Large-momentum effective theory for non-perturbative parton structure

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ALL-thing EFT Lecture 12

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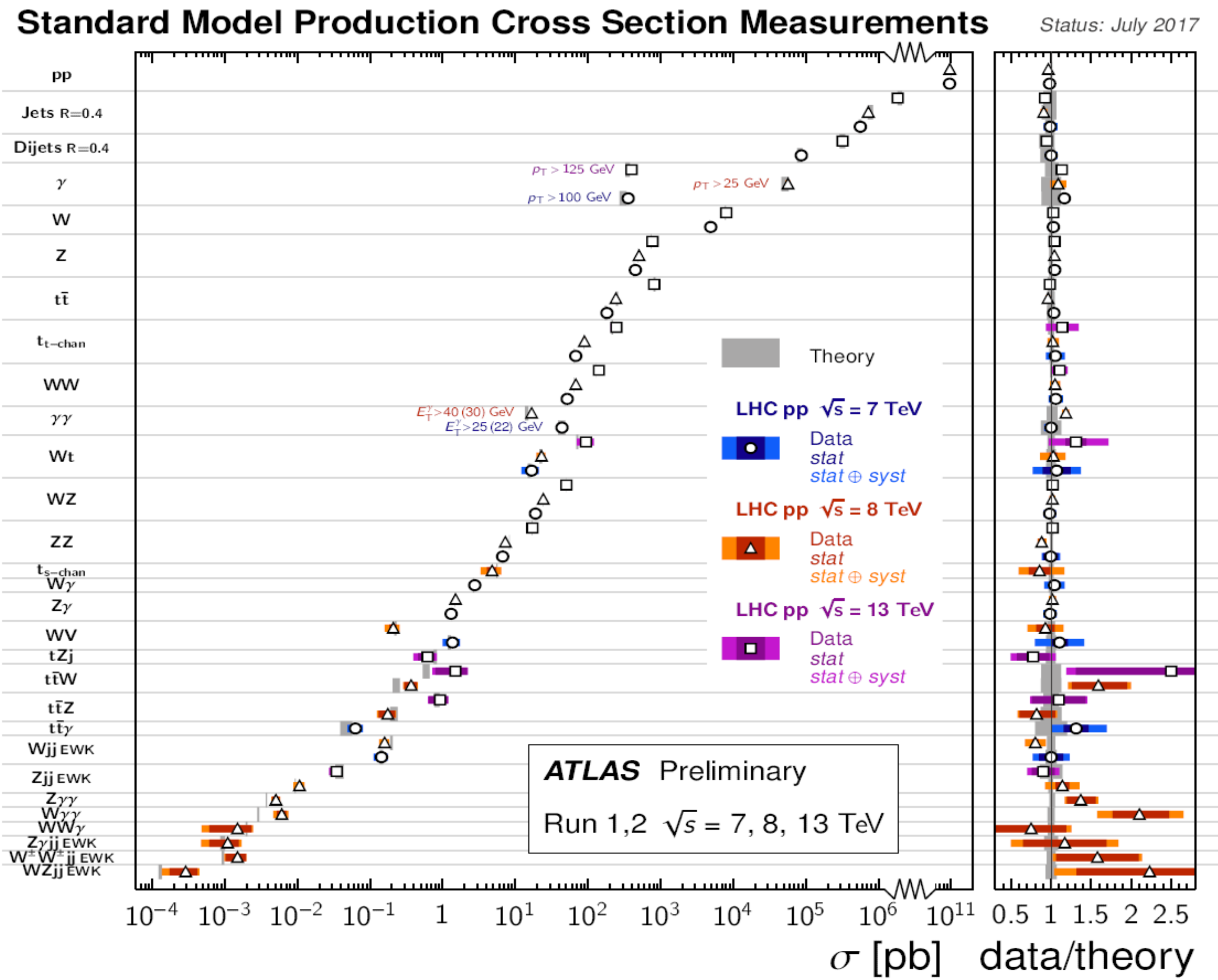
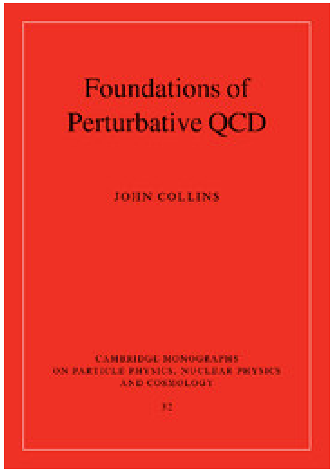
Outline

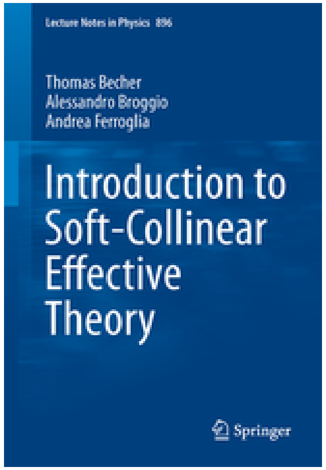
- QCD factorization as an EFT
- Why are partons hard to calculate?
- Back to Feynman: partons in infinite momentum frame
- Large-momentum expansion as an EFT: large-momentum effective theory (LaMET)
- Applications

QCD factorization as EFT


Standard Model Production Cross Section Measurements
Status: July 2017
pp
Jets R=0.4
Dijets R=0.4
γ
W
Z
tt̄
t_t-chan
WW
γγ
Wt
WZ
ZZ
t_s-chan
Wγ
Zγ
WV
tZj
tt̄W
tt̄Z
tt̄γ
Wjj EWK
Zjj EWK
Zγγ
Wγγ
WWγ
Zγjj EWK
W±W±jj EWK
WZjj EWK
pT > 125 GeV
pT > 100 GeV
pT > 25 GeV
E_T^γ > 40 (30) GeV
E_T^γ > 25 (22) GeV
Theory
LHC pp √s = 7 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 8 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 13 TeV
Data
stat
stat ⊕ syst
ATLAS Preliminary
Run 1,2 √s = 7, 8, 13 TeV
10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6 10^11
0.5 1 1.5 2 2.5
σ [pb]
data/theory

EFT within SM

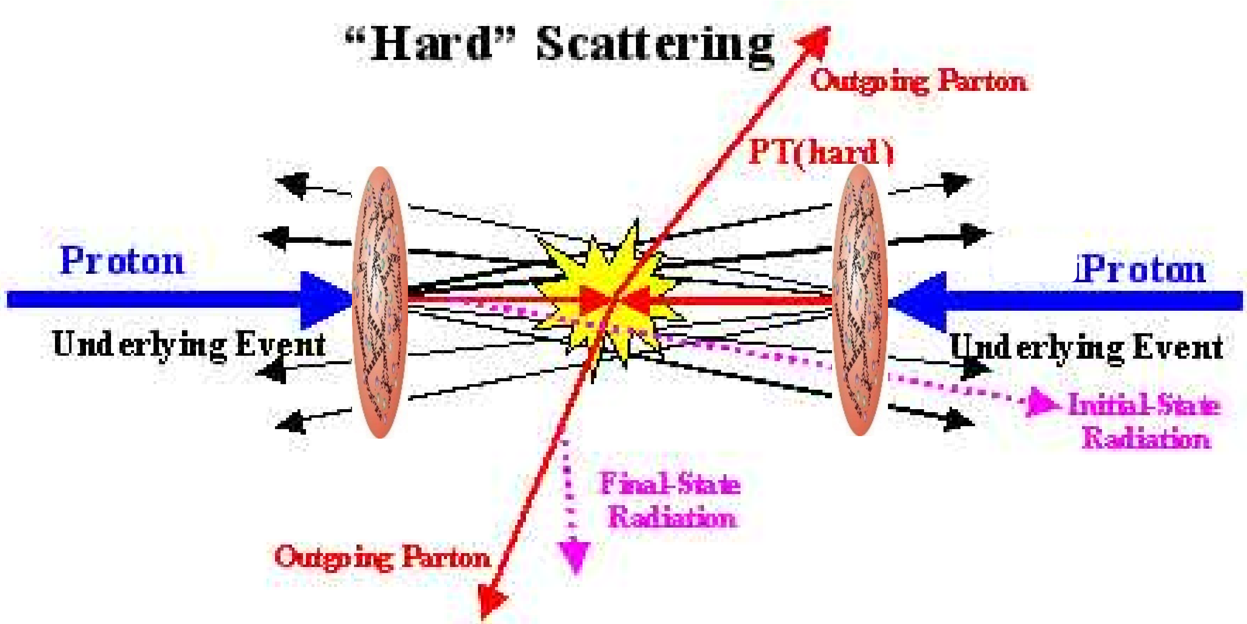




- QCD factorization
- Soft-Collinear Effective Theory
  - All high-energy scale physics can be computed in QCD loop expansion.
  - All hadron-scale physics can be parametrized by “low-energy constants”

Parton distribution functions (distribution amplitudes)
Soft functions
Fragmentation functions
……

QCD factorization



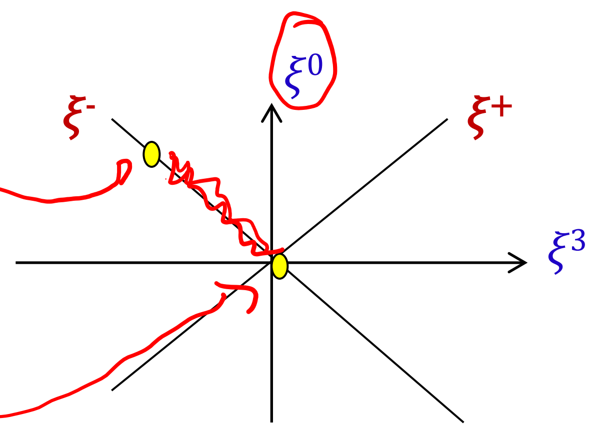
- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a, \mu) f_{b/B}(x_b, \mu) \hat{\sigma} + O(1/Q^2)$$

PDFs as light-front correlations

- Quark and gluon fields are distributed along the light-cone ξ^- direction, where $\xi^{\pm} = (\xi^0 \pm \xi^3)/\sqrt{2}$

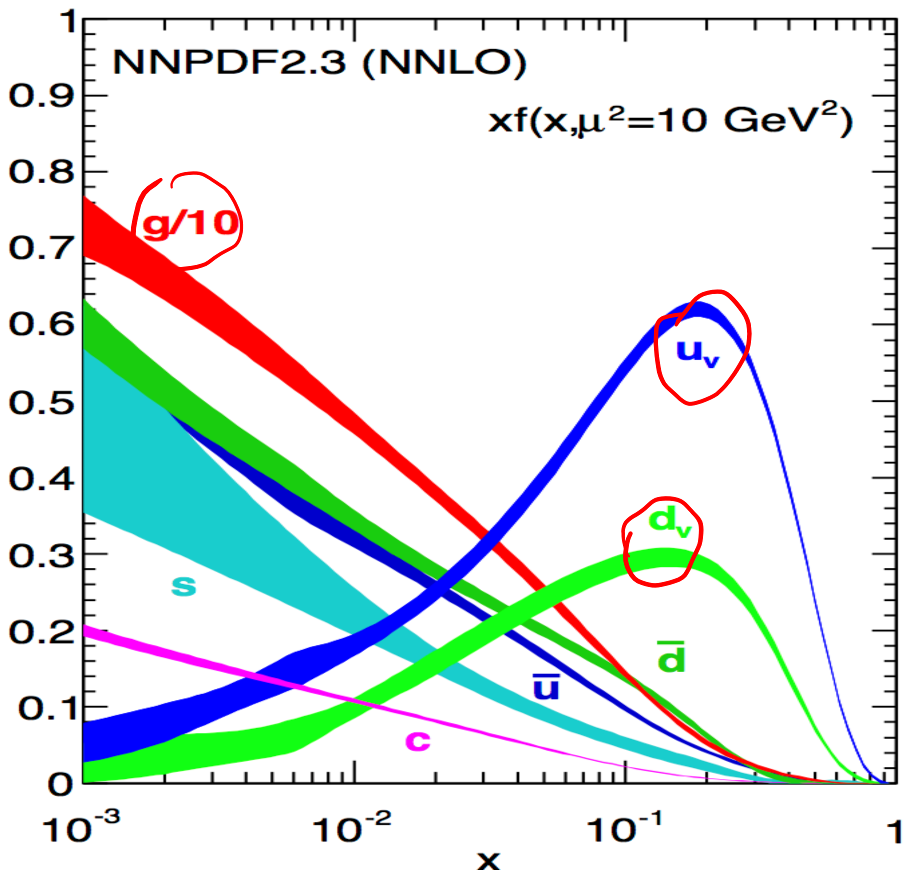
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle \; ,$$



- All “low-energy constants” involve light-front correlations, and can be determined by fitting to data.

Phenomenological PDFs

- Use experimental data (50 yrs) to extract PDFs



CTEQ
NNPDF
MMHT etc

J. Gao, et al, Phys. Rept. 742 (2018) 1-121

How does the parametrization bias the fit?

Why are partons hard to calculate?

Partons as dynamical correlations

- Monte Carlo simulations have not been very successful with quantum real-time dynamics.

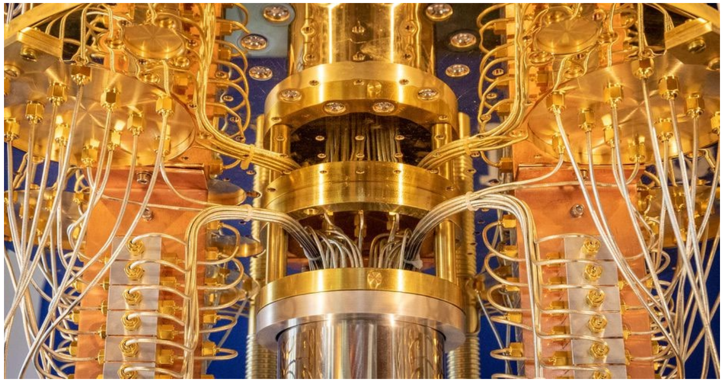
$$\exp(-iHt)$$

an oscillating phase factor!

- "Sign problem": Hubbard model for high Tc.
- Signals are exponentially small!
- Quantum computer?

Kreshchuk et al, 2009.07885

Light-front field theory on current quantum computers



Light-front quantization

- Changes of coordinates (Chang, Ma, Kogut, Soper…)

  take $\xi^{+} = (\xi^{0}+\xi^{3})/\sqrt{2}$ as the new "time"

  and quantize QCD

- Paul A.M. Dirac (1949)

 Forms of Relativistic Dynamics

Rev. Mod. Phys. 21 (1949) 392-399.

 "Front form"

 Light-front Hamiltonian dynamics

Difficulties in LFQ

- All slow-moving stuff in zero-modes (vacuum). Renormalization of the theory is highly non-trivial
- It is a strongly coupled Hamiltonian problem!

There is no demonstration that the weak coupling expansion actually works for QCD.

K. Wilson et. al. Phys. Rev. D49 (1994)

Partons and critical phenomena

- Fourier trans. of PDFs gives a small-x behavior,

  f(x) → $x^{-\alpha}$
- When FT back to position space, one has

  $C(\lambda) \sim \lambda^{\alpha-1}$

  This corresponds to "infinite correlation" length

  $C(\lambda) \sim \exp(-\frac{\xi}{\lambda})$ with $\xi \rightarrow \infty$

No condensed matter theorists directly solve critical phenomena at T=Tc!

Parton physics using lattice QCD

- Lattice QCD (EFT) is a Euclidean field theory.
- One can form local moments to get rid of the time-dependence $\langle x^n \rangle = \int q(x) x^n dx$
  →matrix elements of local operators
  - One can only calculate lowest few moments in practice. H.W. Lin et al, 2018 review
  - Higher moments quickly become difficult.
- Many other parton properties cannot be not related to local operators, e. g. TMDs, soft functions, etc.

Higher-moments through Euclidean $\langle P|J^\mu(0)J^\nu(x)|P\rangle$ Correlators

- To overcome the problems with local operators, It has been proposed to calculate the hadron tensor $W^{\mu\nu}$ through analytical continuation.

  K. F. Liu et al., 1994, 1996, 1999, 2000,..

- Euclidean correlators as Compton amplitude have been proposed to compute higher moments ("OPE with OPE")

  Aglietti, Martinelli, Capitani, Dawson, Chambers et al, Detmold, Lin, et al, 1998, 1999, 2006, 2017,2018, 2020

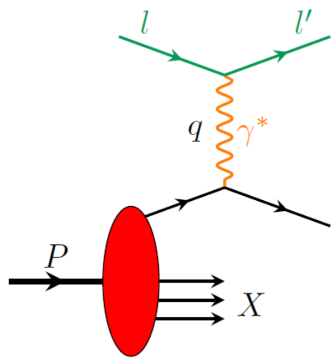
- Current correlators has also been proposed to access higher moments directly through coordinate space OPE

  Braun, Mueller, Bali et al, Ma & Qiu, Suffian et al, 2008, 2017, 2020...

Back to Feynman: Partons in infinite-momentum frame

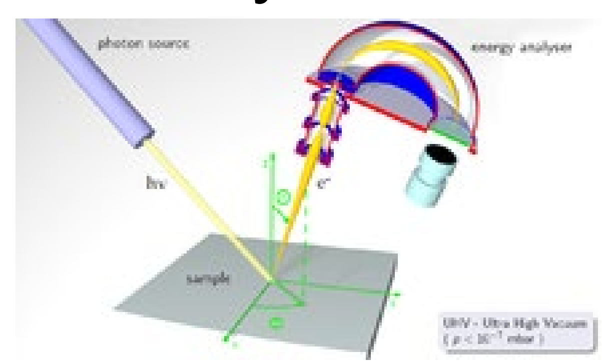
Knock-out reaction in quantum many-body systems

- Electron-proton deep-inelastic scattering (DIS)



- Knock-out scattering in non-relativistic systems
  - e-scattering on atoms
  - ARPES in CM systems
  - Neutron scattering on liquid-^{4}He

…



Momentum distribution in non-relativistic systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega|\hat{\psi}^+(\vec{r})\hat{\psi}(0)|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations. (Quantum Monte Carlo)

Difference between relativistic and non-relativistic systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

  In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$

$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \rightarrow \infty$

Feynman's partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp\, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \to \infty$ limit,

$$f(k^z, P^z) \rightarrow_{P^z \to \infty} f(x) \quad with\ x = \frac{k^z}{P^z},$$

assuming the limit exists!

A Euclidean formulation of partons

- Calculate the Euclidean correlation

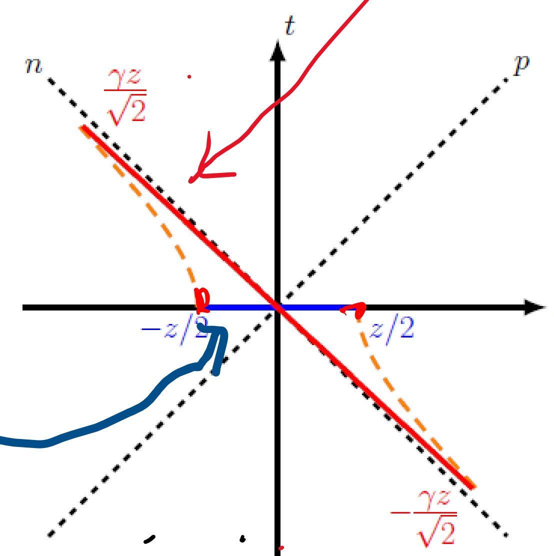
$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z) \Gamma \psi(0) | P^z = \infty \rangle$$

$$\lambda = \lim_{P^z \to \infty, z \to 0} (z P^z).$$

- Parton distribution

$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Minkowski



Relation between two parton formalisms

- Partons in LF correlation formamlism
  - Use LF collinear field operators
  - Parton physics in LF correlations (Heisenberg picture)
  - Independent of external state momentum
- Infinite-momentum-frame parton formalism
  - Use infinite-momentum states to select parton modes
  - Euclidean correlations (Schrodinger picture)
  - Can use different operators: universality class

Large-momentum expansion and EFT

Large momentum approximation

- Approximate p=∞ in

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z)\Gamma\psi(0) | P^z = \infty \rangle$$

by a large P (X. Ji, 2013, 2014)

- Lattice QCD: approximate continuum by discrete points.

cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi/a$

a ~ 0.1 fm → Λ ~ 2 GeV

HQET: $\epsilon = \Lambda_{QCD}/m_Q$

using $m_Q = \infty$ to approximate finite m_Q

m_c =1.5 GeV!

Naïve Large-momentum expansion

- Feynman assumed the $P \to \infty$ limit exists, the limiting process is controlled by expansion,

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

where M is a bound-state scale,

P^z is a large-momentum scale.

- The limit is non-trivial. It must be studied in the context of a QFT (only a field theory can support ∞ momentum modes)

Dimensional analysis

- $\epsilon = \left(\frac{M}{P^z}\right)^2$ is an expansion parameter

  M =1 GeV, P=2 GeV

  $\epsilon = 1/4$

  the expansion may already work.

QFT subtleties

- $f(k^z, P^z)$ is not analytic at $P^z = \infty$ because of the UV cut-off Λ_{UV}
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (QCD factorization)
- Due to asymptotic freedom, the difference is perturbative!

Standard matching using EFT

Large-momentum expansion in QCD

- For finite momentum, one can have a factorization formula for large γ□ $(2-5)$:

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x,\mu) dx + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\right),$$

- **Power counting:**

  Large scales: parton momentum $k^z = yP^z$, hadron remnant momentum $k^z = (1-y)P^z$

  The expansion works away from y ~ 0, 1

Where an EFT?

- Split the Hilbert space into P+Q=1, P is model space and EFT integrates out the dof's in Q.
- LF correlations contain all momentum range, and LaMET P-space contains all modes with momentum between 0 and P^z with

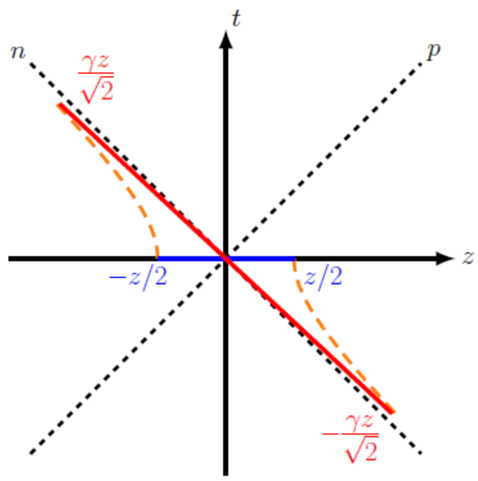
  cutoff $\Lambda_{UV} \gg P^z$.
- Q-Space contains all modes between P^z and ∞

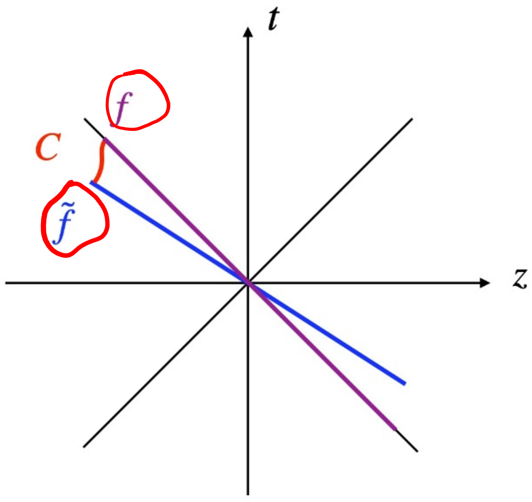
Similar to lattice QCD approximating the continuum theory where some finite lattice spacing effects are accounted for by high-dim operators.

An EFT expansion

- Partons in EFT expansion (all P dependence, a sort of regulator, cancels)

$$f(x,\mu) = \tilde{f}(x,P) \circ C\left(x,\frac{\mu}{P}\right) + \tilde{f}_2(x,P) \circ \frac{C_2\left(x,\frac{\mu}{P}\right)}{P^2} + \ldots$$





Calculating the x-dependence!

- Phenomenologically, the x-dependence of PDFs has been modelled or parametrized in all types of fitting.
- In EFT expansion, x-dependence is obtained *point by point* through calculation.
- The expansion works for x in the middle range (x_{min}, x_{max})

$p^z \to \infty$

[0, 1]

Momentum renormalization group equation

- Mom.dis. $f(k^z, P^z)$ has a non-trivial dependence on P^z (H is frame-dependent).
- At large P^z, this dependence shall be calculable in perturbation theory

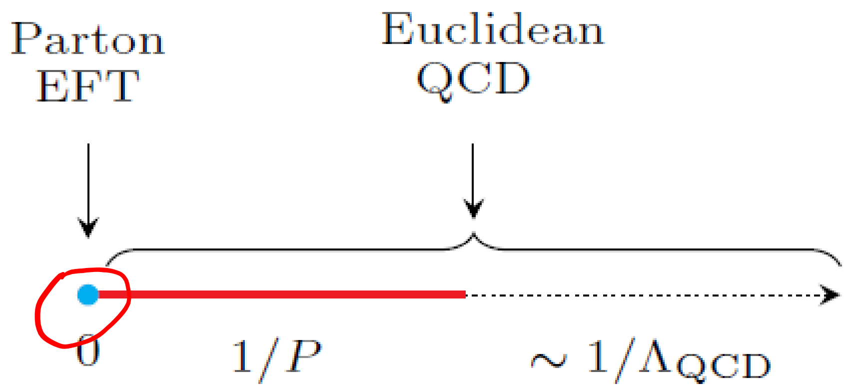
$$\frac{\partial O(P^z)}{\partial \ln P^z} = \gamma_P(\alpha_s) O(P^z) ,$$

Momentum RGE

$$P^z \frac{\partial}{\partial P^z} \tilde{q}(y, P^z, \mu) = \int_0^1 \frac{dt}{|t|} P_{qq}(t) \times \tilde{q}\left(\frac{y}{t}, tP^z, \mu\right) - 2\gamma_F \tilde{q}(y, P^z, \mu) .$$

- DGLAP evolution is related to the change of mom.dis. with different CoM motion.

LaMET and critical phenomena



Universality

- One can practically choose ANY composite operator with arguments $z_1 \ldots z_n$, so long as γ large enough, they give the same collinear or soft physics.
- For different operators, flowing into the fixed point of large momentum will have different rates (which is faster?), but the limit is the same.

Hatta, Ji, Zhao (2017)

Applications

- Approximate light-front correlations by near light-front correlations.
- Simulate operator dynamical correlations through fast moving hadron states.

Application 1: PDFs:

- PDF can be obtained from large momentum limit of a correlation

$$\langle P|\bar{\psi}(z)\Gamma W(z,0)\psi(0)|P\rangle$$

  - Γ can be γ^0or γ^3 or any combination.
  - W is a straight-line Wilson link

This is the starting point of quarsi-PDF,

X. Ji, Phys. Rev. Lett. 110 (2013)

Factorization was conjectured. The full-proved given by Ma and Qiu, PRD98 (2018) 074021

State-of-the-art calculations

- ETMC PRL121(2018) 112001; …
- LP3 PRL121(2018) 242003;…
- LPC PRD 101 (2020) 3, 034020;…
- Jlab & BNL groups:
- Some analyses now have controlled approximations
  - One-loop matching and scale setting, renormalization
  - Excited states, higher twist corrections

ETMC recent result

C. Alexandrou et al, Phys.Rev.Lett. 121 (2018) no.11, 112001

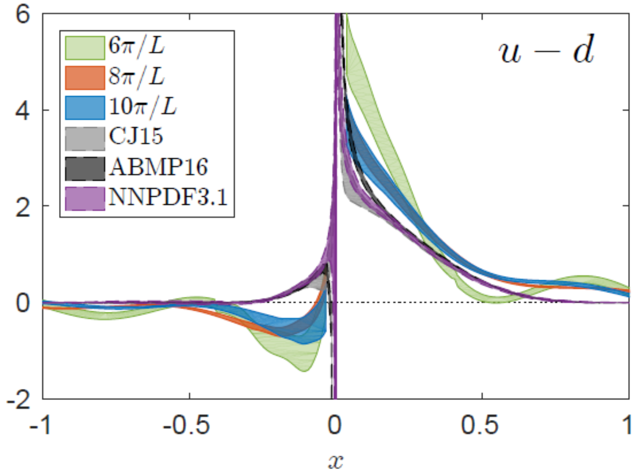


FIG. 4: Comparison of unpolarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), and ABMP16 [39] (NNLO), NNPDF [40] (NNLO) and CJ15 [38] (NLO) phenomenological curves.

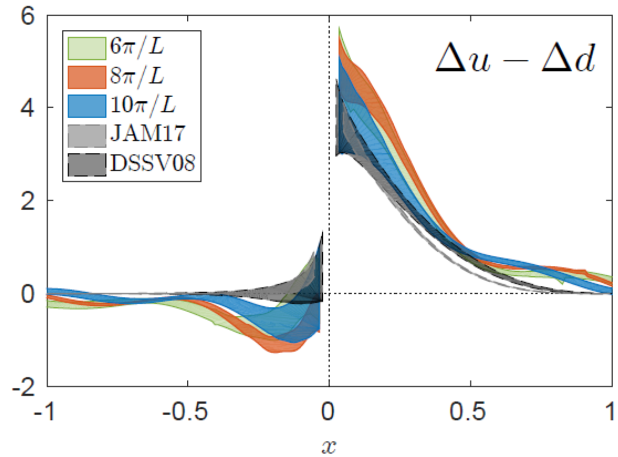


FIG. 5: Comparison of polarized PDF at momenta $\frac{6\pi}{L}$ (green band), $\frac{8\pi}{L}$ (orange band), $\frac{10\pi}{L}$ (blue band), DSSV08 [41] and JAM17 NLO phenomenological data [42].

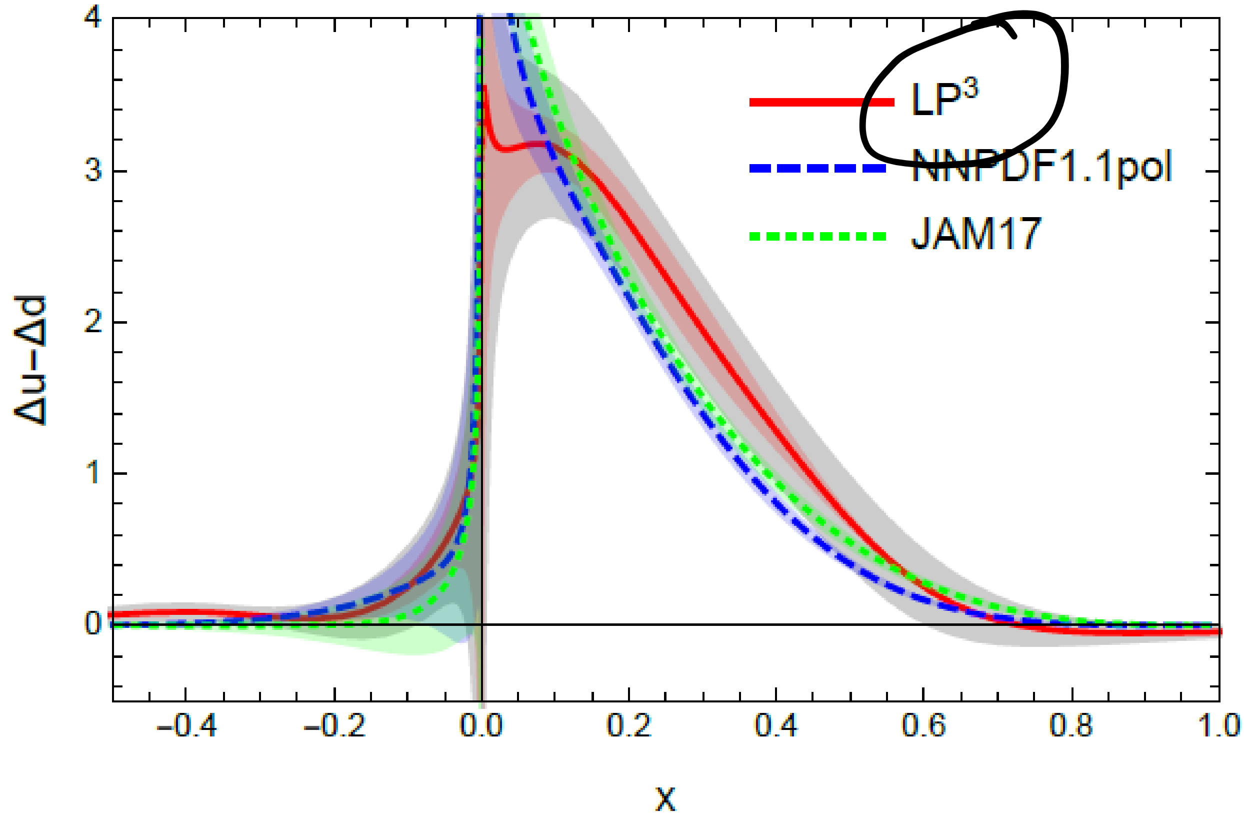
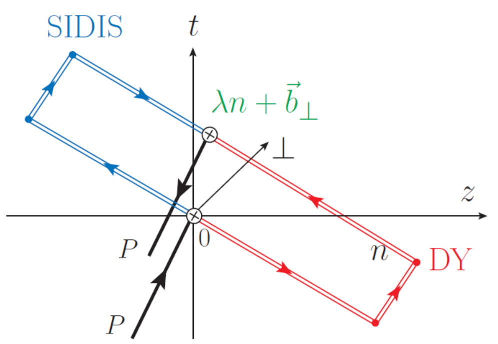
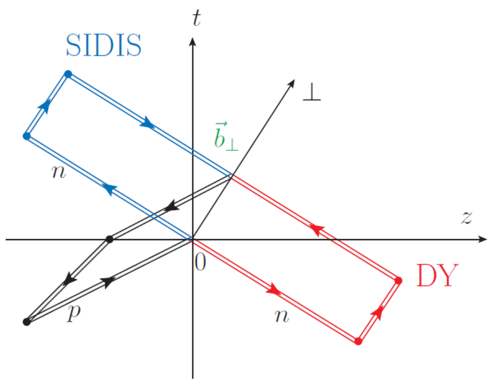


FIG. 21: Proton isovector quark PDFs [177]: The helicity PDF ($P^z = 3.0$ GeV) with red band contains statistic error and grey band further includes systematic error. NNPDF1.1pol [348] and JAM17 [350] are global fits.

Application 2: TMD-PDFs and soft functions

- A very important nucleon observable, many phenomenology related to spin physics (Sivers effect etc).
- It has taken sometime to figure out the correct light-front definition





Echevarria, Idilbi, Scimemi (2013), Collins & Rogers (2013)

Lattice calculations

- Started from A. Schafer et al., much progress; no x dependence has yet been studied.

  Hagler et al, Much et al, Yoon et al. PRD96,094508 (2017)…

- A number of LaMET formulations:

  Ji et al., PRD91,074009 (2015); PRD99,114006(2019)

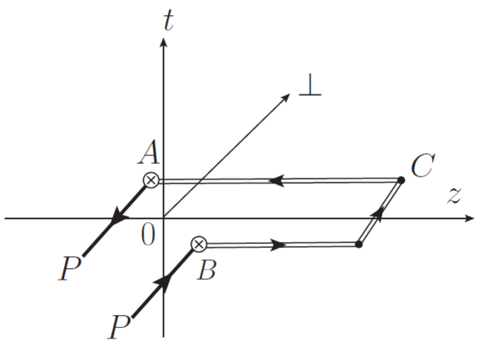
  Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019); arXiv:1910.08569

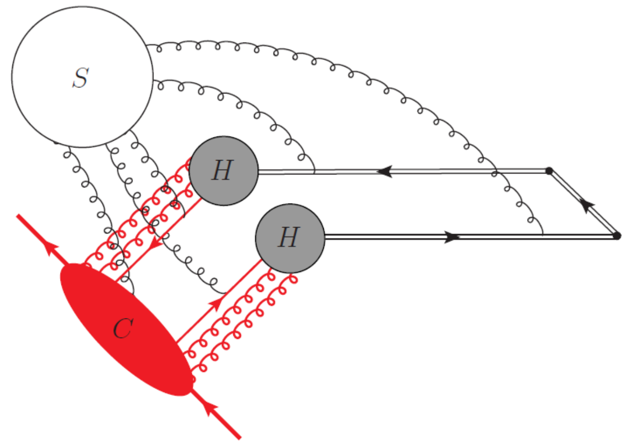
  Collins-Soper evolution kernel can be calculated.

  Soft function can be calculated Ji, Liu, Liu, NPB(2020)

Quasi-TMDPDF and factorization

Ji, Liu, Liu, 1911.03840, PLB



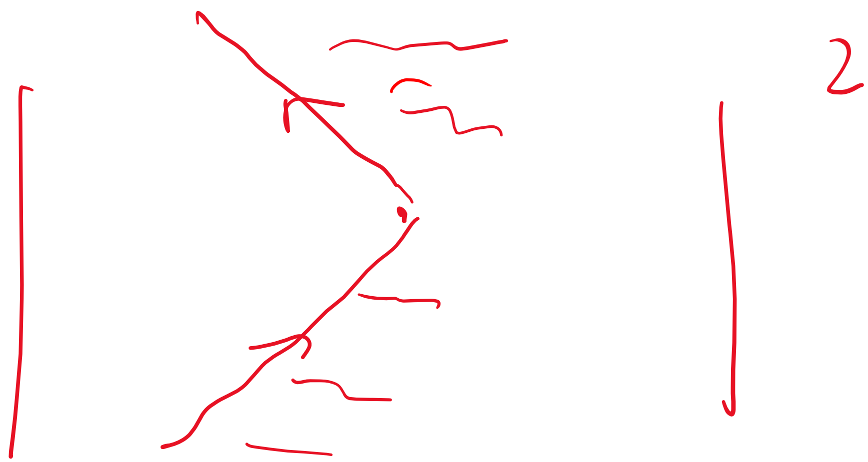


$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)} = H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\rm TMD}(x, b_\perp, \mu, \zeta) + \ldots$$

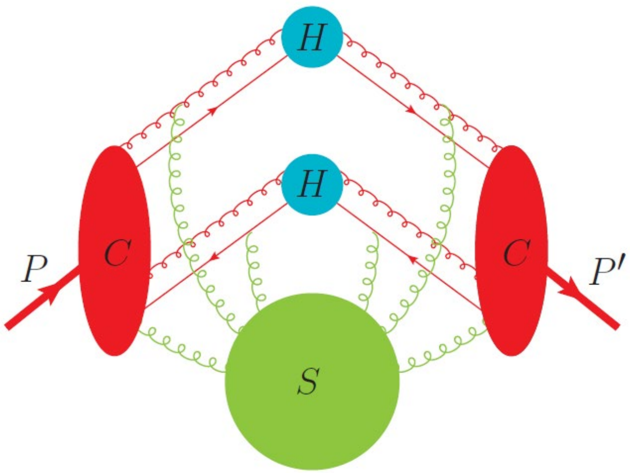
$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\rm cusp} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

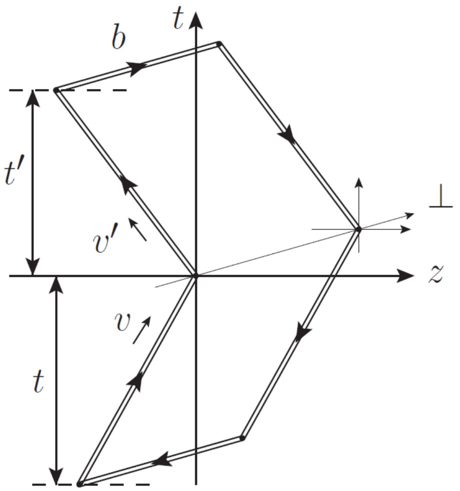
Soft function

- Cross section for two fast oppositely moving charges to radiate



Soft function using LaMET

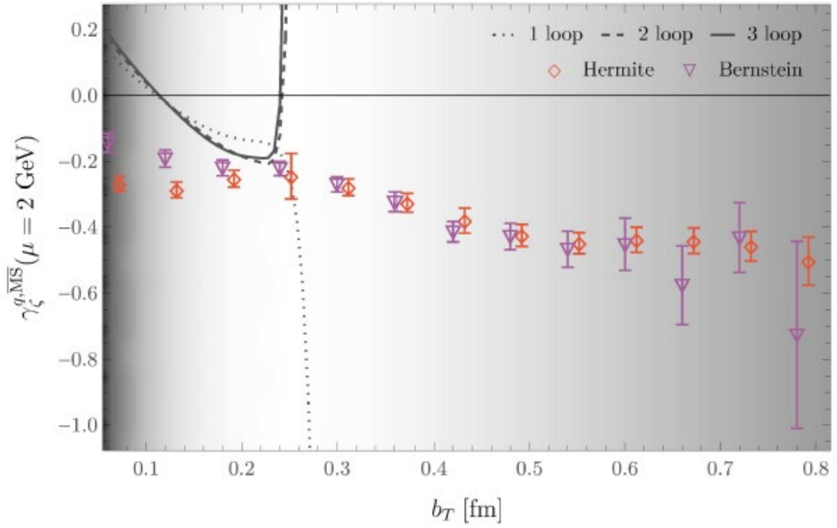


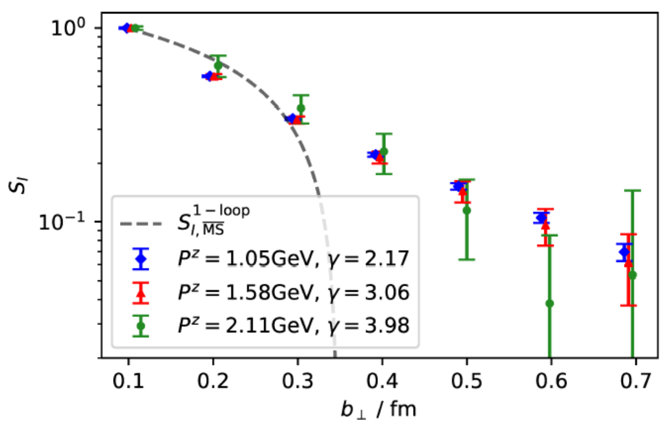


Factorization of form-factor of Light meson

Form-factors of heavy-quark pair

Collins-Soper evolution and soft functions





P. Shanahan, Wagman, Zhao, 2003.06063 [hep-lat]

Q. A. Zhang et al, 2005.14572 [hep-lat]

App. 3 Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji & Liu, to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

Conclusions

- Partons can be calculated on lattice using a large-momentum hadron state.

$$\frac{\Lambda_{QCD}}{P} \ll 1$$

- LaMET3.0 (~ 5% error)

  two-loop matching

  P=3 GeV

  Improved non-pert renormalization
- 1% accuracy in 10-20 years
- Fragmentation functions?