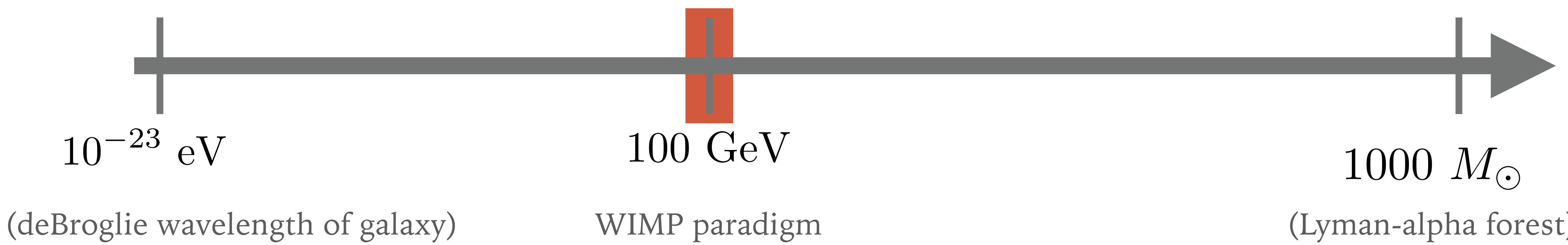
EFT OF DARK MATTER DIRECT DETECTION WITH COLLECTIVE EXCITATIONS

Based on Trickle, Zhang, KZ 2009.13534

Kathryn M. Zurek

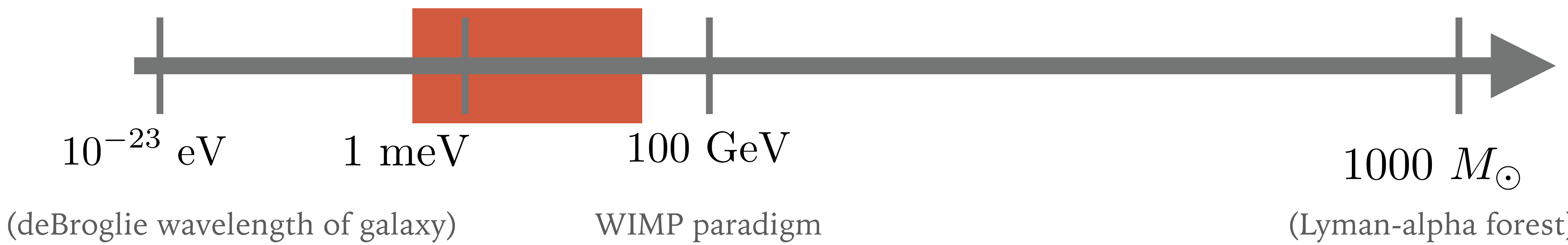
+ work with Hochberg, Pyle, Zhao, Lin, Knapen, Kahn, Lisanti, Coskuner, Mitridate

THE DARK MATTER PANORAMA



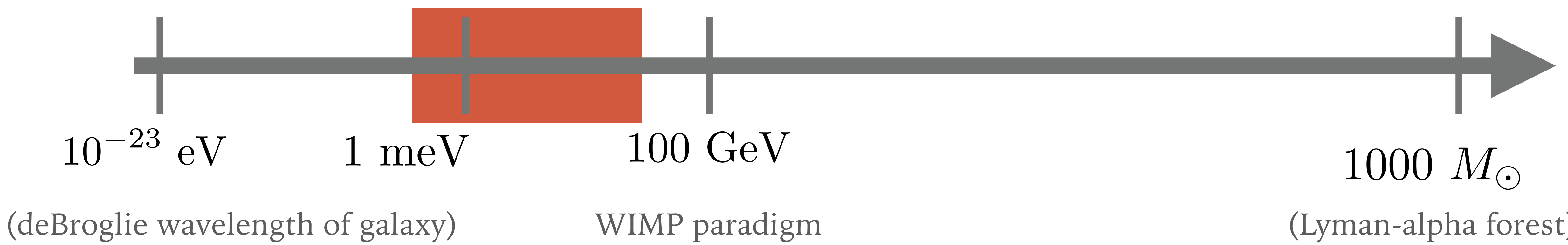
- From an observational standpoint, a wide range of dark matter masses are consistent with data.
- Focused on WIMP largely from arguments based on EFT

THE DARK MATTER PANORAMA

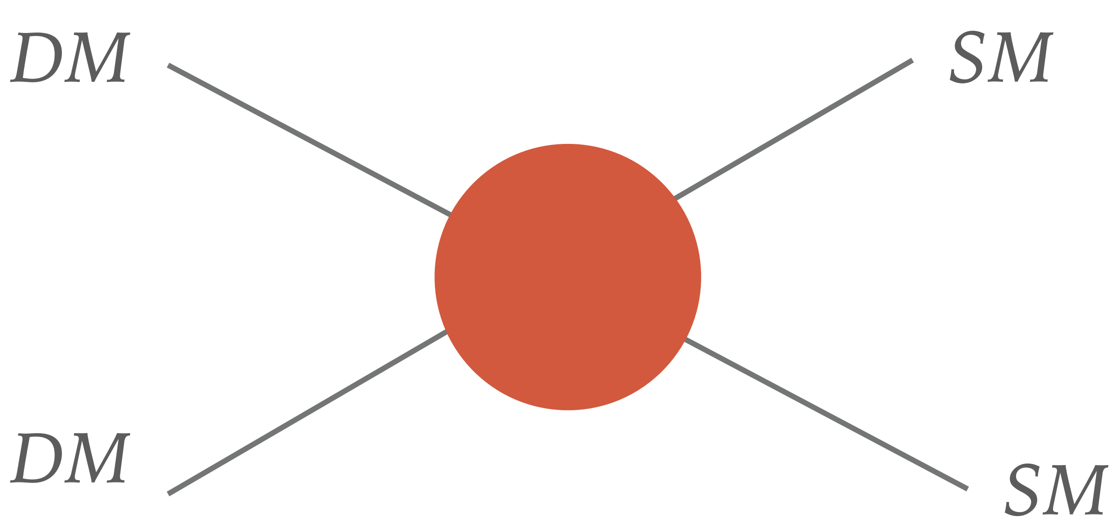


- From an observational standpoint, a wide range of dark matter masses are consistent with data.
- Our discussion will focus on extending the window of observability by 12 OOM in mass utilizing collective excitations in materials
- Why look there?

THE DARK MATTER PANORAMA



- Similar argument as to WIMP based on EFT reasoning
- Dark matter abundance is related to SM interactions

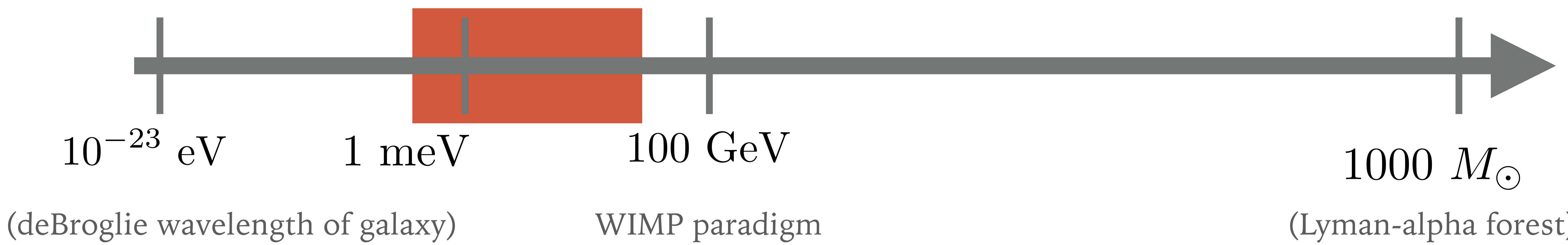


$$n_{DM}\langle\sigma v\rangle \simeq H(T_{\rm fo})$$

$$\rho_{DM} = \rho_{obs}$$

$$\langle\sigma v\rangle \simeq 3\times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$$

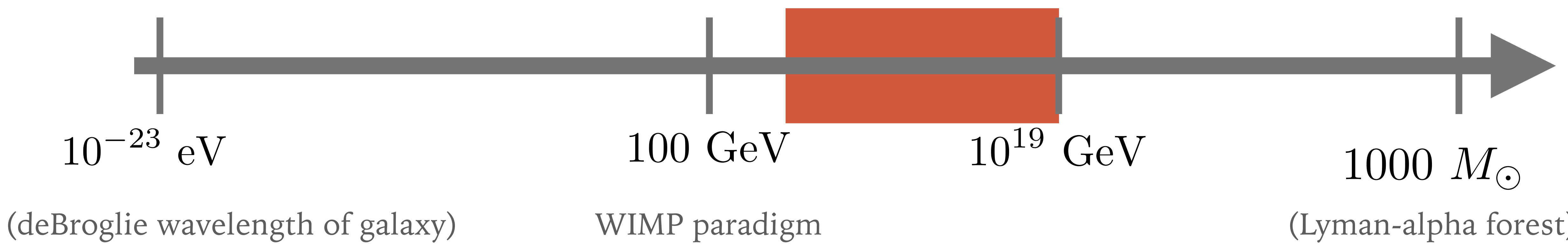
THE DARK MATTER PANORAMA



- Similar argument as to WIMP based on EFT reasoning
- Dark matter abundance is related to SM interactions

$$\sigma_{wk} v_{fo} \simeq \frac{g_{wk}^4 \mu_{XT}^2}{4\pi m_Z^4} \frac{c}{3} \simeq 10^{-24} \frac{\text{cm}^3}{\text{s}} \left(\frac{100 \text{ GeV}}{M} \right)^2$$

THE DARK MATTER PANORAMA



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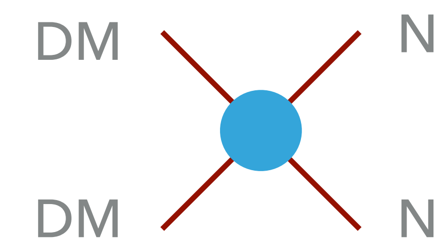
- Heavier dark matter: setting relic abundance through interactions with Standard Model is challenging (NB: exceptions)
- At heavier masses, detection through Standard Model interactions is (generally) not motivated by abundance

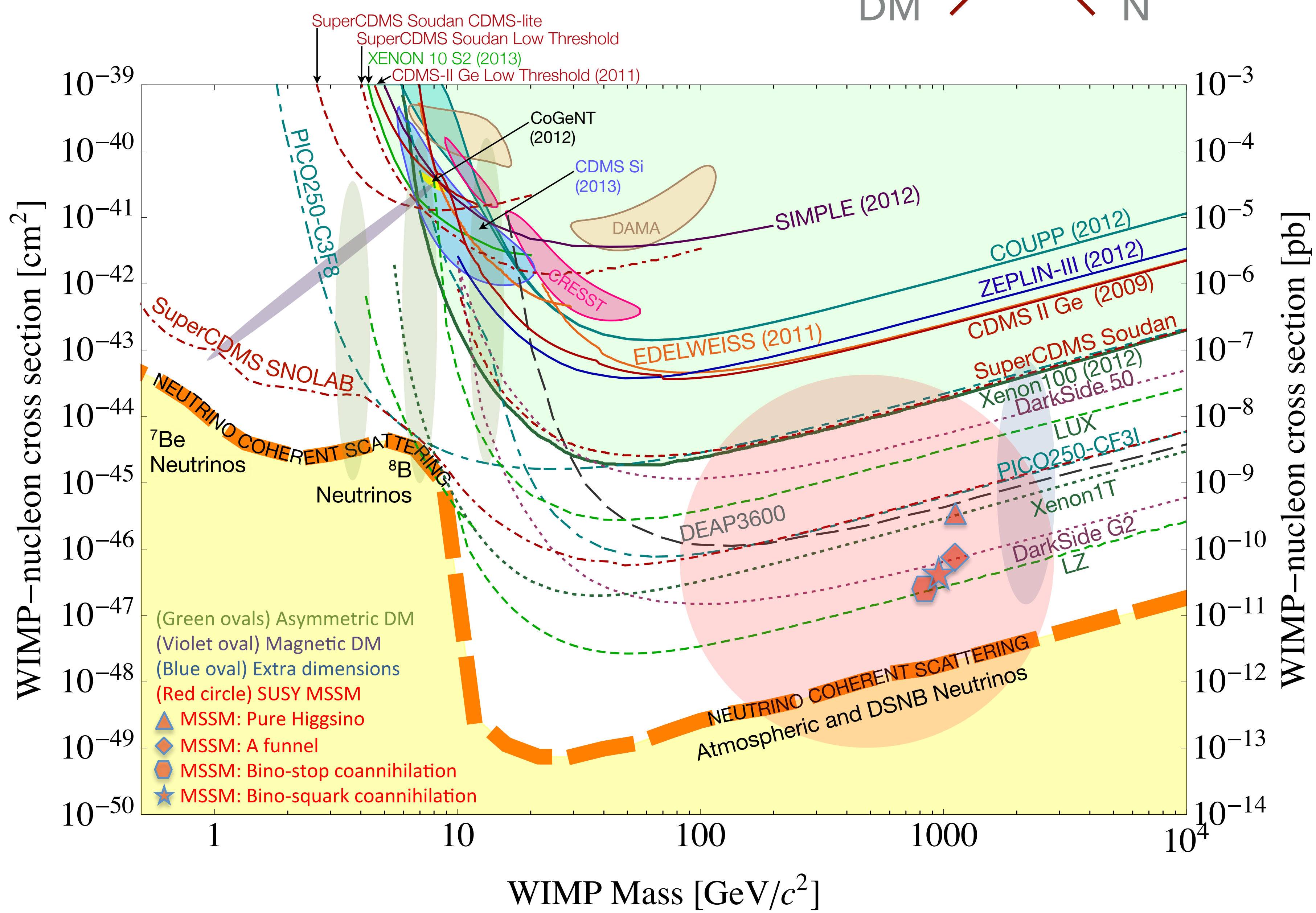
DETECTABLE INTERACTION RATES

- Direct detection searches accordingly focused on weak scale

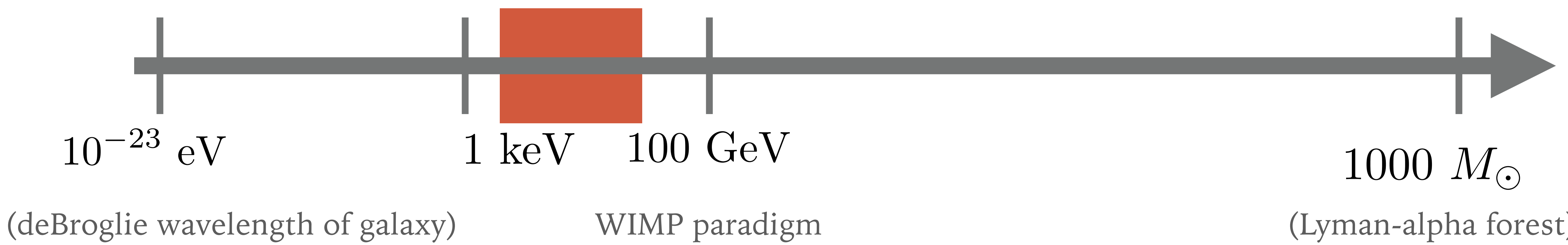


Z-boson interacting dark matter: ruled out

Higgs interacting dark matter: active target



DARK MATTER DETECTION: A FULL COURT PRESS



- Dark sector dynamics are complex and astrophysically relevant.

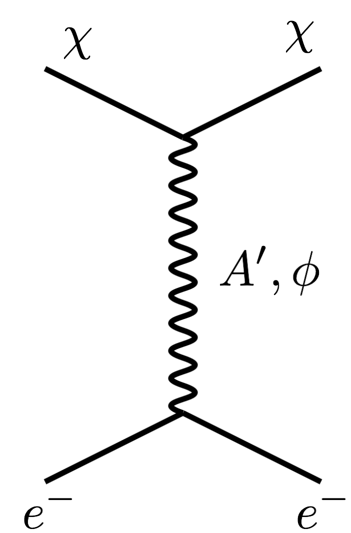
$$\sigma_{str} \simeq \frac{4\pi\alpha_s^2}{M^2} \simeq 10^{-24}\ \mathrm{cm}^2 \left(\frac{1\ \mathrm{GeV}}{M}\right)^2$$

- Abundance may still be set by (thermal) population from SM sector

$$\sigma_{wk} v_{fo} \simeq \frac{g_{wk}^4 \mu_{XT}^2}{4\pi m_Z^4} \frac{c}{3} \simeq 10^{-24} \frac{\mathrm{cm}^3}{\mathrm{s}} \left(\frac{100\ \mathrm{GeV}}{M}\right)^2$$

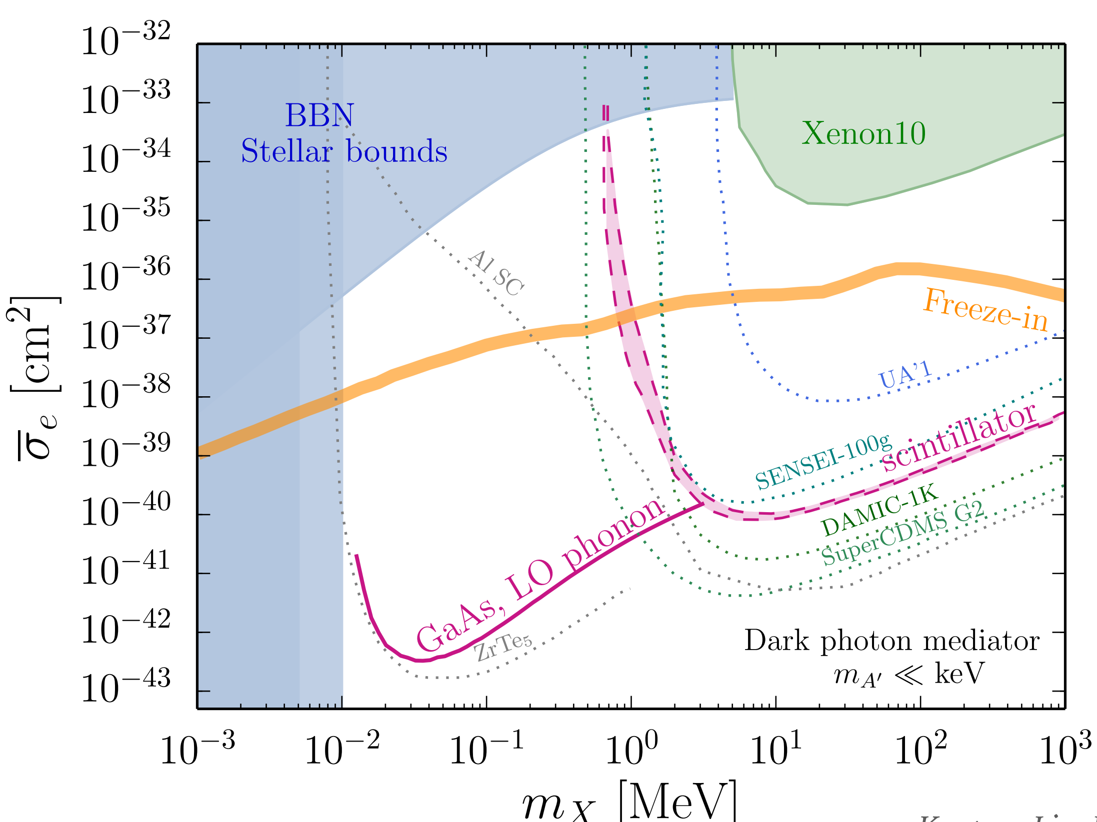
CROSSING SYMMETRY

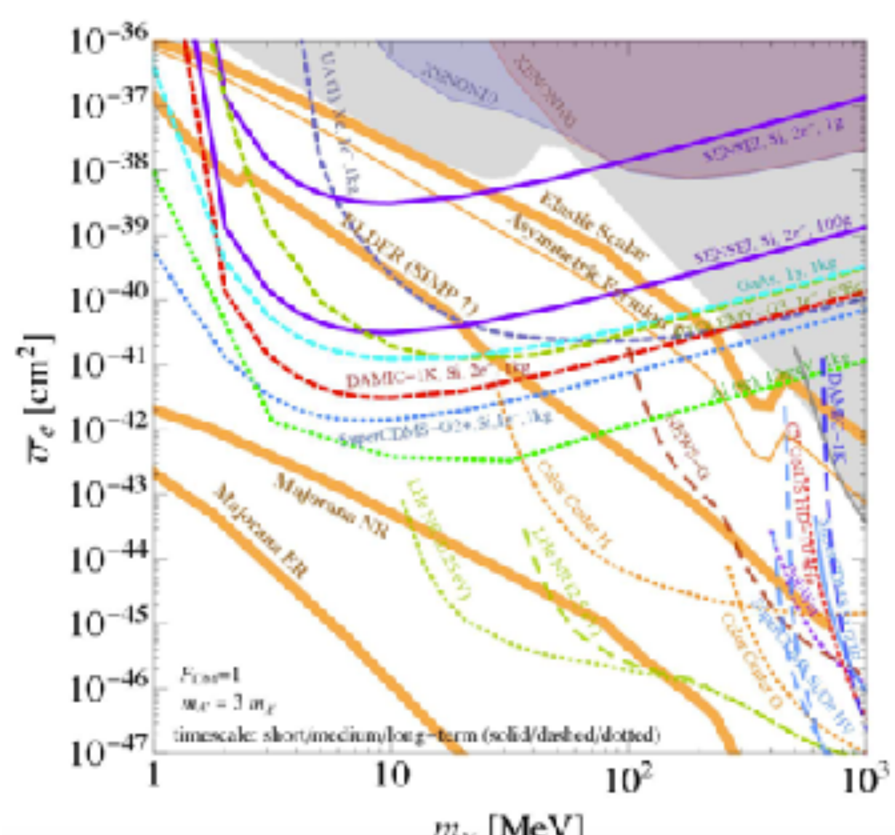
- Utilize DM Abundance and crossing symmetry as guide for interaction rates



Freeze-in

Asymmetric Dark Matter

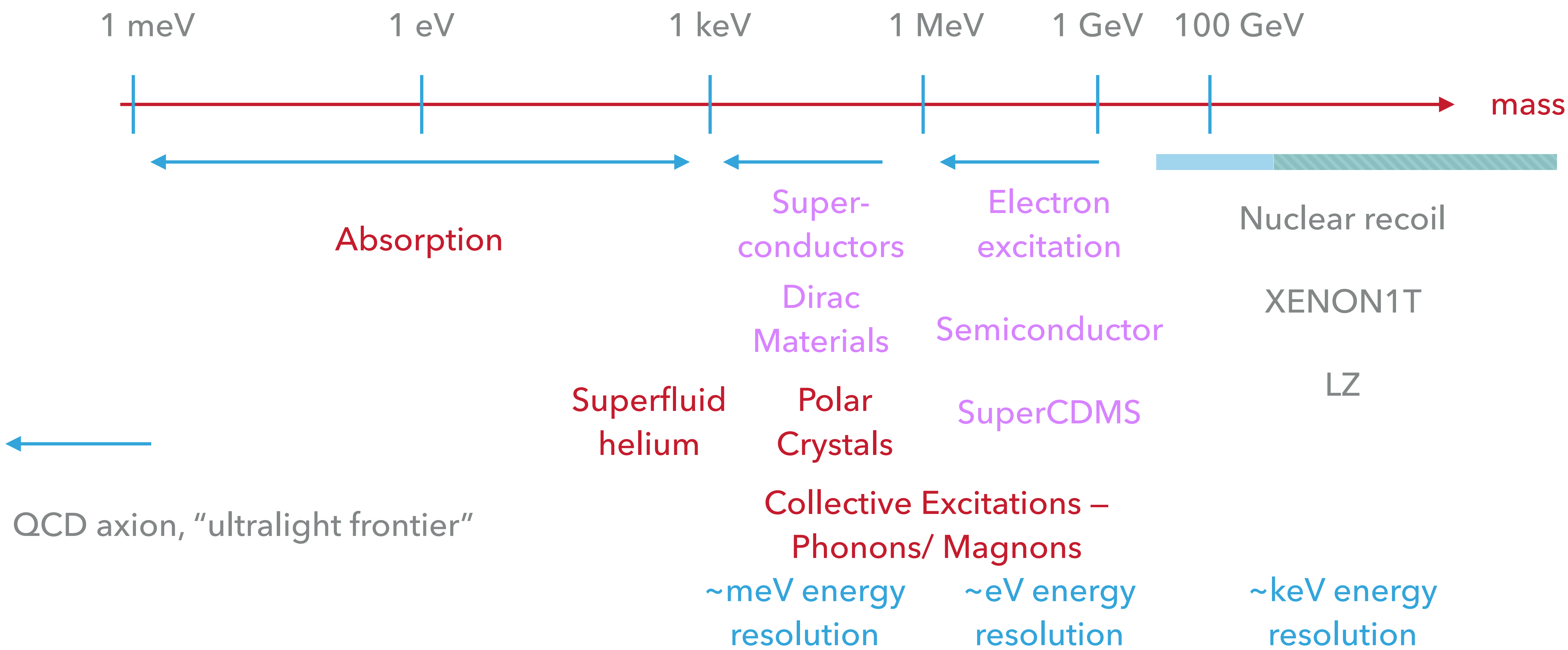




Knapen, Lin, Pyle KZ 1712.06598

US Cosmic Visions 1707.04591

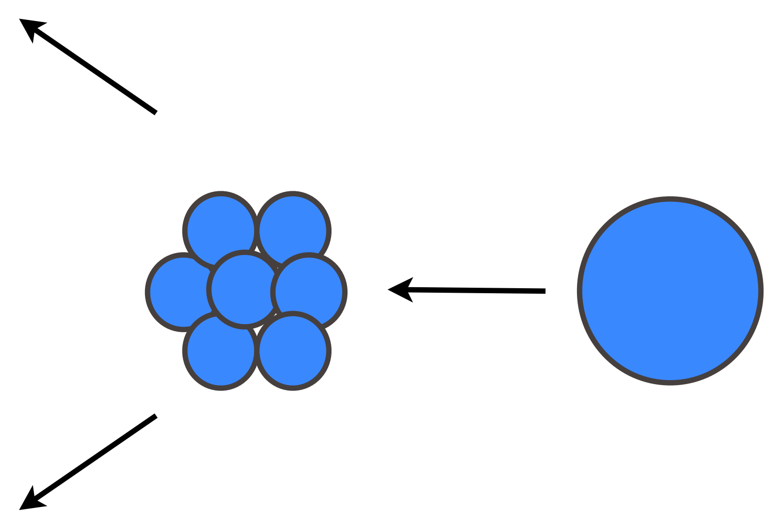
COLLECTIVE PHENOMENA IN MATERIALS



BEYOND BILLIARD BALL SCATTERING

- Nuclear recoil experiments; basis of enormous progress in direct detection

$q,\ E_D$



$v \sim 10^{-3} c$

$$v \sim 300\ \mathrm{km/s} \sim 10^{-3} c$$

$$E_D = \frac{q^2}{2 m_N} \qquad q_{\mathrm{max}} = 2 m_X v$$

LIGHTER TARGETS FOR LIGHTER DARK MATTER

$$E_D = \frac{q^2}{2m_e} \qquad q_{\max} = 2m_X v$$

- In insulators, like xenon

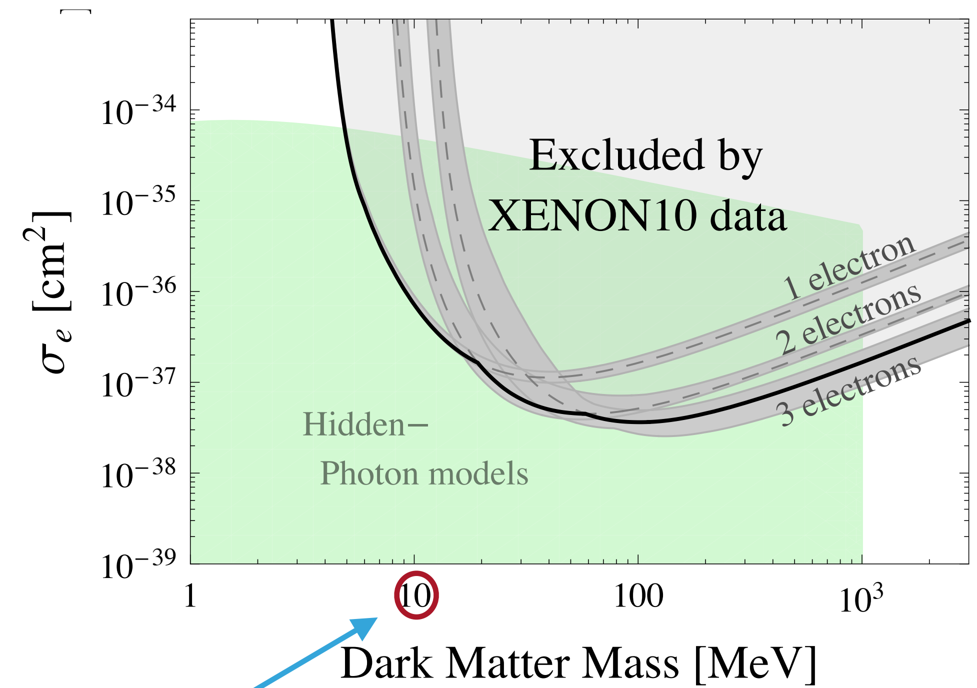
  Tightly bound; ionize for signal

Gap = DM Kinetic Energy

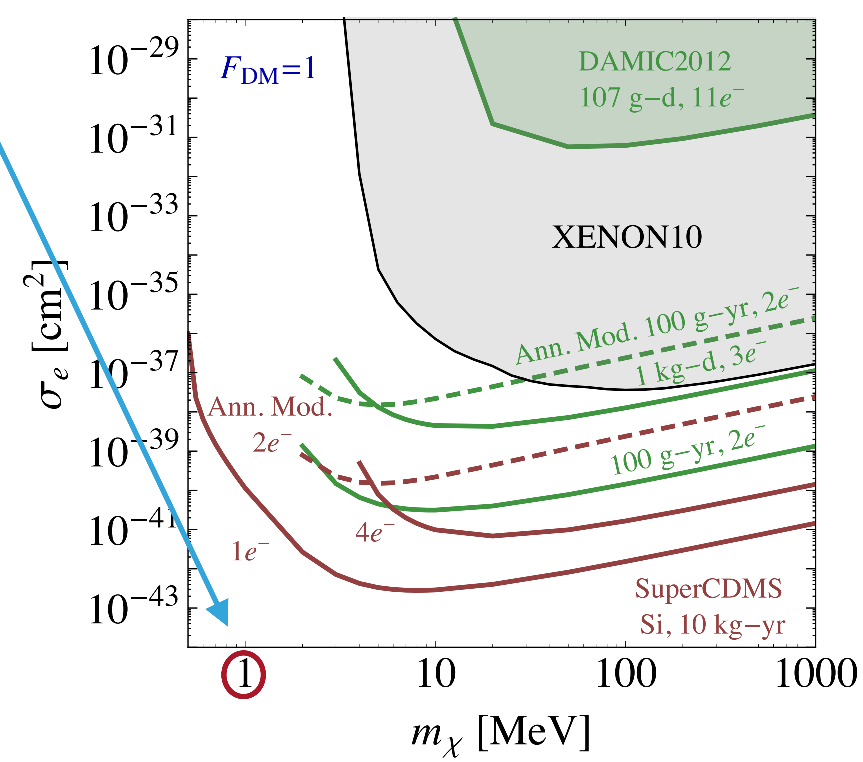
- In semi-conductors, like Ge, Si

  Excite electron to conduction band

P. Sorensen et al 1206.2644

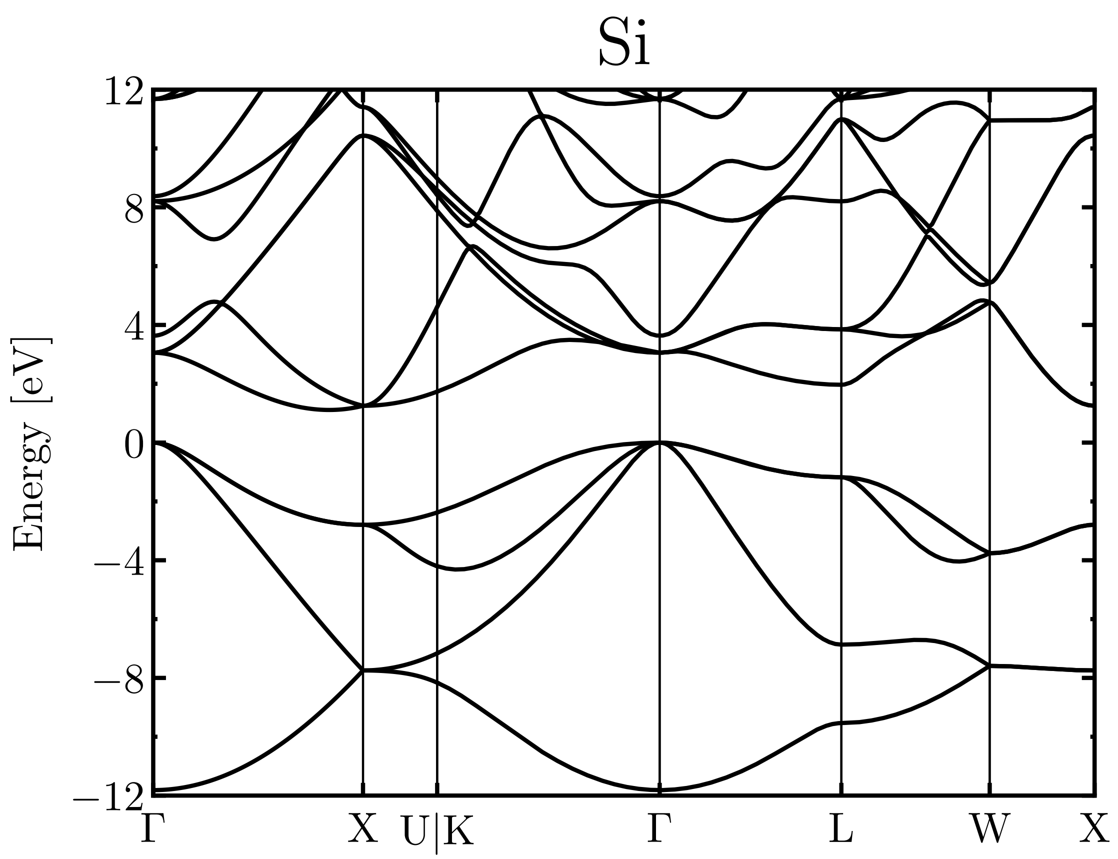


Essig et al 1509.01598

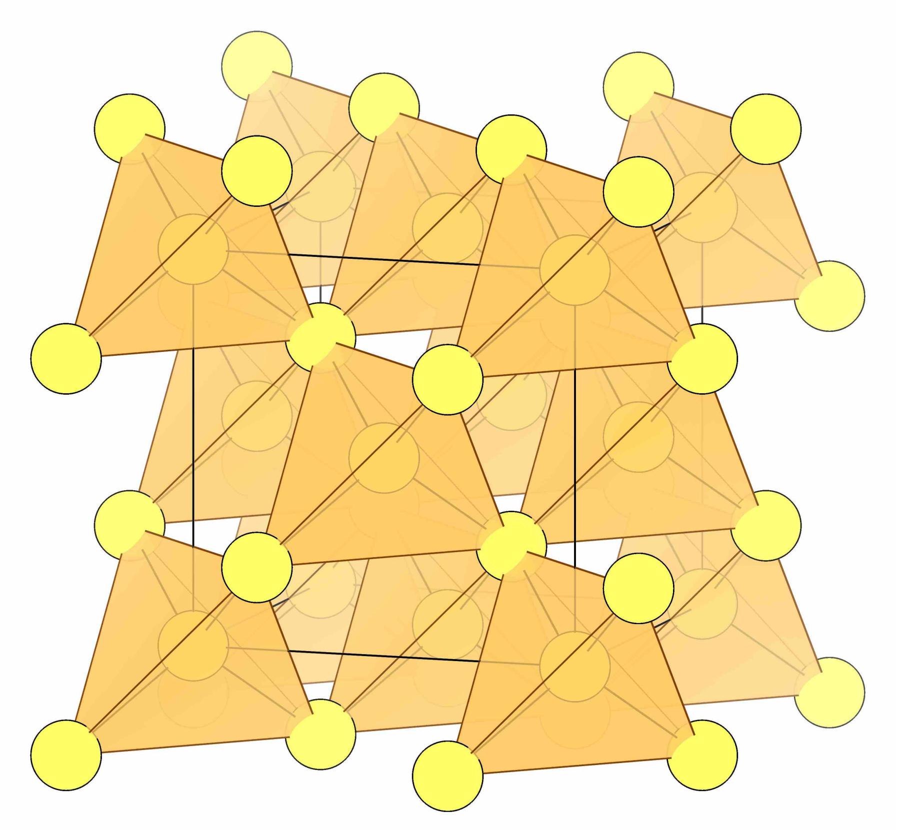


ELECTRONIC STATES IN MATERIALS

- Unless in a metal, electrons in material do not have free dispersions
- The omega-q relation (= dispersion) of the available states is extremely important for determining viability of target

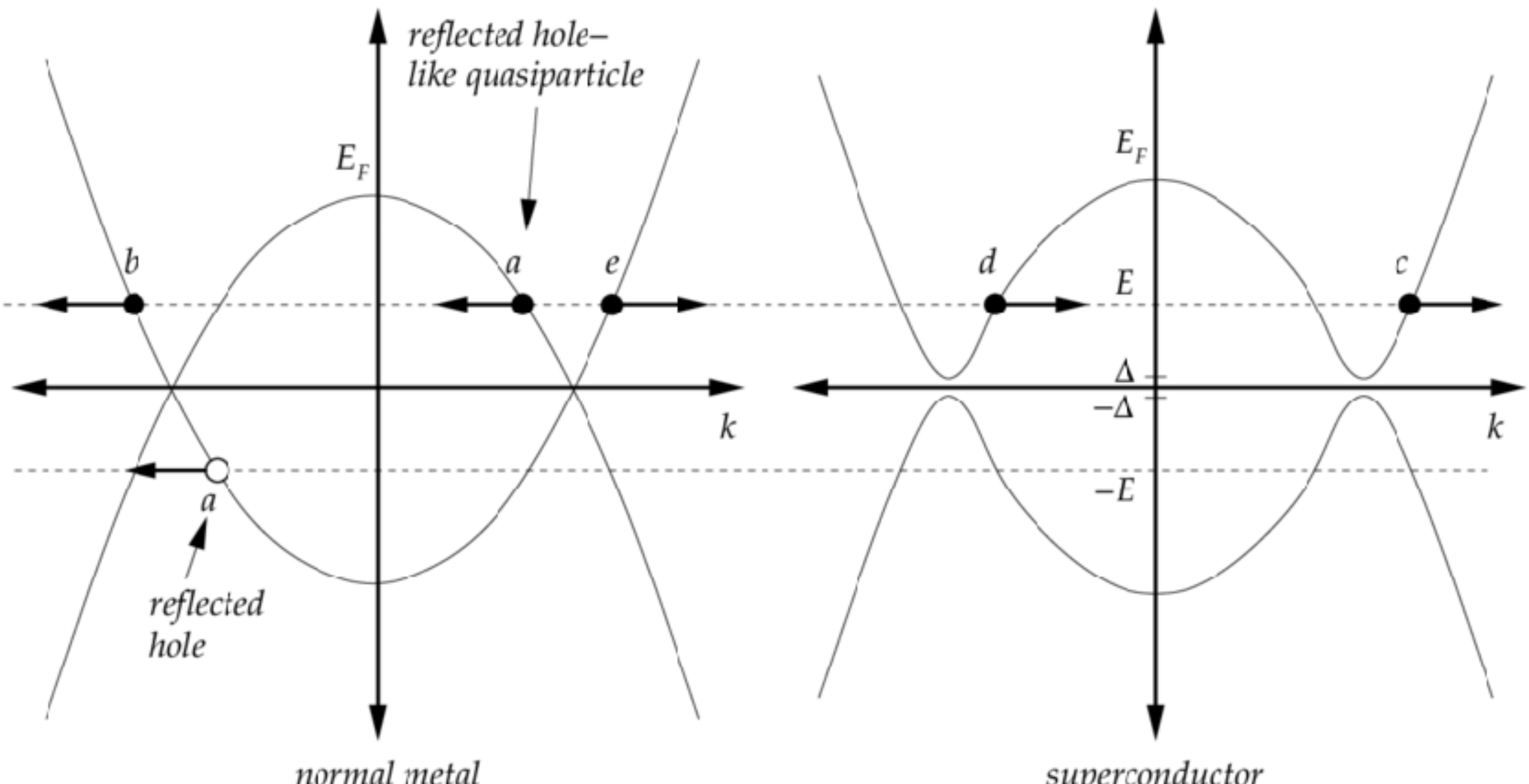


momentum (keV)



ELECTRONIC STRUCTURE IN MATERIALS

- Smaller gap materials are available to access lighter dark matter
- Simplest example is a superconductor — meV gap opens



EFFECTIVE COUPLING TO E-M CURRENT

Hochberg, Pyle, Zhao, KZ 1512.04533
Hochberg, Kahn, Lisanti, KZ et al 1708.08929

- Photon in medium is impacted by screening effects
- This is characterized by the polarization tensor, just like QED

$$J_\mu = -\Pi_{\mu\nu} A^\nu$$

$$\Pi_{\mu\nu} \equiv ie^2 \langle J^\mu_{\rm EM} J^\nu_{\rm EM} \rangle$$

$$\mathscr{L} \supset -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + eJ^\mu_{\rm EM}\left(\tilde{A}_\mu + \varepsilon A'_\mu\right) + g_{\rm D} J^\mu_{\rm DM} A'_\mu + \frac{m^2_{A'}}{2} A'^\mu A'_\mu + \frac{1}{2}\tilde{A}^\mu \Pi_{\mu\nu}\tilde{A}^\nu + \varepsilon \tilde{A}^\mu \Pi_{\mu\nu} A'^\nu$$

$$\mathscr{L} \supset \varepsilon e \frac{q^2}{q^2 - \Pi_{L,T}} A'^{T,L}_\mu J^\mu_{\rm EM}$$

Coskuner, Mitridate, Olivares, KZ 1909.09170

EFFECTIVE COUPLING TO E-M CURRENT

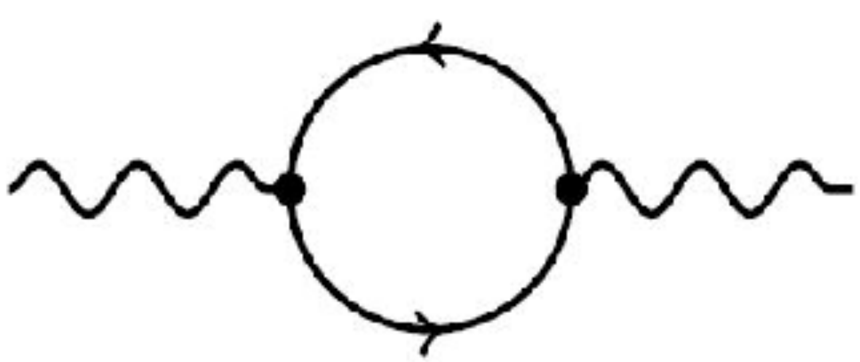
- Polarization tensor is normally recast in terms of dielectric function (you can do this with Maxwell equations)

$$\Pi_{ij} = -i\omega\sigma_{ij}$$
$$\Pi_{i0} = i\sigma_{ij}q^j \, .$$

$$\sigma_{ij} = i\omega(\delta_{ij} - \boldsymbol{\epsilon}_{ij})$$

- Dielectric can be calculated with electron wavefunctions (e.g. Lindhard formula)

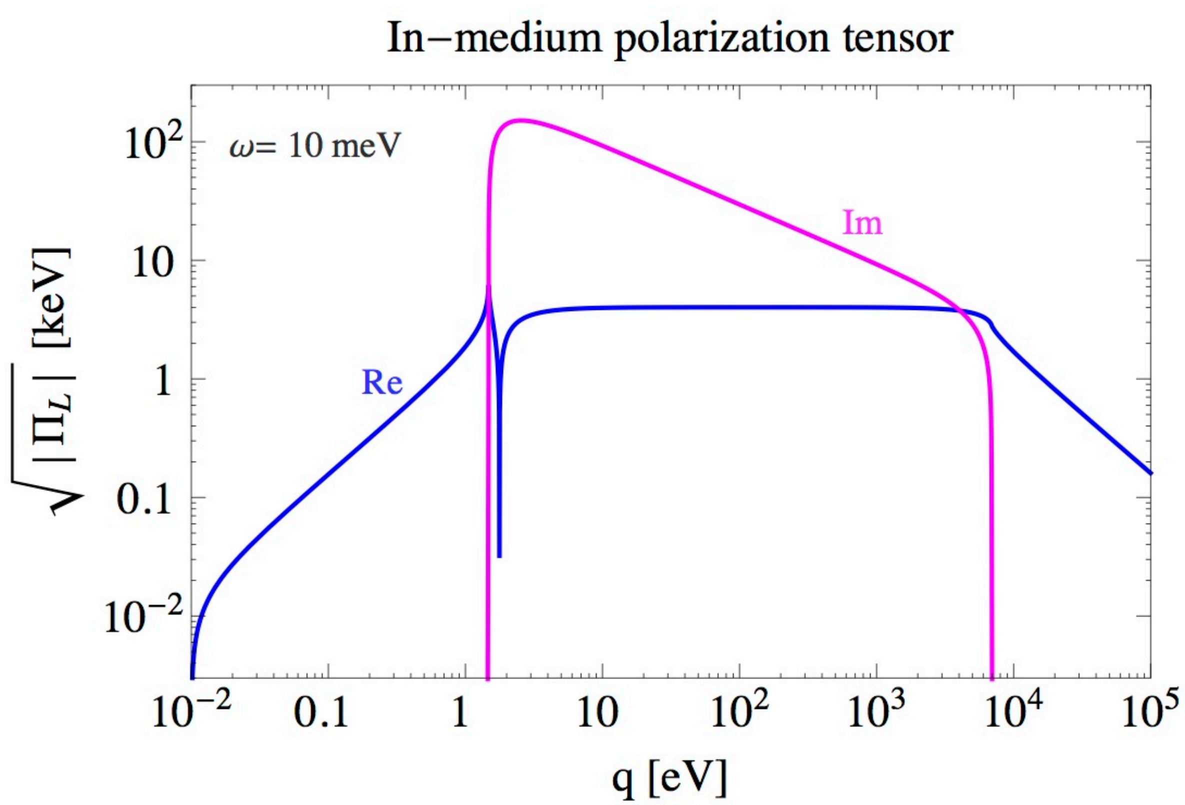
$$\mathrm{Im}[\boldsymbol{\epsilon}_{ii}(\omega)] = \frac{ge^2}{\mathbf{q}^2} \lim_{q\to 0} \sum_{nn'} \int \frac{d^3\mathbf{k}}{(2\pi)^3} 2\pi\, \delta(E_{n'\mathbf{k}} - E_{n\mathbf{k}} - \omega) |f_{[n\mathbf{k}\to n'\mathbf{k}+q\,\hat{\mathbf{e}}_i]}|^2$$



EFFECTIVE COUPLING TO E-M CURRENT

- End result for scattering: $$G^{\mu\nu} = \frac{P_L^{\mu\nu}}{q^2(\hat{\mathbf{q}} \cdot \boldsymbol{\epsilon} \cdot \hat{\mathbf{q}})}$$

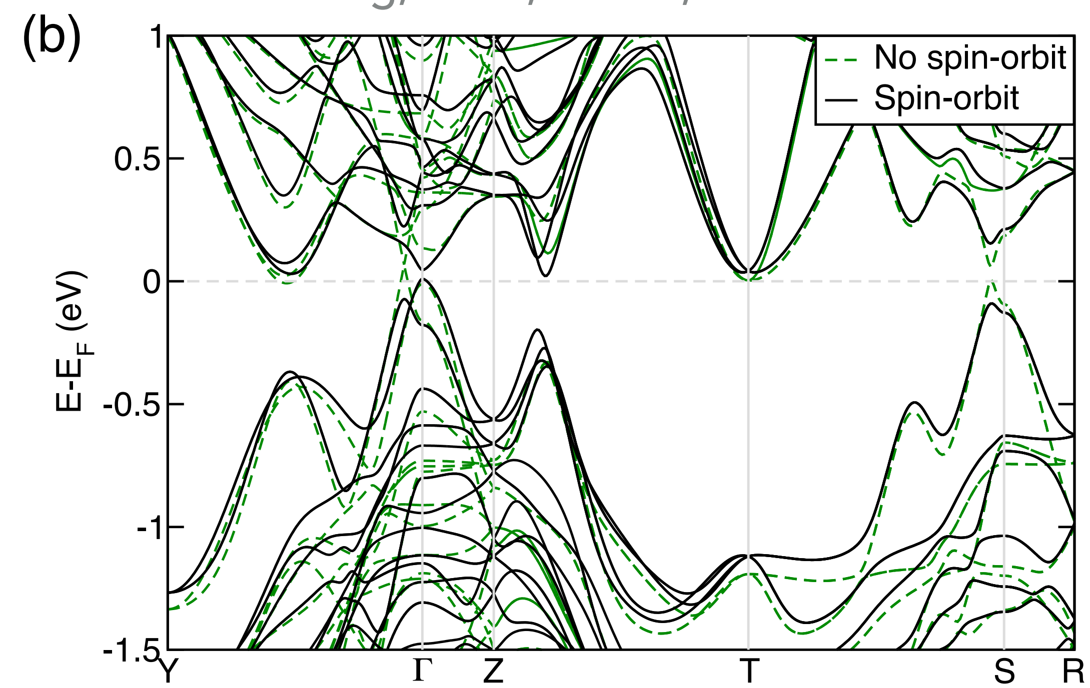
- In-medium effects reduce reach, even for dark photon and scalar mediators. Superconductor:

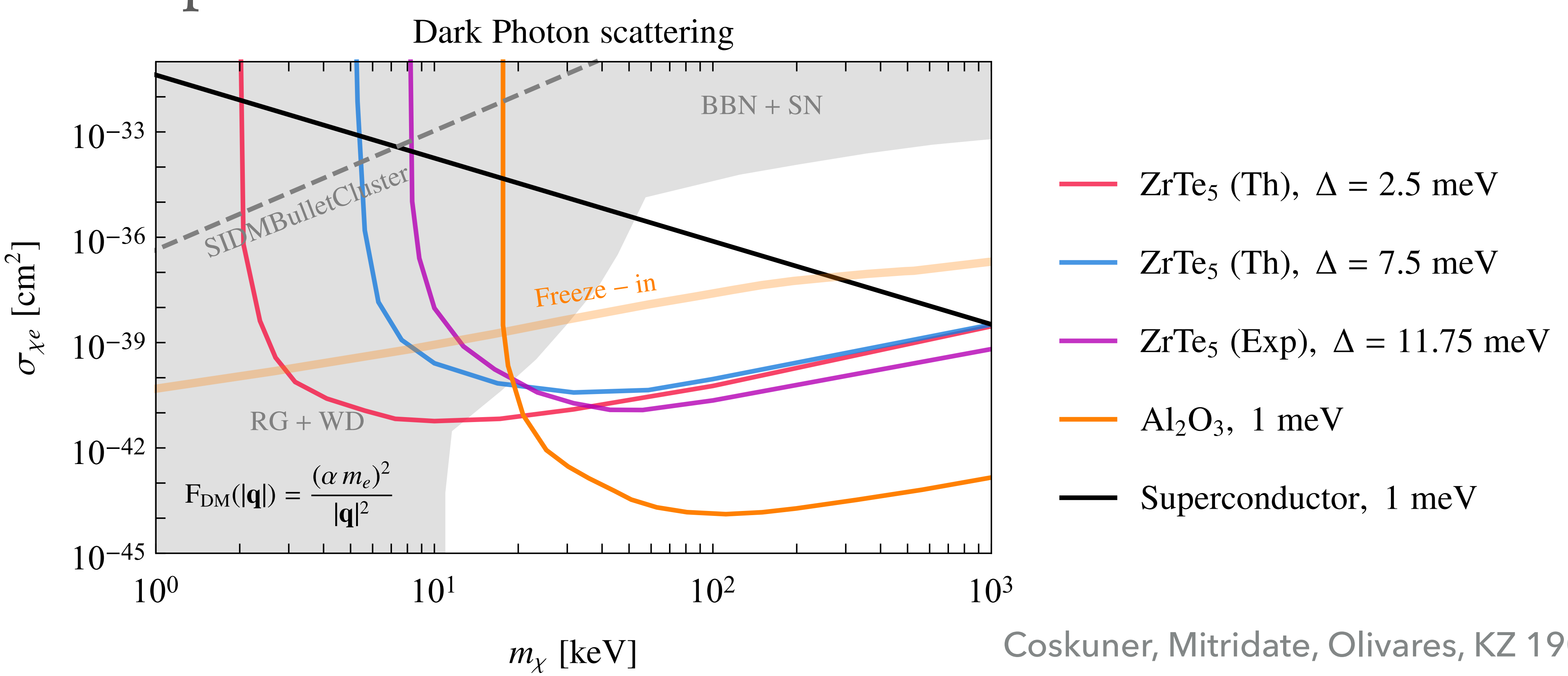


OPTICAL RESPONSE OF "SEMI-METALS"

- Band structure can be "quantum engineered"
- The point-like nature of the density of states at Fermi level implies that screening is less problematic

Hochberg, Kahn, Lisanti, KZ et al 1708.08929





Coskuner, Mitridate, Olivares, KZ 1909.09170

EXCITING COLLECTIVE MODES

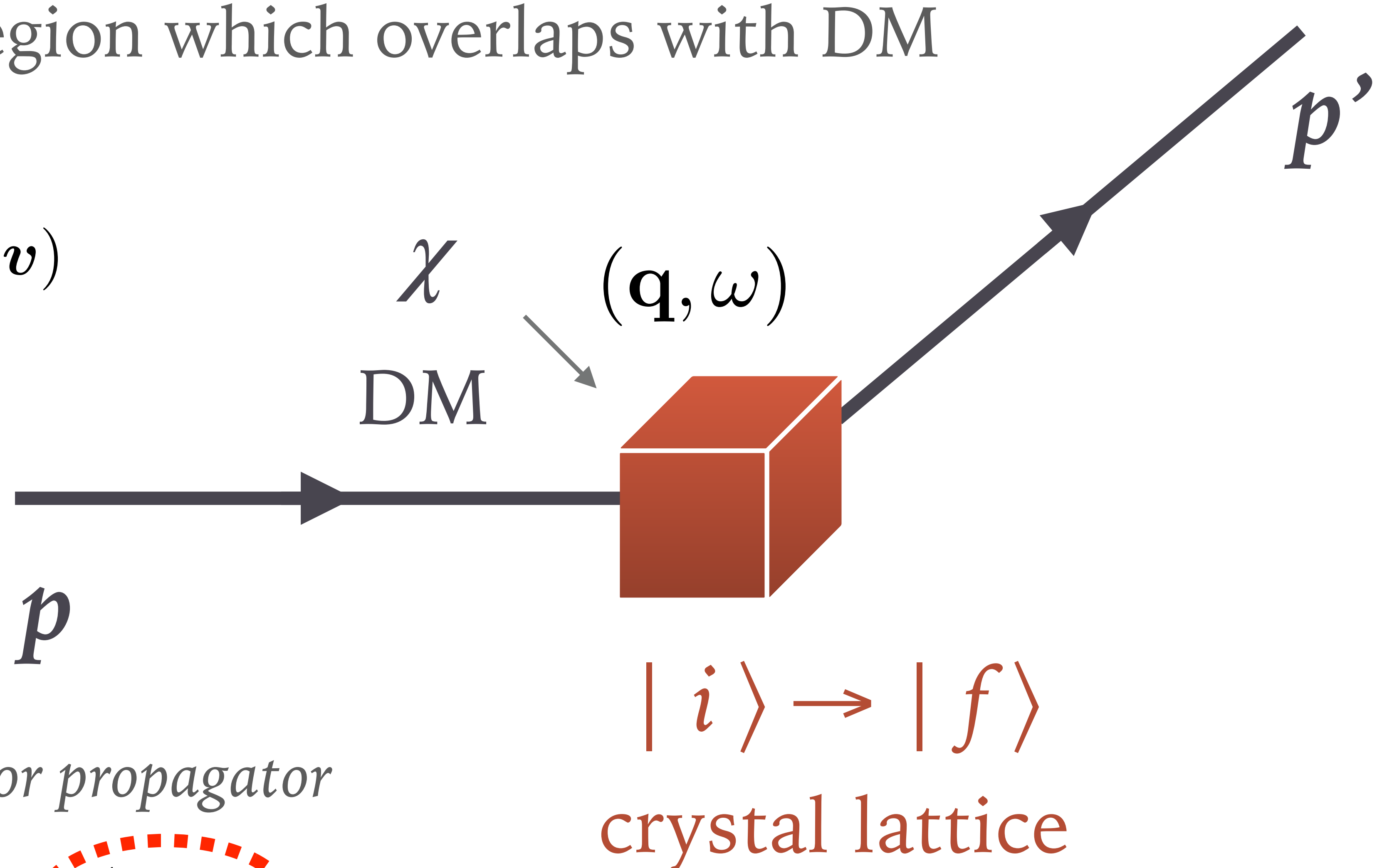
Schutz, KZ 1604.08206, Knapen, Lin, KZ 1611.06228, Knapen, Lin, Pyle, KZ 1712.06598 Griffin, Knapen, Lin, KZ 1807.10291

- Once momentum transfer drops below an keV, deBroglie wavelength is longer than the inter particle spacing in typical materials
- Therefore, relevant d.o.f. in target are no longer individual nuclei or ions
- Must coarse grain to describe DM coupling to “collective excitations”
- Collective excitations = phonon modes, spin waves (magnons)
- Can be applied to just about any material
- Details depend on
  - 1) *nature of collective modes in target material*
  - 2) *nature of DM couplings to target*

DARK MATTER DIRECT DETECTION & KINEMATICS

- Where kinematics is concerned, overarching goal is to find a material with a strong Dynamic Structure Factor in the kinematic region which overlaps with DM

$$R = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\int d^3v\, f_\chi(\boldsymbol{v})\,\Gamma(\boldsymbol{v})$$



Mediator propagator

$$\Gamma(\boldsymbol{v}) = \frac{\pi\overline{\sigma}}{\mu^2}\int\frac{d^3q}{(2\pi)^3}\,\mathcal{F}^2_{\rm med}(q)\,S(\boldsymbol{q},\omega_{\boldsymbol{q}})$$

Dynamic structure factor

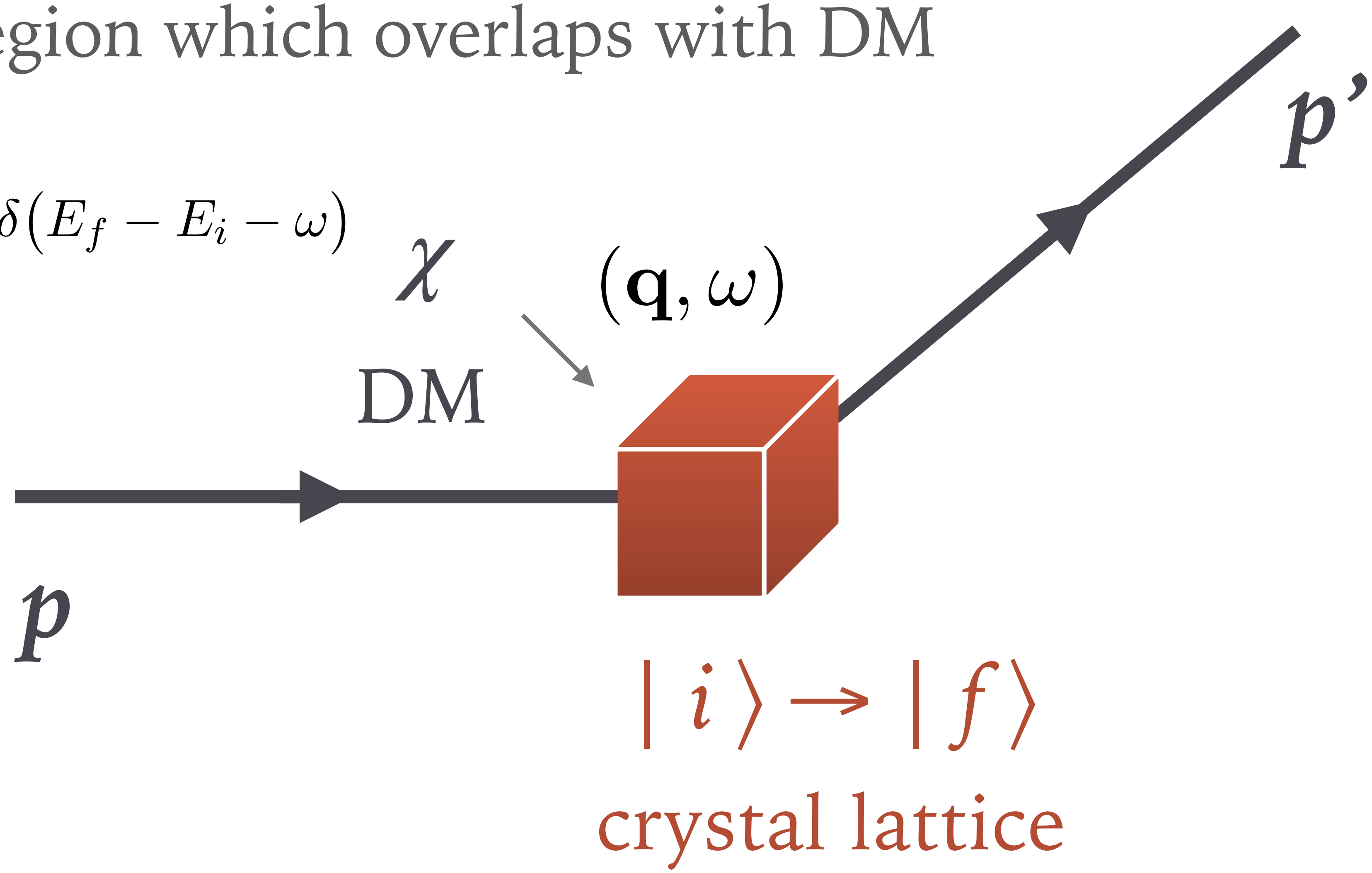
See Trickle, Zhang, KZ 1910.08092

DARK MATTER DIRECT DETECTION & KINEMATICS

- Where kinematics is concerned, overarching goal is to find a material with a strong Dynamic Structure Factor in the kinematic region which overlaps with DM

$$S(\boldsymbol{q},\omega) \equiv \frac{1}{V}\sum_f \left|\langle f|\underbrace{\mathcal{F}_T(\boldsymbol{q})}|i\rangle\right|^2 2\pi\delta\left(E_f - E_i - \omega\right)$$

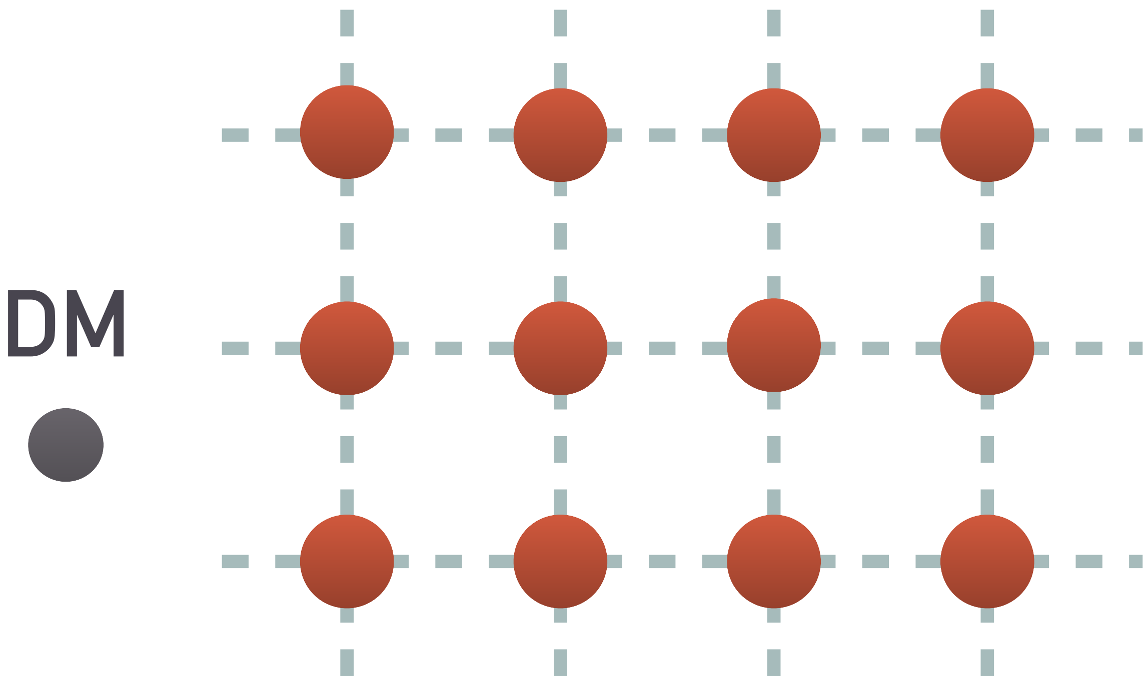
Tabulates the (lattice) potential the incoming DM sees — which in turn depends on the collective modes in the material



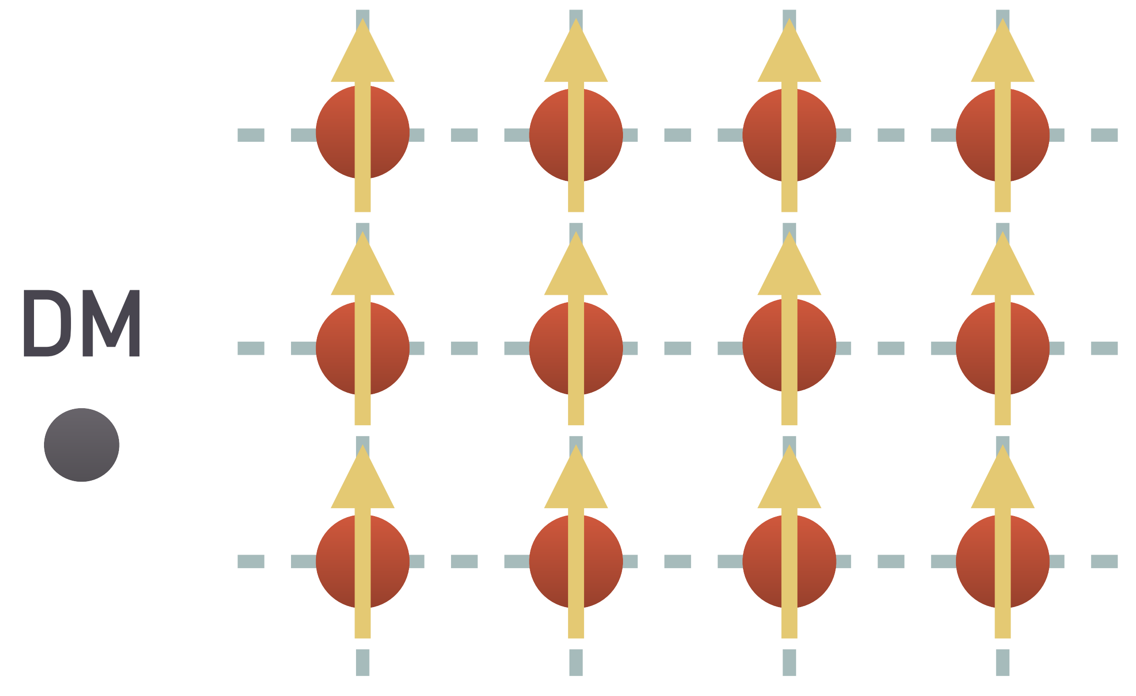
See Trickle, Zhang, KZ 1910.08092

LATTICE DEGREES OF FREEDOM

- Will focus on crystals that have lattice d.o.f.



Phonons

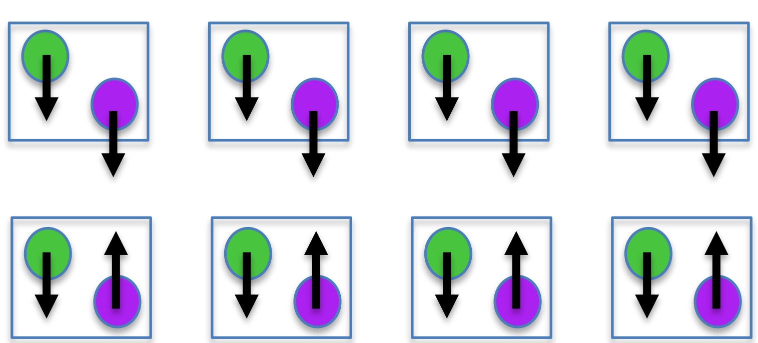


Magnons

Animation credit: Kevin Zhang

Acoustic

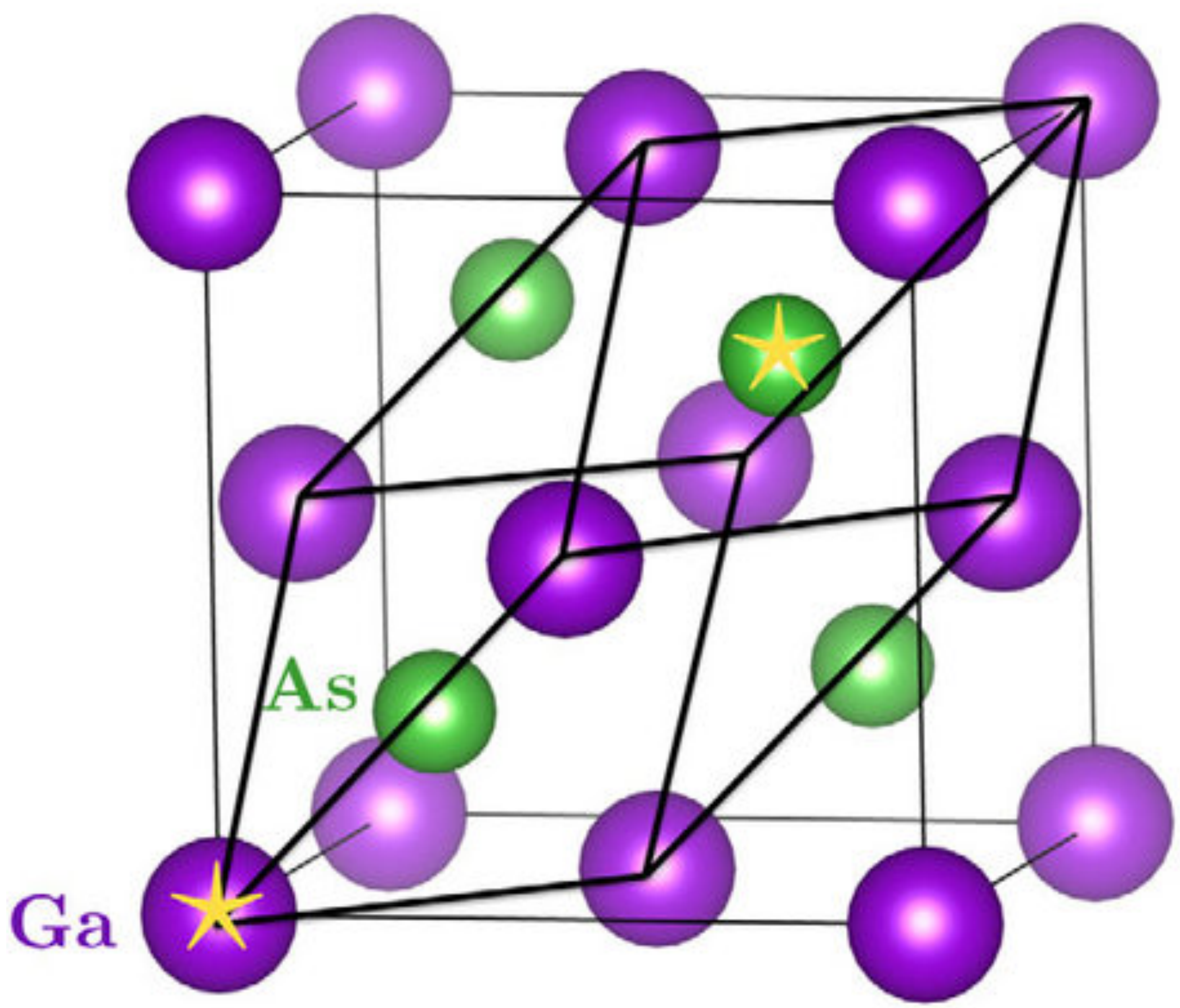
Optical

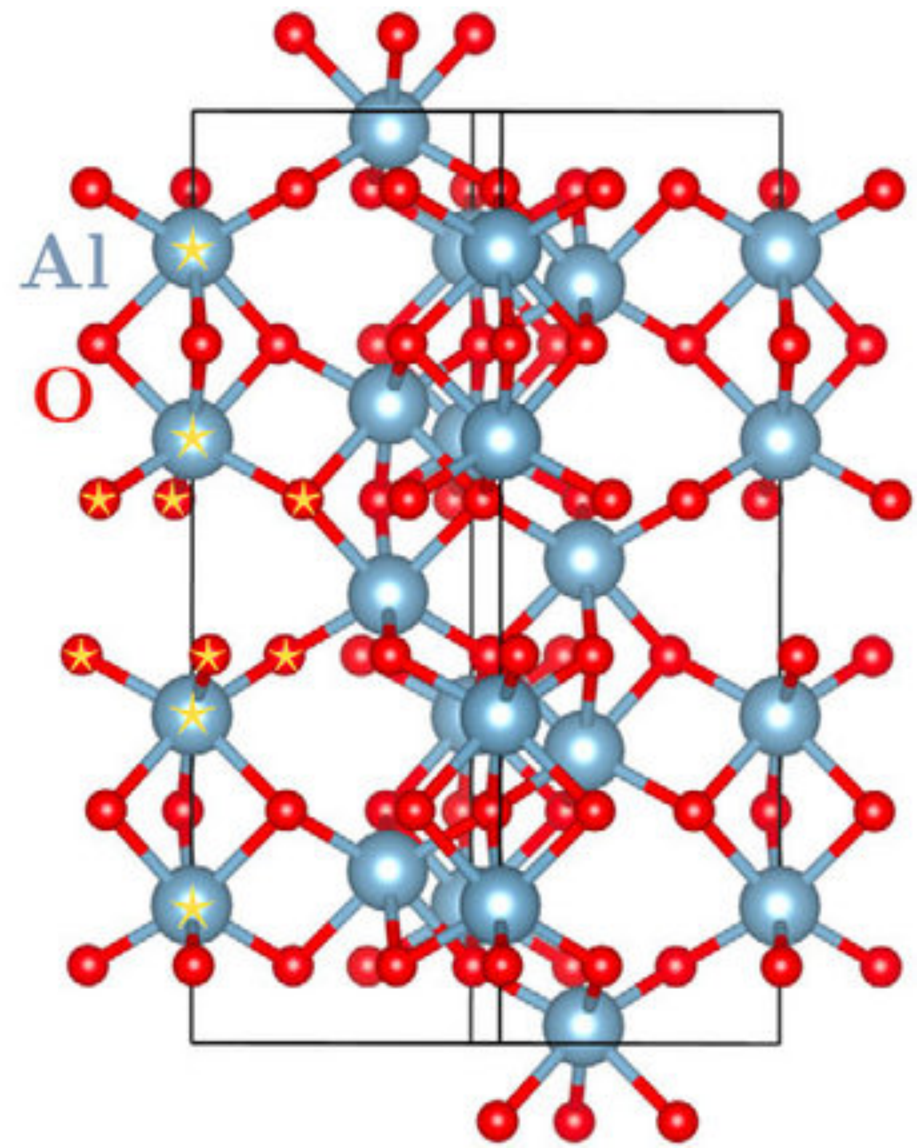


See Trickle, Zhang, KZ 1910.08092

LATTICE DEGREES OF FREEDOM

- Will focus on crystals that have lattice d.o.f.
- Overly simplified; more than one type of ion in a unit cell





DM – COLLECTIVE MODE EFT

- Match relativistic ops onto non-relativistic ops

$$\psi(\boldsymbol{x},t) = e^{-im_\psi t}\,\frac{1}{\sqrt{2}}\begin{pmatrix}\left(1-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{2m_\psi+\varepsilon}\right)\psi^+(\boldsymbol{x},t)\\ \left(1+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{2m_\psi+\varepsilon}\right)\psi^+(\boldsymbol{x},t)\end{pmatrix}$$

Keep leading order in NR expansion

$$\frac{\mathbf{q}}{m_\psi}$$

$$\boldsymbol{v}^\perp \equiv \frac{\boldsymbol{P}}{2m_\chi} - \frac{\boldsymbol{K}}{2m_\psi} = \boldsymbol{v} - \frac{\boldsymbol{k}}{m_\psi} - \frac{\boldsymbol{q}}{2\mu_{\chi\psi}}$$

- Match NR ops onto lattice d.o.f.

$$\boldsymbol{u}_{lj} = \boldsymbol{x}_{lj} - \boldsymbol{x}^0_{lj} = \sum_\nu \sum_{\boldsymbol{k}\in 1\mathrm{BZ}} \frac{1}{\sqrt{2N m_j \omega_{\nu,\boldsymbol{k}}}}\left(\hat{a}_{\nu,\boldsymbol{k}}\,\boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j}\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}} + \hat{a}^\dagger_{\nu,\boldsymbol{k}}\,\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}\,e^{-i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}}\right)$$

- Compute DM excitation rates (apply Fermi's GR)

DM – COLLECTIVE MODE EFT

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$$\psi(\boldsymbol{x},t) = e^{-im_\psi t}\,\frac{1}{\sqrt{2}}\begin{pmatrix}\left(1-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{2m_\psi+\varepsilon}\right)\psi^+(\boldsymbol{x},t)\\ \left(1+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{k}}{2m_\psi+\varepsilon}\right)\psi^+(\boldsymbol{x},t)\end{pmatrix}$$

Keep leading order in NR expansion

$$\frac{\mathbf{q}}{m_\psi}$$

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- Match NR ops onto lattice d.o.f.

$$e^{i\boldsymbol{q}\cdot\boldsymbol{x}_{lj}} = e^{i\boldsymbol{q}\cdot\boldsymbol{x}^0_{lj}}\,e^{-W_j(\boldsymbol{q})}\,\exp\left[\sum_{\nu,\boldsymbol{k}}\frac{i(\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j})\,e^{-i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}}}{\sqrt{2Nm_j\omega_{\nu,\boldsymbol{k}}}}\hat{a}^\dagger_{\nu,\boldsymbol{k}}\right]\exp\left[\sum_{\nu,\boldsymbol{k}}\frac{i(\boldsymbol{q}\cdot\boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j})\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}}}{\sqrt{2Nm_j\omega_{\nu,\boldsymbol{k}}}}\hat{a}_{\nu,\boldsymbol{k}}\right]$$

Lattice form factor

$$W_j(\boldsymbol{q}) = \frac{1}{4Nm_j}\sum_\nu\sum_{\boldsymbol{k}\in 1\mathrm{BZ}}\frac{|\boldsymbol{q}\cdot\boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j}|^2}{\omega_{\nu,\boldsymbol{k}}}$$

- Compute DM excitation rates (apply Fermi's GR)

GOAL OF EFT

Trickle, Zhang, KZ 2009.13534

- To calculate interaction rate with collective excitations from any UV complete DM interaction

Model		UV Lagrangian	NR EFT	Responses
Standard SI		$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{S,\psi}\right)$ or $V_\mu\left(g_\chi J^\mu_{V,\chi} - g_\psi J^\mu_{V,\psi}\right)$	$c_1^{(\psi)} = \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_{\phi,V}}$	N
Standard SD [a]		$V_\mu\left(g_\chi J^\mu_{A,\chi} + g_\psi J^\mu_{A,\psi}\right)$	$c_4^{(\psi)} = \frac{4 g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$	S
Other scalar mediators	P × S	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{S,\psi}\right)$	$c_{11}^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_\phi}$	N
	S × P	$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{P,\psi}\right)$	$c_{10}^{(\psi)} = -\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_\phi}$	S
	P × P	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{P,\psi}\right)$	$c_6^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_\phi}$	S
Multipole DM models	Electric dipole	$V_\mu\Big(g_\chi J^\mu_{\text{edm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N
	Magnetic dipole	$V_\mu\Big(g_\chi J^\mu_{\text{mdm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m^2_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\text{eff}}\frac{\boldsymbol{q}^2}{m_\chi m_\psi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}}\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N, S, L
	Anapole	$V_\mu\Big(g_\chi J^\mu_{\text{ana},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m^2_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}}\frac{\boldsymbol{q}^2}{2m^2_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N, S, L
$(\boldsymbol{L}\cdot\boldsymbol{S})$-interacting		$V_\mu\big(g_\chi J^\mu_{V,\chi} + g_\psi(J^\mu_{\text{mdm},\psi} + \kappa J^\mu_{V2,\psi})\big)$	$c_1^{(\psi)} = (1+\kappa)\frac{\boldsymbol{q}^2}{4m^2_\psi}\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_3^{(\psi)} = \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_4^{(\psi)} = \frac{\boldsymbol{q}^2}{m_\chi m_\psi}\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_6^{(\psi)} = -\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$	N, S, $L \otimes S$

Decomposition carried out previously

Gresham, KZ 1401.3739

Using NR basis of

Fitzpatrick, Haxton, Katz, Lubbers, Xu 1203.3542

GOAL OF EFT

Trickle, Zhang, KZ 2009.13534

- To calculate interaction rate with collective excitations from any UV complete DM interaction

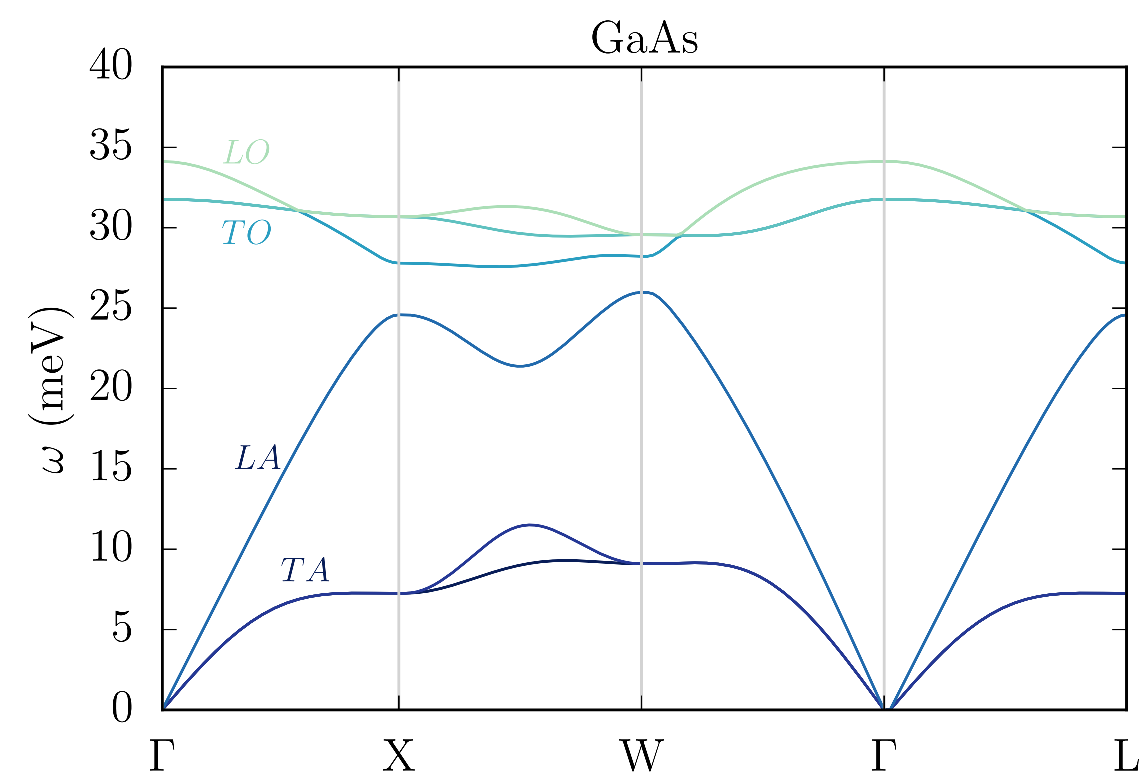
Start simple with standard SI interactions

Understand material effective Hamiltonian and potential

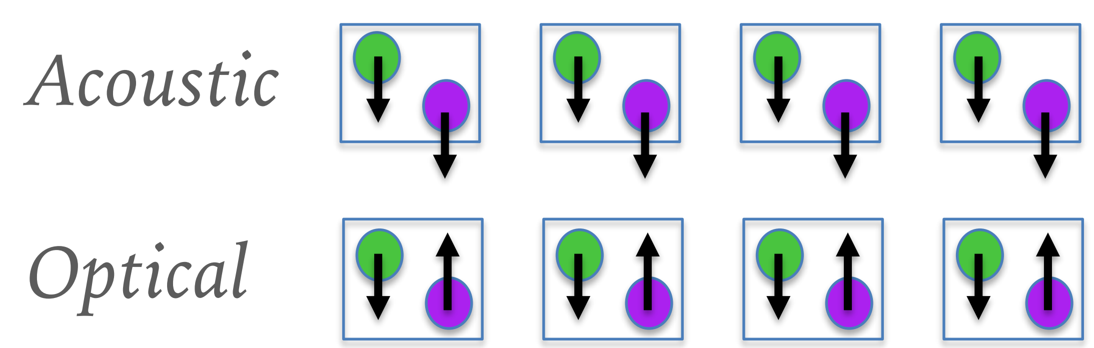
Model		UV Lagrangian	NR EFT	Responses
Standard SI		$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{S,\psi}\right)$ or $V_\mu\left(g_\chi J^\mu_{V,\chi} - g_\psi J^\mu_{V,\psi}\right)$	$c_1^{(\psi)} = \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_{\phi,V}}$	N
Standard SD [a]		$V_\mu\left(g_\chi J^\mu_{A,\chi} + g_\psi J^\mu_{A,\psi}\right)$	$c_4^{(\psi)} = \frac{4 g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$	S
Other scalar mediators	$\text{P} \times \text{S}$	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{S,\psi}\right)$	$c_{11}^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_\phi}$	N
	$\text{S} \times \text{P}$	$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{P,\psi}\right)$	$c_{10}^{(\psi)} = -\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_\phi}$	S
	$\text{P} \times \text{P}$	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{P,\psi}\right)$	$c_6^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_\phi}$	S
Multipole DM models	Electric dipole	$V_\mu\left(g_\chi J^\mu_{\text{edm},\chi} + g_\psi\left(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\right)\right)$	$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N
	Magnetic dipole	$V_\mu\left(g_\chi J^\mu_{\text{mdm},\chi} + g_\psi\left(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\right)\right)$	$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m^2_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{m_\chi m_\psi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N, S, L
	Anapole	$V_\mu\left(g_\chi J^\mu_{\text{ana},\chi} + g_\psi\left(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\right)\right)$	$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m^2_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{2m^2_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_V}$	N, S, L
$(\boldsymbol{L} \cdot \boldsymbol{S})$-interacting		$V_\mu\left(g_\chi J^\mu_{V,\chi} + g_\psi\left(J^\mu_{\text{mdm},\psi} + \kappa J^\mu_{V2,\psi}\right)\right)$	$c_1^{(\psi)} = (1+\kappa) \frac{\boldsymbol{q}^2}{4m^2_\psi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_3^{(\psi)} = \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_4^{(\psi)} = \frac{\boldsymbol{q}^2}{m_\chi m_\psi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$ $c_6^{(\psi)} = -\frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m^2_V}$	N, S, $L \otimes S$

NATURE OF COLLECTIVE OSCILLATIONS OF IONS — PHONONS

- Number of collective modes: 3 x number of ions in unit cell
- 3 of those modes describe in phase oscillation — acoustic phonons — and have a translation symmetry implying gapless dispersion
- The remaining modes are gapped



Momentum transfer

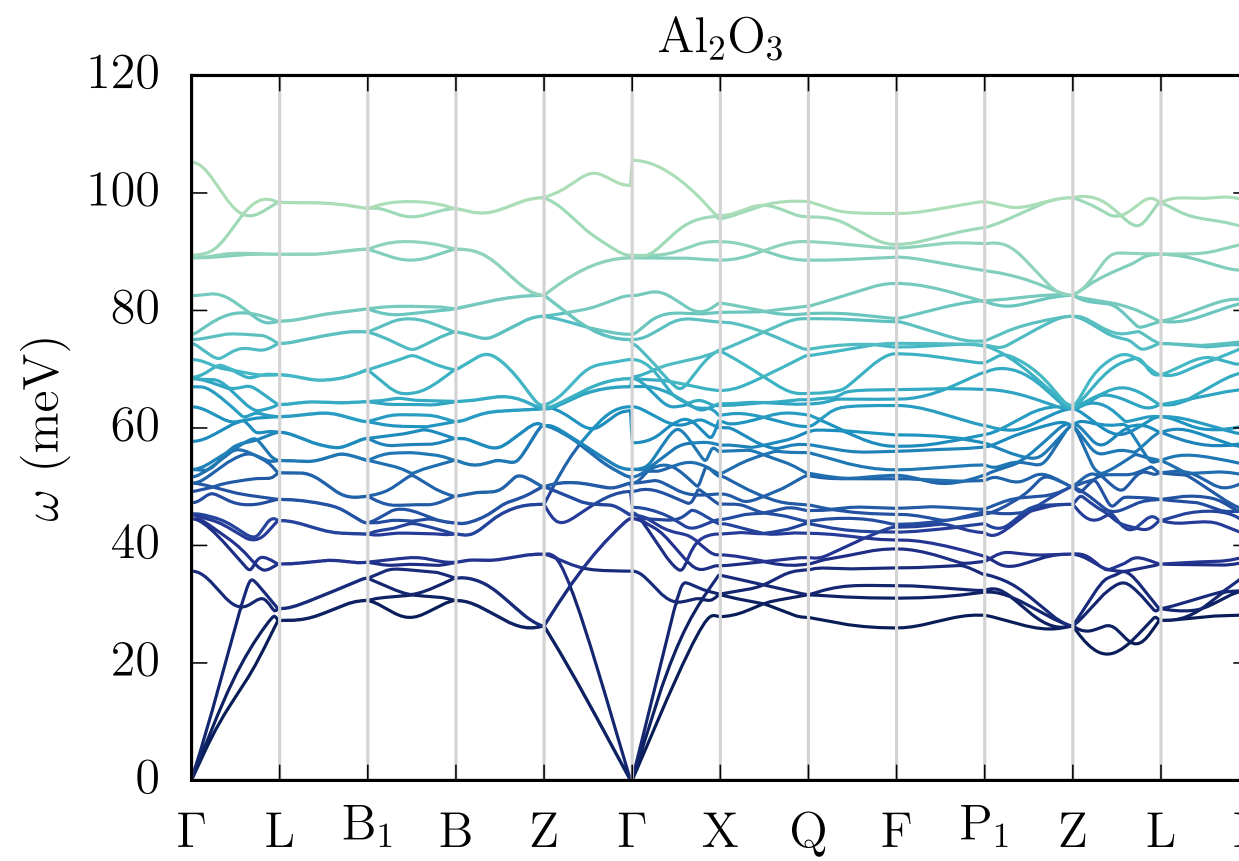


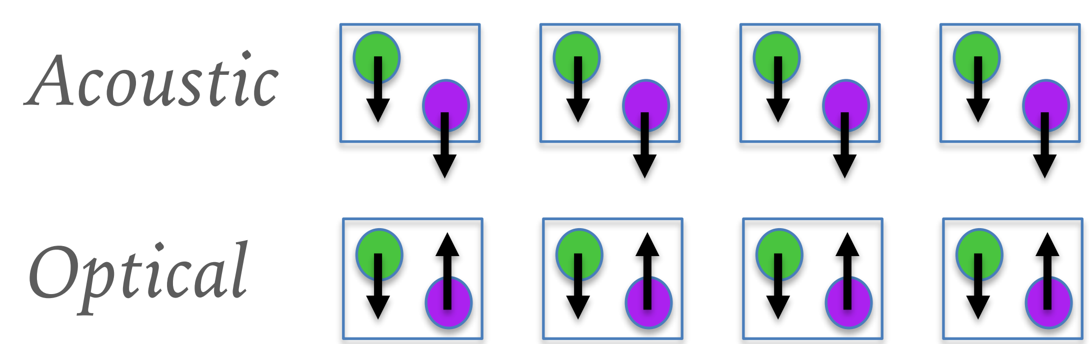
Knapen, Lin, Pyle, KZ 1712.06598 Griffin, Knapen, Lin, KZ 1807.10291

NATURE OF COLLECTIVE OSCILLATIONS OF IONS — PHONONS

- Some materials have an abundance of these modes
- When these gapped modes result from oscillations of more than one type of ion, it sets up an oscillating dipole: Polar Materials
- This oscillating dipole allows to compute an effective interaction and compute the dynamic structure factor

Sapphire





Knapen, Lin, Pyle, KZ 1712.06598 Griffin, Knapen, Lin, KZ 1807.10291

KINEMATICS OF COLLECTIVE MODES

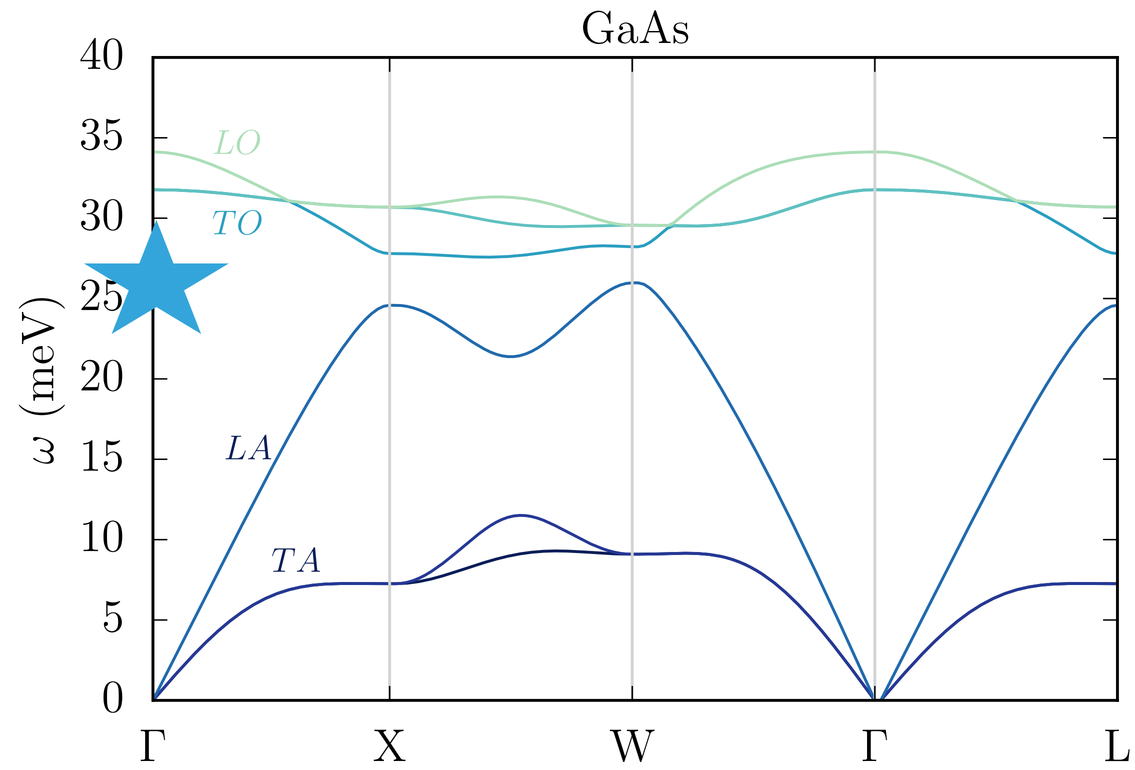
- Each phonon mode is a resonance. The DM needs to be well matched kinematically to the modes to excite large response

$$E_D \sim v_X q$$

vs

$$E_D \sim c_s q$$

$$c_s \ll v_X$$



- Better coupling to gapped modes

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DM - COLLECTIVE MODE EFT

- Match relativistic ops onto non-relativistic ops

(Trivial for SI interactions)

- Match NR ops onto lattice d.o.f.

(Provided by Frohlich Hamiltonian or dynamic structure factor computed by DFT methods)

- Compute DM excitation rates

(Straightforward once one understands the (inelastic) kinematics of the system)

FROHLICH HAMILTONIAN AND EFFECTIVE INTERACTIONS

- For sufficiently simple interactions, the effective interaction is already known, e.g. Frohlich Hamiltonian:

$$\mathcal{H}_I = i\frac{\kappa g_X}{e} C_F \sum_{\mathbf{k},\mathbf{q}} \frac{1}{|\mathbf{q}|}\left[c^\dagger_{\mathbf{q}} a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}} - \text{c.c.}\right] \qquad C_F = e\left[\frac{\omega_{\text{LO}}}{2V_{\text{cell}}}\left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0}\right)\right]^{1/2}$$

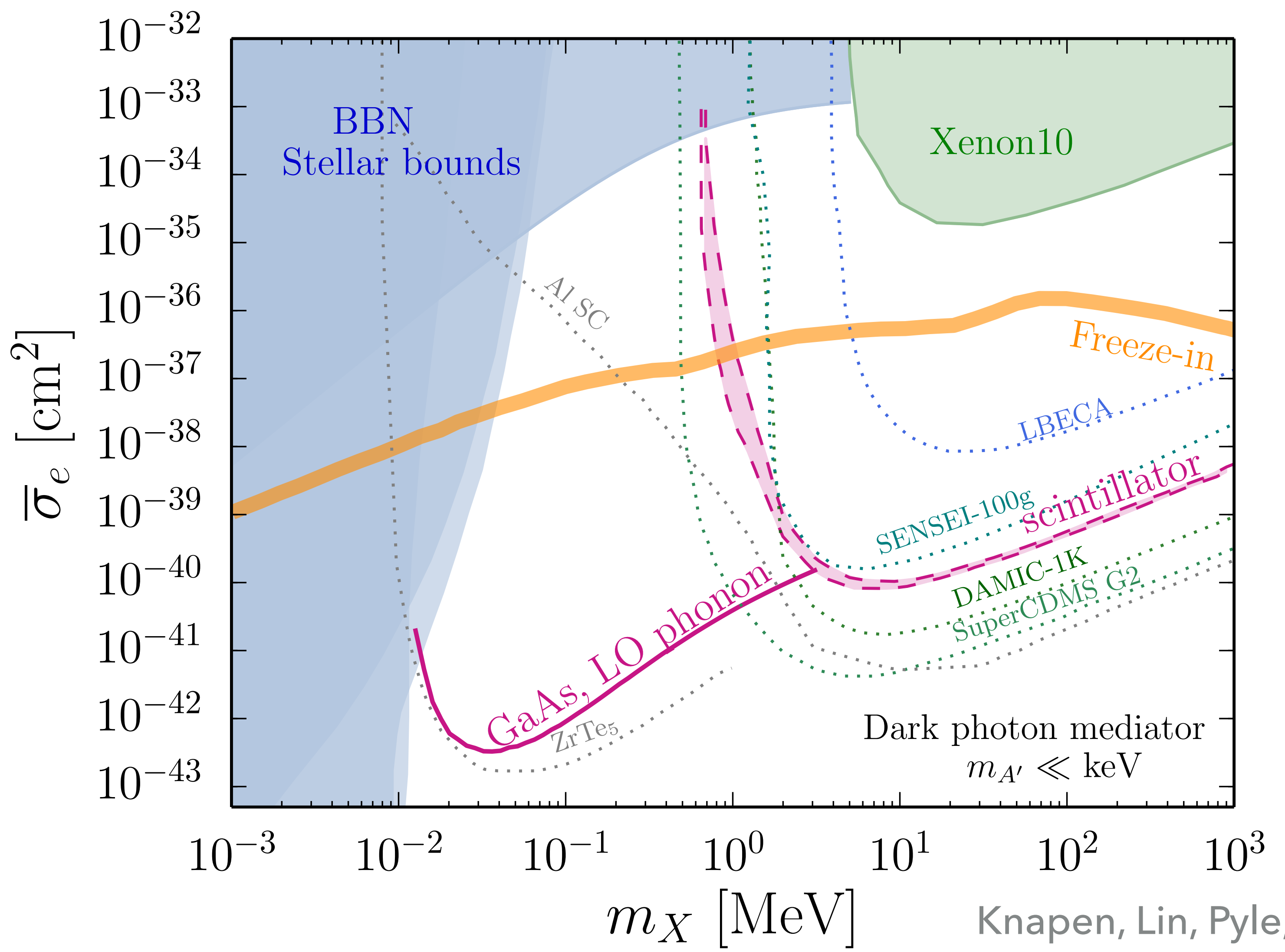
$$|\mathcal{M}_{\mathbf{q}}|^2 = \frac{\kappa^2 g_X^2}{e^2}\frac{C_F^2}{q^2}$$

- Apply Fermi's golden rule: $\Gamma(\mathbf{p_i}) = 2\pi \int \frac{d^3\mathbf{p}_f}{(2\pi)^3}\delta(E_f - E_i - \omega)|\mathcal{M}_{\mathbf{q}}|^2$
- Integrate over phase space:

$$R = \frac{1}{\rho}\frac{\rho_{\text{DM}}}{m_X}\int d^3\mathbf{v} f(\mathbf{v})\Gamma(m_\chi \mathbf{v})$$

FROHLICH HAMILTONIAN AND EFFECTIVE INTERACTIONS

- For sufficiently simple interactions, the effective interaction is already known, e.g. Frohlich Hamiltonian:



Knapen, Lin, Pyle, KZ 1712.06598

FIRST PRINCIPLES DERIVATION

See Trickle, Zhang, KZ 1910.08092
Griffin, Knapen, Lin, KZ 1807.10291

- Phonons are excitations of lattice displacements. Write down in terms of the lattice potential:

$$\langle \nu, \boldsymbol{k} | \, \widetilde{\mathcal{V}}(-\boldsymbol{q}, \boldsymbol{v}) | 0 \rangle = \sum_{l,j} \langle \nu, \boldsymbol{k} | \, e^{i\boldsymbol{q}\cdot\boldsymbol{x}_{lj}} \, \widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q}, \boldsymbol{v}) | 0 \rangle$$

- Now, quantize the lattice displacements:

$$\boldsymbol{u}_{lj} = \boldsymbol{x}_{lj} - \boldsymbol{x}_{lj}^0 = \sum_{\nu=1}^{3n} \sum_{\boldsymbol{k} \in 1\mathrm{BZ}} \frac{1}{\sqrt{2N m_j \omega_{\nu,\boldsymbol{k}}}} \left(\hat{a}_{\nu,\boldsymbol{k}} \, \boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j} \, e^{i\boldsymbol{k}\cdot\boldsymbol{x}_{lj}^0} + \hat{a}_{\nu,\boldsymbol{k}}^\dagger \, \boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j}^* \, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_{lj}^0} \right)$$

- Apply BCH to normal-order phonon creation/annihilation

$$\langle \nu, \boldsymbol{k} | \, \widetilde{\mathcal{V}}(-\boldsymbol{q}, \boldsymbol{v}) | 0 \rangle = \frac{1}{\sqrt{N}} \sum_{\nu,\boldsymbol{k},j} \left[\sum_l \widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q}, \boldsymbol{v}) \, e^{i(\boldsymbol{q}-\boldsymbol{k})\cdot\boldsymbol{x}_{lj}^0} \right] e^{-W_j(\boldsymbol{q})} \, \frac{i(\boldsymbol{q} \cdot \boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j}^*)}{\sqrt{2 m_j \omega_{\nu,\boldsymbol{k}}}}$$

FIRST PRINCIPLES DERIVATION

See Trickle, Zhang, KZ 1910.08092
Griffin, Knapen, Lin, KZ 1807.10291

- Obtain rate from Fermi's golden rule:

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega}\int \frac{d^3q}{(2\pi)^3}\sum_{\nu=1}^{3n} 2\pi\,\delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)\frac{1}{2\omega_{\nu,\boldsymbol{k}}}\left|\sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0}\,\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}}\,\widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v})\right|^2$$

- Frolich Hamiltonian obtained in limit

$$W_j \simeq 0 \qquad \mathbf{G} = 0 \qquad \tilde{V}_j(-\mathbf{q},\mathbf{v}) = -\frac{Z_j^* q^2}{\mathbf{q}\cdot\epsilon_\infty\cdot\mathbf{q}}$$

$$Z_1^* = -Z_2^* \equiv Z^* \qquad |\epsilon_{\mathrm{LO},\mathbf{k},j}| = \sqrt{\mu_{12}/m_j}$$

LO polarization vectors anti-parallel

FIRST PRINCIPLES DERIVATION

See Trickle, Zhang, KZ 1910.08092
Griffin, Knapen, Lin, KZ 1807.10291

- Obtain rate from Fermi's golden rule:

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega} \int \frac{d^3q}{(2\pi)^3} \sum_{\nu=1}^{3n} 2\pi\, \delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big) \frac{1}{2\omega_{\nu,\boldsymbol{k}}} \left| \sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0} \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}} \widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v}) \right|^2$$

$$S(\mathbf{q},\omega)$$

- Dynamic structure factor

- **If** interaction is ordinary SI interaction, can use famous result of Nozieres and Pines

$$S(\omega,\mathbf{k}) = \frac{k^2}{2\pi\alpha_{em}} \frac{1}{1-e^{-\beta\omega}} \text{Im}\left[\frac{-1}{\epsilon_L(\omega,\mathbf{k})}\right]$$

Knapen, Kozaczuk, Lin

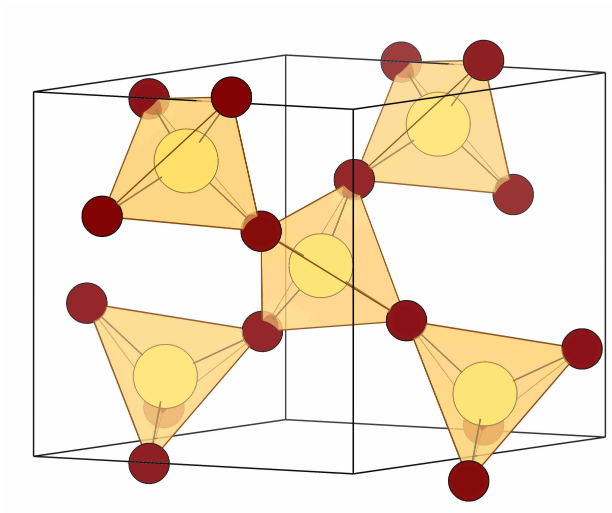
FIRST PRINCIPLES DERIVATION

- Obtain rate from Fermi's golden rule:

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega}\int \frac{d^3q}{(2\pi)^3}\sum_{\nu=1}^{3n} 2\pi\,\delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)\frac{1}{2\omega_{\nu,\boldsymbol{k}}}\left|\sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0}\,\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}}\,\widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v})\right|^2$$

- The inverse lattice vector G maps momentum transfer outside 1BZ back inside it

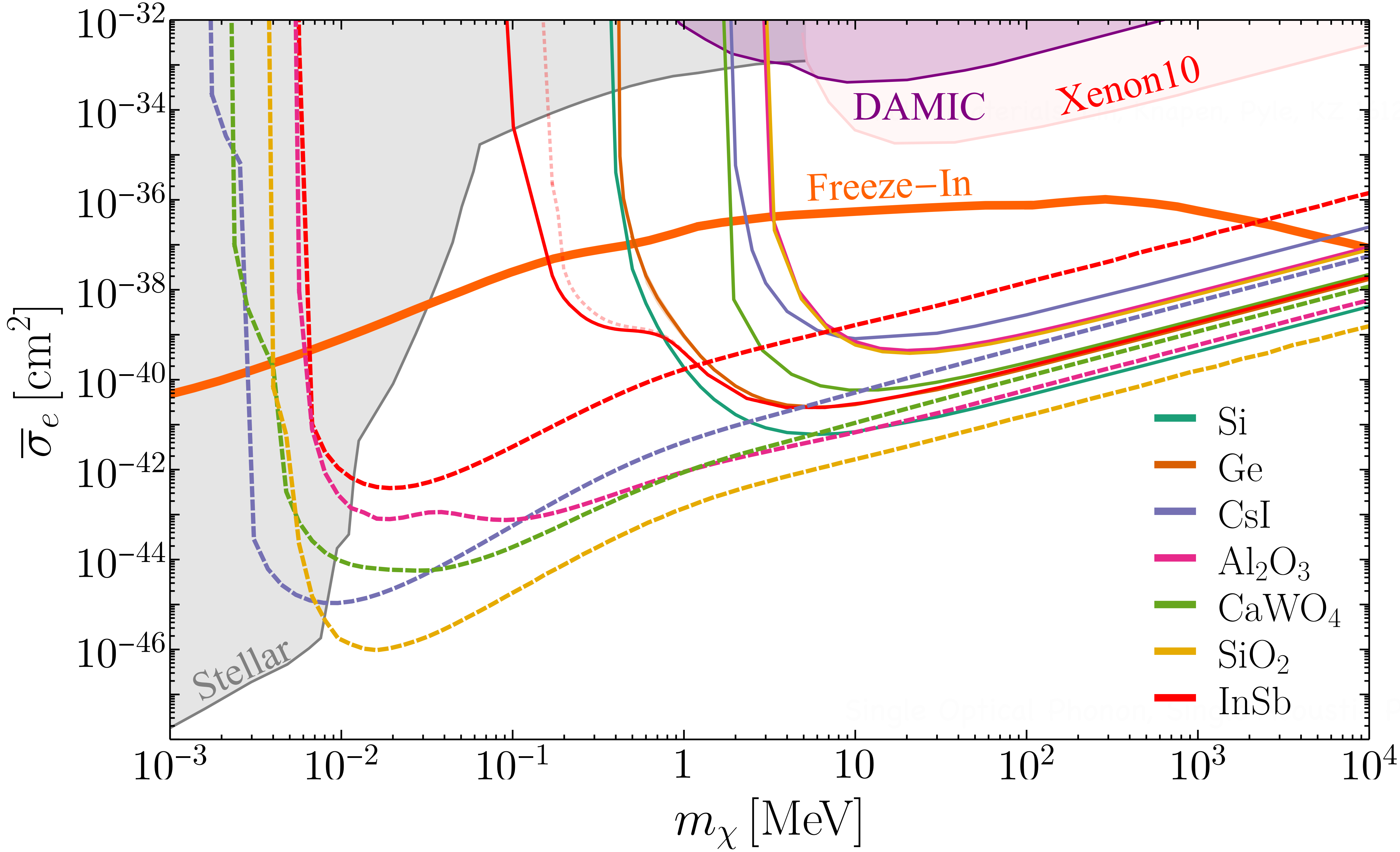
$$\mathbf{q} = \mathbf{k} + \mathbf{G}$$



- Including the inverse lattice vector allows to extend calculation to high DM masses

OPTICAL PHONONS IN POLAR MATERIALS

Griffin, Inzani, Trickle, Zhang, KZ, 1910.10716



Generalize to NR EFT

Trickle, Zhang, KZ 2009.13534

1) MATCH TO NR OPERATORS

Trickle, Zhang, KZ 2009.13534

- Recall we are interested in matrix elements of the form

$$\Gamma(\boldsymbol{v}) = \frac{1}{V} \int \frac{d^3 q}{(2\pi)^3} \sum_{\nu,\boldsymbol{k}} \left| \sum_{l,j} \langle \nu, \boldsymbol{k} | \, e^{i \boldsymbol{q} \cdot \boldsymbol{x}_{lj}} \, \widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q}, \boldsymbol{v}) | 0 \rangle \right|^2 2\pi \, \delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)$$

- We need to calculate the lattice potential in the NR basis

Model		UV Lagrangian	NR EFT	Responses
Standard SI		$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{S,\psi}\right)$ or $V_\mu\left(g_\chi J^\mu_{V,\chi} - g_\psi J^\mu_{V,\psi}\right)$	$c_1^{(\psi)} = \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m^2_{\phi,V}}$	N
Standard SD [a]		$V_\mu\left(g_\chi J^\mu_{A,\chi} + g_\psi J^\mu_{A,\psi}\right)$	$c_4^{(\psi)} = \frac{4 g_\chi g_\psi}{\boldsymbol{q}^2 + m_V^2}$	S
Other scalar mediators	P × S	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{S,\psi}\right)$	$c_{11}^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_\phi^2}$	N
	S × P	$\phi\left(g_\chi J_{S,\chi} + g_\psi J_{P,\psi}\right)$	$c_{10}^{(\psi)} = -\frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_\phi^2}$	S
	P × P	$\phi\left(g_\chi J_{P,\chi} + g_\psi J_{P,\psi}\right)$	$c_6^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_\phi^2}$	S
Multipole DM models	Electric dipole	$V_\mu\Big(g_\chi J^\mu_{\text{edm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	N
	Magnetic dipole	$V_\mu\Big(g_\chi J^\mu_{\text{mdm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{m_\chi m_\psi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	N, S, L
	Anapole	$V_\mu\Big(g_\chi J^\mu_{\text{ana},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{2m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	N, S, L
$(\boldsymbol{L} \cdot \boldsymbol{S})$-interacting		$V_\mu\left(g_\chi J^\mu_{V,\chi} + g_\psi\left(J^\mu_{\text{mdm},\psi} + \kappa J^\mu_{V2,\psi}\right)\right)$	$c_1^{(\psi)} = (1+\kappa) \frac{\boldsymbol{q}^2}{4m_\psi^2} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_V^2}$ $c_3^{(\psi)} = \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_V^2}$ $c_4^{(\psi)} = \frac{\boldsymbol{q}^2}{m_\chi m_\psi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_V^2}$ $c_6^{(\psi)} = -\frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi}{\boldsymbol{q}^2 + m_V^2}$	N, S, $L \otimes S$

1) MATCH TO NR OPERATORS

Trickle, Zhang, KZ 2009.13534

- Recall we are interested in matrix elements of the form

$$\Gamma(\boldsymbol{v}) = \frac{1}{V}\int \frac{d^3q}{(2\pi)^3} \sum_{\nu,\boldsymbol{k}} \left| \sum_{l,j} \langle \nu, \boldsymbol{k}|\, e^{i\boldsymbol{q}\cdot\boldsymbol{x}_{lj}}\, \widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v})|0\rangle \right|^2 2\pi\,\delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)$$

- We need to calculate the lattice potential in the NR basis

Interaction Type	**NR Operators**	**Point-like Response**	**Composite Response**
Coupling to *charge*, $\boldsymbol{v}^\perp$-*independent*	$\mathcal{O}_1^{(\psi)} = \mathbb{1}$ $\mathcal{O}_{11}^{(\psi)} = \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}$	N	-
Coupling to *charge*, $\boldsymbol{v}^\perp$-*dependent*	$\mathcal{O}_5^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_8^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp$	N	L
Coupling to *spin*, $\boldsymbol{v}^\perp$-*independent*	$\mathcal{O}_4^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{S}_\psi$ $\mathcal{O}_6^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_\psi}\big)\big(\boldsymbol{S}_\psi \cdot \frac{\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_9^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\boldsymbol{S}_\psi \times \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{10}^{(\psi)} = \boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}$	S	-
Coupling to *spin*, $\boldsymbol{v}^\perp$-*dependent*	$\mathcal{O}_3^{(\psi)} = \boldsymbol{S}_\psi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_7^{(\psi)} = \boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp$ $\mathcal{O}_{12}^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\boldsymbol{S}_\psi \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_{13}^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp\big)\big(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{14}^{(\psi)} = \big(\boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp\big)\big(\boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{15}^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)\big)\big(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$	S	$L \otimes S$

2) MATCH TO LATTICE D.O.F.

Trickle, Zhang, KZ 2009.13534

- Recall we are interested in matrix elements of the form

$$\Gamma(\boldsymbol{v}) = \frac{1}{V}\int \frac{d^3q}{(2\pi)^3} \sum_{\nu,\boldsymbol{k}} \left| \sum_{l,j} \langle \nu, \boldsymbol{k}|\, e^{i\boldsymbol{q}\cdot\boldsymbol{x}_{lj}}\, \widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v})|0\rangle \right|^2 2\pi\,\delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)$$

- We need to calculate the lattice potential in the NR basis

$$\begin{aligned}
\widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v}) = \sum_{\psi=p,n,e} & c_1^{(\psi)} \langle N_\psi\rangle_{lj} \\
& + c_3^{(\psi)}\left[-\frac{iq}{m_\psi}\,\boldsymbol{v}'\cdot\big(\hat{\boldsymbol{q}}\times\langle \boldsymbol{S}_\psi\rangle_{lj}\big) + \frac{q^2}{2m_\psi^2}\,(\delta^{ik} - \hat{q}^i\hat{q}^k)\big(\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\big)^{ik}\right] \\
& + c_4^{(\psi)}\,\boldsymbol{S}_\chi\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} \\
& + c_5^{(\psi)}\left[\frac{i\boldsymbol{q}}{m_\psi}\cdot(\boldsymbol{v}'\times\boldsymbol{S}_\chi)\langle N_\psi\rangle_{lj} + \frac{q^2}{2m_\psi^2}\,\boldsymbol{S}_\chi\cdot(\mathbb{1} - \hat{\boldsymbol{q}}\hat{\boldsymbol{q}})\cdot\langle \boldsymbol{L}_\psi\rangle_{lj}\right] \\
& + c_6^{(\psi)}\,\frac{q^2}{m_\psi^2}\,(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi)(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}) \\
& + c_7^{(\psi)}\left[\boldsymbol{v}'\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} + \epsilon^{ikk'}\,\frac{iq^{k'}}{2m_\chi}\big(\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\big)^{ik}\right] \\
& + c_8^{(\psi)}\left[(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi)\langle N_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\boldsymbol{S}_\chi\cdot\big(\hat{\boldsymbol{q}}\times\langle \boldsymbol{L}_\psi\rangle_{lj}\big)\right] \\
& + c_9^{(\psi)}\,\frac{iq}{m_\psi}\boldsymbol{S}_\chi\cdot\big(\langle \boldsymbol{S}_\psi\rangle_{lj}\times\hat{\boldsymbol{q}}\big) \\
& + c_{10}^{(\psi)}\,\frac{i\boldsymbol{q}}{m_\psi}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} \\
& + c_{11}^{(\psi)}\,\frac{i\boldsymbol{q}}{m_\psi}\cdot\boldsymbol{S}_\chi\,\langle N_\psi\rangle_{lj} \\
& + c_{12}^{(\psi)}\left[(\boldsymbol{v}'\times\boldsymbol{S}_\chi)\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\big((\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi)\delta^{ik} - \hat{q}^k S_\chi^i\big)\big(\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\big)^{ik}\right] \\
& + c_{13}^{(\psi)}\left[\frac{iq}{m_\psi}(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi)\big(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\big) + \frac{q^2}{2m_\psi^2}\,(\hat{\boldsymbol{q}}\times\boldsymbol{S}_\chi)\cdot\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right] \\
& + c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi)\big(\boldsymbol{v}'\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\big) - \epsilon^{ikk'}\,\frac{q^2}{2m_\psi^2}\,\hat{q}^{k'}(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi)\big(\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\big)^{ik}\right] \\
& + c_{15}^{(\psi)}\left[-\frac{q^2}{m_\psi^2}\big(\hat{\boldsymbol{q}}\cdot(\boldsymbol{v}'\times\boldsymbol{S}_\chi)\big)\big(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\big)\right. \\
& \qquad \left. + \frac{iq^3}{2m_\psi^3}\,\boldsymbol{S}_\chi\cdot(\mathbb{1} - \hat{\boldsymbol{q}}\hat{\boldsymbol{q}})\cdot\langle \boldsymbol{L}_\psi\otimes \boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right],
\end{aligned} \tag{30}$$

LATTICE POTENTIAL

- Recall, displacements contain phonon annihilation and creation operators

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega} \int \frac{d^3q}{(2\pi)^3} \sum_{\nu=1}^{3n} 2\pi\, \delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big) \frac{1}{2\omega_{\nu,\boldsymbol{k}}} \left| \sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0} \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}} \widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v}) \right|^2$$

- Evaluate potential, taking one type of each operator in table on previous page

$$c_1^{(\psi)} \langle N_\psi \rangle_{lj} \qquad c_4^{(\psi)}\, \boldsymbol{S}_\chi \cdot \langle \boldsymbol{S}_\psi \rangle_{lj}$$

$$\widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v}) \supset \sum_\alpha \Big[c_1^{(\psi)} \left\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha} \right\rangle_{lj} + c_4^{(\psi)}\, \boldsymbol{S}_\chi \cdot \left\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha} \boldsymbol{S}_{\psi,\alpha} \right\rangle_{lj}$$

$$+ c_8^{(\psi)}\, \boldsymbol{S}_\chi \cdot \left\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha} \boldsymbol{v}_\alpha^\perp \right\rangle_{lj} + c_3^{(\psi)} \frac{i\boldsymbol{q}}{m_\psi} \cdot \left\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha}\, \boldsymbol{v}_\alpha^\perp \times \boldsymbol{S}_{\psi,\alpha} \right\rangle_{lj} \Big]$$

LATTICE POTENTIAL

- Recall, displacements contain phonon annihilation and creation operators

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega}\int \frac{d^3q}{(2\pi)^3}\sum_{\nu=1}^{3n} 2\pi\,\delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big)\frac{1}{2\omega_{\nu,\boldsymbol{k}}}\left|\sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0}\,\frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}}\,\widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v})\right|^2$$

- Evaluate potential, taking one type of each operator in table on previous page

$$c_3^{(\psi)}\left[\left(\frac{i\boldsymbol{q}}{m_\psi}\times\boldsymbol{v}\right)\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} + \frac{1}{2m_\psi^2}\big(\boldsymbol{q}^2\delta^{ik} - q^i q^k\big)\big(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\big)^{ik}\right]$$

$$\widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v}) \supset \sum_\alpha\Big[c_1^{(\psi)}\,\big\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha}\big\rangle_{lj} + c_4^{(\psi)}\,\boldsymbol{S}_\chi\cdot\big\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha}\boldsymbol{S}_{\psi,\alpha}\big\rangle_{lj}$$

$$+\,c_8^{(\psi)}\,\boldsymbol{S}_\chi\cdot\big\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha}\boldsymbol{v}_\alpha^\perp\big\rangle_{lj} + c_3^{(\psi)}\,\frac{i\boldsymbol{q}}{m_\psi}\cdot\big\langle e^{i\boldsymbol{q}\cdot\boldsymbol{x}_\alpha}\,\boldsymbol{v}_\alpha^\perp\times\boldsymbol{S}_{\psi,\alpha}\big\rangle_{lj}\Big]$$

$$c_8^{(\psi)}\,\boldsymbol{S}_\chi\cdot\left[\left(\boldsymbol{v}-\frac{\boldsymbol{q}}{2m_\chi}\right)\langle N_\psi\rangle_{lj} + \frac{i\boldsymbol{q}}{2m_\psi}\times\langle\boldsymbol{L}_\psi\rangle_{lj}\right]$$

2 NEW RESPONSES — L, LXS

$$\frac{i}{2m_\psi}\langle x^i \overrightarrow{\nabla}^k_\alpha - x^k \overrightarrow{\nabla}^i_\alpha \rangle_{lj} = -\frac{1}{2m_\psi}\epsilon_{ikk'}\langle L^{k'}_\alpha \rangle_{lj}$$

$$\left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj}\right)^{ik} = \langle L^i_\psi S^k_\psi \rangle_{lj} \equiv \sum_\alpha \langle L^i_{\psi,\alpha} S^k_{\psi,\alpha} \rangle_{lj}$$

Appears from gradient in vperp

$$\boldsymbol{v}^\perp_\alpha = \boldsymbol{v} - \frac{\boldsymbol{q}}{2m_\chi} - \frac{(\boldsymbol{k} + \boldsymbol{k}')_\alpha}{2m_\psi} = \boldsymbol{v} - \frac{\boldsymbol{q}}{2m_\chi} + \frac{i}{2m_\psi}\overleftrightarrow{\nabla}_\alpha$$

FOUR CRYSTAL RESPONSES $N, \boldsymbol{S}, \boldsymbol{L}, \boldsymbol{L} \otimes \boldsymbol{S}$

- All four responses generate phonons

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega} \int \frac{d^3q}{(2\pi)^3} \sum_{\nu=1}^{3n} 2\pi\, \delta\big(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\big) \frac{1}{2\omega_{\nu,\boldsymbol{k}}} \left| \sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}_j^0} \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}} \widetilde{\mathcal{V}}_j(-\boldsymbol{q}, \boldsymbol{v}) \right|^2$$

Interaction Type	NR Operators	Point-like Response	Composite Response
Coupling to *charge*, $\boldsymbol{v}^\perp$-*independent*	$\mathcal{O}_1^{(\psi)} = \mathbb{1}$ $\mathcal{O}_{11}^{(\psi)} = \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}$	N	-
Coupling to *charge*, $\boldsymbol{v}^\perp$-*dependent*	$\mathcal{O}_5^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_8^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp$	N	L
Coupling to *spin*, $\boldsymbol{v}^\perp$-*independent*	$\mathcal{O}_4^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{S}_\psi$ $\mathcal{O}_6^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_\psi}\big)\big(\boldsymbol{S}_\psi \cdot \frac{\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_9^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\boldsymbol{S}_\psi \times \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{10}^{(\psi)} = \boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}$	S	-
Coupling to *spin*, $\boldsymbol{v}^\perp$-*dependent*	$\mathcal{O}_3^{(\psi)} = \boldsymbol{S}_\psi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_7^{(\psi)} = \boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp$ $\mathcal{O}_{12}^{(\psi)} = \boldsymbol{S}_\chi \cdot \big(\boldsymbol{S}_\psi \times \boldsymbol{v}^\perp\big)$ $\mathcal{O}_{13}^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp\big)\big(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{14}^{(\psi)} = \big(\boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp\big)\big(\boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$ $\mathcal{O}_{15}^{(\psi)} = \big(\boldsymbol{S}_\chi \cdot \big(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\big)\big)\big(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\big)$	S	$L \otimes S$

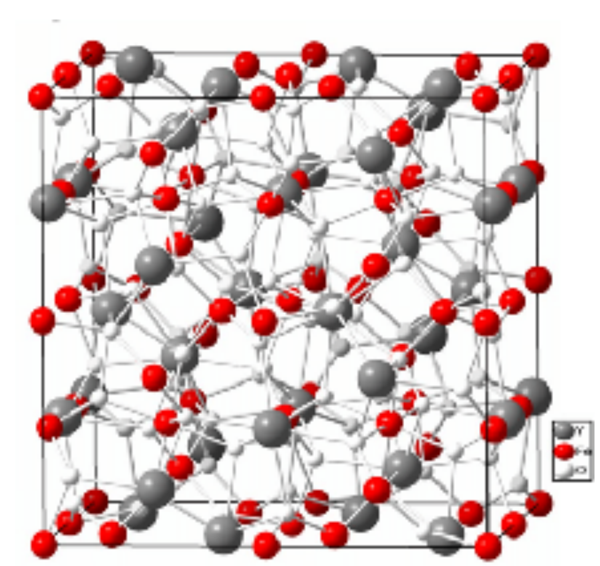
MAGNON COLLECTIVE EXCITATIONS

Trickle, Zhang, KZ 1905.13744

- Magnons couple to S and L responses

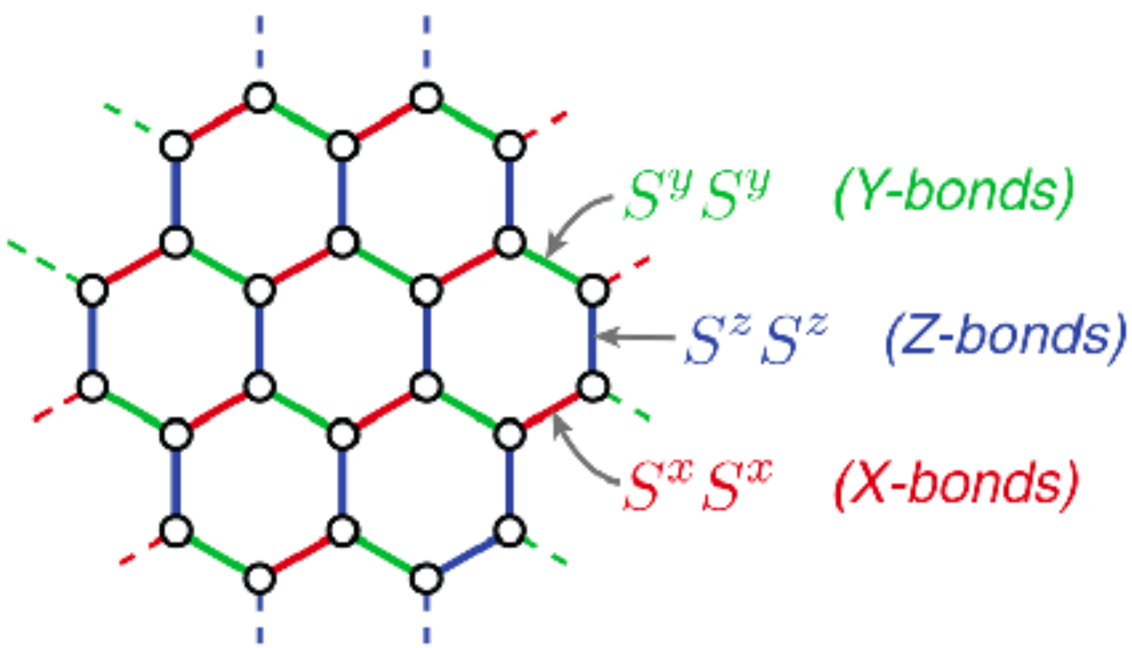
$$\langle \nu, \boldsymbol{k} | \widetilde{\mathcal{V}}(-\boldsymbol{q}, \boldsymbol{v}) | 0 \rangle = \sum_{l,j} e^{i\boldsymbol{q}\cdot\boldsymbol{x}_{lj}} \boldsymbol{f}_j(-\boldsymbol{q}, \boldsymbol{v}) \cdot \langle \nu, \boldsymbol{k} | \boldsymbol{S}_{lj} | 0 \rangle$$

- Project onto ionic spins $\langle \boldsymbol{S}_e \rangle_{lj} \to \lambda_{S,j} \boldsymbol{S}_{lj}$, $\langle \boldsymbol{L}_e \rangle_{lj} \to \lambda_{L,j} \boldsymbol{S}_{lj}$
- Expand in Holstein-Primakoff bosons, Diagonalize spin Hamiltonian (nearest neighbor Heisenberg interaction)
- Need magnetic material to have non-zero spin expectation value over unit cell
- YIG as benchmark



SPIN-ORBIT MATERIALS

- Angular momentum — spin-orbit-entangled Mott insulator
- Effective spins $\lambda_{S,j} = -\frac{1}{3}$, $\lambda_{L,j} = -\frac{4}{3}$



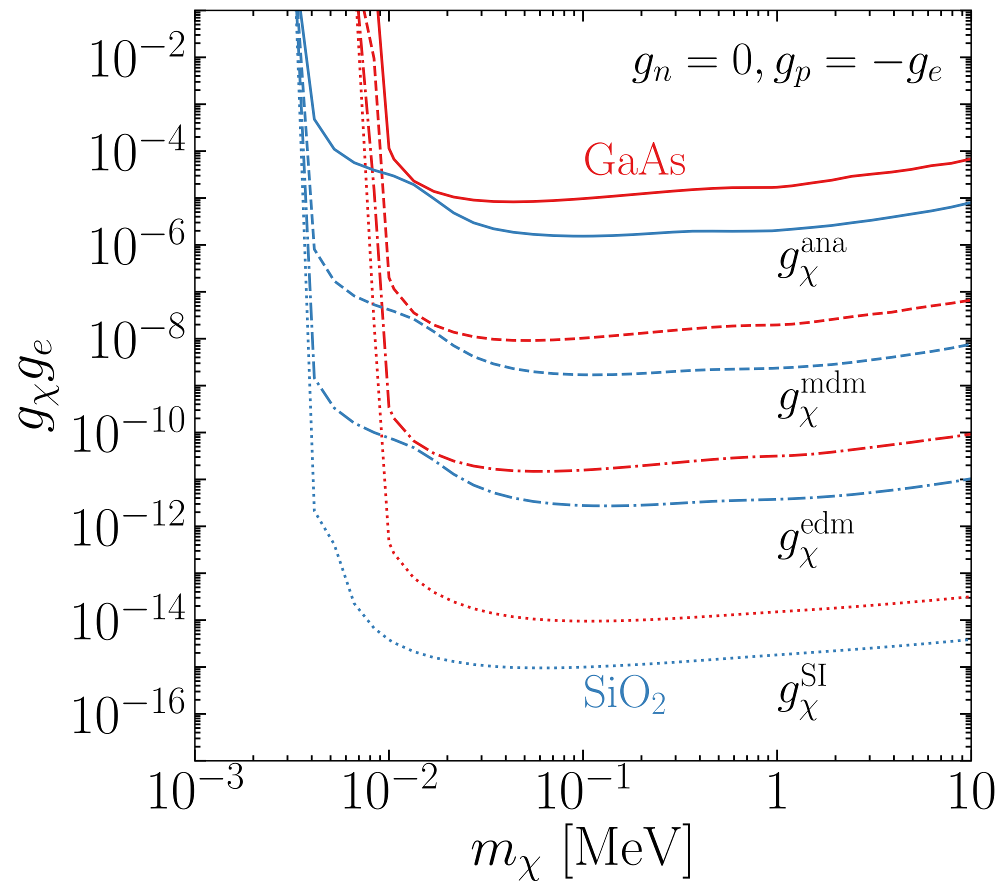
α-$RuCl_3$

- Kitaev material with bond directional coupling
- Antiferromagnetic order

DIPOLE INTERACTIONS

Multipole DM models	Electric dipole	$V_\mu\Big(g_\chi J^\mu_{\text{edm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$	N
	Magnetic dipole	$V_\mu\Big(g_\chi J^\mu_{\text{mdm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m_\chi^2}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\text{eff}}\frac{\boldsymbol{q}^2}{m_\chi m_\psi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}}\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$	$N,\ S,\ L$
	Anapole	$V_\mu\Big(g_\chi J^\mu_{\text{ana},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\text{mdm},\psi}\big)\Big)$	$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m_\chi^2}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}}\frac{\boldsymbol{q}^2}{2m_\chi^2}\frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2+m_V^2}$	$N,\ S,\ L$

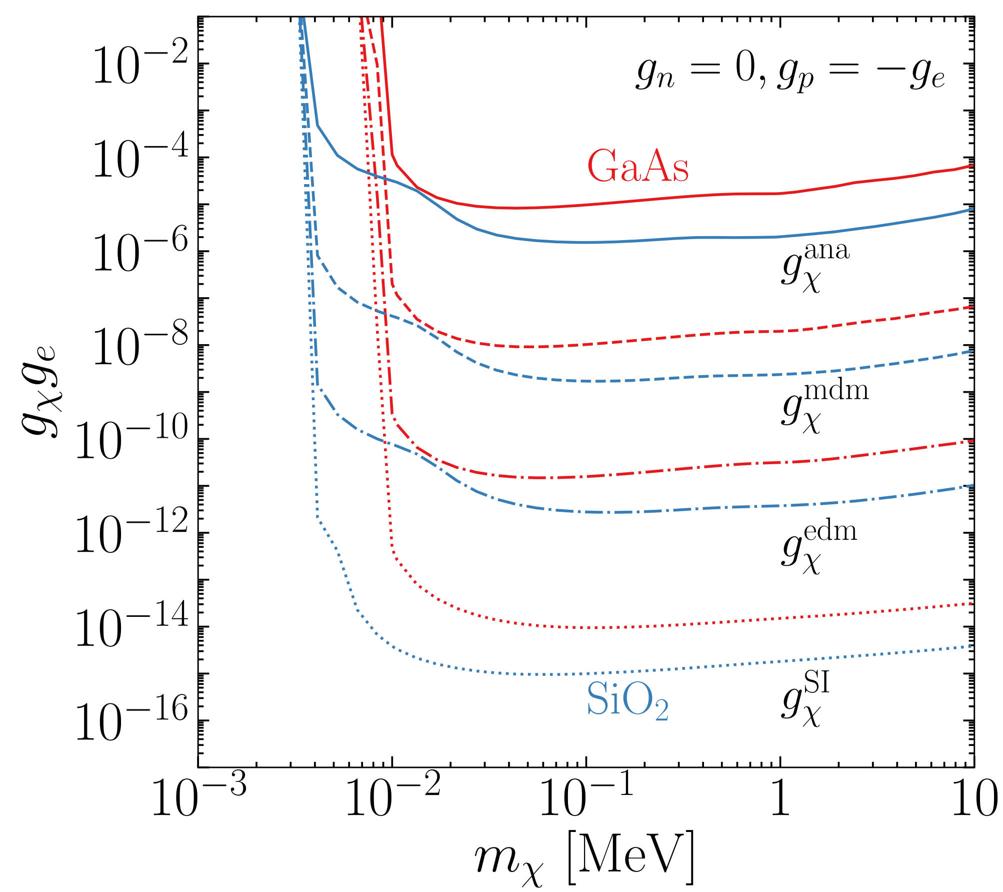
All have N response, probed by phonons



DIPOLE INTERACTIONS

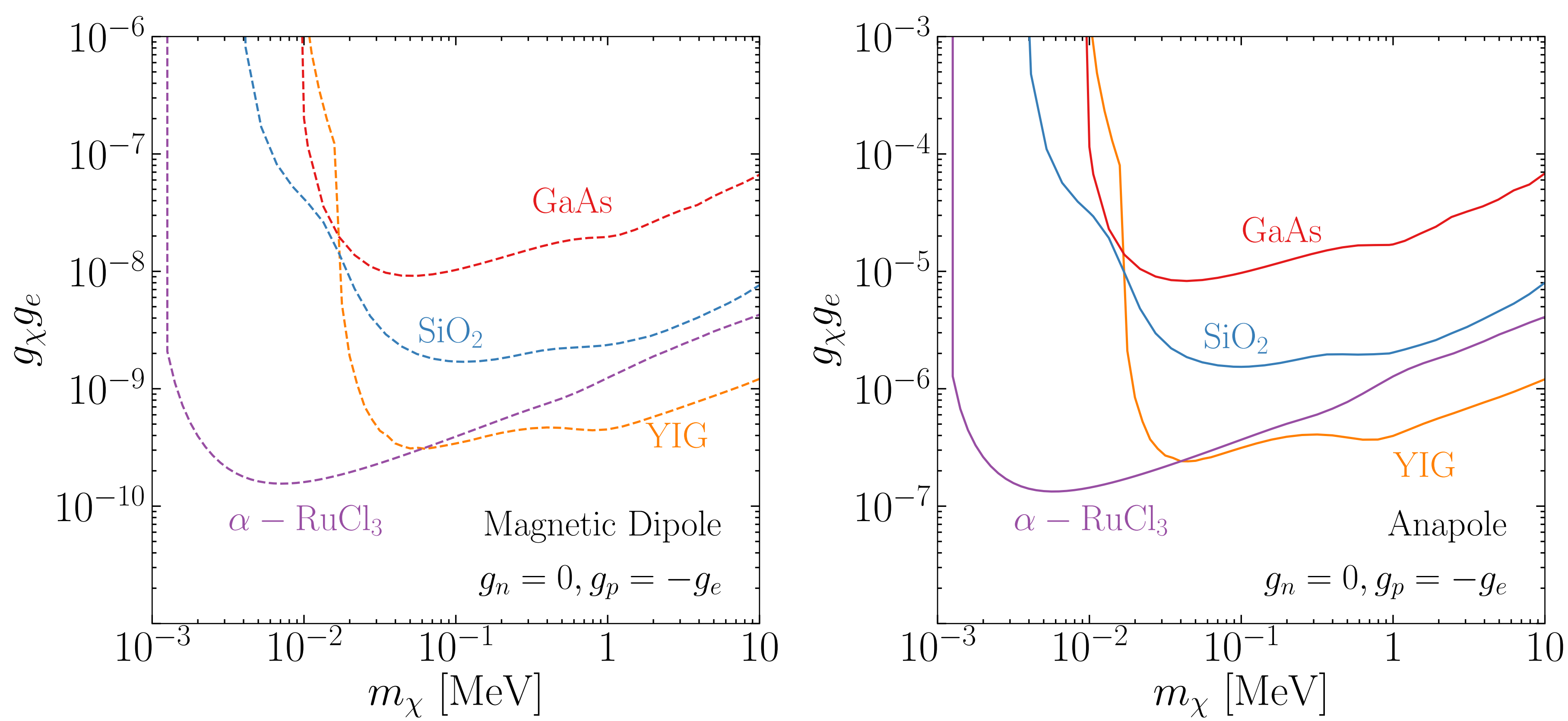
$$
\begin{aligned}
\widetilde{V}_{lj}(-\boldsymbol{q},\boldsymbol{v}) = \sum_{\psi=p,n,e} & c_1^{(\psi)} \langle N_\psi \rangle_{lj} \\
& + c_3^{(\psi)} \left[-\frac{iq}{m_\psi} \boldsymbol{v}' \cdot \left(\hat{\boldsymbol{q}} \times \langle \boldsymbol{S}_\psi \rangle_{lj} \right) + \frac{q^2}{2m_\psi^2} \left(\delta^{ik} - \hat{q}^i \hat{q}^k \right) \left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \right)^{ik} \right] \\
& + c_4^{(\psi)} \boldsymbol{S}_\chi \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \\
& + c_5^{(\psi)} \left[\frac{i\boldsymbol{q}}{m_\psi} \cdot \left(\boldsymbol{v}' \times \boldsymbol{S}_\chi \right) \langle N_\psi \rangle_{lj} + \frac{q^2}{2m_\psi^2} \boldsymbol{S}_\chi \cdot \left(\mathbb{1} - \hat{\boldsymbol{q}}\hat{\boldsymbol{q}} \right) \cdot \langle \boldsymbol{L}_\psi \rangle_{lj} \right] \\
& + c_6^{(\psi)} \frac{q^2}{m_\psi^2} \left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi \right) \left(\hat{\boldsymbol{q}} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \right) \\
& + c_7^{(\psi)} \left[\boldsymbol{v}' \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} + \epsilon^{ikk'} \frac{iq^{k'}}{2m_\chi} \left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \right)^{ik} \right] \\
& + c_8^{(\psi)} \left[\left(\boldsymbol{v}' \cdot \boldsymbol{S}_\chi \right) \langle N_\psi \rangle_{lj} + \frac{iq}{2m_\psi} \boldsymbol{S}_\chi \cdot \left(\hat{\boldsymbol{q}} \times \langle \boldsymbol{L}_\psi \rangle_{lj} \right) \right] \\
& + c_9^{(\psi)} \frac{iq}{m_\psi} \boldsymbol{S}_\chi \cdot \left(\langle \boldsymbol{S}_\psi \rangle_{lj} \times \hat{\boldsymbol{q}} \right) \\
& + c_{10}^{(\psi)} \frac{i\boldsymbol{q}}{m_\psi} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \\
& + c_{11}^{(\psi)} \frac{i\boldsymbol{q}}{m_\psi} \cdot \boldsymbol{S}_\chi \langle N_\psi \rangle_{lj} \\
& + c_{12}^{(\psi)} \left[\left(\boldsymbol{v}' \times \boldsymbol{S}_\chi \right) \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} + \frac{iq}{2m_\psi} \left(\left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi \right) \delta^{ik} - \hat{q}^k S_\chi^i \right) \left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \right)^{ik} \right] \\
& + c_{13}^{(\psi)} \left[\frac{iq}{m_\psi} \left(\boldsymbol{v}' \cdot \boldsymbol{S}_\chi \right) \left(\hat{\boldsymbol{q}} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \right) + \frac{q^2}{2m_\psi^2} \left(\hat{\boldsymbol{q}} \times \boldsymbol{S}_\chi \right) \cdot \langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \cdot \hat{\boldsymbol{q}} \right] \\
& + c_{14}^{(\psi)} \left[\frac{iq}{m_\psi} \left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi \right) \left(\boldsymbol{v}' \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \right) - \epsilon^{ikk'} \frac{q^2}{2m_\psi^2} \hat{q}^{k'} \left(\hat{\boldsymbol{q}} \cdot \boldsymbol{S}_\chi \right) \left(\langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \right)^{ik} \right] \\
& + c_{15}^{(\psi)} \left[-\frac{q^2}{m_\psi^2} \left(\hat{\boldsymbol{q}} \cdot \left(\boldsymbol{v}' \times \boldsymbol{S}_\chi \right) \right) \left(\hat{\boldsymbol{q}} \cdot \langle \boldsymbol{S}_\psi \rangle_{lj} \right) \right. \\
& \qquad \left. + \frac{iq^3}{2m_\psi^3} \boldsymbol{S}_\chi \cdot \left(\mathbb{1} - \hat{\boldsymbol{q}}\hat{\boldsymbol{q}} \right) \cdot \langle \boldsymbol{L}_\psi \otimes \boldsymbol{S}_\psi \rangle_{lj} \cdot \hat{\boldsymbol{q}} \right],
\end{aligned}
$$

$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	N
$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{m_\chi m_\psi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{m_\psi}{m_\chi} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	$N,\ S,\ L$
$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\text{eff}} \frac{\boldsymbol{q}^2}{2m_\chi^2} \frac{g_\chi g_\psi^{\text{eff}}}{\boldsymbol{q}^2 + m_V^2}$	$N,\ S,\ L$



DIPOLE INTERACTIONS — COMPARE SI AND SD REACH

Multipole DM models	Electric dipole	$V_\mu\Big(g_\chi J^\mu_{\mathrm{edm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\mathrm{mdm},\psi}\big)\Big)$	$c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$	N
	Magnetic dipole	$V_\mu\Big(g_\chi J^\mu_{\mathrm{mdm},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\mathrm{mdm},\psi}\big)\Big)$	$c_1^{(\psi)} = \frac{\boldsymbol{q}^2}{4m_\chi^2}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_4^{(\psi)} = \widetilde{\mu}_\psi^{\mathrm{eff}}\frac{\boldsymbol{q}^2}{m_\chi m_\psi}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_5^{(\psi)} = \frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_6^{(\psi)} = -\widetilde{\mu}_\psi^{\mathrm{eff}}\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$	$N,\ S,\ L$
	Anapole	$V_\mu\Big(g_\chi J^\mu_{\mathrm{ana},\chi} + g_\psi\big(J^\mu_{V,\psi} + \delta\widetilde{\mu}_\psi J^\mu_{\mathrm{mdm},\psi}\big)\Big)$	$c_8^{(\psi)} = \frac{\boldsymbol{q}^2}{2m_\chi^2}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$ $c_9^{(\psi)} = -\widetilde{\mu}_\psi^{\mathrm{eff}}\frac{\boldsymbol{q}^2}{2m_\chi^2}\frac{g_\chi g_\psi^{\mathrm{eff}}}{\boldsymbol{q}^2+m_V^2}$	$N,\ S,\ L$



Polar crystals - N response

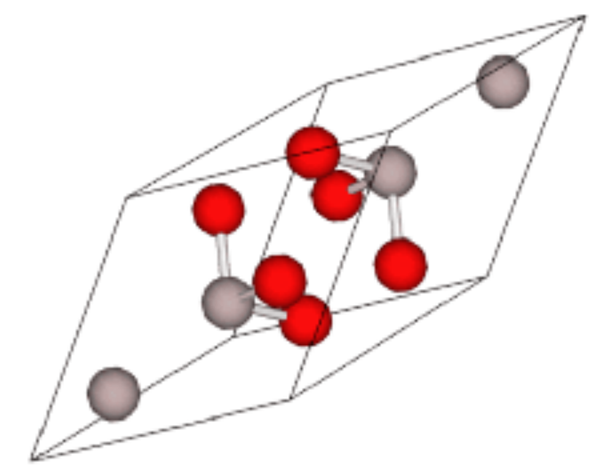
YIG - S response

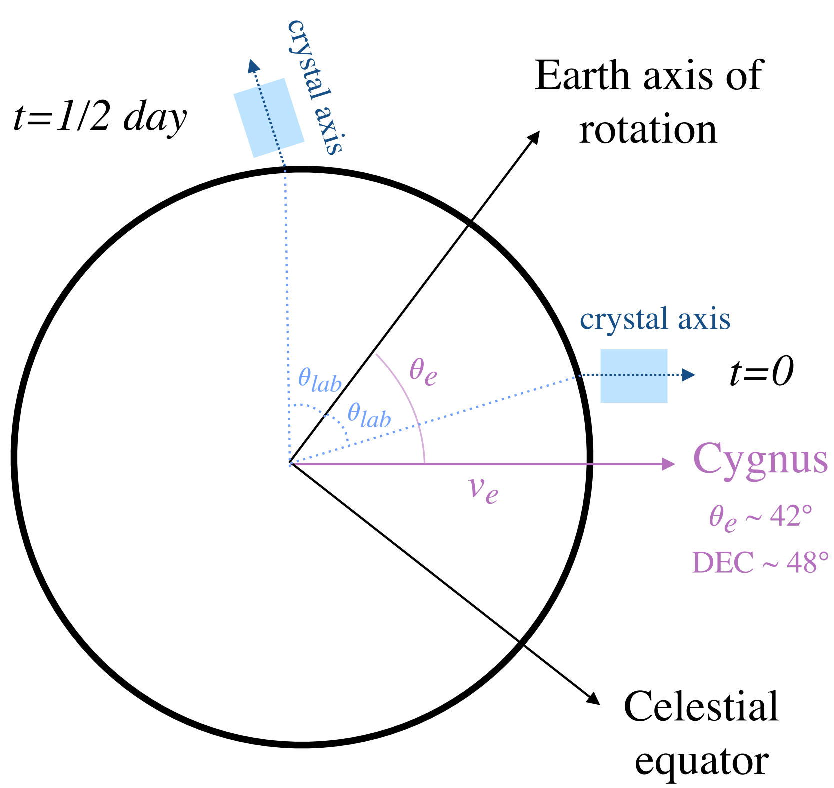
Kitaev material - S&L

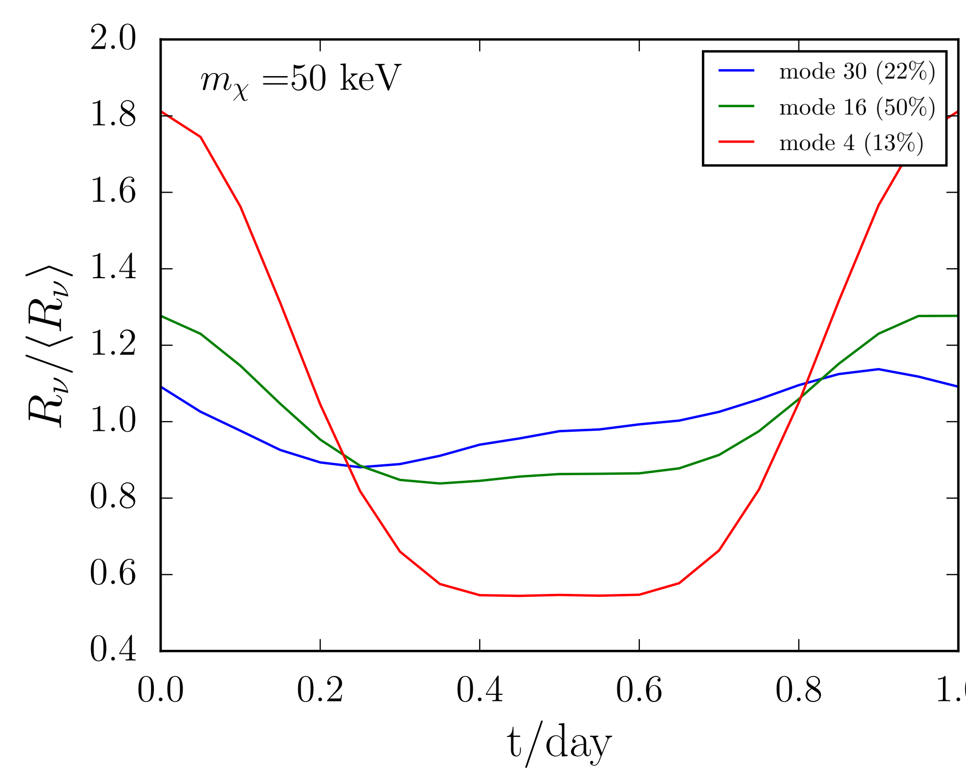
DIRECTIONALITY IN ANISOTROPIC MATERIALS!

Griffin, Knapen, Lin, KZ 1807.10291
Coskuner, Trickle, Zhang, KZ 2102.xxxxx

- Crystal Lattice is not Isotropic
- Especially pronounced in certain materials, like sapphire







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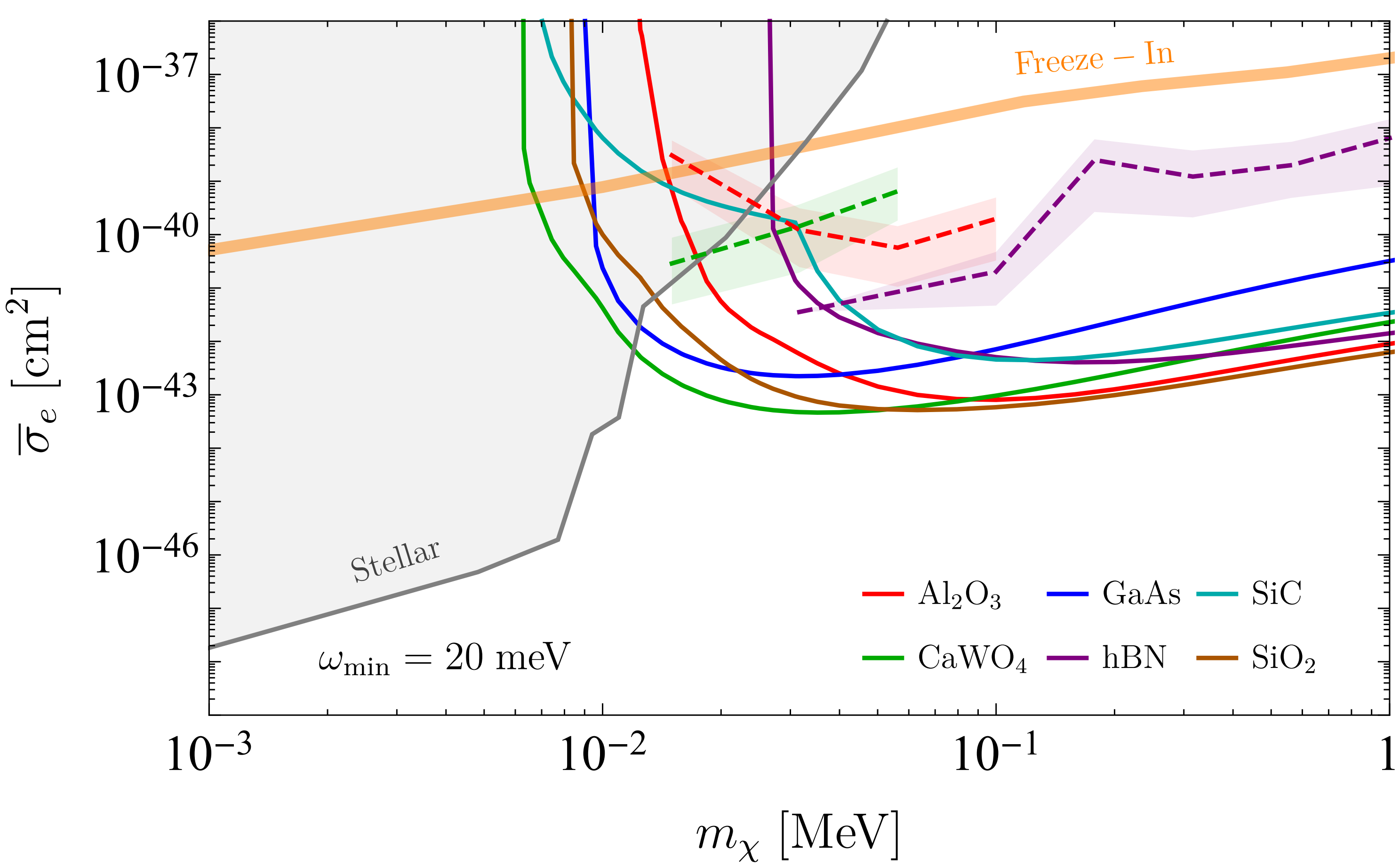

EXPERIMENTAL PROSPECTS

- Sensor to detect phonons coupled to DM “absorber”
- Zero-field read-out of phonons
- Now funded by DoE — TESSERACT (TES with Sub-EV Resolution and Cryogenic Targets)
- For a polar crystal target — Sub-eV Polar Interactions Cryogenic Experiment (SPICE)

Snowmass2021 - Letter of Interest

The TESSERACT Dark Matter Project

Thematic Areas:

- IF1 Quantum Sensors
- IF8 Noble Elements
- CF1 Dark Matter: Particle-like
- CF2 Dark Matter: Wavelike

Contact Information:
Dan McKinsey (LBNL and UC Berkeley) [daniel.mckinsey@berkeley.edu]:
TESSERACT Collaboration

Authors:
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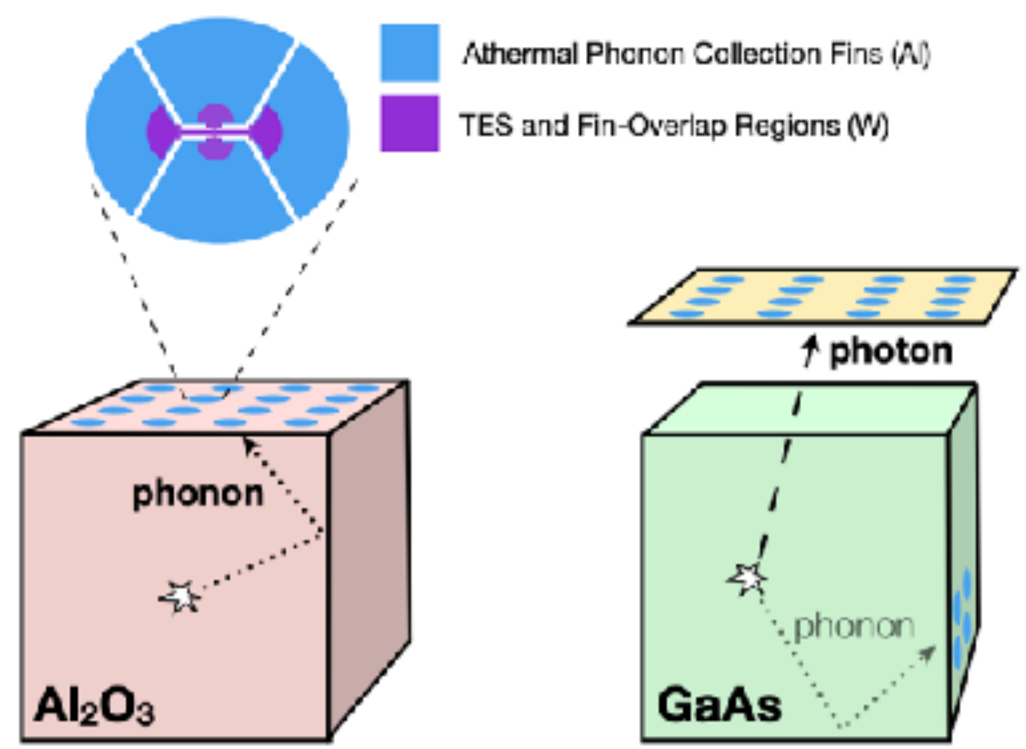

SUMMARY

- Collective excitations provide a novel path to detect light DM
- Theory framework for computing DM interaction rates in materials is now well-developed
- New experiments such as SPICE have broad discovery potential for light DM