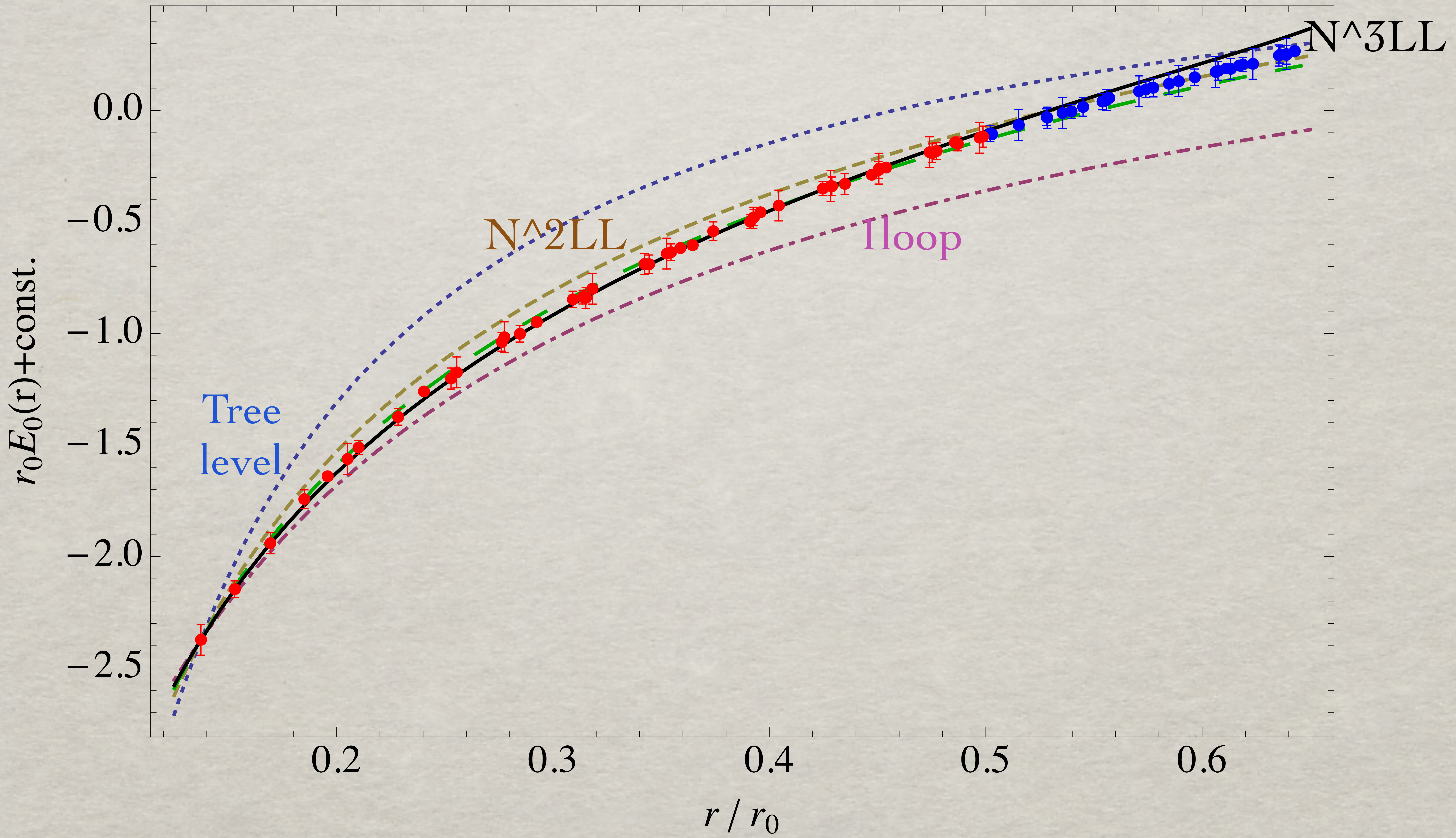

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

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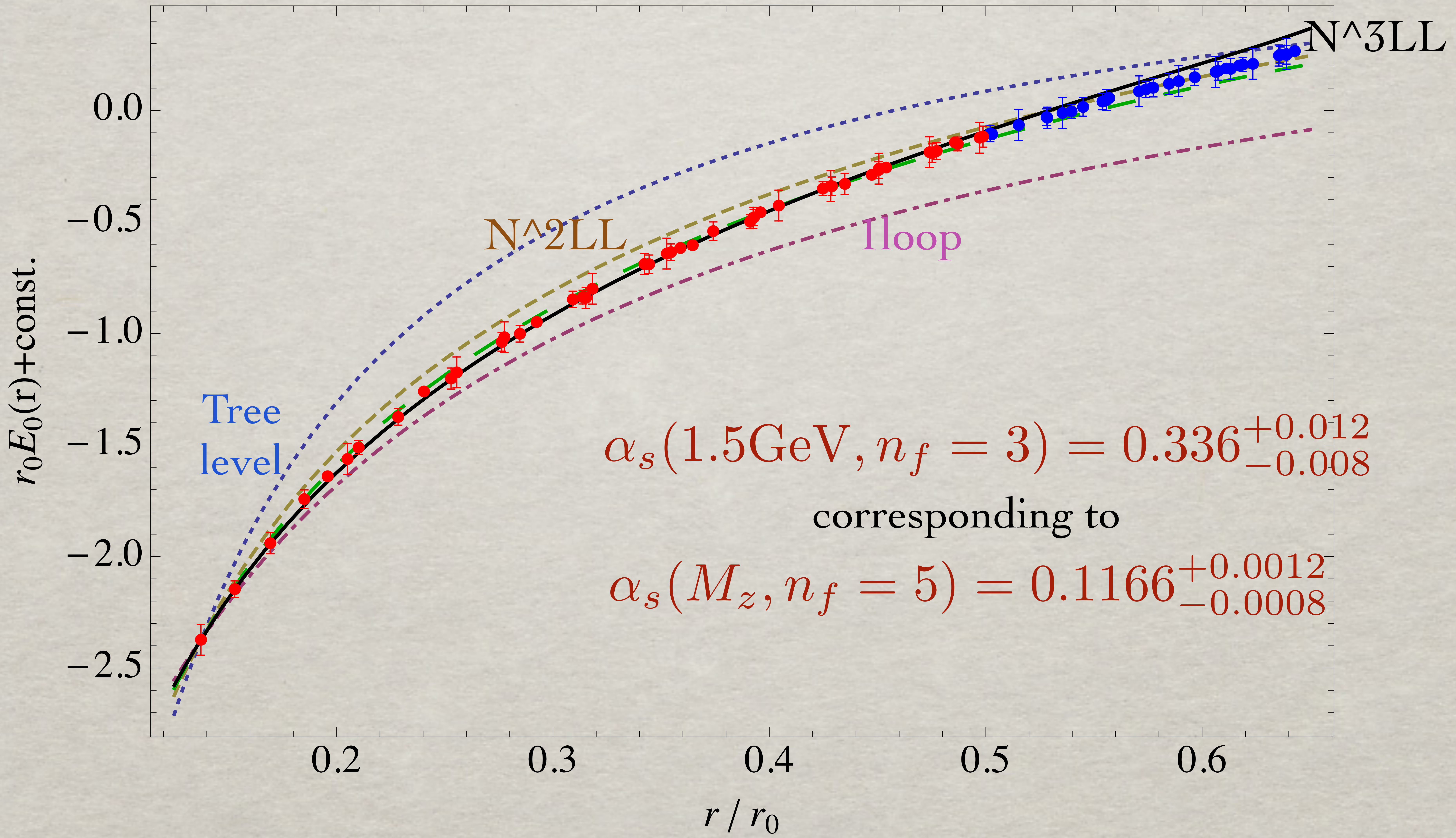

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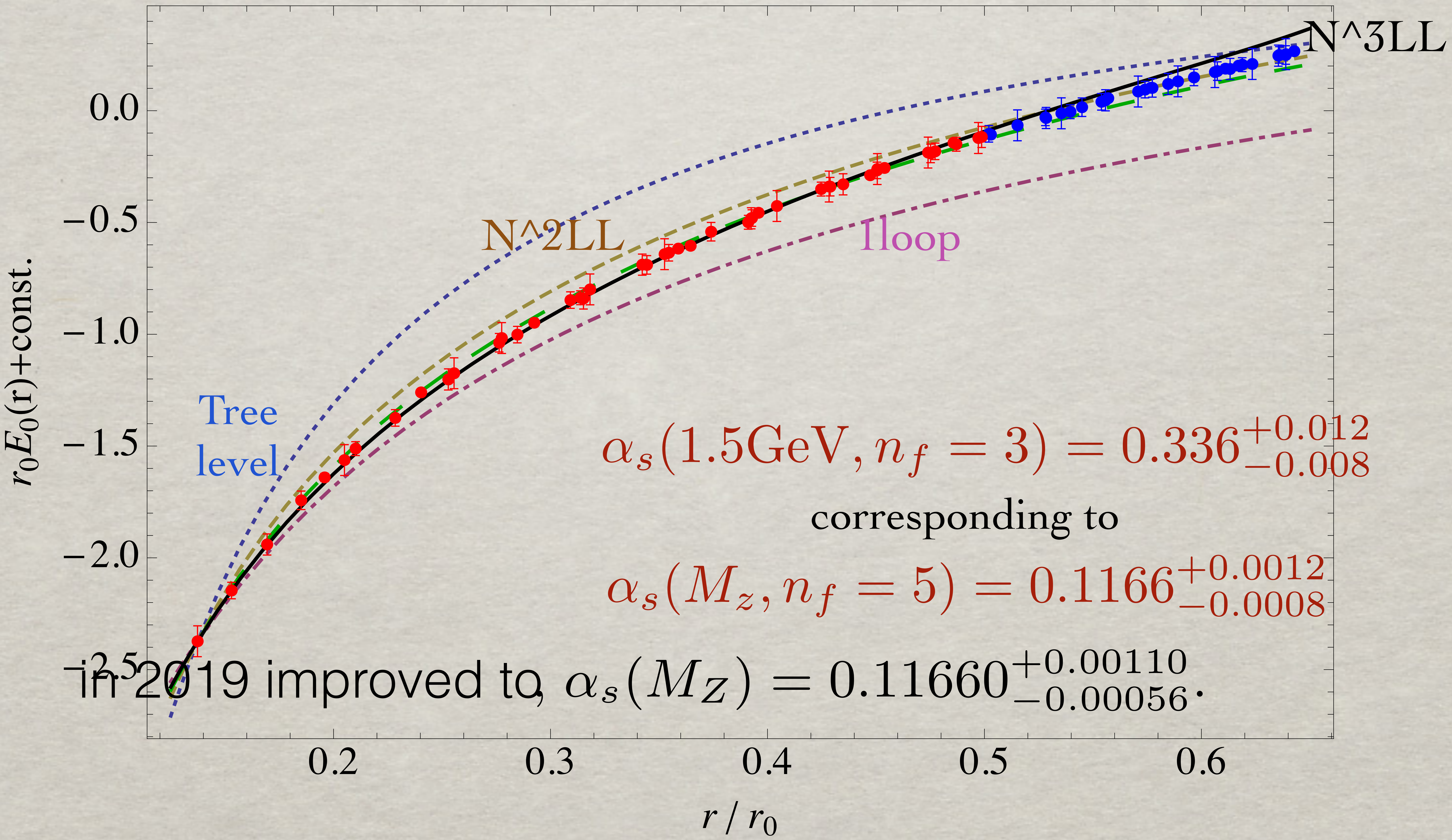


$$\alpha_s(1.5\mathrm{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

corresponding to

$$\alpha_s(M_z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

in 2019 improved to $\alpha_s(M_Z) = 0.11660^{+0.00110}_{-0.00056}$.

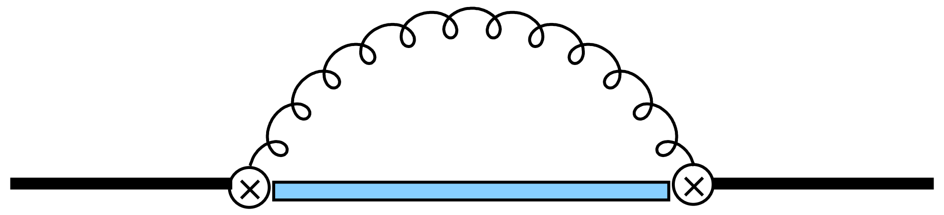
Good convergence to the lattice data

Lattice data less accurate in the unquenched case

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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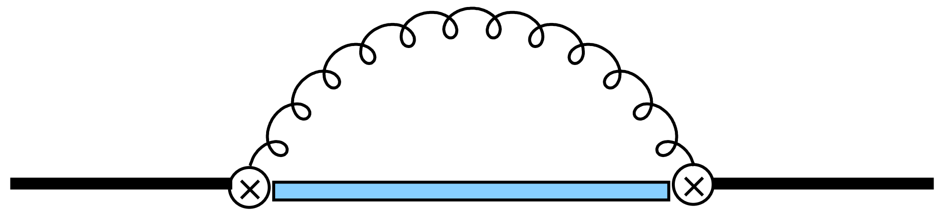
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

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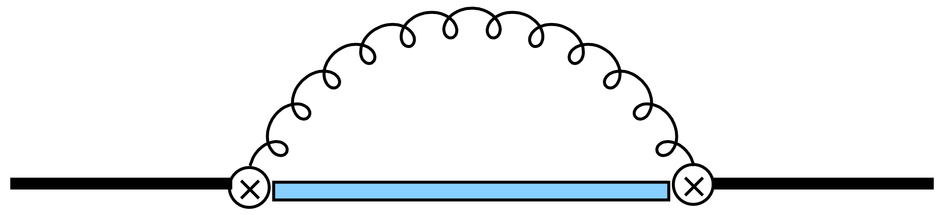
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

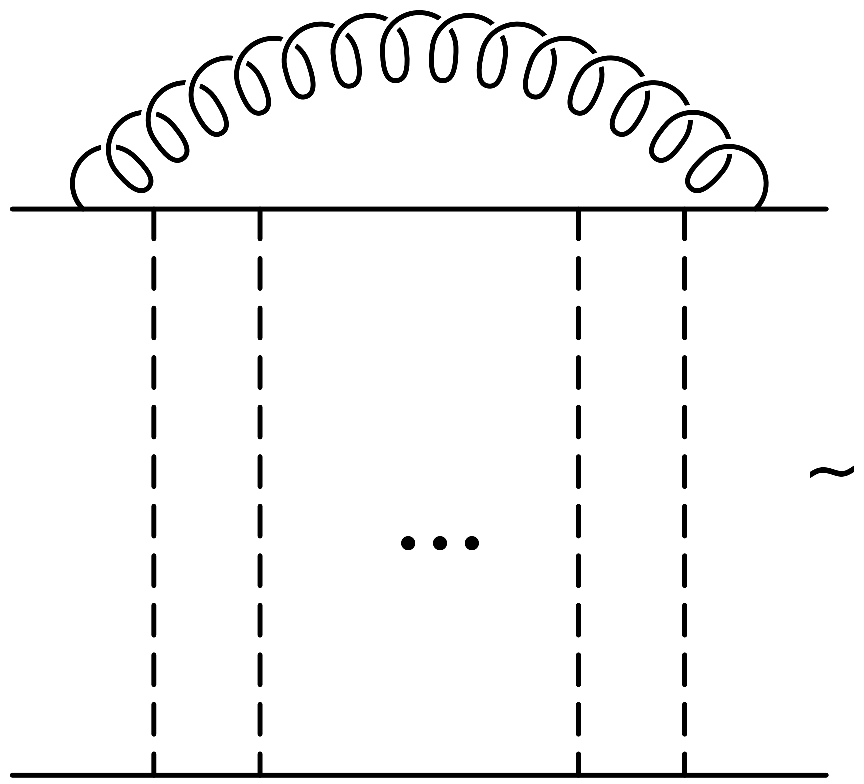
$\sim e^{i\Lambda_{\rm QCD} t}$

$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\,\langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\,\langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

Σ  $\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

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$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
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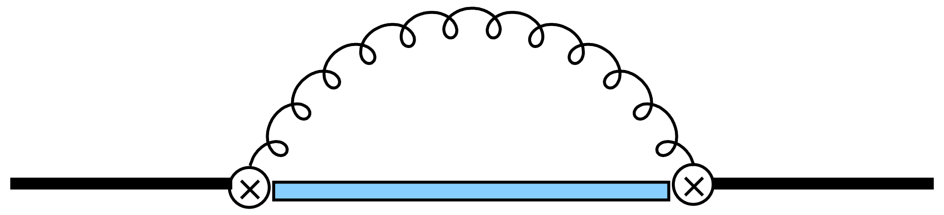
NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n | \frac{p^2}{m} + V_s | n \rangle + \langle n |$$  $$| n \rangle$$

$$\sim e^{i\Lambda_{\rm QCD} t}$$

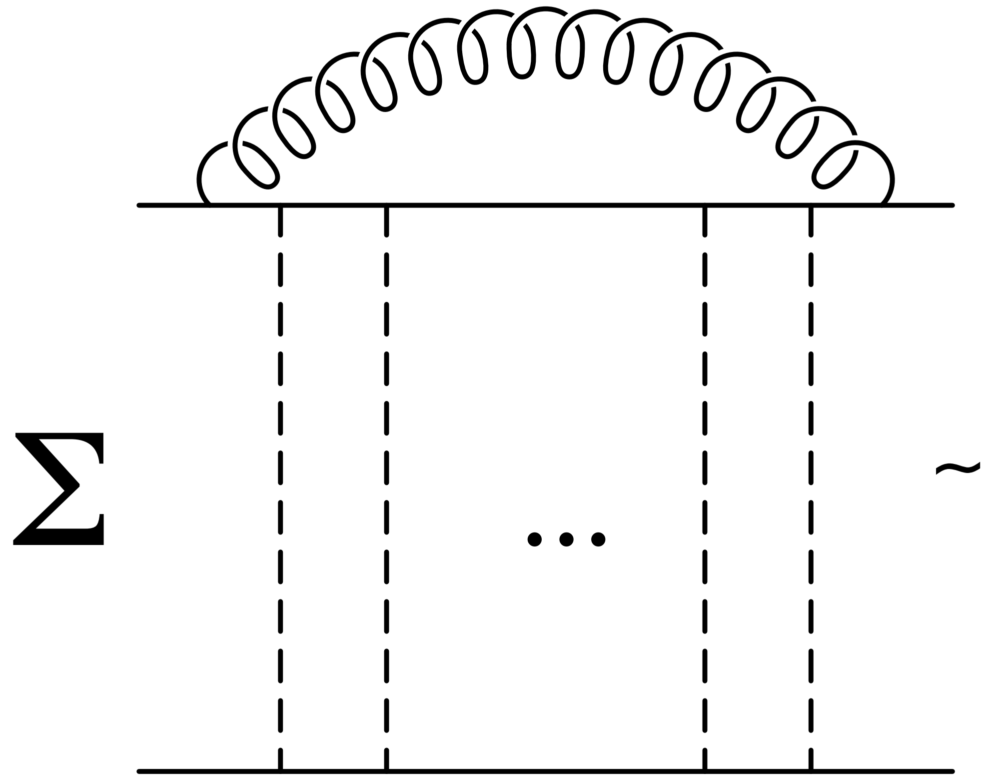
$$E_n = \langle n | H_s(\mu) | n \rangle - i \frac{g^2}{3N_c} \int_0^\infty dt \, \langle n | \mathbf{r} e^{it(E_n^{(0)} - H_o)} \mathbf{r} | n \rangle \, \langle \mathbf{E}(t) \, \mathbf{E}(0) \rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

—>used to extract precise (NNNLO) determination of m_c and m_b

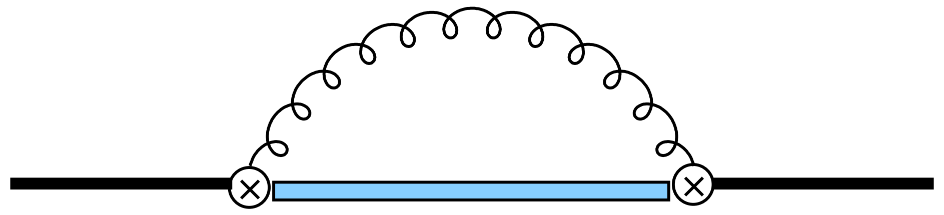


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02   NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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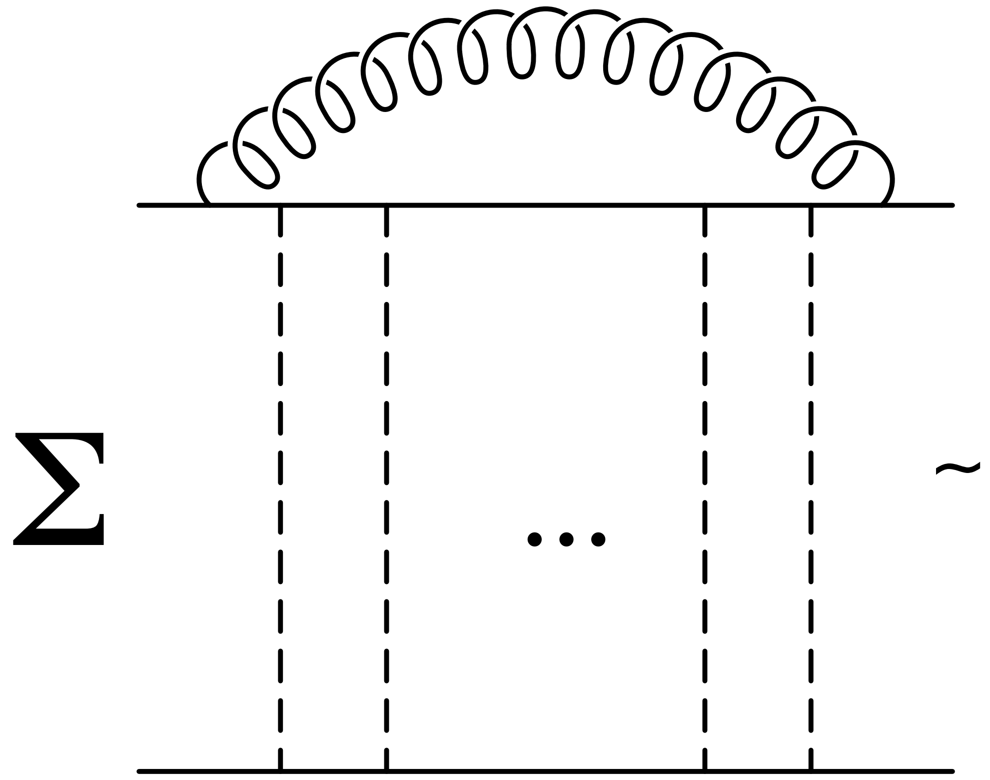


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

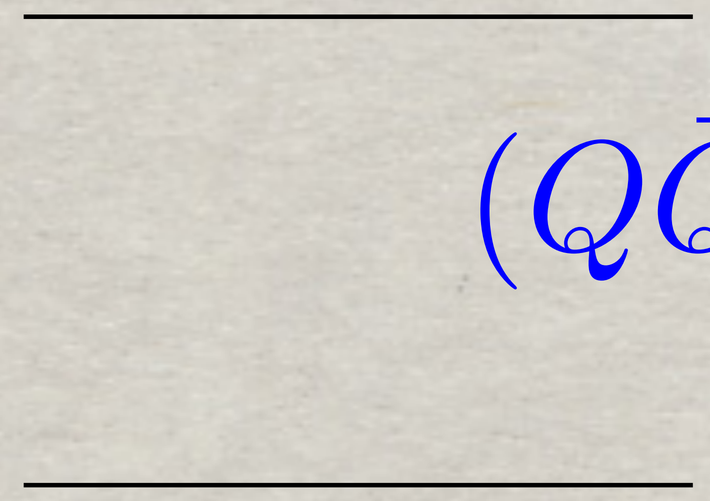
Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

STRONGLY COUPLED PNRQCD: $r \sim \Lambda_{QCD}^{-1}$

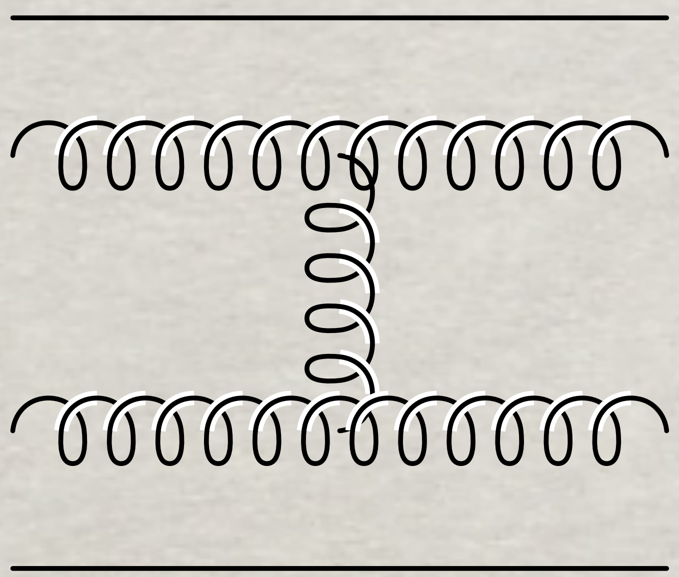
AND THE NONPERTURBATIVE MATCHING

Strongly coupled pNRQCD: Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

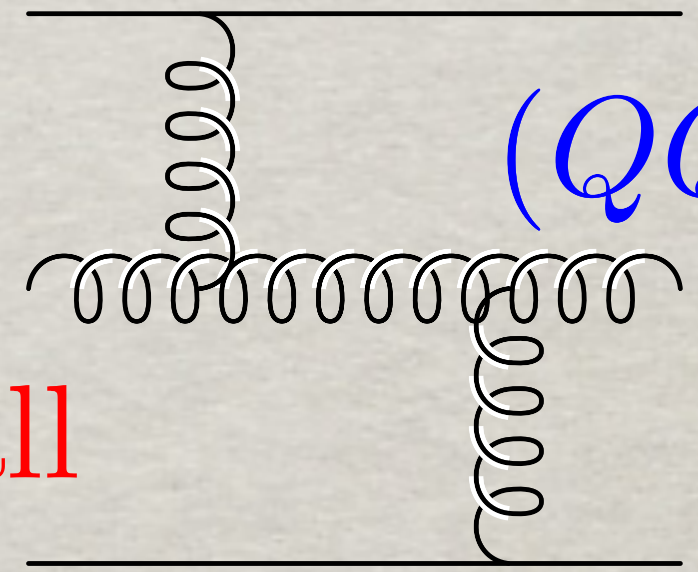
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \mathrm{Glueball}$



$(Q\bar{Q})_8 G$

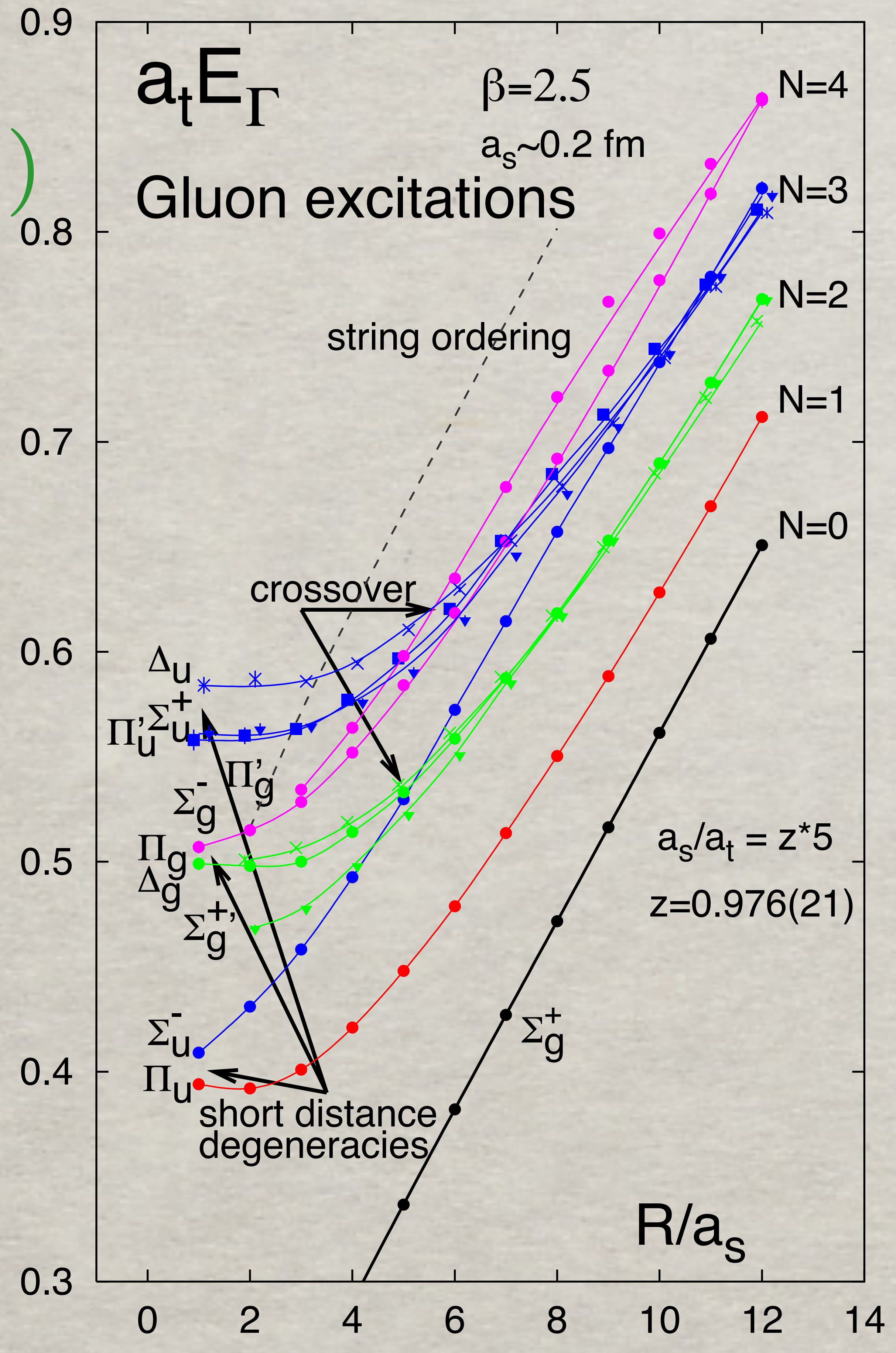
hybrid

with gluons at the scale Λ_{QCD}

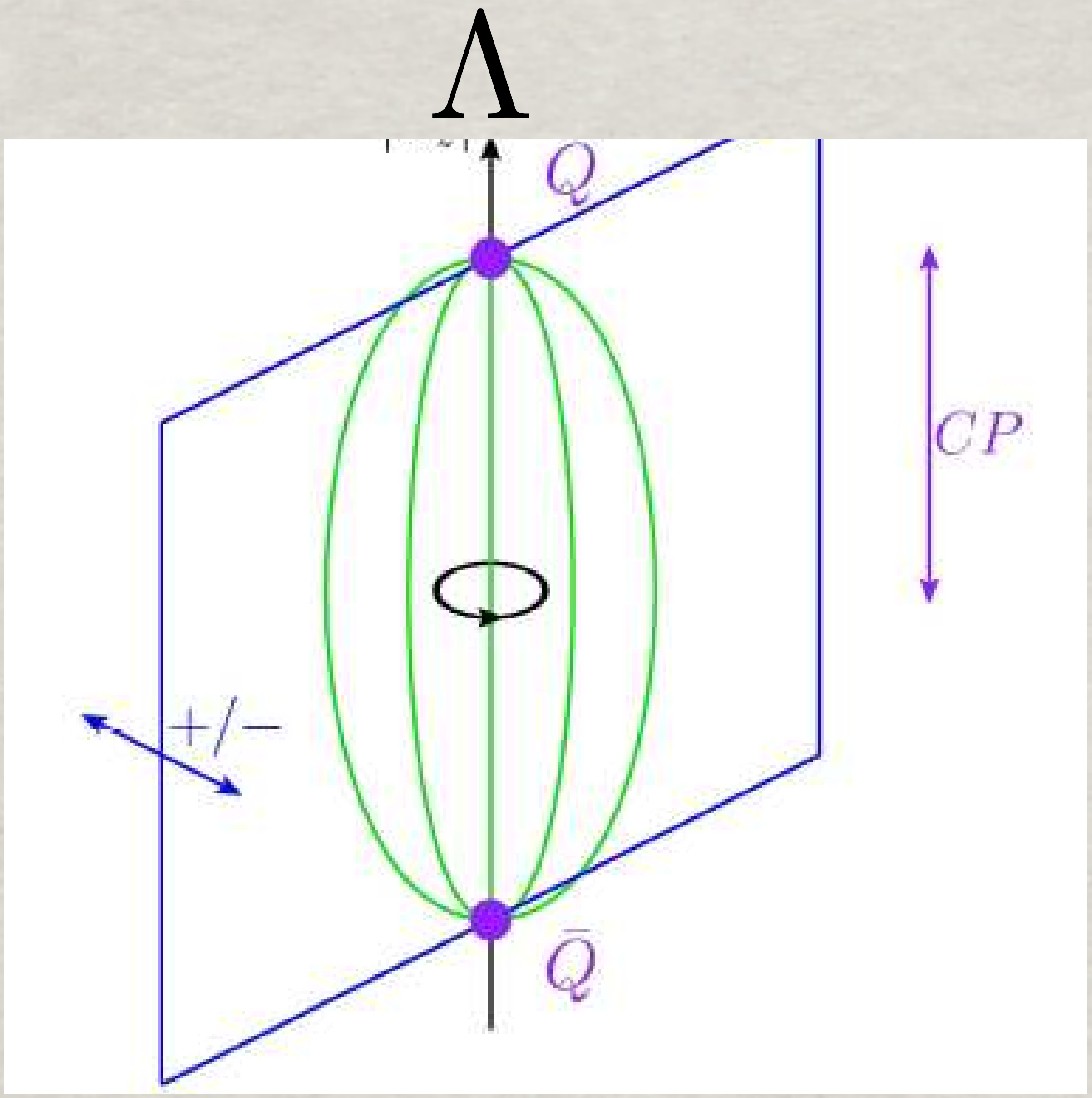
Strongly coupled pNRQCD: Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

Static NRQCD spectrum from lattice QCD

$E_n^{(0)}$



$r = |\mathbf{x}_1 - \mathbf{x}_2|$



Static states classified by symmetry group $D_{\infty h}$
Representations labeled Λ_η^σ $-->n$

- Λ rotational quantum number $|\lambda| = |\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2\ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta\ldots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)

K is the angular momentum of the light degrees of freedom;same symmetry as the diatomic molecule

NRQCD

$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

NRQCD states

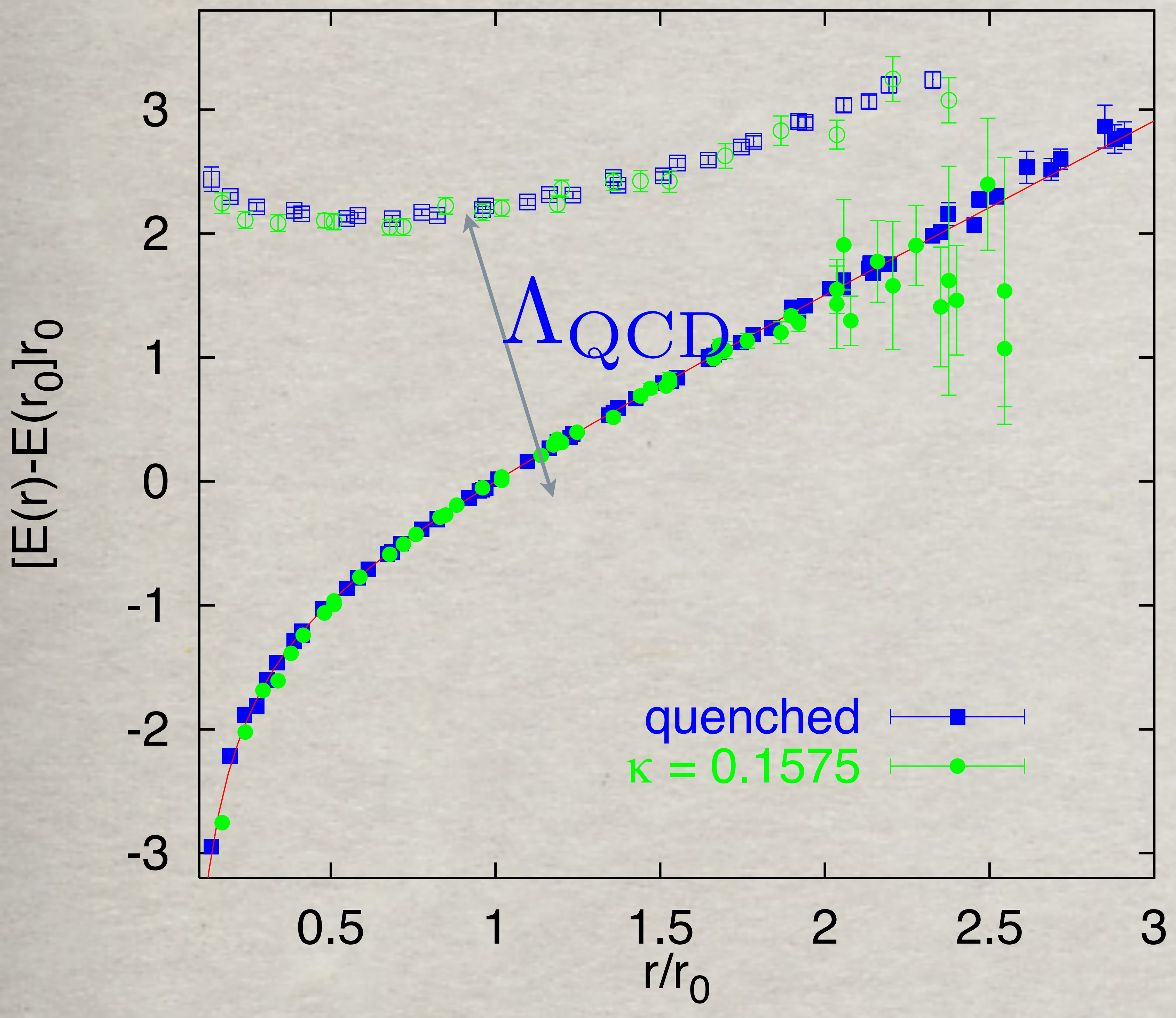
$$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -> \quad |(Q\bar{Q})_1\rangle \rightarrow \text{Quarkonium Singlet}$$

$$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -> \quad |(Q\bar{Q})g^{(n)}\rangle \rightarrow \text{Higher Gluonic Excitations}$$

Juge Kuti Mornigstar 98-06

$mv \sim \Lambda_{QCD}$ • pNRQCD and the potentials come from integrating out all scales up to mv^2

Bali et al. 98

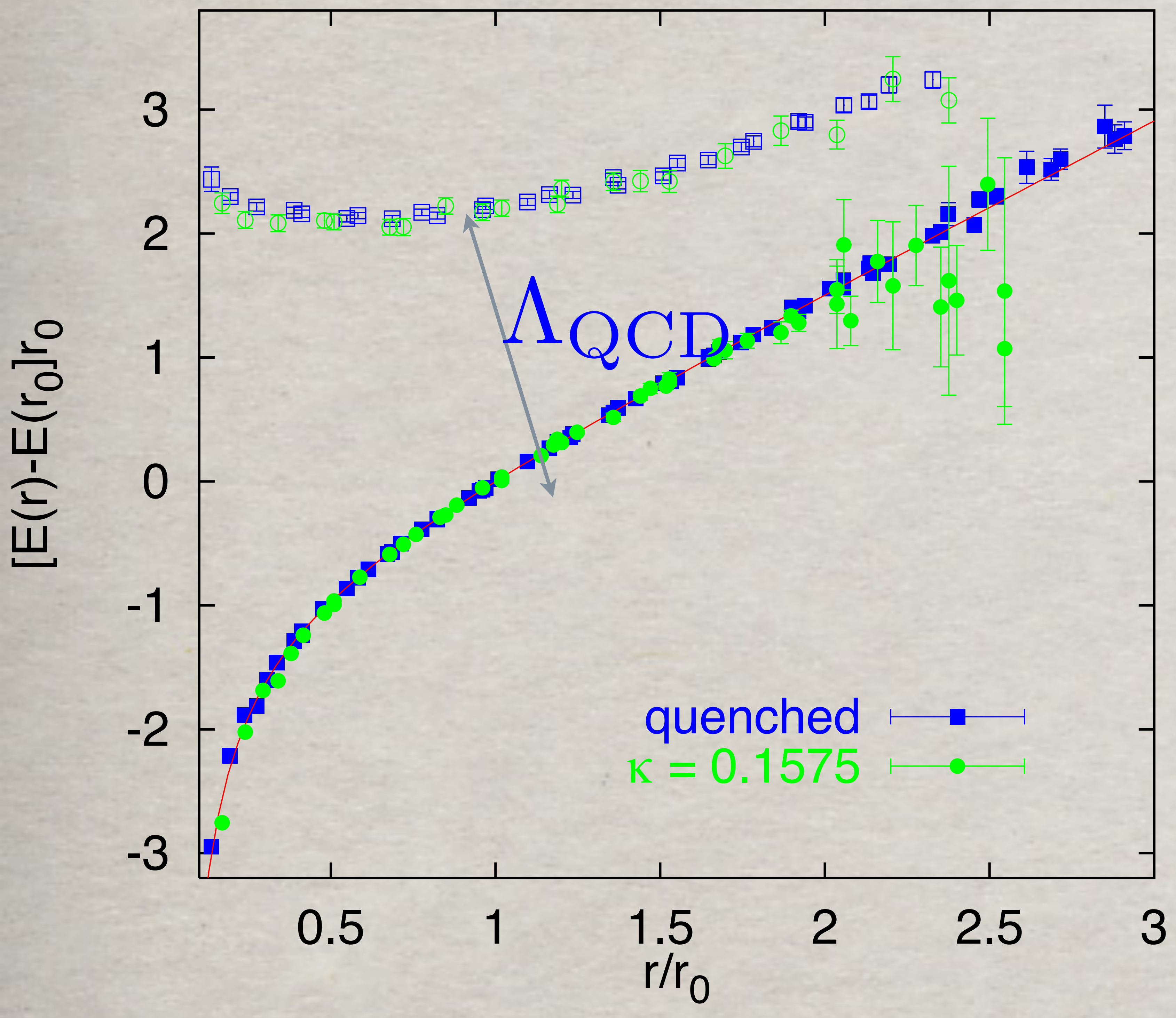


- gluonic excitations develop a gap Λ_{QCD} and are integrated out

Brambilla Pineda Soto Vairo 00

$mv \sim \Lambda_{QCD}$

Bali et al. 98



- pNRQCD and the potentials come from integrating out all scales up to mv^2
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⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

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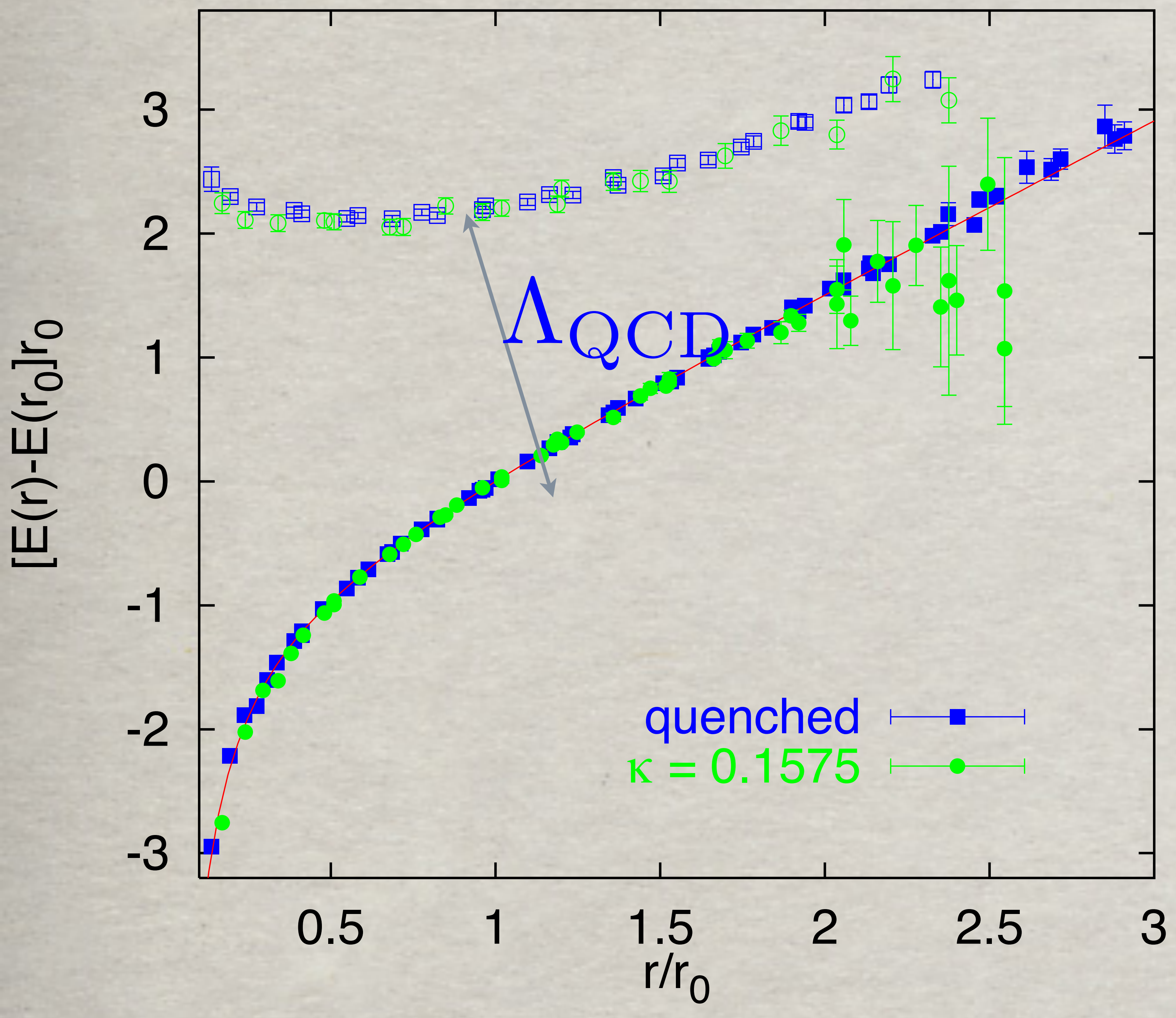

- pNRQCD and the potentials come from integrating out all scales up to mv^2
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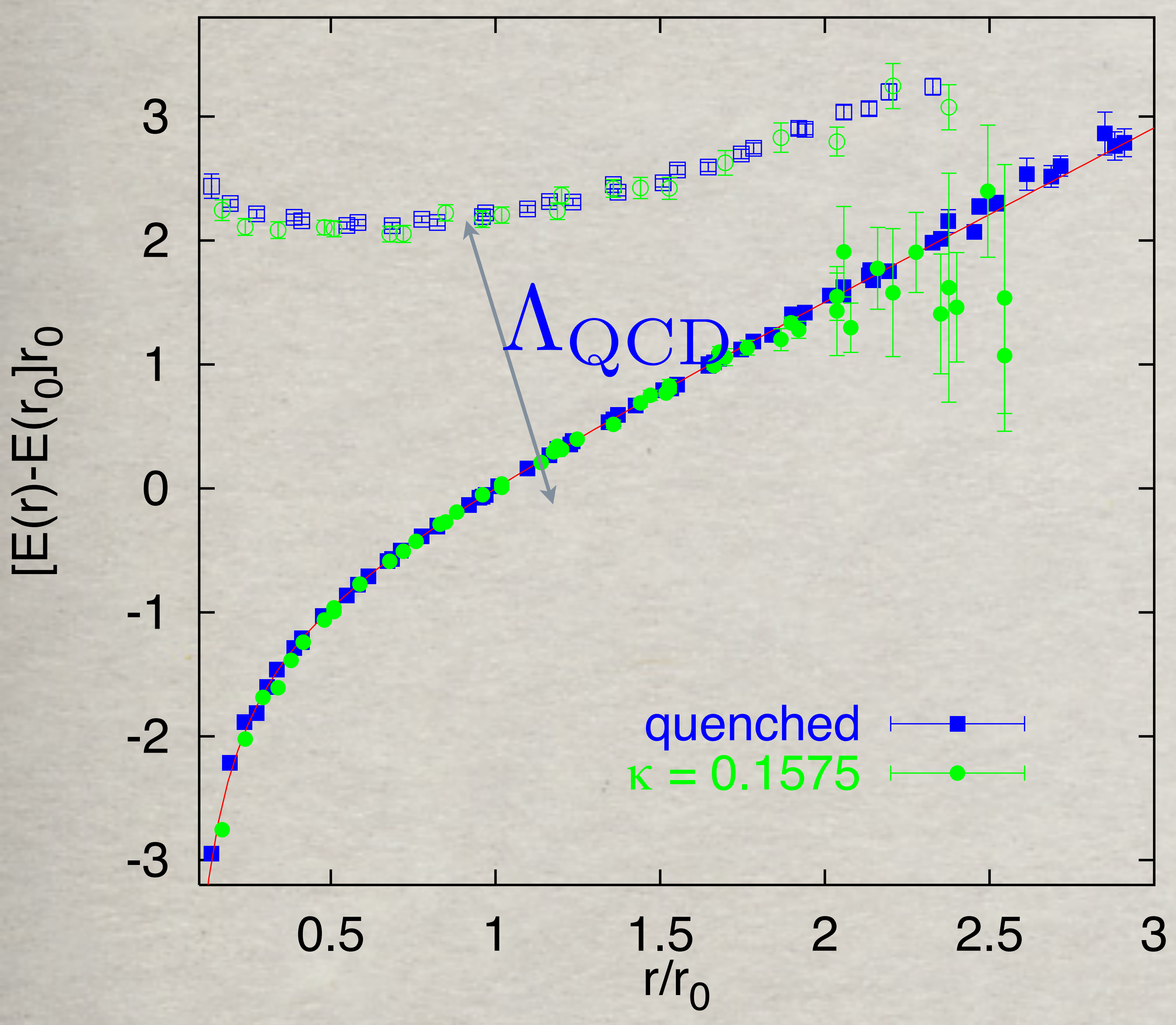
$$\mathcal{L} = {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} \right\}$$

$+\Delta\mathcal{L}(\text{US light quarks})$

Brambilla Pineda Soto Vairo 00

$mv \sim \Lambda_{QCD}$ • pNRQCD and the potentials come from integrating out all scales up to mv^2

Bali et al. 98



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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

• A pure potential description emerges from the EFT

• The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays

• We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops, that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Brambilla Pineda Soto Vairo 00

Strongly coupled pNRQCD: quantum mechanical matching

$$\langle H | \mathcal{H}^{\text{NRQCD}} | H \rangle = \langle nljs | \frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n} | nljs \rangle$$

the matching conditions are :

$$|\underline{0}; \mathbf{x}_1 \mathbf{x}_2\rangle -> S^\dagger(\mathbf{x}_1 \mathbf{x}_2)|\text{vac}\rangle$$

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expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$$\overset{\text{NRQCD}}{\mathcal{H}} = \mathcal{H}^{(0)} + \frac{\delta\mathcal{H}^{(1)}}{m} + \frac{\delta\mathcal{H}^{(2)}}{m^2} + \frac{\delta\mathcal{H}^{(3)}}{m^3} + \frac{\delta\mathcal{H}^{(4)}}{m^4} + \ldots$$

$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\,\frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum_{n_f}\bar{q}\,i\mathbf{D}\cdot\boldsymbol{\gamma}\,q$$

$$\delta\mathcal{H}^{(1)} = -\int d^3\mathbf{x}\,\psi^\dagger\left(\frac{\mathbf{D}^2}{2} + c_F\,g\,\mathbf{S}\cdot\mathbf{B}\right)\psi + \text{antip.}$$

$$|H\rangle \rightarrow |\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle \otimes |nljs\rangle$$

$$|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle = |\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} + \sum_{n\neq 0}\int d^3z_1\,d^3z_2|\underline{n};\mathbf{z}_1,\mathbf{z}_2\rangle^{(0)}$$

$$\times\frac{{}^{(0)}\langle\underline{n};\mathbf{z}_1,\mathbf{z}_2|\delta\mathcal{H}^{(1)}|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}}{E_0^{(0)}(z) - E_n^{(0)}(x)} + \ldots$$

Strongly coupled pNRQCD: quantum mechanical matching

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$$\times\frac{{}^{(0)}\langle\underline{n};\mathbf{z}_1,\mathbf{z}_2|\delta\mathcal{H}^{(1)}|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}}{E_0^{(0)}(z) - E_n^{(0)}(x)} + \ldots$$

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \ldots$$

$$V^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\ln\langle\,\Box\,\rangle$$

given in terms of gauge invariant Wilson loops

$$\Box = \exp\left\{ig\oint_{r\times T} dz^\mu A_\mu\right\}$$

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023
Pineda Vairo PRD 63 (2001) 054007

spin dependent 1/m^2 potential

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \Box^{\,E(t)\,E} \right\rangle$$

$$V_{\rm SD}^{(2)} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \Box^{\,E(t)}_{\,i\;j\;B} \right\rangle \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2) \;\; |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \Box_{\,i\;j} \right\rangle - \frac{2c_F - 1}{2} \nabla^k V^{(0)} \right) \mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2) \;\; |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\left\langle \Box_{\,i\;j} \right\rangle - \frac{\delta_{ij}}{3} \left\langle \Box \right\rangle \right) \left(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}}) \right) \;\; |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \left\langle \Box \right\rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv} \right) \delta^{(3)}(\mathbf{r}) \right) \mathbf{S}_1 \cdot \mathbf{S_2} \;\; |V_S$$

$c_F = 1 + \alpha_{\rm s}/\pi(13/6 + 3/2 \ln m/\mu) + \ldots)$, $d_{sv,vv} = O(\alpha_{\rm s}^2)$ from NRQCD.

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$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle \Box_{i\ j} \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1\cdot\mathbf{S}_1 + (1 \leftrightarrow 2)\ |V^{(1)}_{LS}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \Box_{i\ j} \rangle - \frac{\delta_{ij}}{3} \langle \Box \rangle \right)\left(\mathbf{S}_1\cdot\mathbf{S_2} - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S_2}\cdot\hat{\mathbf{r}})\right)\ |V_T$$

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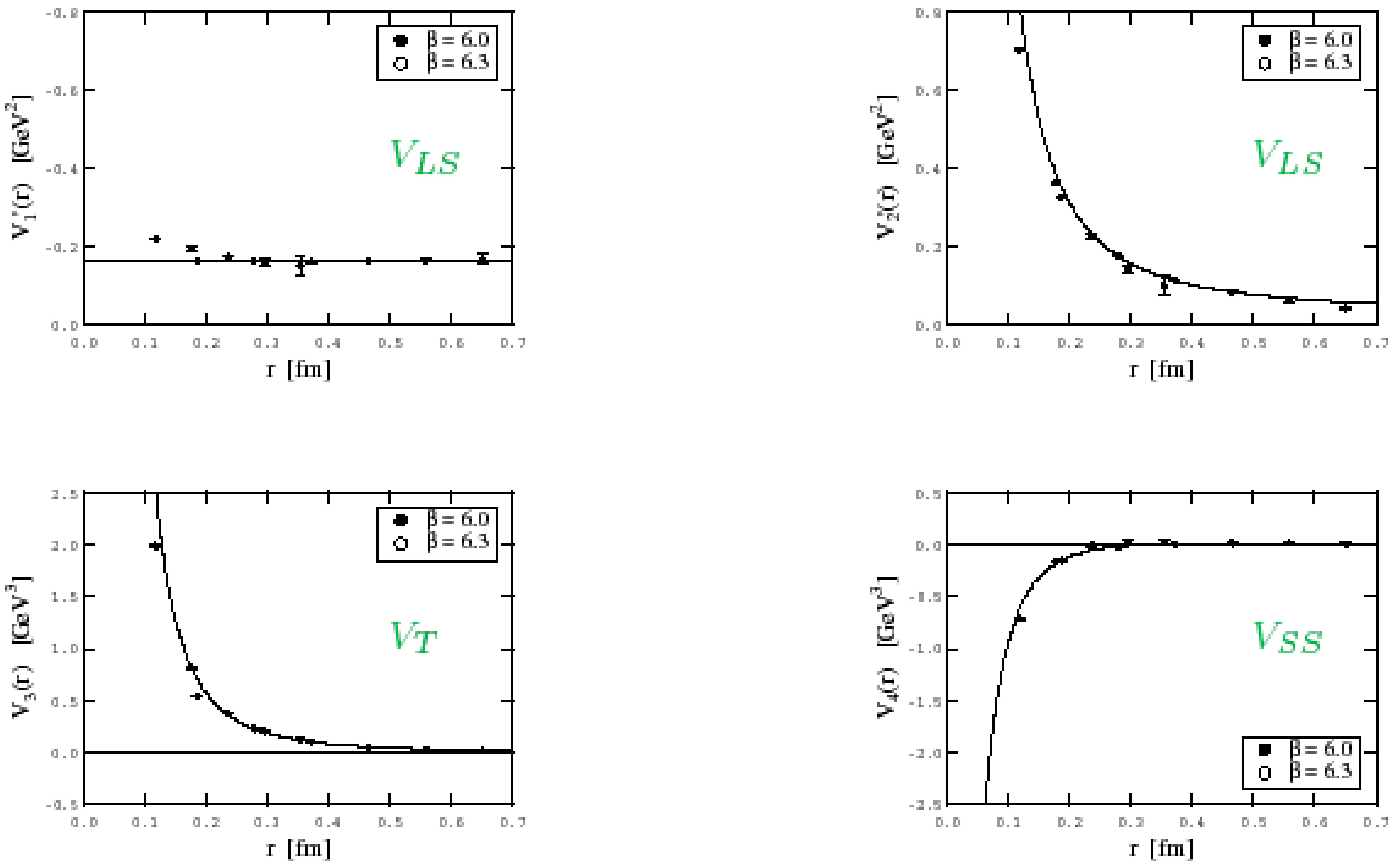
$c_F = 1 + \alpha_{\rm s}/\pi(13/6 + 3/2\ln m/\mu) + \ldots),\ d_{sv,vv} = O(\alpha_{\rm s}^2)$ from NRQCD.

- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour

- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

Lattice evaluation of the spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

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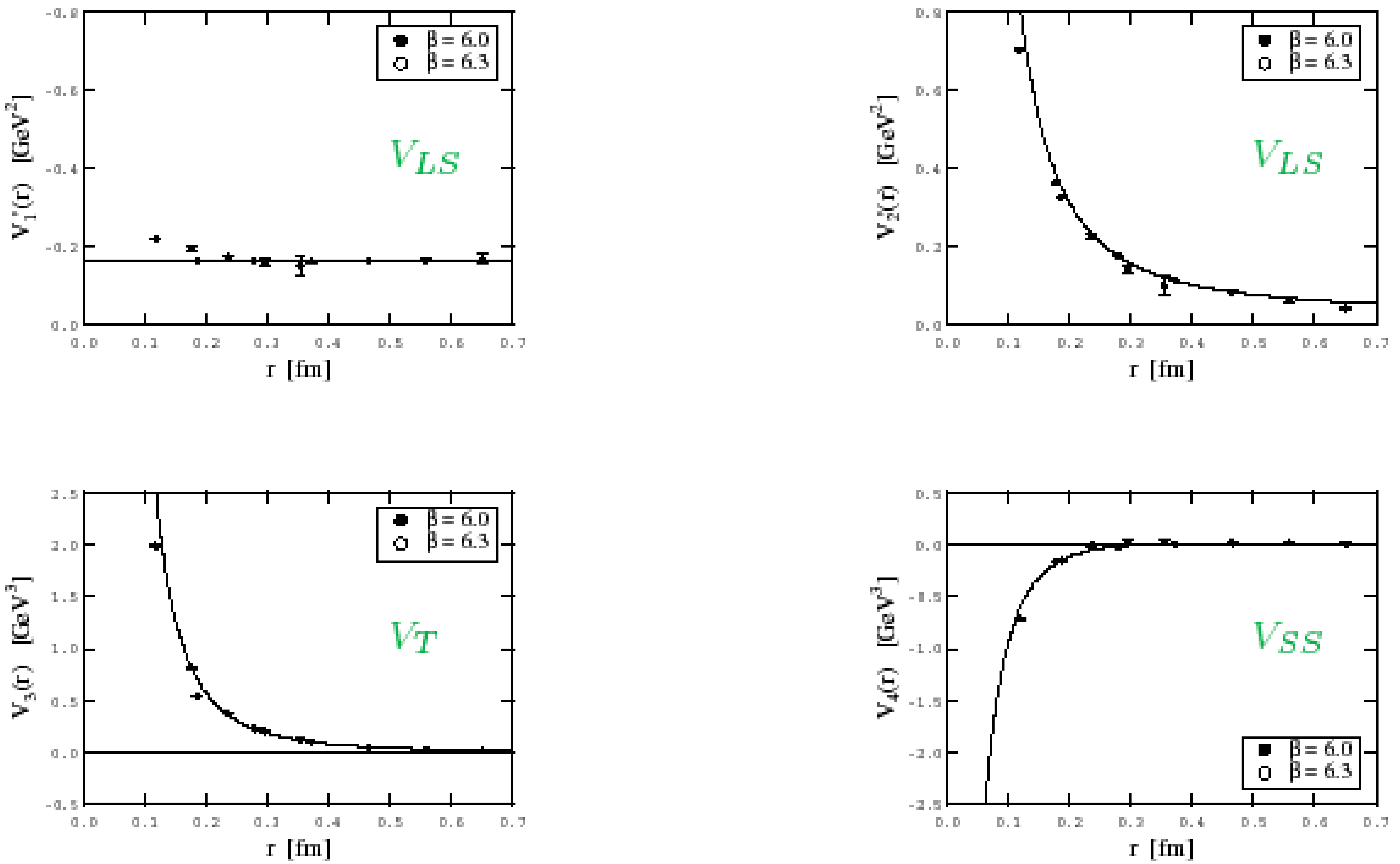


Koma Koma Wittig 05, Koma Koma 06

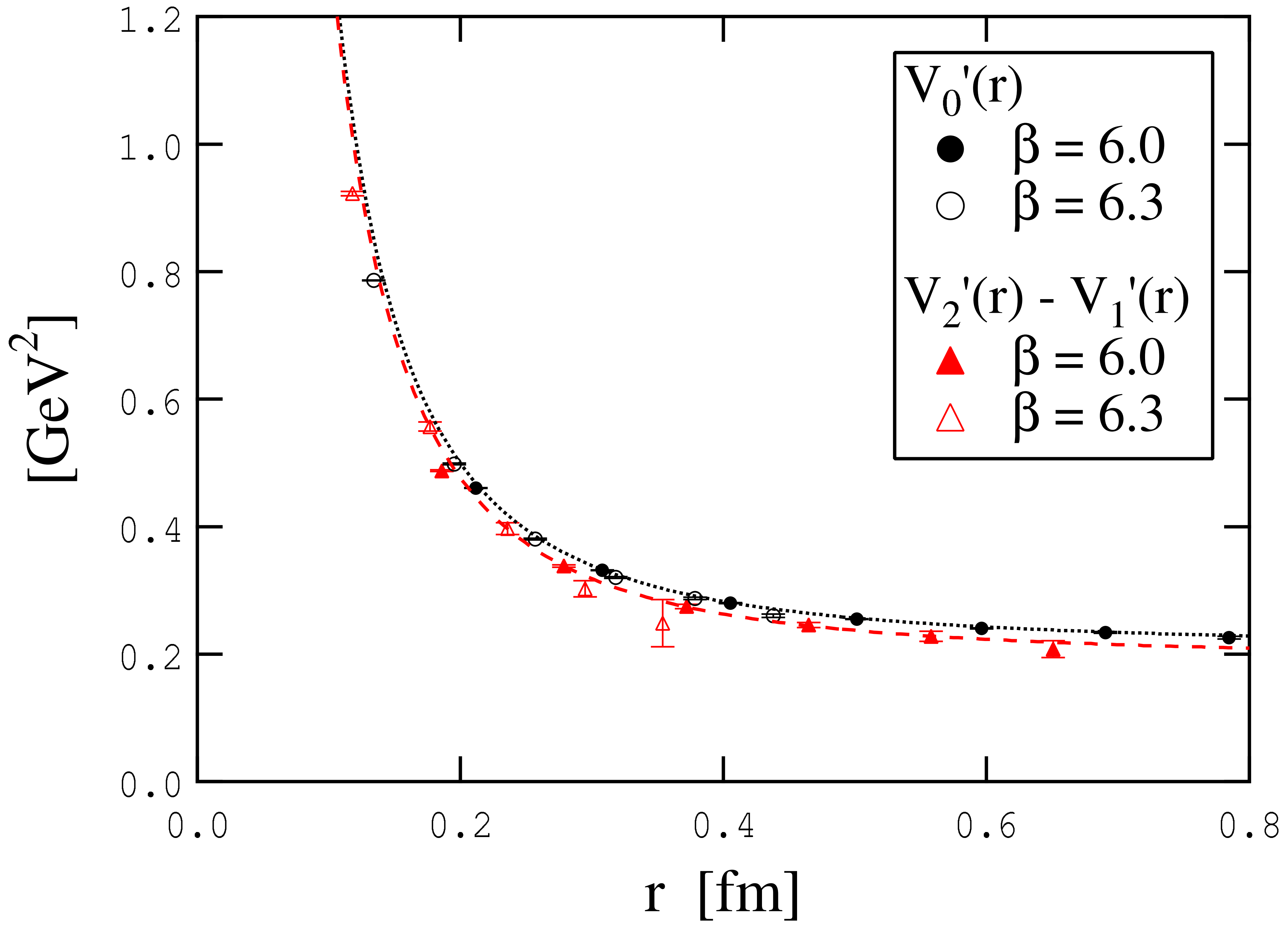
Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, Vairo 2014

Lattice verification of the Poincaré constraints

A lattice verification of $V_{LS}^{(1)}(r) - V_{LS}^{(2)}(r) + \frac{1}{2r} V^{(0)\prime}(r) = 0$:



The exact relations help also in guiding the continuum limit

○ M.Koma Y.Koma NPB 769 (2007) 79

Applications of strongly coupled pNRQCD include: Quarkonium Production at LHC

Intense work in the theory community, within QCD, NRQCD and SCET,

Qiu, Nayak, Sterman, Butenschon Kniehl , Bodwin , Hee Soh, Chung, J. Lee, Kuang Ta Chao, Y. Q. Ma, Gong Wang, Fleming, Mehen, Yu Jia, Braaten, Lansberg, Leibovich, Rothstein…

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NRQCD factorization formula for quarkonium production valid for large p_T

Bodwin Braaten Lepage 1995

cross section
$$\sigma(H) = \sum_n F_n \langle 0|\mathcal{O}_n^H|0\rangle.$$

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short distance coefficients
partonic hard scattering cross section
convoluted with parton distribution

long distance matrix elements (LDME)
give the probability of a qqbar pair with certain quantum number to evolve into a final quarkonium H

they are vacuum expectation values of four fermion operators with color singlet and color octet contributions and a projection over quarkonium plus X in the middle

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One problem is the proliferation of LDMEs:
nonperturbative objects
that cannot evaluated on the lattice
and should be extracted from the data,
they depend on the considered quarkonium state

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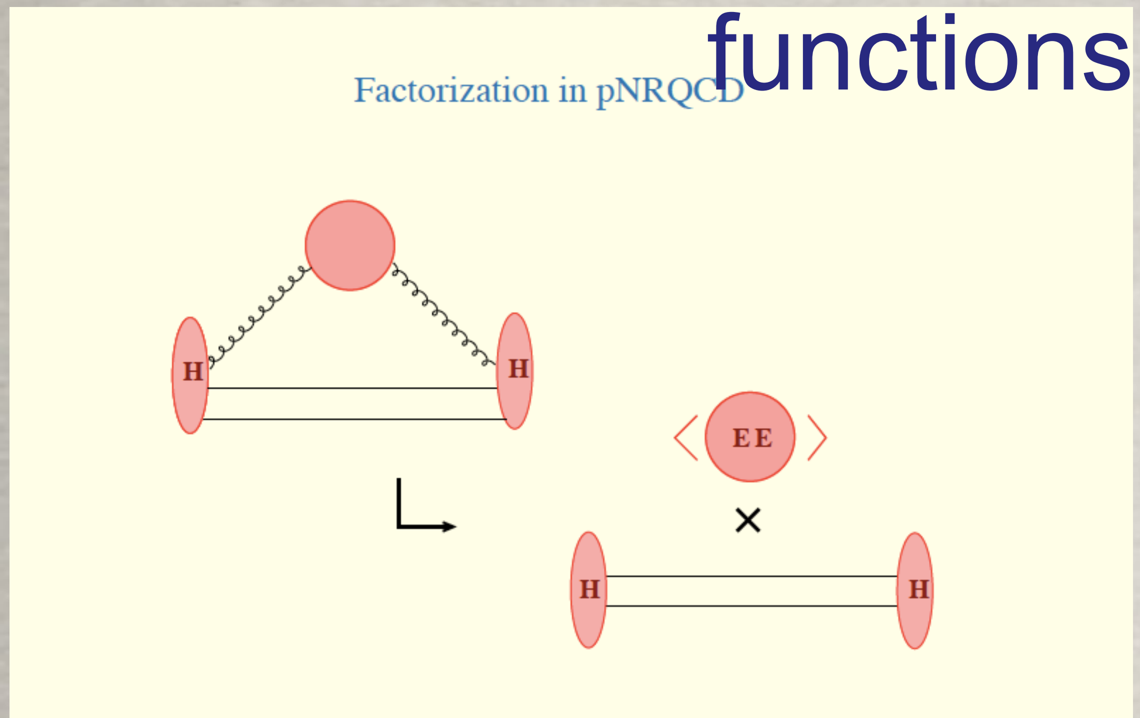
Factorization of LDMEs in pNRQCD : the NRQCD LDMEs are factorized in terms of wave functions and universal nonperturbative correlators depending only on the glue



- The number of nonperturbative unknowns is reduced by half
- The nonperturbative unknowns are correlators of gluonic fields that can be calculated on the lattice

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

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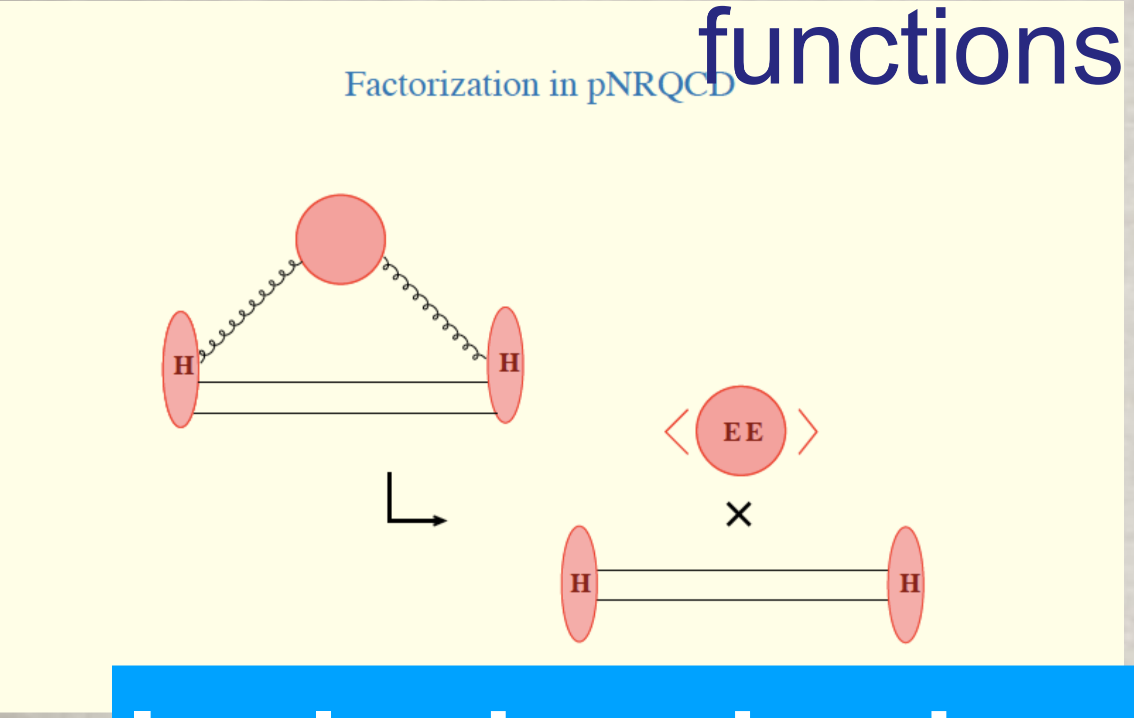

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Inclusive hadroproduction of p wave quarkonia

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$$
$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle.$$

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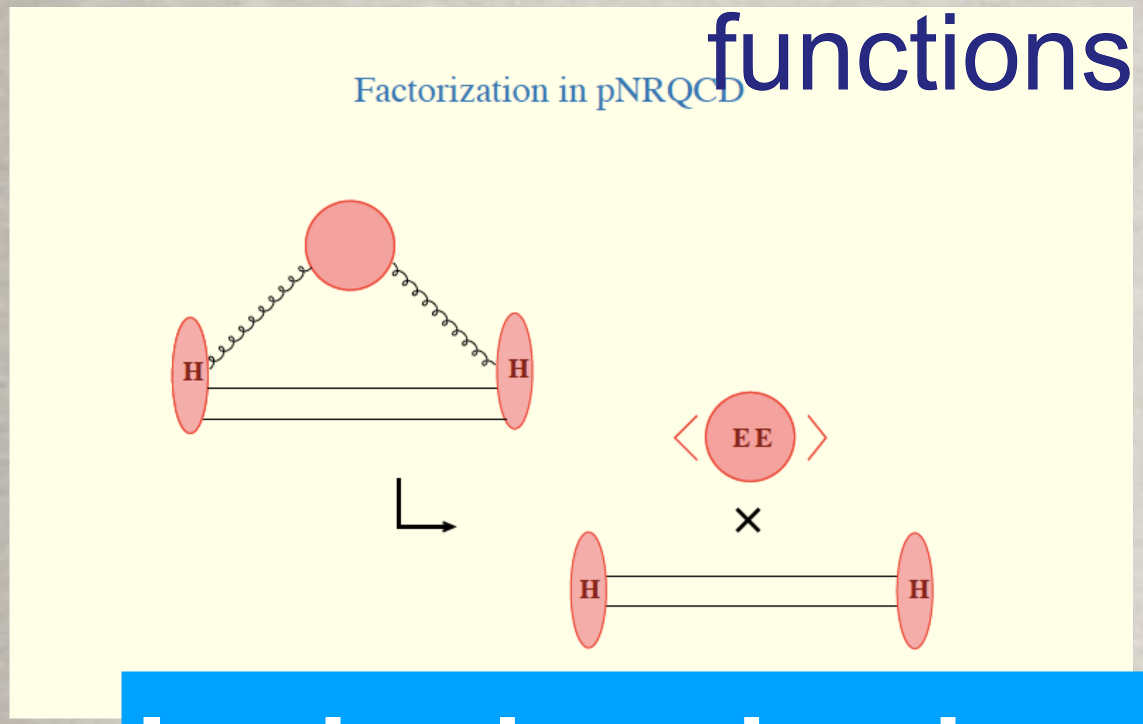

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$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}$$

$$\frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$$

$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}$$

$$\frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$

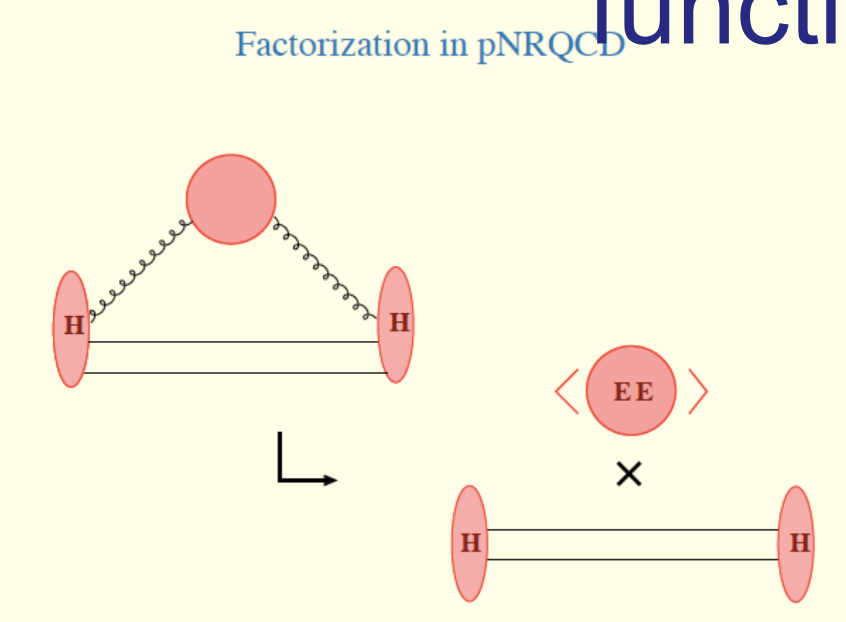
- The dimensionless correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE with Wilson lines Φ extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty t\,dt \int_0^\infty t'\,dt' \langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger da}(0,t)gE^{d,i}(t)gE^{e,i}(t')\Phi_0^{ec}(t',0)\Phi_\ell^{bc}|\Omega\rangle$$

- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

Factorization of LDMEs in pNRQCD : the NRQCD LDMEs are factorized in terms of wave functions and universal nonperturbative correlators depending only on the glue



- The number of nonperturbative unknowns is reduced by half
- The nonperturbative unknowns are correlators of gluonic fields that can be calculated on the lattice

Inclusive hadroproduction of p wave quarkonia

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})} \quad \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$$

$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})} \quad \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$

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- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

$\mathcal{E}$ is a universal quantity that ***does not depend on quark flavor or radial excitation***. Determination of $\mathcal{E}$ directly leads to ***determination of all*** χ_{cJ} ***and*** $\chi_{bJ}(nP)$ ***cross sections, as well as*** h_c ***and*** h_b ***production rates***.

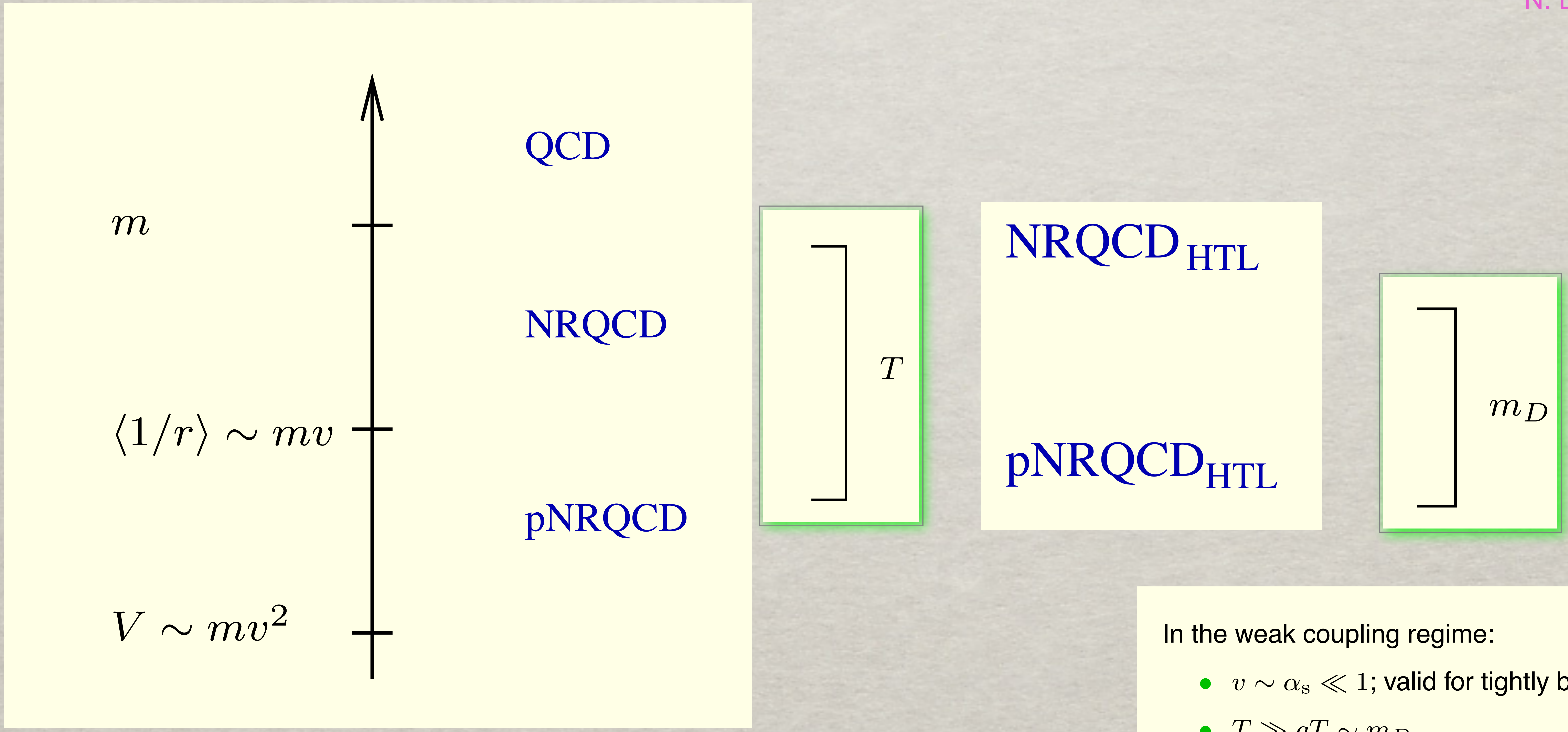
->good description of data at ATLAS and CMS

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

Notice: additional scales smaller than m can be integrated out combining with other EFTs

Example: quarkonium in thermal medium, T<m, the thermal medium has scales T and m_d=gT=> integrate out T produces Hard Thermal EFT (HTL)

N. B., Ghiglieri,Petreczky, Vairo 08



if T > E the potential has thermal effects

In the weak coupling regime:

- $v \sim \alpha_s \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ, ...
- $T \gg gT \sim m_D$.

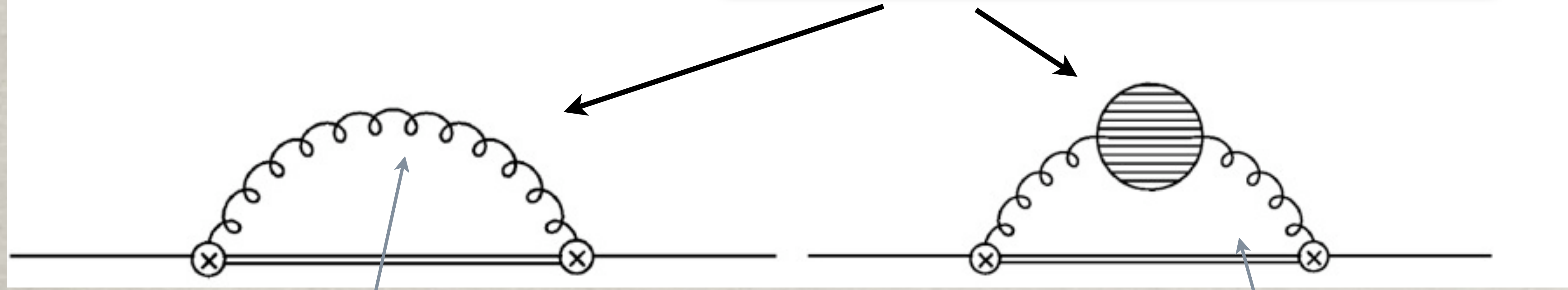
Effects due to the scale Λ_{QCD} will not be considered.

The potential V(r,T) dictates throught the Schroedinger equation the real time evolution of the QQbar pair in the medium-> use pNRQCD to define and calculate it

$\mathrm{Re}V_S\,(r,T)$

$\mathrm{Im}V_S\,(r,T)$

thermal width of $Q\bar{Q}$



Singlet-to-octet
N.B Ghiglieri, Petreczky, Vairo 2008
(gluo dissociation)

Landau damping
Laine et al 07, Escobedo Soto 07
(inelastic parton scattering)

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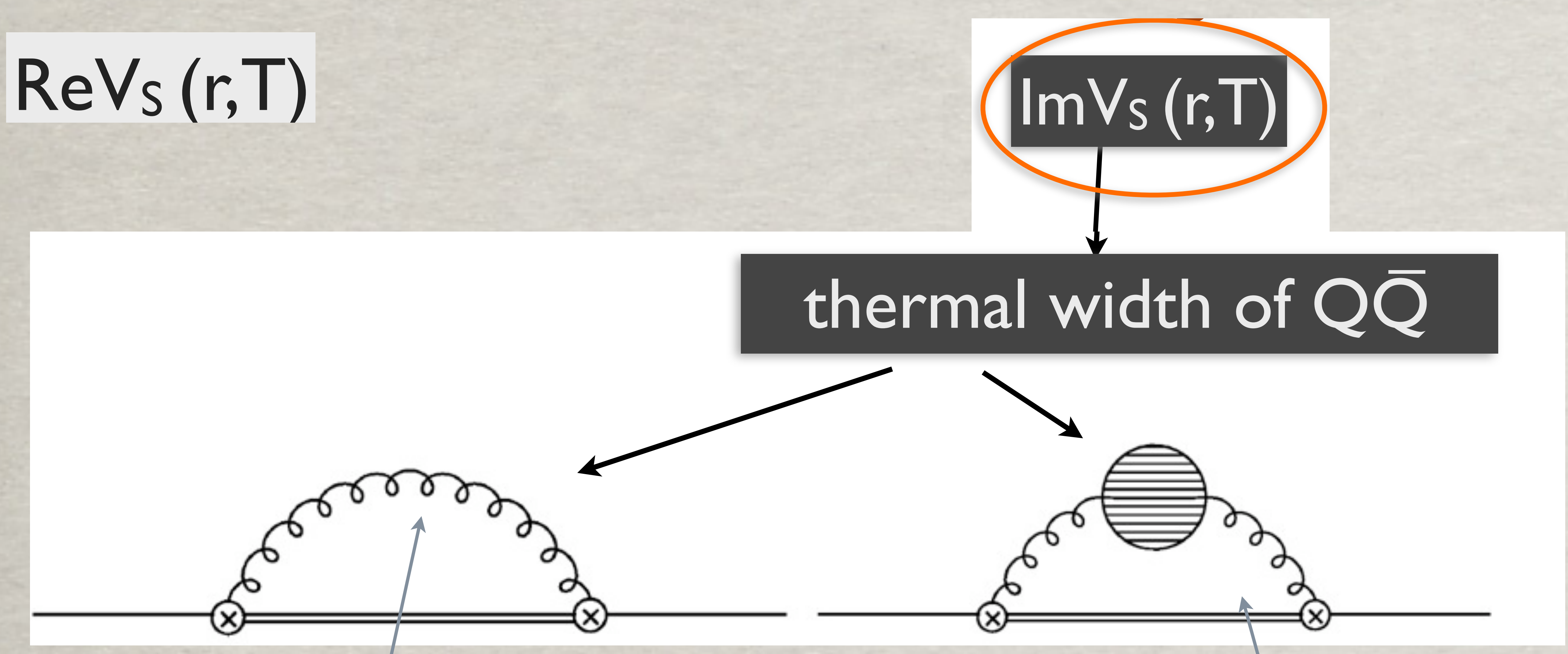

Singlet-to-octet

N.B Ghiglieri, Petreczky, Vairo 2008

(gluo dissociation)

Landau damping

Laine et al 07, Escobedo Soto 07

(inelastic parton scattering)

The EFT produces:

=> T effects can be different from screening

=> dissociation of quarkonium is driven by the imaginary parts of the potentials instead than by Debye screening

=> a technology to calculate systemically thermal energies and widths: spectrum of quarkonium at finite T at alpha_s^5

The Frontier of the NR Bound State:

THE FRONTIER OF THE NR BOUND STATE:

NR PAIRS AND LIGHT DEGREES FREEDOM:
X, Y, Z EXOTICS OBSERVED AT COLLIDERS

BO (BORN-OPPENHEIMER) EFT

N ,B., Berwein, Tarrus, Vairo 1510.04299, Oncala, Soto 1702.03900, N. B., Krein, Tarrus, Vairo, 1707.09647, Soto, Tarrus, 2005.00552, N.B., W.K. Lai, Segovia, Tarrus, Vairo 1805.07713, 1908.11699

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NR PAIRS IN NONEQUILIBRIUM EVOLUTION IN A MEDIUM (QGP, EARLY UNIVERSE) THAT TRIGGER DECAYS AND RECOMBINATIONS

PNREFT PLUS OPEN QUANTUM SYSTEM: LINBLAD EQUATION

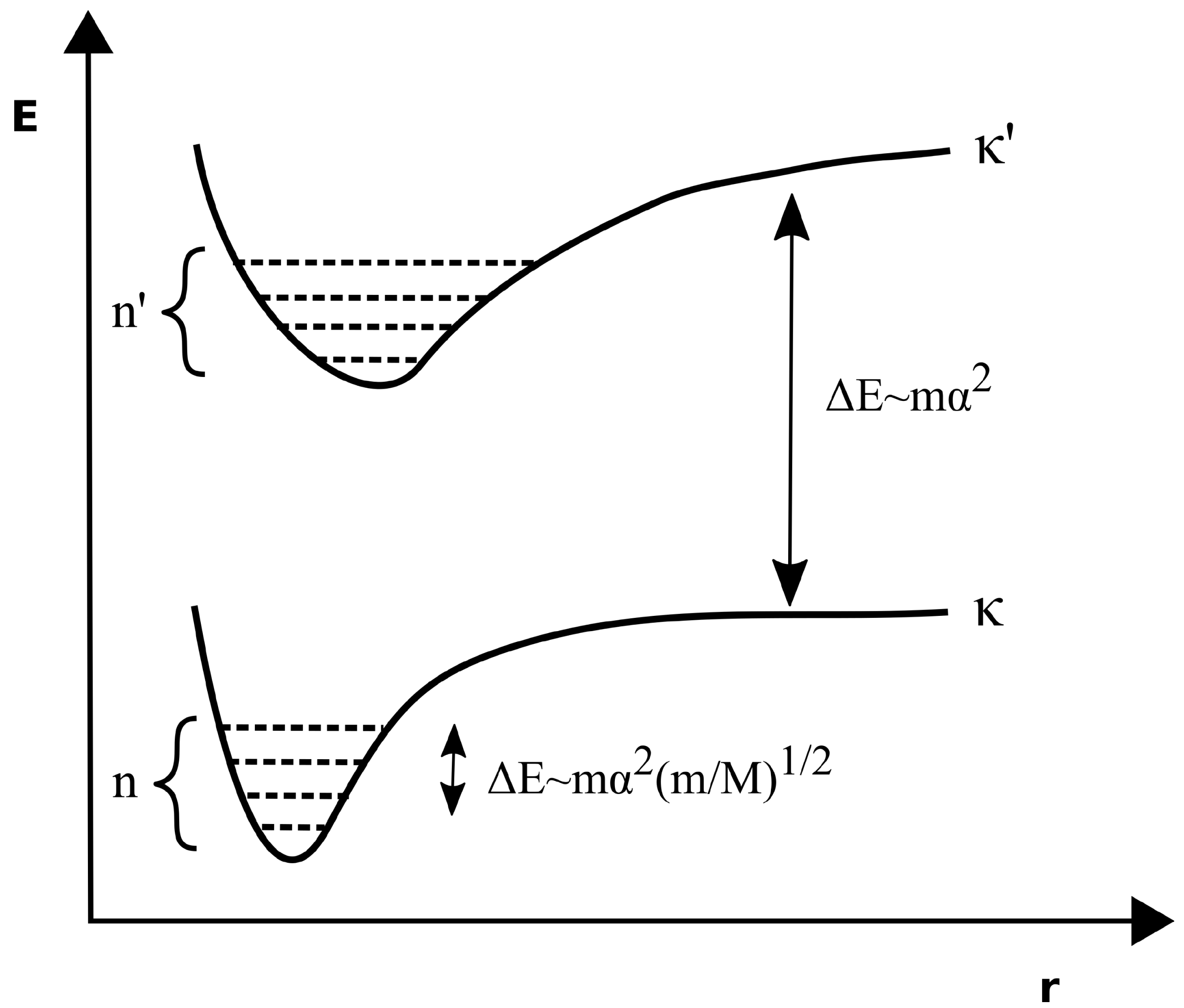
N.B; Escobedo, Soto, Vairo. 1711.04515, 1612.07248; N.B. Escobedo, Vairo, Vander Griend 1903.08063; N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240; Yao, Mehen 2009.02408, 1811.07027; Sharma 2020…

BOEFT: EFT for nonrelativistic pairs and light d.o.f.

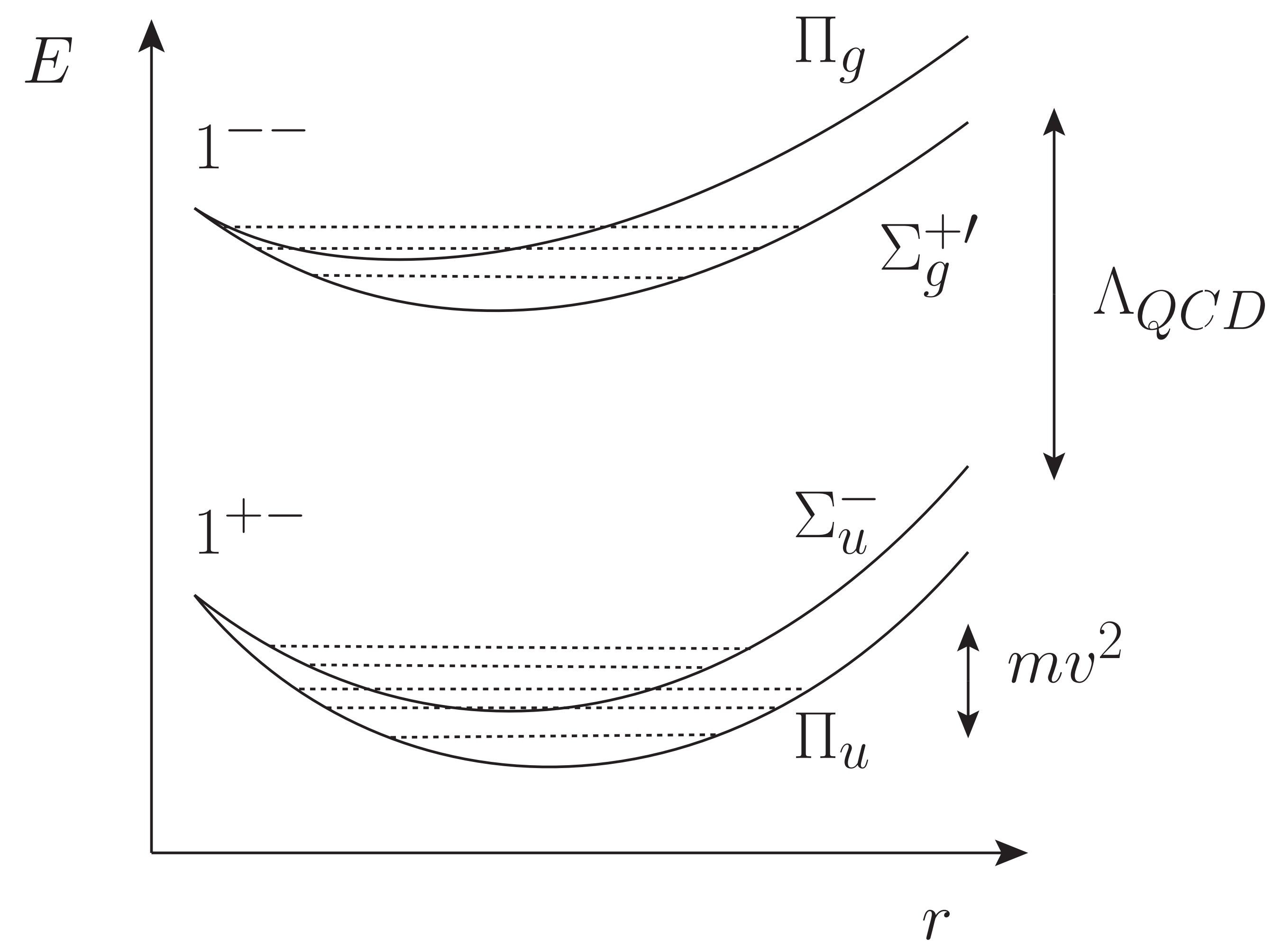
Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ($Q\bar{Q}g$ states)or tetraquarks ($Q\bar{Q}q\bar{q}$ states):

- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential V_κ between static sources, where κ labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials V_κ by solving the corresponding Schrödinger equation.

This picture goes also under the name of Born-Oppenheimer approximation. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called Born–Oppenheimer EFT (BOEFT).



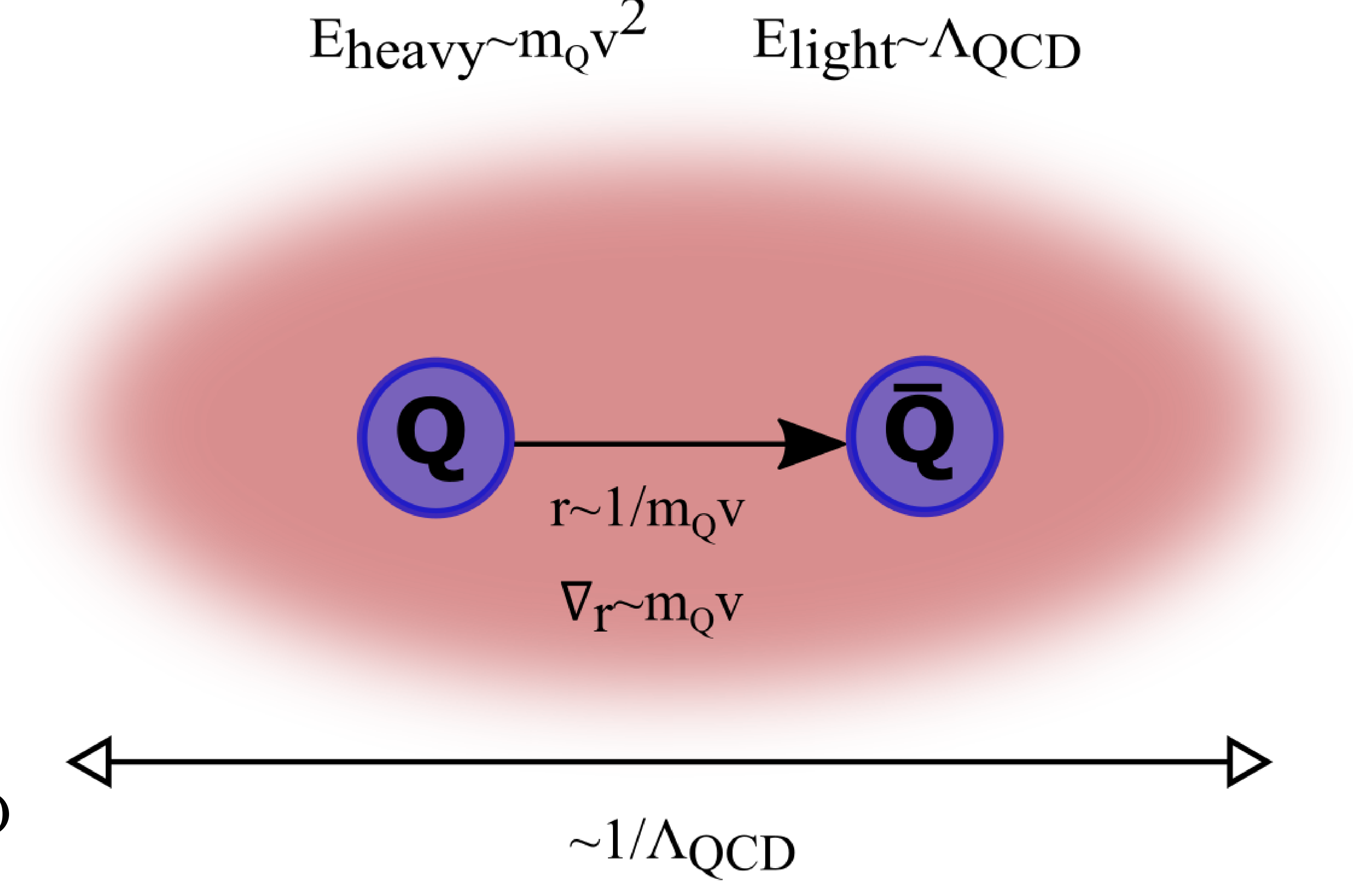
Michael et al. 1983,
Juge, Kuti, Mornigstar 1997, 1998,
Braaten,Langsmack, Smith 2014



hybrids

two different scales

$\Lambda_{QCD} \gg mv^2$



analogous to

$E_{electrons} \gg E_{nuclei}$

in QED

we proceed to integrate out 1/r and then Λ_{QCD}

(the two scales can also be integrated out simultaneously see Soto, Tarrus 2020)

Λ_{QCD} is nonperturbative but we can use the lattice static energies

hybrids

two different scales

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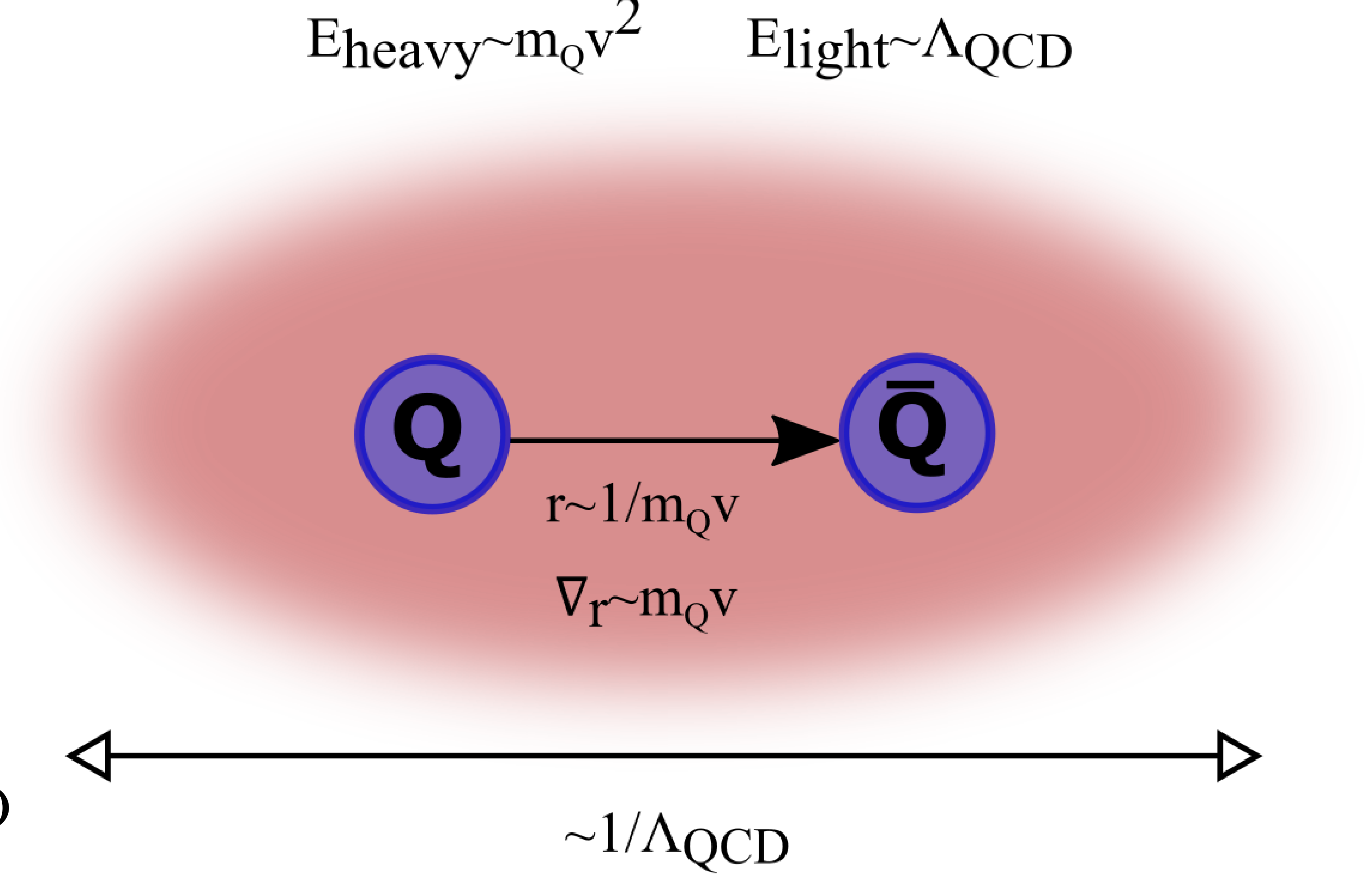

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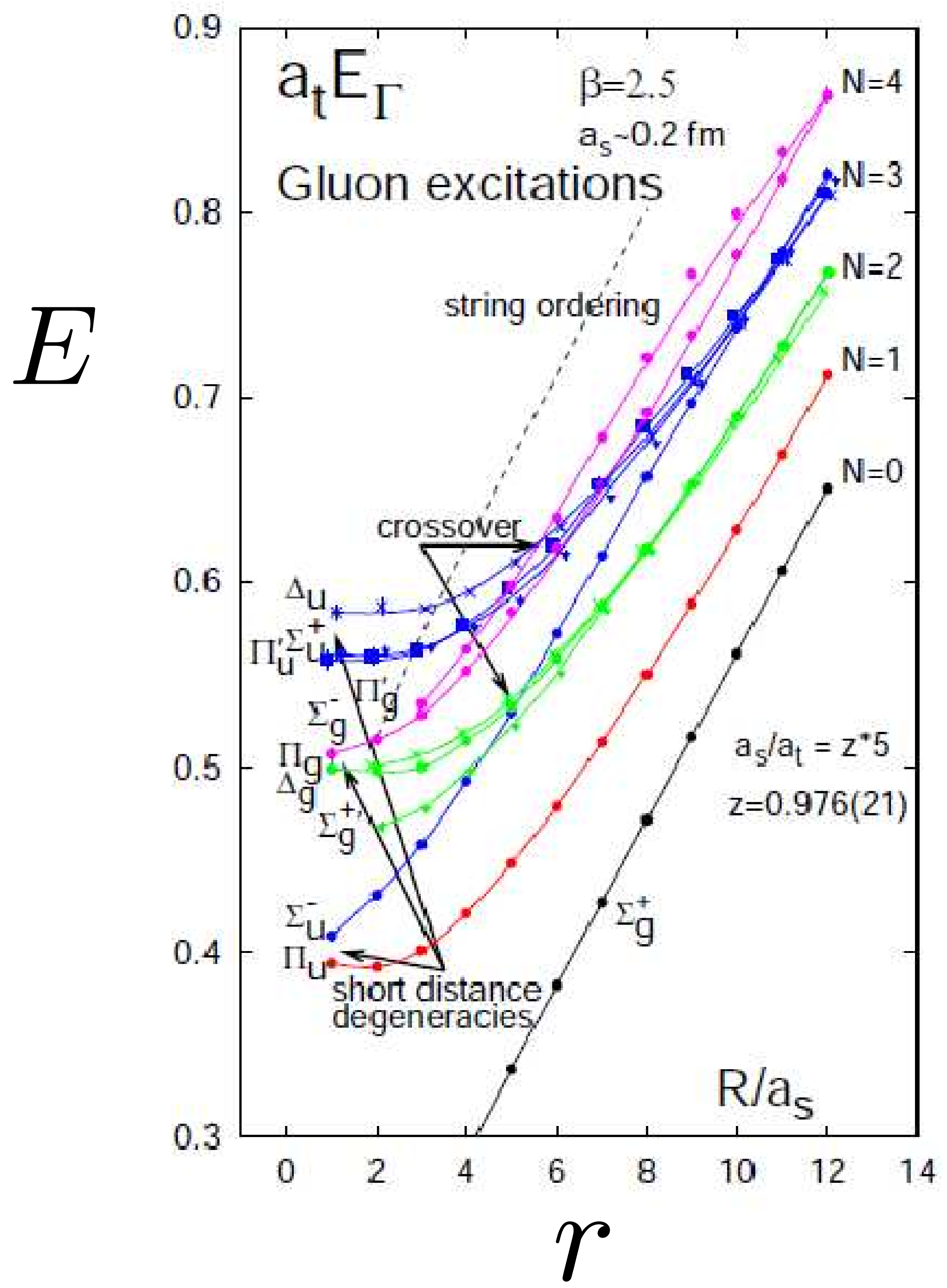
d out

20)

Λ_{QCD} is nonperturbative but we can use the lattice static energies



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

Juge Kuti Morningstar PRL 90 (2003) 161601

Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 03450

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

○ Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
Oncala Soto PRD 96 (2017) 014004
Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0\Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda\left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda\left[\dfrac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

Spectrum: general consideration

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}$, E_{Π_u}
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}$, E_{Π_u}

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

we can calculate the structure of the hybrids multiplets

- Braaten PRL 111 (2013) 162003
  Braaten Langmack Smith PRD 90 (2014) 014044

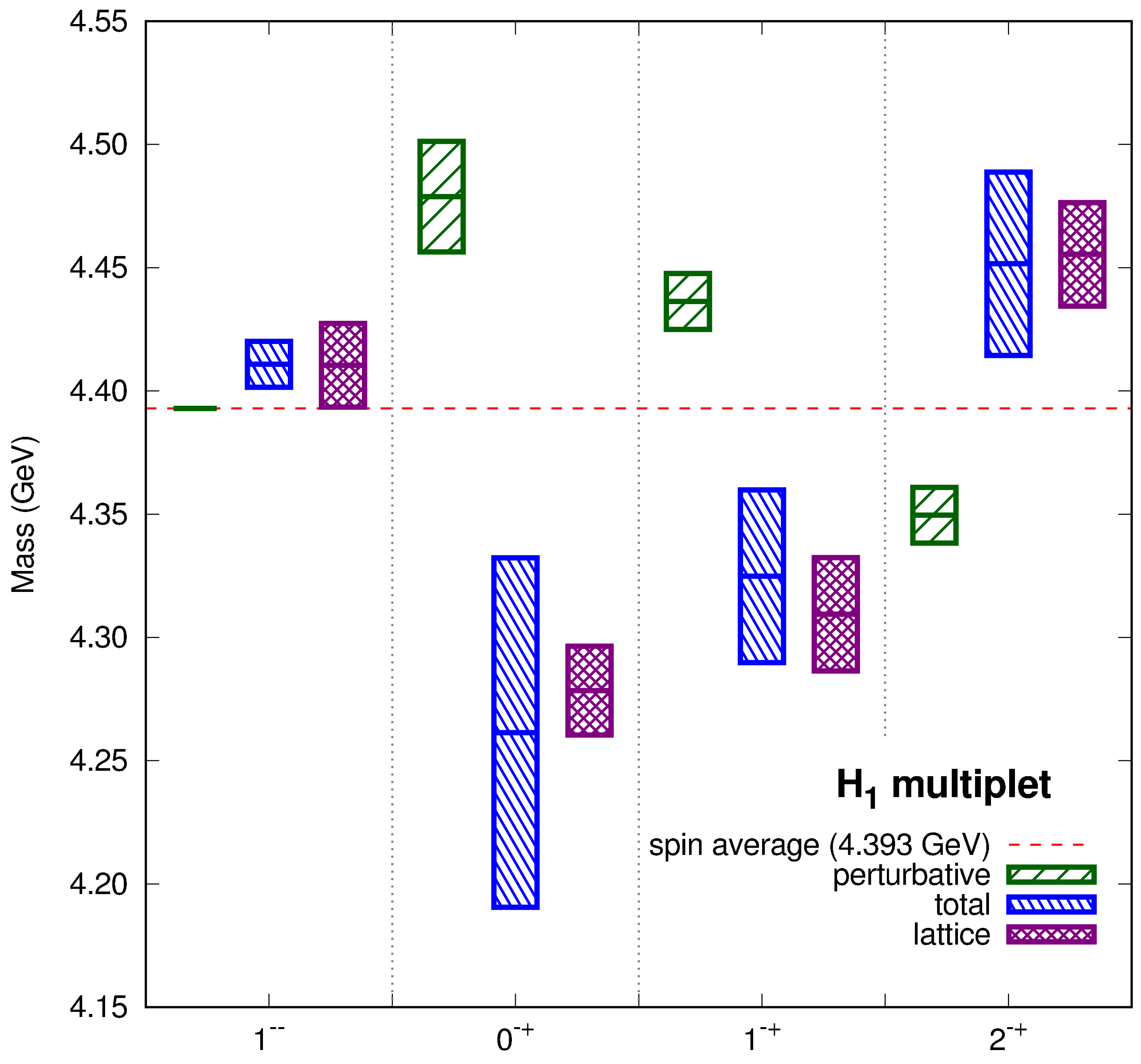
Hybrid spin-dependent potentials at order $1/m$ and $1/m^2$

$$
\begin{aligned}
V^{(1)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) &= V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} \\
&+ V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S}+\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right] \\
V^{(2)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) &= V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S}+V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j}+S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} \\
&+ V^{(2)}_{LS\,c}(r)\left[\hat{r}_{\lambda}\cdot\boldsymbol{r}\left(\boldsymbol{p}\times\boldsymbol{S}\right)\cdot\hat{r}_{\lambda'}+\hat{r}_{\lambda}\cdot\left(\boldsymbol{p}\times\boldsymbol{S}\right)\hat{r}_{\lambda'}\cdot\boldsymbol{r}\right] \\
&+ V^{(2)}_{S^2}(r)\boldsymbol{S}^{2}\delta_{\lambda\lambda'}+V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'}+V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2}+S^{i}_{2}S^{j}_{1}\right)
\end{aligned}
$$

$\left(K^{ij}\right)^{k}=i\epsilon^{ikj}$ is the angular momentum of the spin one gluons
and $\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Charmonium H_1 hybrid spin splittings



fix the nonperturbative unknowns from a charmonium lattice hybrid calculation

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

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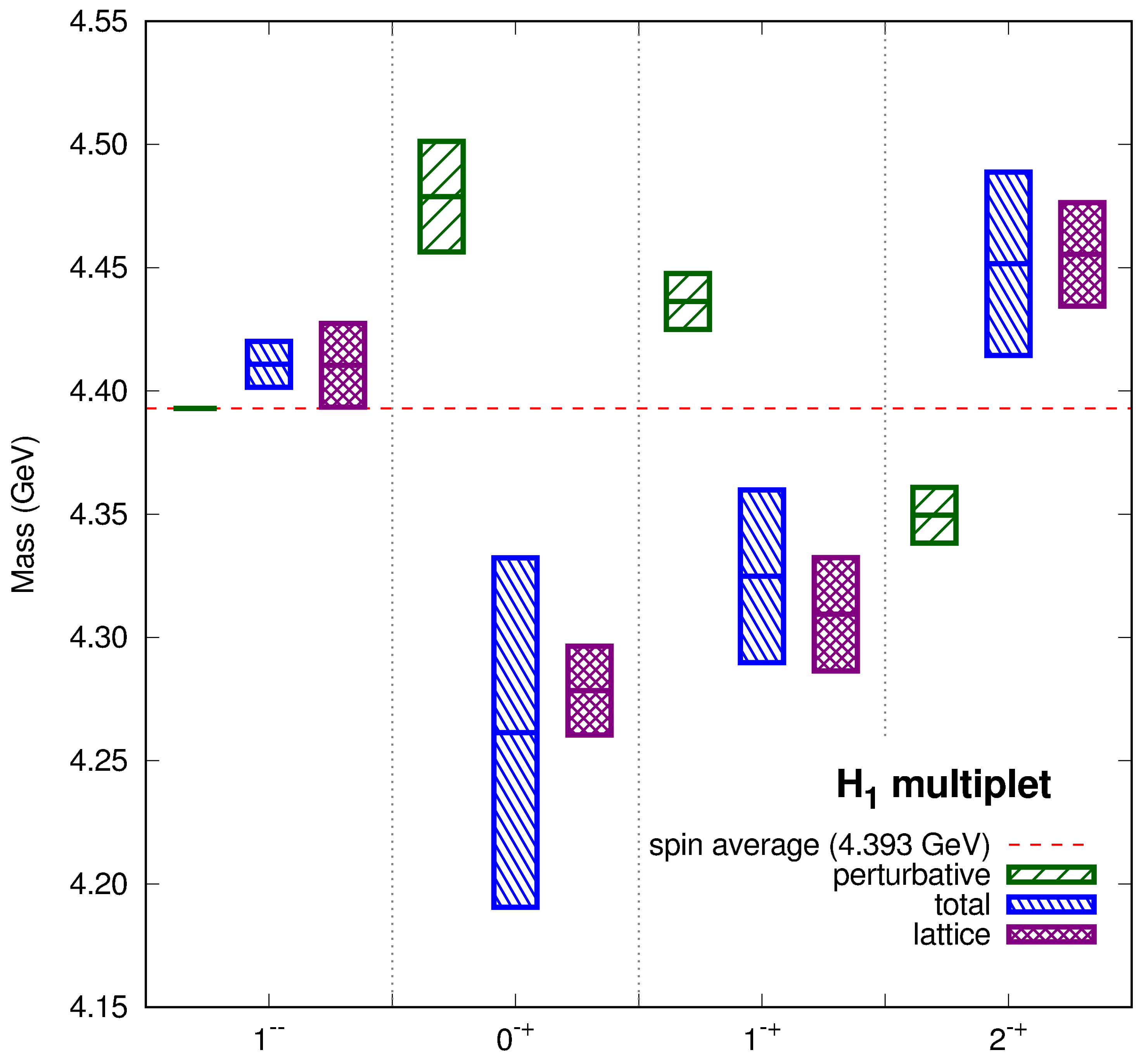

predict
the bottomonium
hybrid splittings

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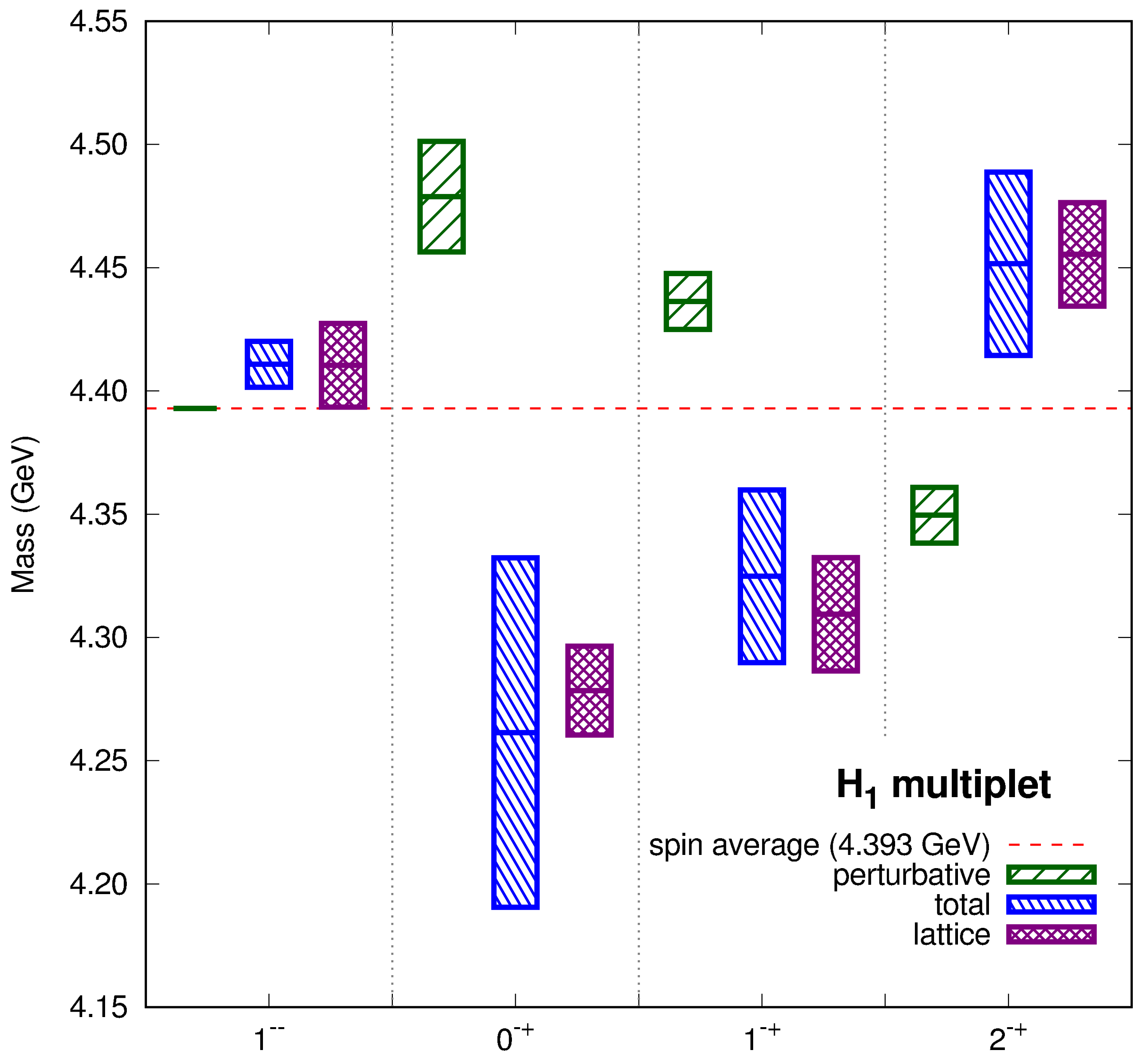

predict
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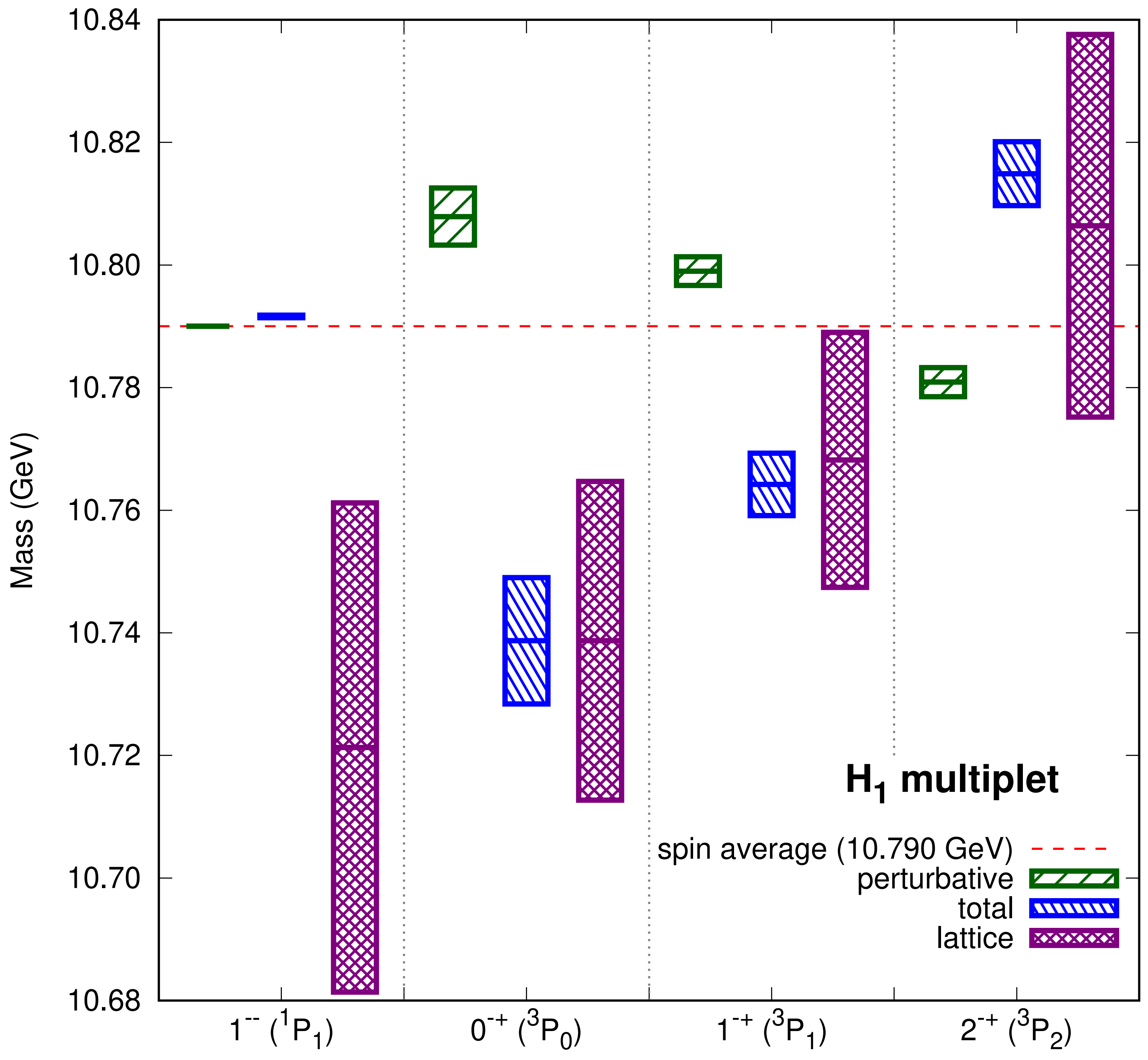

fix the nonperturbative unknowns from
a charmonium lattice hybrid calculation

blue EFT predictions,
violet actual lattice calculation

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

- Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV
  unpublished plot by J. Segovia

nonequilibrium evolution of quarkonium in the fireball with pNRQCD +open quantum systems+lattice

nonequilibrium evolution of quarkonium in the fireball with pNRQCD +open quantum systems+lattice

work in the hierarchy and in real time formalism

$$M \gg \frac{1}{r} \sim M\alpha_{\rm s} \gg T \sim gT \gg \text{any other scale}, \qquad v \sim \alpha_{\rm s}$$

use a Coulombic quarkonium to test the strongly coupled plasma

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- Subsystem: heavy quarks/quarkonium
- Environment: quark gluon plasma

N.B., J. Soto, M. Escobedo, A. Vairo 2016, 2018 (1612.07248, 1711.04515)

We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\langle \mathbf{r}', \mathbf{R}' | \rho_s(t';t) | \mathbf{r}, \mathbf{R} \rangle \equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, S^\dagger(t,\mathbf{r},\mathbf{R}) S(t',\mathbf{r}',\mathbf{R}')\}$$

$$\langle \mathbf{r}', \mathbf{R}' | \rho_o(t';t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} \equiv \mathrm{Tr}\{\rho_{\text{full}}(t_0)\, O^{a\dagger}(t,\mathbf{r},\mathbf{R}) O^b(t',\mathbf{r}',\mathbf{R}')\}$$

$t_0 \approx 0.6$ fm is the time formation of the plasma.

nonequilibrium evolution of quarkonium in the fireball with pNRQCD +open quantum systems+lattice

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$t_0 \approx 0.6$ fm is the time formation of the plasma.

The system is in non-equilibrium because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although the number of heavy quarks is conserved: $\mathrm{Tr}\{\rho_s\} + \mathrm{Tr}\{\rho_o\} = 1$.

nonequilibrium evolution of quarkonium: density matrix evolution

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\frac{d\rho_o(t;t)}{dt} = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

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\frac{d\rho_o(t;t)}{dt} &= -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) \\
&\quad + \Xi_{oo}(\rho_o(t;t), t)
\end{aligned}
$$

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$
\begin{aligned}
-i\Sigma_{s,o}(t) + i\Sigma^\dagger_{s,o}(t) &= 2\,\mathrm{Re}\,(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t) \\
\Sigma_{s,o}(t) + \Sigma^\dagger_{s,o}(t) &= -2\,\mathrm{Im}\,(-i\Sigma_{s,o}(t)) = \Gamma_{s,o}(t)
\end{aligned}
$$

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$$\frac{d\rho_o(t;t)}{dt} = -i[h_o,\rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t),t) + \Xi_{oo}(\rho_o(t;t),t)$$

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$\begin{aligned} -i\Sigma_{s,o}(t) + i\Sigma^\dagger_{s,o}(t) &= 2\,\mathrm{Re}\,(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t) \\ \Sigma_{s,o}(t) + \Sigma^\dagger_{s,o}(t) &= -2\,\mathrm{Im}\,(-i\Sigma_{s,o}(t)) = \Gamma_{s,o}(t) \end{aligned}$$

- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.

nonequilibrium evolution of quarkonium: Linblad equations

If $E \ll T \sim m_D$ the Lindblad equation for a strongly coupled plasma reads

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \left(C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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the sQGP is characterised by two nonperturbative parameters (transport coefficients) kappa and gamma that must be calculated on the lattice

κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0})\rangle$$

$$\gamma = \frac{g^2}{18}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\, \langle \mathrm{T}\, E^{a,i}(s,\mathbf{0})\, \phi^{ab}(s,0)\, E^{b,i}(0,\mathbf{0})\rangle$$

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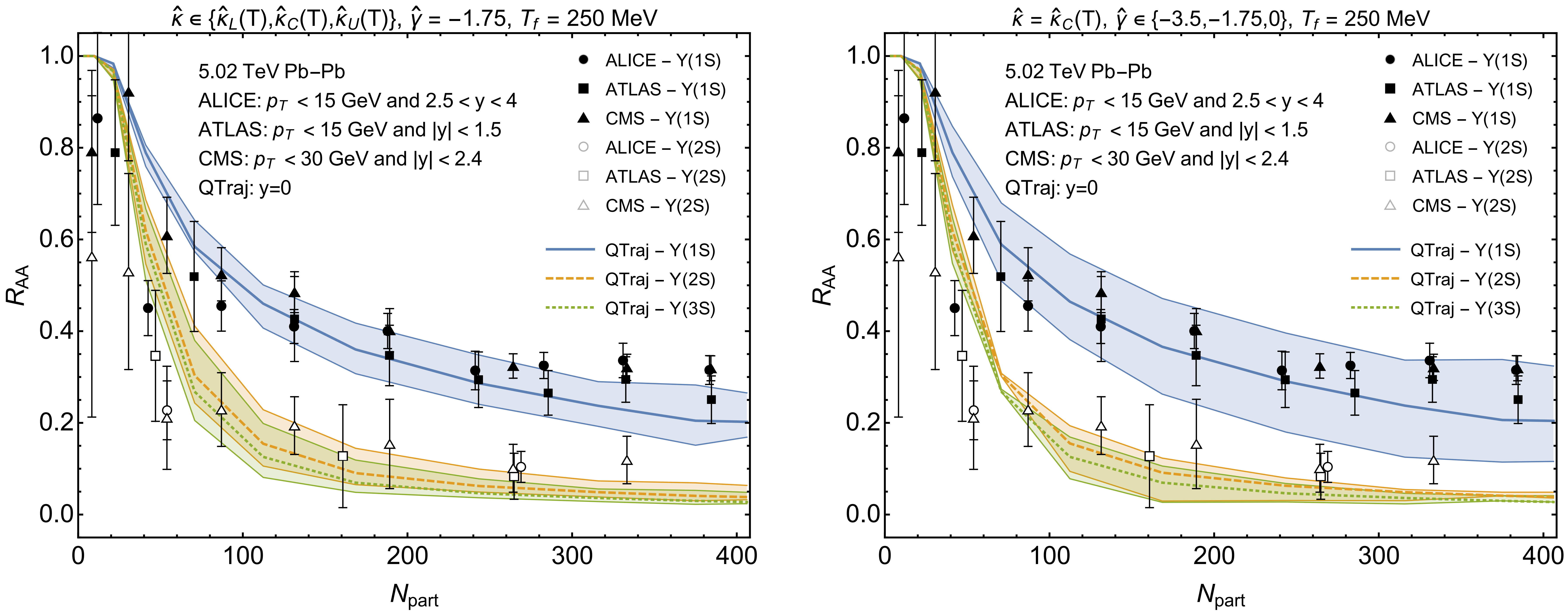
the EFT allows to use lattice QCD equilibrium calculation to study the non equilibrium evolution! EFT is intermediate layer to non equilibrium

nonequilibrium evolution of quarkonium in medium: nuclear modification factor

We compute the nuclear modification factor R_{AA}:

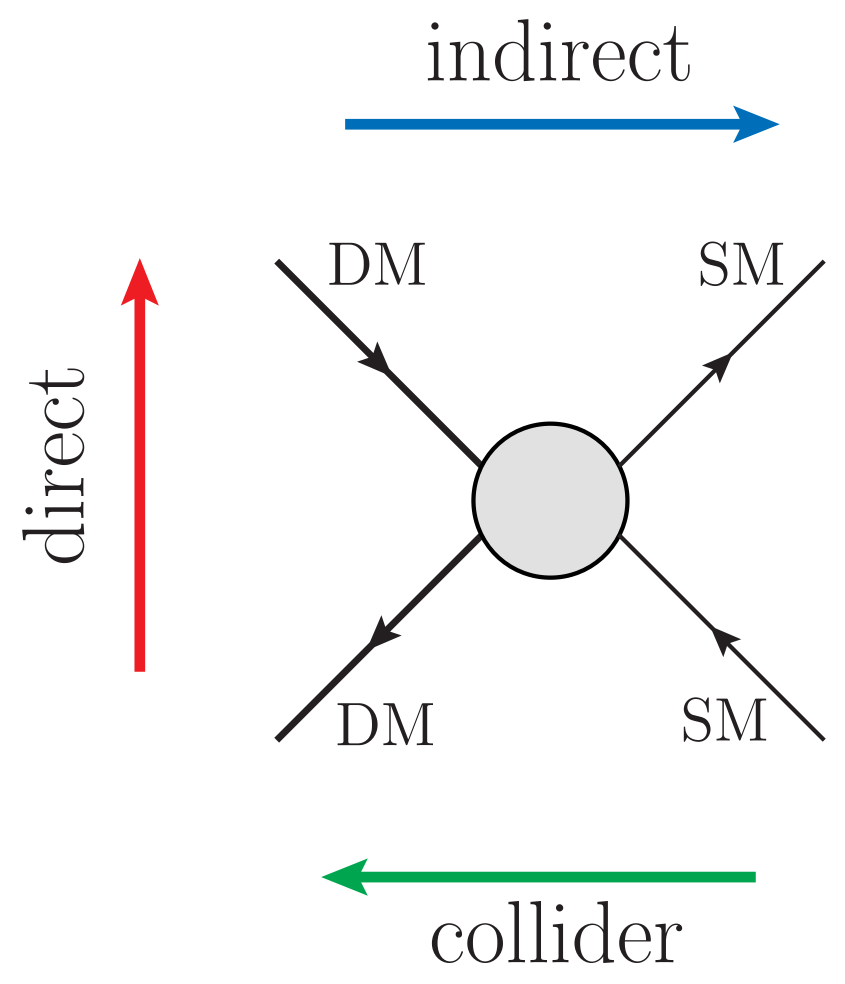
$$R_{AA}(nS) = \frac{\langle n, \mathbf{q}|\rho_s(t_F;t_F)|n, \mathbf{q}\rangle}{\langle n, \mathbf{q}|\rho_s(0;0)|n, \mathbf{q}\rangle}$$



R_AA of singlet Bottomonium in comparison to ALICE, ATLAS and CMS data, left plot variation in kappa, right plot variation in gamma

N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240

These results and approach could be applied to the study of the non equilibrium evolution of dark matter annihilation and formation in the early universe and after



- DM as a particle: many candidates
- Any model has to comply with

$$\Omega_{\mathrm{DM}} h^2(M_{\mathrm{DM}}, M_{\mathrm{DM'}}, \alpha_{\mathrm{DM}}, \alpha_{\mathrm{SM}}) = 0.1200 \pm 0.0012$$

Thermal freeze-out

- Boltzmann equation for DM (χ)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

- relevant processes $\chi\chi \leftrightarrow \mathrm{SM}\,\mathrm{SM}$
- $\langle \sigma v \rangle$: input from particle physics with $v \sim \sqrt{T/M} < 1$

$$\langle \sigma v \rangle \approx \langle a + bv^2 + \ldots \rangle \Rightarrow \boxed{\langle \sigma v \rangle^{(0)} \approx \frac{\alpha^2}{M^2}}$$

Outlook

pNREFT gives us a powerful tool to address NR bound and threshold states in QFT

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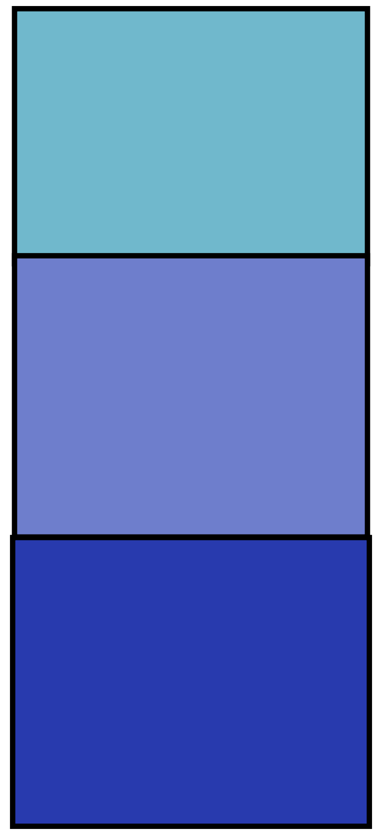
pNREFT is a flexible and versatile tool that could be applied to the realm of atomic, condensed matter, gravitation…. physics wherever NR states play a role pNRQCD shows that the type of low energy (nonperturbative) factorized effects depend on the size of the physical system

$$r \ll \frac{1}{\Lambda_{QCD}}$$

lowest quarkonia states

The EFT factorizes the low energy nonperturbative part.
Depending on the physical system:



$\langle F^2(0) \rangle$ $\quad \Upsilon(1S)$, ... local condensates

$\Lambda_{\rm QCD} \sim t$ $\quad \langle F(t)F(0) \rangle$ $\quad$ annihilations, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ... non local condensates

$\Lambda_{\rm QCD} \sim loop$ $\quad \langle e^{ig \oint dx A(x)} \rangle$ $\quad$ long range $c\bar{c}$, $b\bar{b}$, hybrids, glueballs, ... Wilson loops

The more extended the physical object, the more we probe the non-perturbative vacuum.

excited quarkonia states

$$r \sim \frac{1}{\Lambda_{QCD}}$$

$$r \ll \frac{1}{\Lambda_{QCD}} \qquad T$$

quarkonia in a hot medium

$$r \sim \frac{1}{\Lambda_{QCD}}$$

quarkonia and exotics close and above threshold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THESE OBJECTS SHOULD BE CALCULATED ON THE LATTICE
which is why we founded the TUMQCD lattice collaboration dedicates to EFTs calculations

Notice: pNREFT is the lowest energy EFT for a single NR system but in the interaction between two NR systems more energy scales can be integrated out giving interaction potentials:

WEFT the EFT for bound-state—bound state-interaction

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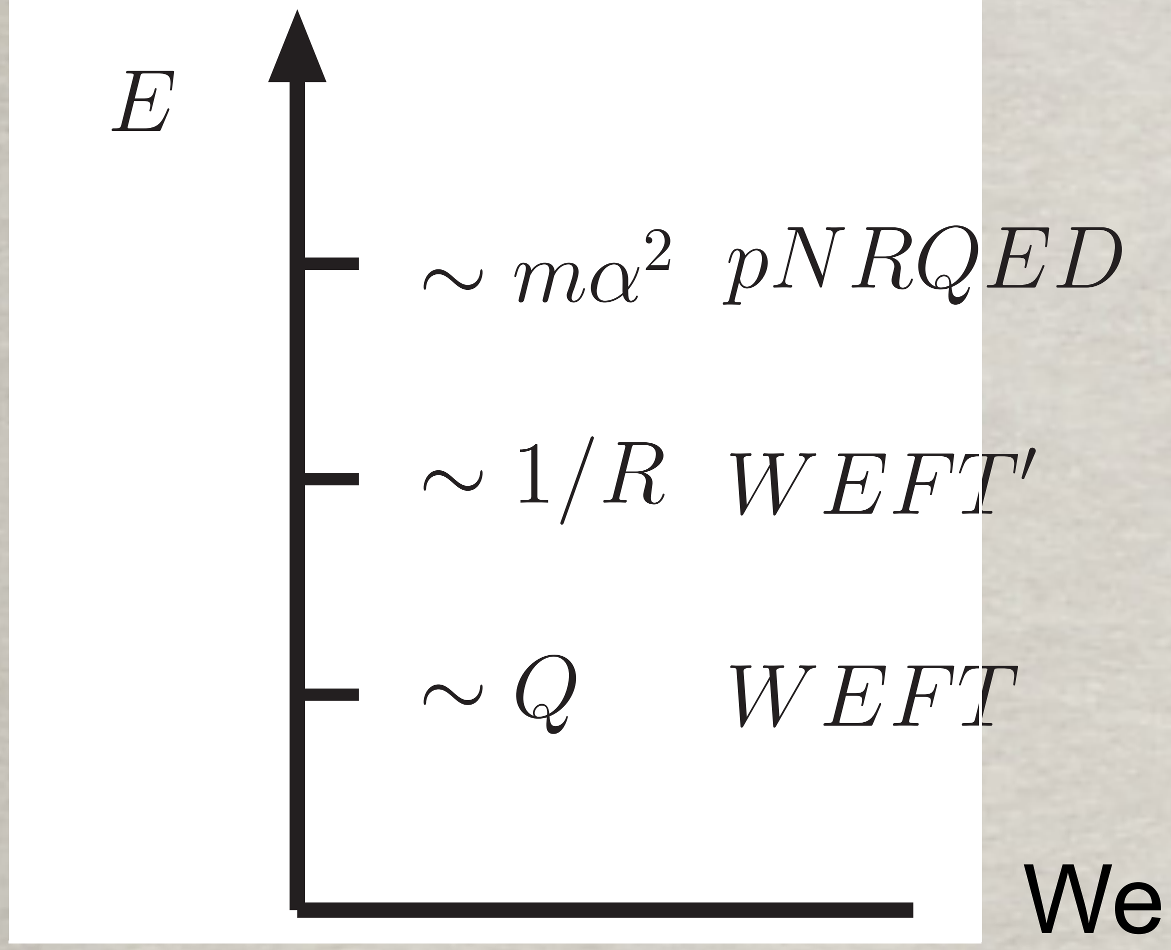
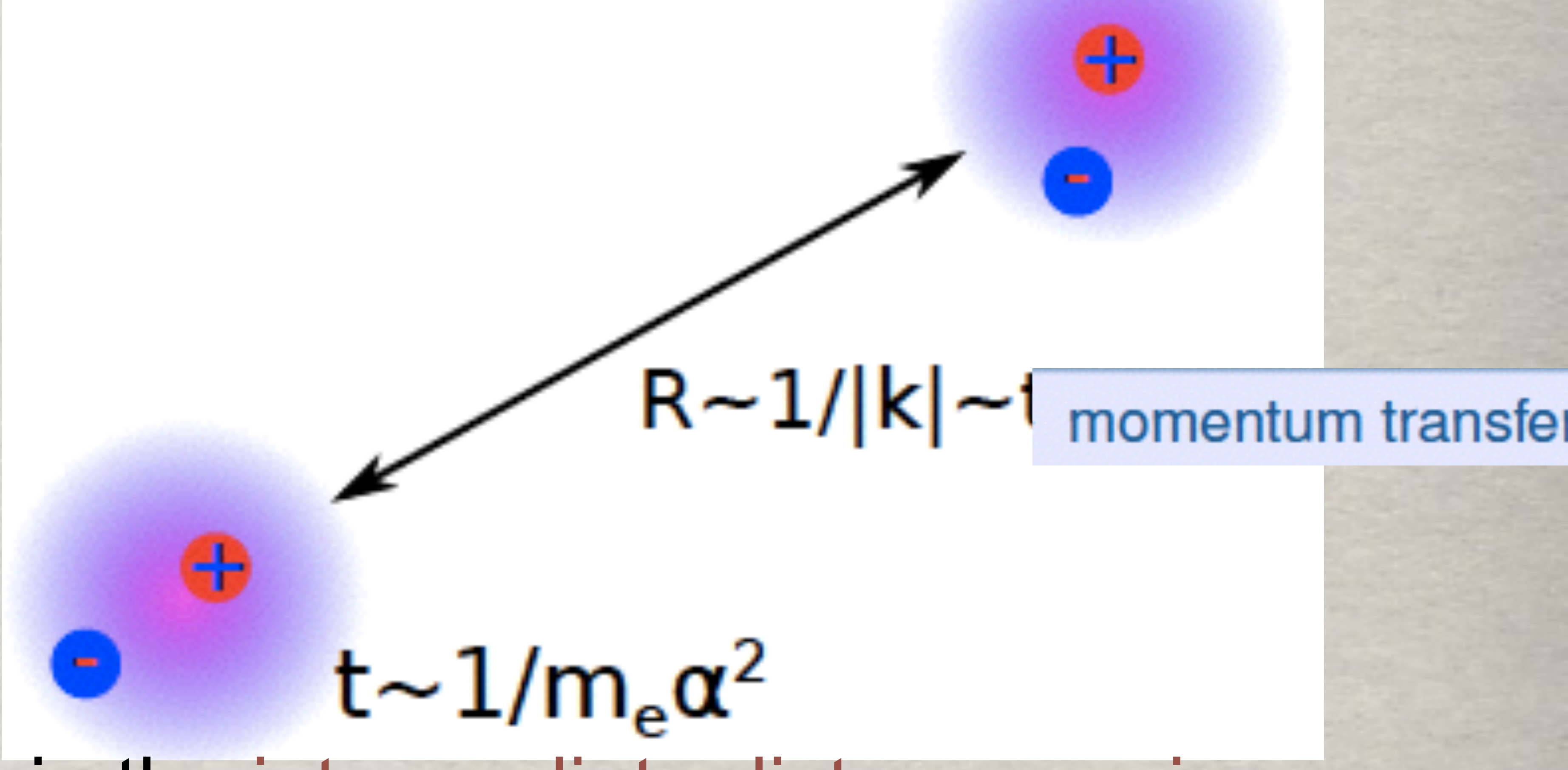
- Consider van der Waals interactions between two hydrogen atoms in the ground state at the distance R

N.B., V. Shtabovenko, J. Tarrus, A. Vairo 1704.03476



WEFT is matched to pNREFT

Van der Walls EFT



We have obtained the van der Waals potential also in the intermediate distance region (limits for short and large distance reproduce London and Casimir Polder)

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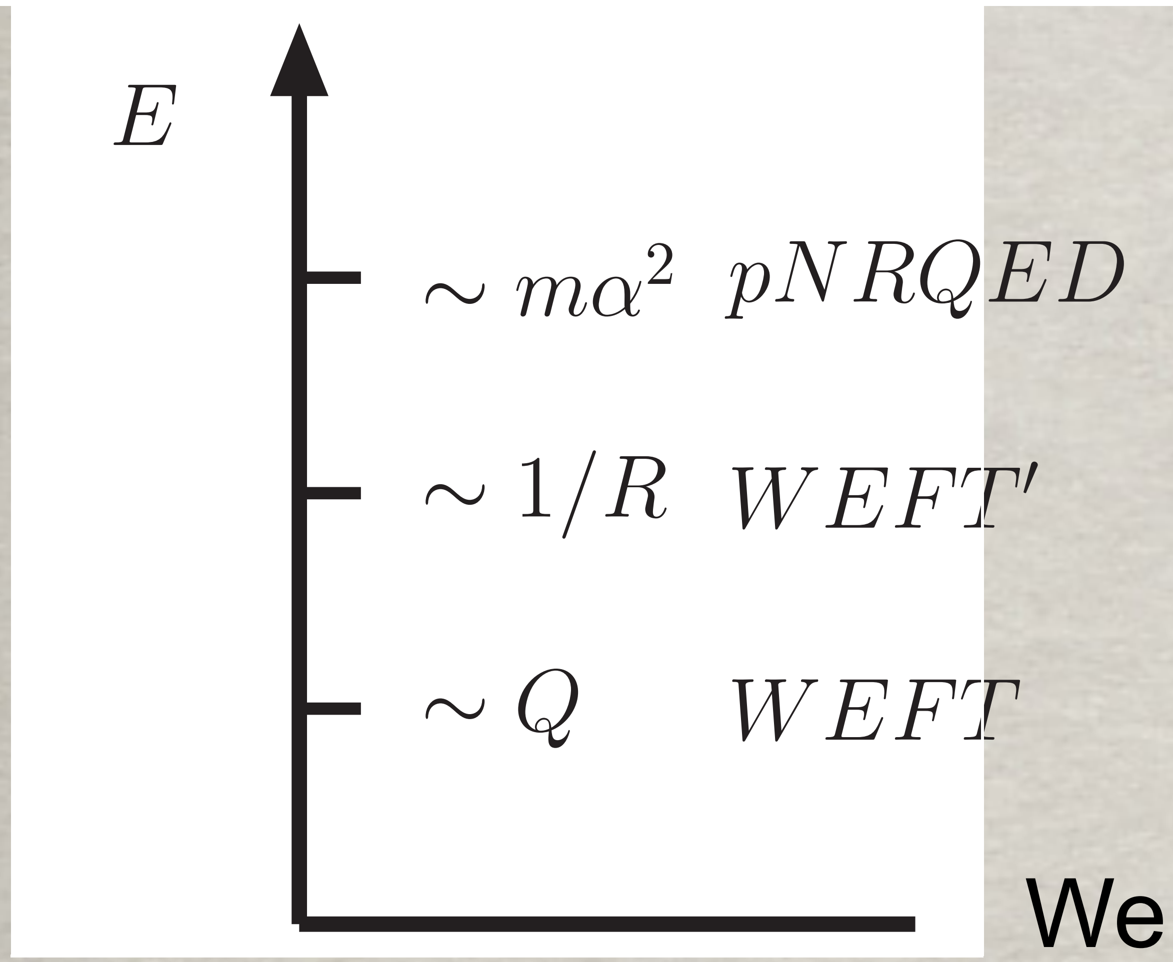

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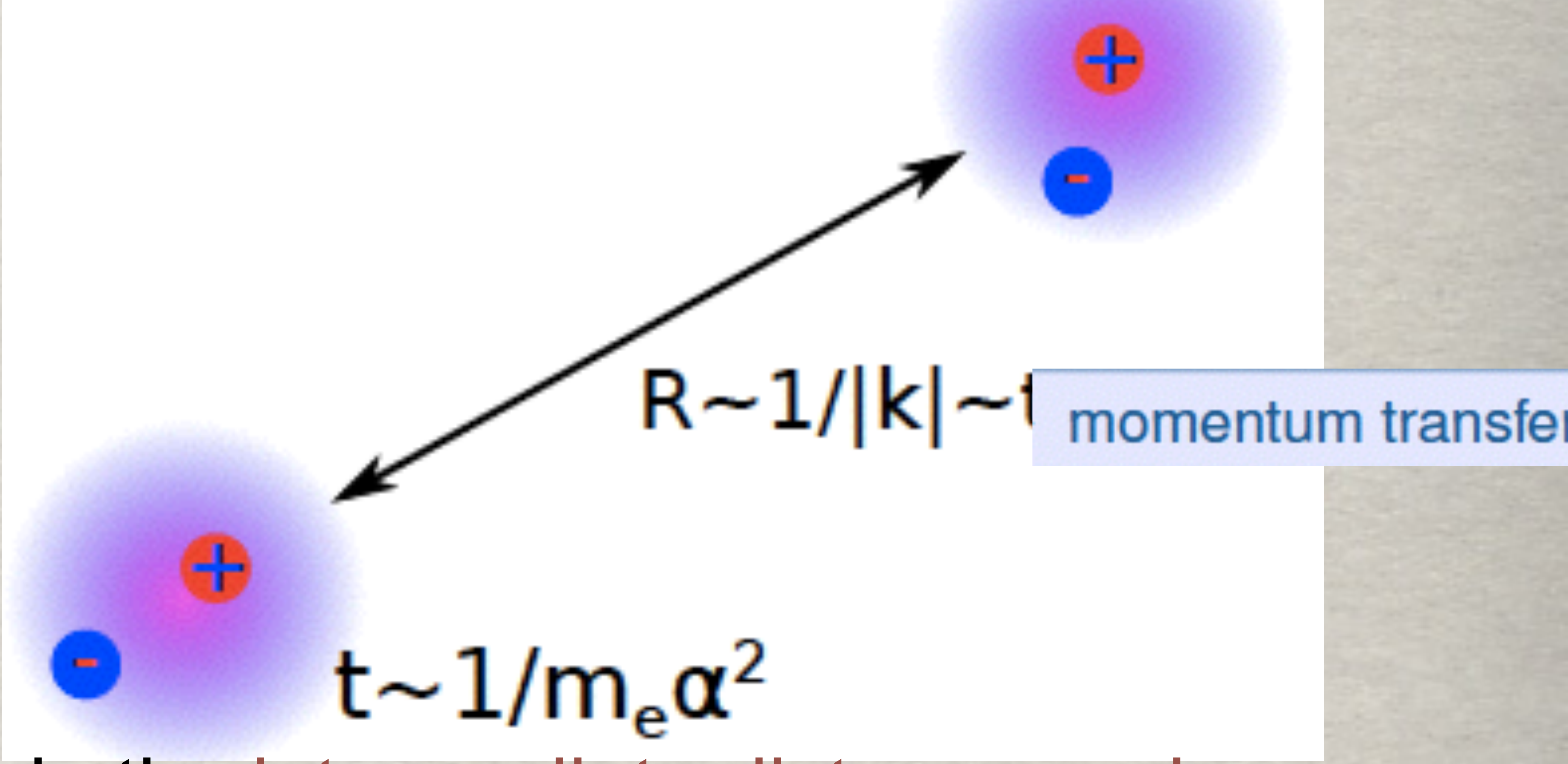

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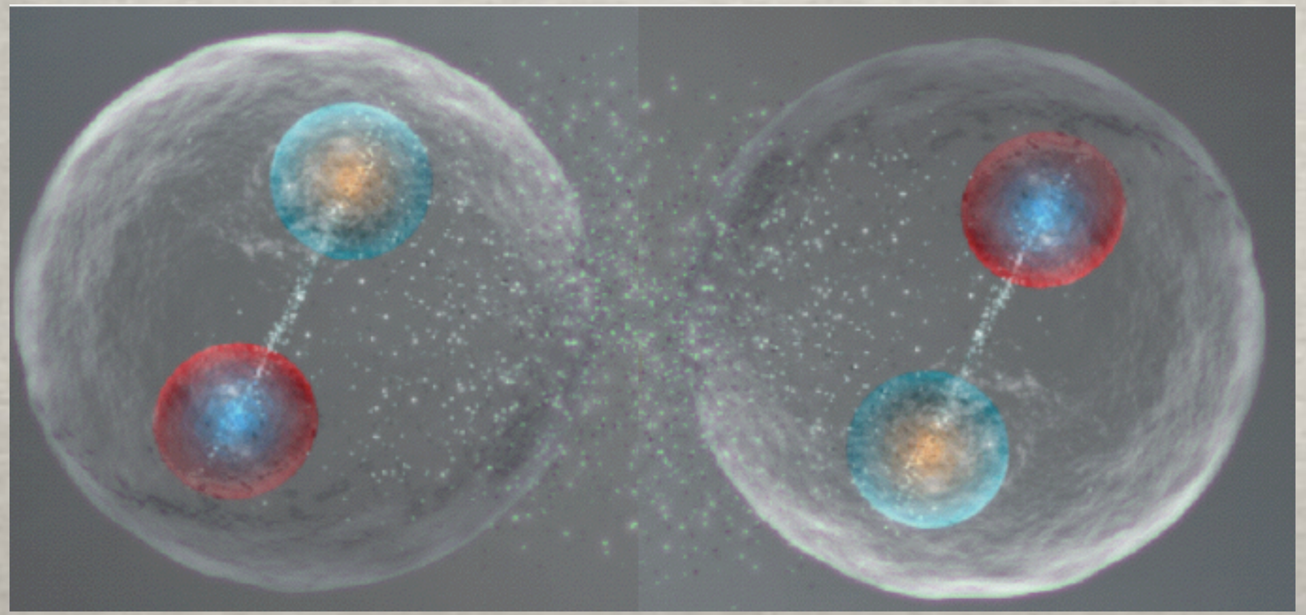
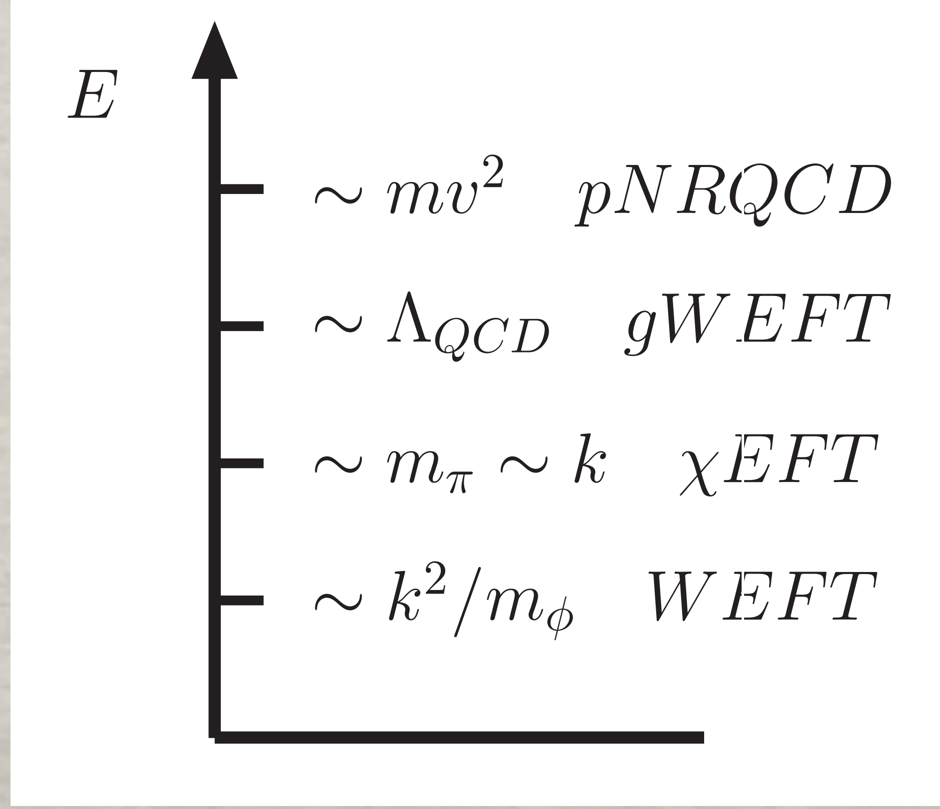
we obtained WEFT in QCD for $\eta_b - \eta_b$



Chromopolarizability & color van der Waals forces

N.B., G. Krein, J. Tarrus, A. Vairo 2015



more scales to integrate out in QCD