EFT for BH horizons

“All Things EFT,”
January 6, 2021
NBI

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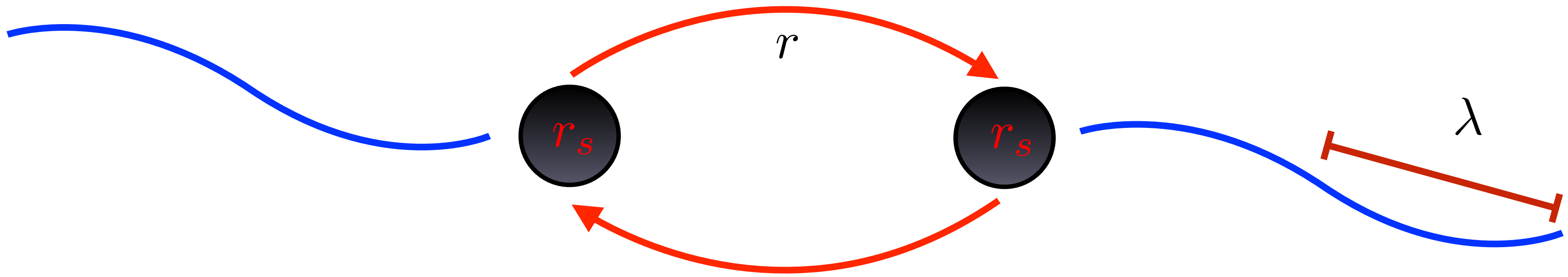
Yale U.

(mostly) based on:
arXiv:1912.01650,
2007.00726,
2007.00726, (w/ I. Rothstein)
2012.14869 (w/ Li+Rothstein)

Black hole binary inspirals

Gravitational dynamics of radiating classical BH (or NS) binary systems in the non-relativistic limit is experimentally relevant (LIGO/VIRGO,...)



Even for $v \ll 1$, the non-linear nature of GR makes this a difficult problem, involving a hierarchy of length scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \ll r \ll \lambda$$

$$r \sim r_g/v^2$$

$$\lambda \sim r/v \sim r_g/v^3$$

"correlated scales"

Experiments will be sensitive to at least v^6 = "3PN" corrections beyond Newtonian gravity (Thorne et al 1994). (5PN considered feasible). Numerical GR results also motivate computing higher order corrections.

In the NR limit $v/c \ll 1$ these scales are correlated:

$$r \sim r_g/v^2 \qquad\qquad \lambda \sim r/v \sim r_g/v^3$$

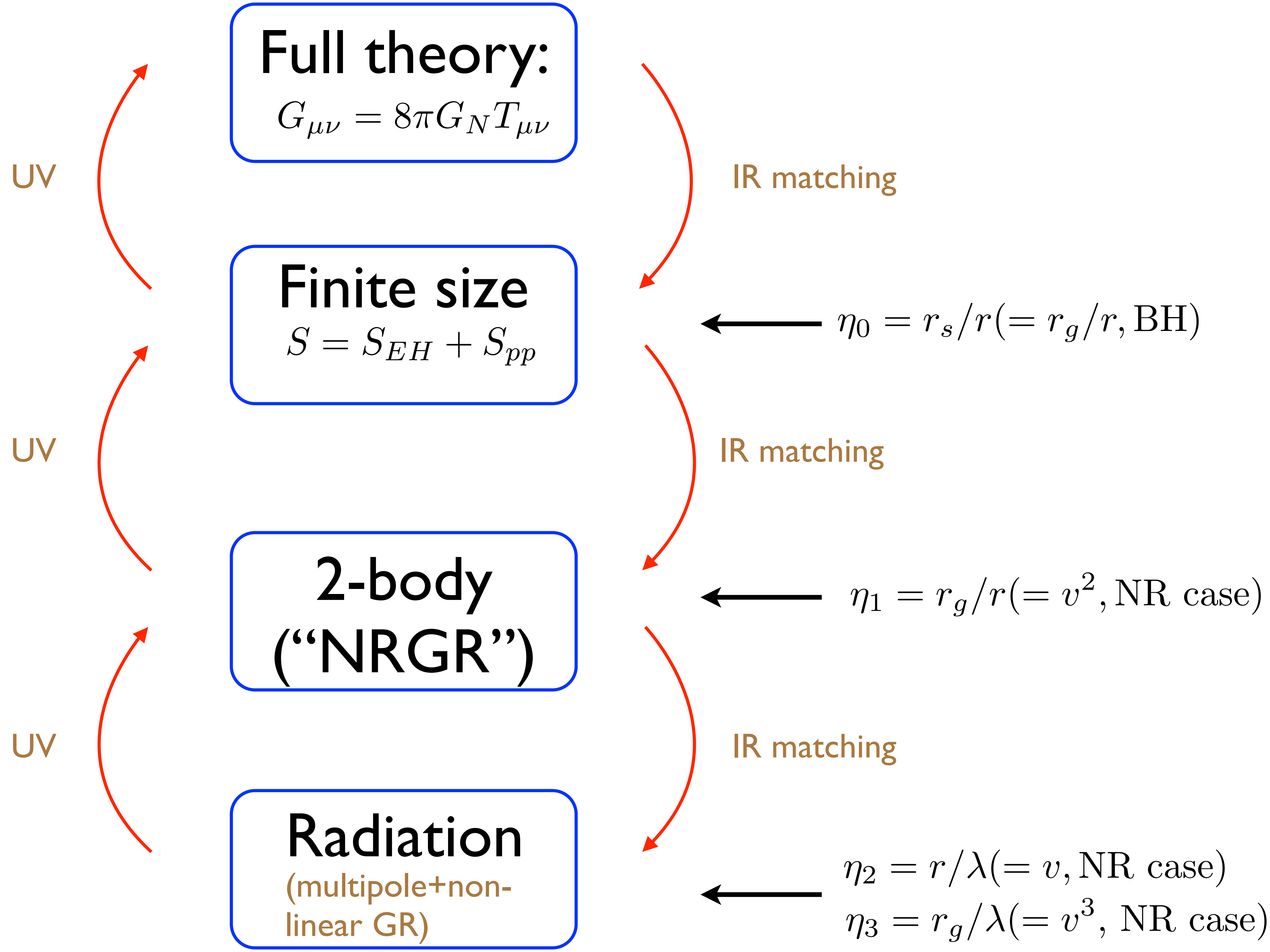
Thus at a fixed order in velocity ("Post-Newtonian expansion"), physics effects from all these scales may appear.

→ Treat each scale separately, by constructing a tower of gravity Effective Field Theories

(WG+I. Rothstein, 2004)

The correct set of EFTs for the binary system has properties in common w/ its gauge theory counterparts (HQET, NRQED/NRQCD,…)

Tower of gravity EFTs:



Independent EFTs with distinct expansion parameter coincide in PN limit.
UV divergence in EFT_{i+1} corresponds to IR effect in EFT_i

Finite size effects:

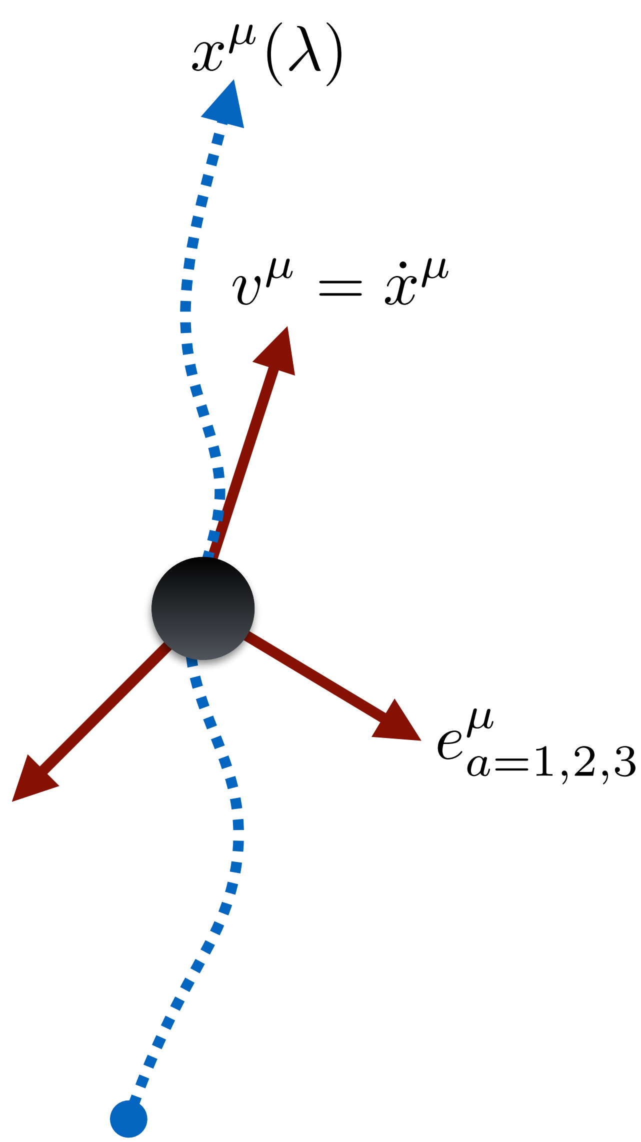
What can we learn about the internal structure of compact objects from binary inspirals at LIGO?

In the inspiral phase, binary constituents can be treated as point-like. Finite size effects encoded in an worldline EFT coupled to gravity

DOFs:

$x^\mu(\lambda) =$ worldline CM coordinate

$e^\mu_{a=1,2,3}(\lambda) =$ local frame (SPIN)



Symmetries:

Diff. invariance $x^\mu \mapsto x^\mu + \xi^\mu(x)$

Worldline RPI $\lambda \mapsto \lambda'(\lambda)$

Local $SO(3)$ rotations acting on (for BH only) e^μ_a

EFT for gravity coupled to BH, in the point particle limit:

$(\hbar = c = 1)$

$(m_{Pl}^2 = 1/(32\pi G_N))$

$S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R(x)$

$$S = S_{EH} + S_{pp}$$

The most general (mod. e.o.m's) point particle Lagrangian consistent with symmetries (ignoring spin, assume parity invariance), organized in a derivative expansion:

$$S_{pp} = \underbrace{-m \int d\tau}_{\mathcal{O}(\partial^0 g)} + c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + \underbrace{c_B \int}_{\mathcal{O}(\partial^4 g)} d\tau B_{\mu\nu} B^{\mu\nu} + \cdots$$

w/ $E_{\mu\nu} = R_{\mu\alpha\nu\beta} v^\alpha v^\beta$ = "electric" curvature tensor

$B_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\rho\sigma\lambda} v^\rho v^\alpha R_{\nu\alpha}{}^{\sigma\lambda}$ = "magnetic" curvature tensor

(Note: $\mathcal{O}(\partial^2 g)$ terms, eg $\int d\tau g^{\mu\nu} R_{\mu\nu}$ are redundant due to source free eom $R_{\mu\nu} = 0$).

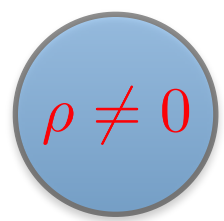
The curvature couplings

$$S_{pp} \supset c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu}$$

describe the $\ell = 2$ linear tidal response of the compact object to external gravitational fields.

E.g, Newtonian spherical self-gravitating star (radius R). No external gravitational field:

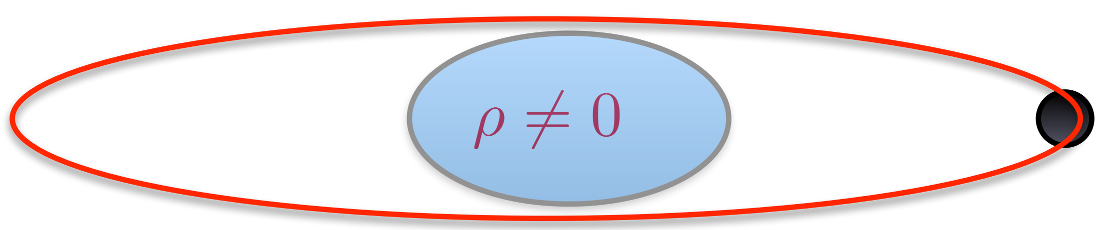
$\Phi = 0$



$$Q_{ij} = \int d^3\vec{x}\rho\left(x^i x^j - \text{trace}\right) = 0$$

Turn on weak external perturbation:

$\Phi \neq 0$



$$\delta Q_{ij} = -\frac{2}{3}k\ \frac{R^5}{G_N} E_{ij}$$

$$E_{ij} = \partial_i \partial_j \Phi$$

w/ dimensionless (gravitational) “Love number” $k \sim \mathcal{O}(1)$ that depends on fluid eqn. of state.

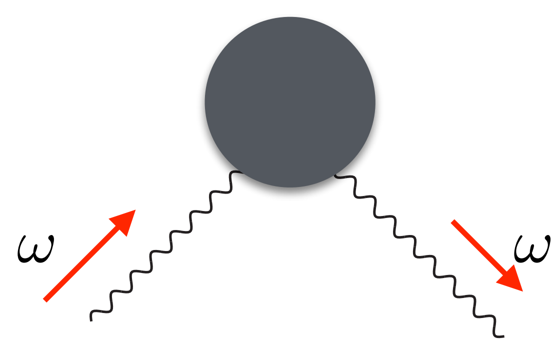
In the point particle EFT, the induce quadrupole moment is:

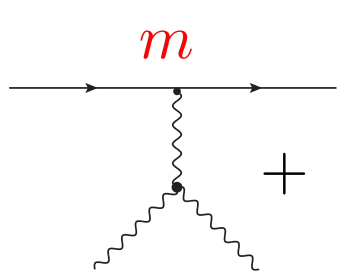
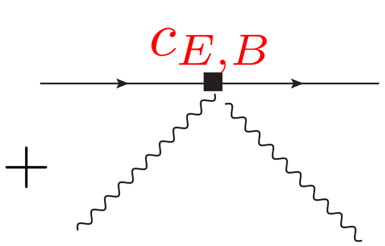
$$\delta Q_{ij} = -\frac{\delta}{\delta E_{ij}} S_{pp} = -2 c_E E_{ij} \qquad \text{vs.} \qquad \delta Q_{ij} = -\frac{2}{3} k \, \frac{R^5}{G_N} E_{ij}$$

so we expect on dimensional grounds:

$$c_E \sim R^5 / G_N$$

Same scaling also holds for relativistic (compact) objects. Eg elastic graviton+BH scattering amplitude: $(r_s \omega \ll 1)$

Full theory: $i\mathcal{A} =$  $= r_s f(r_s \omega)$

EFT: $i\mathcal{A} =$  $+ \cdots +$  $+ \cdots$

$$\sim \frac{c_{E,B}}{m_{Pl}^2} \omega^4$$

$\Rightarrow$ $c_{E,B} \sim r_s^5 / G_N$

from matching (WG, '06)

Given that $c_E \sim R^5/G_N$, we expect finite size/tidal corrections to potentials and radiation to scale as

$$\text{Tidal effects} \sim \left(\frac{R}{r}\right)^5 = \left(\frac{R}{r_s}\right)^5 \times v^{10}$$

which is formally a 5PN effect. Specifically

Black hole: $R = r_s$ ➡ Tidal effects at 5PN

Neutron start: $R \sim \mathcal{O}(10) \times r_s$ ➡ Enhancement by a factor of $\sim 10^5$

(Flanagan+Hinderer, 2007)

(in fact the NS/NS inspiral event GW17087 at LIGO has already placed very crude constraints on the neutron star tidal coefficients…)

For neutron stars, c_E depends on the EOS and has been calculated numerically in (Flanagan+Hinderer, 2007)

For the case of Schwarzschild black holes in $d=4$, the tidal response in the full theory $R_{\mu\nu}=0$ has been computed analytically by

Damour+Nagar, 2009

Binnington+Poisson, 2009

Kol+Smolkin, 2011

Steinhoff et al 2013

while the EFT side corrections were shown to vanish in Kol+Smolkin, 2011. The result for BH's in $d=4$

$$c_E^{BH} = c_B^{BH} = 0$$

so no (static) tidal response at $\ell=2$ (and likely also for $\ell > 2\ldots$)

Recently extended to Kerr (spinning) BH, H.S Chia 2010.07300

(However, non-static finite size effects, eg dissipation at the BH horizon, are non-vanishing…)

Calculating the Binary waveform

In principle, the radiation field measured by observers at infinity

$$\lim_{r\to\infty} r h_{ij}^{TT}(\vec{x},t)$$

encodes all the relevant physical info about the binary (masses,spins, multipole moments,QNM frequencies,...). In perturbation theory, it is most conveniently computed by recasting Einstein's equations in the form (eg Weinberg 1972)

$$h_{\mu\nu}(x) = \frac{1}{2m_{Pl}} \int_k \frac{e^{-ik\cdot x}}{k^2} \left[\tilde{T}^{\mu\nu}(k) - \frac{1}{2}\eta_{\mu\nu}\tilde{T}^{\sigma}{}_{\sigma}\right]$$

(deDonder gauge)

$$\tilde{T}^{\mu\nu} = T^{\mu\nu}_{pp} + T^{\mu\nu}_{g} =$$ EM pseudotensor $$\partial_\mu \tilde{T}^{\mu\nu} = 0$$

$$\sim h\partial^2 h + h^2\partial^2 h + \cdots$$

$$S_{pp} = -m\int d\tau + c\int d\tau R^2_{\mu\nu\alpha\beta} + \cdots$$

The radiation field at infinity has a simple relation to the pseudo-tensor evaluated on-shell

$$h_{\pm}(t,\vec{n}) = \frac{4G_N}{r}\int \frac{d\omega}{2\pi} e^{-i\omega t} \epsilon^{*ij}_{\pm}(k)\tilde{T}_{ij}(k)$$

(Weinberg 1972)

w/

$$k^{\mu} = \omega(1, \vec{n} = \frac{\vec{x}}{r})$$

$$k^2 = 0$$

In practice computing higher order terms in perturbation theory $(v \ll 1)$ is difficult for two reasons:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

Many physically relevant scales

Gravitational radius: $r_g = 2G_N M$

Physical radius: $r_s (= r_g \text{ for BH})$

Orbital scale: r

Radiation wavelength λ

$$r_g \sim r_s \gg r \gg \lambda$$

all correlated to the perturbative expansion parameter

$$r \sim r_g / v^2 \qquad \lambda \sim r/v \sim r_g / v^3$$

These challenges can be ameliorated by employing some tools from QFT:

Many terms in the expansion of $\tilde{T}^{\mu\nu}(x)$ at high orders in $h_{\mu\nu}$

→ Organize the expansion in terms of Feynman diagrams

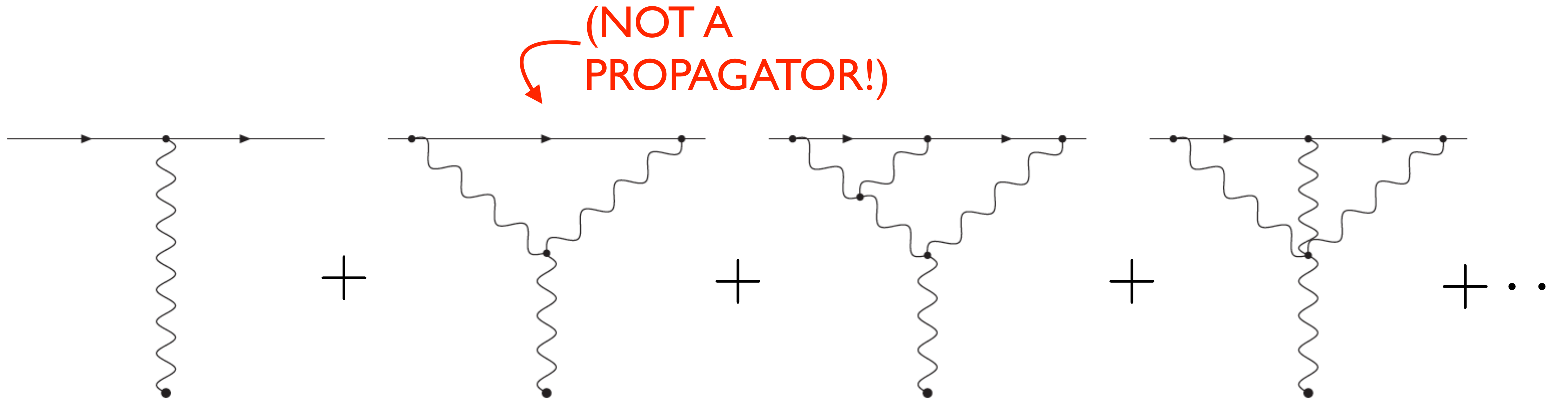
Many physically relevant scales

→ Treat each scale separately, by constructing a tower of gravity Effective Field Theories

$$h_{\mu\nu} = h_{\mu\nu}^{potential} + h_{\mu\nu}^{rad}$$

The types of Feynman diagrams that are relevant are of the same type as in Duff's (1973) perturbative construction of the Schwarzschild solution:

$h_{\mu\nu} =$

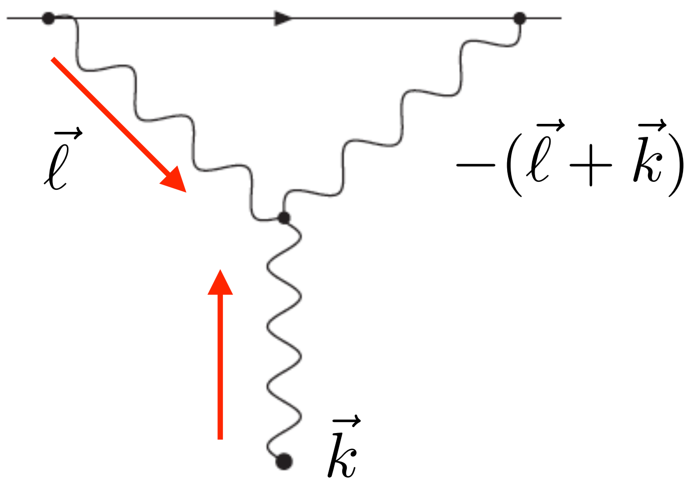


$$g_{00} = 1 - \frac{2G_N m}{r} + 2\left(\frac{G_N m}{r}\right)^2 + 2\left(\frac{G_N m}{r}\right)^3 + \cdots$$

$$g_{ij} = -\delta_{ij}\left[1 + \frac{2G_N m}{r} + 5\left(\frac{G_N m}{r}\right)^2 - \frac{2}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots\right]$$

$$+ \frac{x_i x_j}{r^2}\left[7\left(\frac{G_N m}{r}\right)^2 - \frac{4}{3}\left(\frac{G_N m}{r}\right)^3 + \cdots\right]$$

These are tree diagrams coupled to classical (particle sources). Despite being tree, they have the same structure as loop Feynman integrals in QFT. E.g.,



$$\sim I = \int \frac{d^{d-1}\ell}{(2\pi)^{d-1}} \frac{1}{\vec{\ell}^2} \frac{1}{(\vec{k}+\vec{\ell})^2} [\text{Numerator}]$$

with interesting UV and IR structure (though at high orders in PT).

Note: We use dimensional regularization to handle both UV and IR divergences

Diagramatics:

We now assume that internal scales have been integrated out. We have a system of gravitationally bound point particles:

$$S = S_{EH} + S_{pp}$$

where now,

$$S_{pp} = -\sum_{a=1,2} \int d\tau_a m_a + \sum_{a=1,2} \int d\tau_a \left(c_a^E E_{\mu\nu}^2 + c_a^B B_{\mu\nu}^2 \right) + \cdots$$

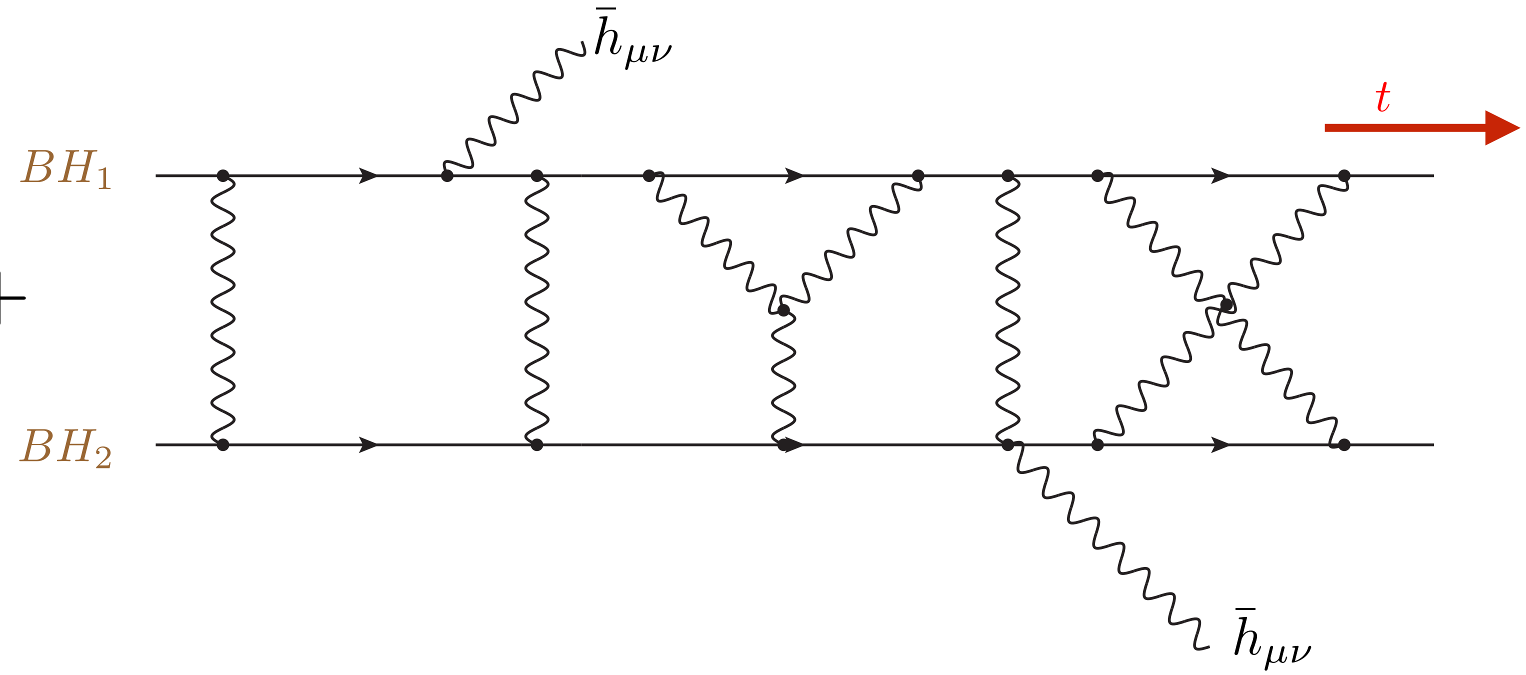
ignoring spin,

The gravitational "Wilson line"

$$W = \exp i\Gamma[\bar{h}, x_a] = \int [\mathcal{D}h_{\mu\nu}]_{\text{b.c's}} e^{iS[h,\bar{h},x_a]}$$

generates all the observables of the (classical) binary system.
Diagrammatically:

$$W = \cdots +$$



$$+ \cdots$$

$$= e^{\sum(\text{BH irreducible diagrams})}$$

where we split up the metric into a background field and a "fluctuating part":

$$g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} + h_{\mu\nu}$$

background → $\bar{h}_{\mu\nu}$ fluctuation → $h_{\mu\nu}$

and integrate out fluctuations.

For example,

$$\Gamma[\bar{h}=0, x_a] = \int dt L(\mathbf{x}_a(t), \dot{\mathbf{x}}_a(t)) =$$ two-body Lagrangian

generates the equations of motion for the BH trajectories

The linear term in the background defines an effective energy-momentum tensor:

$$\Gamma[\bar{h}=, x_a] = \cdots + \frac{1}{2m_{Pl}} \int d^4x T^{\mu\nu}(x) \bar{h}_{\mu\nu} + \cdots$$

$$\partial_\mu T^{\mu\nu}(x) = 0$$

(Ward id. for diff invariance)

which can be used to compute radiation at infinity

In particular, with standard in/out (Feynman) b.c.'s, graviton emission amplitude is

$$\mathcal{A}_{h=\pm 2}(k) = \int d^4x e^{ik\cdot x} \epsilon^*_{\mu\nu}(h,k) T^{\mu\nu}(x)$$

and the graviton emission rate over $T \to \infty$

$$d\Gamma_h(\mathbf{k}) = \frac{1}{T} \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} |\mathcal{A}_h(\mathbf{k})|^2,$$

yield time-averaged energy and momentum emission rates:

$$\langle \dot{P}^\mu \rangle_{h=\pm 2} = \int k^\mu d\Gamma_h(\mathbf{k}),$$

$$\langle \dot{\mathbf{J}} \rangle = 2 \int \mathbf{n} d\Gamma_{h=2}(\mathbf{k}) - 2 \int \mathbf{n} d\Gamma_{h=-2}(\mathbf{k}),$$

Using in/in boundary conditions (as in cosmology) gives instantaneous observables, e.g. radiation field at infinity: (Galley and Tiglio, 2009)

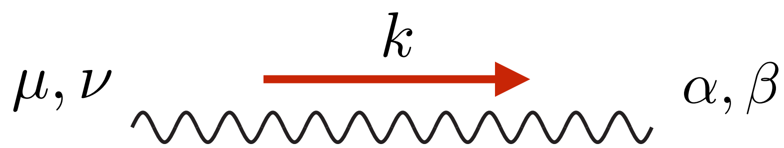
$$h_{\mu\nu}(\mathbf{x} \to \infty, t) = \int d^4y D^{\mathrm{ret}}_{\mu\nu;\alpha\beta}(x-y) T^{\alpha\beta}(y)$$

which yields the time-dep. waveform seen in the detector.

To compute the generating function W one could use standard covariant Feynman rules obtained by expanding $S_{EH} = -2m_{Pl}^2 \int d^4x \sqrt{g} R$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{Pl}$$

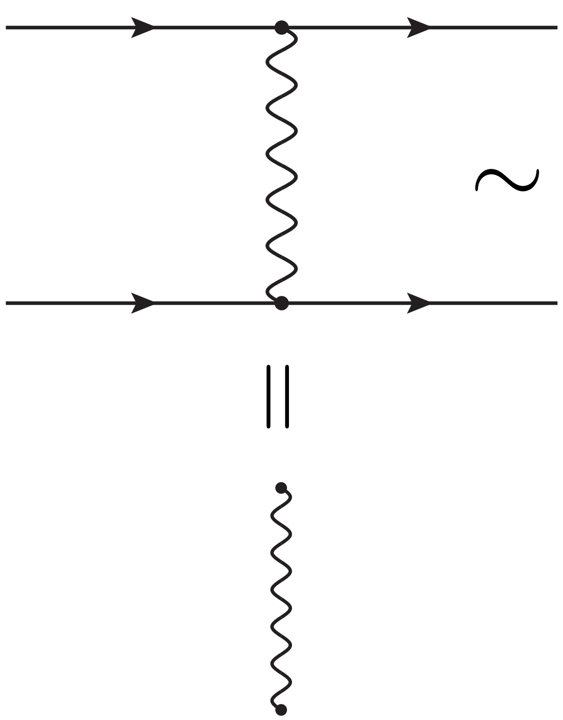
w/ e.g



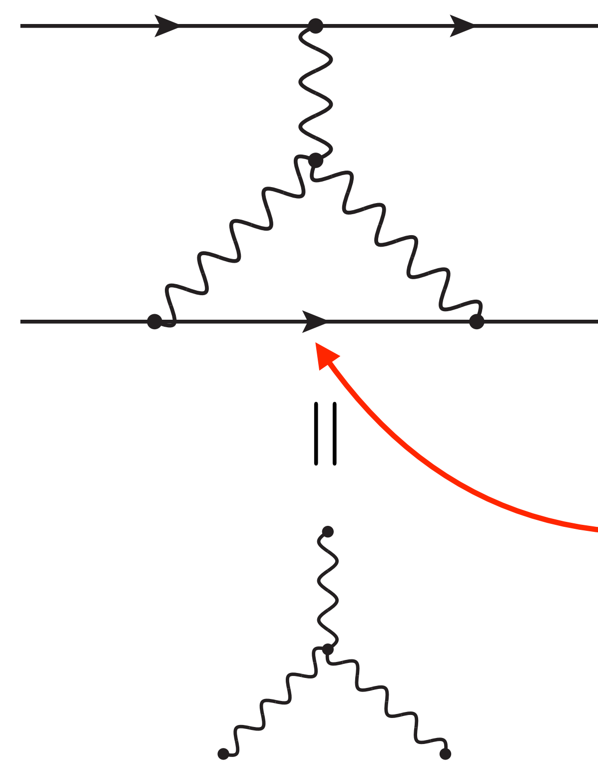
$$= \frac{i}{k^2} P_{\mu\nu;\alpha,\beta}$$

(Feynman gauge)

However, these Feynman rules are not optimal optimal for the NR limit $v \ll 1$
The diagrams don't have manifest power counting in the exp. parameter:



$\sim v^2 + v^4 + \cdots$



$\sim v^?$

(NOT A PROPAGATOR!)

The problem is that the diagrams involve momentum integrals over all momentum regions. However, for NR kinematics, two momentum space configurations dominate:

"potential": $(E \sim 0, \vec{p} \sim 1/r)$ (off-shell)

"radiation": $(E \sim v/r, \vec{p} \sim v/r)$

The solution to this problem is well known from NRQED/NRQCD and HQET. Decompose graviton into distinct momentum modes and "pull out" short scales:

$$\partial_\mu H_{\mathbf{k}} \sim \frac{v}{r} H_{\mathbf{k}}$$

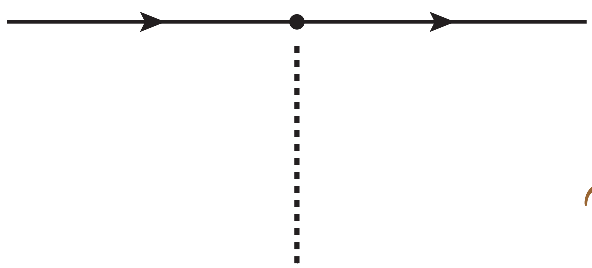
$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x) + \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} H_{\mathbf{k}\,\mu\nu}(x^0)$$

$$\partial_\mu \bar{h} \sim \frac{v}{r}\bar{h}$$

$$\mathbf{k} \sim \frac{1}{r}$$

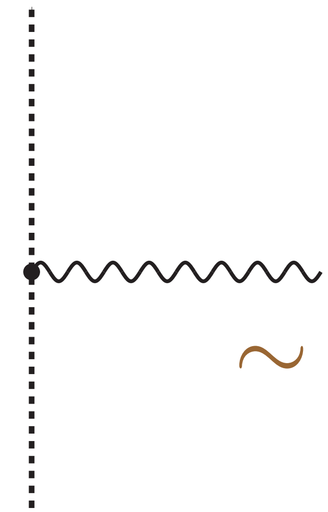
The radiation mode can be regarded as long wavelength background field in which potential gravitons propagate

In addition, need to multipole expand the couplings of the radiation mode to the particles and to the potentials. This yields an effective Lagrangian with manifest power counting in velocity:



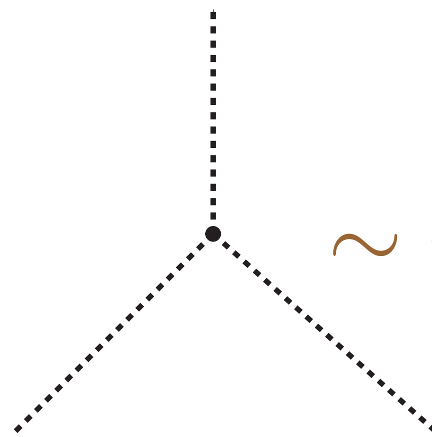
$\sim \sqrt{L}$

Pt. particle-Newton potential interaction:



$\sim v^{5/2}/\sqrt{L}$

Radiation-potential interaction



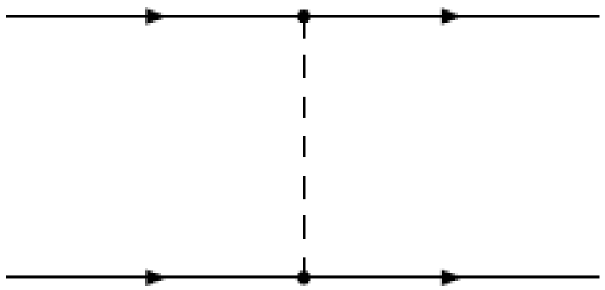
$\sim v^2/\sqrt{L}$

Potential graviton cubic self-interaction

By connecting vertices together, generate the 2-body potentials and the interactions of matter with radiation.

Drop (for now) quantum corrections suppressed by powers of $\sim \hbar/L \ll 1$

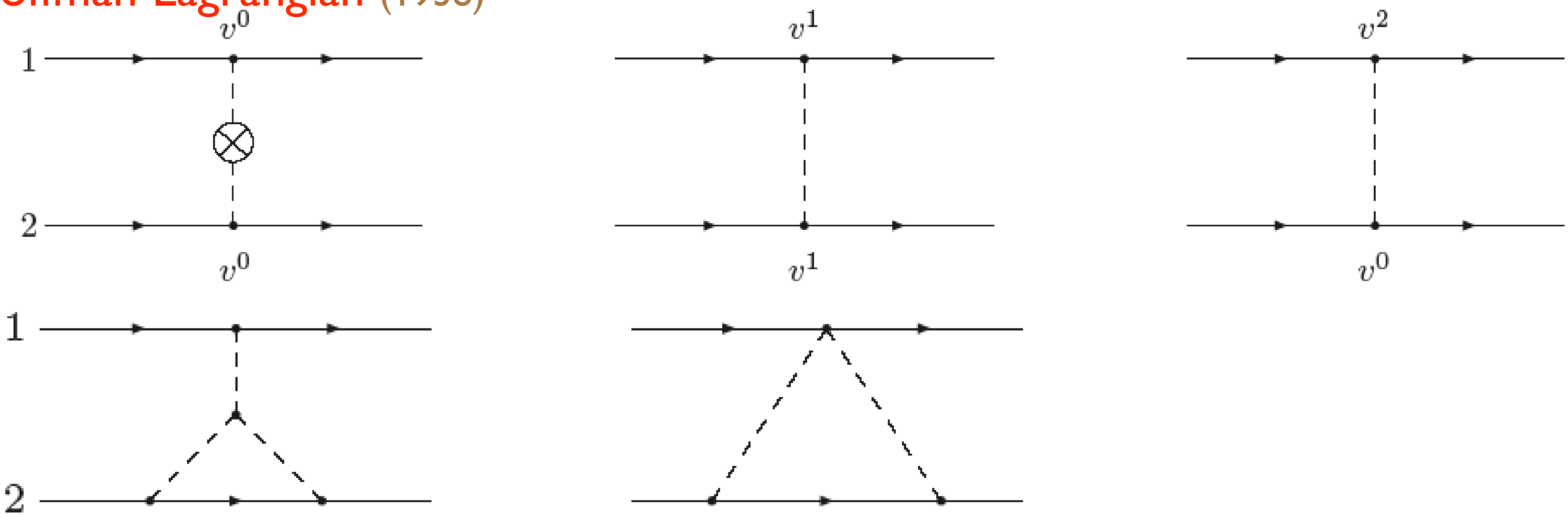
Leading order:
Newton
(1687)



(a)

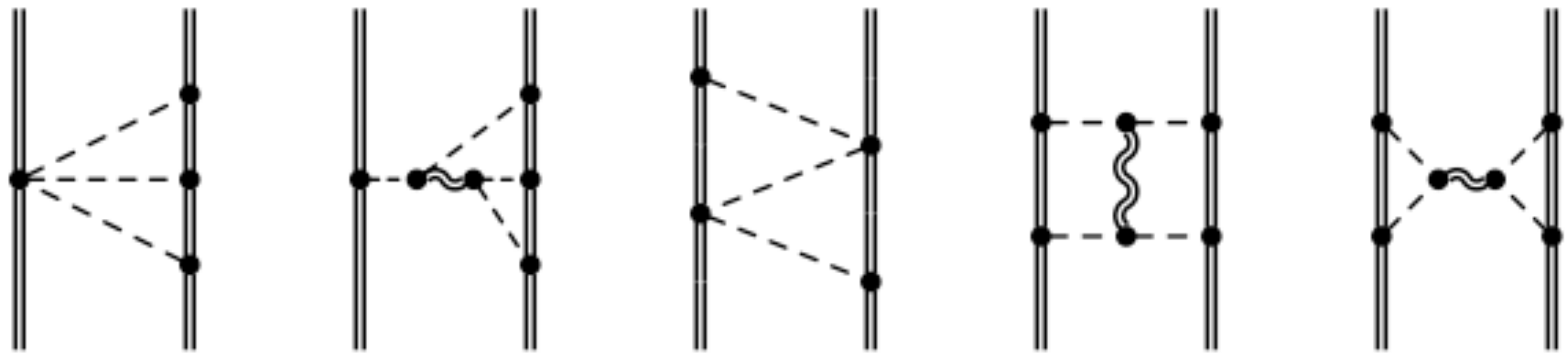
$$L = \frac{1}{2}\sum_a m_a \vec{v}_a^{\,2} + \frac{G_N m_1 m_2}{r}$$

Next-to-leading (1PN): Einstein-Infeld
Hoffman Lagrangian (1938)



$$L_{EIH} = \frac{1}{8}\sum_a m_a \vec{v}_a^{\,4} + \frac{G_N m_1 m_2}{2r}\left[3(\vec{v}_1^{\,2}+\vec{v}_2^{\,2}) - 7\vec{v}_1\cdot\vec{v}_2 - (\vec{v}_1\cdot n)(\vec{v}_1\cdot n)\right]$$
$$-\frac{G_N^2 m_1 m_2}{2r^2}$$

2PN (1981-2002): Some of the diagrams are (Gilmore+Ross, PRD 2008)



$$
\begin{aligned}
L_{2PN} =& \frac{m_1 \mathbf{v}_1^6}{16} \\
&+ \frac{G m_1 m_2}{r}\left(\frac{7}{8}\mathbf{v}_1^4 - \frac{5}{4}\mathbf{v}_1^2 \mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{4}\mathbf{v}_1^2 \mathbf{n}\cdot\mathbf{v}_1 \mathbf{n}\cdot\mathbf{v}_2 + \frac{3}{16}\mathbf{v}_1^2\mathbf{v}_2^2 + \frac{1}{8}(\mathbf{v}_1\cdot\mathbf{v}_2)^2\right. \\
&\qquad \left. - \frac{1}{8}\mathbf{v}_1^2(\mathbf{n}\cdot\mathbf{v}_2)^2 + \frac{3}{4}\mathbf{n}\cdot\mathbf{v}_1 \mathbf{n}\cdot\mathbf{v}_2 \mathbf{v}_1\cdot\mathbf{v}_2 + \frac{3}{16}(\mathbf{n}\cdot\mathbf{v}_1)^2(\mathbf{n}\cdot\mathbf{v}_2)^2\right) \\
&+ G m_1 m_2\left(\frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}\mathbf{v}_2^2 + \frac{3}{2}\mathbf{a}_1\cdot\mathbf{v}_1 \mathbf{n}\cdot\mathbf{v}_2 - \frac{7}{4}\mathbf{a}_1\cdot\mathbf{v}_2 \mathbf{n}\cdot\mathbf{v}_2 - \frac{1}{8}\mathbf{a}_1\cdot\mathbf{n}(\mathbf{n}\cdot\mathbf{v}_2)^2\right) \\
&+ G m_1 m_2 r\left(\frac{15}{16}\mathbf{a}_1\cdot\mathbf{a}_2 - \frac{1}{16}\mathbf{a}_1\cdot\mathbf{n}\mathbf{a}_2\cdot\mathbf{n}\right) \\
&+ \frac{G^2 m_1 m_2^2}{r^2}\left(\frac{7}{4}\mathbf{v}_1^2 + 2\mathbf{v}_2^2 - \frac{7}{2}\mathbf{v}_1\cdot\mathbf{v}_2 + \frac{1}{2}(\mathbf{n}\cdot\mathbf{v}_1)^2\right) \\
&+ \frac{G^3 m_1 m_2^3}{2r^3} + \frac{3G^3 m_1^2 m_2^2}{2r^3} + (1 \leftrightarrow 2),
\end{aligned}
$$

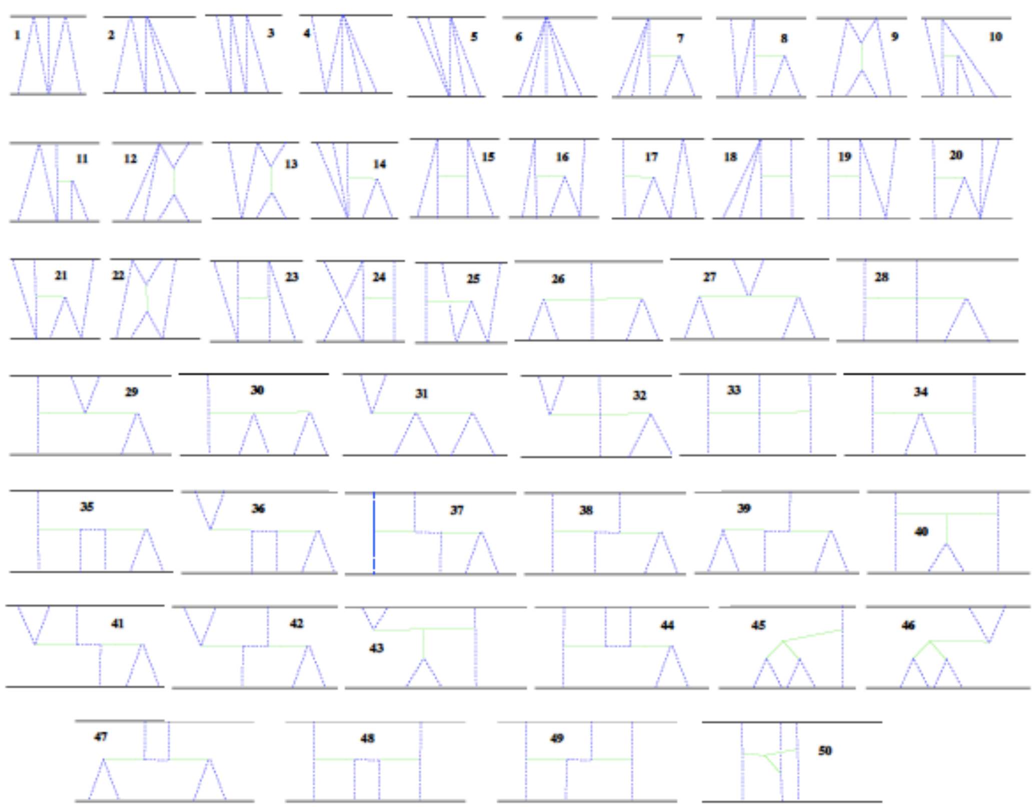
reducible to one-loop integrals via IBP:

$$
\int \frac{d^{d-1}\mathbf{k}}{(2\pi)^{d-1}} \frac{1}{[(\mathbf{k}+\mathbf{p})^2]^\alpha [\mathbf{k}^2]^\beta}
$$

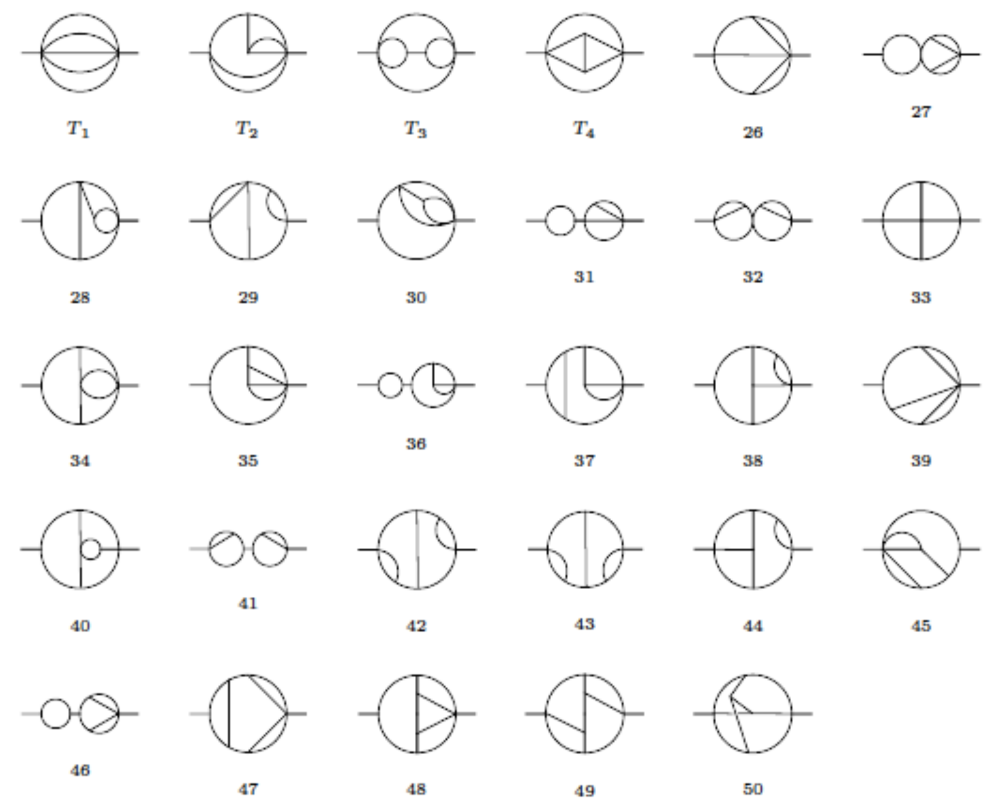
(simplification of PT via field redefs: B. Kol+M. Smolkin, 2007-2008.)

4PN order (Foffa, Mastrolia,Sturani,Sturm;2016) (Damour et al; Blanchet et al 2015)

Reduces to 5 master integrals via IBP identities:



2-body graviton exchange diagrams



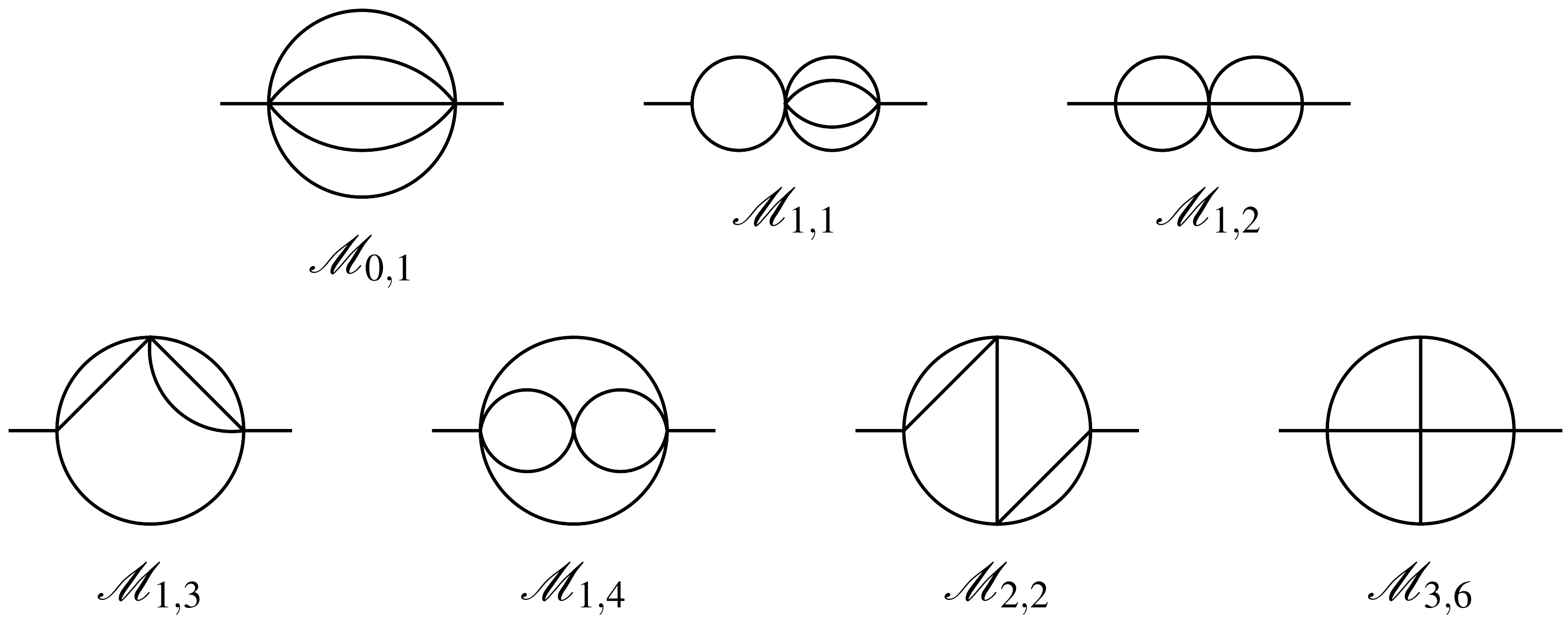
Equivalent 2-pt fns.

Static 2-body Lagrangian:

$$\sum_{a=1}^{50} \mathcal{L}_a = \frac{3}{8}\frac{G_N^5 m_1^5 m_2}{r^5} + \frac{31}{3}\frac{G_N^5 m_1^4 m_2^2}{r^5} + \frac{141}{8}\frac{G_N^5 m_1^3 m_2^3}{r^5} \,.$$

5PN order Static potential has been computed by Foffa, Mastrolia,Sturani,Sturm,Bobadilla; 2019.

Reduces to 7 master integrals, corresponding to 3D massless Feynman integrals:



Result for static (velocity independent) 2-body potential:

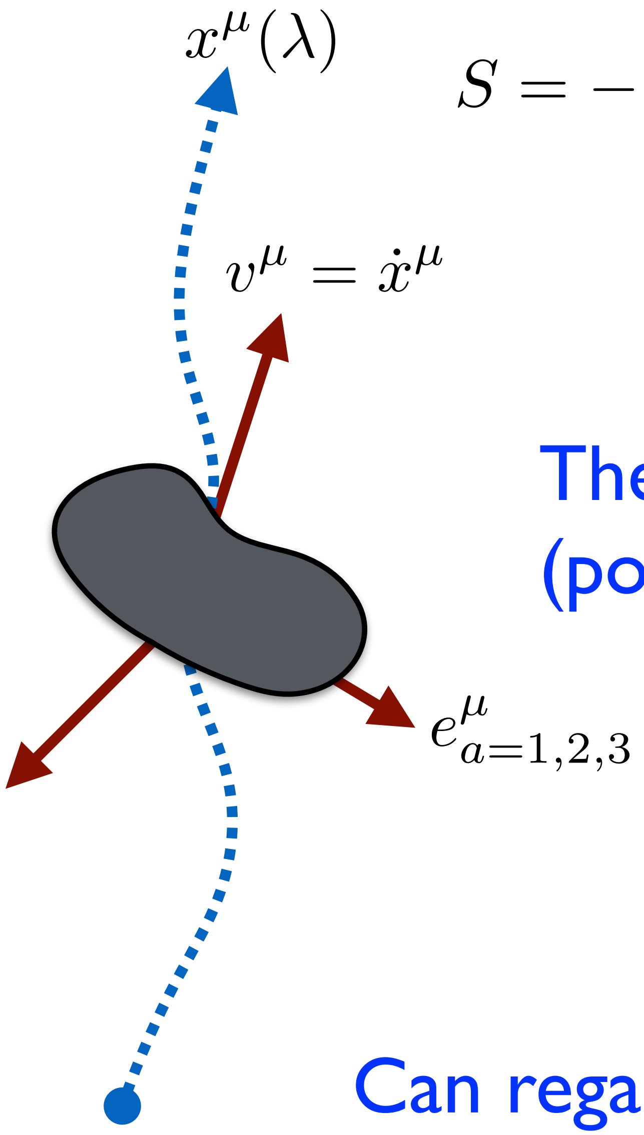
$$V_{\text{static}}^{(5\text{PN})} = \frac{5}{16}\frac{G_N^6 m_1^6 m_2}{r^6} + \frac{91}{6}\frac{G_N^6 m_1^5 m_2^2}{r^6} + \frac{653}{6}\frac{G_N^6 m_1^4 m_2^3}{r^6} + (m_1 \leftrightarrow m_2).$$

EFTIII: Radiation

(Double expansion: $\eta_2 = r/\lambda \sim v$
$\eta_3 = r/r_g \sim v^3$)

(WG+Ross, PRD 2010)

This is a field theory of radiation coupled to a point object with multipole moments. Most general diff. invariant action:



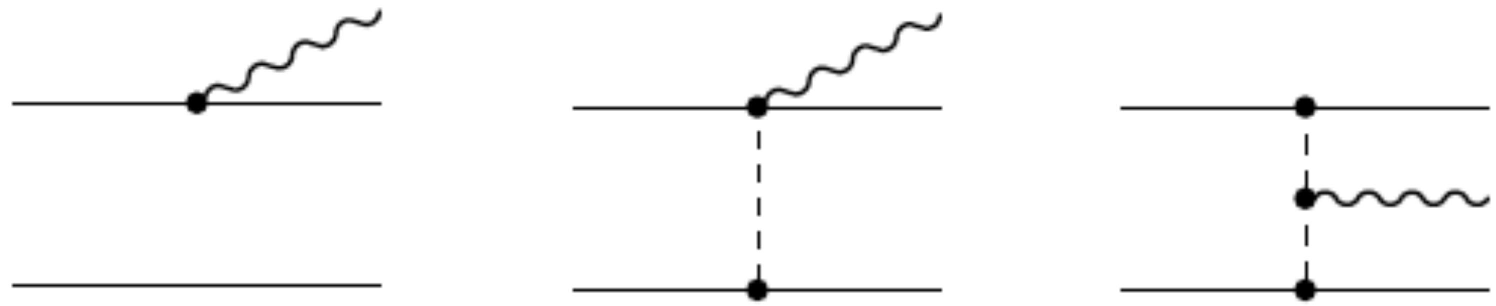
$$S = -\int d\tau(\lambda) m(\lambda) - \int dx^\mu L_{ab}(\lambda)\, \omega_\mu^{ab}(x(\tau)) + \frac{1}{2}\int d\tau(\lambda) I_{ab}(\lambda) E^{ab}(x(\tau))$$
$$-\frac{2}{3}\int d\tau J_{ab}(\lambda) B^{ab}(x) + \frac{1}{6}\int d\tau I_{abc}(\lambda) \nabla^c E^{ab}(x) + \cdots$$

The time evolution of the moments arises from short dist. (potentials) as well as radiative corrections (radiation reaction).

Can regard the moments as time-dependent Wilson coefficients (coupling constants). Radiative corrections in the EFT will generate RG flows for them.

Matching and RG Running in the Radiation Sector

Matching:



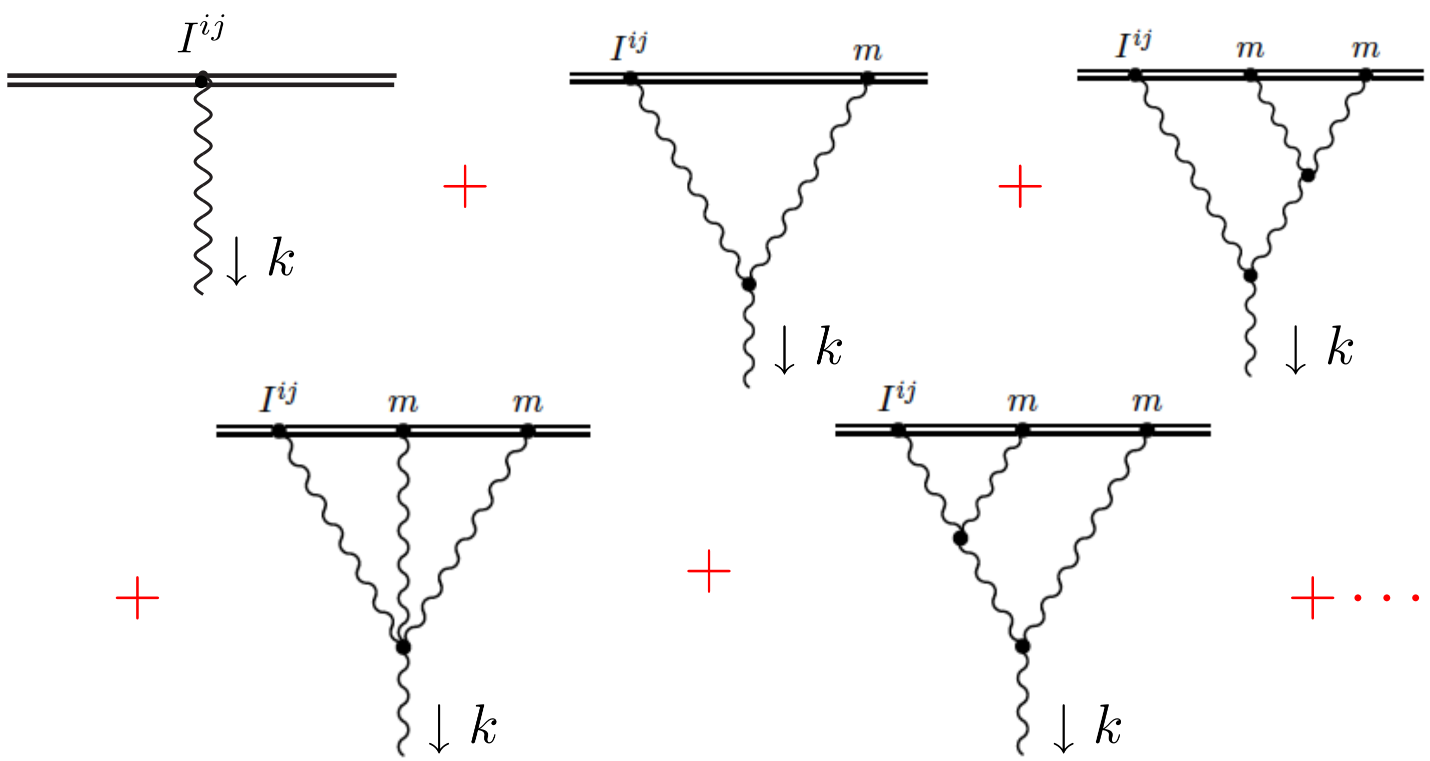
$q^\mu \to 0$

Quadrupole moment

$$I_{ij} = \sum_a m_a \mathbf{x}_a^i \mathbf{x}_a^j \left[1 + \frac{3}{2}\mathbf{v}_a^2 - \sum_b \frac{G_N m_b}{|\mathbf{x}_a - \mathbf{x}_b|}\right] + \frac{11}{42}\sum_a m_a \frac{d^2}{dt^2}(\mathbf{x_a}^2 \mathbf{x}_a^i \mathbf{x}_a^j)$$
$$-\frac{4}{3}\sum_a m_a \frac{d}{dt}(\mathbf{x_a}\cdot\mathbf{v}_a \mathbf{x}_a^i \mathbf{x}_a^j) - \text{traces} + \mathcal{O}(v^4)$$

Quadrupole Radiation:

$h_{ij}^{TT} =$



$$\frac{\dot{E}_{3PN,LL}}{\dot{E}_{LO}} = -\frac{1712}{105} v^6 \ln v$$

UV divergences in EFT renormalize the multipole moments. Logs of velocity can be "re-summed" using the renormalization group.

Recent results in the radiation sector of the EFT:

2PN Radiation: Leibovich, Maia, Rothstein, Yang, 1912.12546.

Logarithms in radiation at 6, 7PN: Blanchet, Foffa, Larrouturou, Sturani, 1912.12359

Tail+memory effects at NLO (2.5,5PN): Foffa, Sturani, 1907.02869.

Radiative corrections to conservative dynamics at 4PN: Foffa, Sturani, 1903.05113; Foffa, Porto, Rothstein, Sturani, 1903.05118.

Radiation reaction for spinning objects at 4PN: Maia, Galley, Leibovich, Porto, 1705.07934, 1705.07938.

EFT for BH Horizons

(WG+ Rothstein, 2005; 2020)

The results on tidal coefficient suggests that finite size effects are absent for black holes.

But formalism outlined so far neglects dissipation, ie absorption of energy and angular momentum by the compact objects themselves.

On general grounds, dissipation implies the existence of low frequency modes (eg NS: hydro modes,…. BH: horizon absorption) not captured by the point particle EFT

$$S_{pp} = -m\int d\tau + c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu} + \cdots$$

eg, for a Schwarschild black hole, the spectrum contains an infinite tower of modes labeled by $SO(3)$. In this case there are some zero modes:

Mode	Freq.	J^P
$m(\lambda)$	0	0
$x^\mu(\lambda)$	0	1^+
$\omega_{ij}(\lambda)$ (spin)	0	1^-

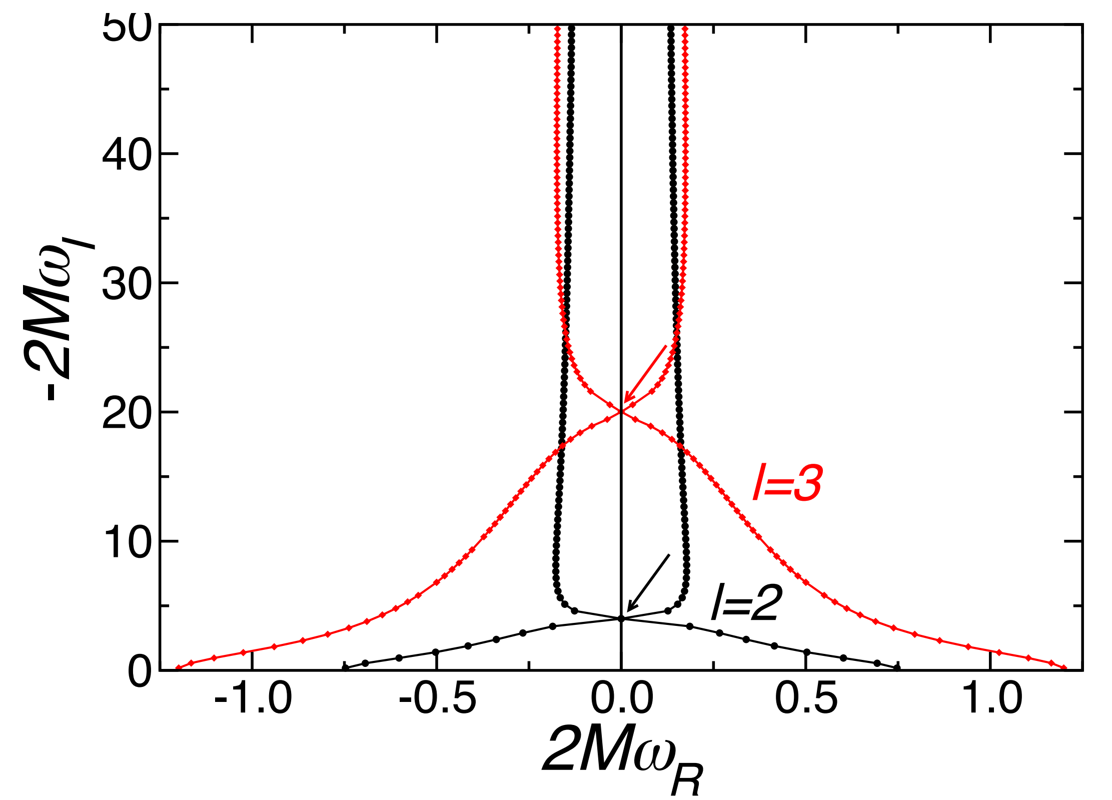
there are also an infinite tower of "quasinormal modes"...

n	$\ell = 2$		$\ell = 3$		$\ell = 4$	
0	0.37367	-0.08896 i	0.59944	-0.09270 i	0.80918	-0.09416 i
1	0.34671	-0.27391 i	0.58264	-0.28130 i	0.79663	-0.28443 i
2	0.30105	-0.47828 i	0.55168	-0.47909 i	0.77271	-0.47991 i
3	0.25150	-0.70514 i	0.51196	-0.69034 i	0.73984	-0.68392 i

Table 1: *The first four QNM frequencies* (ωM) *of the Schwarzschild black hole for* $\ell = 2, 3$, *and* 4 *[135]. The frequencies are given in geometrical units and for conversion into kHz one should multiply by* $2\pi(5142Hz) \times (M_\odot/M)$.

(from Kokkotas and Schmidt, gr-qc/9909058).

which are increasingly "broad resonances," rather than "quasiparticles":



Schwarzschild QNMs for $\ell = 2, 3$

(Berti et al CQG (2009)):

Even though the form of the internal spectrum depends on the details of the internal structure, can incorporate the effects of dissipation in a model independent w/o the need to explicitly track the light DOFs

The idea is to treat the compact object as $R \to 0$ as an "atom", i.e a worldline with local operators coupled to gravitons. For a spherical symmetric object, the leading interactions with gravitons take the form

$$S_{int} = -\int d\tau(\lambda) Q^E_{ab}(\lambda) E^{ab}(x) - \int d\tau(\lambda) Q^B_{ab}(\lambda) B^{ab}(x).$$

With operators $Q^{ab}_E(\lambda),\ Q^{ab}_B(\lambda)\quad \ldots$ acting on the Hilbert space of internal states. These are gravitational analogs of the EM dipole interaction

$$H_{em} = -\hat{\vec{p}} \cdot \vec{E}$$

Microscopic properties are then encoded in the correlation functions

$$\langle Q^{E,B} \cdots Q^{E,B} \rangle$$

which can be related to observable quantities of the compact object.

Example: Classical graviton absorption and power dissipation in binary systems

Consider an compact object of mass M . Graviton absorption amplitude in the object's rest frame:

$$i\mathcal{A}(g_h(k) + M \to X) = \langle X|Te^{-i\int dt H_{int}}|k, h; M\rangle$$

$$\approx -\int dt\langle X|Q^E_{ij}(t)|M\rangle\langle 0|E_{ij}(t,0)|k,h\rangle + (E \leftrightarrow B)$$

absorption cross section is $\sigma_{abs}(\omega) = \lim_{T\to\infty} \frac{1}{T} \cdot \frac{1}{2\omega} \sum_X |\mathcal{A}(g(k) + M \to X)|^2$

then, assuming unitarity (even for BHs!):

$$\sum_X |X\rangle\langle X| = \mathbb{I}$$

→

$$\sigma_{abs}(\omega) = \frac{\omega^3}{8m_{Pl}^2} \int dt e^{-i\omega t} \epsilon_{ij}(k)\epsilon^*_{rs}(k) \left[\langle M|Q^E_{rs}(0)Q^E_{ij}(t)|M\rangle + \langle M|Q^B_{rs}(0)Q^B_{ij}(t)|M\rangle\right],$$

where the 2-pt. correlators are in the initial state of the compact object

$$\langle Q^E(0)Q^E(x^0)\rangle = \langle M|Q^E(0)Q^E(x^0)|M\rangle$$

(alternatively, initial state could be mixed/thermal)

Matching to the full theory

For the case of black holes, the low frequency $\sigma_{abs}(\omega)$ can be calculated analytically, by finding the graviton wavefunctions in the BH background:

$$\Box_{BH} h_{\mu\nu} = 0$$

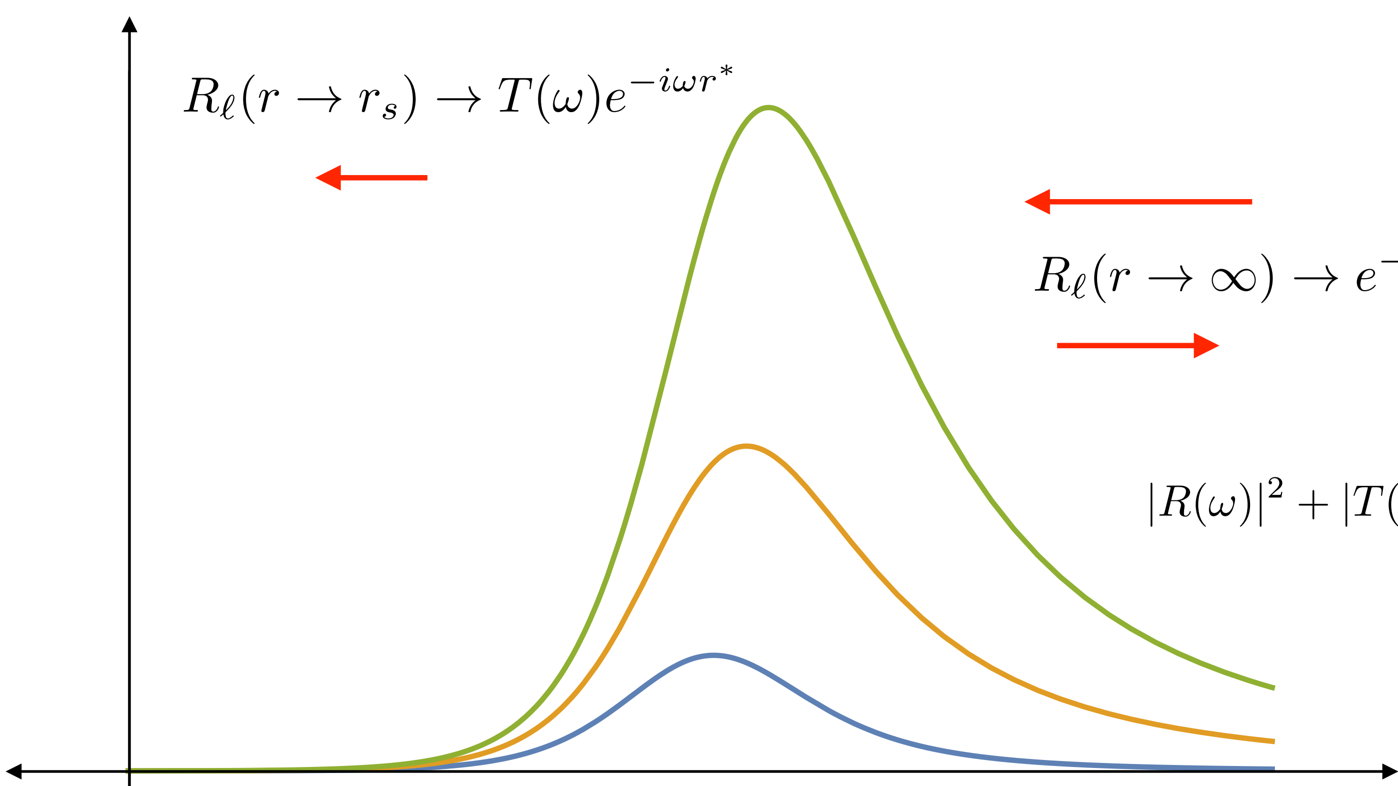
$$h_{\mu\nu}(x) = e^{-i\omega t}\frac{R_\ell(r)}{r}Y^{\ell m}_{\mu\nu}(\Omega)$$

$$\left(-\frac{d^2}{dr^{*2}} + V_\ell(r)\right)R_\ell(r) = \omega^2 R_\ell(r)$$

$$V_\ell(r) = \left(1-\frac{r_s}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{3r_s}{r^3}\right)$$

Schrodinger eqn for radial modes
="Regge-Wheeler" eqn.

BCs for scattering:



$$\sigma_{abs}(\omega) \sim |T(\omega)|^2$$

(QNMs: Same eqn. but purely outgoing bc's at the horizon and infinity)

These absorption coefficients were computed by Page (1975) for massless particles of arbitrary spin in the case of Kerr black holes:

$$\sigma_s(\omega) = \pi\omega^{-2}\sum_{l,m}\Gamma_{s\omega lmp} \underset{\omega\to 0}{\sim} \begin{cases} A, & s=0 \\ 2\pi M^2, & s=\frac{1}{2} \\ \frac{4}{9}A(3M^2-a^2)\omega^2, & s=1 \\ \frac{16}{225}A(5M^2+\frac{5}{2}M^2a^2+a^4)\omega^4, & s=2. \end{cases}$$

Using his result we can match the two-point functions in the case $s = 2$

$$\int dte^{i\omega t}\langle M|Q^{E,B}_{ij}(t)Q^{E,B}_{kl}(0)|M\rangle = \frac{1}{2}A_+(\omega)\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right),$$

and

$$\sigma^{\ell=2}_{abs}(\omega) = \frac{\omega^3}{4m^2_{Pl}}A_+(\omega) \approx 4\pi r_s^2\left[\frac{(r_s\omega)^4}{45}+\mathcal{O}(r_s\omega)^6\right]$$

$\Rightarrow$

$$A_+(\omega>0) = \frac{1}{2G_N}\frac{r_s^6\omega}{45}+\mathcal{O}(r_s^8\omega^3).$$

$$A_+(\omega<0) = 0$$ IF NO EMISSION FROM BH

"Boulware state"

Dissipative particle mechanics

Same correlation functions encode dissipative effects in binary dynamics. First consider the case of a BH in a background gravitational field, with $r_s \ll \mathcal{R} =$ curvature scale. Pt. particle action:

$$S = S_{pp} + S_X$$

Hamiltonian form of pt. particle action:

$$S_{pp} = -\int dx^\mu p_\mu + \frac{1}{2}\int d\lambda e \left(g^{\mu\nu} p_\mu p_\nu - m^2\right)$$

$ds = e d\lambda$
(RPI worldline parameter)

Action for worldline localized DOFs:

$$S_X = \int d\lambda e L_X(X, e^{-1}\dot{X}) - \int d\lambda e Q^E_{\mu\nu}(X,e) E^{\mu\nu}(x,p) - \int d\lambda e Q^B_{\mu\nu}(X,e) B^{\mu\nu}(x,p)$$

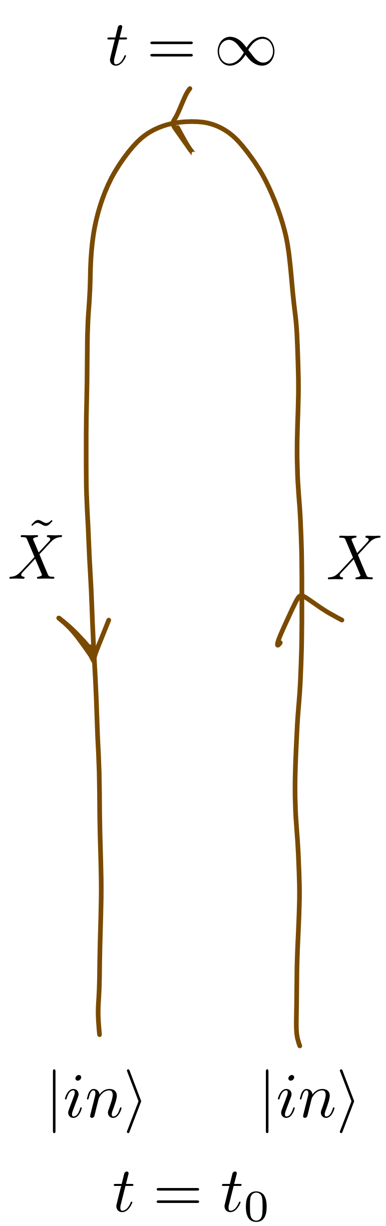
$$+ \cdots$$

explicit form of the internal Lagrangian $L_X = L_X(X, e^{-1}\dot{X})$ is not important.

Schwinger-Keldysh variational principle:

We treat the internal modes quantum mechanically. Since we are solving an initial value problem, the correct formulation of the EFT is the IN-IN (closed time path) formalism (see Galley+Tiglio, 2009)

Effective action for orbital DOFs $\chi = (x^\mu, p_\mu)$ obtained by integrating out internal modes from the Schwinger-Keldysh path integral



$$\exp\left[i\Gamma[\chi, e; \tilde{\chi}, \tilde{e}]\right] = \int DX D\tilde{X} \exp\left[iS[\chi, X, e] - iS[\tilde{\chi}, \tilde{X}, \tilde{e}]\right]$$

Equations of motion extremize the Schwinger-Keldysh action:

$$\frac{\delta}{\delta\chi(\lambda)}\Gamma[\chi, e; \tilde{\chi}, \tilde{e}]\bigg|_{\tilde{\chi},\tilde{e}=\chi,e} = 0.$$

For the system $S = S_{pp} + S_X$, variation of the IN-IN action yields:

$\dfrac{\delta}{\delta e}$:

$$p^2 = m^2 + 2\langle H_X + H_{int}\rangle$$

$$H_X = -\frac{\delta}{\delta e}\int d\lambda e L_X(X, e^{-1}\dot{X}) = \dot{X}\frac{\partial L_X}{\partial \dot{X}} - L_X = \text{Internal Hamiltonian}$$

$$H_{int} = \frac{\delta}{\delta e}\int d\lambda e\left(Q^E_{\mu\nu}E^{\mu\nu} + Q^B_{\mu\nu}B^{\mu\nu}\right) = \text{Tidal interaction}$$

(encodes transfer of energy bet. orbital and internal modes)

$\dfrac{\delta}{\delta x^\mu}$:

$$\frac{D}{Ds}p^\mu \equiv \frac{dx^\rho}{ds}\nabla_\rho p^\mu = \langle Q^E_{\rho\sigma}\rangle \nabla^\mu E^{\rho\sigma} + \langle Q^B_{\rho\sigma}\rangle \nabla^\mu B^{\rho\sigma} \qquad p^\mu = \frac{dx^\mu}{ds}$$

(tidal force on CM momentum)

Here, the expectation value of an operator is defined as

$$\langle \mathcal{O}[X]\rangle = \int DX D\tilde{X} e^{iS[x,e,X] - iS[x,e\tilde{X}]}\mathcal{O}[X]$$

For example, the presence of background curvature induces a non-zero expectation value for the quadrupole operators. In the linear response approximation (drop curvature squared terms)

$$\langle Q^E_{\mu\nu}(s)\rangle = \int ds' G^{E;ret}_{\mu\nu;\rho\sigma}(s-s') E^{\rho\sigma}(x(s')) + \mathcal{O}(E^2),$$

with the Green's function

$$G^{E;ret}_{\mu\nu,\rho\sigma}(s-s') = -i\theta(s-s')\langle[Q^E_{\mu\nu}(s), Q^E_{\rho\sigma}(s')]\rangle$$

is the retarded (causal) Green's function of the operator $Q^E_{\mu\nu}$

(similar equation holds for the magnetic curvature and quadrupole…)

Wightman vs. Retarded Green's function.

In particular, consider a BH at rest at the origin placed in a background tidal field. The induced quadrupole moment is then

$$\langle Q_{ij}(t)\rangle = -2c_E\bar{E}_{ij}(t,0) + \int_{-\infty}^{\infty} dt' G^R_{ij,rs}(t-t')\bar{E}_{rs}(t',0),$$

where I have also included the contribution of a term $S_{pp} \supset c_E \int d\tau E_{\mu\nu}E^{\mu\nu} + c_B \int d\tau B_{\mu\nu}B^{\mu\nu}$

In frequency space, can think of this as an "AC" (freq. dep.) Love number.

$$\langle Q_{ij}(\omega)\rangle = -2c_E(\omega)\bar{E}_{ij}(\omega,0),$$

$$c_E(\omega) = c_E - \frac{1}{2}G^R(\omega) = c_E - \frac{1}{2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{A_+(\omega') - A_+(-\omega')}{\omega - \omega' + i\epsilon}.$$

"dynamical Love number"

In the last line, I used a dispersion relation to related retarded correlator to $\langle QQ\rangle$. Given the matching result $A_+(\omega) = \theta(\omega)r_s^6\omega/90G_N$ as well as the vanishing of the static Love number yields a prediction for the response fn.

$$c_E(\omega)|_{classical} = \frac{ir_s^6\omega}{360G_N} + \mathcal{O}(r_s^8\omega^2).$$

(tune $\mathrm{Re}c_E(\omega\to 0) = 0$ to ensure vanishing static Love number)

(see also Steinhoff et al, 2013)

Stated covariantly, this implies that the transverse traceless response is

$$\langle Q^E_{\mu\nu}(s)\rangle = \frac{r_s^6}{180 G_N}\left(P_\mu{}^\rho P_\nu{}^\sigma - \frac{1}{3}P_{\mu\nu}P^{\rho\sigma}\right)\dot{E}_{\rho\sigma}(x(s)) + \cdots$$

$$\langle Q^B_{\mu\nu}(s)\rangle = \frac{r_s^6}{180 G_N}\left(P_\mu{}^\rho P_\nu{}^\sigma - \frac{1}{3}P_{\mu\nu}P^{\rho\sigma}\right)\dot{B}_{\rho\sigma}(x(s)) + \cdots ,$$

$$\dot{E}_{\mu\nu} = \dot{x}^\sigma \nabla_\sigma E_{\mu\nu}$$

$$\dot{B}_{\mu\nu} = \dot{x}^\sigma \nabla_\sigma B_{\mu\nu}$$

$$P^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

So the tidal friction on a black hole moving in a background field is

$$\frac{D}{Ds}p^\mu \approx \frac{r_s^6}{180 G_N}\left[\dot{E}_{\rho\sigma}\nabla^\mu E^{\rho\sigma} + \dot{B}_{\rho\sigma}\nabla^\mu B^{\rho\sigma}\right]$$

and thus the BH gains mass at rate,

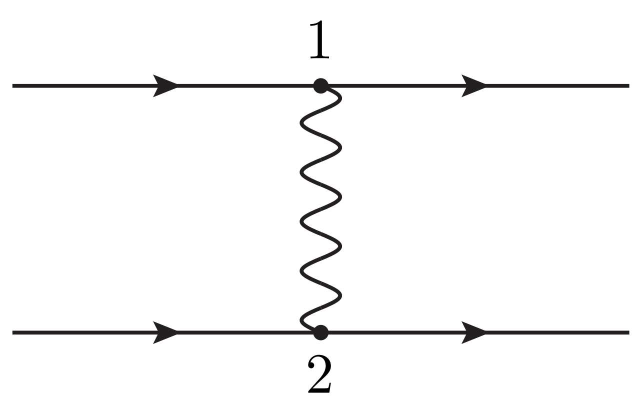
$$\dot{M} = \frac{1}{M}p\cdot\frac{D}{Ds}p \approx \frac{16}{45}(G_N^5 M^6)\left(\dot{E}_{\mu\nu}\dot{E}^{\mu\nu} + \dot{B}_{\mu\nu}\dot{B}^{\mu\nu}\right)$$

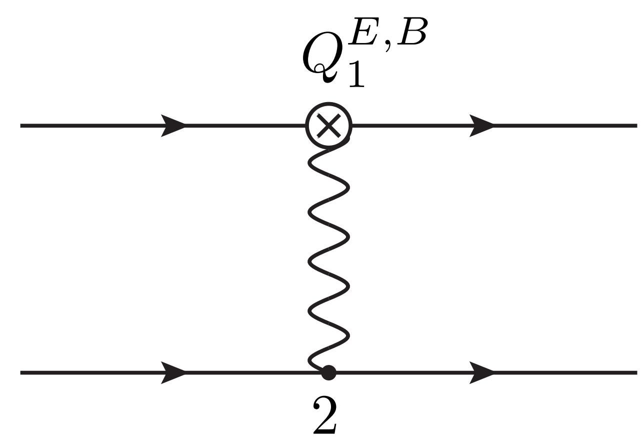
consistent with (time averaged) results in D'Eath (1975), Poisson (2004)

Dissipation in PN and PM binary black hole systems

Now we assume a separation of scales $G_N E/b \ll 1$ (PM) or $v^2 \sim G_N M/r \ll 1$ (PN), in the two-body system, so that the gravitational interaction is perturbative.

In this case, we can view the grav. force as being mediated by the exchange of an off-shell ("potential") graviton. (On-shell radiation is higher order in the power counting)

$S_{int}[x_\alpha, p_\alpha] =$ 

$+ \cdots +$ 

$+ \cdots$

$$D(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} \frac{i}{k^2}$$

(a) (b)

$$(a) = 8\pi i G_N \int ds_1 ds_2 \left[(p_1 \cdot p_2)^2 - \frac{1}{2} p_1^2 p_2^2 \right] D(x_{12}) =$$ LO grav. interaction

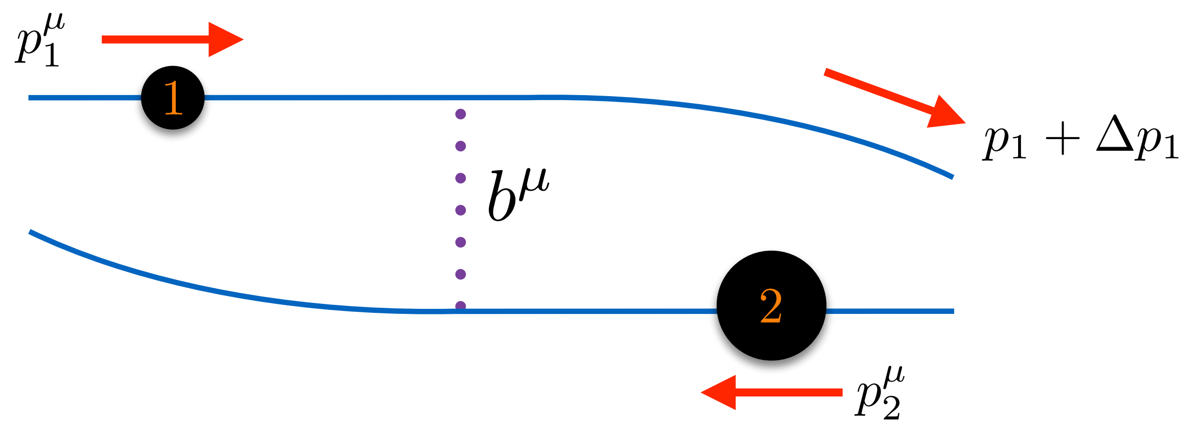
$$(b) = 8\pi i G_N \sum_{\alpha=1,2} \int ds_1 ds_2 \left[Q^{E,\alpha}_{\mu\nu}(s_\alpha) T_{E,\alpha}{}^{\mu\nu,\rho\sigma} + Q^{B,\alpha}_{\mu\nu}(s_\alpha) T_{B,\alpha}{}^{\mu\nu,\rho\sigma} \right] \partial_\rho \partial_\sigma D(x_{12})$$

$=$ grav. monopole/quadrupole interaction

tensor constructed out of $p_{1,2}$

Applications of the BH response function I: (WG+Rothstein, 2020)

Inelastic BH-BH scattering in the PM limit $G_N E/b \ll 1$



The momentum deflection of each particle follows from varying S_{int} ($S_{0,\alpha}$ = free particle action)

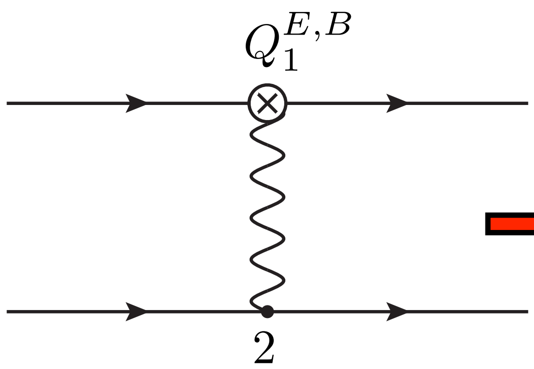
$$\Delta p_{\mu,\alpha} = \int ds \frac{d}{ds} p_{\mu,\alpha}(s) = \int ds \frac{\delta}{\delta x^\mu_\alpha(s)} S_{0,\alpha} = -\int ds \left\langle \frac{\delta}{\delta x^\mu_\alpha(s)} S_{int}[\chi, X] \right\rangle .$$

It is useful to split $\Delta p^\mu = \Delta p^\mu_{el} + \Delta p^\mu_{in}$.

By definition, the elastic part preserves the mass of each particle, $(p + \Delta p_{el})^2 = p^2$, and can be computed by standard PM or scattering amplitude methods,

$$\Delta p_1^\mu = -\Delta p_2^\mu = -\frac{4 G_N m_1 m_2}{b^2} \frac{(v_1 \cdot v_2)^2 - \frac{1}{2}}{\sqrt{(v_1 \cdot v_2)^2 - 1}} b^\mu + \cdots$$

The leading order inelastic contribution to Δp^μ is from the quadrupole operators:



$$P(\gamma) = 21\gamma^4 - 14\gamma^2 + 1$$

$$\Delta p^\mu_{1,in} = -\frac{5\pi}{16}\frac{G_N^7 m_1^4 m_2^4}{b^7}\frac{P(v_1\cdot v_2)}{\sqrt{(v_1\cdot v_2)^2-1}}\left[\frac{m_1^2}{m_2^2}(v_1-(v_1\cdot v_2)v_2)^\mu - (1\leftrightarrow 2)\right],$$

which is a 6PM(!) effect.

Some simple consequences of this result:

Change in BH mass:

$$\Delta m_1^2 \approx 2p_1\cdot\Delta p_{1,in} = \frac{5\pi}{16}\frac{G_N^7 m_1^7 m_2^2}{(b^2)^{7/2}}P(\gamma)\sqrt{\gamma^2-1} > 0$$

consistent with Hawking's area thm.

Distribution of final state BH masses/areas: $(\gamma \gg 1)$

$$\frac{d\sigma}{d\Delta m_1^2} = 2\pi\left|b\frac{\partial b}{\partial\Delta m_1^2}\right| \approx \left[\frac{225\pi^9}{33614}\right]^{1/7}(G_N m_1)^2\gamma^{10/7}m_2^{4/7}(\Delta m_1^2)^{-9/7}, \qquad \frac{1}{A_1}\cdot\frac{d\sigma}{d\log\Delta A_1} \approx \frac{1}{7}\left[\frac{(105\pi)^2}{2^{29}}\right]^{1/7}\gamma^{10/7}\left[\frac{A_2}{\Delta A_1}\right]^{2/7}$$

Correction to CM frame scattering angle at 8PM:

$$\frac{\Delta\chi_{in}}{\chi_{1PM}} = \frac{5\pi}{16}\left(\frac{G_N m_1 m_2}{J}\right)^7 P(\gamma(\hat{s}))\left[\frac{1}{m_1^3}+\frac{1}{m_2^3}+\gamma(\hat{s})\left(\frac{m_1^5+m_2^5}{m_1^4 m_2^4}\right)\right]\frac{|\vec{p}|^6}{\sqrt{\hat{s}}}$$

$$E_{CM} = \sqrt{\hat{s}} \qquad J = |\vec{p}|b \qquad \gamma(\hat{s}) = (\hat{s}^2 - m_1^2 - m_2^2)/\,m_1 m_2 \qquad |\vec{p}| = m_1 m_2\sqrt{\gamma^2-1}/\sqrt{\hat{s}}$$

Applications of the BH response function II:

Horizon friction in non-relativistic BH/BH binary system. Action in NR limit

$$S_{int} \approx -G_N m_1 m_2 \int dt \left(\frac{Q^{ij}_{E,1}(t)}{m_1^2} + \frac{Q^{ij}_{E,2}(t)}{m_2^2} \right) \partial_i \partial_j \frac{1}{|\vec{x}(t)|},$$

Variation of the IN-IN action gives dissipative force:

$$\vec{F}_1(t) = \frac{\delta}{\delta \vec{x}_1(t)} \Gamma[\vec{x}, \tilde{\vec{x}}]\Big|_{\vec{x}=\tilde{\vec{x}}} = -G_N m_1 m_2 \left\langle \frac{Q^{jk}_{E,1}(t)}{m_1^2} + \frac{Q^{jk}_{E,2}(t)}{m_2^2} \right\rangle \nabla \partial_j \partial_k \frac{1}{|\vec{x}(t)|} = -\vec{F}_2(t),$$

From the retarded response function, in the $|\vec{v}| \ll 1$ limit: (consistent with Endlich+Penco 2015)

6.5PN rad reaction force

$$\vec{F}_1(t) = -\vec{F}_2(t) = -\frac{32}{5} \frac{G_N^7 (m_1 m_2)^2 (m_1^4 + m_2^4)}{|\vec{x}|^8} \left[\vec{v} + \frac{2\vec{v} \cdot \vec{x}_{12}}{|\vec{x}|^2} \vec{x} \right],$$

Dissipated mechanical energy and angular momentum:

$$\frac{d}{dt} E_h = \sum_{\alpha=1}^{2} \vec{v}_\alpha \cdot \vec{F}_\alpha = -\frac{32}{5} G_N^{-1} \left(\frac{G_N M}{|\vec{x}|} \right)^8 \left(\frac{\mu}{M} \right)^2 \left(1 - \frac{2\mu}{M} + \frac{2\mu^2}{M^2} \right) \left[\vec{v}^2 + \frac{2(\vec{x} \cdot \vec{x})^2}{|\vec{x}|^2} \right]$$

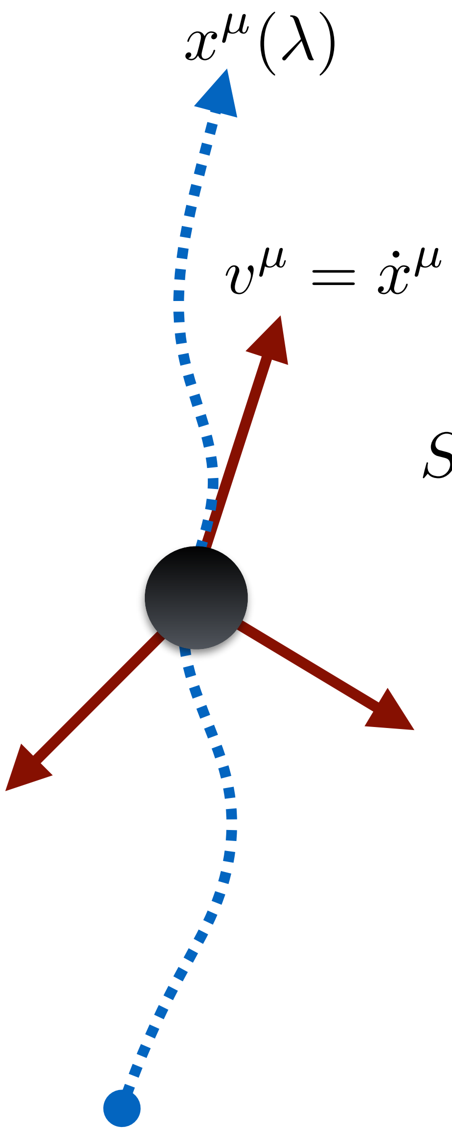
$$\frac{d}{dt} \vec{L}_{CM} = \sum_{\alpha=1}^{2} \vec{v}_\alpha \times \vec{F}_\alpha = \frac{64}{5} G_N^7 \mu M^6 \left(1 - \frac{2\mu}{M} + \frac{2\mu^2}{M^2} \right) \frac{\vec{v} \cdot \vec{x}}{|\vec{x}|^{10}} \vec{L}_{CM}$$

(WG+Rothstein, 2006; agrees w/ Poisson '95 at $\mu \ll M$)

Note: $P_{abs}/P_{quad} \sim v^8$ is a (small) 4PN effect. Absorption enhanced to v^5 for rotating black holes (Tagoshi et al '97, Poisson '04)

The spinning case

(WG+Li,Rothstein arXiv:2012, previous work: Porto, 2008, Penco+Endlich 2016)



In the point particle limit, the action is now (Regge-Hansen 1974; Porto el al; Levi et al)

$$S_{pp} = -\int dx^\mu e^a{}_\mu p_a + \frac{1}{2}\int d\lambda S^{ab}\Omega_{ab} + \frac{1}{2}\int d\lambda e\left(p_a p^a - m^2\right) + \int d\lambda e \lambda_a S^{ab} p_b,$$

The indices $a = 0,1,2,3$ correspond to quantities measured in the local (rotating) frame of the particle. The relation between the local frame and the global coordinates is encoded in a dynamical orthonormal frame e^a_μ localized on the worldline

$$\eta_{ab} e^a_\mu e^b_\nu = g_{\mu\nu}(x(s)) \qquad g_{\mu\nu}(x(s)) e^\mu_a e^\nu_b = \eta_{ab},$$

$$\dot{e}^a{}_\mu \equiv \dot{x}^\sigma \nabla_\sigma e^a{}_\mu = \Omega^a{}_b e^b{}_\mu \qquad \Omega^{ab} = -\Omega^{ba}$$

The multipole operators are now regarded as diffeo scalars living on the particle wordline

$$S_{int} = -\int d\lambda e Q^E_{ab}(X,e)E^{ab}(x,p) - \int d\lambda e Q^B_{ab}(X,e)B^{ab}(x,p)$$

$$E_{ab} = e^{\mu}{}_a e^{\nu}{}_b E_{\mu\nu} \qquad\qquad B_{ab} = e^{\mu}{}_a e^{\nu}{}_b B_{\mu\nu}$$

The most general correlator can be expressed in a basis of local tensors invariant under rotations about the axis defined by the spin. These consist of the independent powers of the generator of local rotations J_3 about the spin axis.

$$\langle Q^{ab}(s)Q^{cd}(s')\rangle = \sum_{k=0}^{4} A^{+}_{k}(s-s')\langle a,b|J_3^k|c,d\rangle,$$

w/

$$\langle a,b|J_3|c,d\rangle = \frac{1}{2}\left[\langle a|c\rangle[J_3]^b{}_d + \langle a|d\rangle[J_3]^b{}_c + \langle b|c\rangle[J_3]^a{}_d + \langle b|d\rangle[J_3]^a{}_c\right], \qquad (J_3^{\dagger} = J_3)$$

$$\langle a|b\rangle = \delta^a{}_b - p^a p_b/p^2 \qquad\qquad [J_3]^a{}_b = is^c\epsilon_c{}^a{}_b = \frac{i}{M}p^c s^d \epsilon_{cd}{}^a{}_b$$

$s^a =$ unit spin vector $\quad (s_a s^a = -1)$

$\langle a,b|J_3|c,d\rangle$ should be viewed as the 5×5 matrix that generates rotations on the $\ell = 2$ (STF tensors) representation of $SO(3)$. It has been normalized to have eigenvalues $m = \pm 2, \pm 1, 0$

To extract the five form factors $A_k^+(s-s')$, we match to the probability that a Kerr BH absorbs a single graviton of energy ω and angular momentum quantum numbers $(\ell = 2, m, h = \pm 2)$

(Page 1975)

$$p(1 \to 0) \approx \frac{16}{225\pi} A_H (G_N M)^4 \omega^5 \left[1 + (m^2 - 1)\chi^2\right] \left[1 + \frac{1}{4}(m^2 - 4)\chi^2\right] \theta(\omega - m\Omega_H)\,(\omega - m\Omega_H)$$

($\omega - m\Omega_H < 0$ corresponds instead to superradiant emission from the BH)

vs the EFT result, computed in the rest frame of the point particle, which rotates with constant angular velocity Ω:

$$e_a^\mu = \delta_a^0 \qquad\qquad e^i{}_a = \begin{pmatrix} \cos\Omega t & -\sin\Omega t & 0 \\ \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

One finds

$$p(1 \to 0) \approx \frac{\pi^2}{5(m_{Pl} M)^2} \left| \int ds \langle X | Q_E^{ab}(t) | M \rangle \langle a, b | \ell = 2, m \rangle \lambda(t) e^{im\Omega t} \right|^2 + \text{mag}$$

graviton wavefn.

$$= \frac{4}{5} G_N \omega^5 \sum_{j=0}^{4} m^j \left(A_{E,j}^+(\omega - m\Omega) + A_{B,j}^+(\omega - m\Omega) \right)$$

(freq space correlators)

note shifted frequency $\omega \to \omega - m\Omega$ reflects transformation to the co-rotating frame

Comparing the Kerr BH to the EFT requires that $\Omega = \Omega_H$ and $A_k^+(\omega) \sim i\theta(\omega)\omega$

$$A^+_{0,E}(\omega) = A^+_{0,B}(\omega) = \frac{2A_H}{45\pi G_N}(G_N M)^4(1-\chi^2)^2\theta(\omega)\omega,$$
$$A^+_{2,E}(\omega) = A^+_{2,B}(\omega) = \frac{A_H}{18\pi G_N}(G_N M)^4\chi^2(1-\chi^2)\theta(\omega)\omega,$$
$$A^+_{4,E}(\omega) = A^+_{4,B}(\omega) = \frac{A_H}{90\pi G_N}(G_N M)^4\chi^4\theta(\omega)\omega.$$

Using a (Kramers-Kronig) dispersion relation* the retarded response is then

$$\begin{aligned}\langle Q_E^{ab}(s)\rangle &= \int ds' G^{ab}_{R,Ecd}(s-s')E_{cd}(s') \\ &= \frac{A_H}{45\pi G_N}(G_N M)^4\langle a,b|(1-\chi^2)^2 + \frac{5}{4}\chi^2(1-\chi^2)J_3^2 + \frac{1}{4}\chi^4 J_3^4|c,d\rangle\frac{d}{ds}E_{cd}(x(s)),\end{aligned}$$

(in obtaining this result, we have also used the result of H.S Shen, 2010.07300, to fix the time-reversal even part of the response)

Note that the time derivative is

$$\frac{d}{ds}E_{ab} = e^{-1}\frac{d}{d\lambda}E_{ab} = e_a{}^\mu e_b{}^\nu\left(\frac{dx^\rho}{ds}\nabla_\rho\right)E_{\mu\nu} - e^{-1}\Omega_a{}^c E_{cb} - e^{-1}\Omega_b{}^c E_{ac}.$$

which includes the transformation to the co-rotating frame, $\dot{e}^a{}_\mu = \Omega^a{}_b e^b{}_\mu$

Check: $\dot{M}$, $\dot{S}$ of Kerr BH in a background field (curvature scale $= \mathcal{R}$)

$$\frac{\delta}{\delta x^\mu}\Gamma = 0 \;\Rightarrow\; \frac{D}{Ds}p^\mu = -\frac{1}{2}R^\mu{}_{\lambda\rho\sigma}\frac{dx^\lambda}{ds}S^{\rho\sigma} + e^a{}_\rho e^b{}_\sigma\left[\langle Q^E_{ab}\rangle\nabla^\mu E^{\rho\sigma} + \langle Q^B_{ab}\rangle\nabla^\mu B^{\rho\sigma}\right]$$

$$\Rightarrow\; \frac{d}{ds}M^2 = 2e^a{}_\rho e^b_\sigma\left[\langle Q^E_{ab}\rangle(p\cdot\nabla)E^{\rho\sigma} + \langle Q^B_{ab}\rangle(p\cdot\nabla)B^{\rho\sigma}\right] \neq 0$$

$$\frac{\delta}{\delta e^a{}_\mu}\Gamma = 0 \;\Rightarrow\; \frac{D}{Ds}S^{\mu\nu} = \frac{dx^\nu}{ds}p^\mu - \frac{dx^\mu}{ds}p^\nu + 2e^\mu{}_a e^\nu{}_b\left[\langle Q^E_{cd}\rangle\frac{\delta}{\delta\theta_{ab}}E_{cd} + \langle Q^B_{cd}\rangle\frac{\delta}{\delta\theta_{ab}}B_{cd}\right]$$

$$\Rightarrow\; \frac{d}{ds}S^2 = 4\langle Q^E_{ab}\rangle E^{bc}S^a{}_c + 4\langle Q^B_{ab}\rangle B^{bc}S^a{}_c \neq 0.$$

inserting the response fns.

$$\frac{d}{d\tau}M \approx \frac{8(G_N M)^5}{45 G_N}\chi\epsilon^{\mu\nu}{}_\lambda s^\lambda\left[(1+3\chi^2)E_{\mu\rho}\dot{E}^\rho{}_\nu + \frac{15}{4}\chi^2 E_{\mu\rho}s^\rho\dot{E}_{\nu\sigma}s^\sigma\right] + \text{magnetic} + \mathcal{O}(G_N M/\mathcal{R}).$$

$$\frac{d}{d\tau}S \approx -\frac{2}{45 G_N}(G_N M)^5\chi\left[8(1+3\chi^2)E_{\rho\sigma}E^{\rho\sigma} + 3(4+17\chi^2)E_{\lambda\rho}E^\lambda{}_\sigma s^\rho s^\sigma + 15\chi^2(E_{\rho\sigma}s^\rho s^\sigma)^2\right] + \text{magnetic}.$$

$(\Omega_H \gg \mathcal{R}^{-1})$

$$\frac{d}{d\tau}M \approx \frac{16}{45 G_N}(G_N M)^6\left[\dot{E}_{\rho\sigma}\dot{E}^{\rho\sigma} + \dot{B}_{\rho\sigma}\dot{B}^{\rho\sigma}\right] + \mathcal{O}(\chi)$$

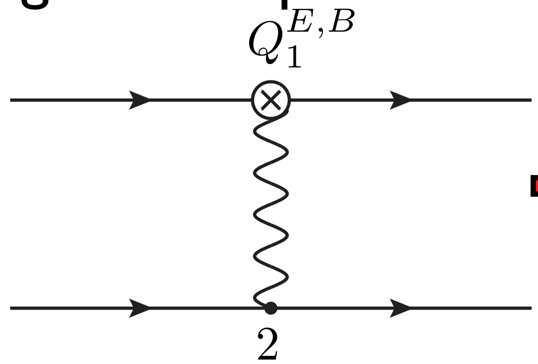
$$\frac{d}{d\tau}S \approx -\frac{8}{45 G_N}(G_N M)^6\epsilon^{\mu\nu}{}_\lambda s^\lambda\left[E_{\mu\rho}\dot{E}^\rho{}_\nu + B_{\mu\rho}\dot{B}^\rho{}_\nu\right]$$

$(\Omega_H \ll \mathcal{R}^{-1})$

in agreement w/ D'Eath (1975), Poisson (2004), Chantziioannou et al (2012,2016)

Application: PN Equations of motion for near extremal BH/BH binary

Integrate out potential exchange in NR limit:



$$S_{int} \approx -G_N m_1 m_2 \int dt \left[\frac{Q^{ab}_{E,1}(t)}{m_1^2} {e_1}^i{}_a {e_1}^j{}_b + (1 \leftrightarrow 2) \right] \partial_i \partial_j \frac{1}{|\vec{x}(t)|},$$

Force and torque from variation of Schwinger-Keldysh action:

$$\vec{F}_1(t) = \frac{\delta}{\delta \vec{x}_1(t)} \Gamma[\vec{x}, \tilde{\vec{x}}; e_{1,2}, \tilde{e}_{1,2}] \Big|_{\vec{x}=\tilde{\vec{x}}; e_{1,2}=\tilde{e}_{1,2}} = -\vec{F}_2(t) \qquad \frac{d}{dt} \vec{S}_1^i = e^i_{1a} \epsilon^{abc} \frac{\delta}{\delta {\theta_1}^{bc}} \Gamma[\vec{x}, \tilde{\vec{x}}; e_{1,2}, \tilde{e}_{1,2}] \Big|_{\vec{x}=\tilde{\vec{x}}; e_{1,2}=\tilde{e}_{1,2}}$$

For near extremal (max. rotating) BHs, w/ $\chi = S/G_N M^2 \sim \mathcal{O}(1)$

$$\vec{F}_1(t) = -\vec{F}_2(t) = -\frac{8}{5} \frac{G_N^5 m_1^3 m_2^2}{|\vec{x}|^7} \left[1 + 3\chi_1^2 - \frac{15}{4} \chi_1^2 \left(\vec{s}_1 \cdot \frac{\vec{x}}{|\vec{x}|} \right)^2 \right] \frac{\vec{x}}{|\vec{x}|} \times \vec{S}_1 + (1 \leftrightarrow 2)$$

(5PN non-conservative force)

$$\frac{d}{dt} \vec{S}_1 = -\frac{8}{5} \frac{G_N^5 m_1^3 m_2^2}{|\vec{x}|^6} \left[1 + 3\chi_1^2 - \frac{15}{4} \chi_1^2 \left(\vec{s}_1 \cdot \frac{\vec{x}}{|\vec{x}|} \right)^2 \right] \left[\vec{S}_1 - \frac{\vec{S}_1 \cdot \vec{x}}{\vec{x}^2} \vec{x} \right].$$

(4PN spin precession)

Leads to new result for 2.5 PN energy flux into/from orbital degrees of freedom

$$\frac{d}{dt} E = \sum_a \vec{v}_a \cdot \vec{F}_a \approx \frac{8}{5} \frac{G_N^5 m_1^2 m_2}{|\vec{x}|^8} (m_1 + m_2) \left[1 + 3\chi_1^2 - \frac{15}{4} \chi_1^2 \left(\vec{s}_1 \cdot \frac{\vec{x}}{|\vec{x}|} \right)^2 \right] \vec{S}_1 \cdot \vec{L} + (1 \leftrightarrow 2).$$

enhanced relative to $\chi = 0$ by $v^{-3} \gg 1$. Note that $\dot{E} > 0$ is allowed in the super-radiant regime corresponding to energy extraction from the BH (Penrose process)

Hawking emission and EFT

WG+Rothstein, arXiv:1912.13435
arXiv:2007.00731

In principle, same methods can be applied to capture long distance effects of emission by black hole horizon. To understand how this effects the Wightman functions $\langle Q^{E,B} Q^{E,B} \rangle$ we match the EFT to the particle emission/absorption probabilities of the BH in the "Unruh state"

Probability to emit n particles in normalized wavepacket, starting with m particles (same wavepacket) in the initial state:

(Bekenstein+Meisels;
Pananganden+Wald, 1977)

$$p_\ell(m \to n) = \frac{(1-x)x^n(1-|R_\ell|^2)^{n+m}}{(1-x|R_\ell|^2)^{n+m+1}} \sum_{k=0}^{\min(n,m)} \frac{(n+m-k)!}{k!(n-k)!(m-k)!} \left[\frac{(|R_\ell|^2 - x)(1-x|R_\ell|^2)}{x(1-|R_\ell|^2)^2} \right]^k$$

Boltzmann factor $x = e^{-\beta_H \omega}$ $\qquad |R_\ell(\omega)|^2 = 1 - |B_\ell(\omega)|^2$

In the limit $r_s \omega \ll 1$

$$p(1 \to 0) \approx p(0 \to 1) \approx \frac{|B^h_{\ell=2,m}(\omega)|^2}{\beta_H \omega} \approx \frac{(r_s\omega)^6}{225\pi} \quad \text{(full theory)}$$

$$p(1 \to 0) \approx \frac{4}{5} G_N \omega^5 \left(A^E_+(\omega) + A^B_+(\omega)\right)$$

$$p(0 \to 1) \approx \frac{4}{5} G_N \omega^5 \left(A^E_+(-\omega) + A^B_+(-\omega)\right)$$

(EFT) $\Longrightarrow$ $A^E_+(\omega) = A^B_+(\omega) \approx \frac{1}{360\pi} \frac{r_s^5}{G_N}$

Note that in the presence of Hawking emission, Wightman fns are enhanced by $(r_s\omega)^{-1} \gg 1$ relative to the classical case (no $1/M_{Pl}$ suppression)! Nevertheless, Hawking emission is suppressed in classical processes, i.e. $G^{Hawk.}_{ret}(\omega) = G^{class.}_{ret}(\omega) + \mathcal{O}(\hbar\omega/M_{Pl})^2$

Application: Inelastic gravitational scattering off BH mediated by virtual Hawking graviton exchange.

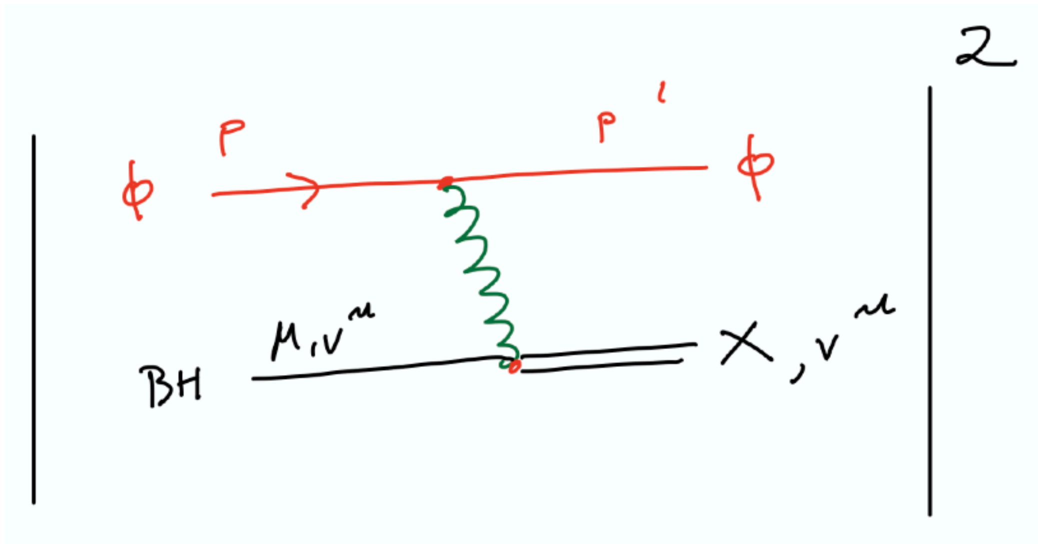
Wightman functions can be used to predict inclusive scattering cross section $\phi + BH \to \phi + BH'$.
We assume that

$$m_\phi \gg kT_H \sim 1/r_s \qquad q \ll 1/r_s$$

$$m_\phi \ll M_{BH} \qquad M_{BH} \gg M_{Pl}$$

but otherwise generic kinematics. In this limit the inelastic scattering process is dominated by off-shell Hawking graviton exchange with the BH

$$\sum_X |\mathcal{A}(\phi(p) + BH \to \phi(p') + X)|^2 = \sum_X$$



$$\frac{d^2\sigma}{dq^2 d(q\cdot v)} \approx \frac{7G_N r_s^5}{270\pi[(v\cdot p)^2 - m^2]}\left[(v\cdot p)^4 - m^2 (v\cdot p)^2\left(1 - \frac{12}{7}\frac{(v\cdot q)^2}{q^2}\right) + \frac{1}{7}m^4\left(1 - 3\frac{(v\cdot q)^2}{q^2} + 6\frac{(v\cdot q)^4}{q^4}\right)\right]$$

(In BH rest frame $v^\mu = (1, 0)$)

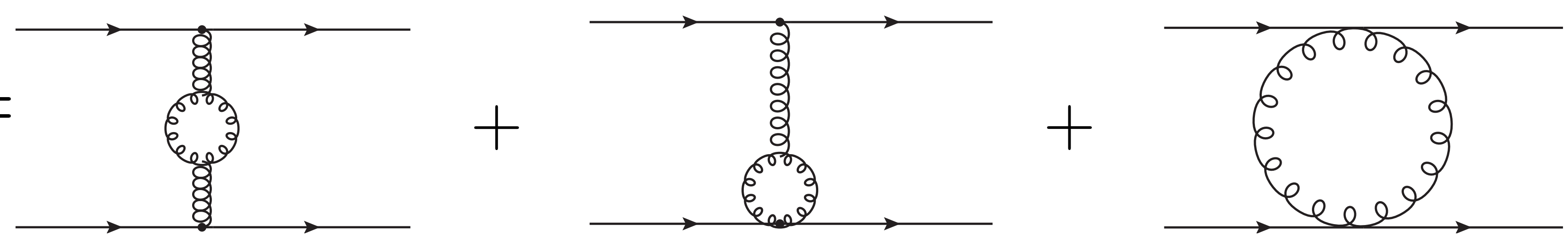
$$\frac{d^2\sigma}{dq^2 d(q\cdot v)} \approx \frac{7G_N r_s^5}{270\pi[(v\cdot p)^2 - m^2]}\Bigg[(v\cdot p)^4 - m^2(v\cdot p)^2\left(1 - \frac{12}{7}\frac{(v\cdot q)^2}{q^2}\right) + \frac{1}{7}m^4\left(1 - 3\frac{(v\cdot q)^2}{q^2} + 6\frac{(v\cdot q)^4}{q^4}\right)\Bigg]$$

(In BH rest frame $v^\mu = (1,0)$)

Suppressed over elastic scattering by a factor of $\sim \frac{q^2}{M_{Pl}^2}(r_s q)^3$ relative to LO elastic scattering off BH's Newtonian gravitational field.

This is same order in q^2/M_{Pl}^2 as the correction to elastic scattering from one-loop graviton vacuum polarization, a new calculable perturbative quantum gravity effect…

$$\mathcal{A}_{el}|_{\mathcal{O}(\hbar^1)} =$$



$$\sim \frac{q^2}{M_P^2 l}\cdot \mathcal{A}_{el}|_{\mathcal{O}(\hbar^0)}$$

't Hooft + Veltman (1974)
Donoghue (1995)

Conclusions

Proliferation of length scales in the compact binary inspiral problem motivates an EFT description

Black hole finite size effects, eg dissipation, can be included systematically in this EFT. For spinning BHs these can be enhanced in the superradiant regime $\Omega_H \gg \omega_{orbit}$.

→ New results at 2.5PN for dE/dt in BH/BH binary inspirals.

Quantum effects associated with the horizon can also be described in this EFT.

→ New calculable quantum gravity effects at $\mathcal{O}(q^2/M_{Pl}^2)$ due to off-shell Hawking exchange.