UV/IR

and

EFT

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Glory of Wilsonian Paradigm

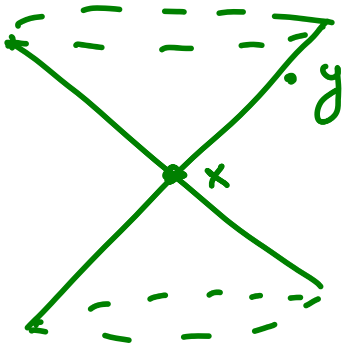
Decoupling of UV + IR

$\vec{x}$ $\vec{y}$ $(\vec{x}-\vec{y})^2$ tiny $\longleftrightarrow$ Need Microscope to Probe Short Dist!

* Fundamentally EUCLIDEAN
* Born out in CONDENSED MATTER

"The World is Not a Crappy Metal"

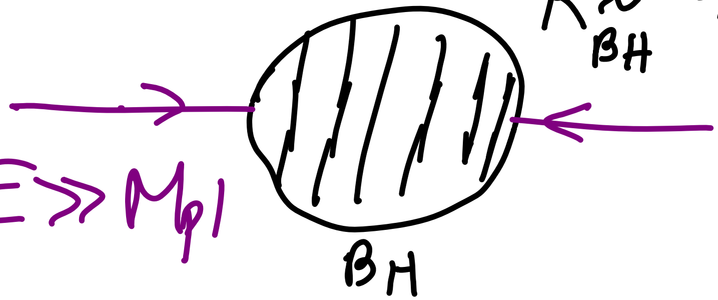
(A) LORENTZIAN



$(x-y)^2$ near zero can be probed at macroscopic distances close to light cone

{ex Bounds on e.g. $c(\partial\varphi)^4$ $c>0$ from no superlum.}

(B) GRAVITATIONAL



$E \gg M_{pl}$

$R_{BH} \sim E/M_{pl}^2$ grows

We know HE states, BH thermo $\longleftrightarrow$ IR consistency

{ex No Glob symm $\rightarrow$ WGC ...}

[Not to mention]

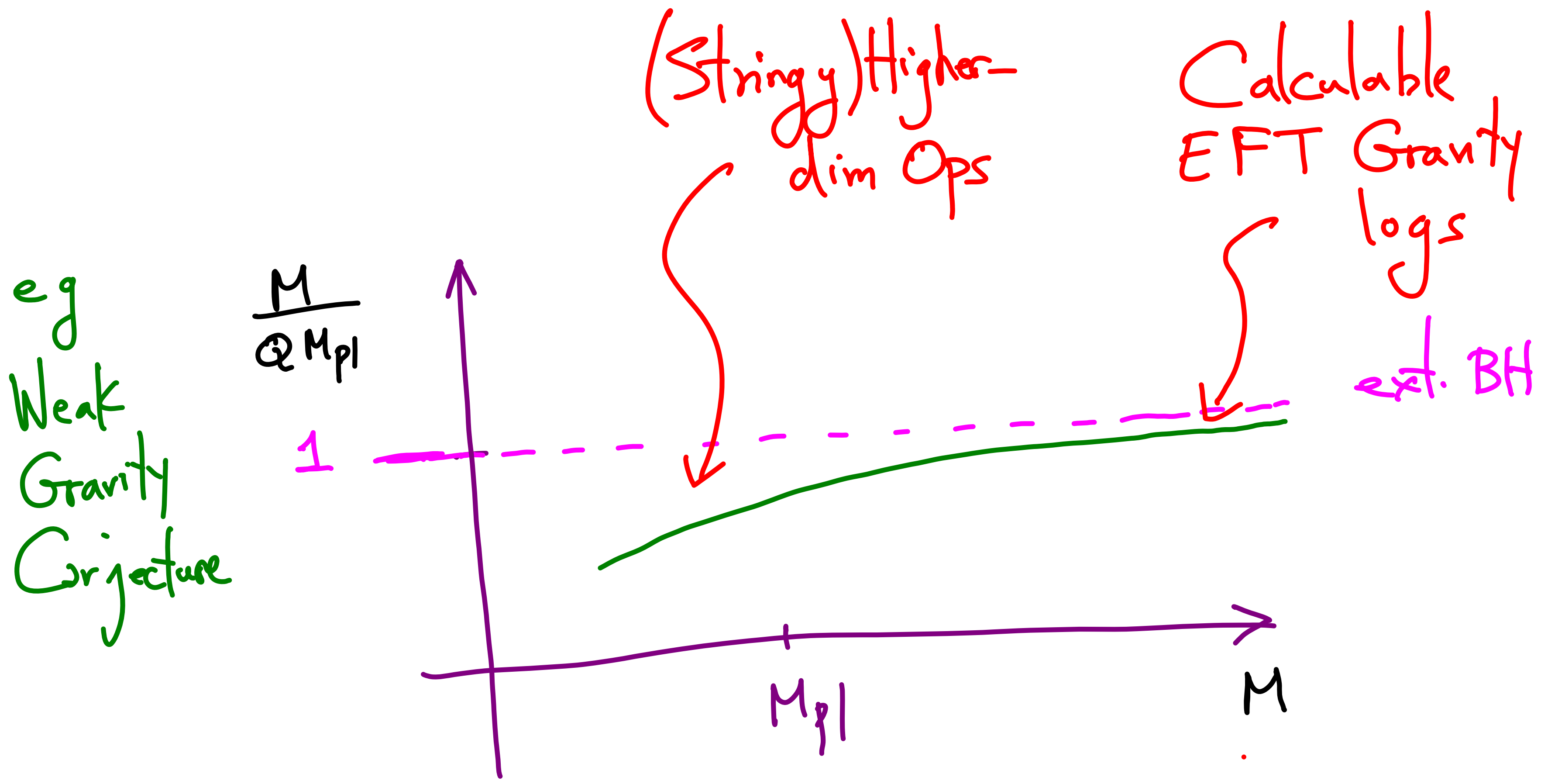
(C) The Spectacular Failure of the Wilsonian notion of Naturalness

– which works beautifully in CM! –

for $\Lambda_{cc} + m_h^2$

{Dramatically} {If you turn into a pumpkin @ midnight, it's 11.45 pm....}.

... (A) + (B) are clearly related...



Three Frontiers in UV/IR-EFT

(I) "Positive Geometry" of EFT:
"EFThedra" + beyond

(II) IR $\longleftrightarrow$ UV Completion + Strings

(III) Celestial Amplitudes + Anti-Wilsonian Paradigm

EFT-hedra

Locality / Causality
Unitarity
$\longrightarrow$
Analyticity
Positivity

vs.

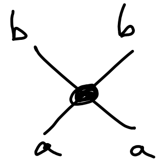
Locality / Causality
Unitarity
$\longleftarrow$
Positive Geometry
(Amplituhedra, Associahedra, "Clusterhedra"...)

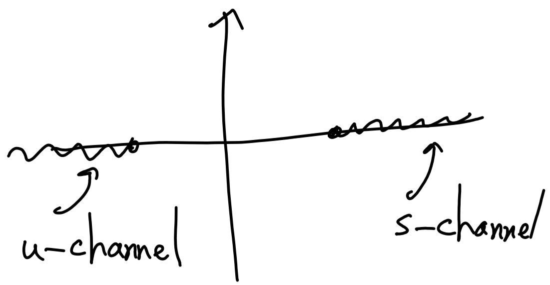
Locality/Causality
Unitarity
→
Analyticity
Positivity
←
Infinitely
more Hidden
Positivity
[EFThedra]

?

vs.

Locality/Causality
Unitarity
←
Positive Geometry
(Amplituhedra, Associahedra,
"Clusterhedra"...)

Dispersive Representation of 



Fixed $|t| \ll M^2$ [Avoid Anom. thresholds]

* With gap $\mathcal{A}(s,t) < s \log^2 s$ (From assuming analyticity in t, $\mathcal{A} < s^N$)
* Grav. theory w/ weak coupling $\mathcal{A} < \frac{s^{2-\varepsilon}}{t}$ (Eikonal/Shockwave)

$$\mathcal{A}(s,t) = \mathcal{A}_0(t) \cdot \mathcal{A}_1(t)\, s + \int dM^2 \sum_{J} \overbrace{P_J(M^2)}^{>0}\, G_J\left(1+\frac{2t}{M^2}\right) \times \left[\frac{1}{\underbrace{M^2-s}_{>0}} + \frac{1}{\underbrace{M^2-u}_{>0}}\right]$$

EFT: $A(s,t) = \sum_{D,q} a_{D,q}\, s^{D-q} t^{q}$

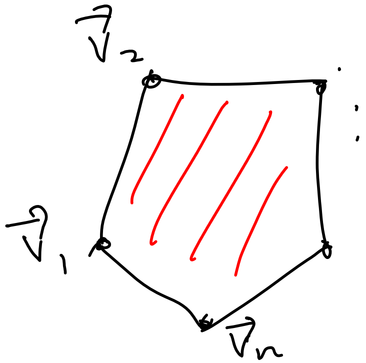
$$
\begin{array}{ccccc}
a_{0,0} & a_{1,0} & a_{2,0} & a_{3,0} & a_{4,0} \\
 & a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} \\
 & & a_{2,2} & a_{3,2} & a_{4,2} \\
 & & & a_{3,3} & a_{4,3} \\
 & & & & a_{4,4}
\end{array}
\quad \cdots\cdots \quad
\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\}
$$

Are forced to lie in "EFThedron" — ∞'ly many linear + non-linear constraints

"Hankel"

"Gegenbauer Polytope"

Polytopes 101



$$\vec{A} = \frac{w_1 \vec{v}_1 + \cdots + w_n \vec{v}_n}{w_1 + \cdots + w_n} \qquad \boxed{w_a > 0}$$

Better Projectively: $A^I = \begin{pmatrix} 1 \\ \vec{A} \end{pmatrix}, \; V_i^I = \begin{pmatrix} 1 \\ \vec{v}_i \end{pmatrix}$

$$A^I = w_1 V_1^I + \cdots + w_n V_n^I \qquad \text{"Convex Hull"}$$

Check in A is inside? "Face" description $A^I W_I^{(a)} \geq 0$

Facet structure completely captured by $\langle V_{a_1} \cdots V_{a_D} \rangle \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$

Very Special Class of Polytopes

$$\begin{array}{c} \\ D \updownarrow \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} V_1 & V_2 & \cdots & V_n \end{pmatrix} \end{array} \longleftarrow \underline{\underline{\text{Ordering}}}$$

$\langle V_{a_1} \cdots V_{a_D} \rangle > 0$ for $a_1 < \cdots < a_D$

Matrix: "Total Positivity", "Positive Grassmannian"

Vectors, vertices of "Cyclic Polytope" = "$k=1$ Amplituhedron"

Know all Facets!

$$\langle A\, V_i V_{i+1} V_j V_{j+1} \cdots V_k V_{k+1} \rangle \geq 0$$

Surprise: Hidden Total Positivity In

$$\frac{P_n}{m_n^2 - s}, \quad G_J[\cos\theta]$$

Locality + Unitarity

s-channel Hankel : $\mathcal{A}(s) = \sum_{M^2} \frac{p(M^2)}{M^2 - s} = \sum_n a_n s^n$

$$* \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} \longrightarrow \mathbb{A}_{ij} = a_{i+j} = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots \\ a_1 & a_2 & a_3 & \cdots \\ a_2 & a_3 & a_4 & \cdots \end{pmatrix}$$

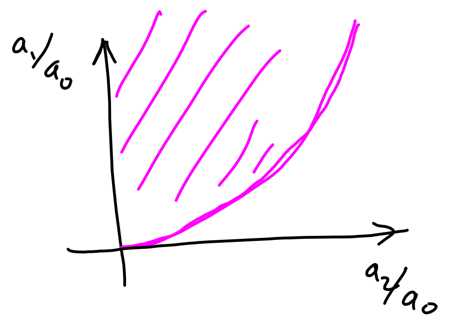
Positivity of M^2, $p(M^2)$
$\Updownarrow$
All minors of $\mathbb{A}$ are positive

$\Big\{$ * If the gap M_* is also given, working in $M_* \to 1$ units,

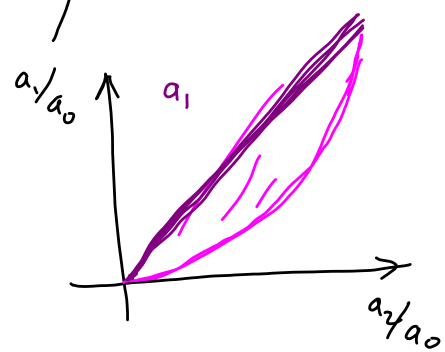
$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix}, \begin{pmatrix} a_1 - a_2 \\ a_2 - a_3 \\ a_3 - a_4 \\ \vdots \end{pmatrix}, \begin{pmatrix} (a_2 - a_3) - (a_3 - a_4) \\ (a_3 - a_4) - (a_4 - a_5) \\ \vdots \end{pmatrix}$$

all have positive Hankel matrices $\Big\}$

ex :



$a_2 a_0 - a_1^2 > 0$, $a_{2,1,0} > 0$



Also $\frac{a_1}{a_0} - \frac{M_*^2 a_2}{a_0} > 0$

Gegenbauer Polytope

s-channel ex: $$\sum_{M^2} \frac{P_J(M^2)\; G_J\left[1+\frac{2t}{M^2}\right]}{M^2-s} = \sum_D \sum_{J,q=0}^{D} P_{J,D}\, s^{D-q} t^q\, G_J^{[q]}(1).$$

So, $\begin{pmatrix} a_{D,0} \\ a_{D,1} \\ \vdots \\ a_{D,D} \end{pmatrix}$ lie in convex hull of $\begin{pmatrix} G_J^{(0)} \\ G_J^{(1)} \\ \vdots \\ G_J^{(D)} \end{pmatrix}$ = cyclic polytope!

* Reason is a hidden positivity of $\mathbb{G}_{Jq} = G_J^{(q)}(1)$:

$$\mathbb{G} = \begin{pmatrix} G_0^{(0)} & G_1^{(0)} & G_2^{(0)} \\ \vdots & G_1^{(1)} & G_2^{(1)} \\ & \vdots & G_2^{(2)} \\ & & \vdots \end{pmatrix}$$ has all minors positive!

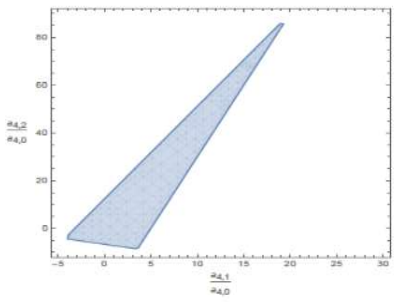
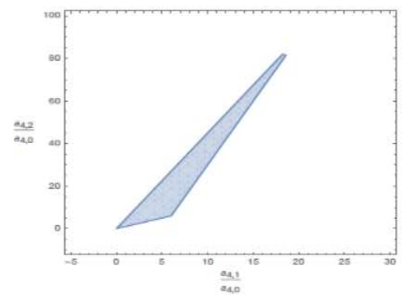
* (Equivalent to a result known since 60's, true for any orthogonal polynomials wrt a positive measure)

* But there is even more hidden Positivity! [+ useful for EFThedron]

configuration, where it's identical helicity in the s-channel, the amplitude for the D^8F^4 and D^8R^4 operator takes the form:

$$\langle 12\rangle^{2h}[34]^{2h}(a_{4,0}s^4+a_{4,1}s^3t+a_{4,2}s^2t^2\cdots)\,. \tag{1.10}$$

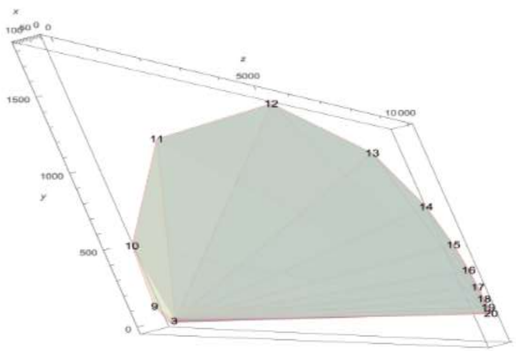
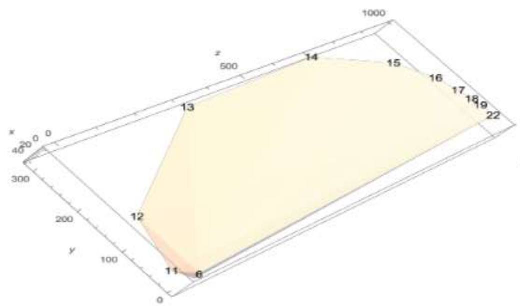
where $h=1,2$ for photon and graviton respectively. The allowed region for $\frac{a_{4,1}}{a_{4,0}}$, $\frac{a_{4,2}}{a_{4,0}}$ is given as:

photon  graviton 

Note that the allowed region are bounded. Considering instead $(+,-,-,+)$ helicity configuration, the $D^{16}F^4$ and $D^{16}R^4$ operator leads to the amplitude

$$\langle 23\rangle^{2h}[14]^{2h}(a_{8,0}z^8+a_{8,2}z^6t^2+a_{8,4}z^4t^4+a_{8,2}z^2t^6+\cdots)\,. \tag{1.11}$$

where $z=\frac{t}{2}+s$ and the even dependence on z is reflecting the $2\leftrightarrow 3$ symmetry of the helicity configuration. The coefficients $\frac{a_{8,2}}{a_{8,0}}$, $\frac{a_{8,4}}{a_{8,0}}$, $\frac{a_{8,6}}{a_{8,0}}$ are bounded as:

photon  graviton 

It is also important to note that, while the EFThedorn places extremely constraints on the effective field theory expansion, sensible effective field theories do not appear to populate the entire region allowed by the EFThedron, but cluster close to its boundaries. The reason is likely that the physical constraints we have imposed, while clearly necessary, are still not

Simpler Cousin of Amplituhedron

$$a_{D,q} = C_{D,s}\, G_{s,q}$$

Positive

Non-linear Hankel Positivity

Simpler Cousin of Amplituhedron

$$a_{\alpha I} = C_{\alpha, a} \, Z_{a, I}$$

Positive

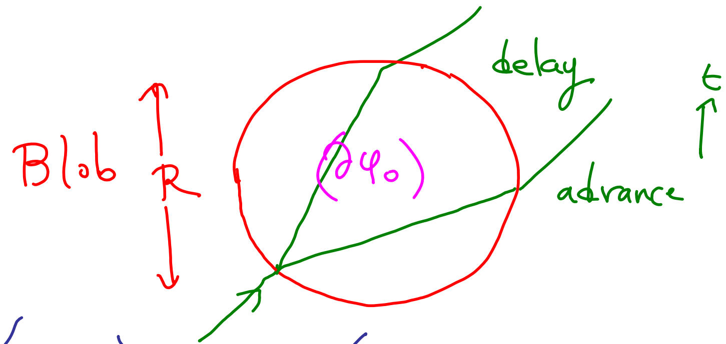
Non-linear Grassmannian Positivity

I. Gelfand ~ '90s: "You Polytope people are doing trivial things. You should generalize polytopes into Grassmannians!"

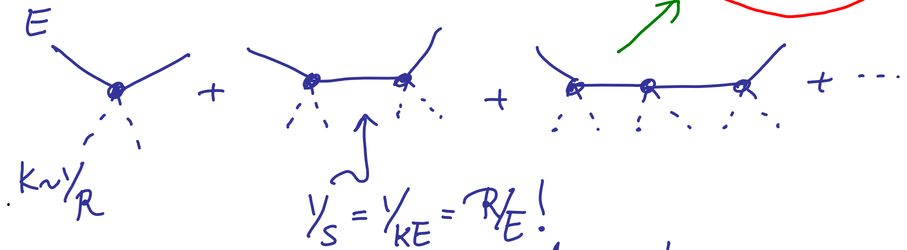
Polytopes $\longrightarrow$ EFThedron $\longrightarrow$ Amplituhedra

Comments

(1) Causality in more detail:



$$\langle B,E|S|B,E\rangle =$$



$$\rightarrow \sum_m \binom{N}{m}\left(iA(s{=}kE)\frac{k}{E}\right)^m \longrightarrow e^{i\frac{AkN}{E}} = e^{i\delta(E)=\frac{A(s)}{s}(\partial\varphi_0)^2 k^2}.$$

Time delay $\Longrightarrow \dfrac{\partial}{\partial E}\delta(E)<0 \Longrightarrow \boxed{\dfrac{\partial}{\partial s}\left(\dfrac{-A(s)}{s}\right)>0}$

This follows from dispersive representation (for $A(s=0)\geq 0$, tacit above)

$$-A(s) = -A(s=0) + \int dM^2\, p(M^2)\left[\frac{1}{M^2-s}+\frac{1}{M^2+s}-\frac{2}{M^2}\right]$$

$$\partial/\partial s\left[\left(\frac{1}{M^2-s}+\frac{1}{M^2+s}-\frac{2}{M^2}\right)\frac{1}{s}\right] = \frac{2(M^2+s^2)}{M^4(M^2-s)^2} > 0\,!$$

(2) A big missing ingredient in all these arguments, is that by using fixed-t dispersion relations, we can't input crucial constraints of UV completion — softness of HE, fixed angle scattering.

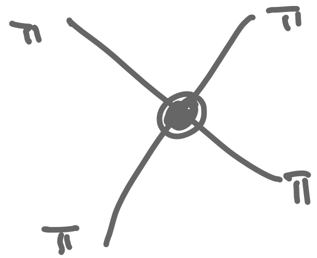
Among other things — in all sensible UV examples, the contribution to the discA from higher spins is suppressed for large spin — but this is not reflected in any of our analysis....

Strings + IR Challenge

of UV Completion

Q: Why is gravity, the most obvious force of daily macro life, hardest to UV complete, while much more hidden weak interactions were discovered + UV completed in ~ 70 years?

A: Another Aspect of UV-IR!



$$\mathcal{A}^{LE} \sim -\frac{1}{f^2}(s+t)$$

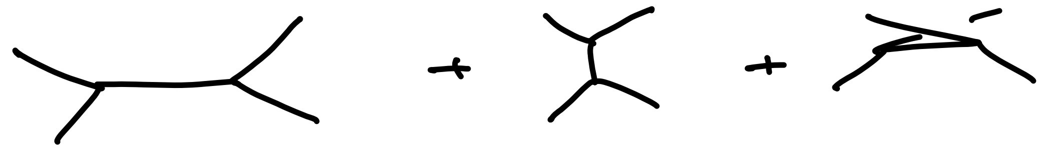
$\Big\downarrow$ UV complete

$$\mathcal{A} \sim -\frac{1}{f^2}\left(\frac{s}{1-s/M^2} + \frac{t}{1-t/M^2}\right)$$

Correct sign residue

Higgs Pole

TRIVIAL TREE UV COMPLETION

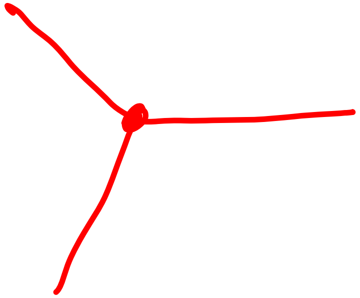


$$\mathcal{M} \sim g^2\left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right)$$

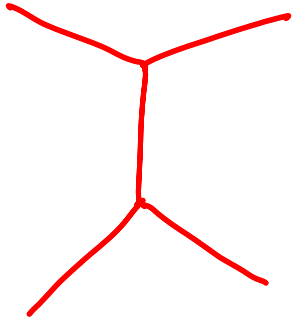
$$g^2\left(\frac{1}{s(1-s/M^2)} + \frac{1}{t(1-t/M^2)} + \frac{1}{u(1-u/M^2)}\right)$$

No!

Massless Residues $> 0 \overset{UV}{\underset{improv}{\Longrightarrow}}$ Some Massive Residues Must Be Negative!



$\Longrightarrow$



IR Poles

3 pt Amp.

$\Longrightarrow$ Impossible to Tree UV complete with finite # of particles/poles!

Need INFINITE TOWER of MASSES + SPINS

Bottom-Up Tripping Over Strings

* $$\mathcal{M}^{\text{Grav}}_{1^+2^-3^+4^-} = \frac{G_N \langle 24\rangle^4 [13]^4}{s\,t\,u} \longrightarrow \left(\quad\right) \cdot \frac{N}{\prod (s-m_i^2)(t-m_i^2)(u-m_i^2)}$$

* $N(s,t,u)$ should have <u>zeroes</u> to kill $\frac{1}{(t-m_i^2)}\frac{1}{(-m_j^2-m_i^2-t)}$ when $s \to m_j^2$.

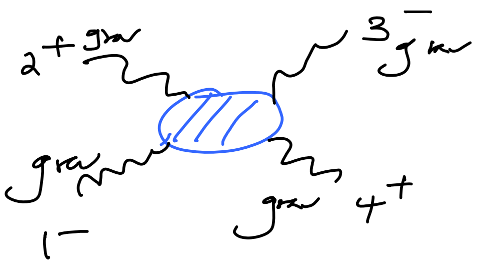
* Minimal Ansatz $\mathcal{N} = \prod (s+r_j)(t+r_j)(u+r_j)$

$\Longrightarrow$ $\{r_i\}$ should contain $\{m_i^2, m_i^2+m_j^2\}$

$\Longrightarrow$ $\{m_i^2\} = M_s^2\, i$ Linear spectrum!

* $$\mathcal{M} = G_N \langle 24\rangle^4 [13]^4 \frac{\prod_{j=1}^{\infty}(s+j)(t+j)(u+j)}{\prod_{j=0}^{\infty}(s-j)(t-j)(u-j)}$$

$$= G_N \langle 24\rangle^4 [13]^4 \frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$ Virasoro-Shapiro



$$A = g^2 \langle 13\rangle^4 [24]^4 \frac{\Gamma(-s)\,\Gamma(-t)\,\Gamma(-u)}{\Gamma(1+s)\,\Gamma(1+t)\,\Gamma(1+u)}$$

Q: Is this "unique" weakly coupled (only poles) UV completion of gravity?

Real Miracle: Positivity/Unitarity for Open Superstring

$$\prod_{i=1}^{N-1}\left[\cos\theta - \frac{(N-2i)}{N}\right] = \sum_{s} c_s\, G_s^{(d)}[\cos\theta], \quad d \leq 9$$
$$c_s \geq 0$$

Note already @ $N=3$:

$$\left[\cos\theta - \frac{1}{3}\right]\left[\cos\theta + \frac{1}{3}\right] = \left(\cos^2\theta - \frac{1}{9}\right) = \underbrace{\left(\cos^2\theta - \frac{1}{d}\right)}_{\text{spin 2}} + \underbrace{\left(\frac{1}{d} - \frac{1}{9}\right)}_{\text{spin 0}}$$

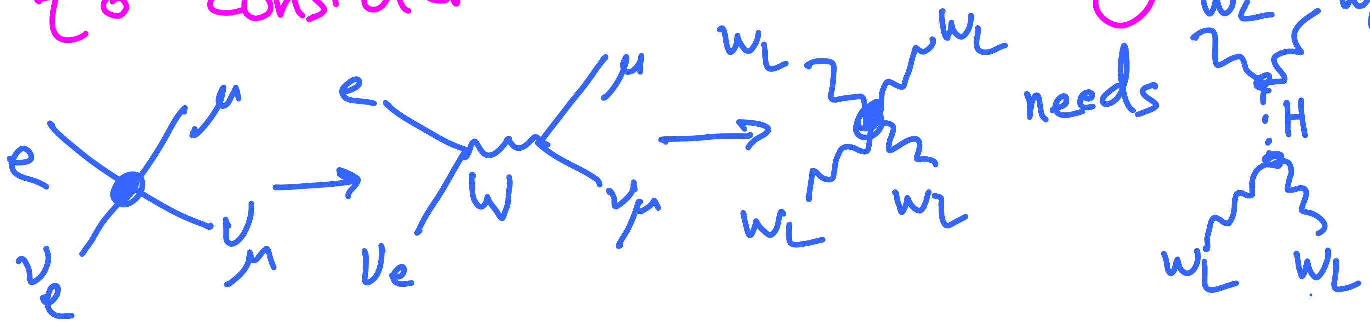
$\Longrightarrow$ spin 0 ghost unless $d \leq 9$, critical dim!

A: At least only checking consistency of massless scattering, No:

e.g. $$\mathcal{A} = \langle 13\rangle^4 [24]^4 \left(\frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} + \epsilon \frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(2+s)\Gamma(2+t)\Gamma(2+u)} + \cdots \right)$$

for $1 \geq \epsilon \geq 0$

Need to consider massive consistency too

{ c.f 

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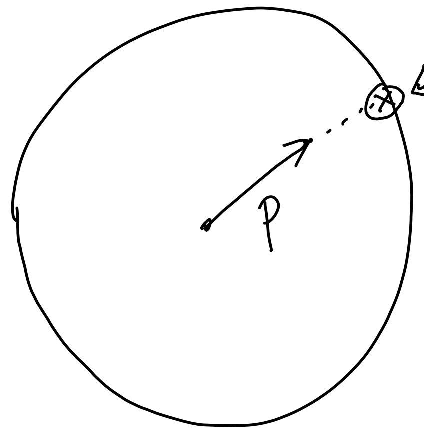
Celestial Amplitudes

and the

Anti-Wilsonian Paradigm

Null Momenta $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$



z on (Riemann) Celestial Sphere

$\updownarrow$

$$\lambda_\alpha = \sqrt{2\omega}\begin{pmatrix} 1 \\ z \end{pmatrix}, \quad \tilde{\lambda}_{\dot{\alpha}} = \pm\sqrt{2\omega}\begin{pmatrix} 1 \\ \bar{z} \end{pmatrix}$$

Lorentz $SL(2,\mathbb{C})$:

$$\sqrt{\omega}\begin{pmatrix} 1 \\ z \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\sqrt{\omega}\begin{pmatrix} 1 \\ z \end{pmatrix} = e^{i\theta}\sqrt{\omega'}\begin{pmatrix} 1 \\ z' \end{pmatrix},$$

$$z' = \frac{az+b}{cz+d}, \quad \omega' = |cz+d|^2\omega, \quad e^{2i\theta} = \frac{cz+d}{\bar{c}\bar{z}+\bar{d}} \quad \text{[little group]}$$

Natural to consider not momentum eigenstates,

but Boost Eigenstates:

$$|\Delta, z\rangle \equiv \int_0^\infty \frac{d\omega}{\omega}\, \omega^{\Delta}\, |\omega, z\rangle$$

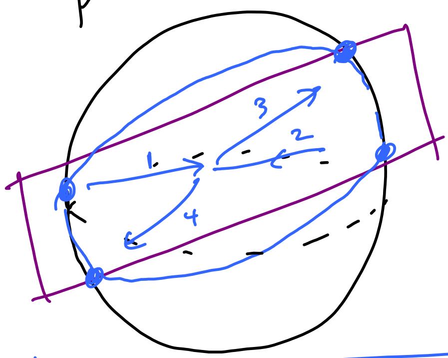
$$|\Delta, z\rangle \xrightarrow{\text{Lorentz}} |cz+d|^{2\Delta} \left|\Delta, \frac{az+b}{cz+d}\right\rangle$$

(Eigenstates of Boosts in z-direction)

Scattering Amplitude for Boost Eigenstates

$$\mathcal{A}(\Delta_i, z_i) = \int_0^\infty \frac{d\omega_i}{\omega_i}\, \omega_i^{\Delta_i}\, \mathcal{A}(\omega_i, z_i) \quad \text{[Mellin transform]}$$

4 pts : $SL(2) \Longrightarrow$ dependence only on cross-ratio $z = \frac{z_{13} z_{24}}{z_{12} z_{34}}$



Momentum Conservation $\longrightarrow$ 1,2,3,4 co-circular

$\longrightarrow \delta(z - \bar{z})$

$z = -\frac{1}{2}(1+\cos\theta)$, $\omega = E_{CM}$

Fixed Angle Amp

$\Delta_1 + \dots + \Delta_4$

$$\boxed{\mathcal{A}(\omega_i, z_i) = (\text{hel. dep})\, \delta(z - \bar{z}) \times \int_0^\infty \frac{d\omega}{\omega}\, \omega^{\beta}\, M(\omega, z)}$$

....The Study of Celestial Amplitudes has already clarified, unified + revealed many aspect of IR/"Soft" Amplitudes ....

Boost Eigenstate Scattering is Perfect Probe of UV-IR

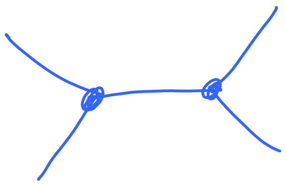
* Maximally violates Wilsonian intuition — we are scattering states with arbitrarily high energies!

* "EFT" amplitudes don't make sense — must deal with UV completion issue from the get-go — this is good!

$\left\{ \text{e.g. } A(\omega) \sim \omega^p, \ \int_0^\infty \frac{d\omega}{\omega}\, \omega^{(\beta+p)} \text{ ill-defined for any } p>0 \right\}$

UV Completion $\longleftrightarrow$ Analytic Structure in β-plane

Ex:



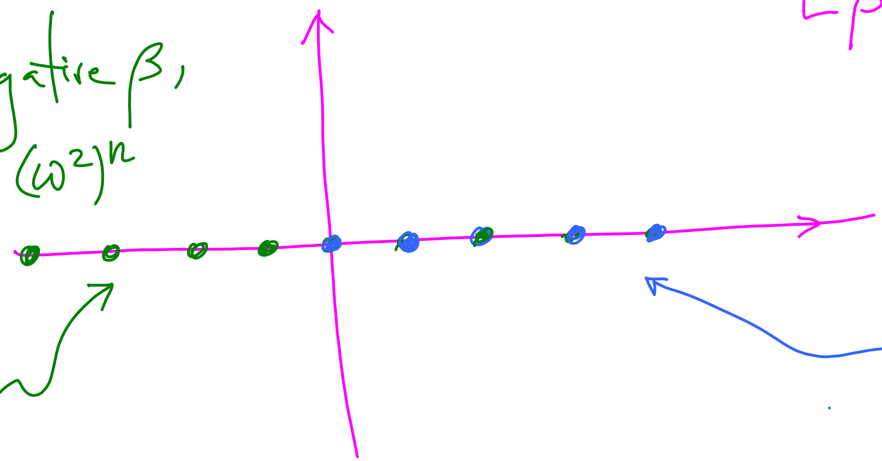
$$\mathcal{M} = -\frac{\lambda M^2}{s - M^2} \qquad (\lambda \equiv g^2/M^2)$$

$$A(\beta) = \int_0^\infty \frac{d\omega}{\omega}\, \omega^\beta \mathcal{M} = \frac{\lambda \pi M^\beta}{1 - e^{i\pi\beta}}$$

$L\beta$

Poles at negative β,

$$\mathcal{M}^{IR} = \sum a_n^{IR}(z)\,(\omega^2)^n \qquad \mathcal{M}^{UV} = \sum a_m^{UV}(z)\,\omega^{-2m}$$

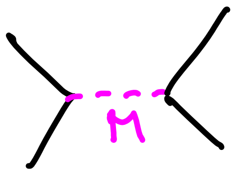


$$\frac{a_n^{IR}(z)}{\beta + 2n} \qquad \frac{a_m^{UV}(z)}{\beta - 2m}$$

Residues are coeff of high-dim ops!

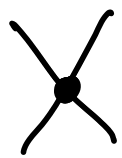
power-law fall off in UV

Anti-Wilsonian



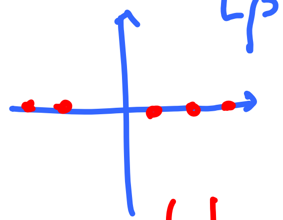
$$\mathcal{M}^{\text{full}} = -\frac{\lambda M^2}{s-M^2}$$

$$\Big\downarrow\ M \to \infty$$

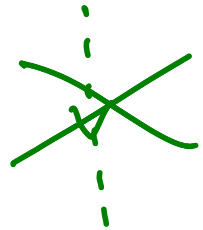


$$\mathcal{M}^{\text{EFT}} = \lambda$$

$$\mathcal{A}^{\text{full}}(\beta) = \frac{\lambda M^\beta}{1-e^{2i\pi\beta}}$$



Analytic



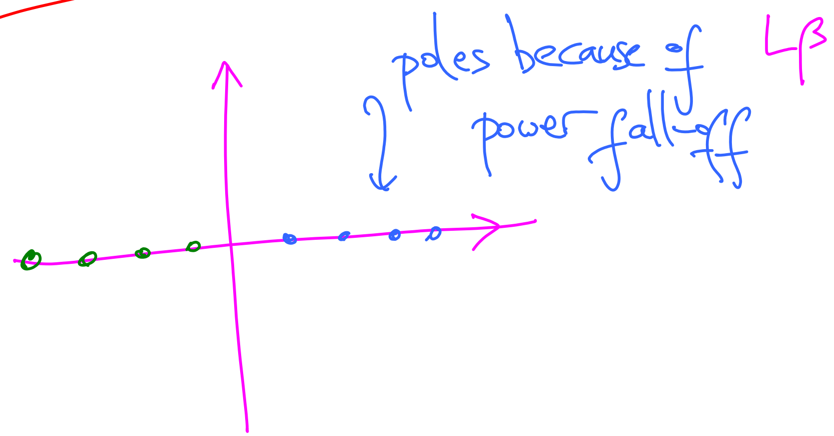
$$\mathcal{A}^{\text{EFT}}(\beta) = \delta(\beta)$$

NOT ANALYTIC

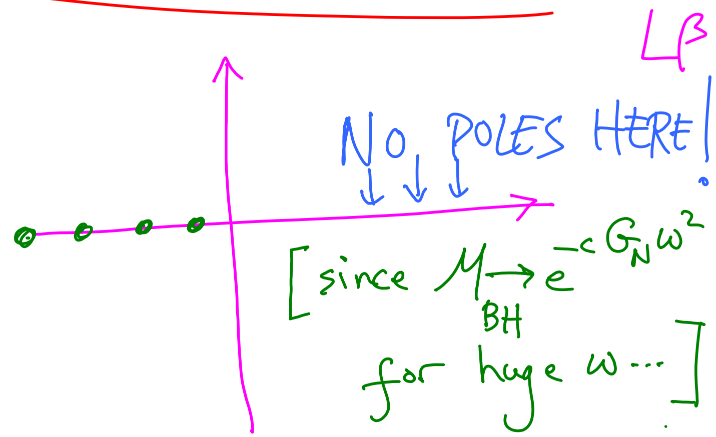
$$\int_0^{\omega_*} \frac{d\omega}{\omega}\, \omega^{\beta}\, \omega^{+2n} \longrightarrow \frac{1}{\beta+2n} \quad \text{IR}$$

$$\int_{\omega_*}^{\infty} \frac{d\omega}{\omega}\, \omega^{\beta}\, \omega^{-2m} \longrightarrow \frac{1}{\beta-2m} \quad \text{UV}$$

Radical Difference Between Quantum Gravity + Field Theory



Field Theory



Strings ⋯> Gravity

(Analog of absence of "Bulk Point Singularity" in AdS)

Log-running of higher-dim ops $\longrightarrow$ Higher order poles in β

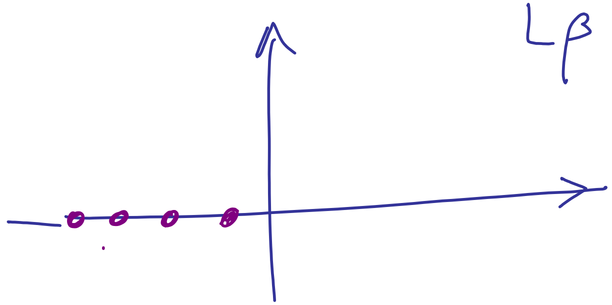
$$\mathcal{M} = \sum_{r \leq m,n} a_{n,m,r}(z)\, \omega^{2n} (G_N \omega^2)^m \log^r(\omega/\Lambda_{UV})$$

(massless loops in low-E thry)

$$\int_0^{\omega_*} \frac{d\omega}{\omega}\, \omega^a \log^k \omega \sim \frac{\partial^k}{\partial a^k} \int_0^{\omega_*} \frac{d\omega}{\omega}\, \omega^\beta \sim \frac{1}{a^{(1+k)}}$$

$$A(\beta \to -2N, z) = \sum_{r=0}^{(N-1)} \frac{1}{(\beta+2N)^r} \sum_n a_{n,m=N-n,r}(z)$$

Only sing are (multiple) poles in β:



Residues are (running) EFT coefficients!

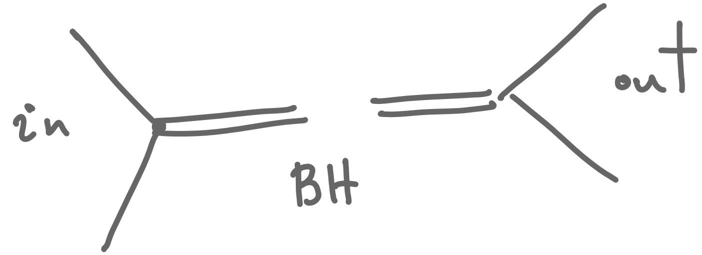
Large β-behavior

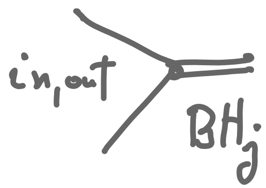
* $\beta \to \infty$, $\displaystyle\int_0^\infty \frac{d\omega}{\omega}\, \omega^\beta M(\omega, z)$ dominated by large ω.

In quantum gravity where $M(\omega \to \infty) \sim e^{-c G_N \omega^2}$ Dies Exponentially

$$\left| A(\beta \to \infty) \right|^2 \to G_N^{-\beta}\, \Gamma(\beta)$$ Grows Exponentially

Chaotic vs. Smooth



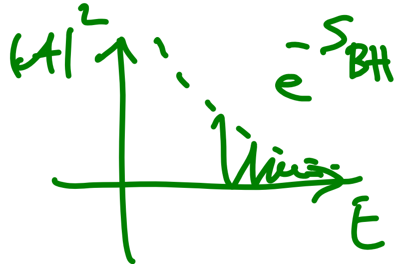


$$\sim e^{i\varphi^{(j)}_{in,out}}\, e^{-S_{BH}/2}$$

* $\sigma^{tot}_{BH} \sim \sum_j \left| {}_{in} \succ\!=_j \right|^2 \sim \underbrace{e^{+S_{BH}}}_{N_{BH}} \times \left(e^{-S_{BH}/2}\right)^2 \sim 1.$

* $M(in \to out) \sim \sum_j \underbrace{e^{i(\varphi^{(j)}_{in} - \varphi^{(j)}_{out})}}_{\text{Random Phases}}\, e^{-S_{BH}}$,

$\Longrightarrow |M(in \to out)|^2 \sim e^{-S_{BH}}$ but Chaotic

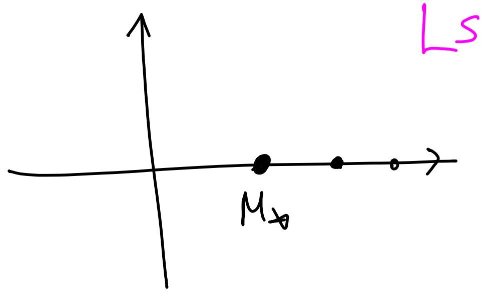


* But $A(\beta)$ is <u>averaging</u> over E + is smooth!

Large β-behavior

$\beta \to -\infty$: $\quad A(\beta) \longrightarrow \dfrac{M_*^{\beta}\,\beta^{-(1+q)}}{\left(1-e^{i\pi\beta}\right)}$

{ where M_* is the gap to the massive states



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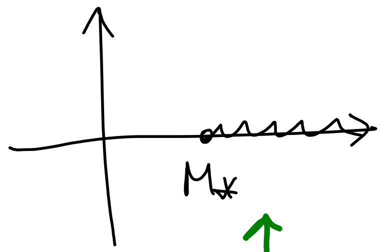
or



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$\sigma \sim (s-M_*)^q$ near threshold }

What determines $A(\beta, z)$?

* Poles in β $\longleftrightarrow$ EFT couplings,
  constrained by EFThedron.

* $\beta \to \infty$ prescribed by BH physics,
  Leading $\beta \to -\infty$ controlled by $\dfrac{M_*^{\beta}}{1-e^{\pi i \beta}}$ the gap

* So we know about poles + leading behavior @ ∞, ... what else is needed?

... Something more <u>is</u> needed, to reflect reasonable analyticity back in momentum space.

e.g. can shift $A(\beta) \to A(\beta) + c\mu^{\beta}$ with $\mu > M_*$.

$$\mathcal{M}(\omega) \to \mathcal{M}(\omega) + c\,\delta(s-\mu^2)$$

Horribly non-analytic in s!