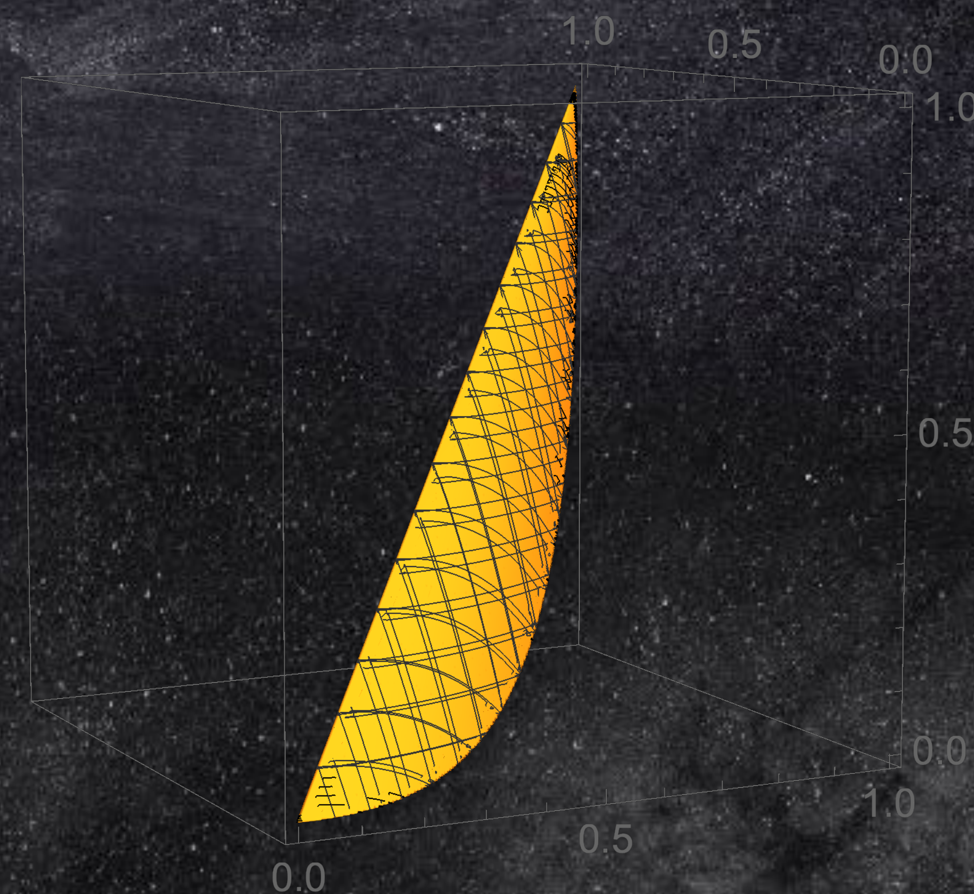
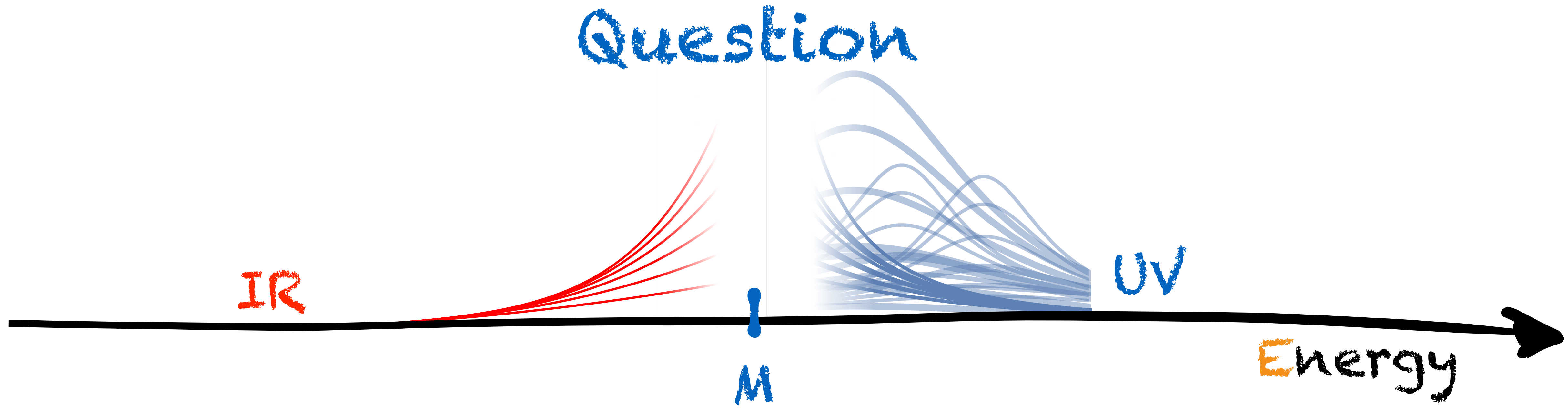
Positive Moments for Scattering Amplitudes



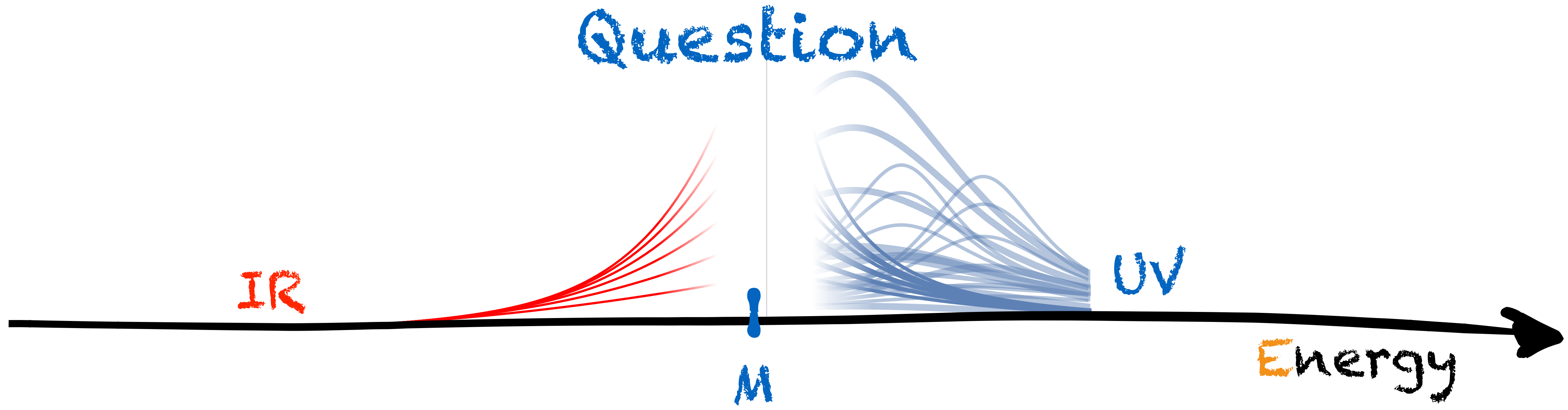
Francesco Riva
(Geneva University)

with Bellazzini, Elias-Miro, Rattazzi, Riembau
(and parts with Sgarlata and Serra)



Effective Field Theories: at the frontier with unknown/incalculable

(SM EFT, Quantum Gravity, Strong Coupling,...)



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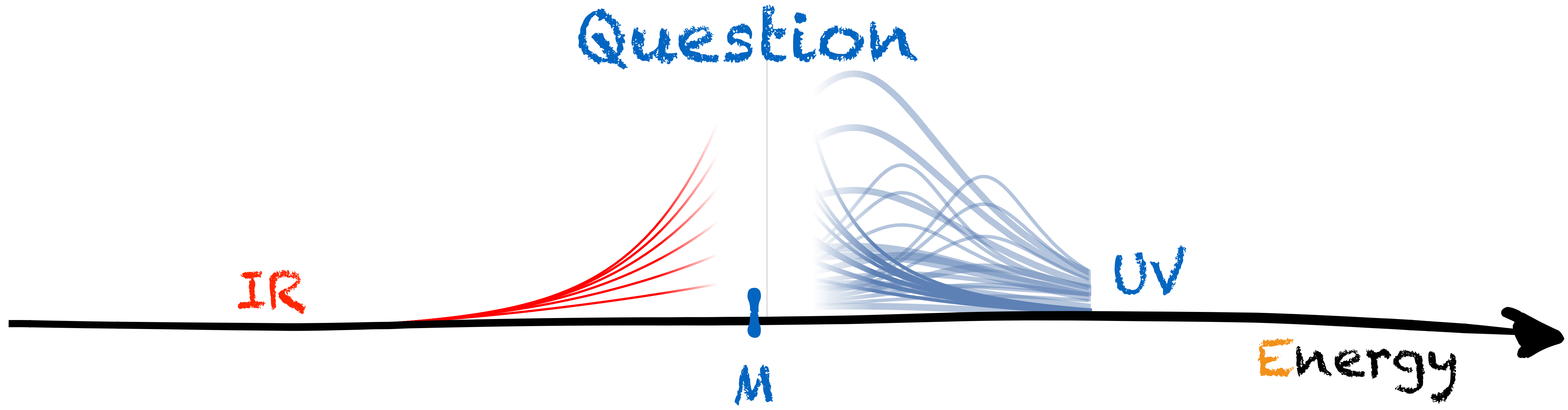
Model Independence ⟷ Broadest possible hypotheses



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Different E-behaviours IR consistent!

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

(e.g. tree-level)

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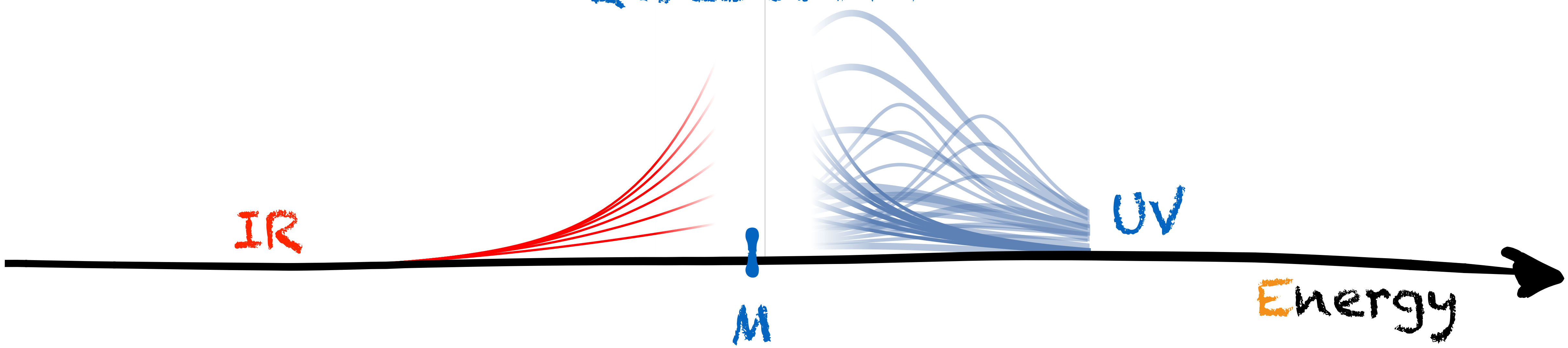

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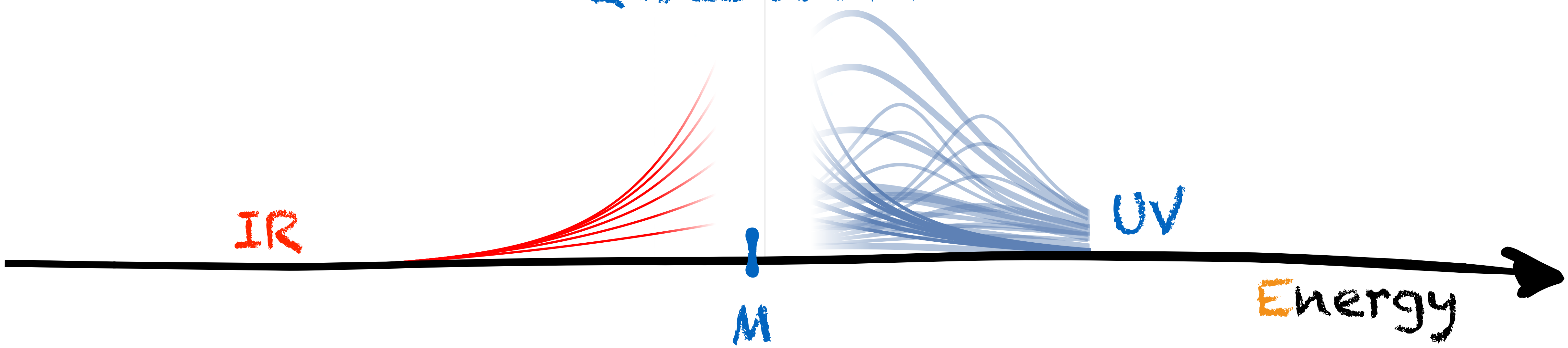

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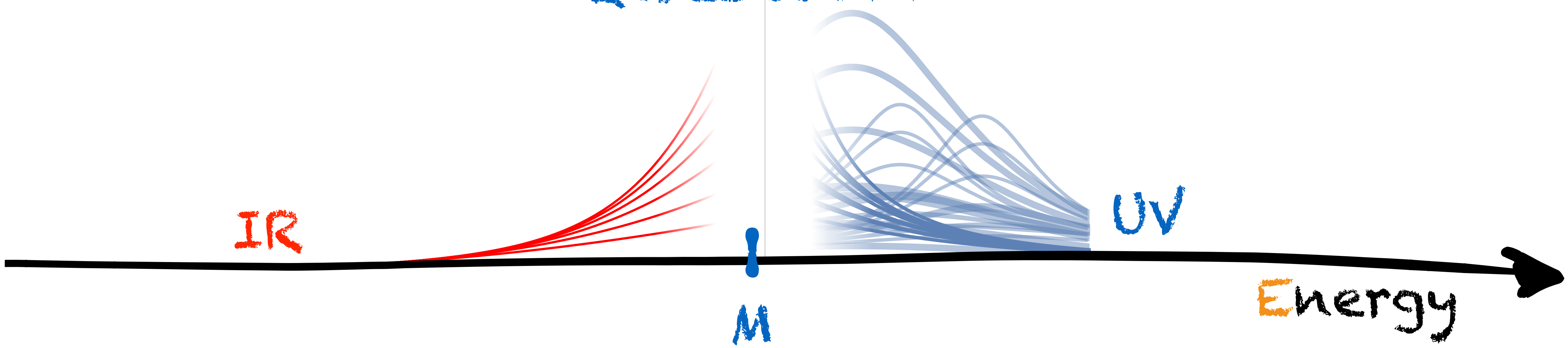

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Which IR theories are Causal and Unitary in UV?

Notation/Outline

$$A_{2\to 2}(s,t) = c_0 + \boldsymbol{c_2} s^2 + \boldsymbol{c_{2,1}} s^2 t + c_4 s^4 + \cdots + \boldsymbol{c_{n,m}} s^n t^m$$

(e.g. tree-level)

Notation/Outline

powers of t →

powers of s ↓

$\boldsymbol{c_2}$	$\boldsymbol{c_{2,1}}$	$c_{2,2}$	$\cdots$
$\boldsymbol{c_4}$	$c_{4,1}$	$c_{4,2}$	$\cdots$
c_6	$c_{6,1}$	$c_{6,2}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	

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Notation/Outline

1. IR

powers of t

powers of s

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2. UV → IR

(2 →2 amplitude, dispersion relation, arcs, moments)

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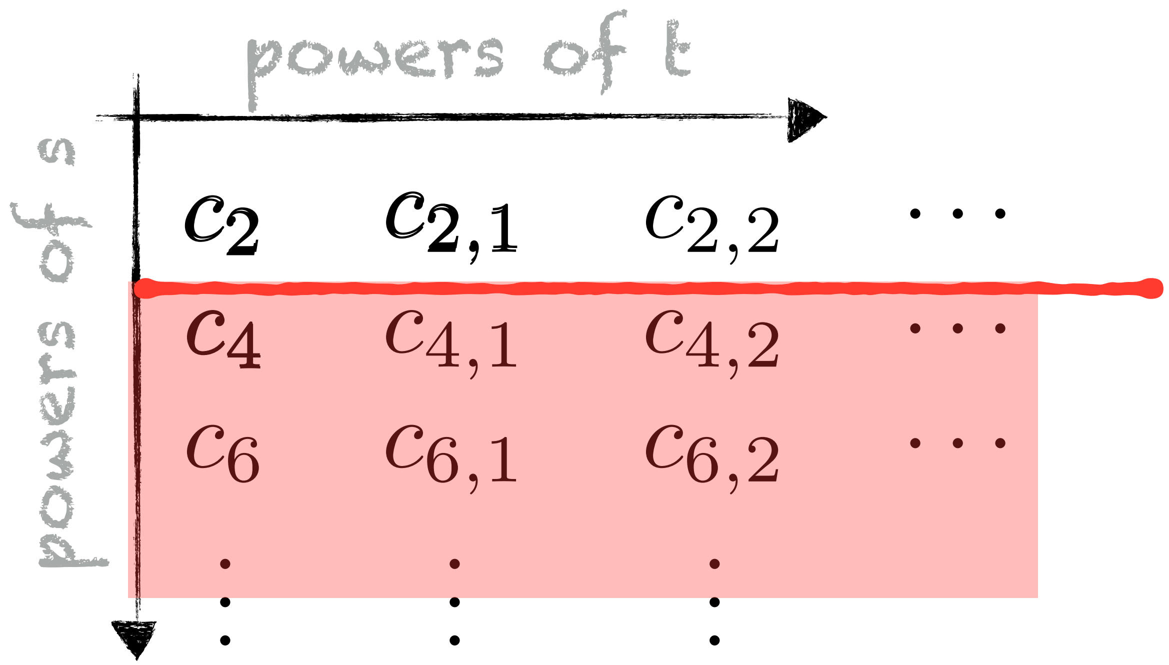

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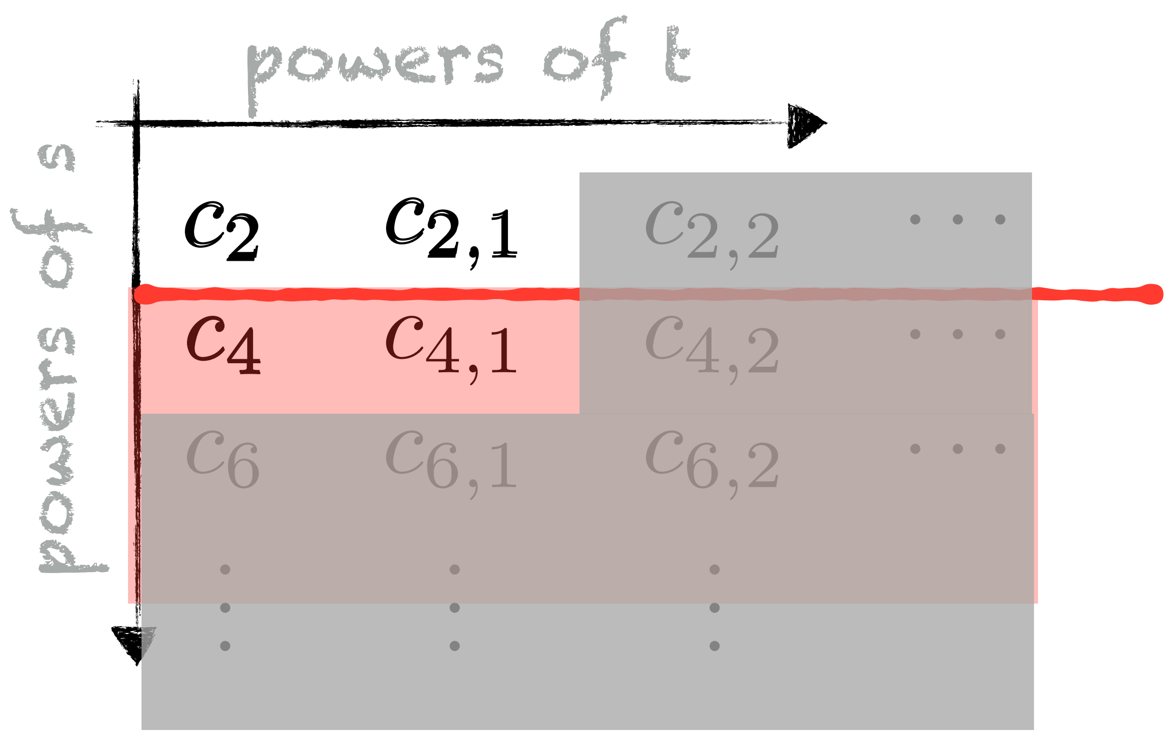

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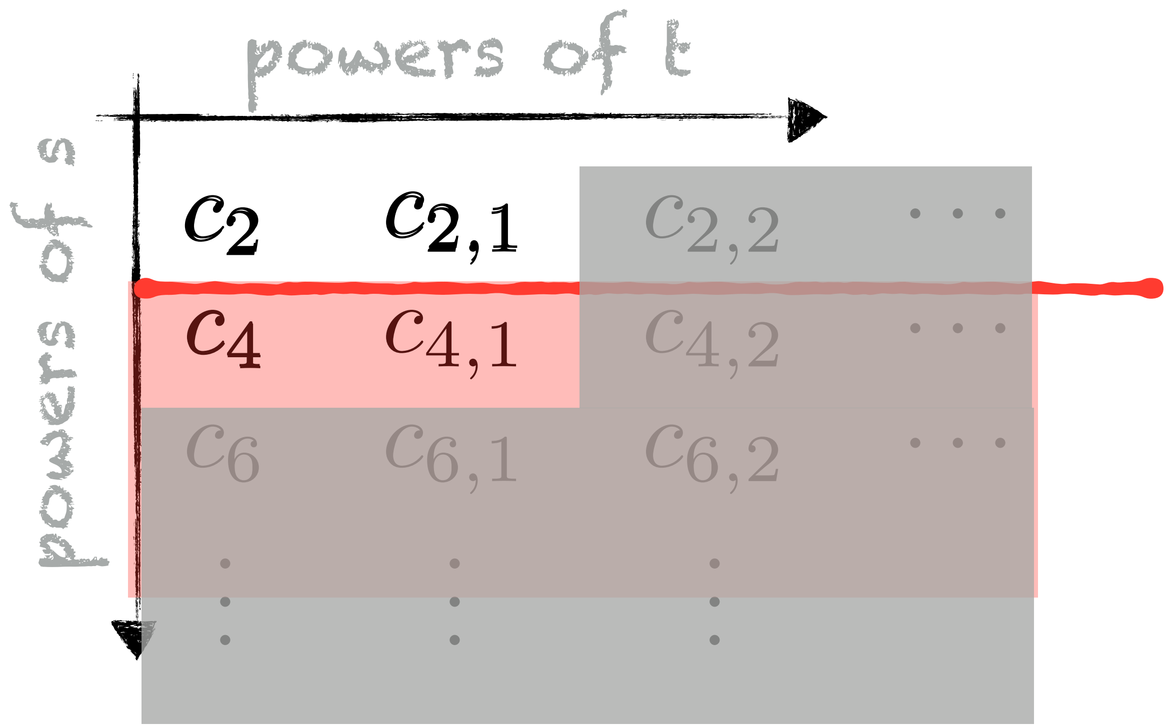

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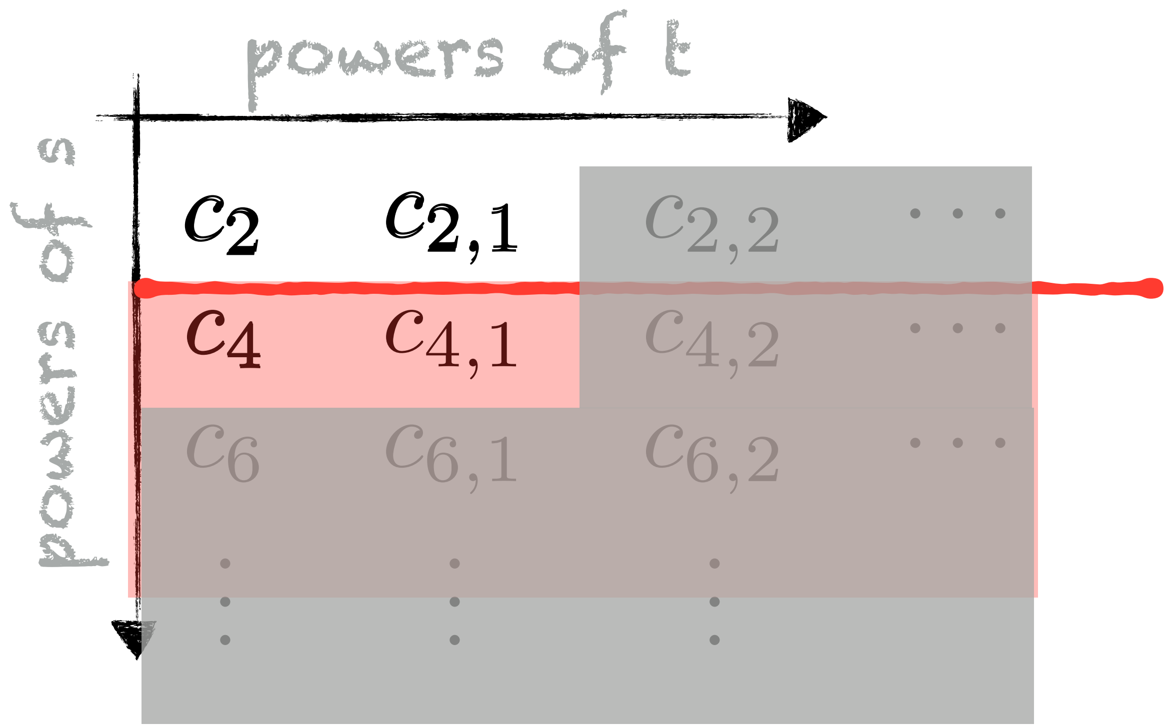

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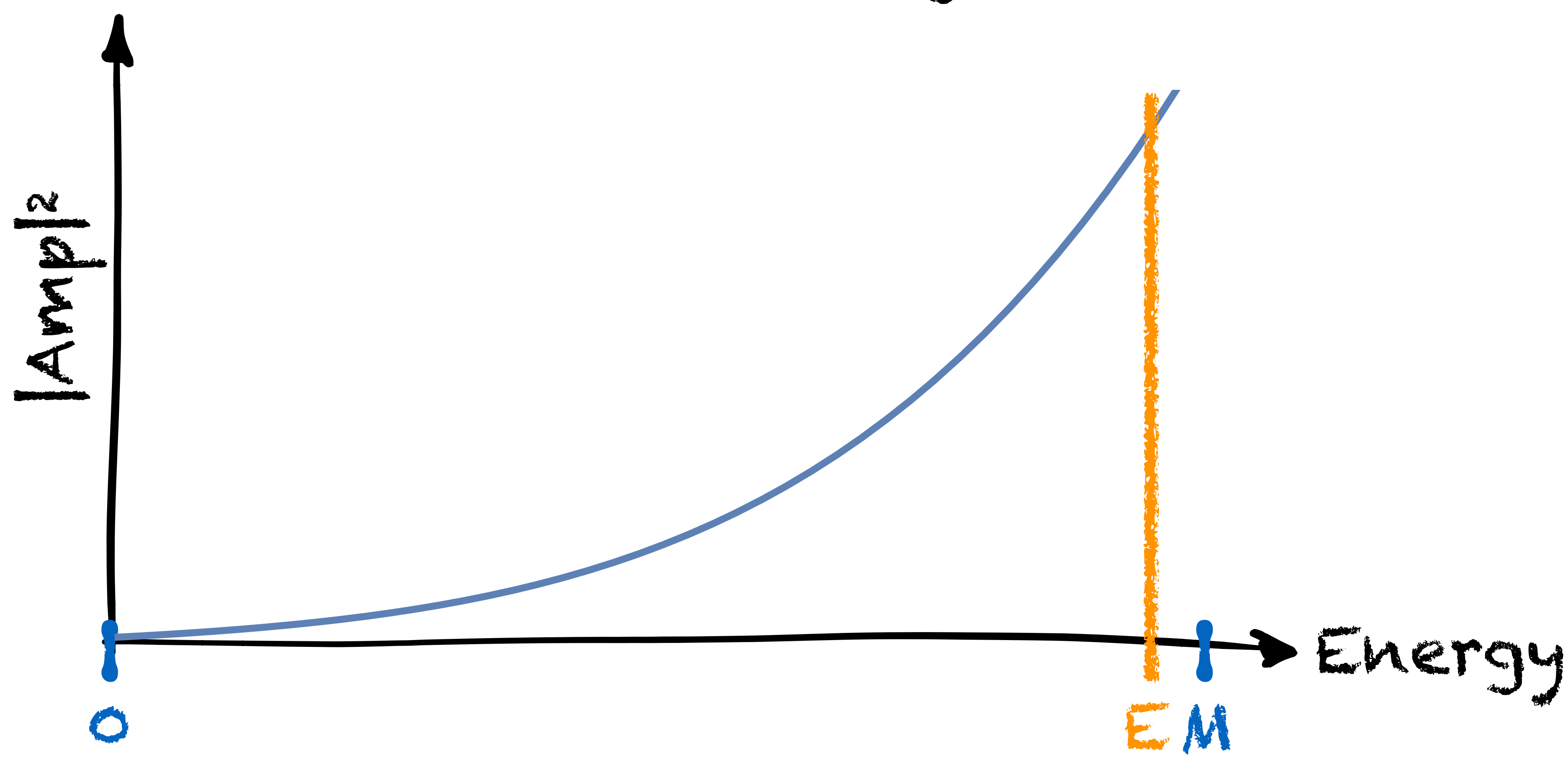
Field Content: for most of the talk, a single scalar π

Symmetries: sometimes shift symmetries $\pi \to \pi + \alpha + \beta x \cdots + \gamma x^n$

1. IR

Effective Field Theory

2>2 scattering



generic UV theory generates generic coefficients at E=M

$$A = c_0 + c_1 \frac{E^2}{M^2} + c_2 \frac{E^4}{M^4} + c_3 \frac{E^6}{M^6} + c_4 \frac{E^8}{M^8} + \cdots$$

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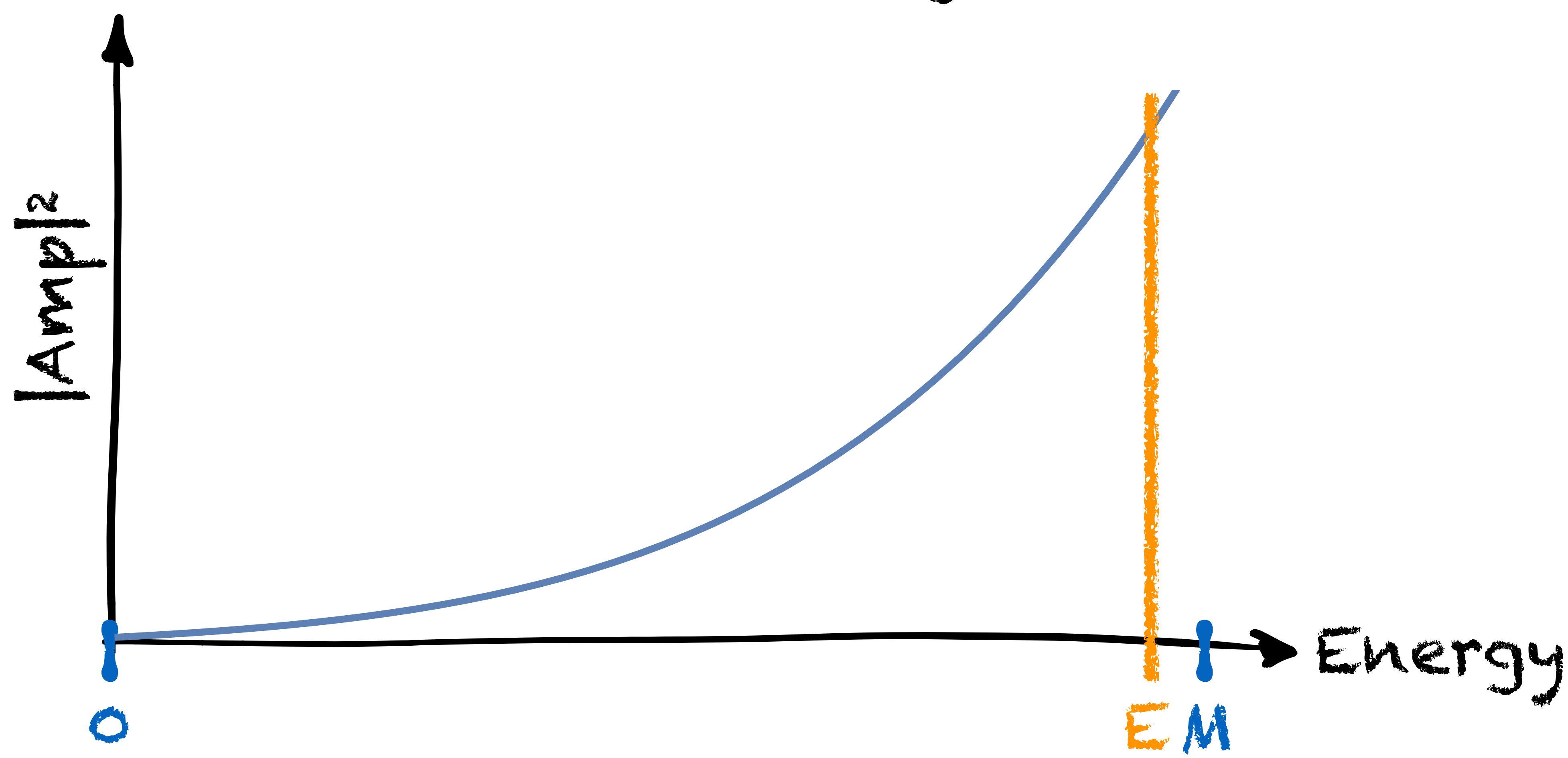

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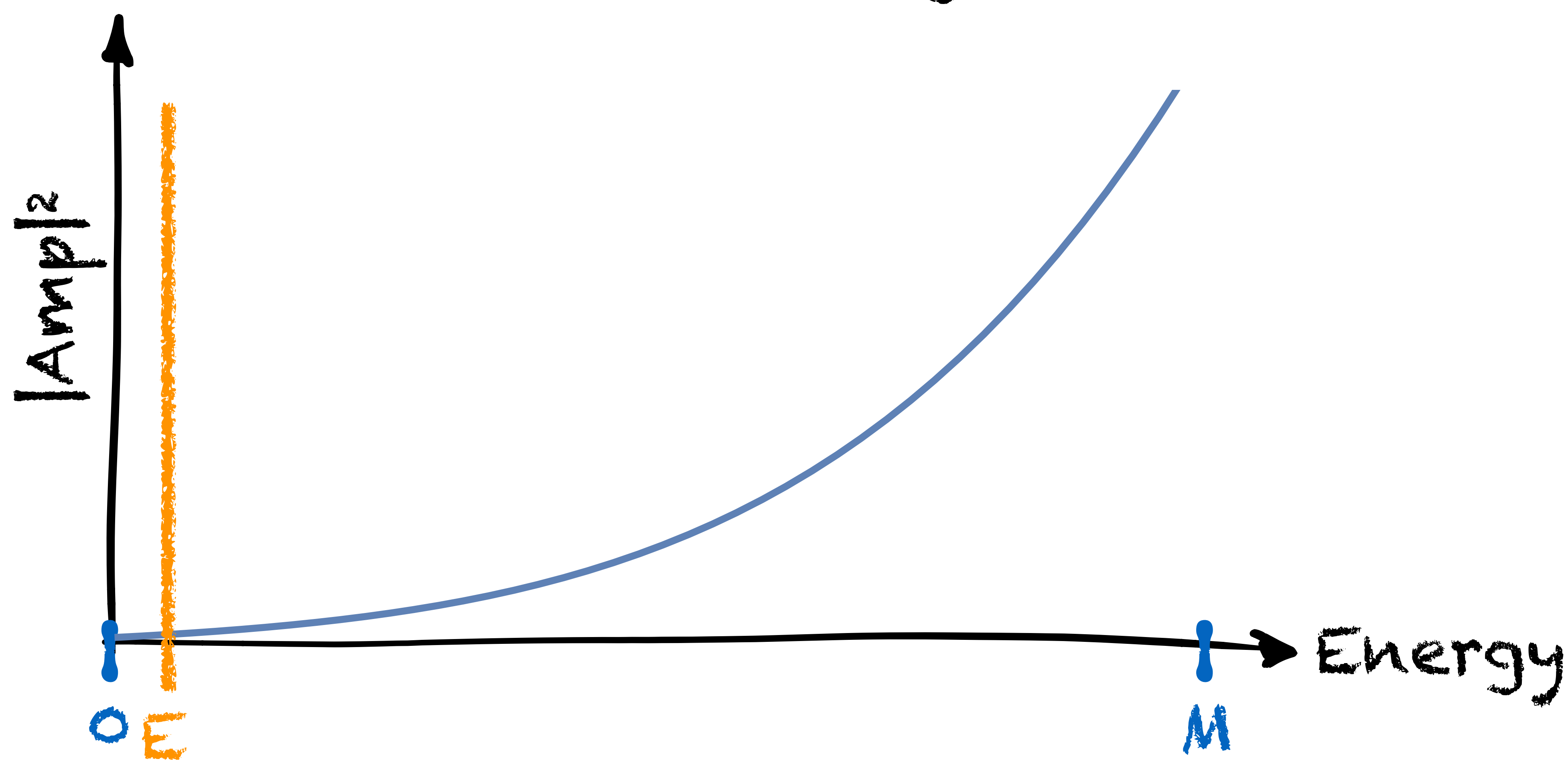

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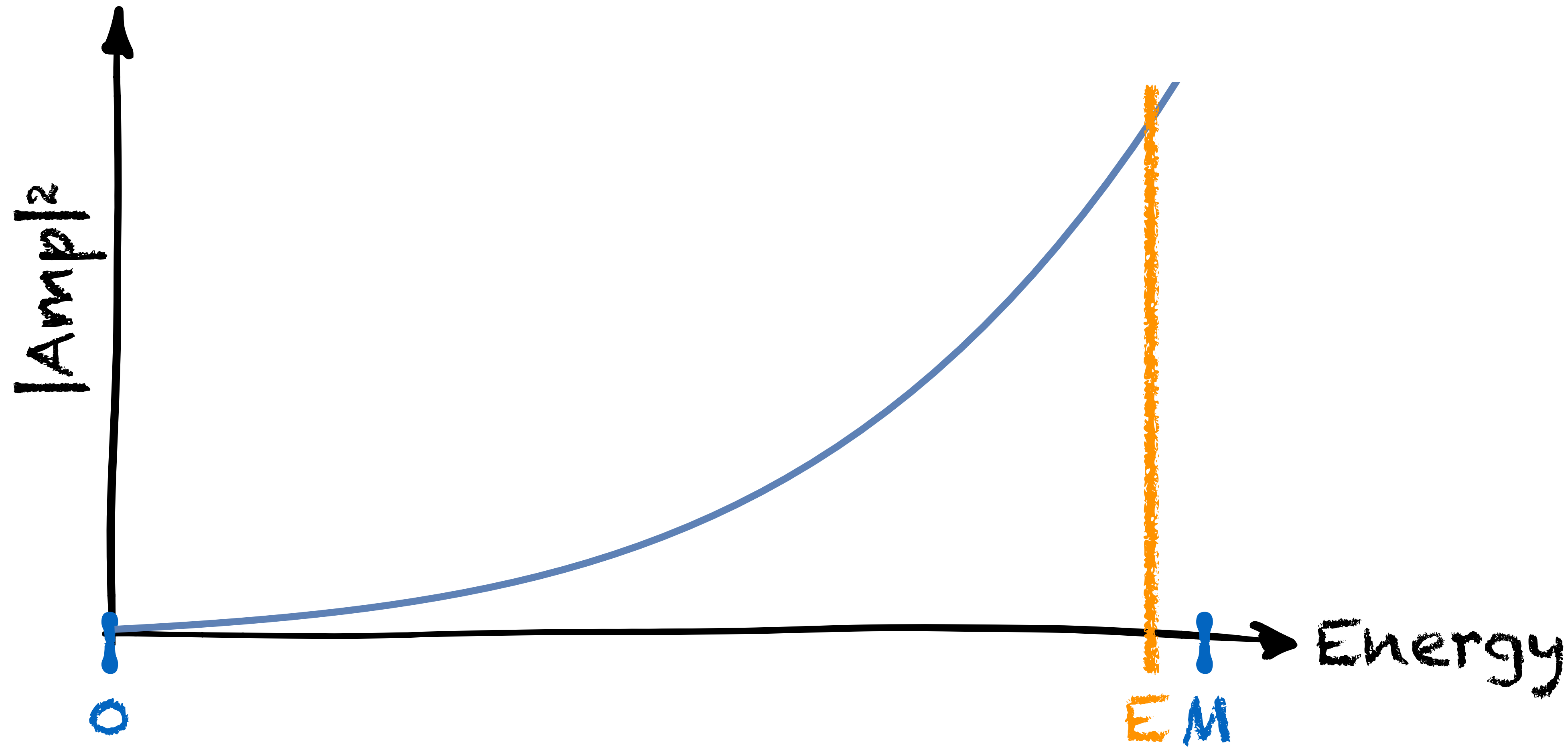

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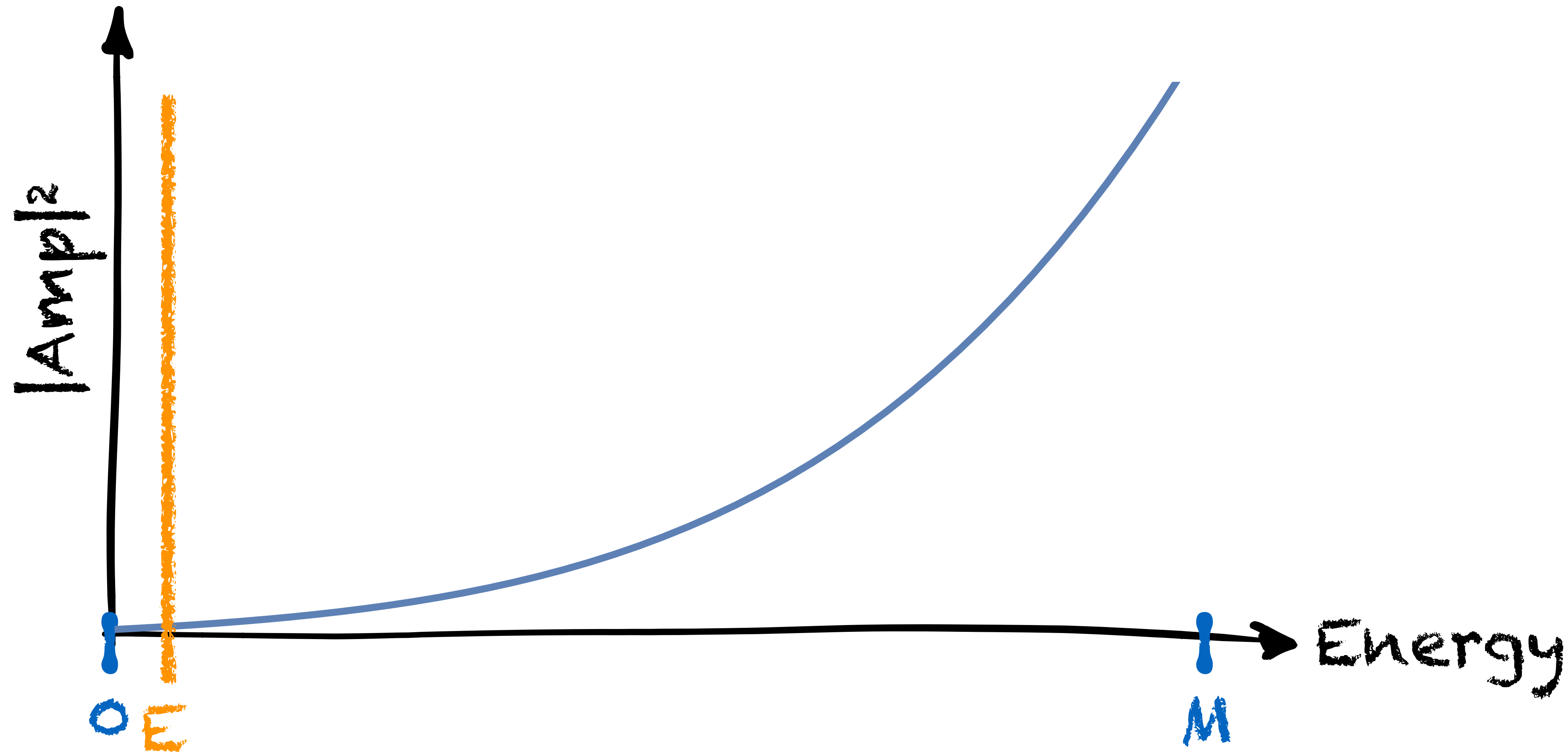

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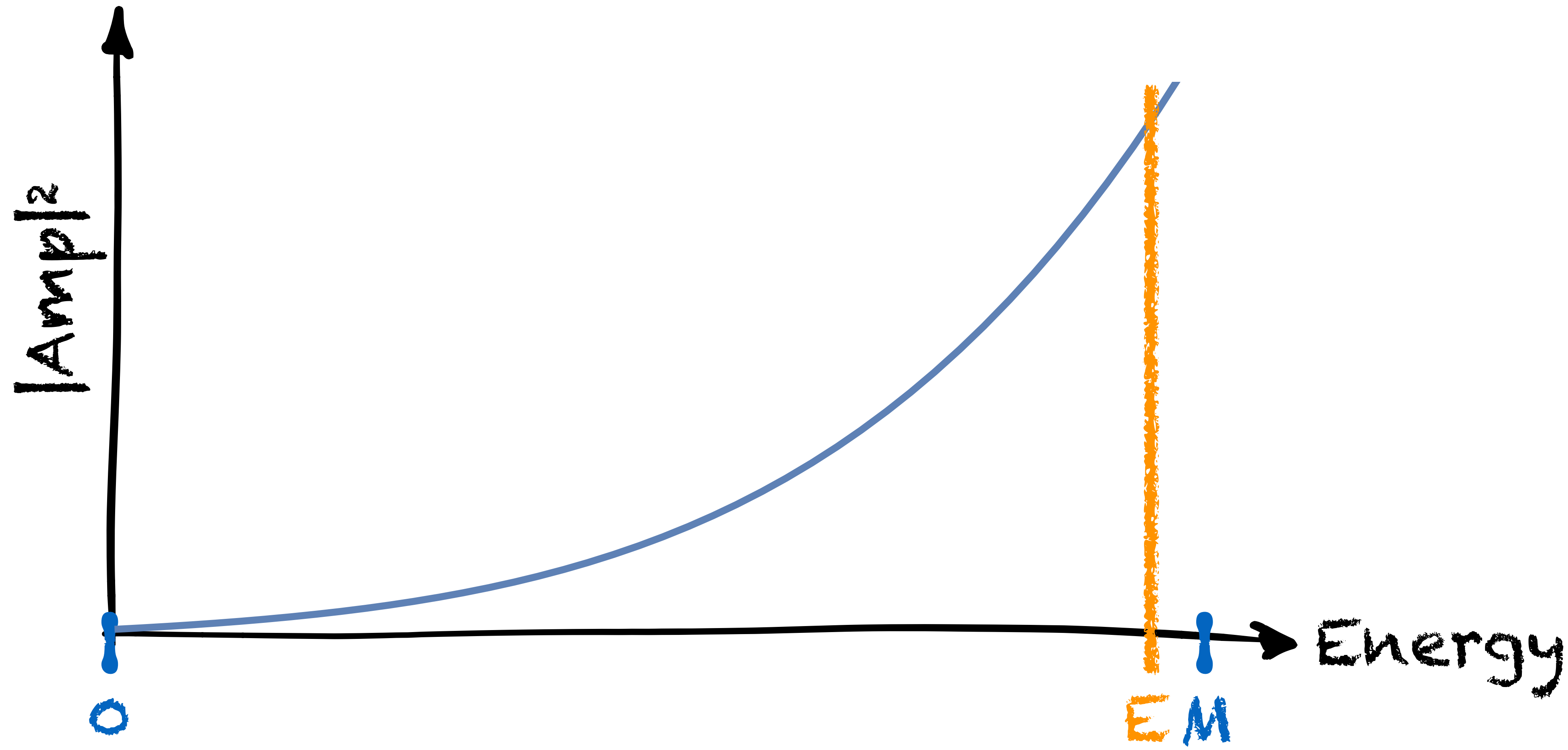

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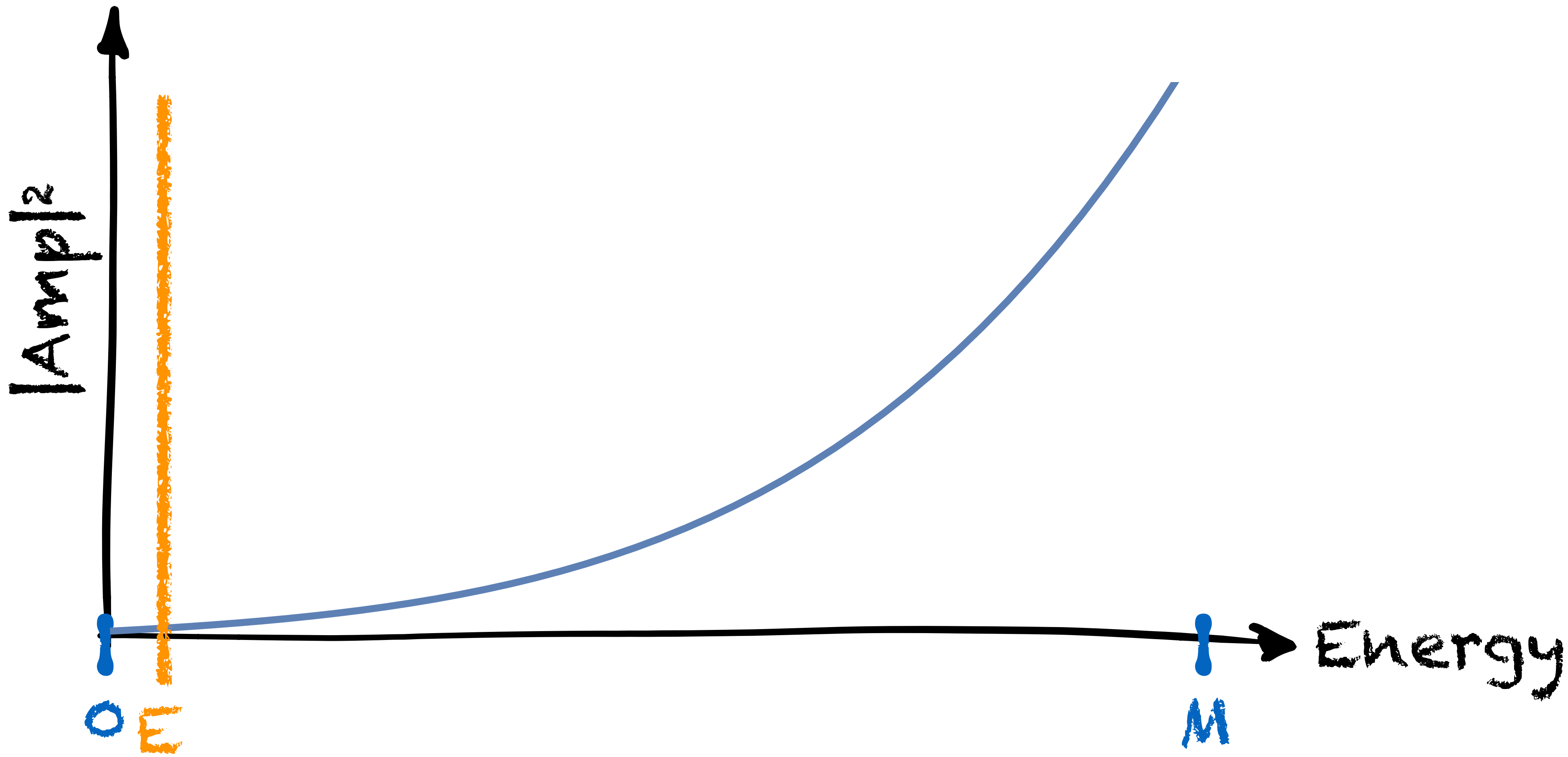

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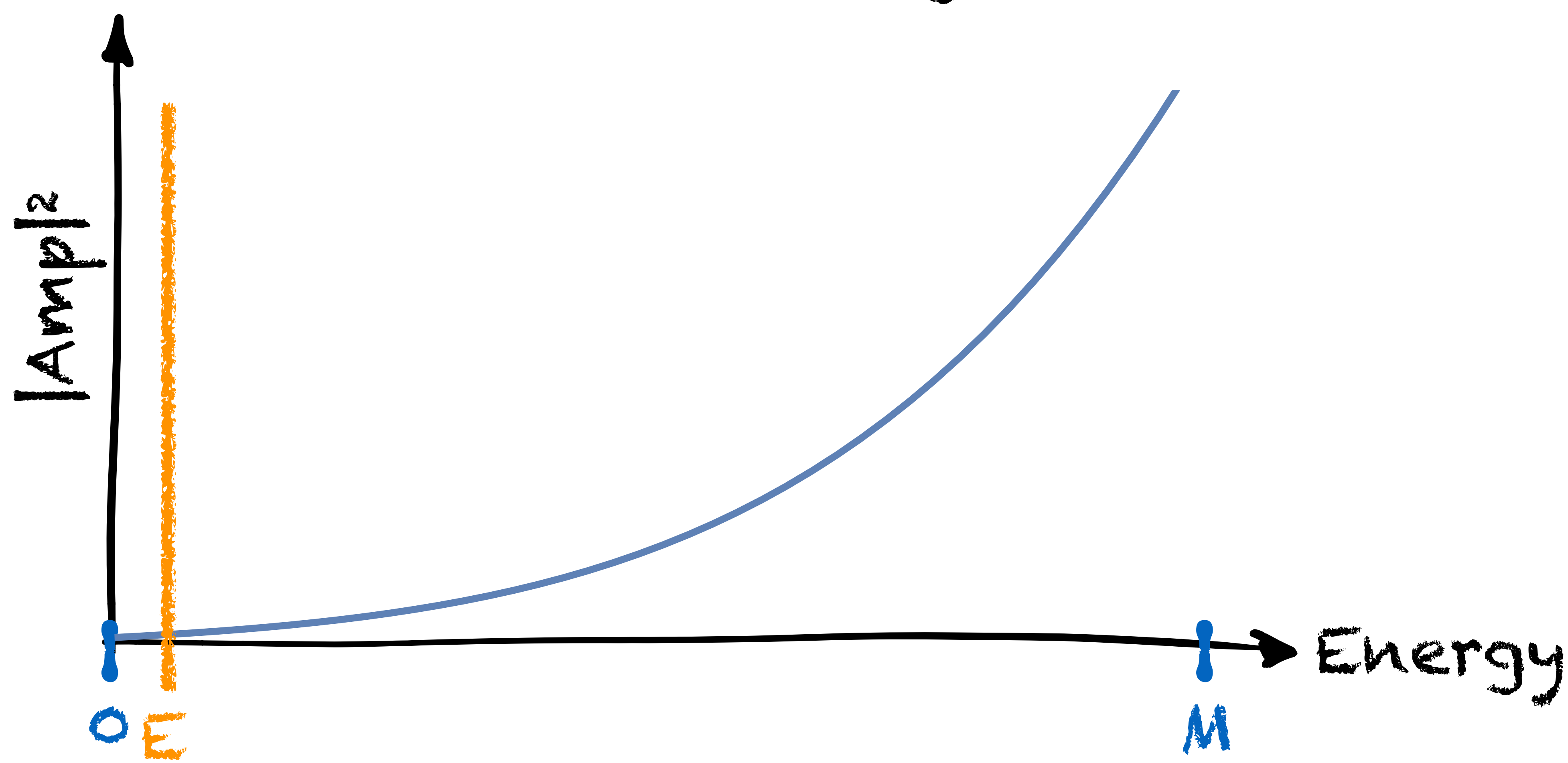

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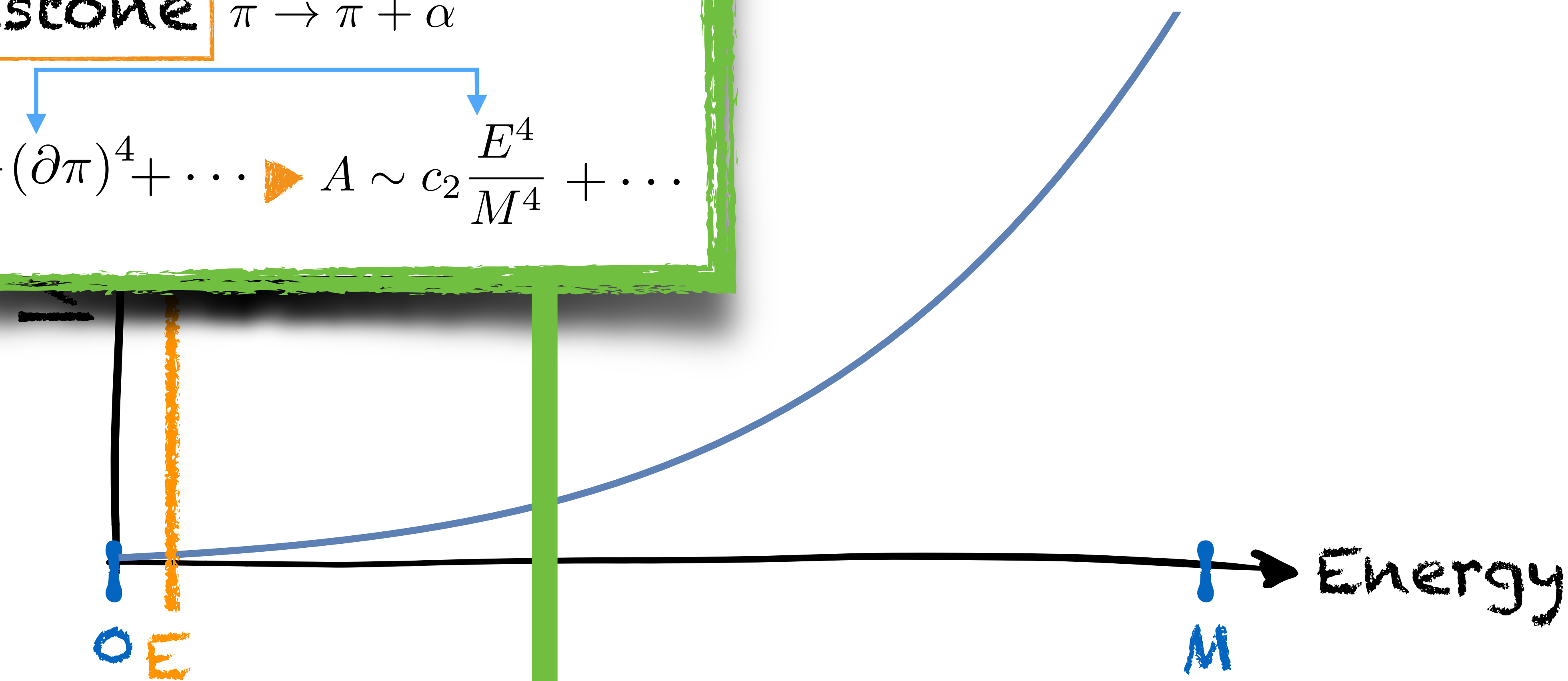
Goldstone $\pi \to \pi + \alpha$

$$\mathcal{L} = \frac{c_2}{M^4}(\partial\pi)^4 + \cdots \quad \blacktriangleright \quad A \sim c_2 \frac{E^4}{M^4} + \cdots$$



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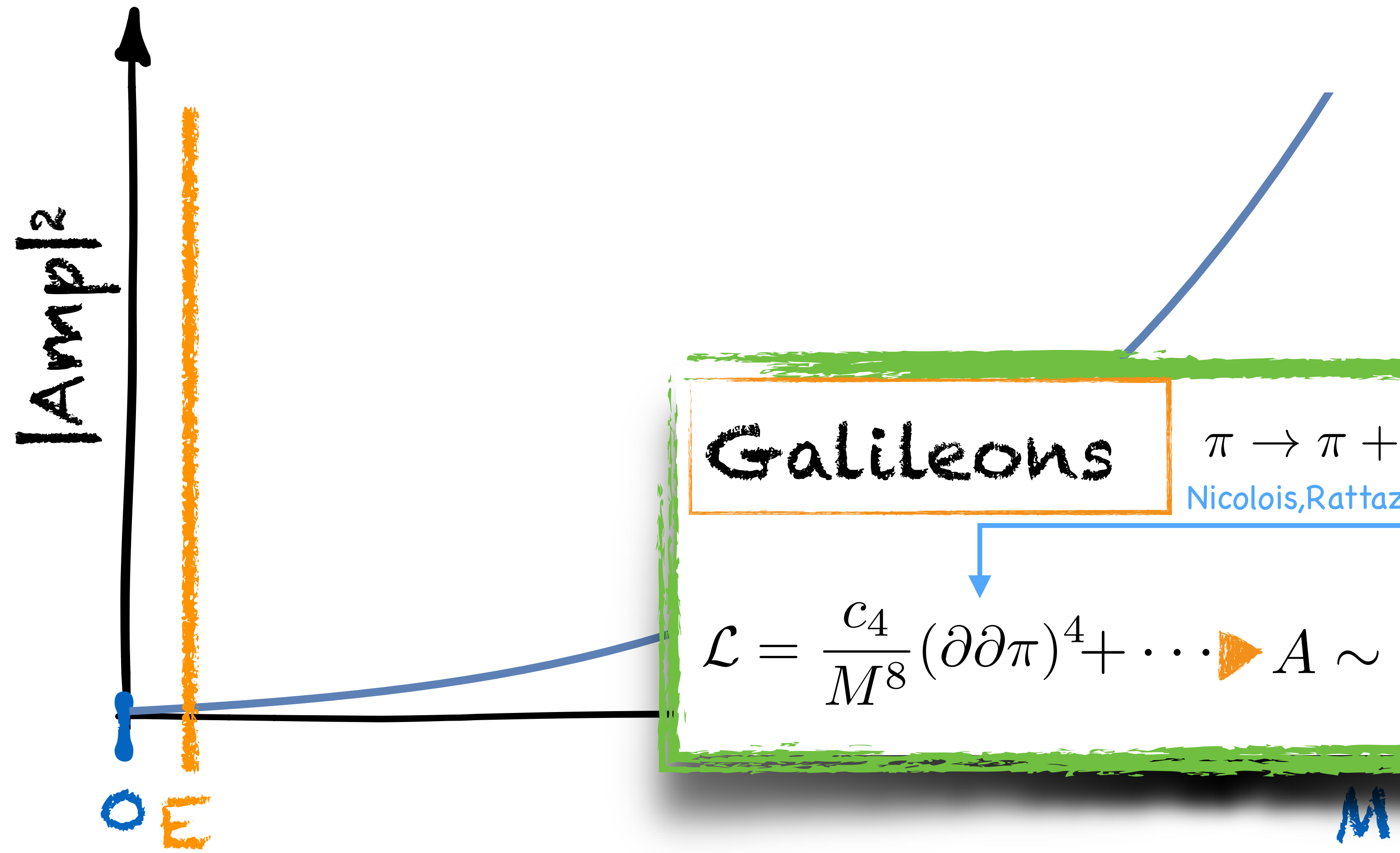

Galileons $\pi \to \pi + \alpha + \beta_\mu x^\mu$

Nicolois,Rattazzi,Trincherini'08

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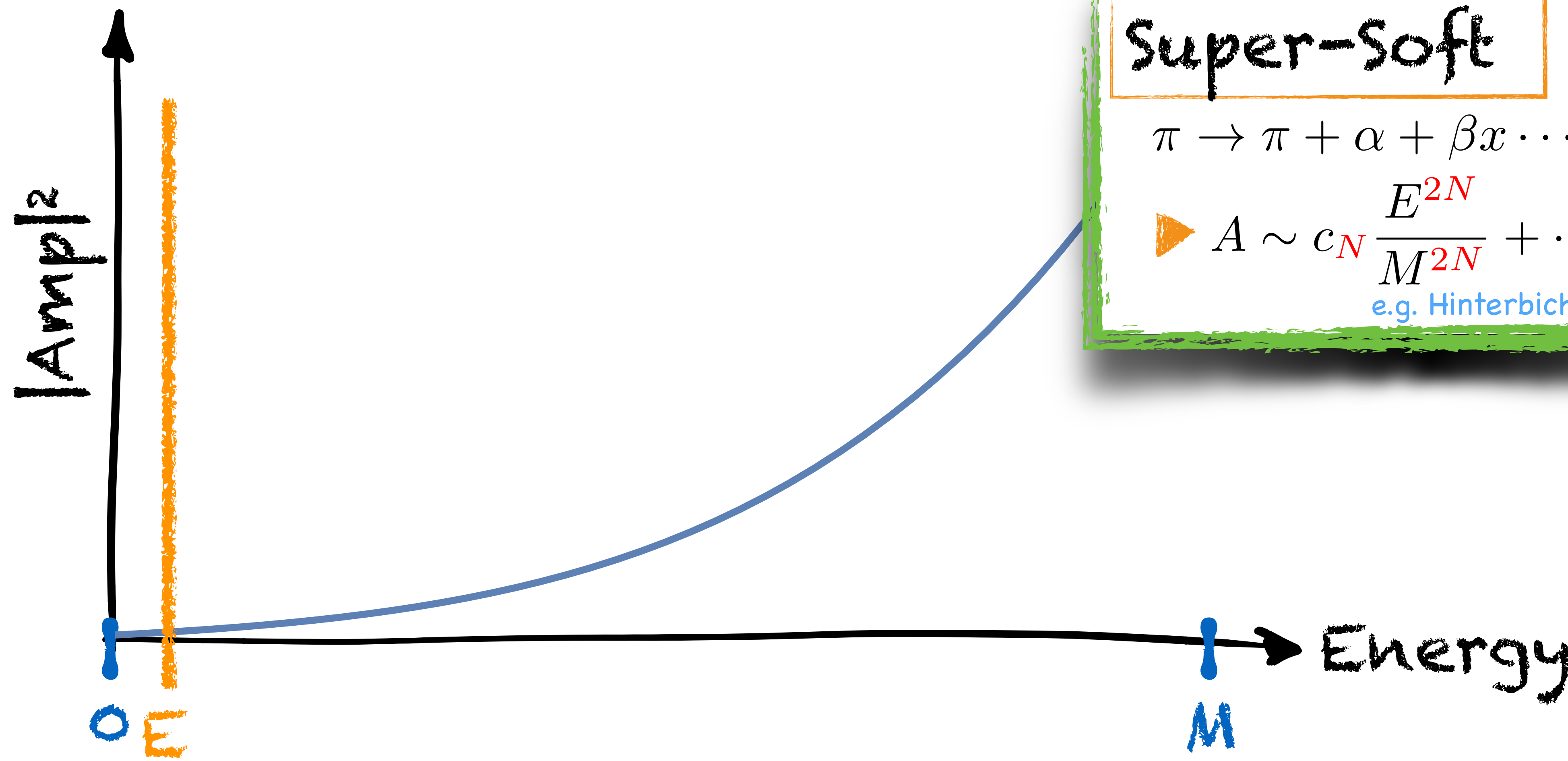

Super-Soft

$$\pi \to \pi + \alpha + \beta x \cdots + \gamma x^n$$

▶ $A \sim c_N \dfrac{E^{2N}}{M^{2N}} + \cdots$

e.g. Hinterbichler,Joyce'14

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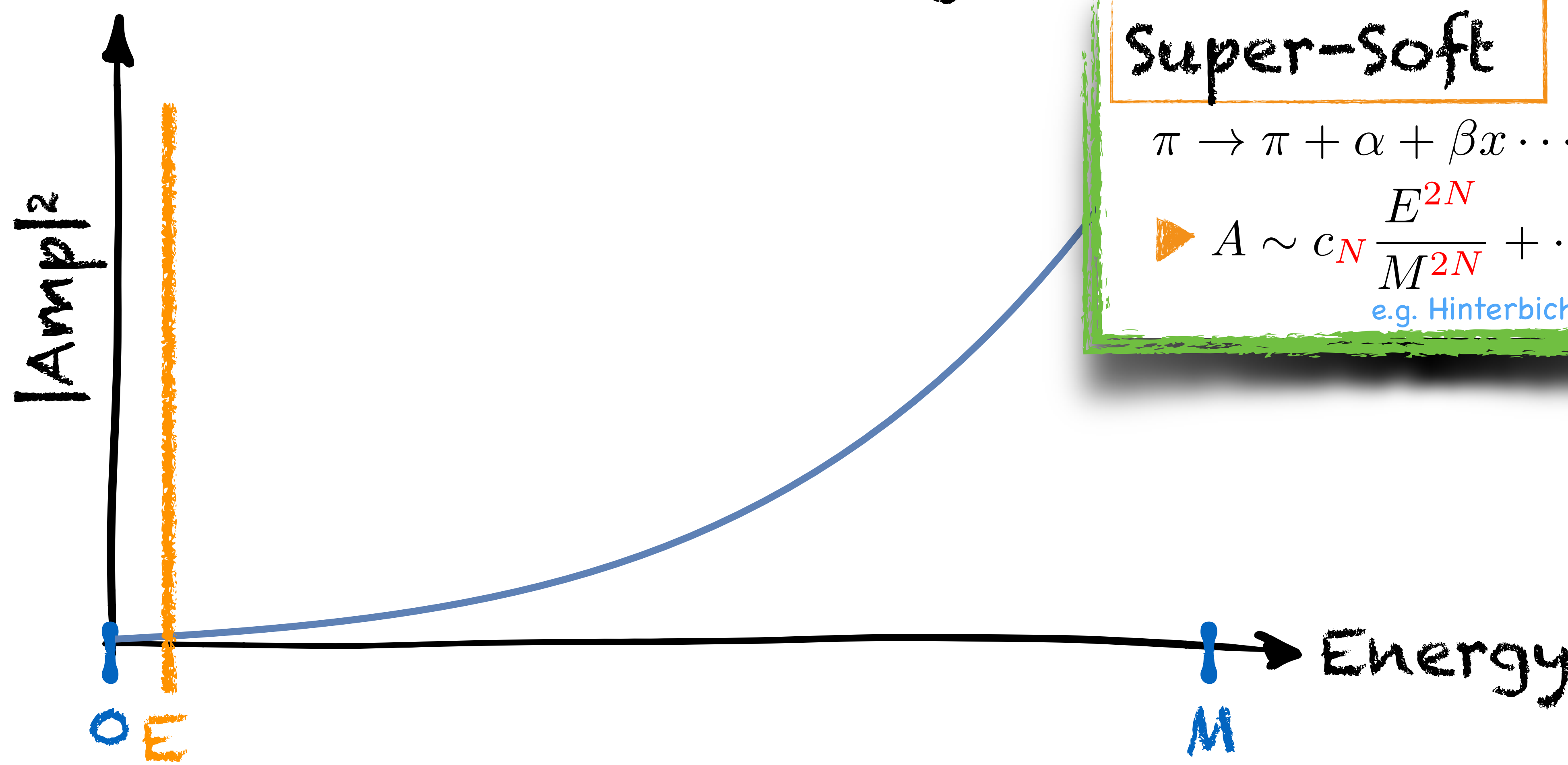

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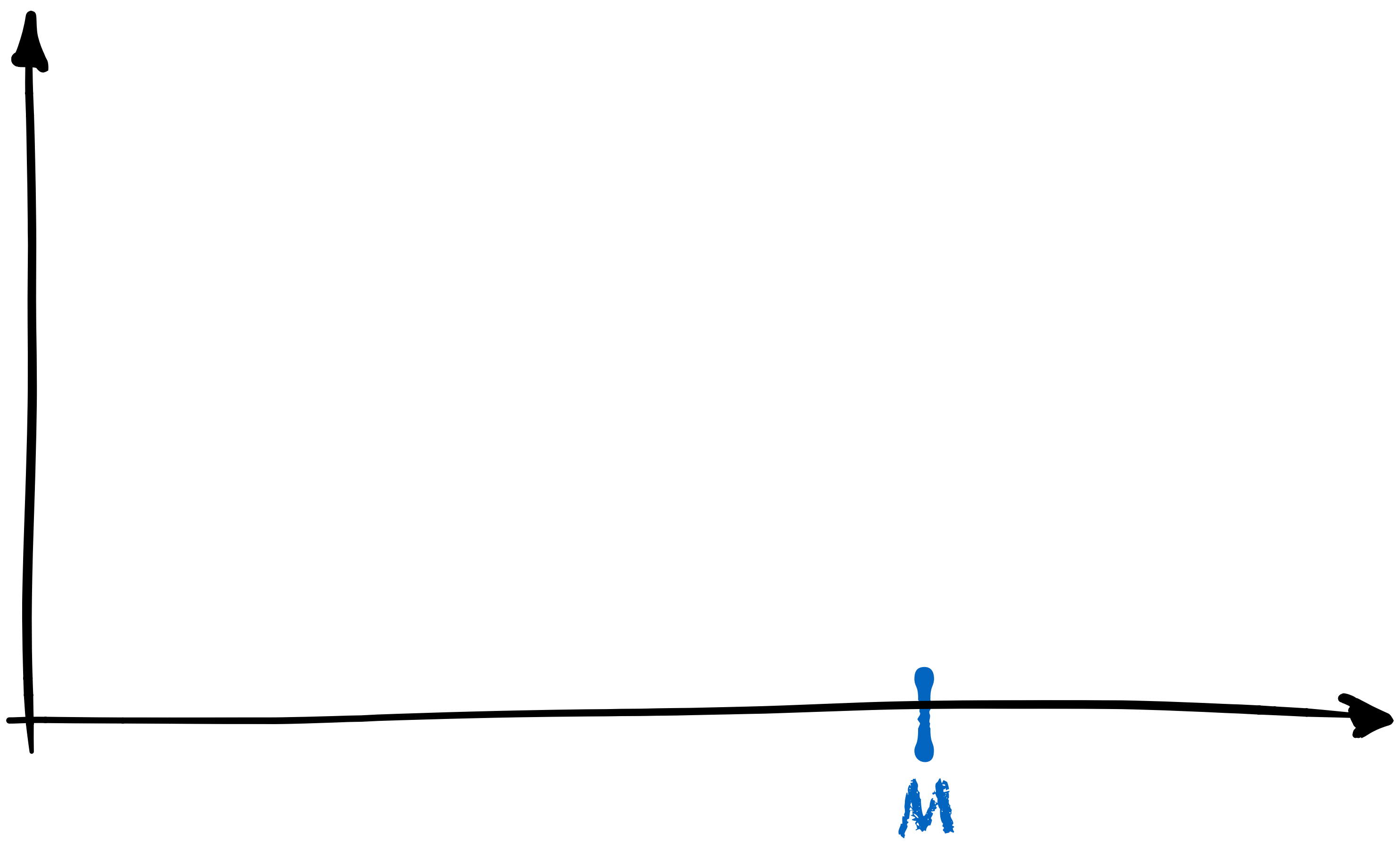
Can supersoft theories be UV completed in QFT?

2. UV → IR

UV-IR Connection

Froissart,Martin',...60s
Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,
...

Forward Scattering $t \sim \theta^2 = 0$ ⟶



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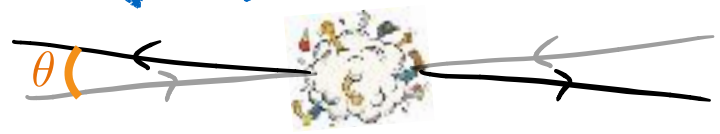
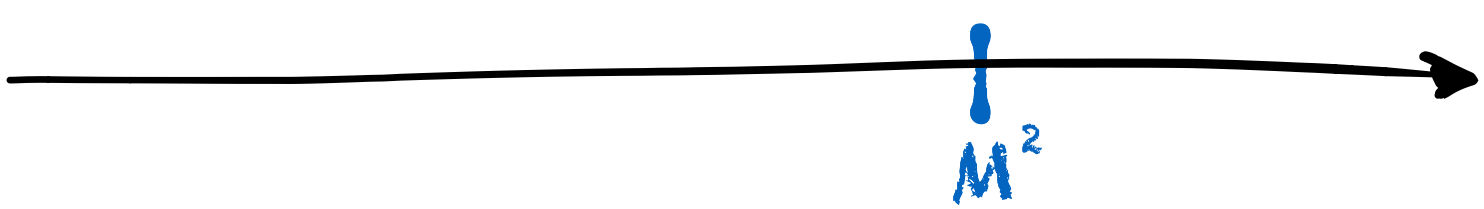


Total energy $s = E^2$



s = Energy$_{cm}^2$

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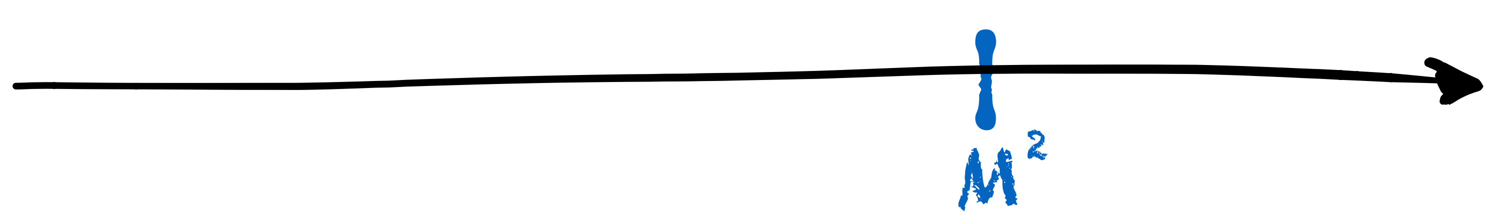
Physical Properties

Causality

Unitarity



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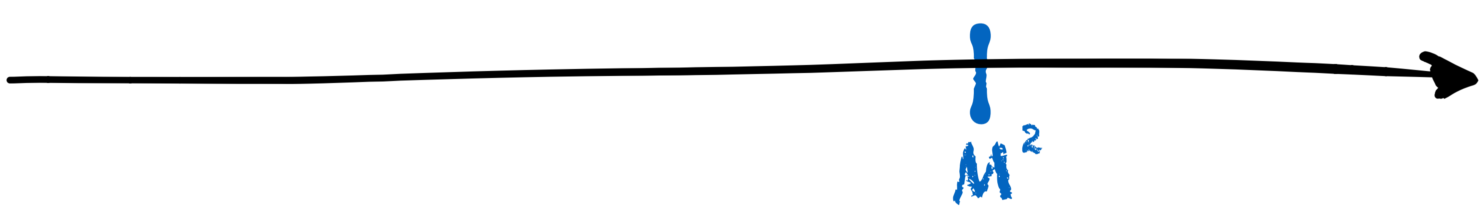
Physical Properties

Mathematical Properties of 2→2 forward amplitude $A(s)/s^n$

Causality ▶ Analytic in $s \in \mathbb{C}/phys$

Unitarity ▶ Positive across $s \in \mathbb{R}$

(optical theorem)



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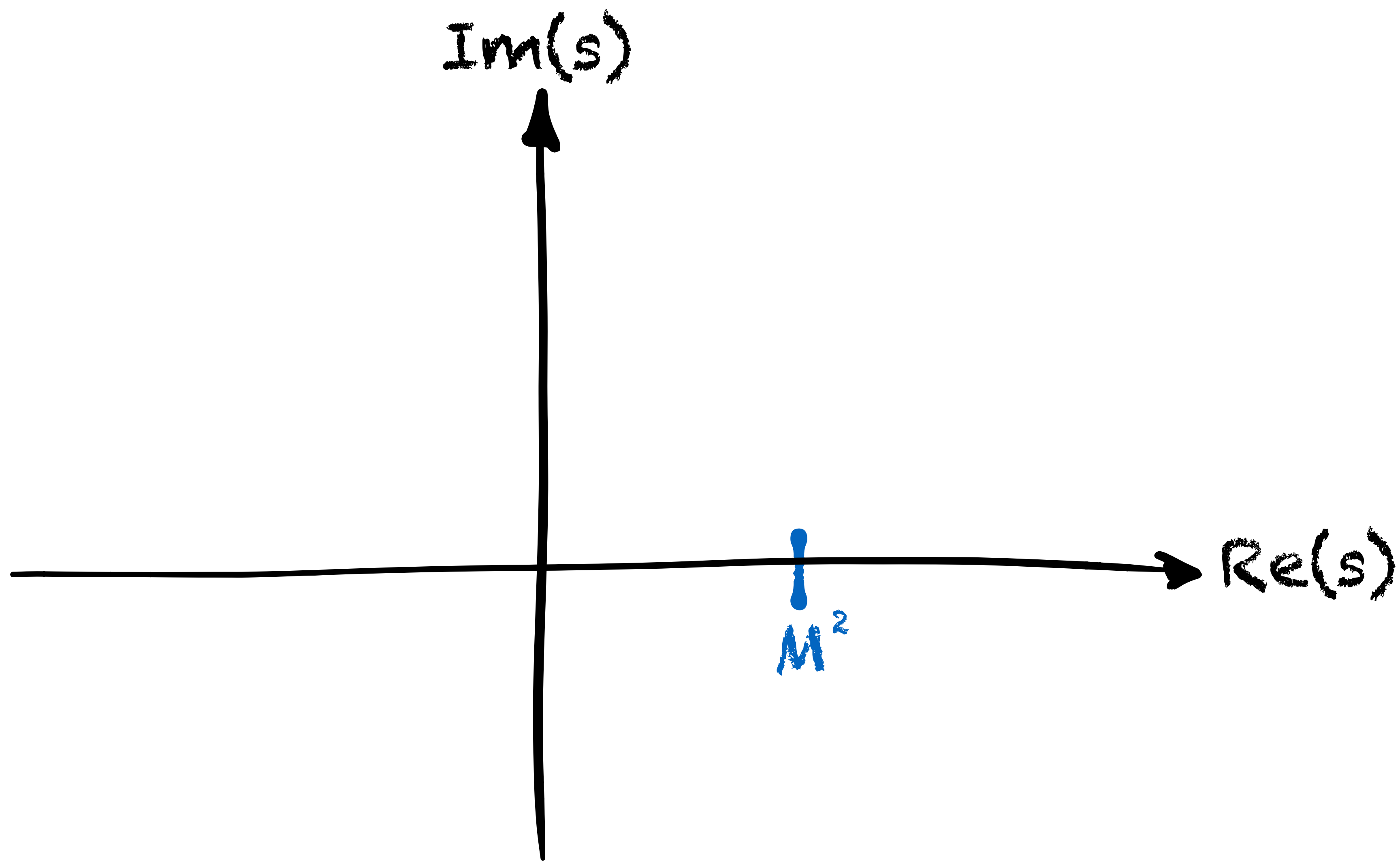

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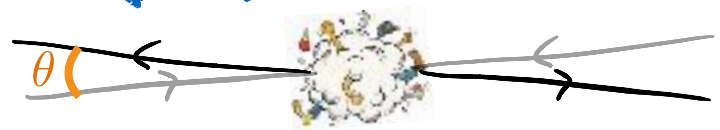

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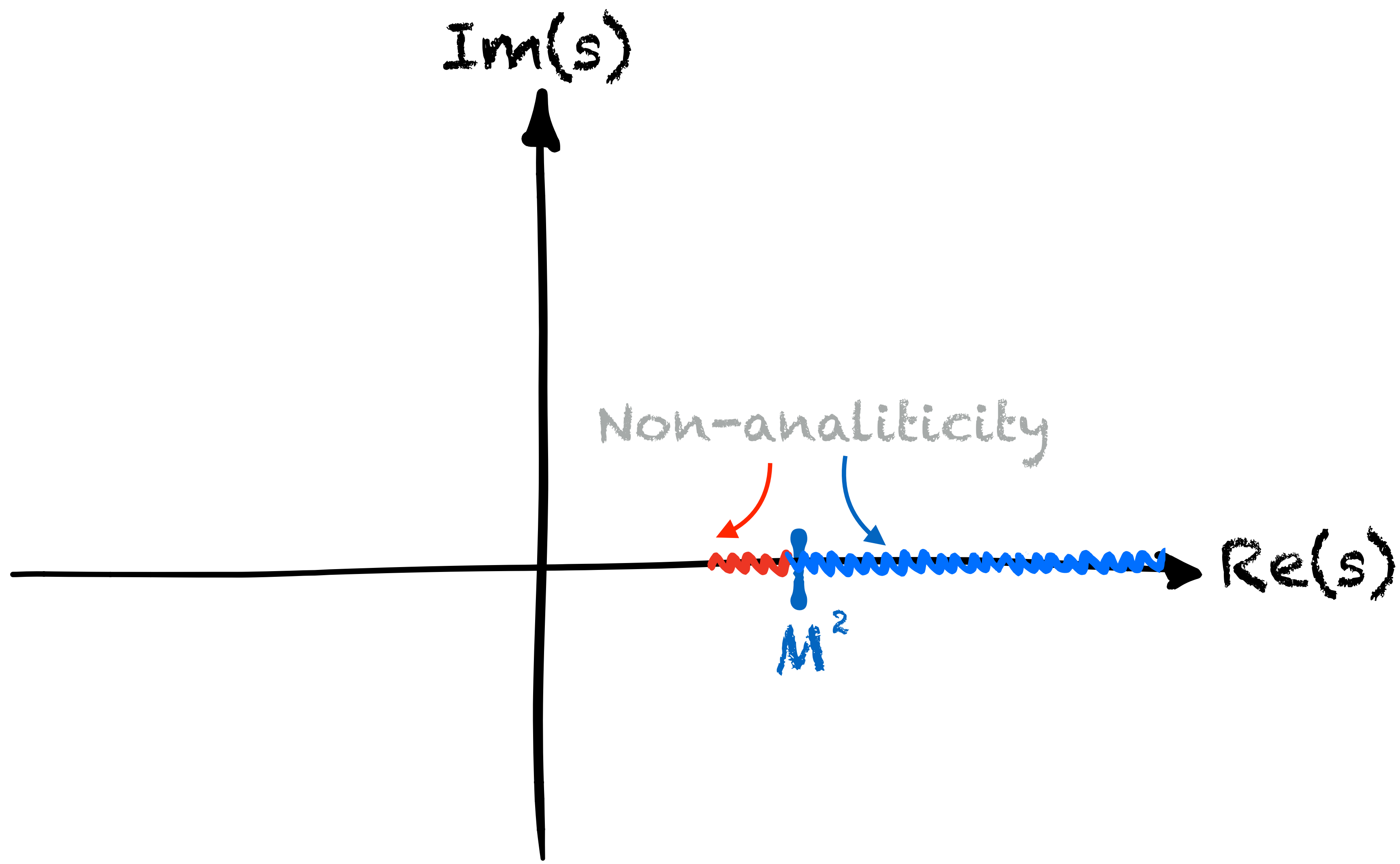

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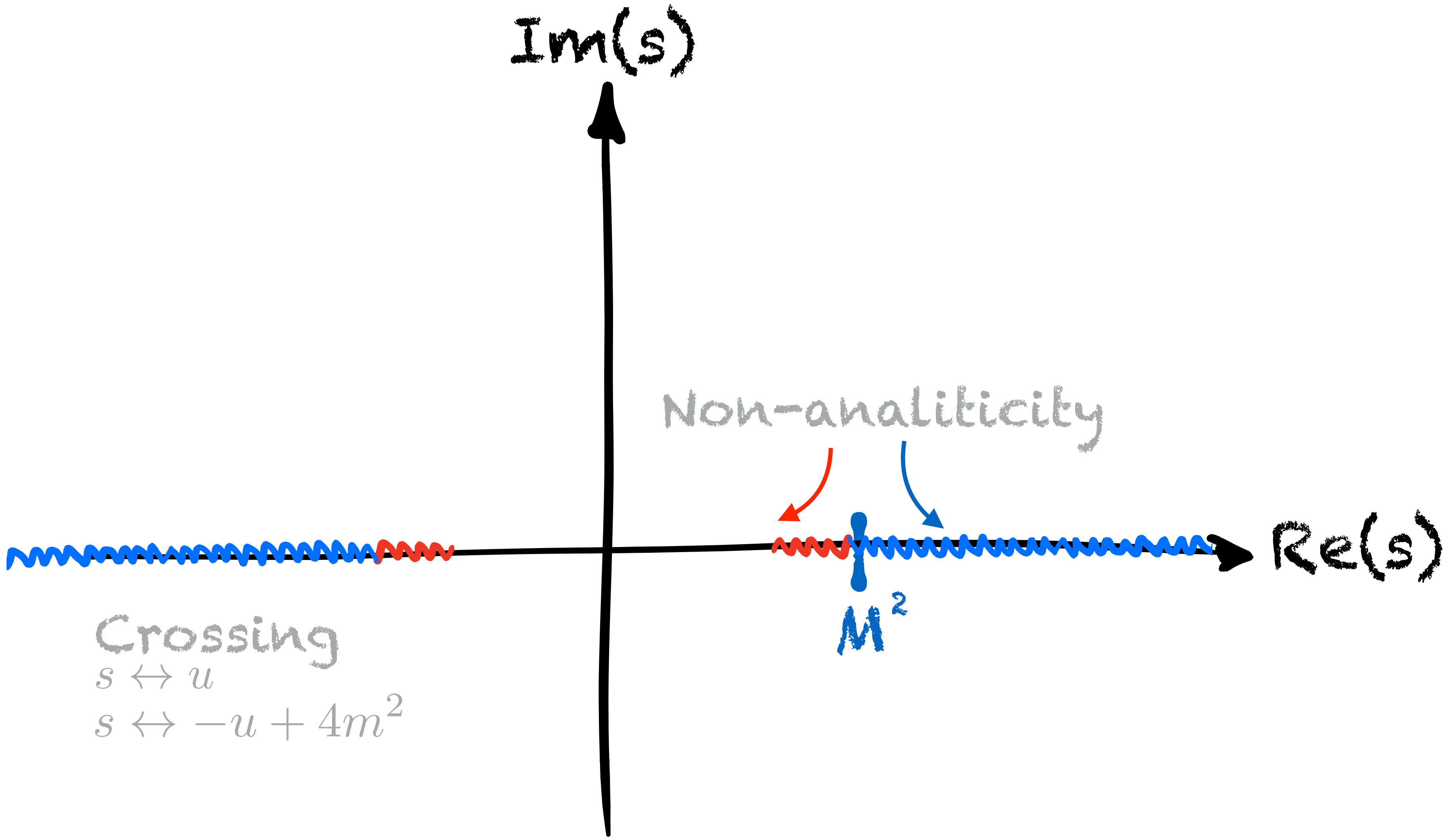

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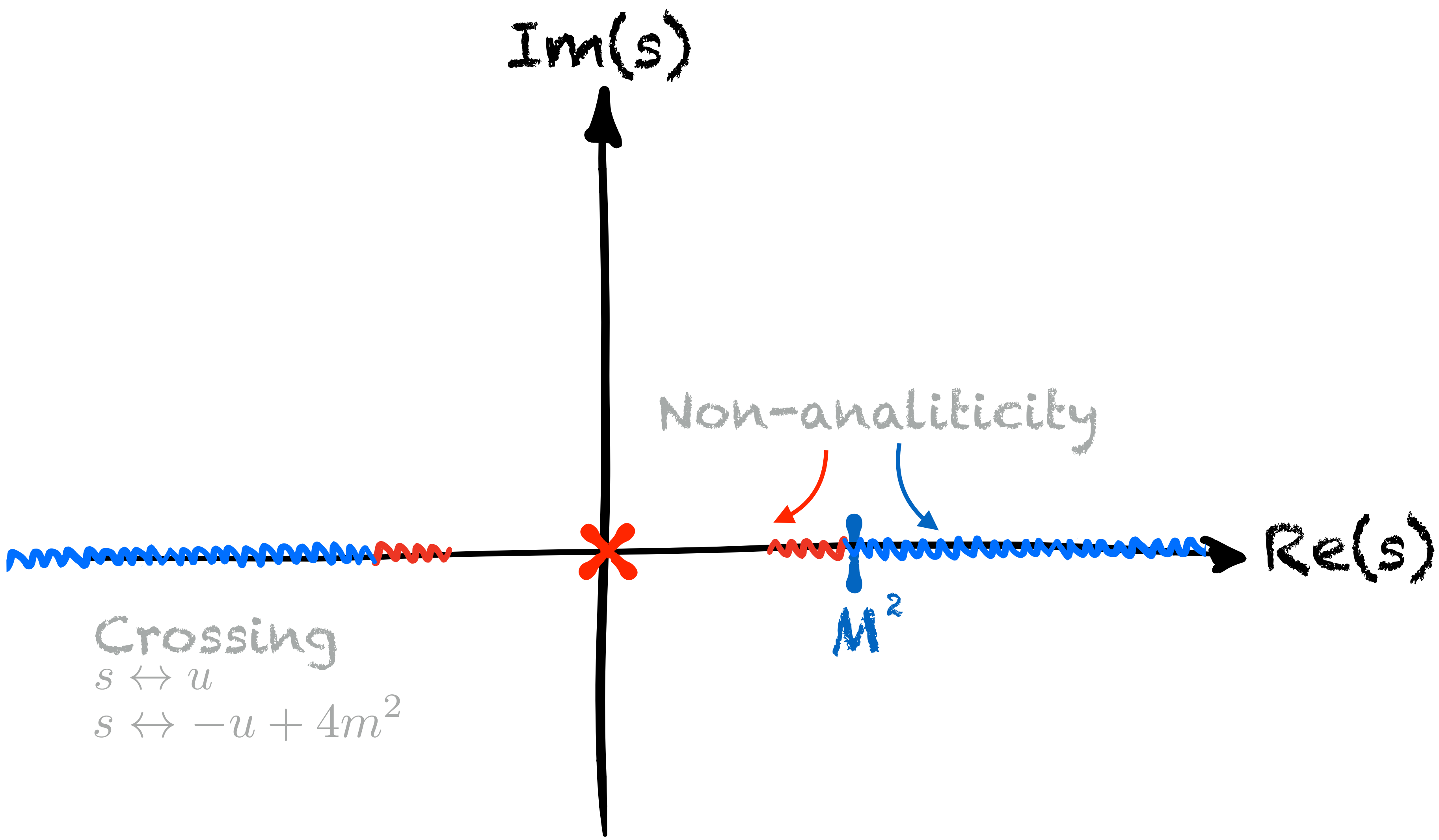

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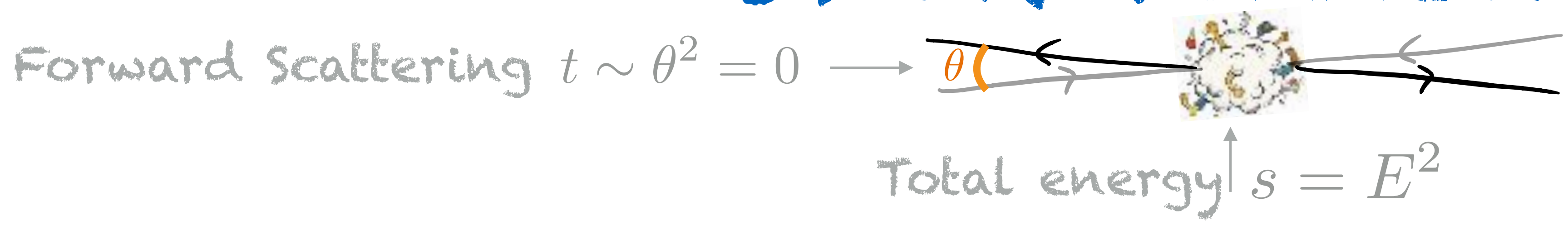
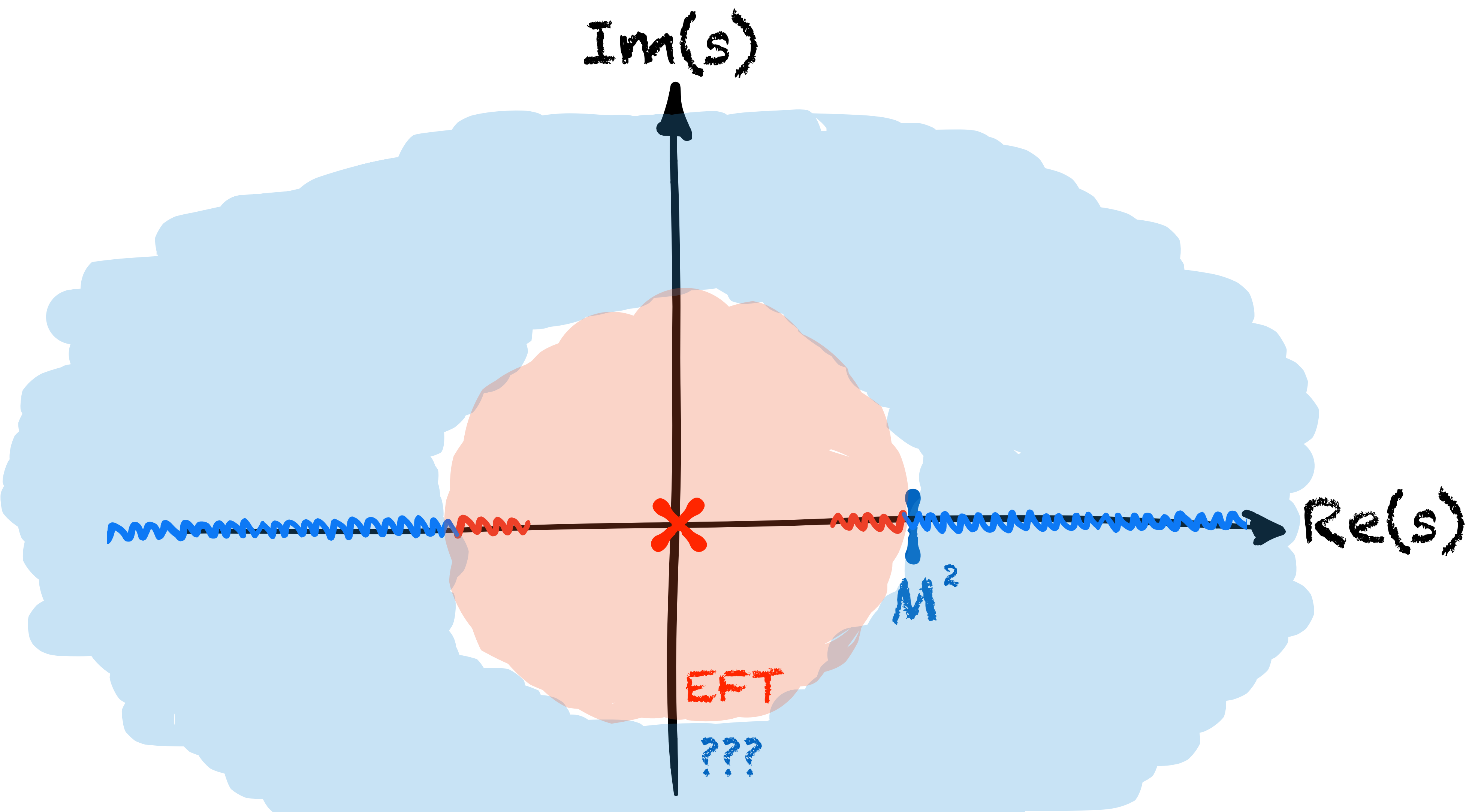


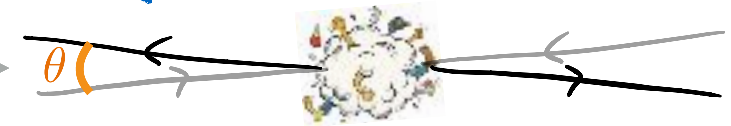
Physical Properties		Mathematical Properties of 2→2 forward amplitude $A(s)/s^n$
Causality	▶	Analytic in $s \in \mathbb{C}/phys$
Unitarity	▶ (optical theorem)	Positive across $s \in \mathbb{R}$



UV-IR Connection

Froissart,Martin',...60s
Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,
...

Forward Scattering $t \sim \theta^2 = 0 \longrightarrow$



Total energy $s = E^2$

Physical Properties

Mathematical Properties
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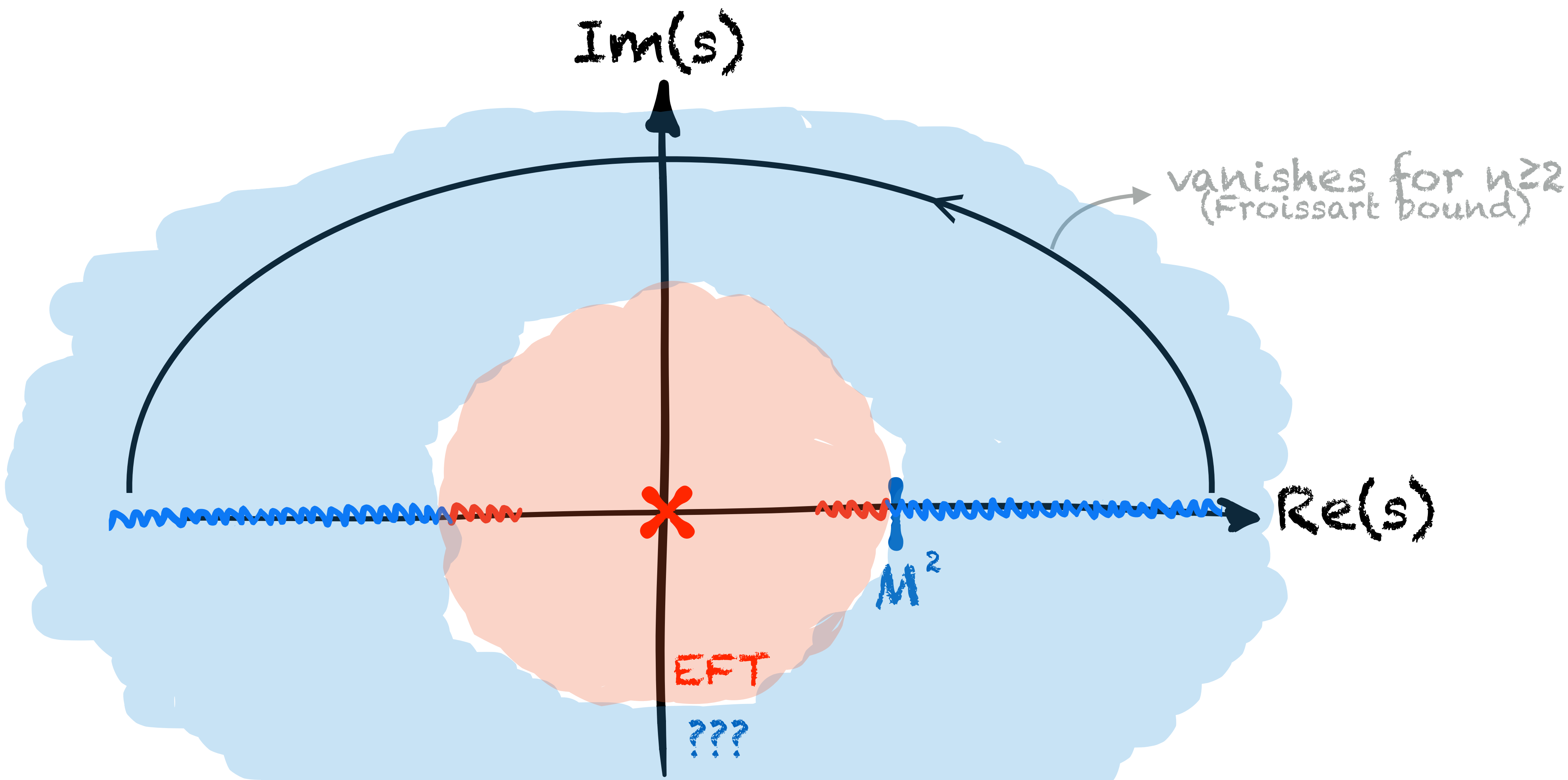

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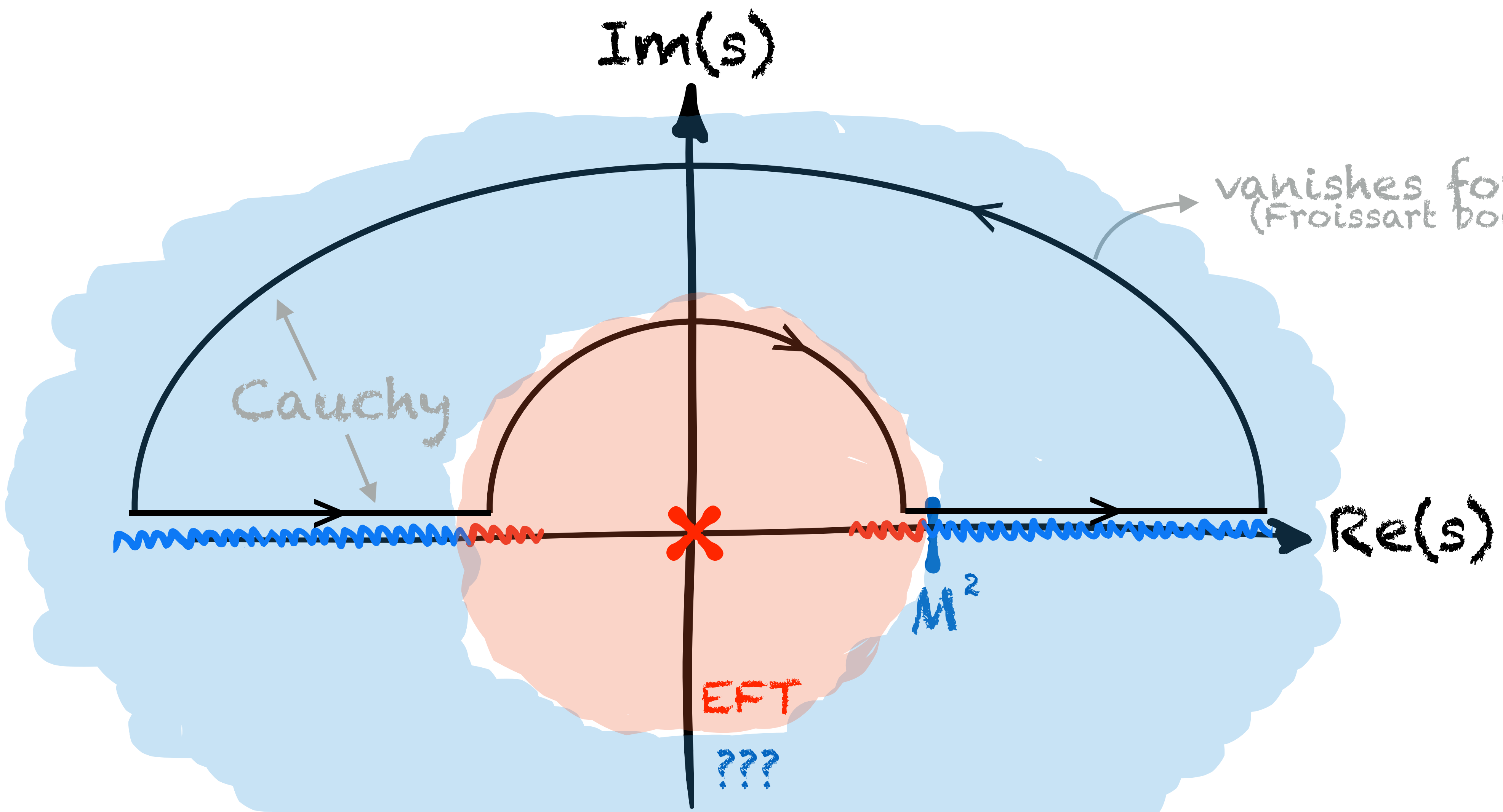

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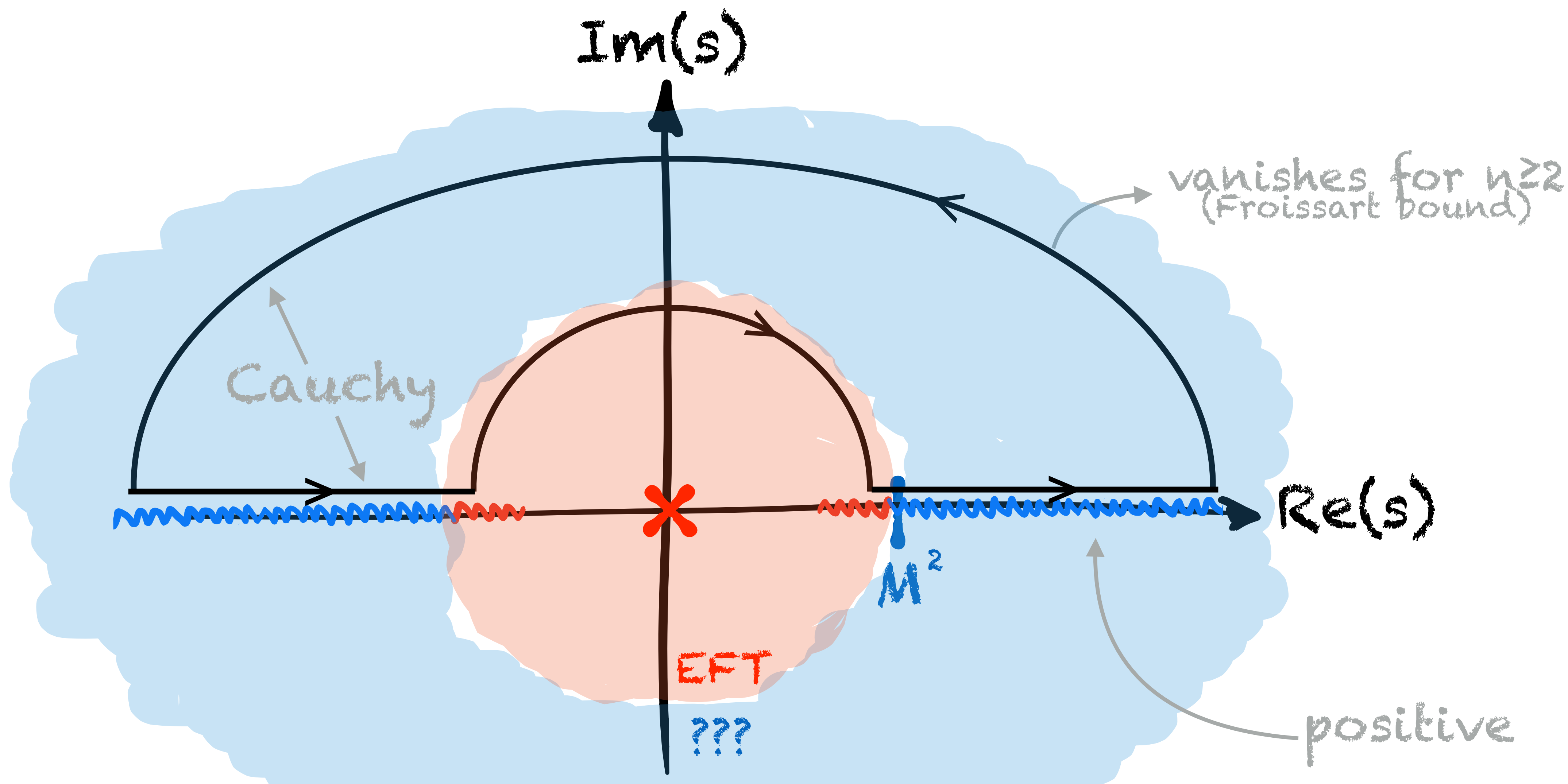
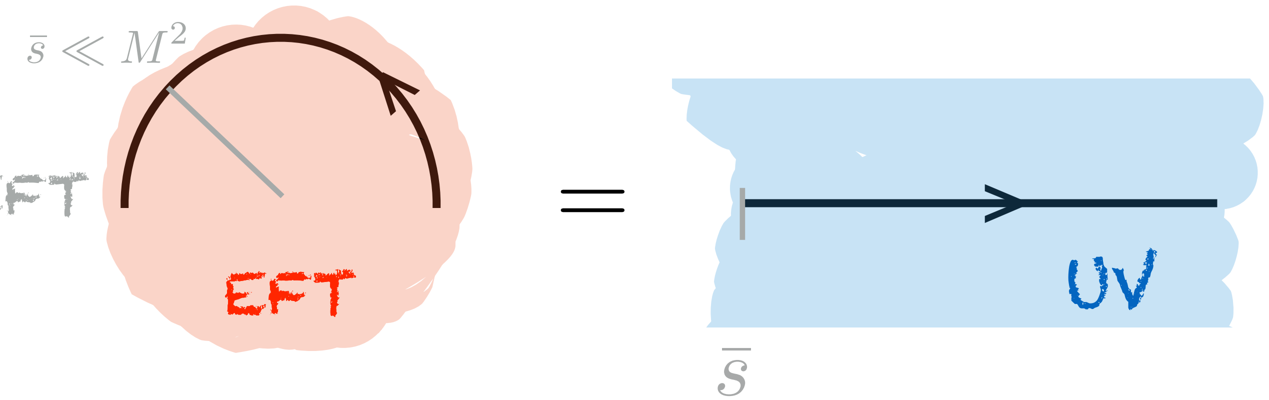


Arcs: UV-IR Connection

$\overline{S}$ = any energy within EFT



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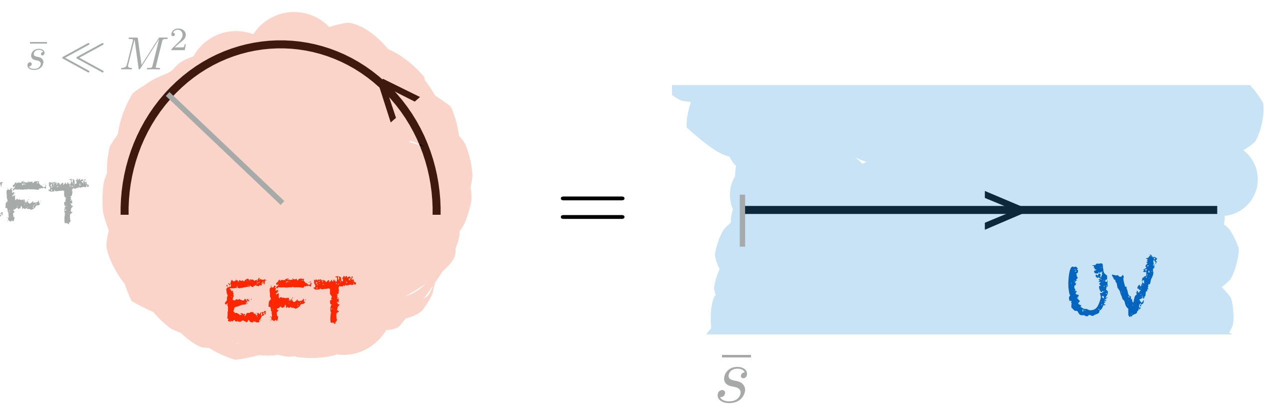

Arcs: $\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{\bar{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^{n+1}} = \frac{2}{\pi} \int_{\bar{s}}^{\infty} ds \frac{\text{Im} A(s)}{s^{n+1}}$

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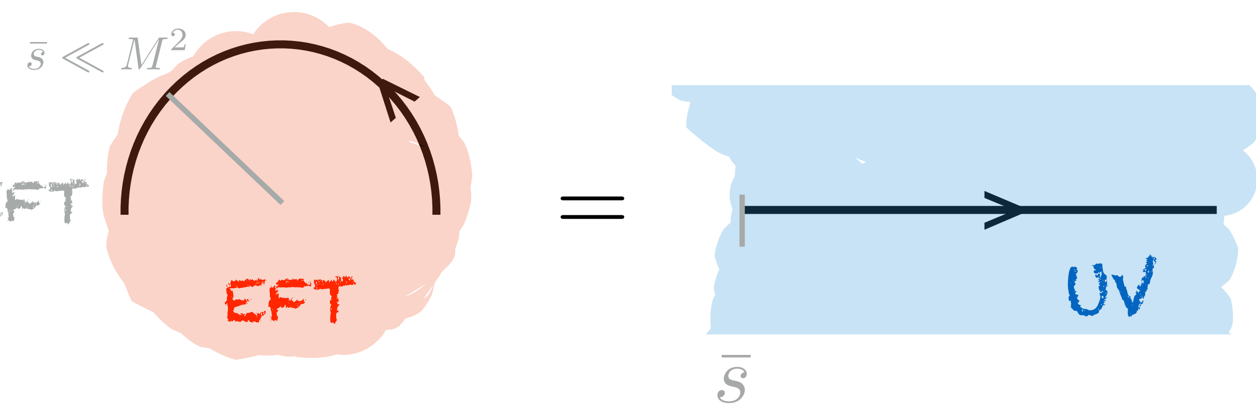

positive

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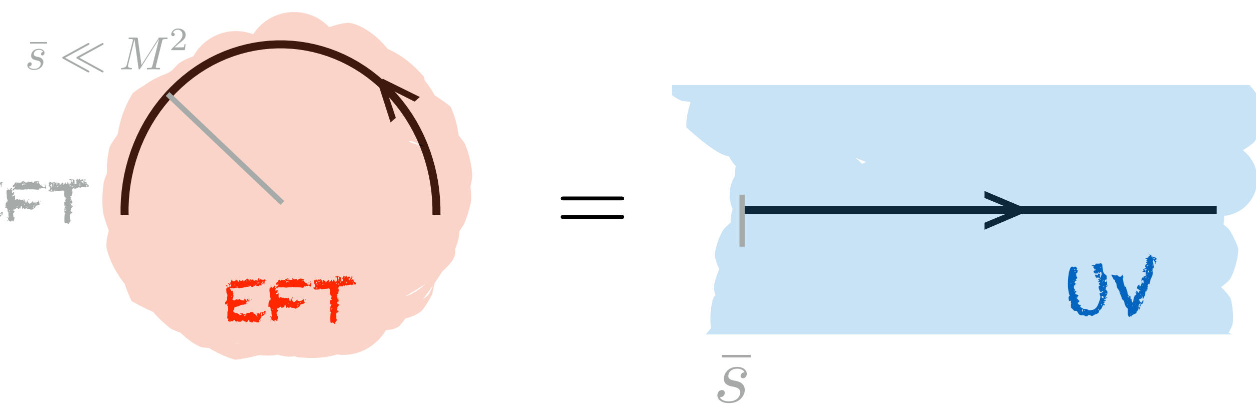

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$$\mathcal{A}_n > 0 \quad (n \geq 2)$$

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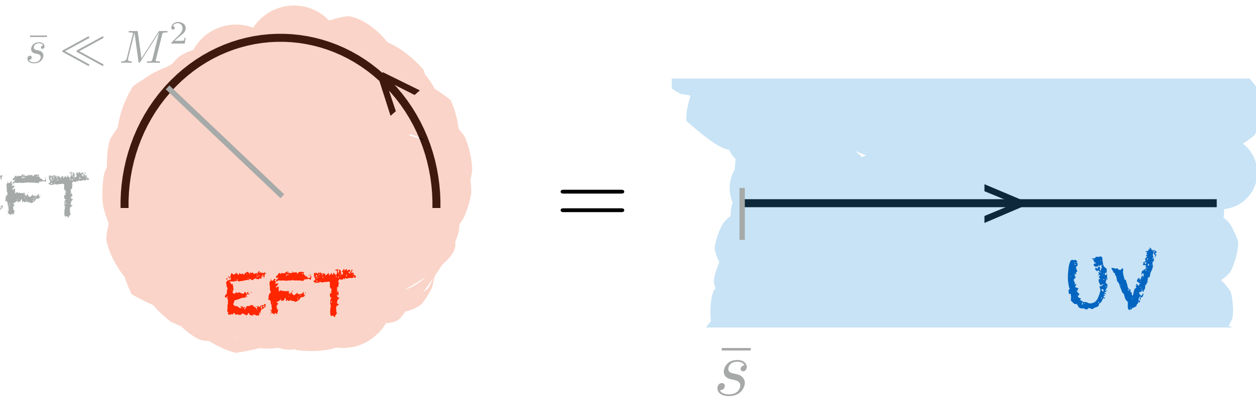

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Calculable in EFT

(e.g at tree-level $\mathcal{A}_n = c_n$, the energy coefficients in EFT)

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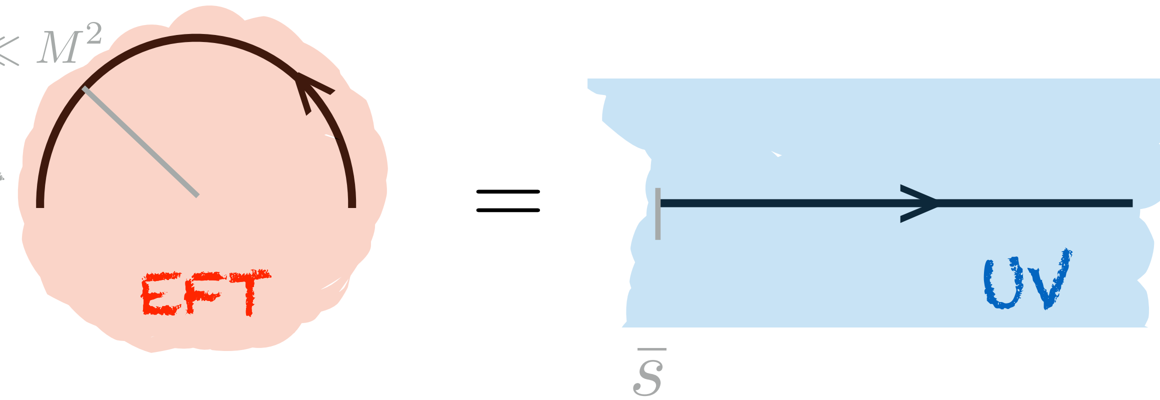

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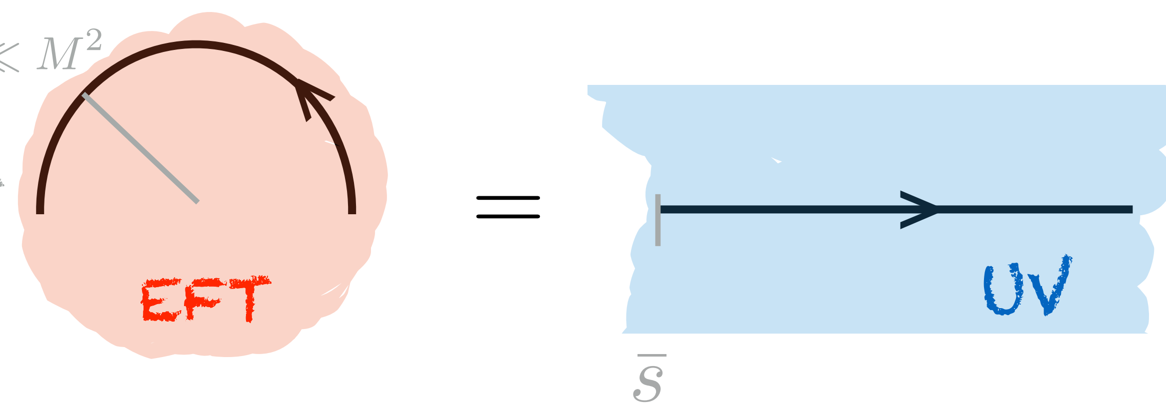

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Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,

- Consistency condition for EFTs

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

positive

$$\frac{2}{\pi}\int_{\bar{s}}^{\infty} ds\, \frac{\mathrm{Im}A(s)}{s^{n+1}}$$

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Moments

$$\int_0^1 d\mu(x) x^n$$

variables change ←

positive ↓

$$\frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\text{Im}A(s)}{s^{n+1}}$$

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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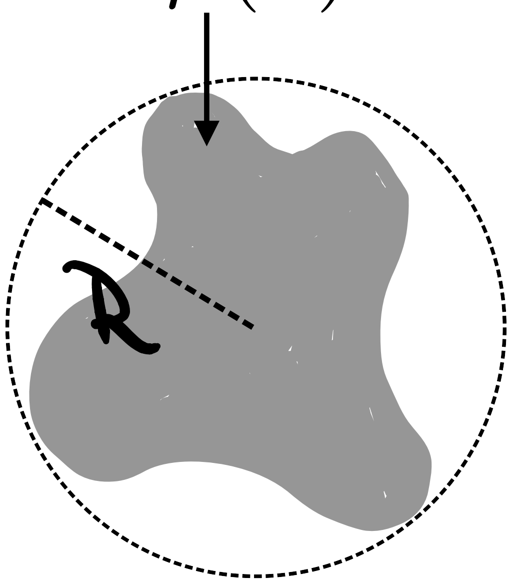
positive (↓ $\mathrm{Im} A(s)$)

Moments appear everywhere in physics...

e.g. stones $\quad d\mu(x)$ = mass distributions



More UV-IR Connections

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Moments

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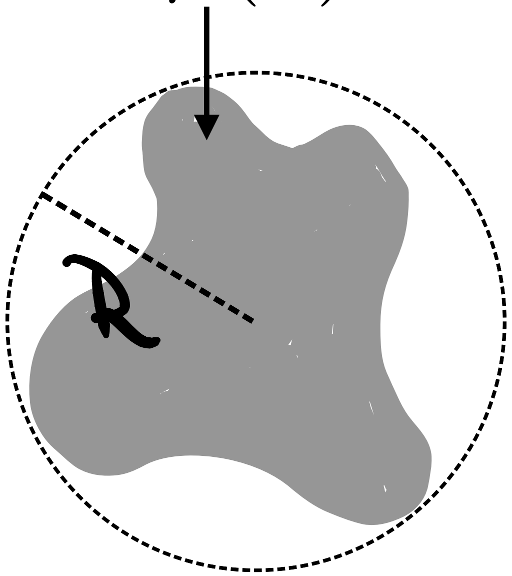

n=0: total mass M (sets units)

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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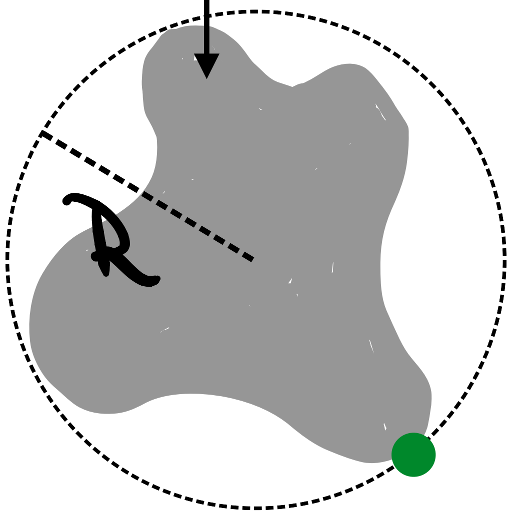

n=0: total mass M (sets units)

n=1: centre of mass <RM

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Moments

$$\int_0^1 d\mu(x)x^n \quad \xleftarrow{\text{variables change}} \quad \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\text{Im}A(s)}{s^{n+1}}$$

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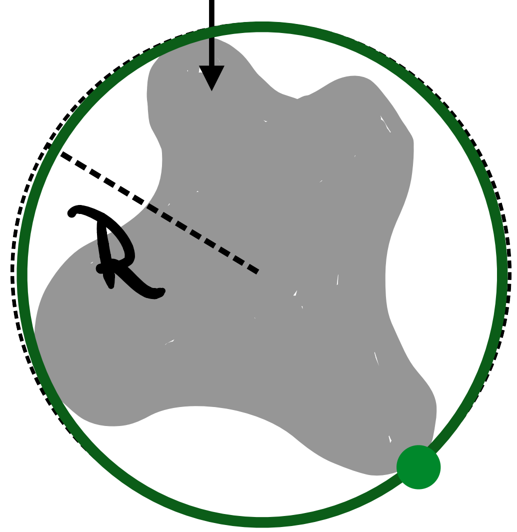

n=0: total mass M (sets units)

n=1: centre of mass $<R\,M$

n=2: moment of inertia $<R^2 M$

More UV-IR Connections

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Moments

$$\int_0^1 d\mu(x) x^n$$

← variables change

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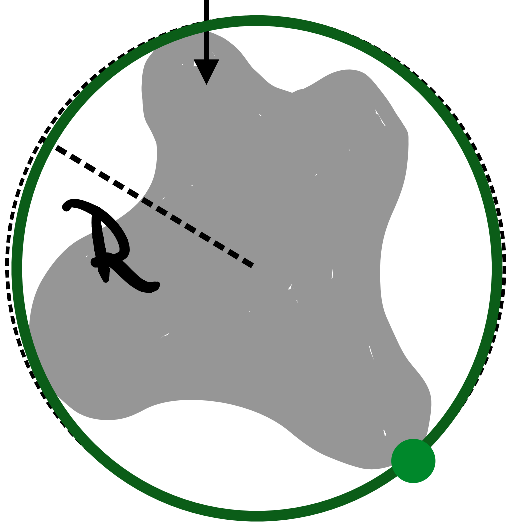

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n=1: centre of mass <RM

n=2: moment of inertia <R²M

→ Bounded

What bounds do moments satisfy?

All Bounds ⟷ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT arcs ∼ Wilson coef.	⟷ $\mathcal{A}_n = \int_0^1 d\mu(x) x^n$	Bounds on Moments

All Bounds ↔ All Positive Polynomials

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Bounds on EFT arcs ∼ Wilson coef.	⟷ $\mathcal{A}_n = \int_0^1 d\mu(x) x^n$	Bounds on Moments	⟷	Positive polynomials in [0,1]

$$p_N(x) = \sum_{i=0}^{N} \alpha_i x^i > 0$$

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Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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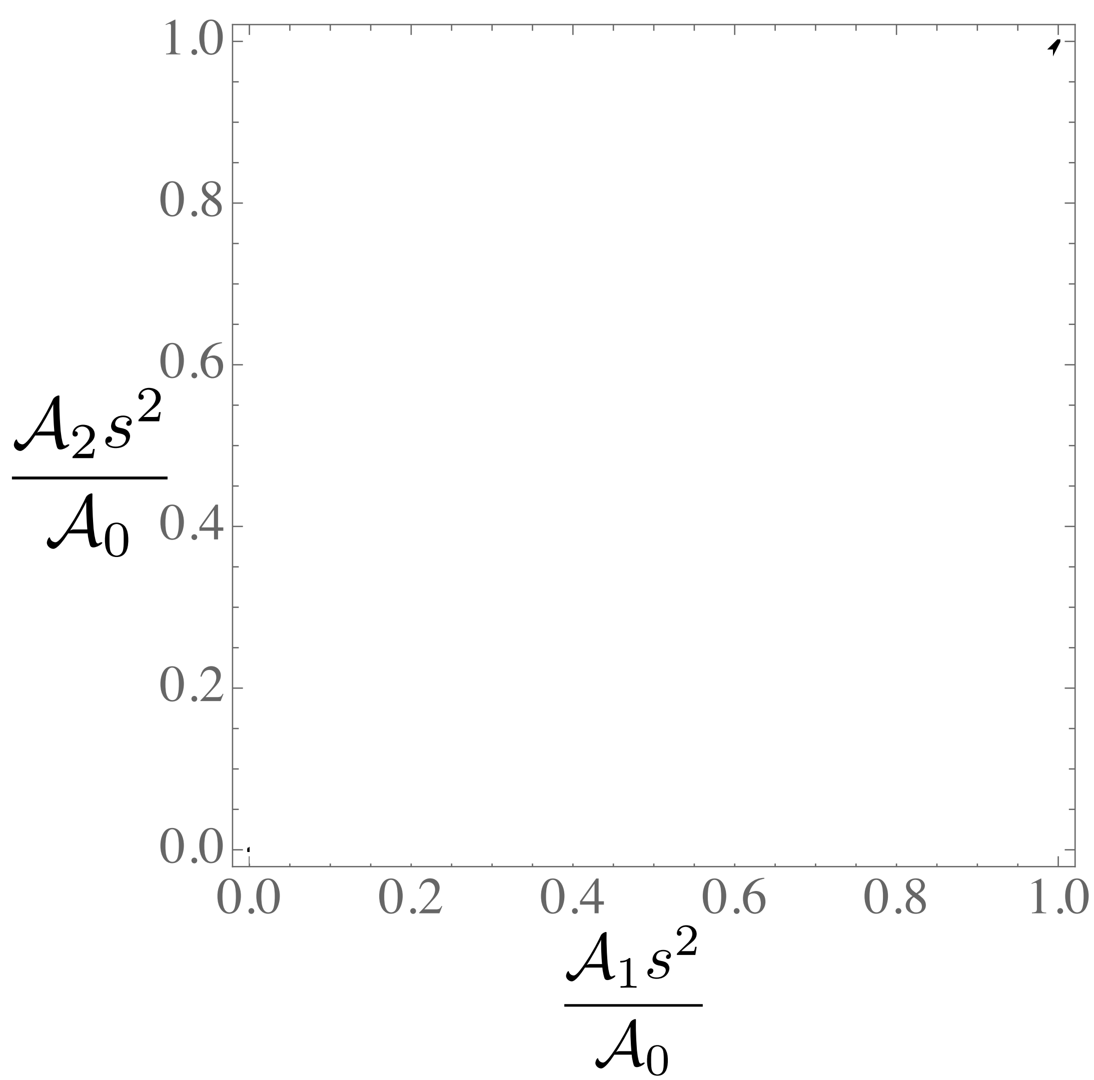
For the first 3 arcs:



All Bounds ⟷ All Positive Polynomials

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Bounds on EFT arcs
∼ Wilson coef.

⟷

$\mathcal{A}_n = \int_0^1 d\mu(x) x^n$

Bounds on Moments

⟷

Positive polynomials in [0,1]

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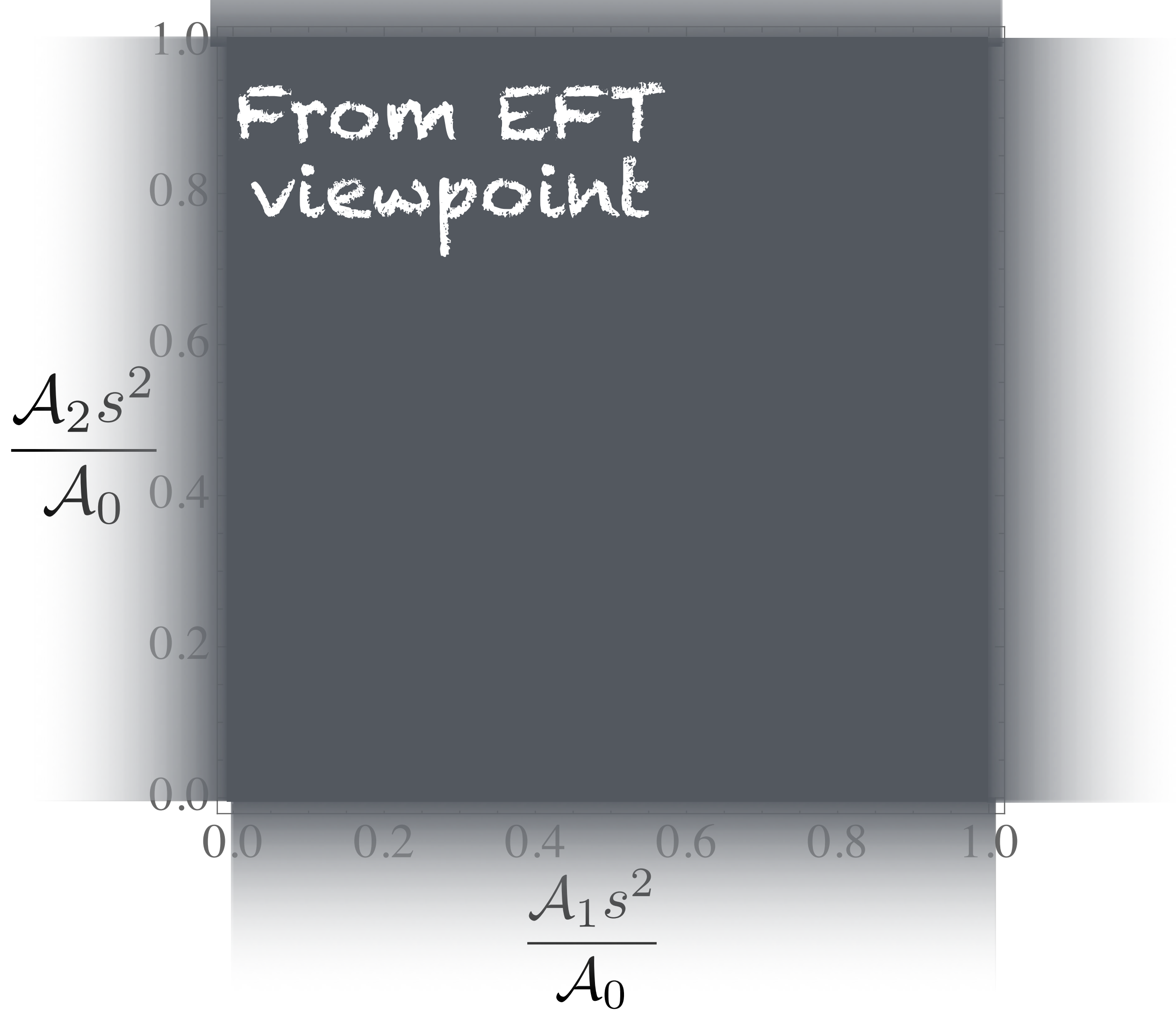

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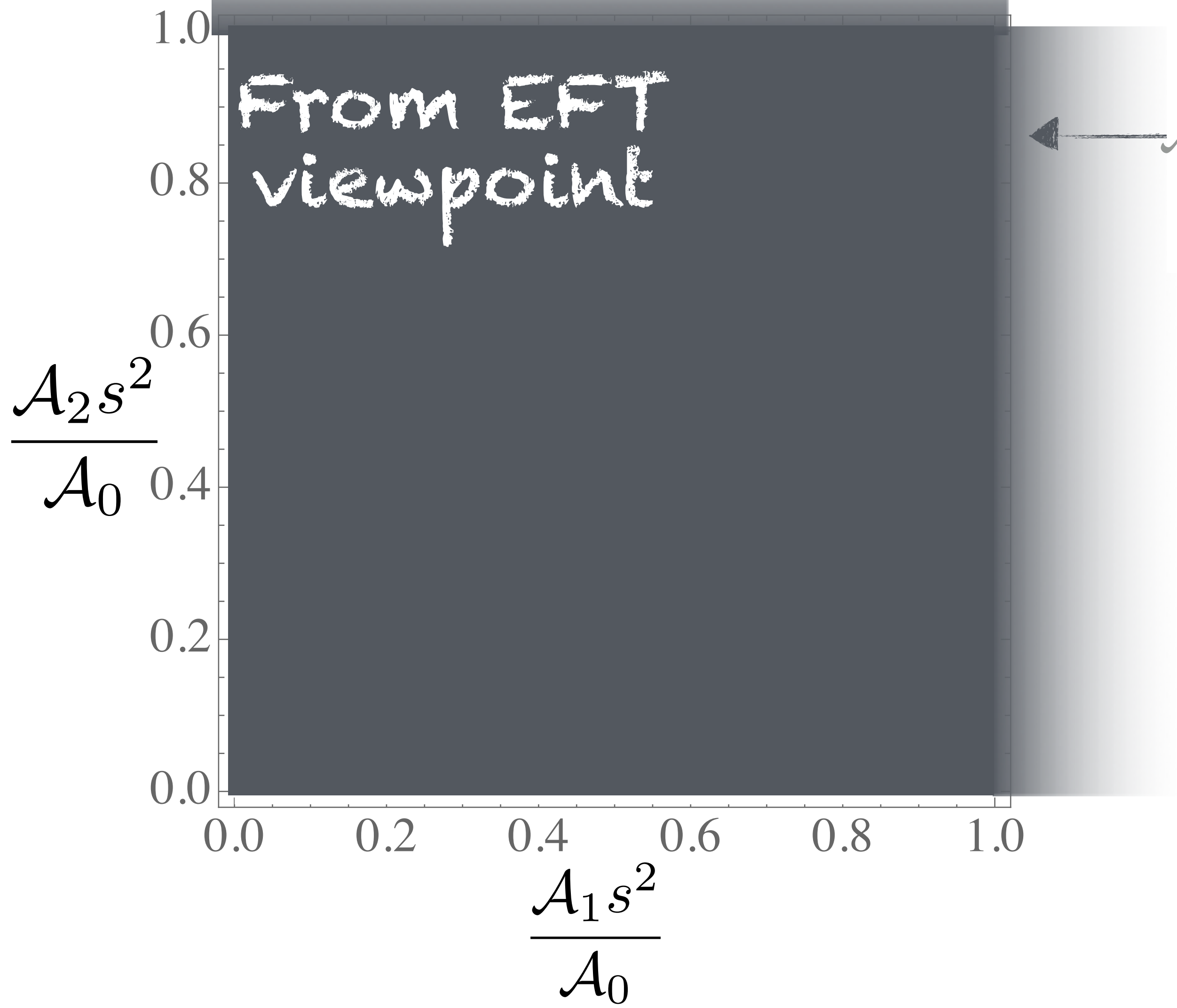

Polynomials:

$\longleftarrow \mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2 > 0 \longleftarrow 1 \quad x \quad x^2$

All Bounds ↔ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT arcs ∼ Wilson coef. ⟷ $\mathcal{A}_n = \int_0^1 d\mu(x) x^n$ **Bounds on Moments** ⟷ **Positive polynomials in [0,1]**

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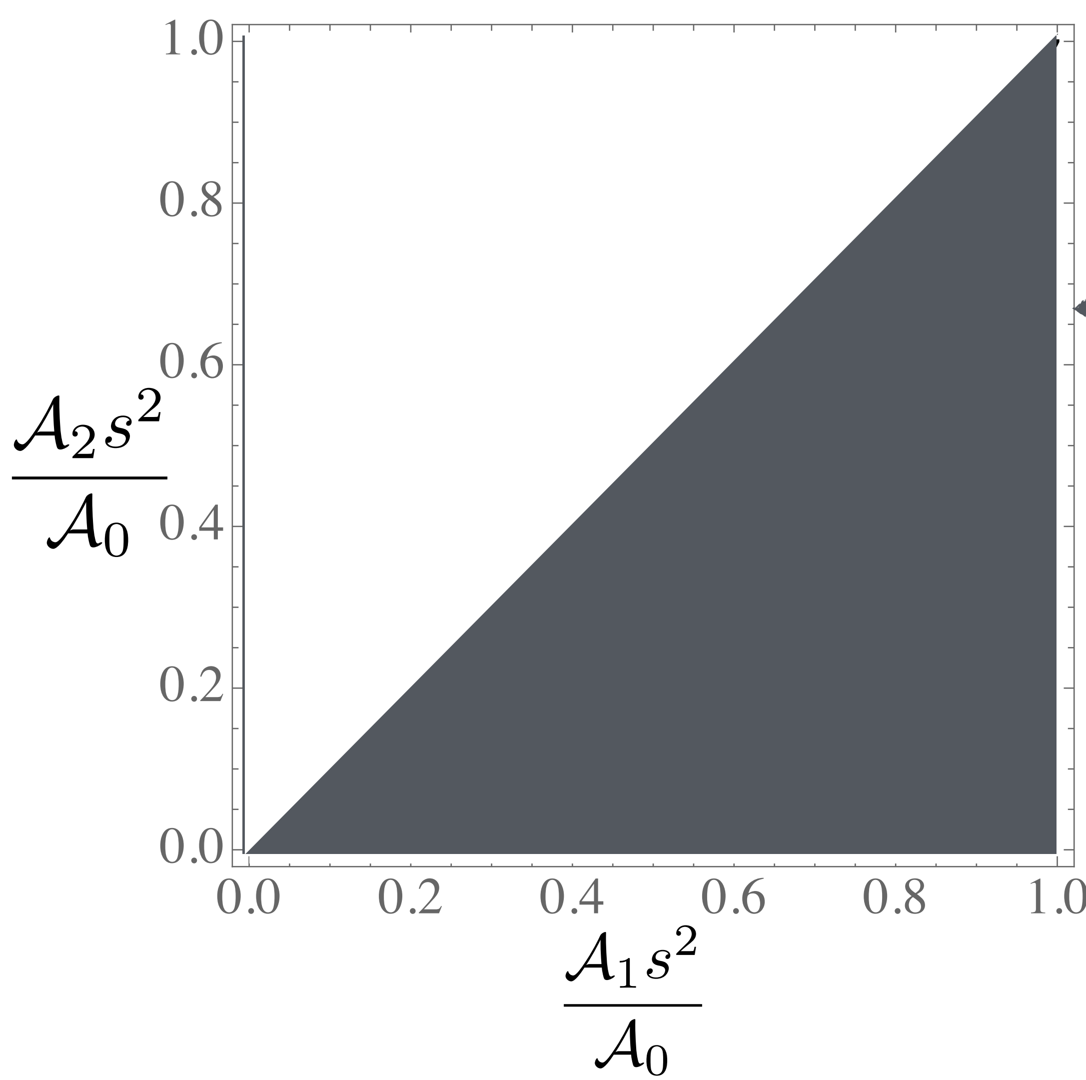

Polynomials:

$\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2 > 0$ ⟵ $1 \quad x \quad x^2$

$\mathcal{A}_0 > s^2 \mathcal{A}_1$, $\mathcal{A}_1 > s^2 \mathcal{A}_2$ ⟵ $1 - x \quad x(1-x)$

All Bounds ↔ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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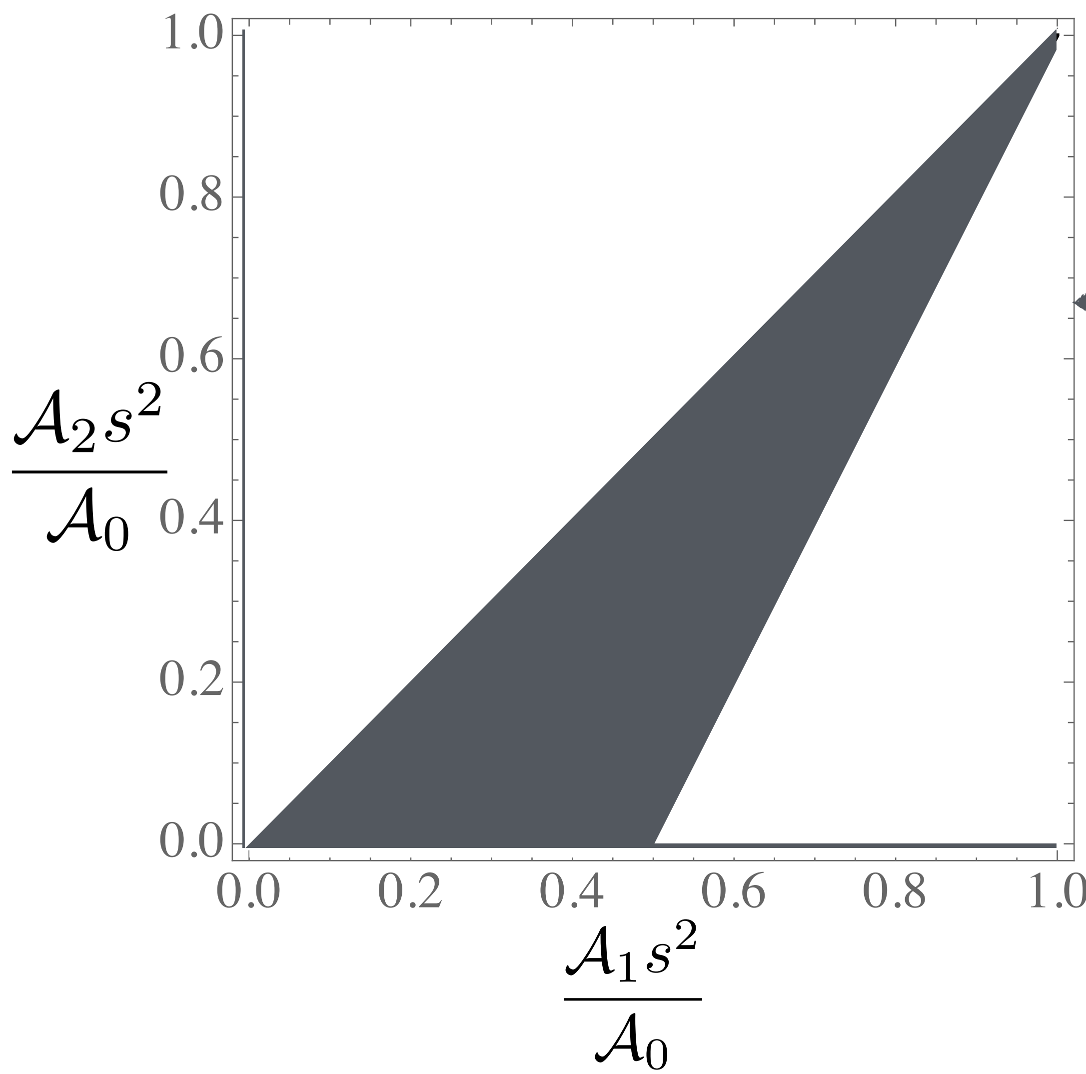

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$x^3 + x(1-x)^2$

All Bounds ↔ All Positive Polynomials

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Bounds on EFT arcs ~ Wilson coef. ⟷ **Bounds on Moments** ⟷ **Positive polynomials in [0,1]**

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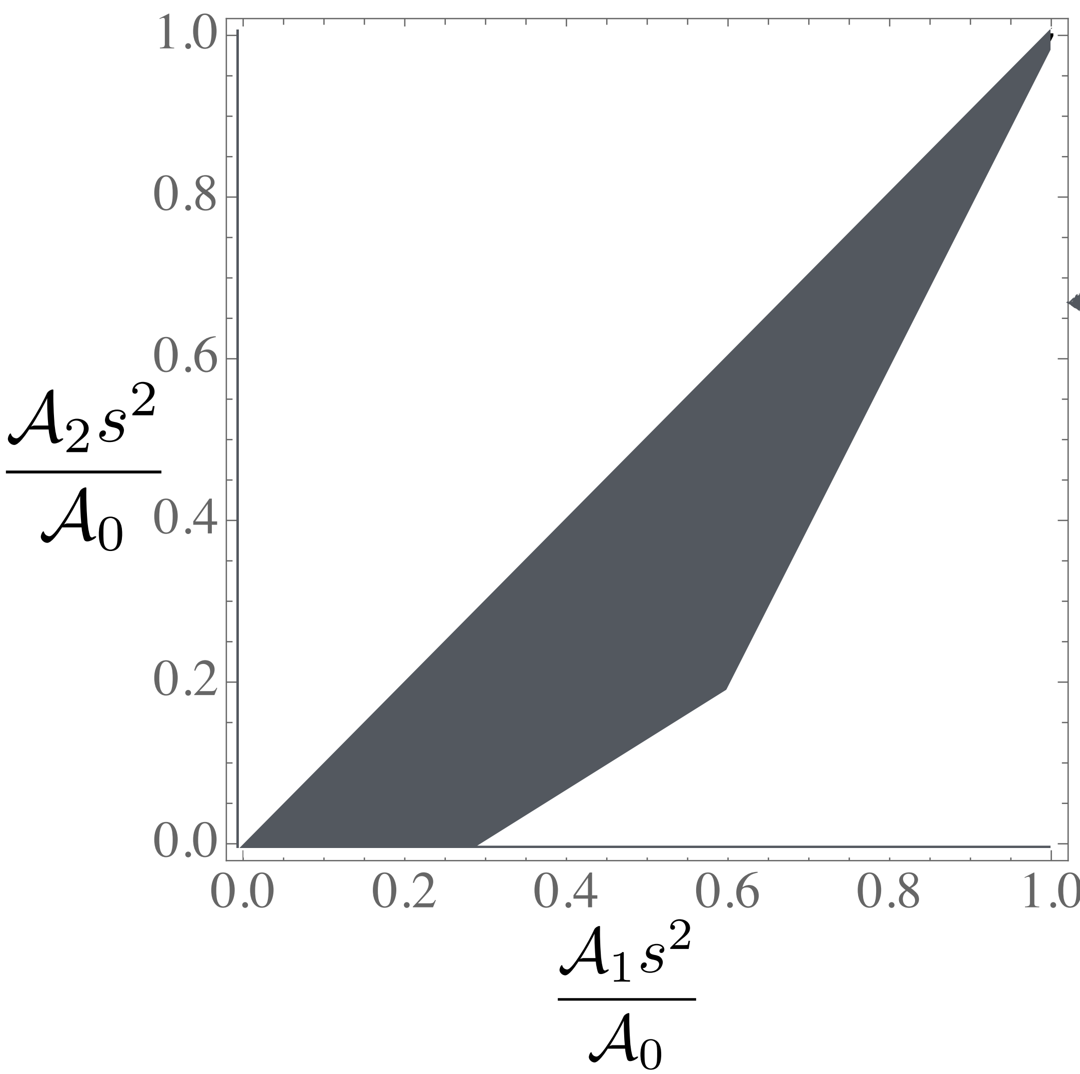

Polynomials:

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Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

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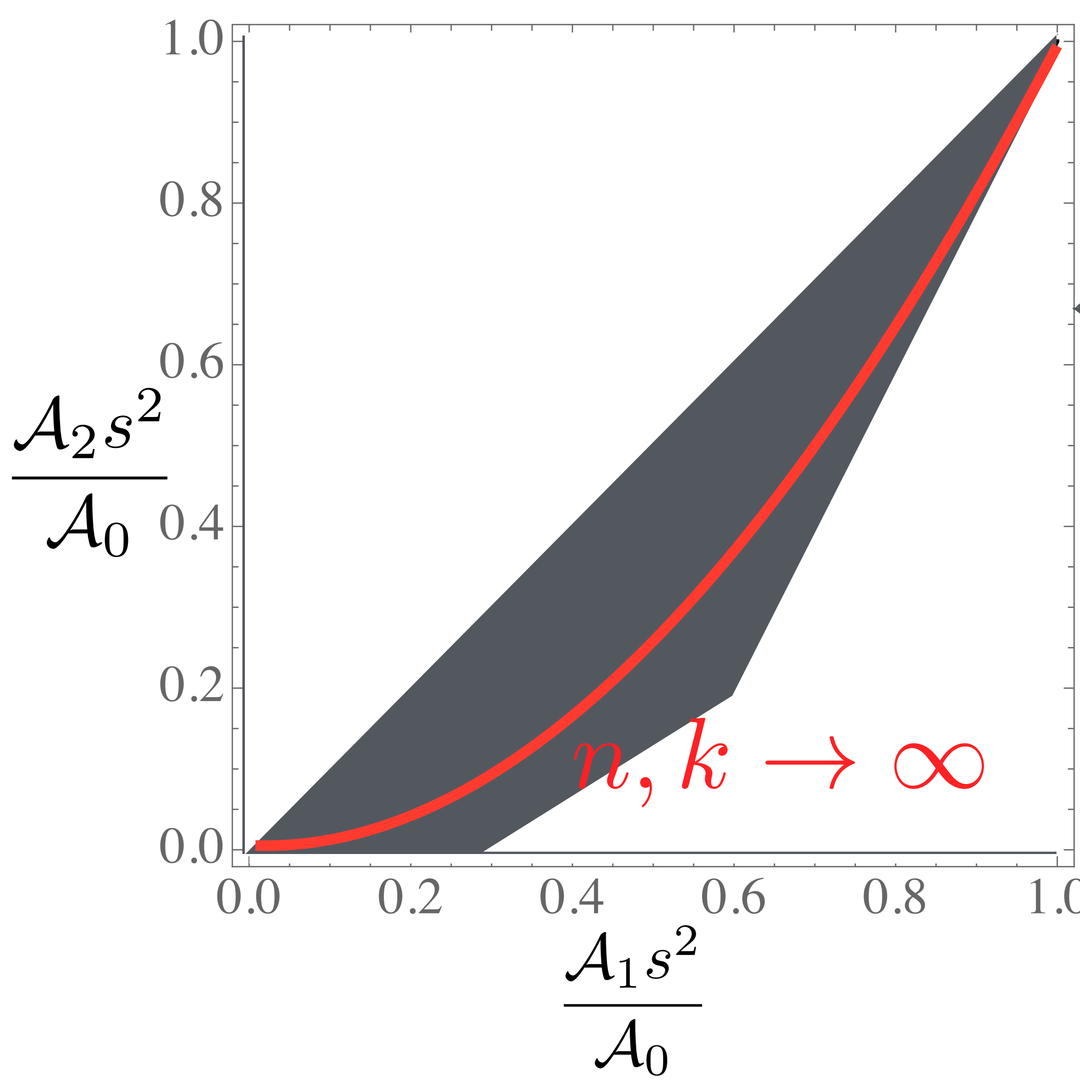

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$x^3 + x(1-x)^2$

$\ldots$

All polynomials:

$$p = x^n (1-x)^k$$

(Bernstein)

All Bounds ⟷ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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Better: finite number of bounds for finite number of arcs

(recurring situation in EFT)

All Bounds ↔ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT arcs ~ Wilson coef.	⟷	Bounds on Moments	⟷	Positive polynomials in [0,1]

$\mathcal{A}_n = \int_0^1 d\mu(x) x^n$

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Better: finite number of bounds for finite number of arcs

(recurring situation in EFT)

Optimal Bounds involving only N_{max} Arcs ⟵ Polynomials with $N \leq N_{max}$

$$p = q_1^2 + x q_2^2 + (1-x) q_3^2 + x(1-x) q_4^2$$

All Bounds ⟷ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT arcs ∼ Wilson coef.	⟷ $\mathcal{A}_n = \int_0^1 d\mu(x) x^n$	Bounds on Moments	⟷	Positive polynomials in [0,1]
$\sum_{i=0}^{N} \alpha_i \mathcal{A}_i > 0$	⟵	$\int_0^1 p_N(x) d\mu(x) > 0$	⟵	$p_N(x) = \sum_{i=0}^{N} \alpha_i x^i > 0$

Better: finite number of bounds for finite number of arcs

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$$\int_0^1 q_1(x)^2 d\mu(x)$$

All Bounds ⟷ All Positive Polynomials

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Bounds on EFT arcs ∼ Wilson coef.	⟷ $\mathcal{A}_n = \int_0^1 d\mu(x) x^n$	Bounds on Moments	⟷	Positive polynomials in [0,1]

$$\sum_{i=0}^{N} \alpha_i \mathcal{A}_i > 0 \quad \longleftarrow \quad \int_0^1 p_N(x) d\mu(x) > 0 \quad \longleftarrow \quad p_N(x) = \sum_{i=0}^{N} \alpha_i x^i > 0$$

Better: finite number of bounds for finite number of arcs

(recurring situation in EFT)

Optimal Bounds involving only N_{max} Arcs ⟵ Polynomials with $N \leq N_{max}$

$$p = q_1^2 + x q_2^2 + (1-x) q_3^2 + x(1-x) q_4^2$$

$$\sum_{i,j=0}^{N_{max}/2} \alpha_i \mathcal{A}_{i+j} \alpha_j > 0 \quad \longleftarrow \quad \int_0^1 q_1(x)^2 d\mu(x)$$

Must be Positive Definite

All Bounds ⟷ All Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

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Must be Positive Definite

For Nmax=4 $$0 \prec \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_1 & \mathcal{A}_2 & \mathcal{A}_3 \\ \mathcal{A}_2 & \mathcal{A}_3 & \mathcal{A}_4 \end{pmatrix} \equiv H_4^0$$ **Hankel Matrix**

Optimal Bounds for EFTs

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

All Optimal conditions involving N arcs only, written as Hankel Matrices:

e.g. $$H_4^0 \equiv \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_1 & \mathcal{A}_2 & \mathcal{A}_3 \\ \mathcal{A}_2 & \mathcal{A}_3 & \mathcal{A}_4 \end{pmatrix}$$

$$\begin{aligned} H_N^0 &\succ 0 \\ H_N^1 &\succ 0 \\ H_{N-1}^0 - \hat{s}^2 H_N^1 &\succ 0 \\ H_{N-1}^1 - \hat{s}^2 H_N^2 &\succ 0 \end{aligned}$$

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any energy within EFT

...up to 3 arcs...

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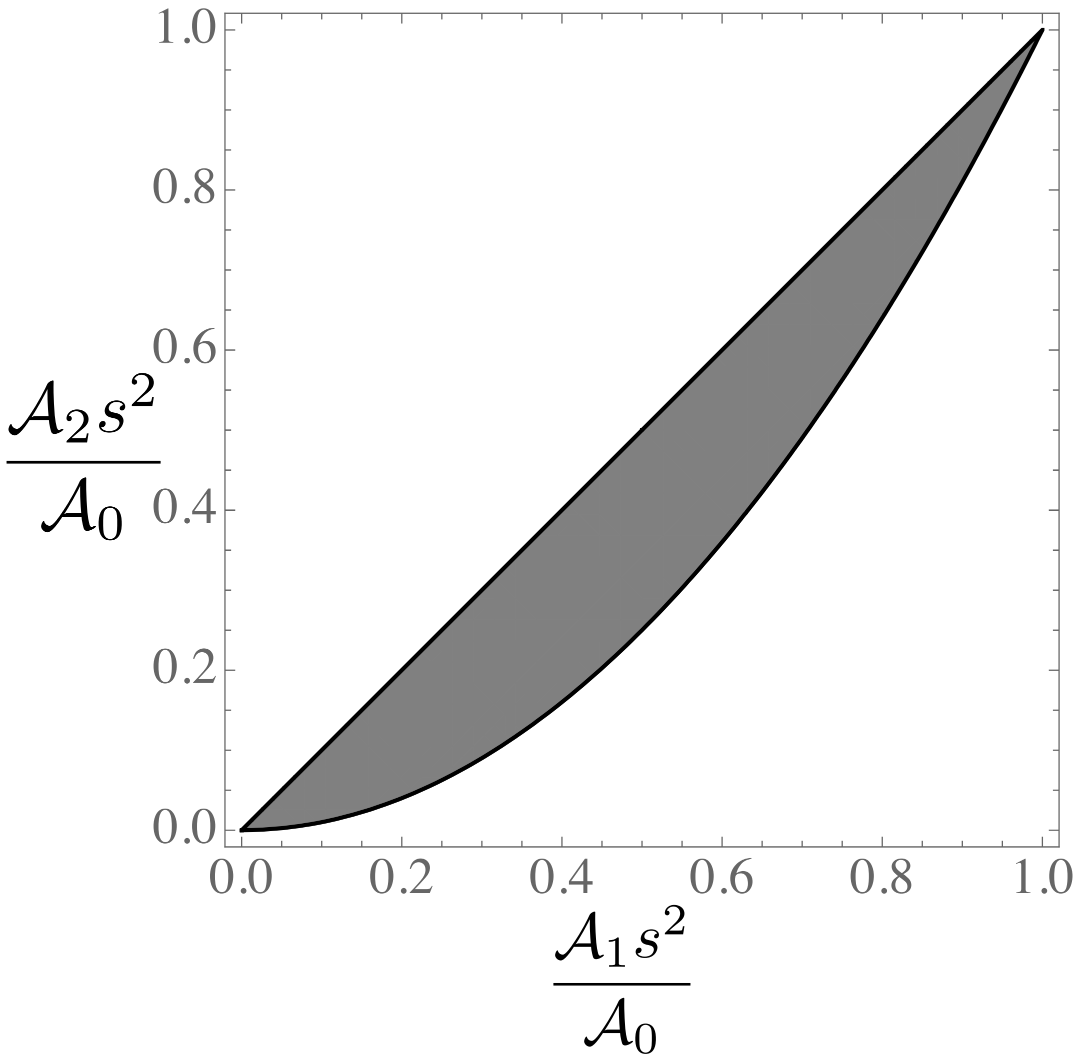

Optimal Bounds for EFTs

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$\hat{s}^2$ → any energy within EFT

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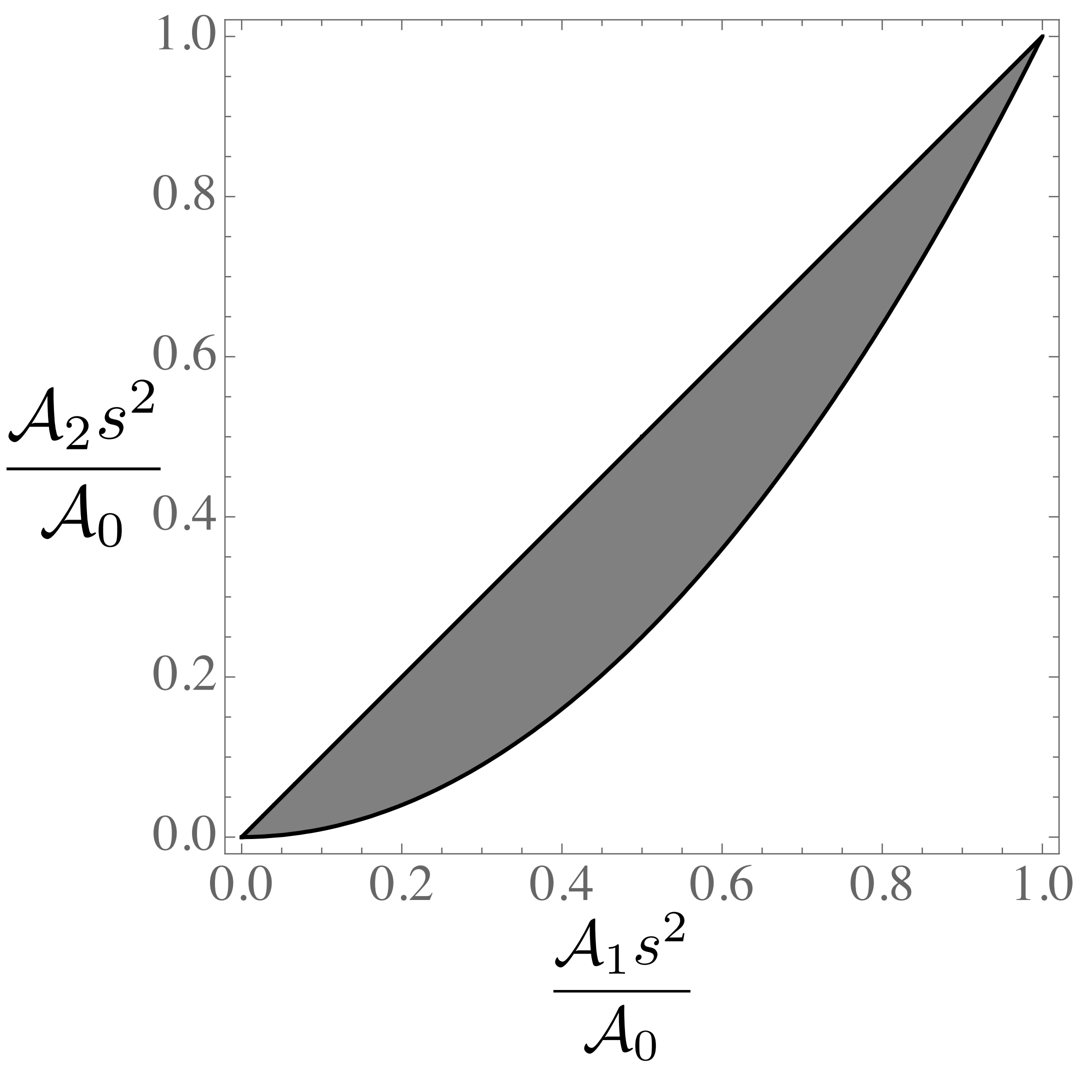

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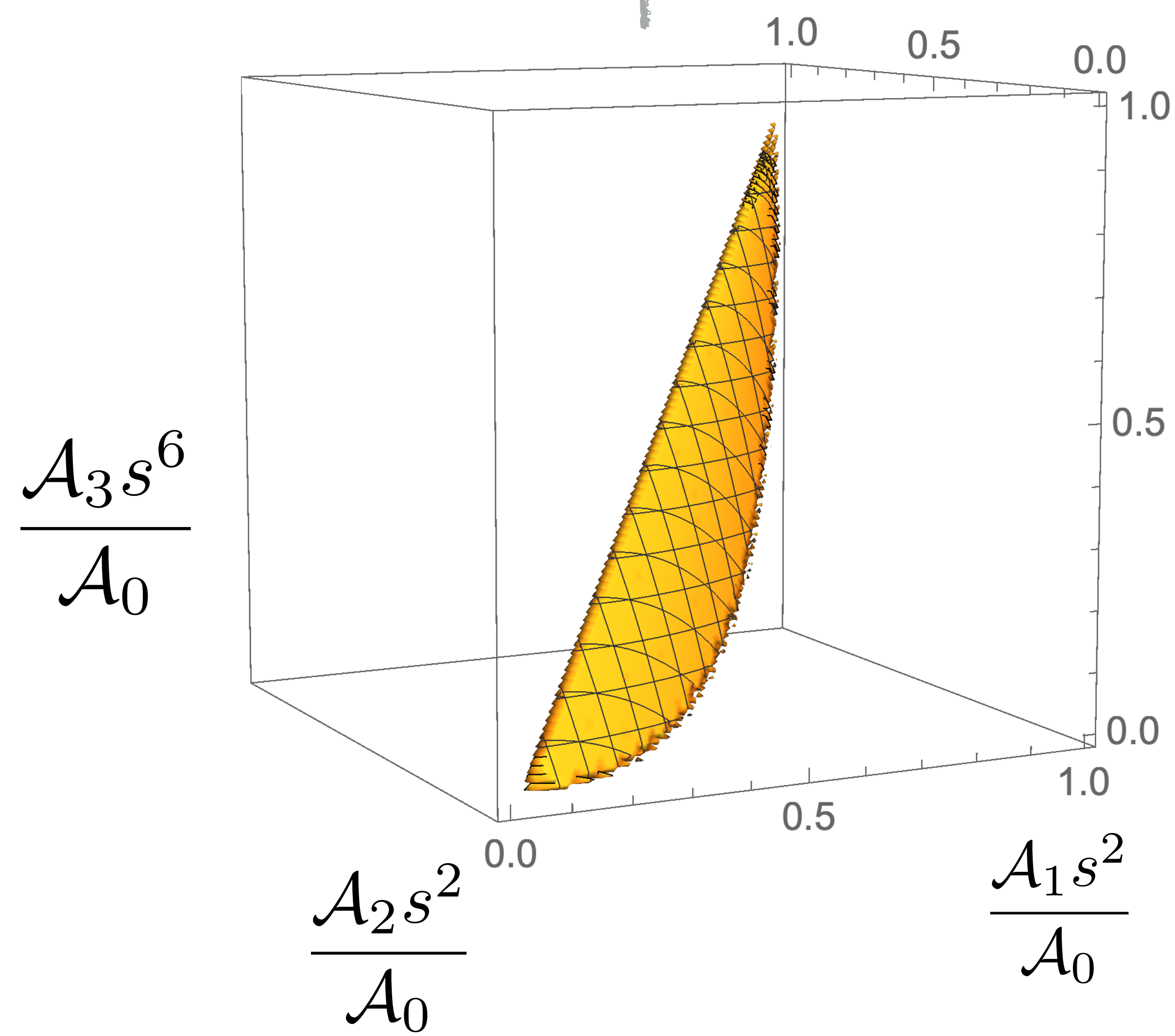

3. Applications at tree-level

1. Super-Softness

Forward Limit, tree-level:

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + c_6 s^6 + c_8 s^8 + \cdots$$

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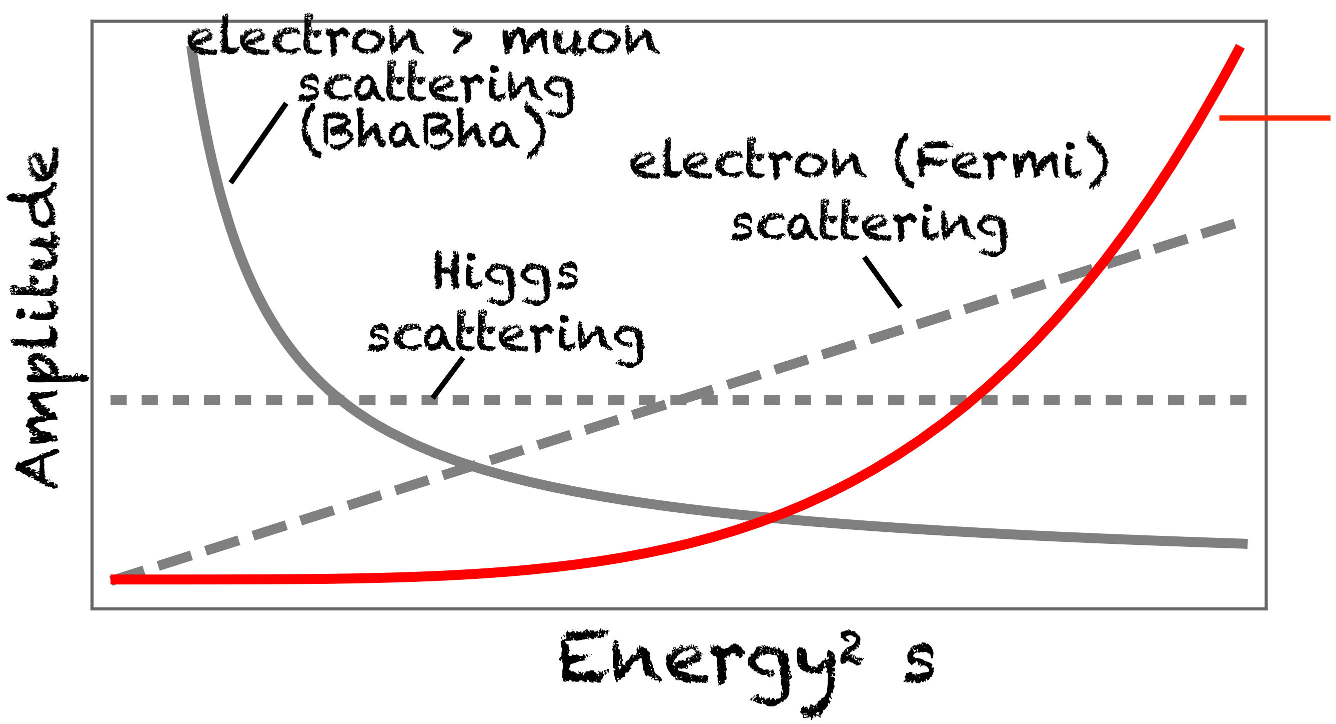

is this possible?

(it's natural and IR consistent, see intro)

1. Super-Softness

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

see also
Englert, Giudice, Greljo, McCullough'19,
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Arcs in tree-level approximation:

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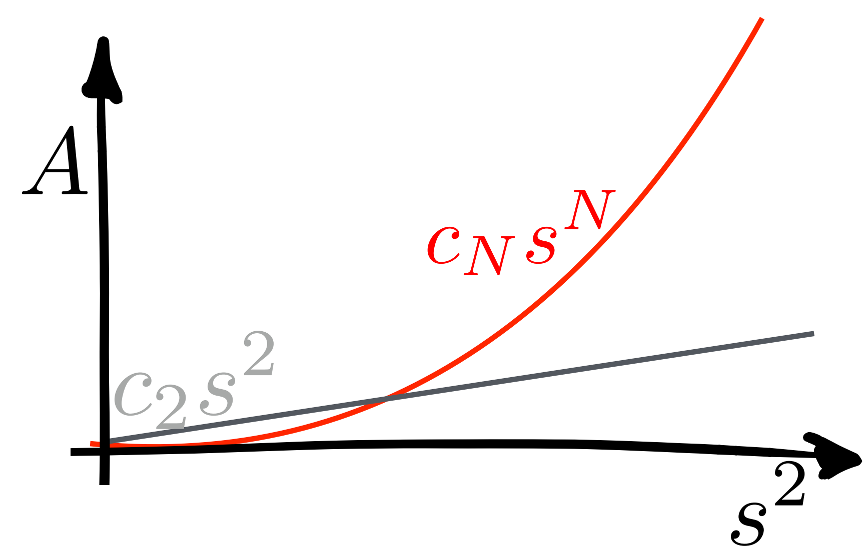

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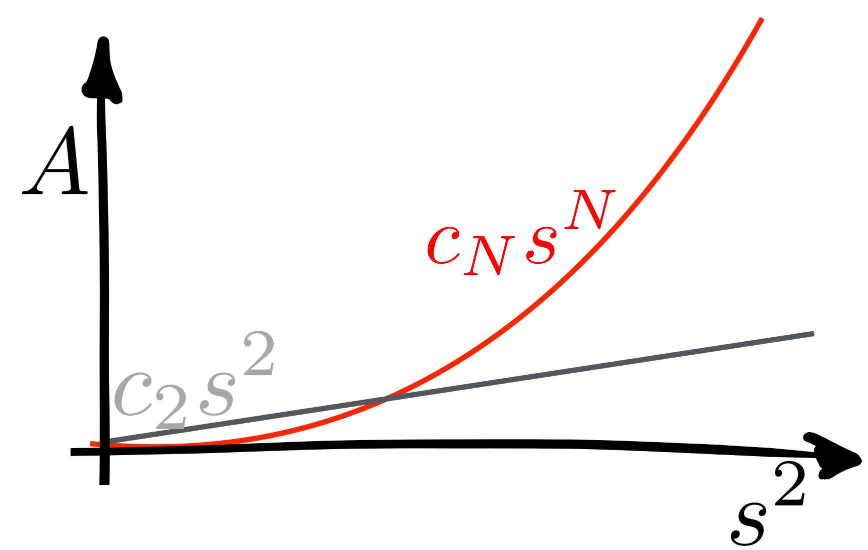

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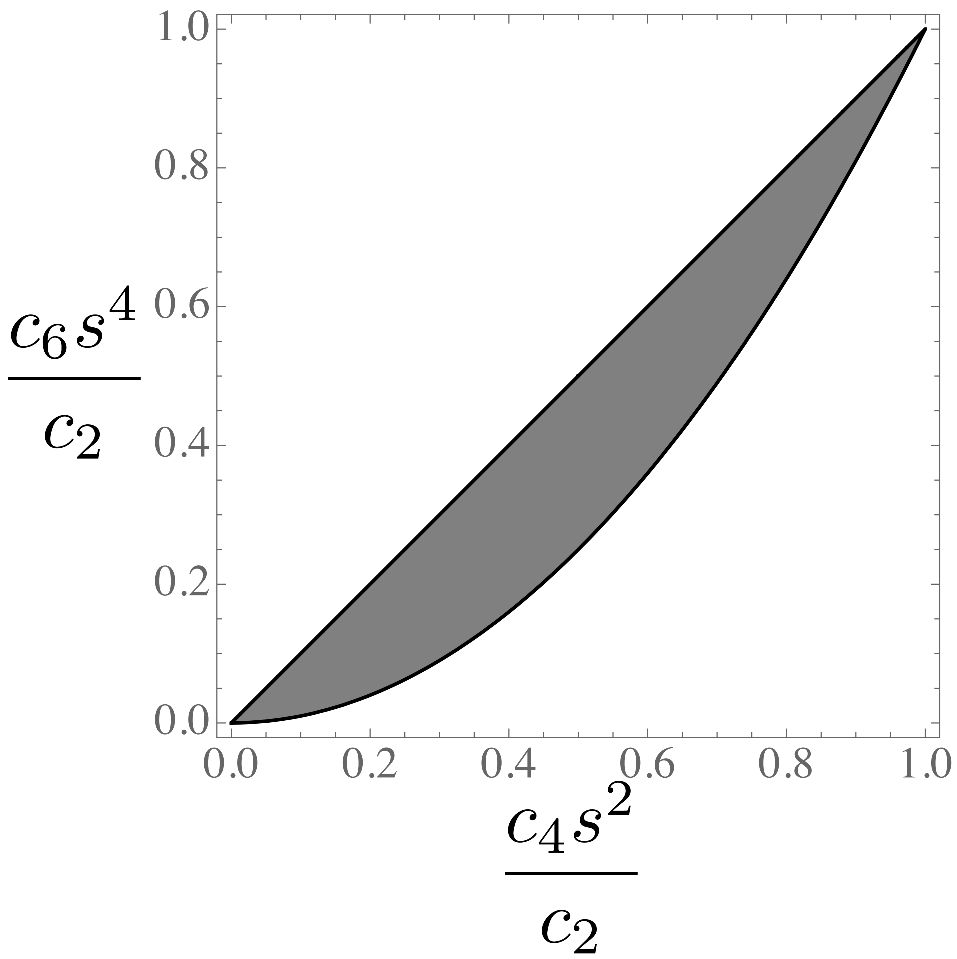

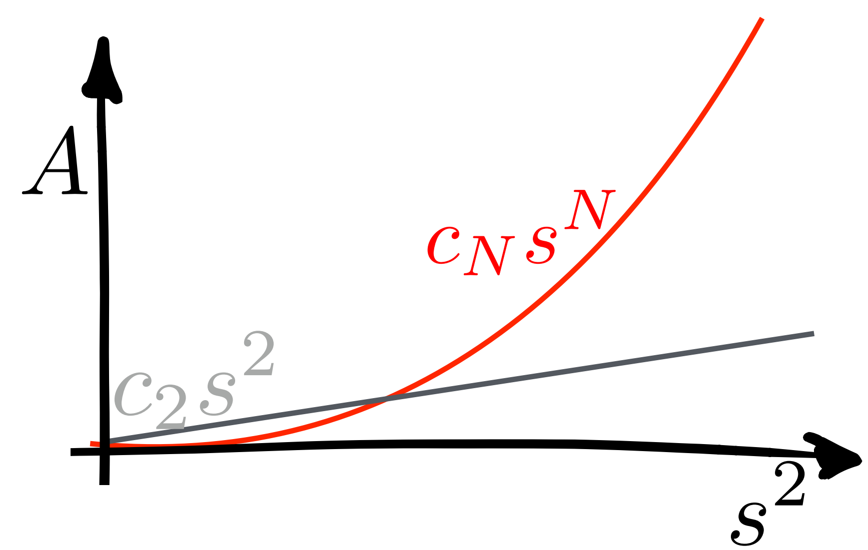

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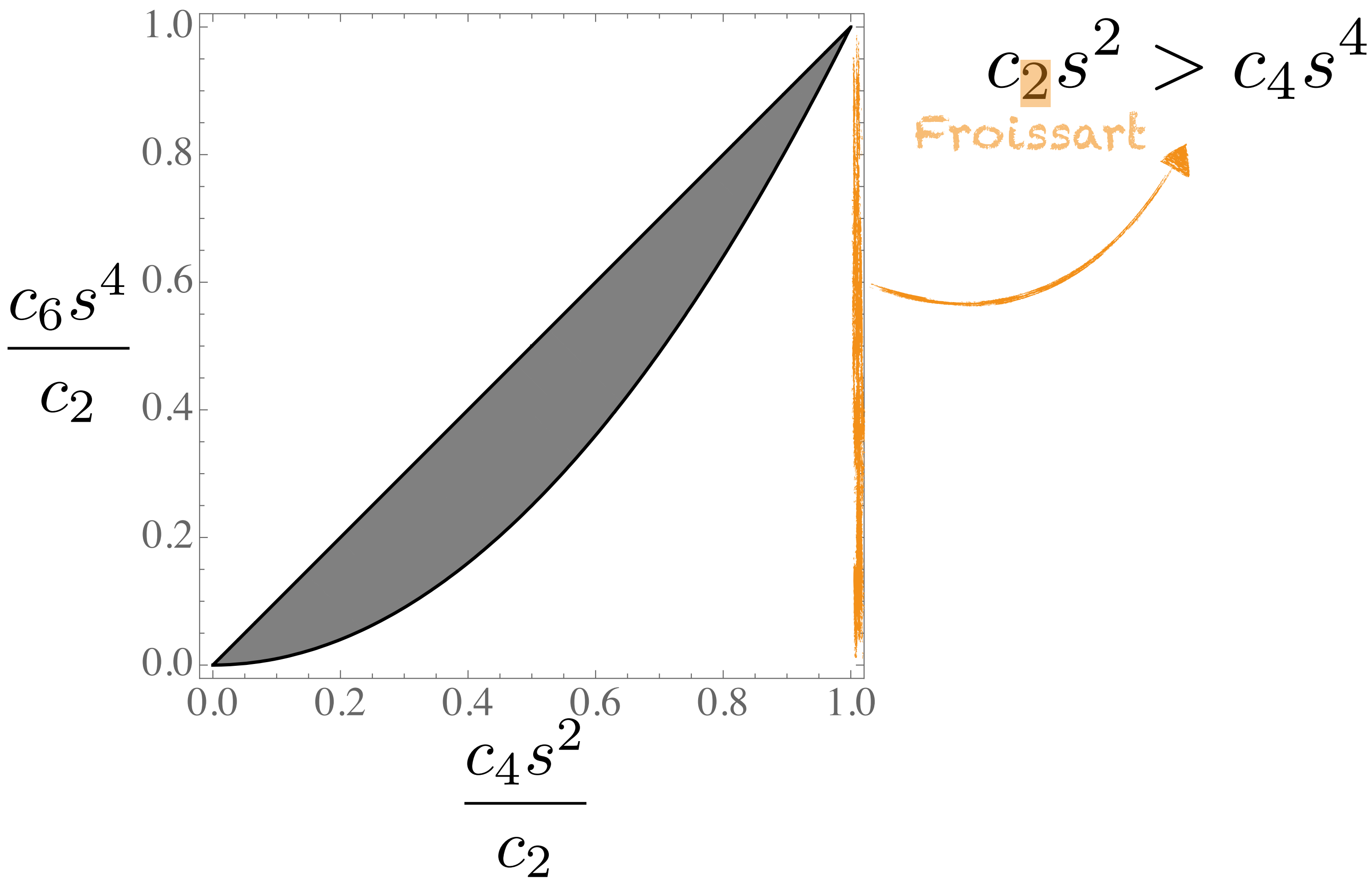

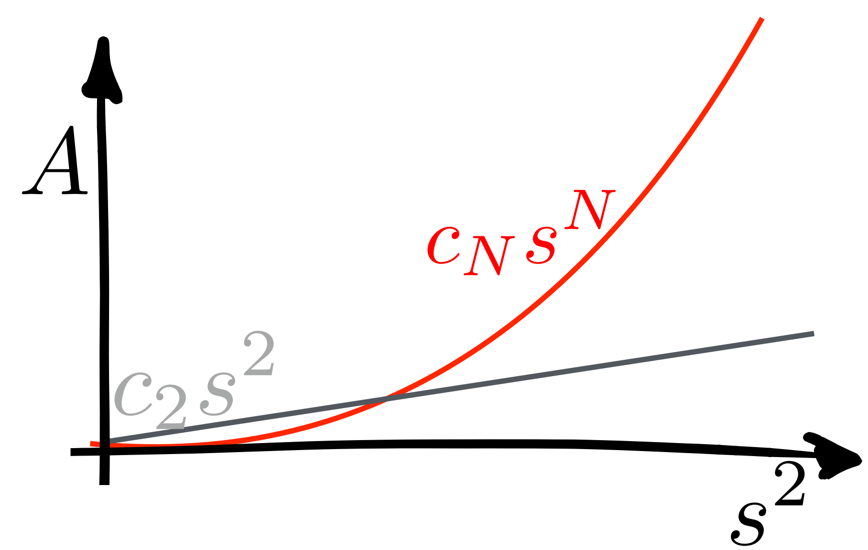

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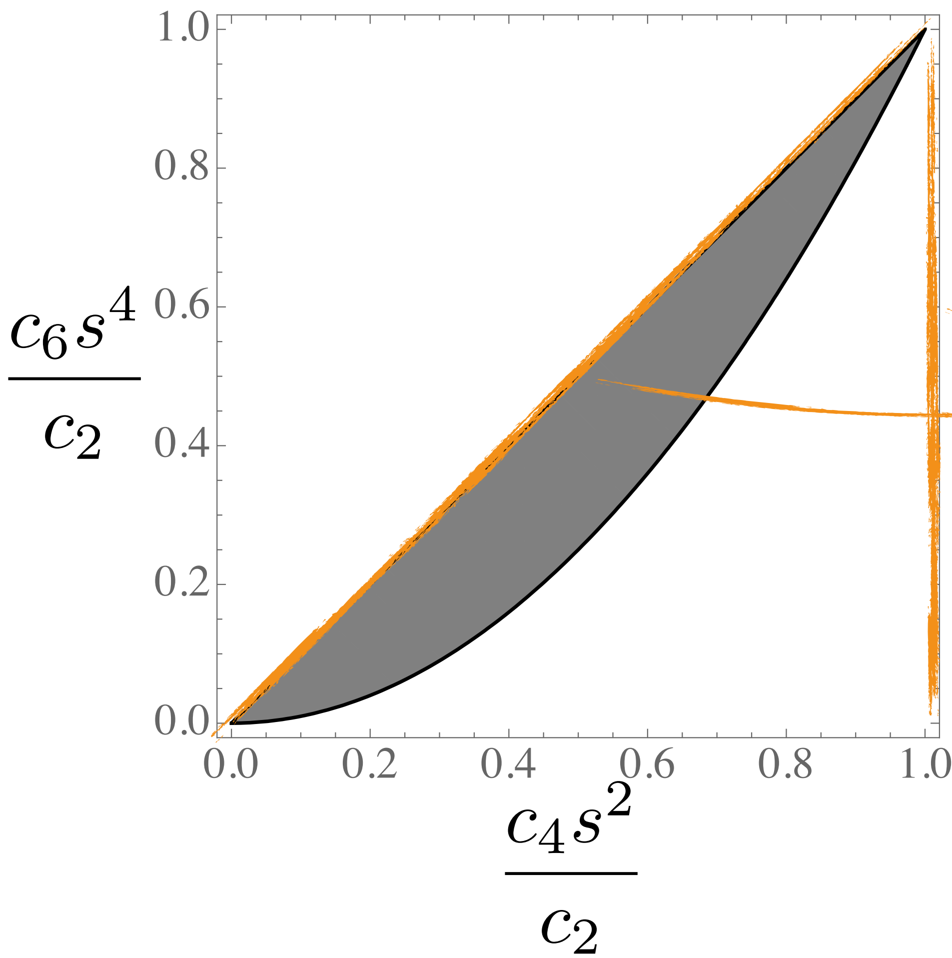

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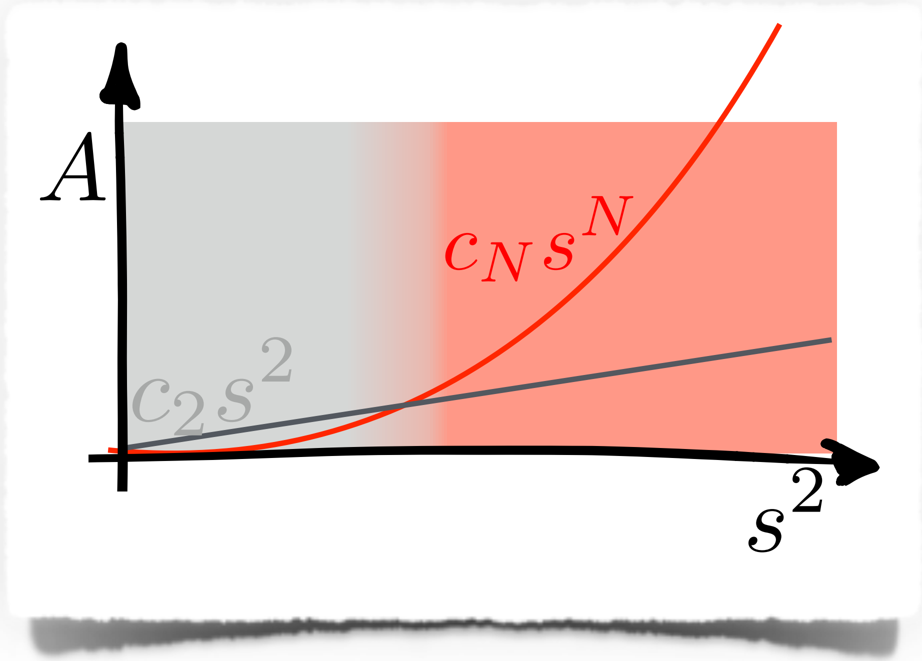
Froissart



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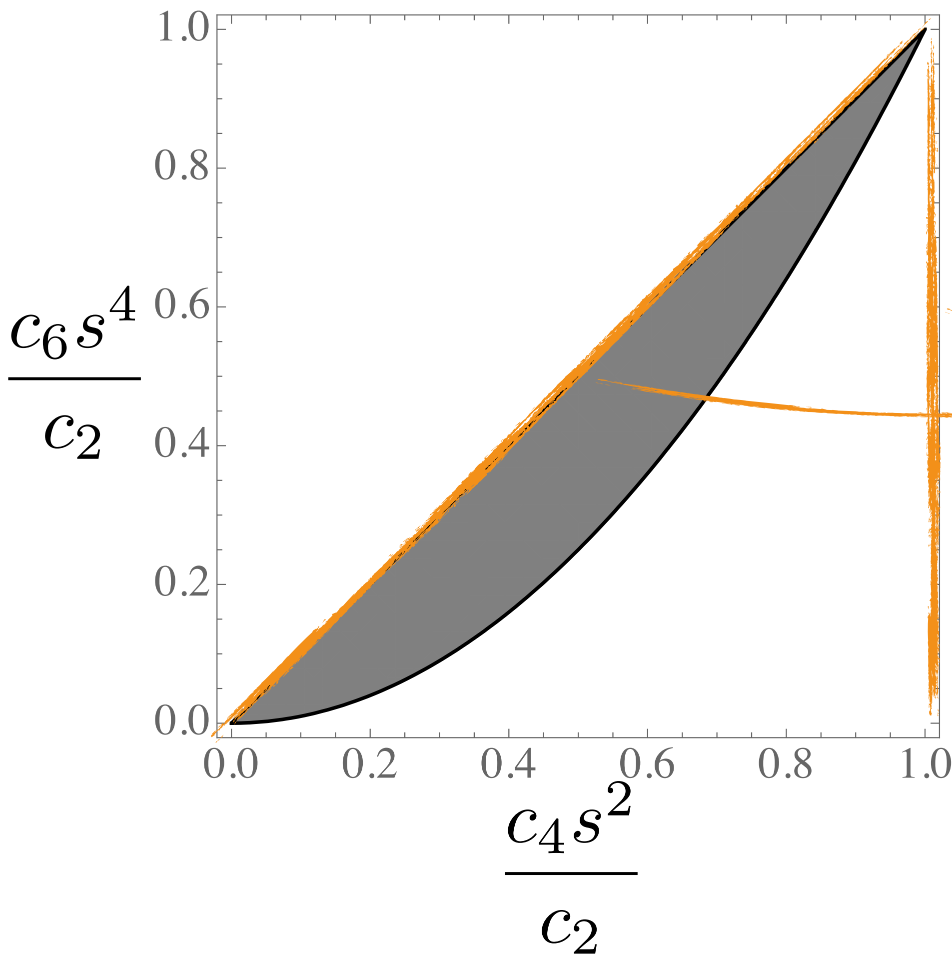

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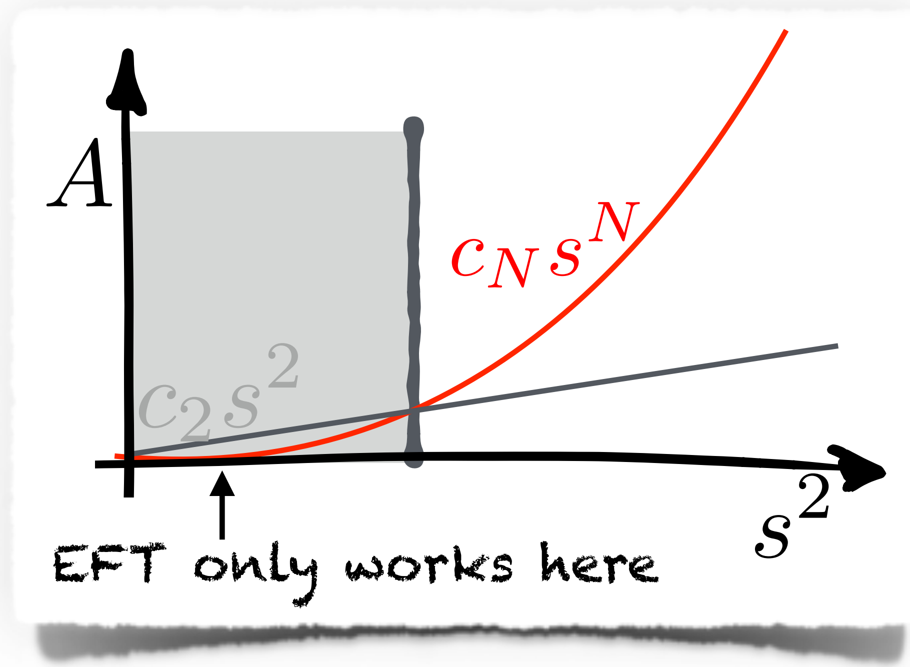

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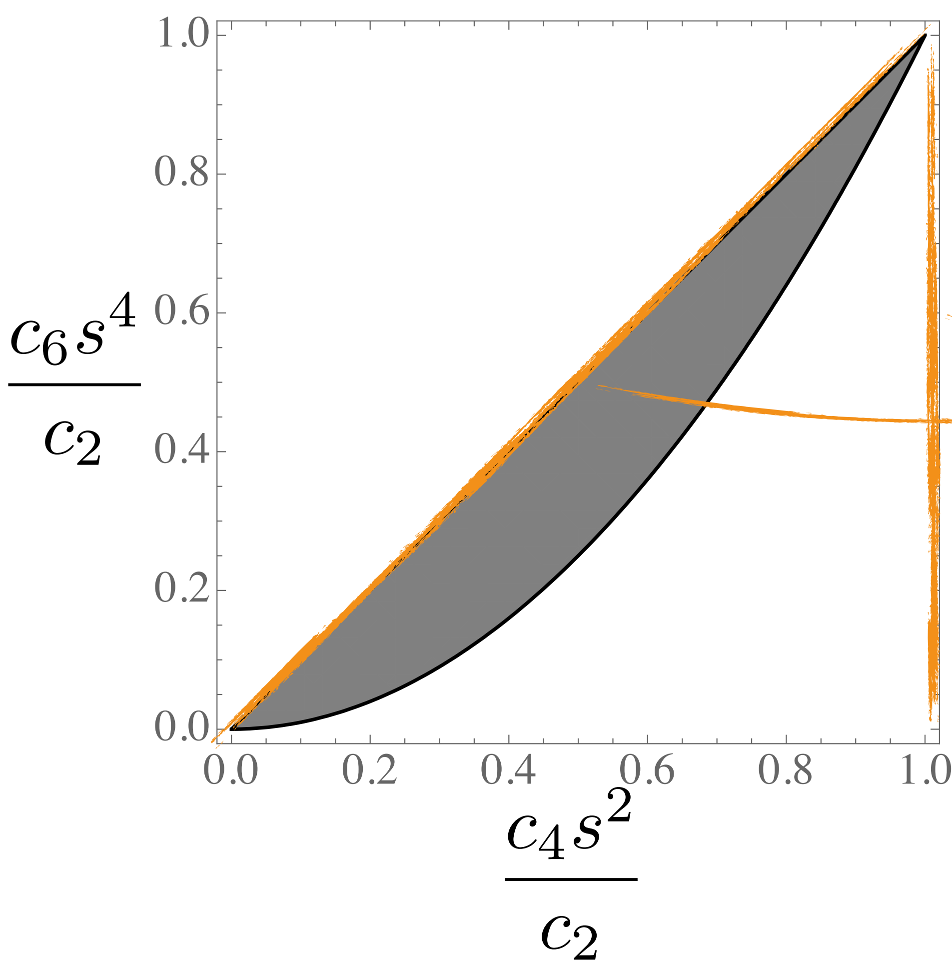

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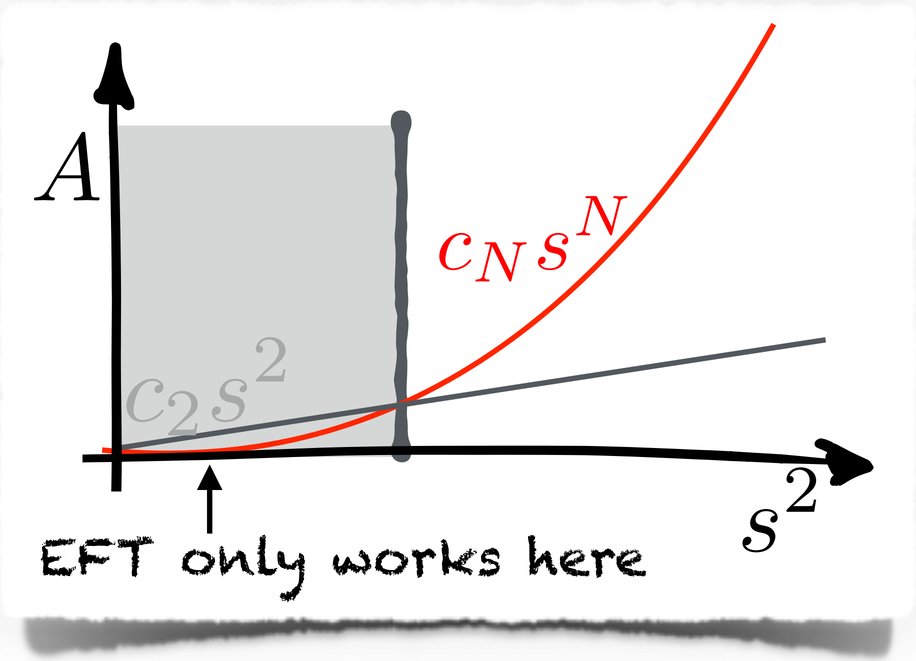
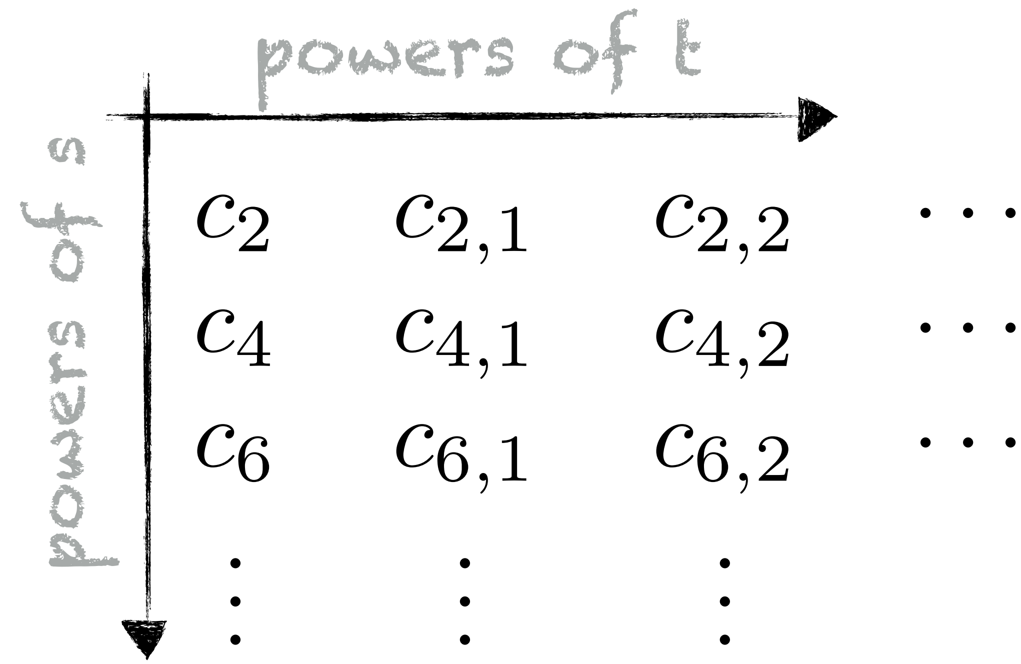


Optimal bounds involving s, have info on theory cutoff!

Supersoft theories have low cutoff... so low that supersoftness unobservable!


powers of t
powers of s
c_2 $c_{2,1}$ $c_{2,2}$ $\cdots$
c_4 $c_{4,1}$ $c_{4,2}$ $\cdots$
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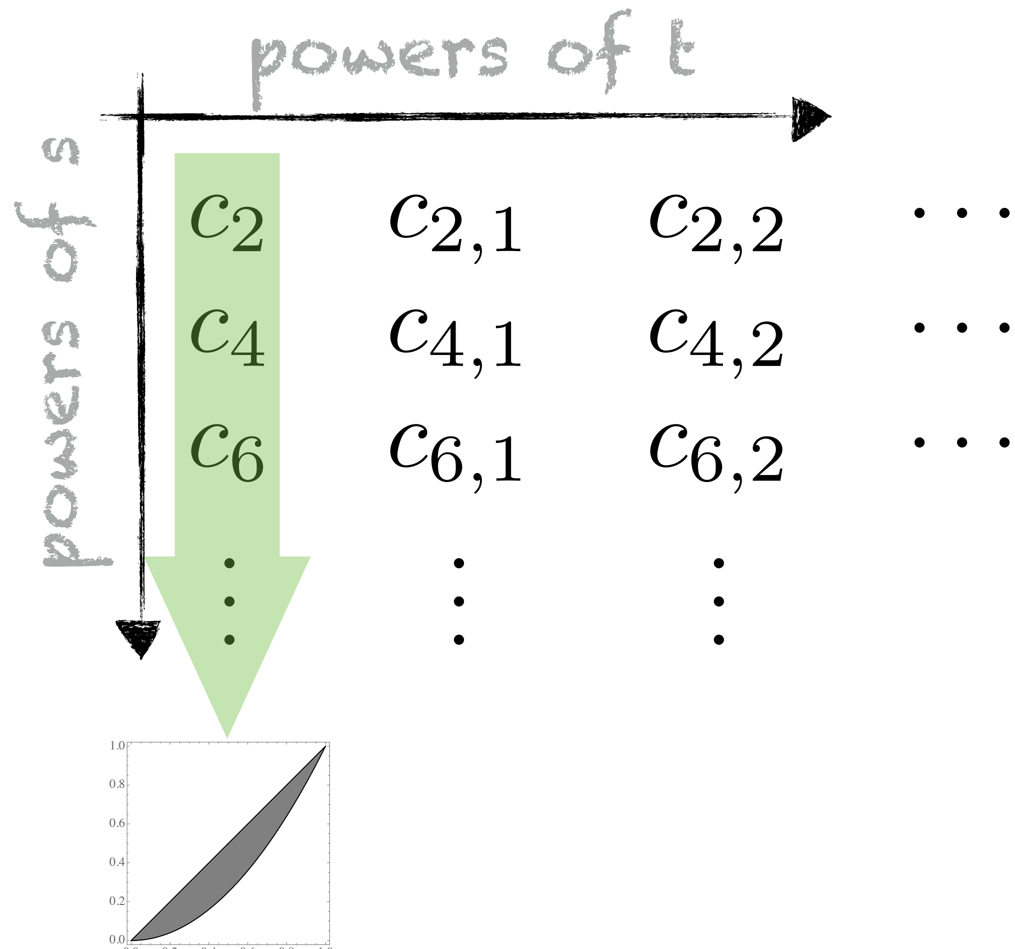
$\vdots$ $\vdots$ $\vdots$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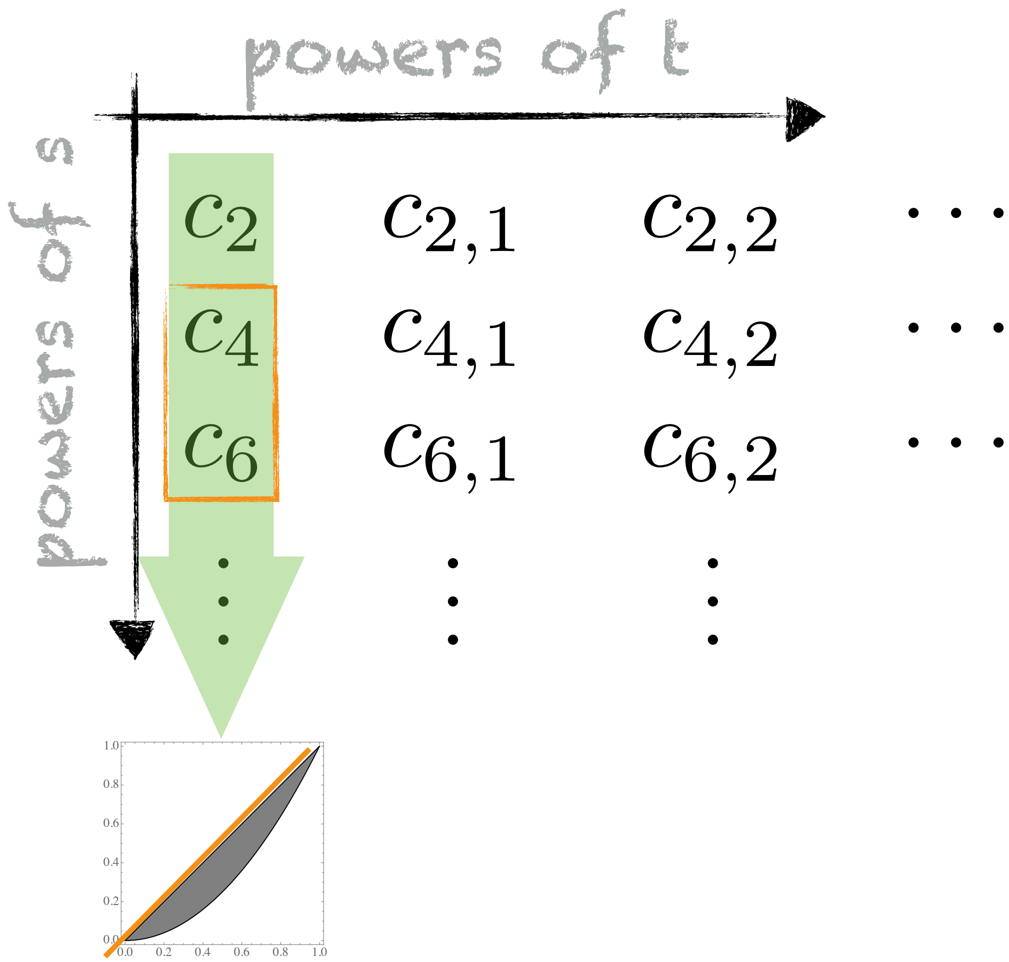
$\vdots$ $\vdots$ $\vdots$
0.0 0.2 0.4 0.6 0.8 1.0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c_2 $c_{2,1}$ $c_{2,2}$ $\cdots$
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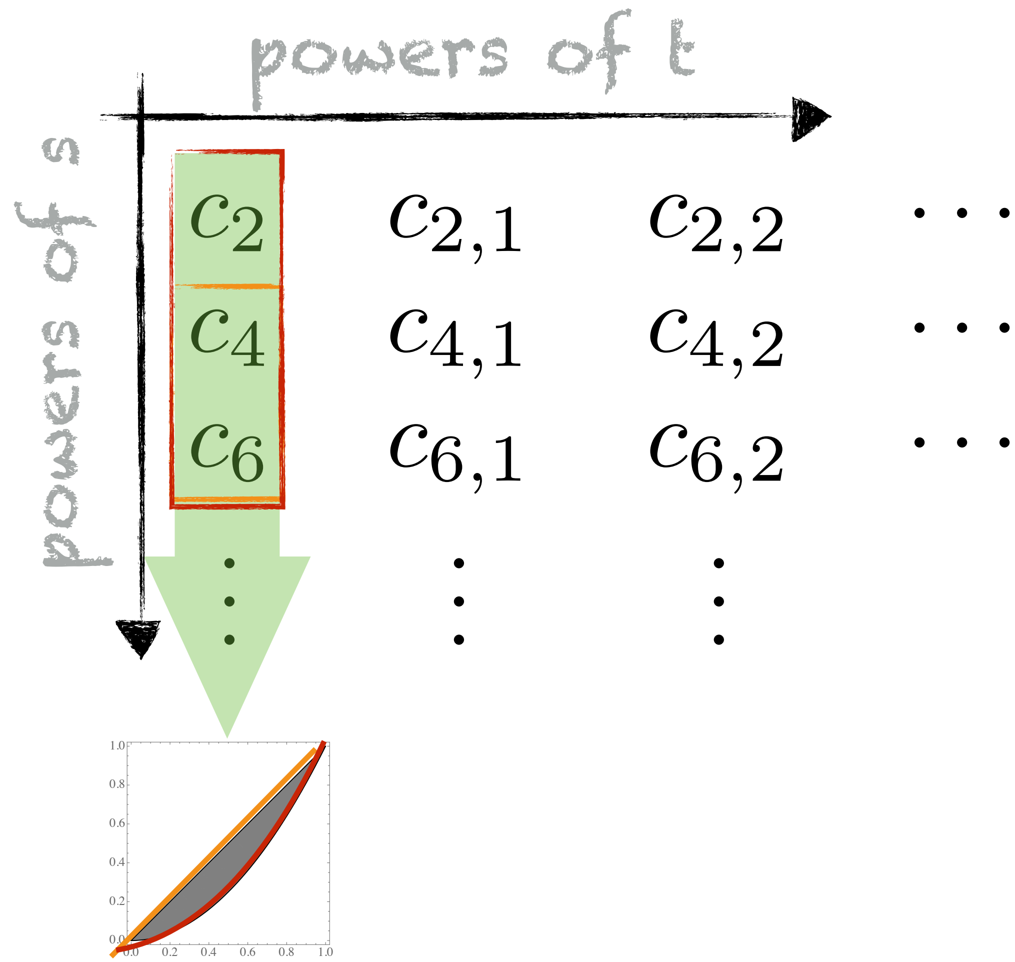
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$\vdots$ $\vdots$ $\vdots$
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0

2. Massive Higher Spin

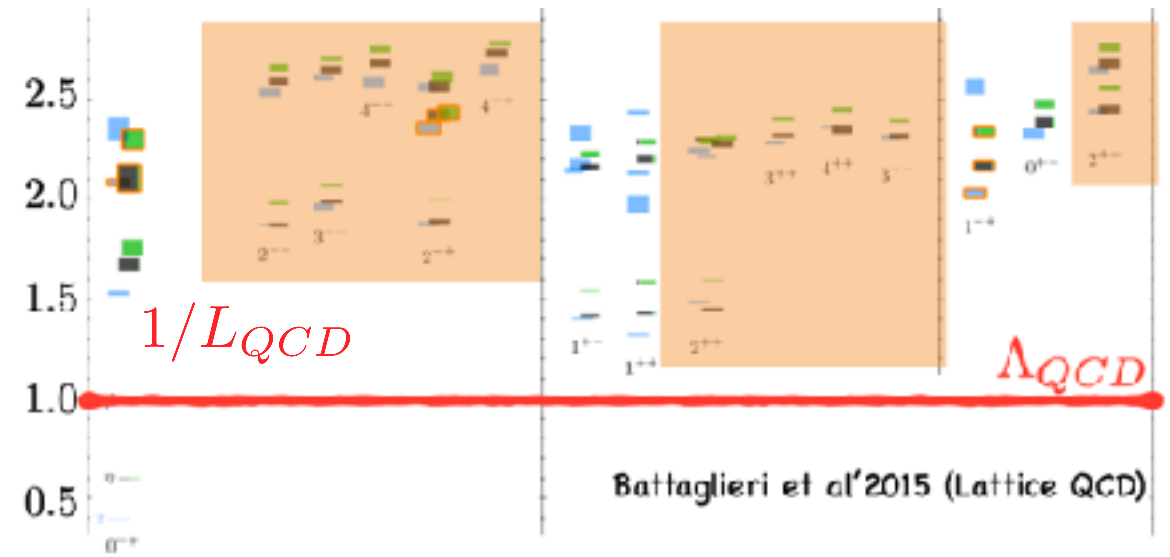
Bellazzini,Serra,Sgarlata,FR'19

$$\Phi^{\mu_1 \cdots \mu_J}$$

Higher Spin resonances exist in QCD, Nuclei/atoms, Strings,...

(J>2)

$$m_{HS} \gtrsim \frac{1}{L_{HS}}$$



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Can there be lighter HS states?

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Can there be lighter HS states?

Interactions grow with $E \lesssim \dfrac{1}{L_{HS}}$

$$A(\Phi\Phi \to \Phi\Phi) \propto \frac{s^2}{m_{HS}^4} + \cdots + \frac{s^{2J}}{m_{HS}^{4J}}$$

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$\propto \quad c_2 \qquad\qquad c_{2J}$

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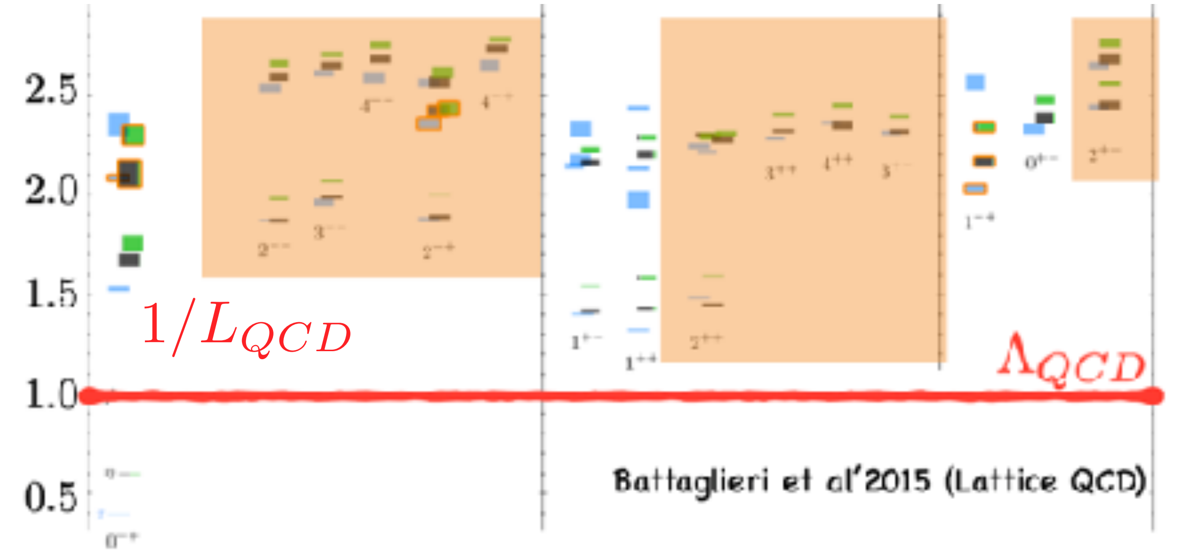

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$$m_{HS} \gtrsim \frac{1}{L_{HS}} \longleftarrow \propto \quad c_2 > \bar{s}^{2J-2} c_{2J}$$

Higher Spin always heavier than their size^{-1}

3. Finite-t supersoftness and Galileons

Beyond forward:

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

3. Finite-t supersoftness and Galileons

Beyond forward:

Galileon Nicolois,Rattazzi,Trincherini'08

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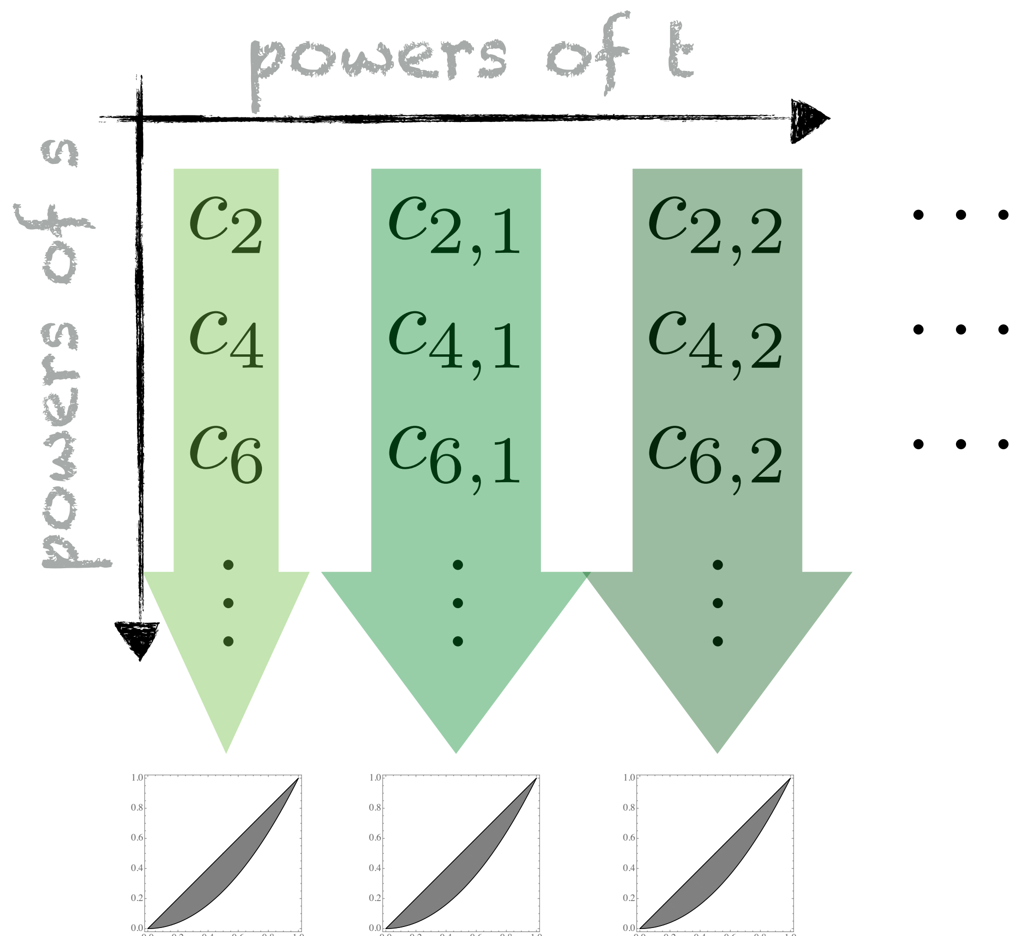
$\partial_t^q \text{Im}A|_{t=0} > 0$ Martin'65



3. Finite-t supersoftness and Galileons

Beyond forward:

Galileon Nicolois,Rattazzi,Trincherini'08

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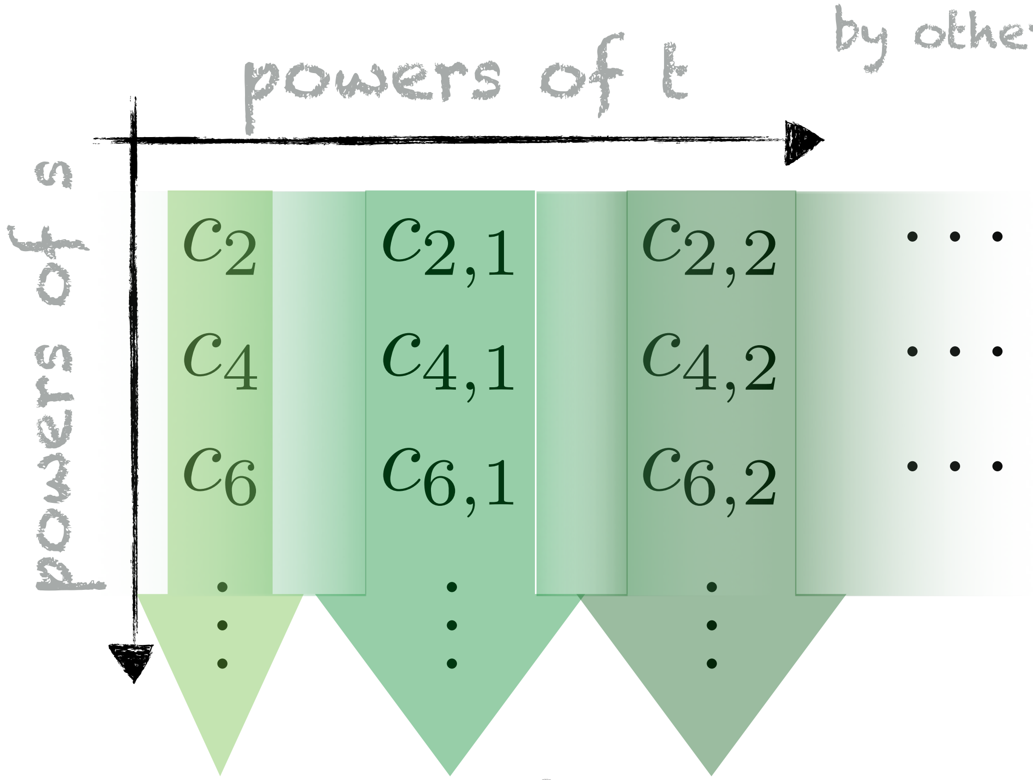
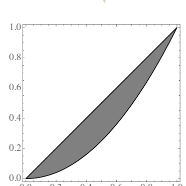
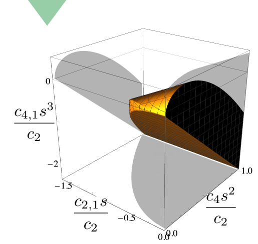
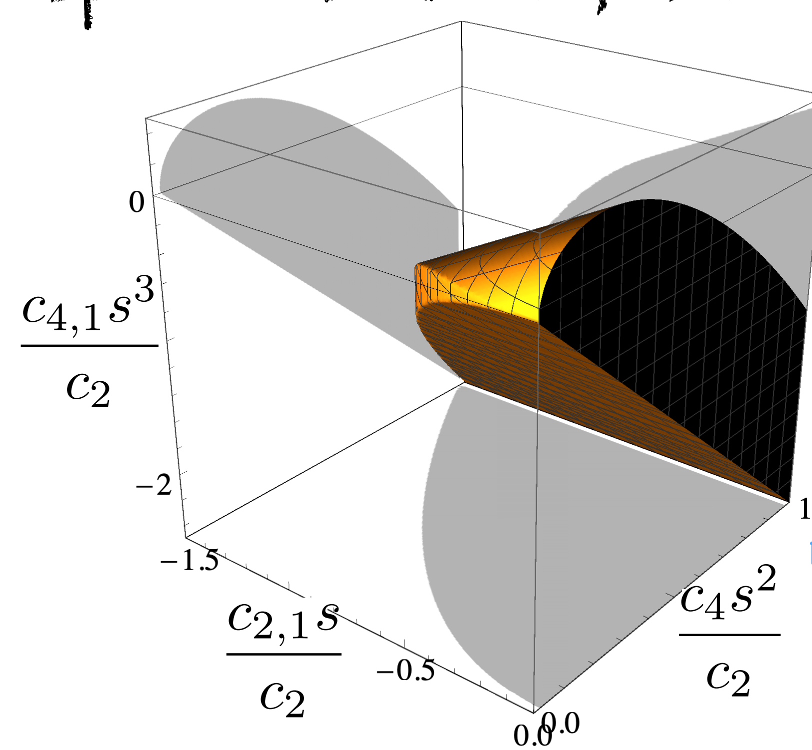
Negative, but limited by other moments







Optimal bounds for single t-derivative:



Bellazzini,Elias-Miro, Rattazzi,Riembau,FR'20

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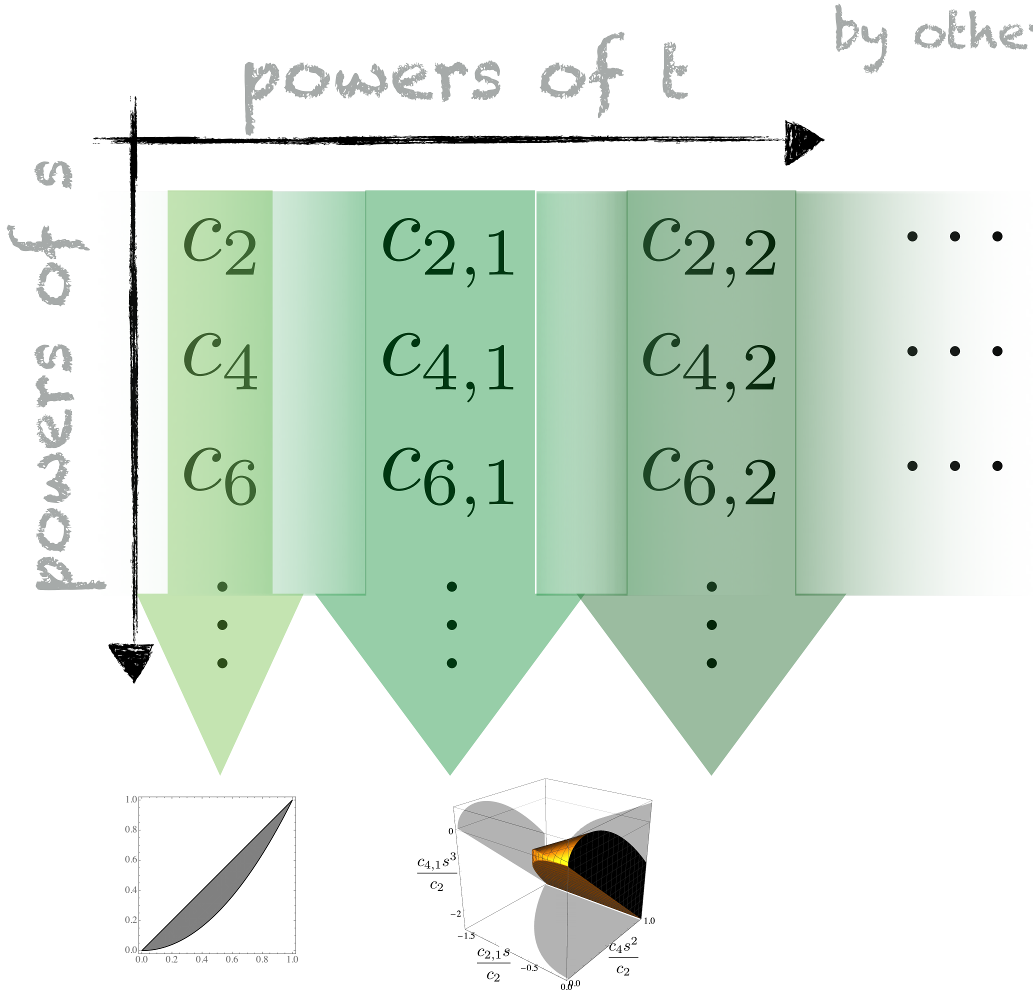

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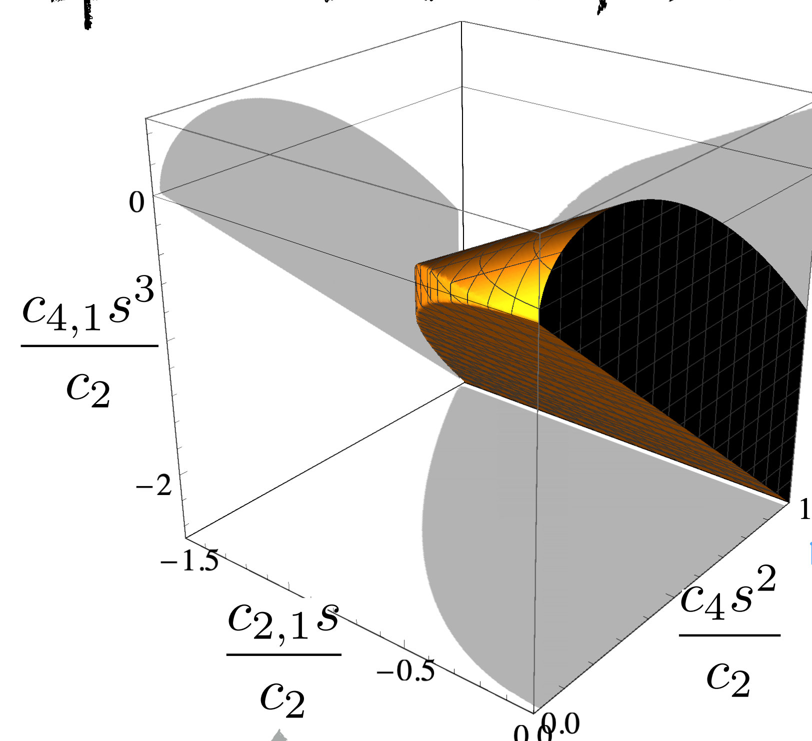

Bellazzini,Elias-Miro, Rattazzi,Riembau,FR'20

Can be slightly negative $c_{2,1} > -\frac{3}{2}\sqrt{c_4 c_2}$

also deRham,Melville,Tolley,Zhou'17 : $> -3c_2/2s$

3. Finite-t supersoftness and Galileons

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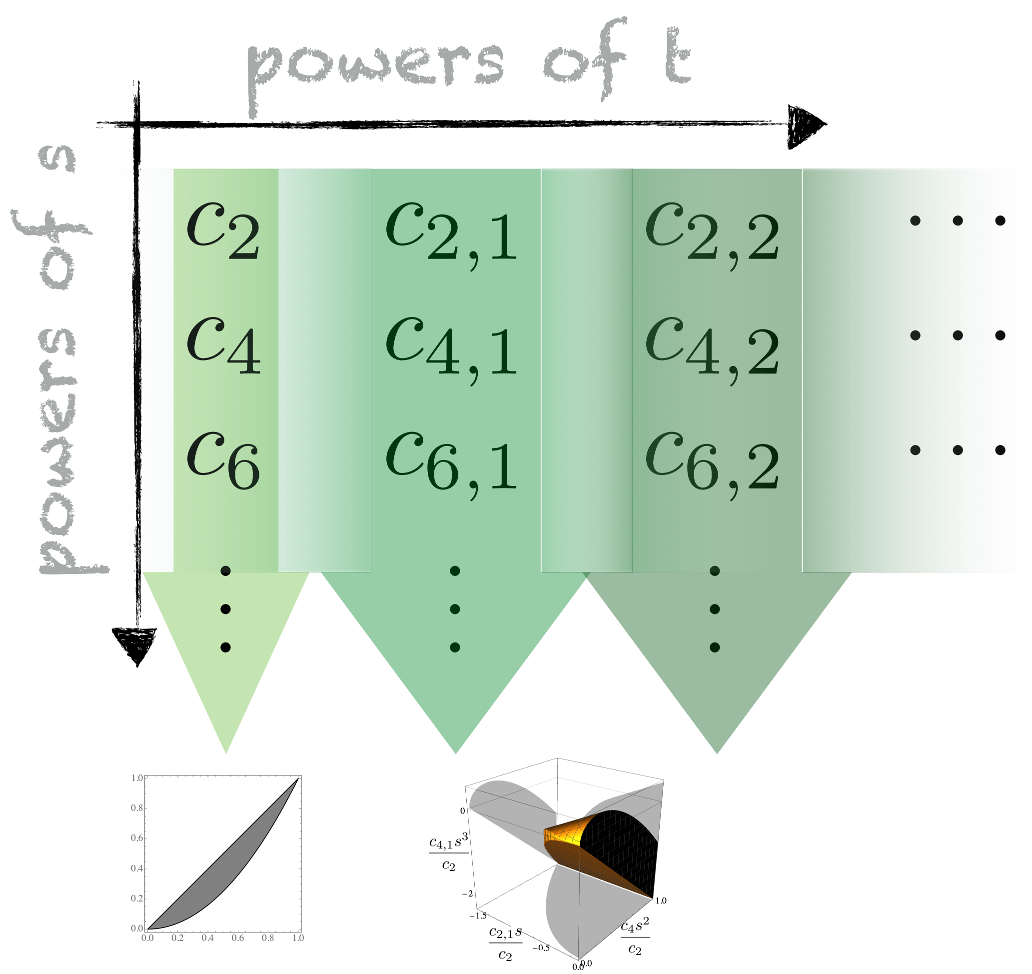

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Arkani-Hamed,Huang[2],2020

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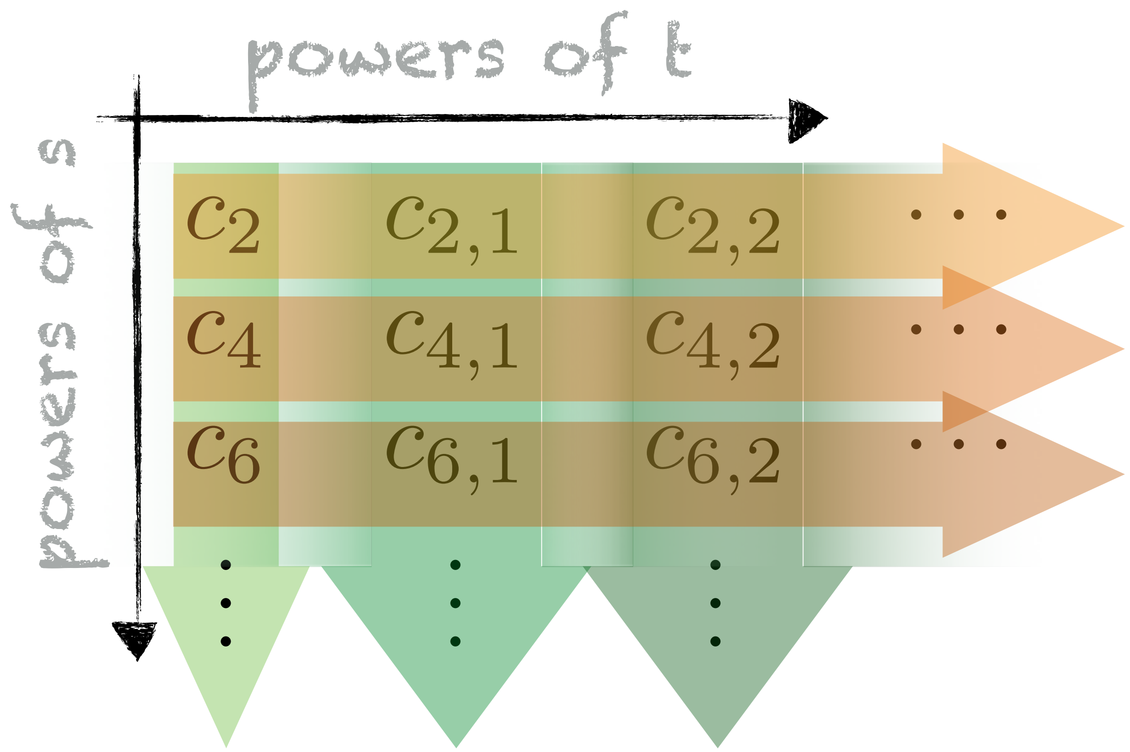

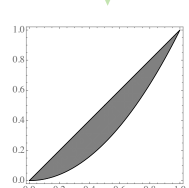

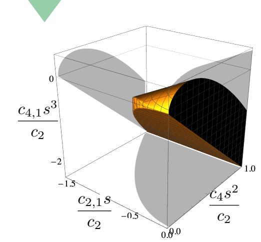

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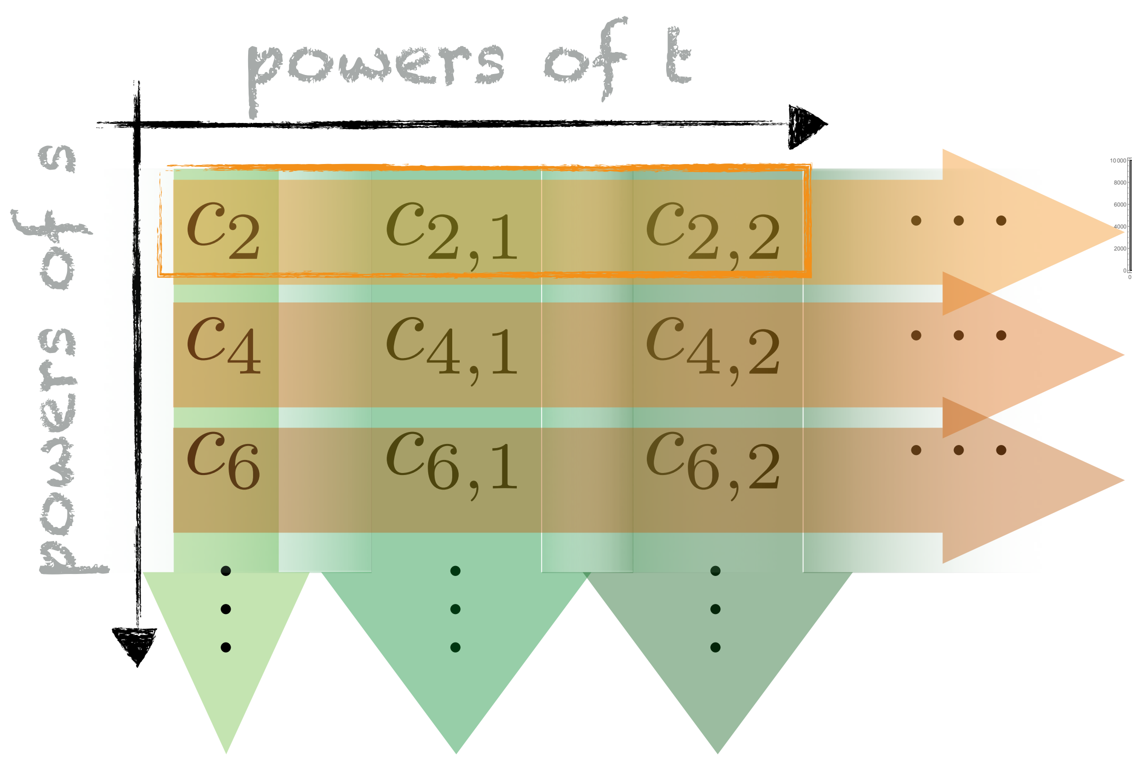
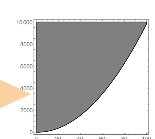
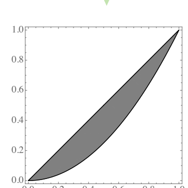
$$\sim \int_0^\infty d\mu(l) l^{2q}$$

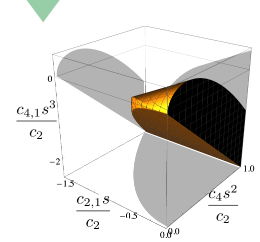
Moments in l

Bellazzini et al, to appear









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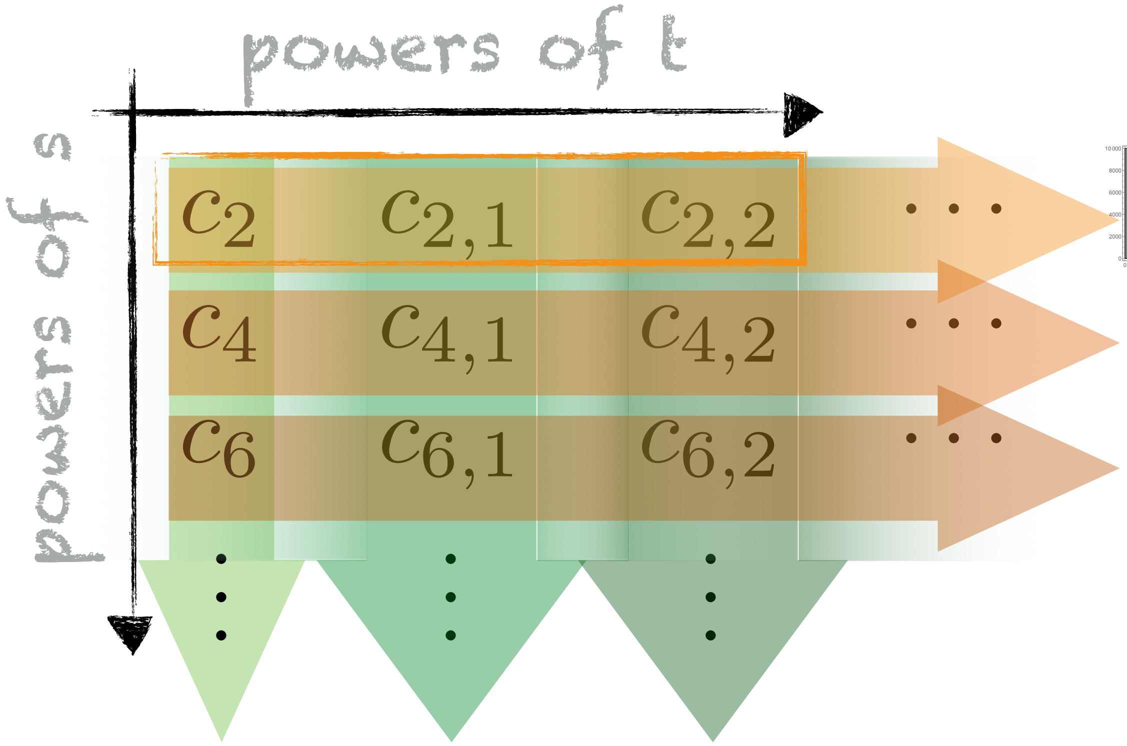

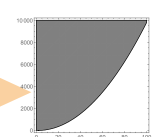

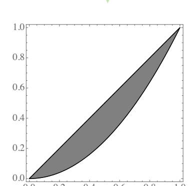

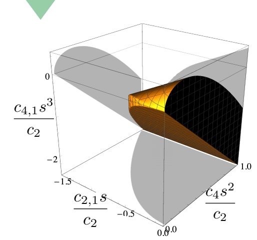

At tree-level* x-ing symmetry implies

$$A(s,t) = \cdots + g_4 (s^2 + t^2 + u^2)^2 + \cdots$$

$$c_{2,2} = 2c_4$$

3. Finite-t supersoftness and Galileons

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Moments in l

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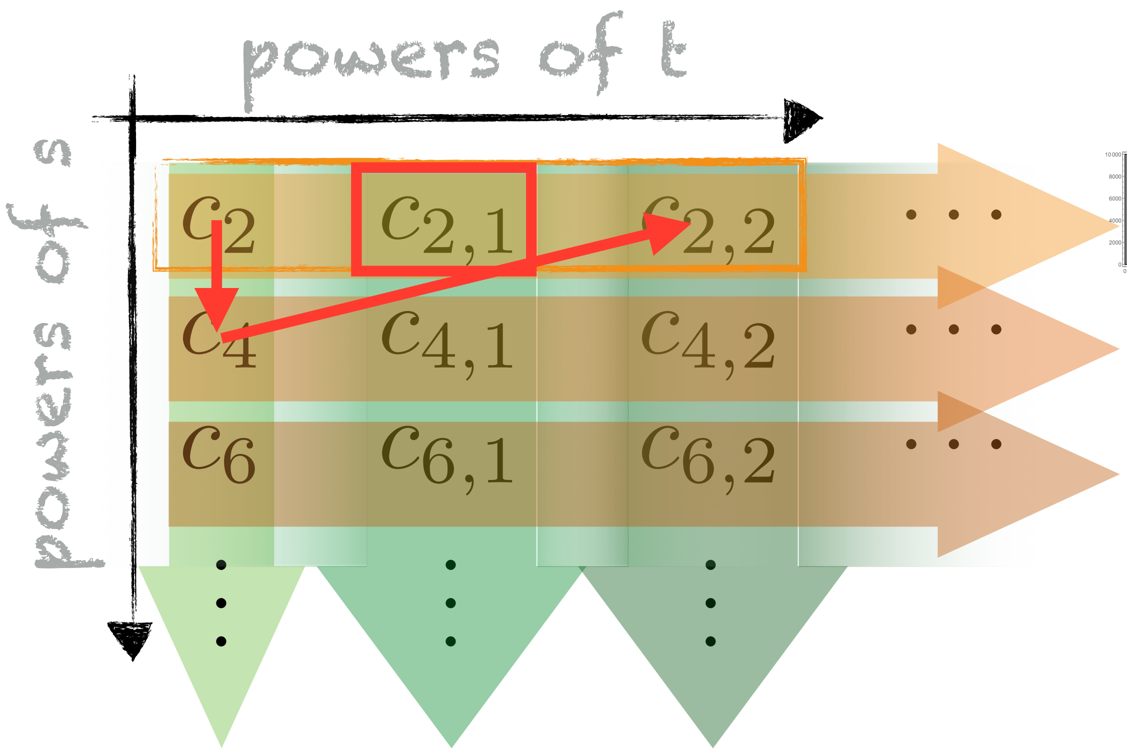

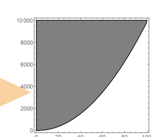

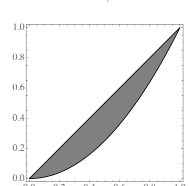

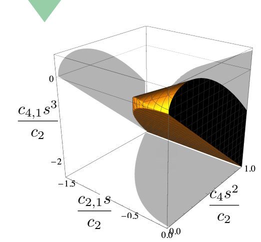

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➤ $$-\frac{3}{2}c_2 < c_{2,1}s < 8c_2$$

Tolley,Wang,Zhou'20
Caron-Huot,vanDuong'20

Galileons tightly bounded!

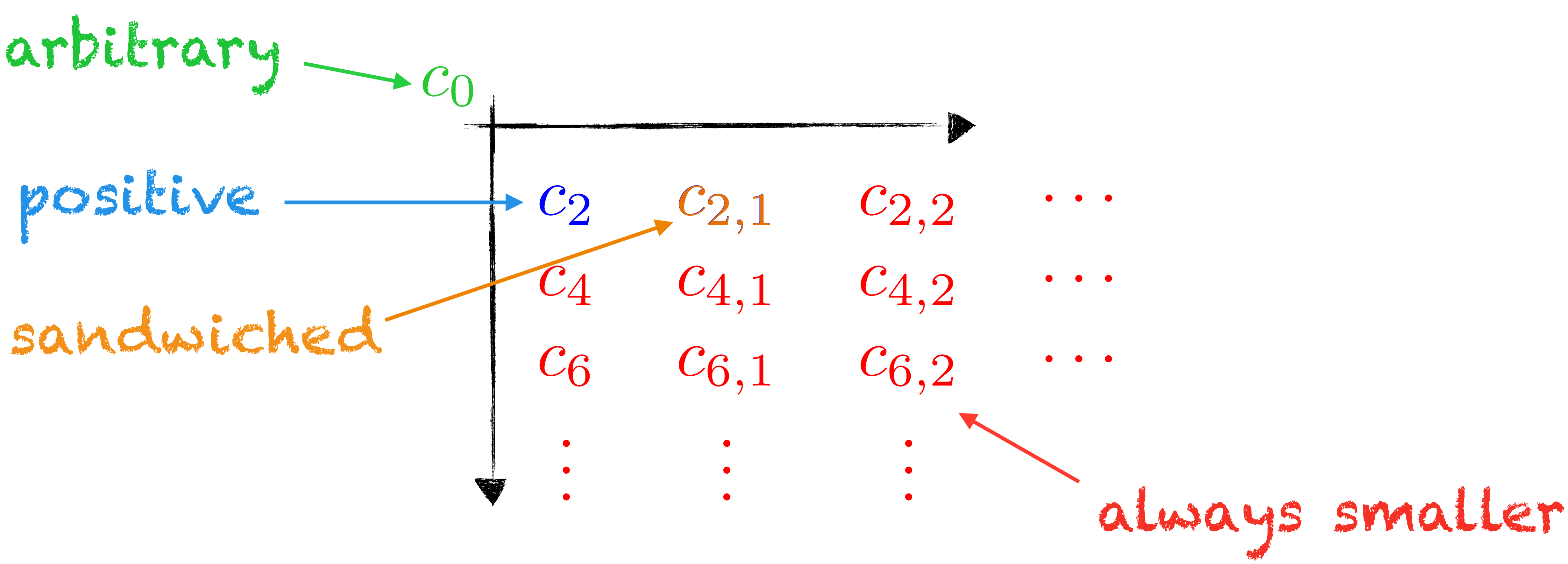
* See loop effects later

Super-Softness

Tree-level, beyond forward:

$$A(s,t) = \sum_{p,q} c_{p,q} s^p t^q = \ c_0 \ + \ c_2 s^2 \ + \ c_{2,1} s^2 t \ + \cdots$$

Of ∞ many coefficients, only 3 can lead the amplitude:



4. Beyond Tree

(for simplicity: only for Goldstone Boson)

IR Effects alter Bounds

Change relation Wilson coeff. ⟷ arcs (on which bounds apply)

Running

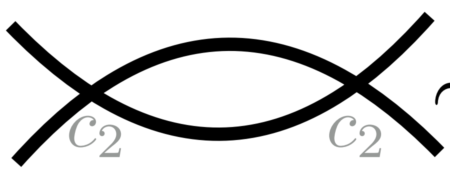
Collinear Divergences

IR Effects alter Bounds

Change relation Wilson coeff. ⟷ arcs (on which bounds apply)

Running

Collinear Divergences



$$\sim \frac{c_2^2 s^4}{16\pi^2} \log \frac{s}{\mu}$$

IR Effects alter Bounds

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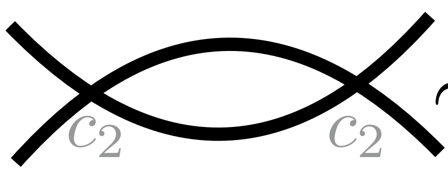
Running



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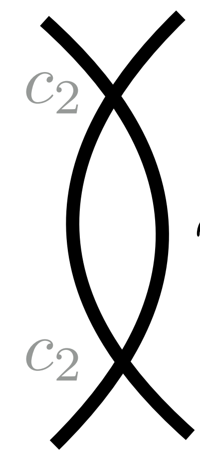
Collinear Divergences



$$\sim \frac{c_2^2 s^2 t^2}{16\pi^2} \log \frac{s}{t}$$

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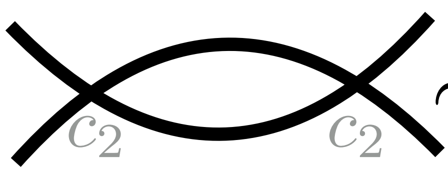

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$$\delta \mathcal{A}_n \sim \frac{c_2^2}{16\pi^2 s^{2n-2}}$$

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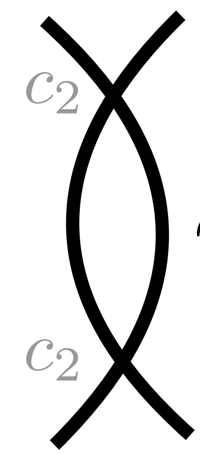

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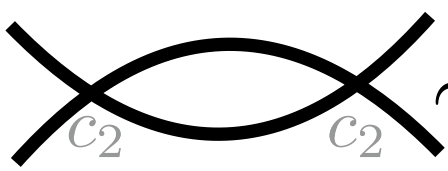

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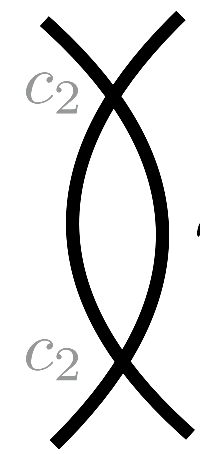

$$\sim \frac{c_2^2 s^2 t^2}{16\pi^2} \log \frac{s}{t}$$

k powers of t →

n powers of s ↓

c_2	$c_{2,1}$	$c_{2,2}$	$\cdots$
c_4	$c_{4,1}$	$c_{4,2}$	$\cdots$
c_6	$c_{6,1}$	$c_{6,2}$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	

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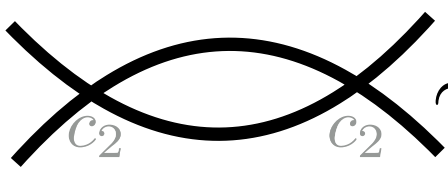

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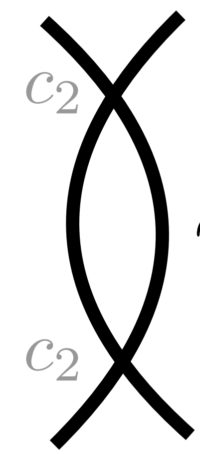

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$$\delta \partial_t^k \mathcal{A}_n \sim \left(\frac{c_2^2}{16\pi^2} \log \frac{s}{m^2} \right)^{k-2}$$

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c_2	$c_{2,1}$	$c_{2,2}$	$\cdots$
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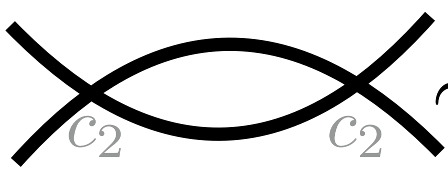

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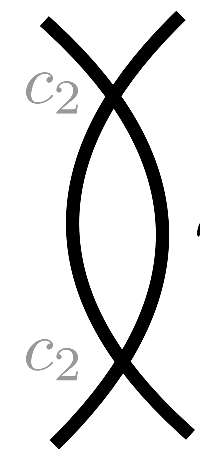

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$\vdots$	$\vdots$	$\vdots$	

Running

EFT Wilson coefficients run: Do bounds apply for $c_n(s)$?

$$A(s) = c_2 s^2 + s^4 \underbrace{\left[c_4 + \beta_4 \log(-is)\right]}_{c_4(s)} - i\pi s^5 \beta_5/2 + s^6 \underbrace{\left[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)\right]}_{c_6(s)} + \cdots$$

s

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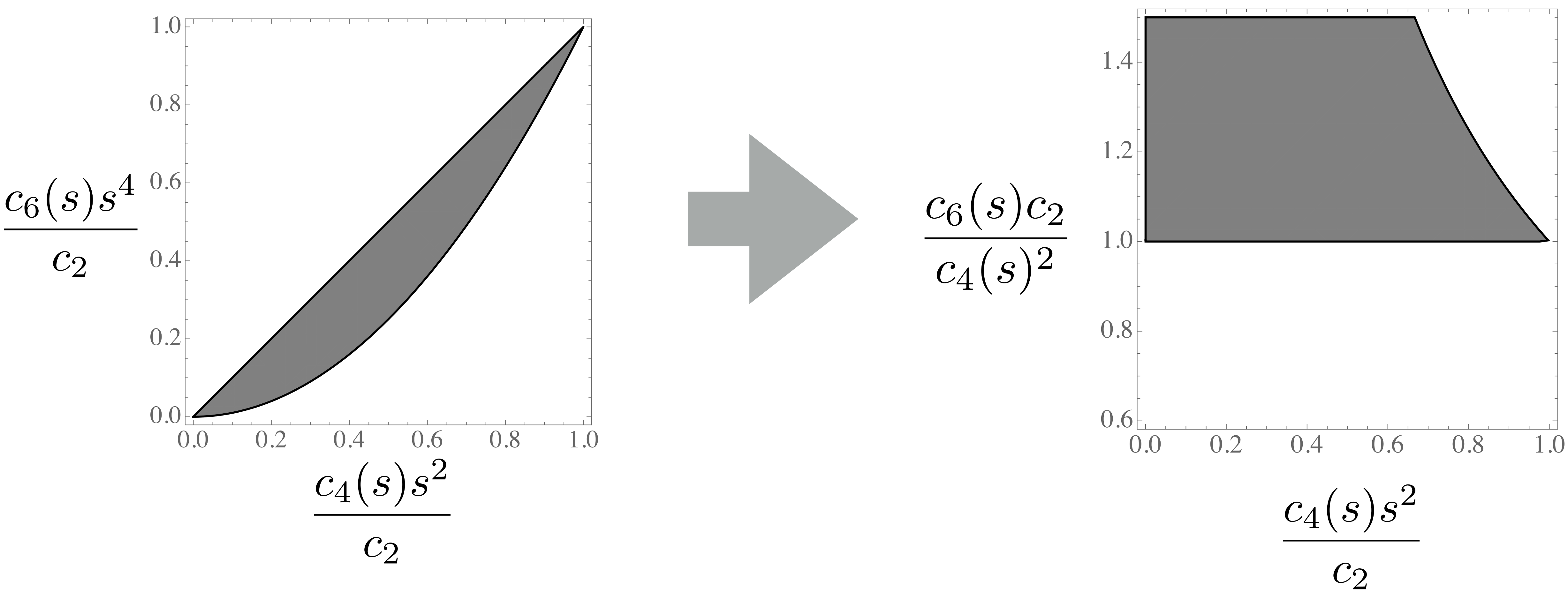

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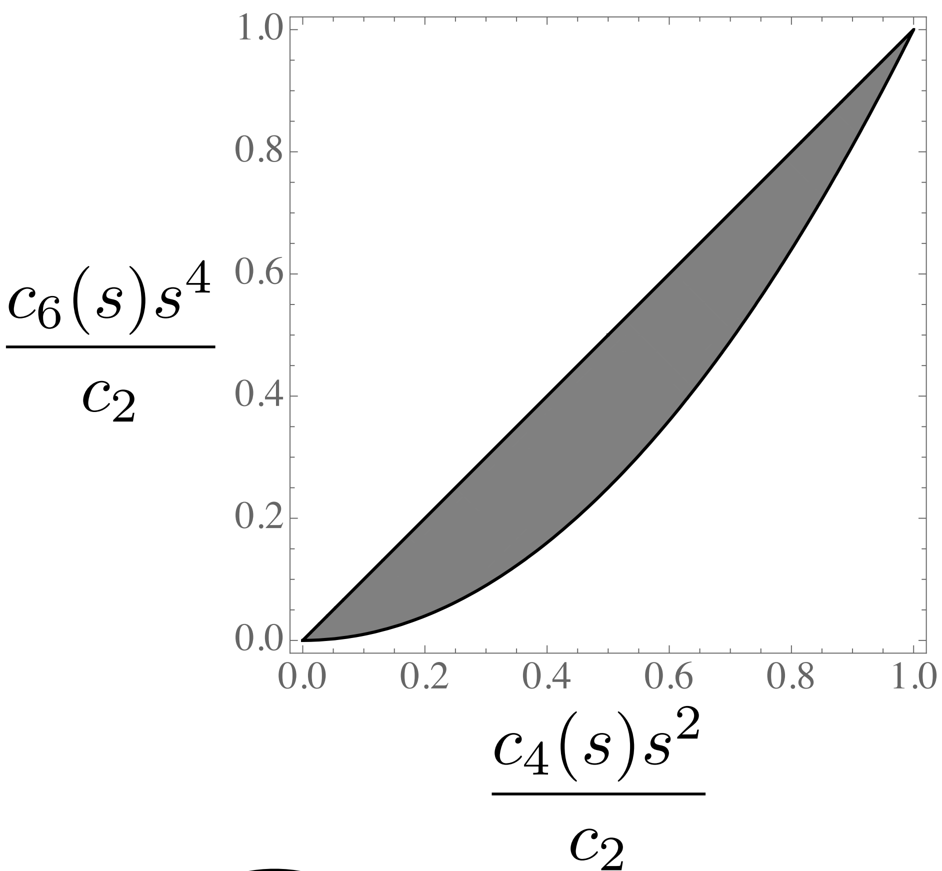

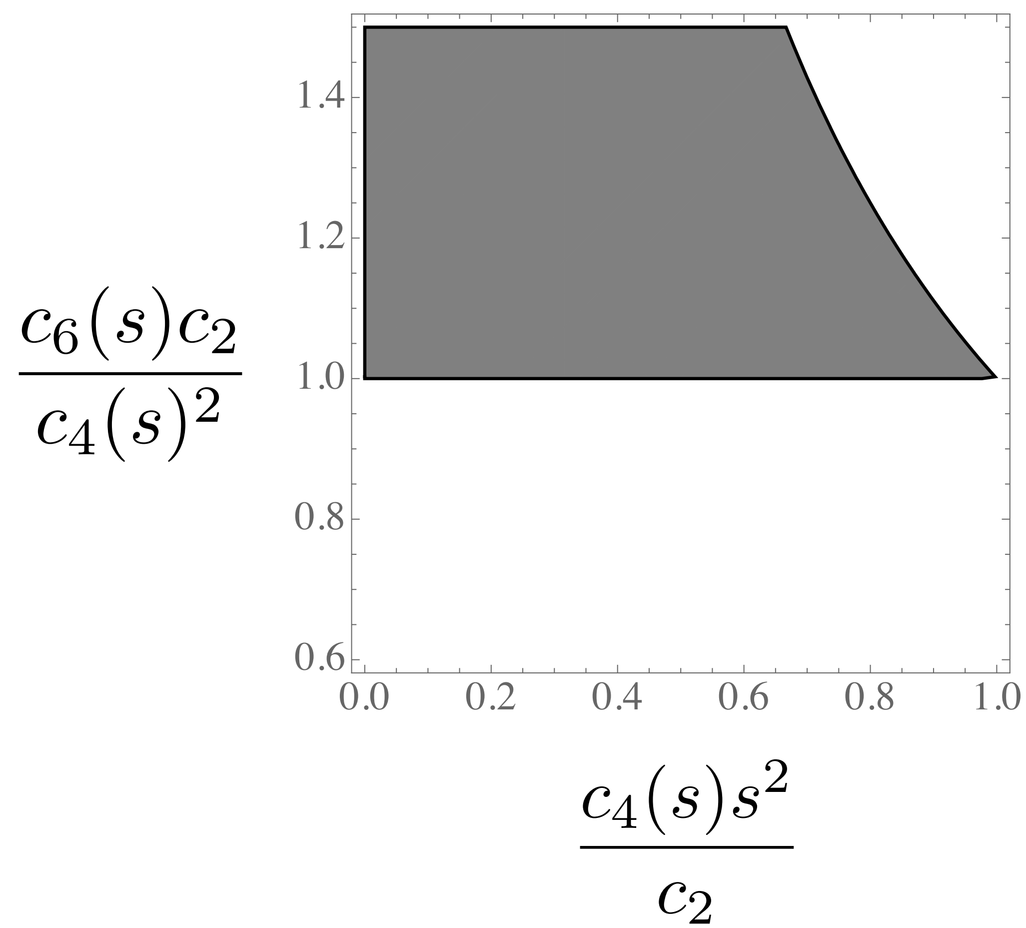

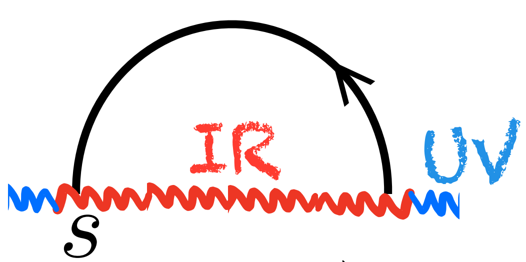

Arcs: suitable to access running coefficients

Running

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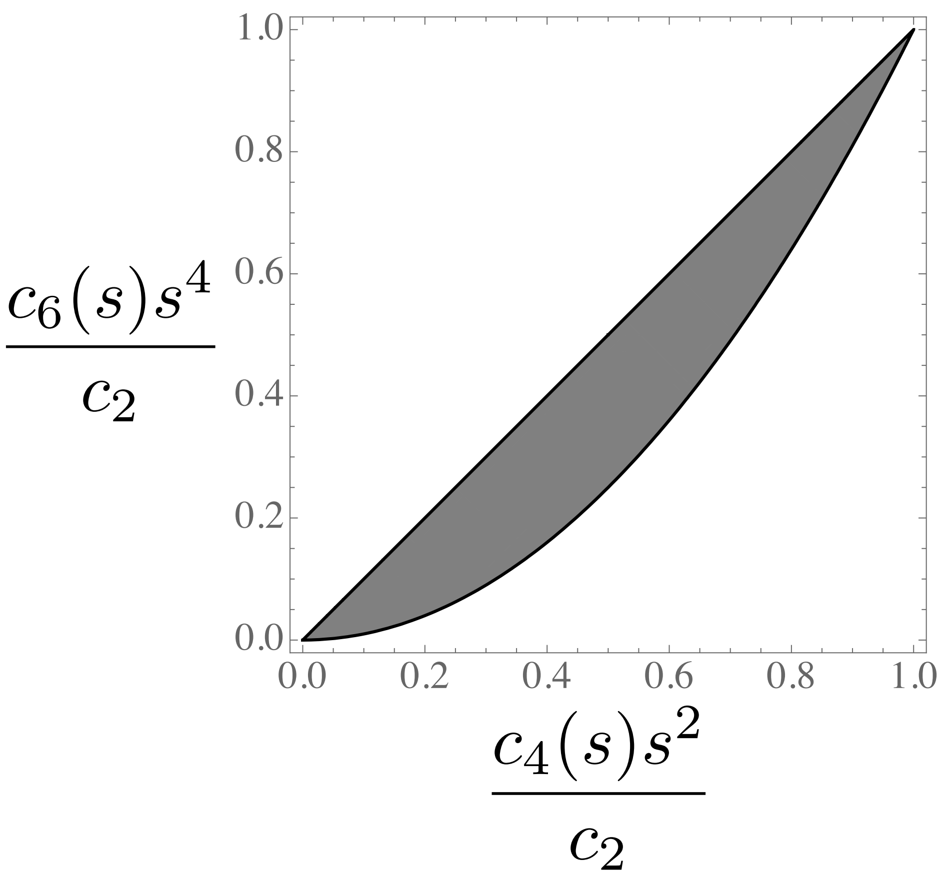

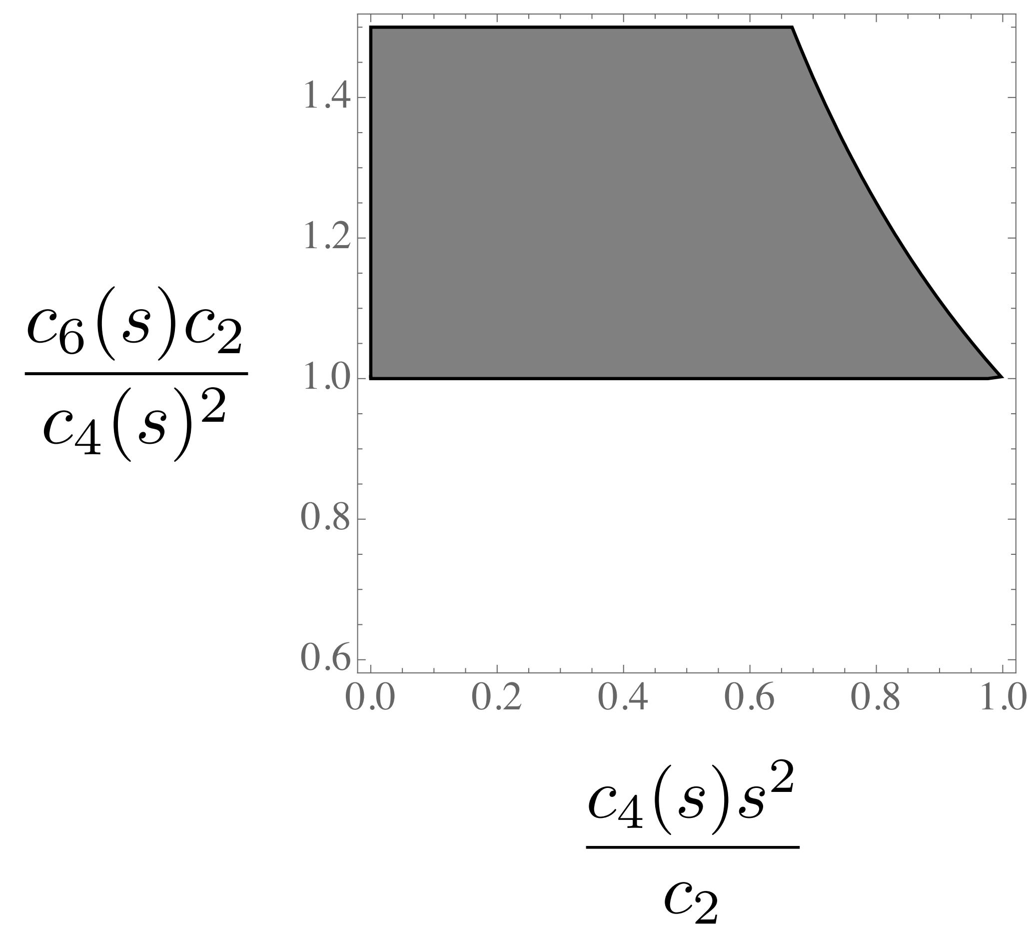

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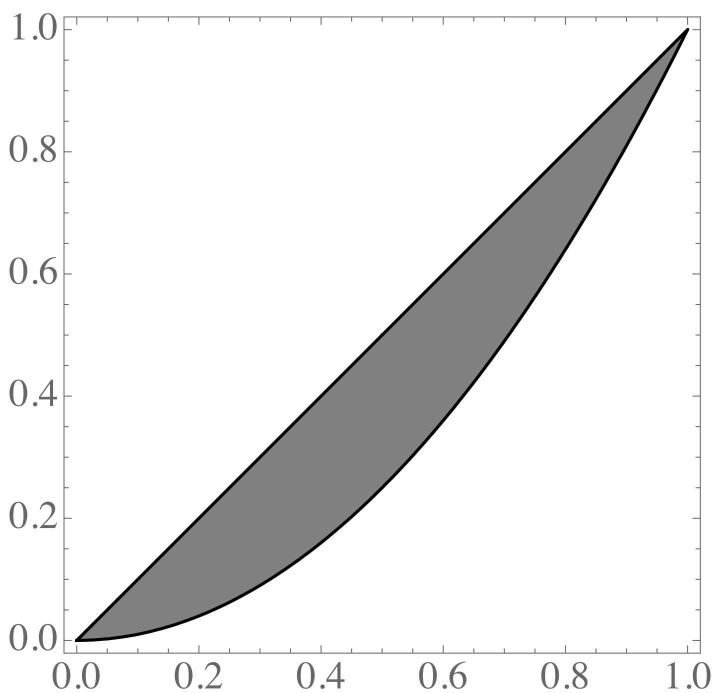

$\dfrac{c_4(s)s^2}{c_2}$

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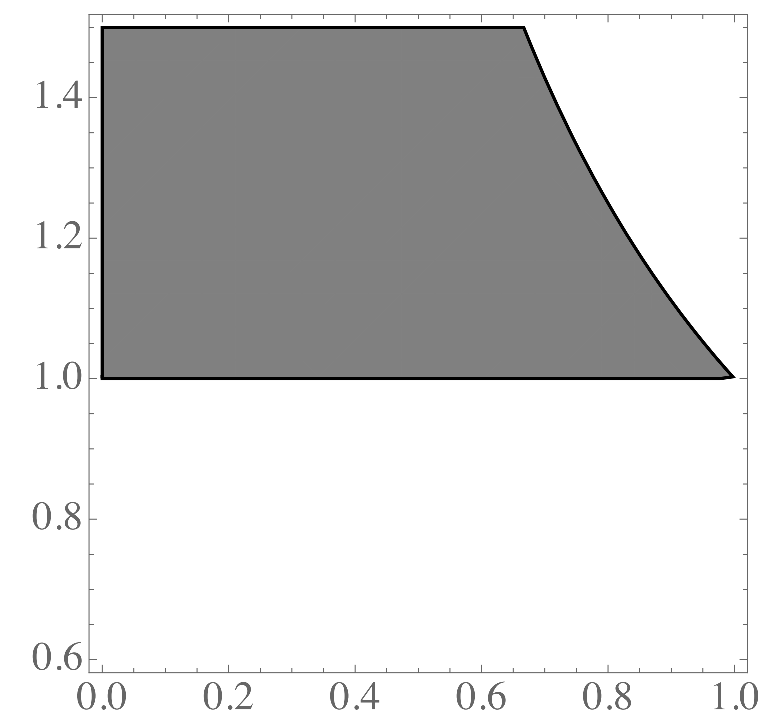

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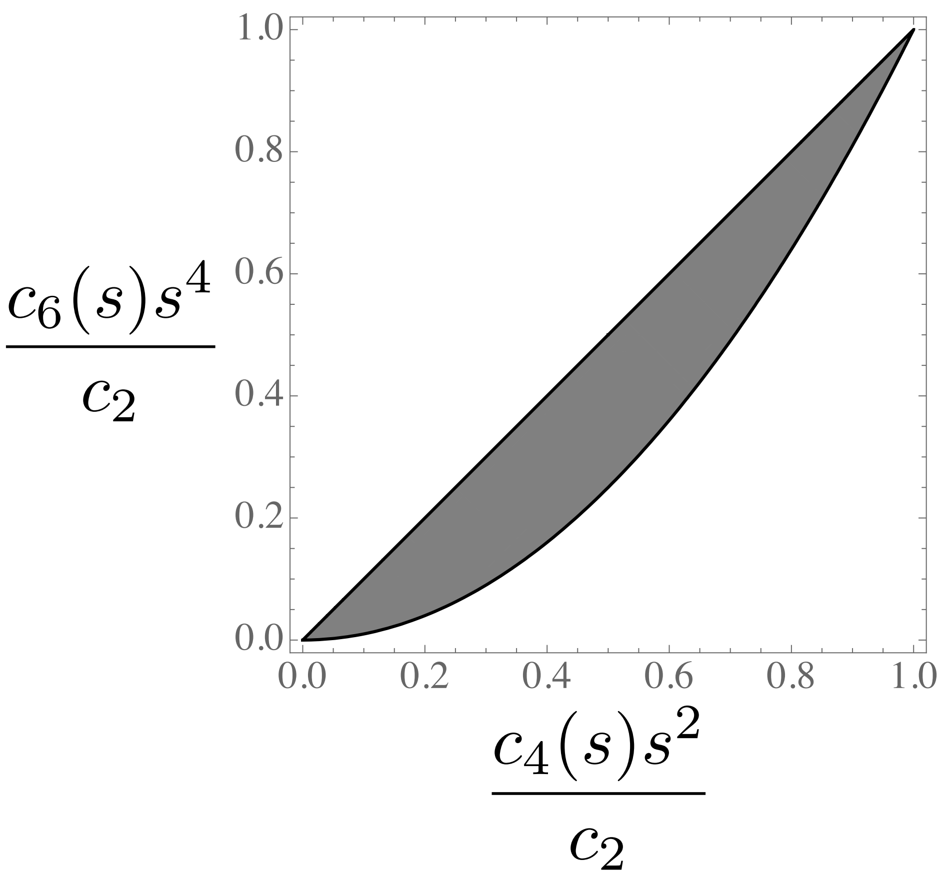

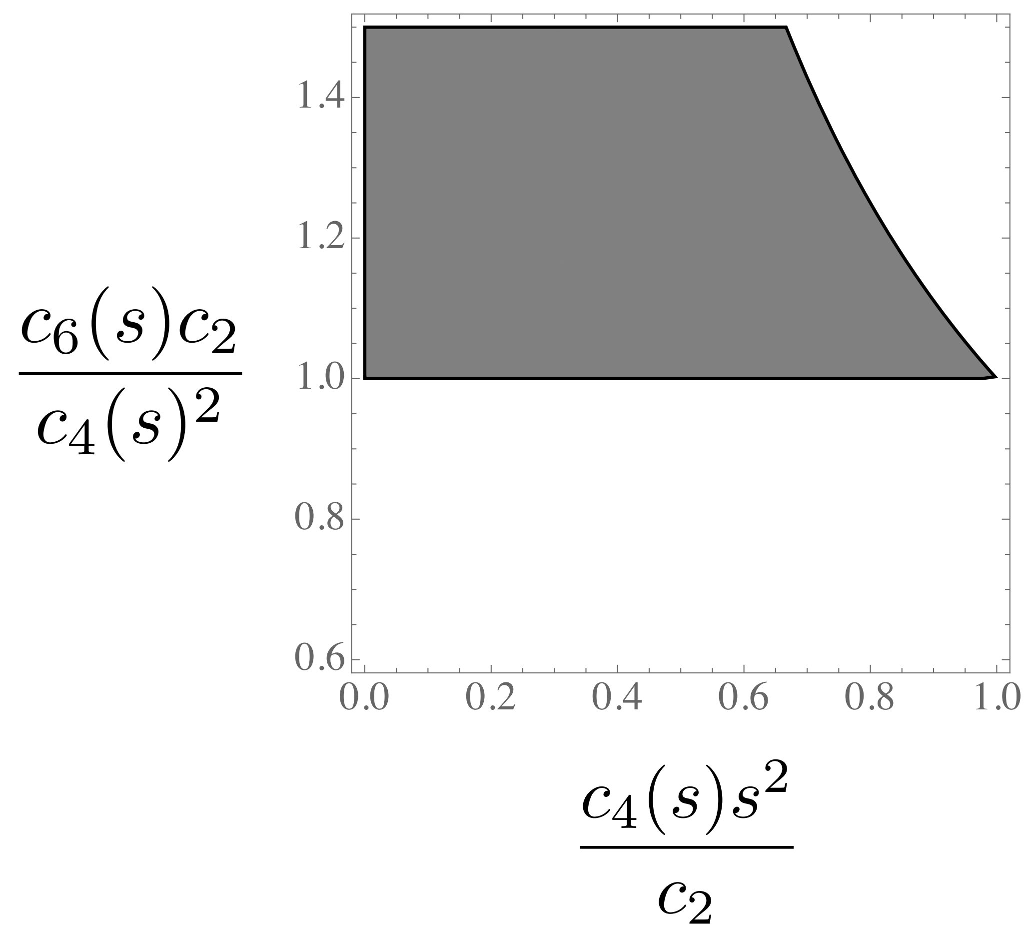

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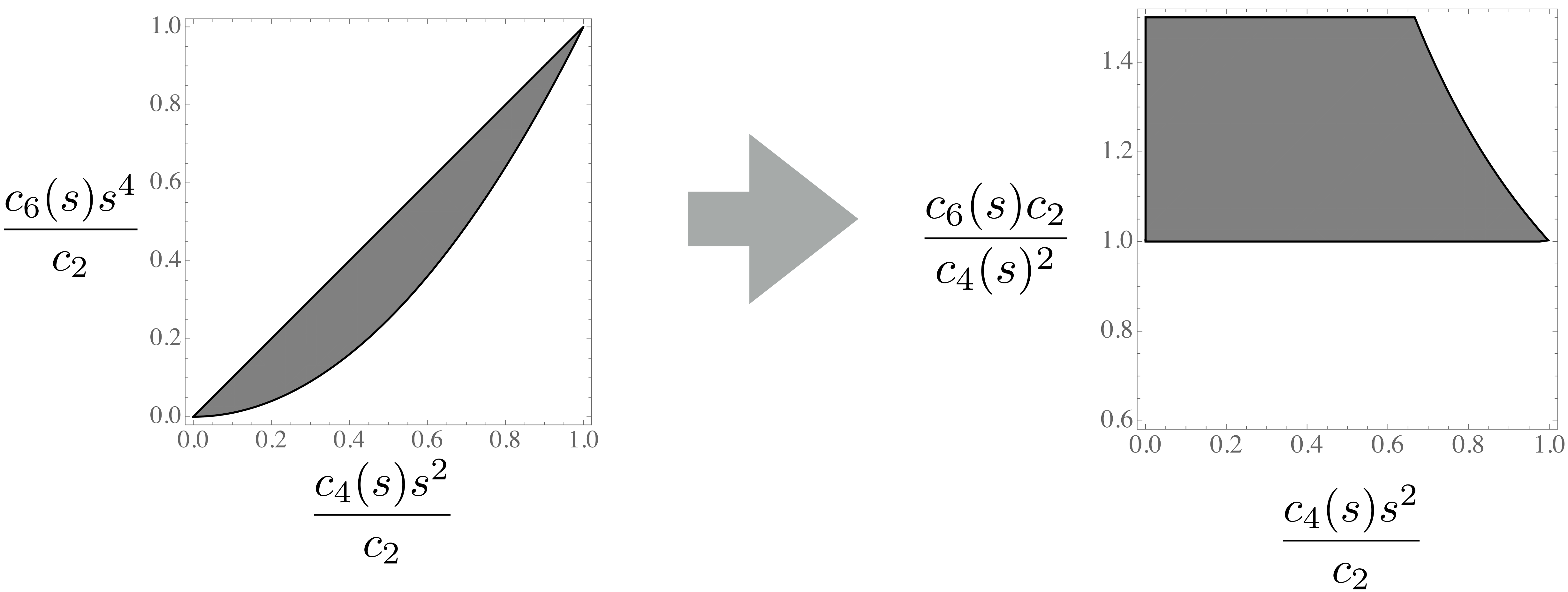

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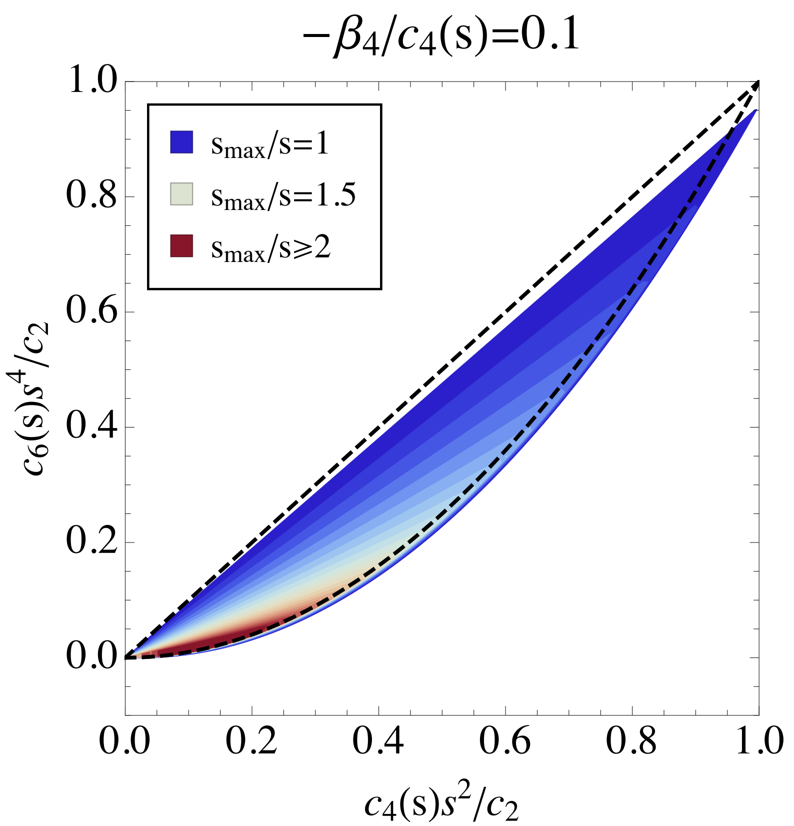

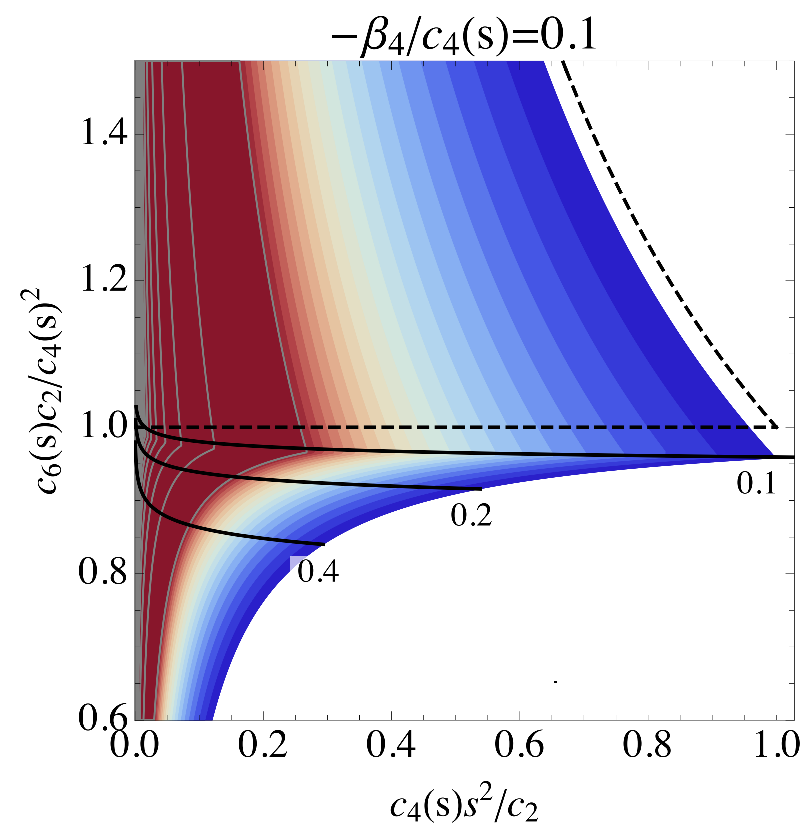

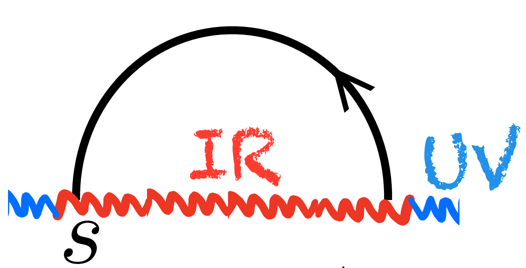

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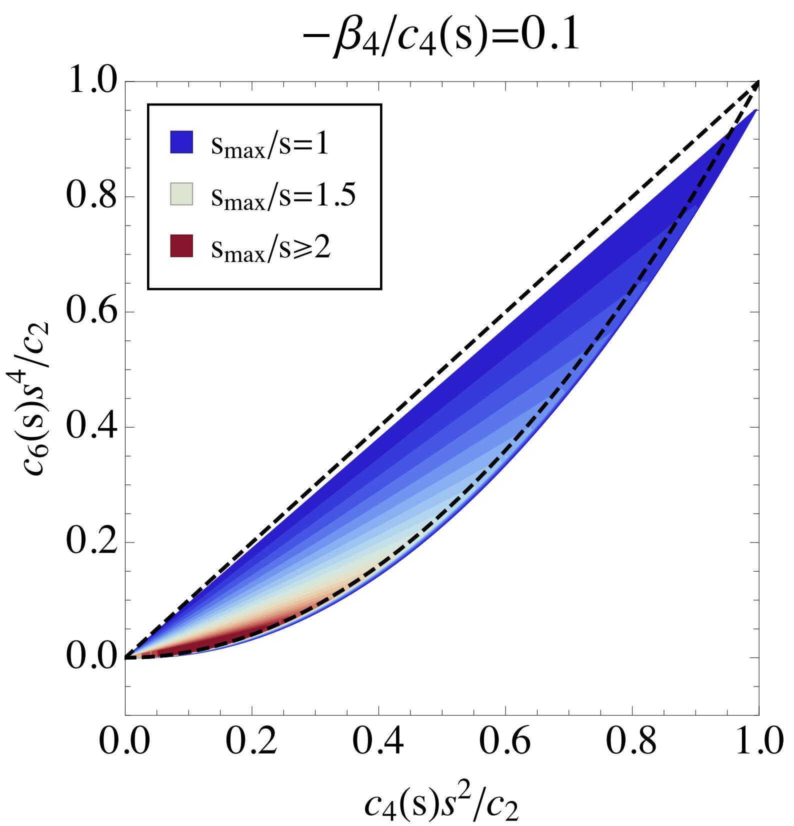

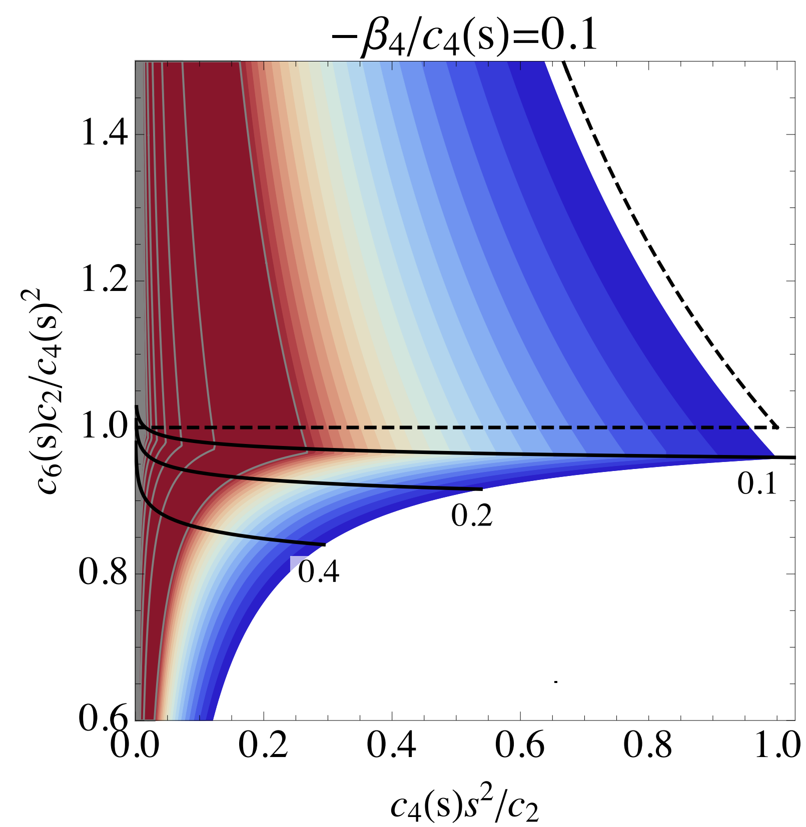

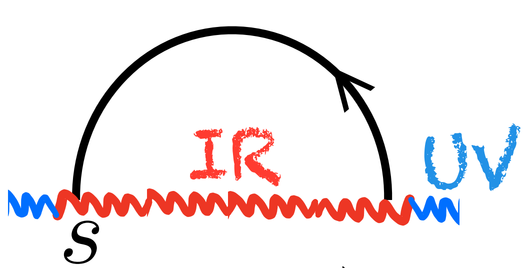

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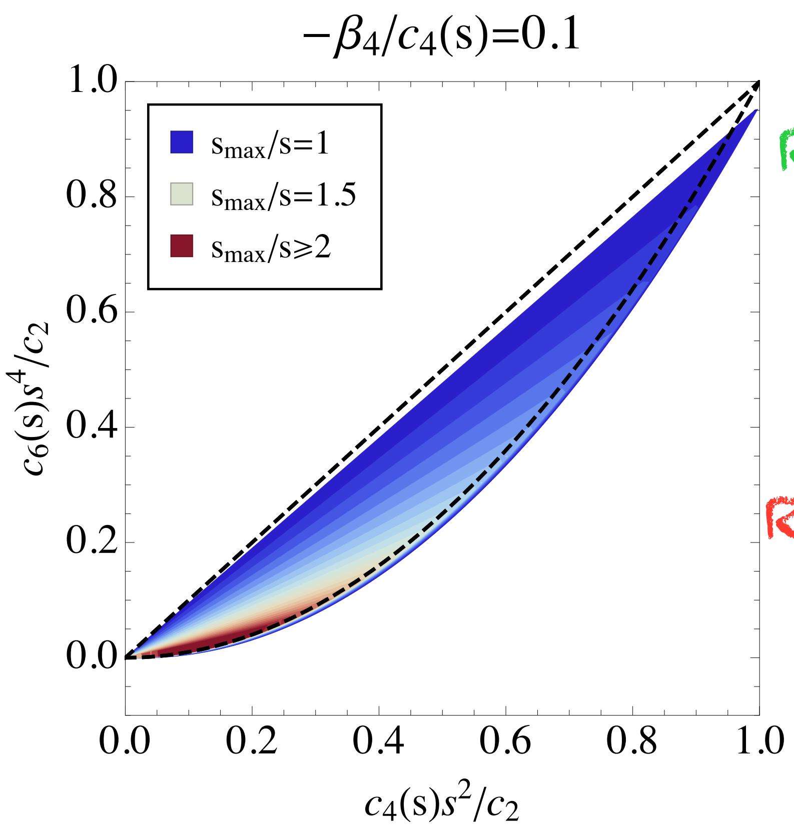

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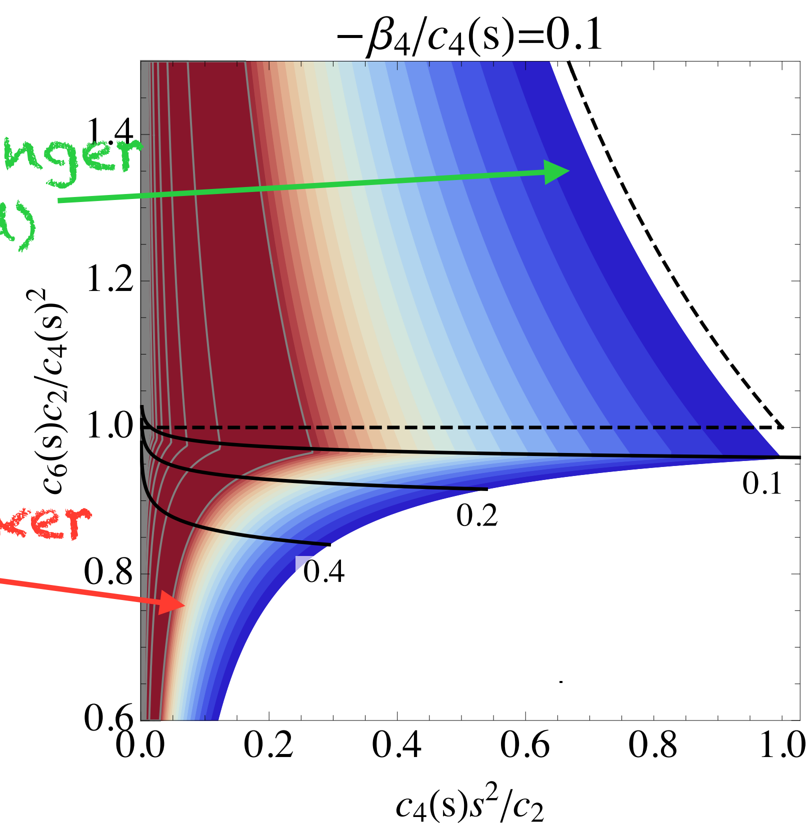

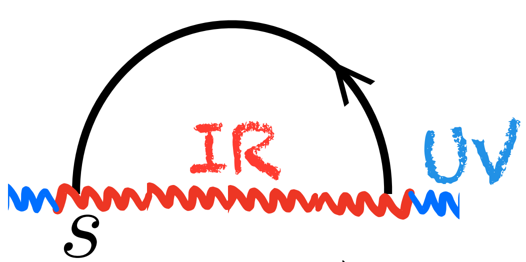

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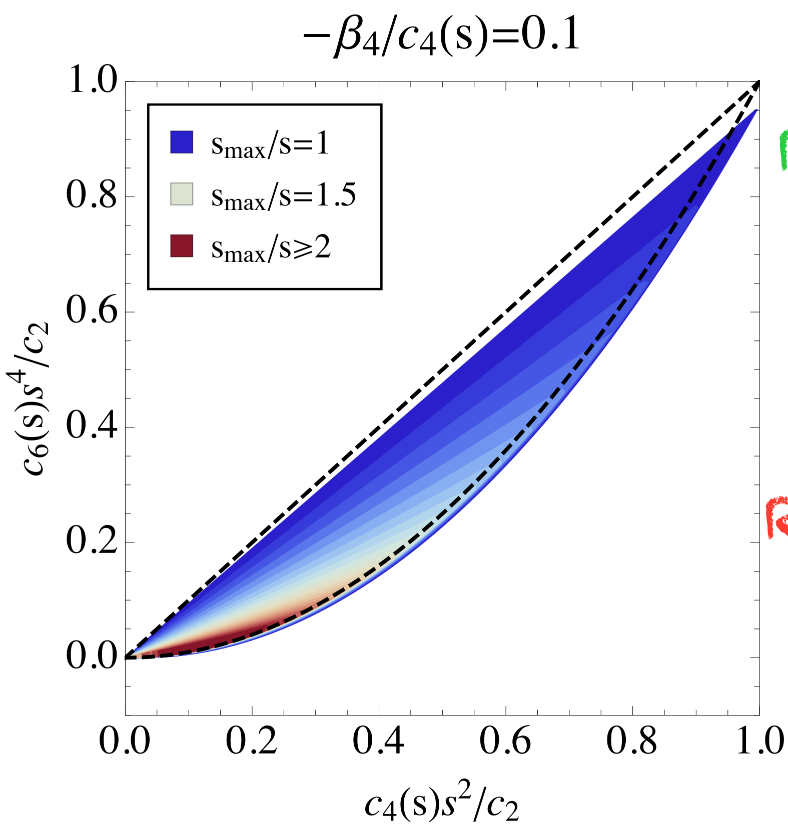

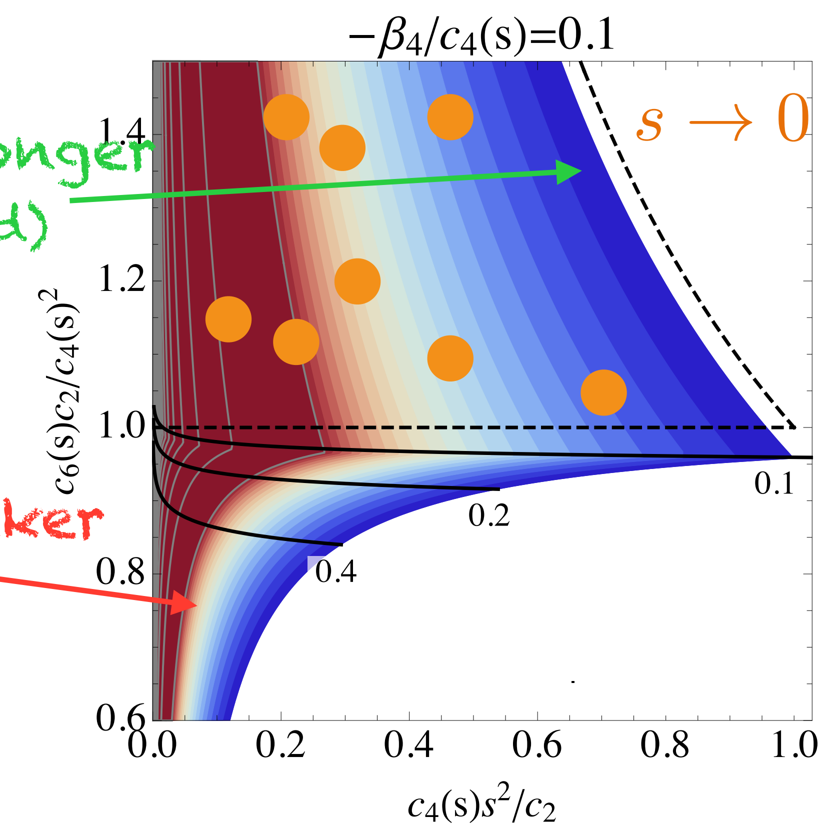

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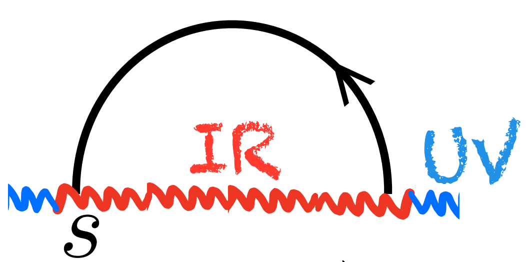

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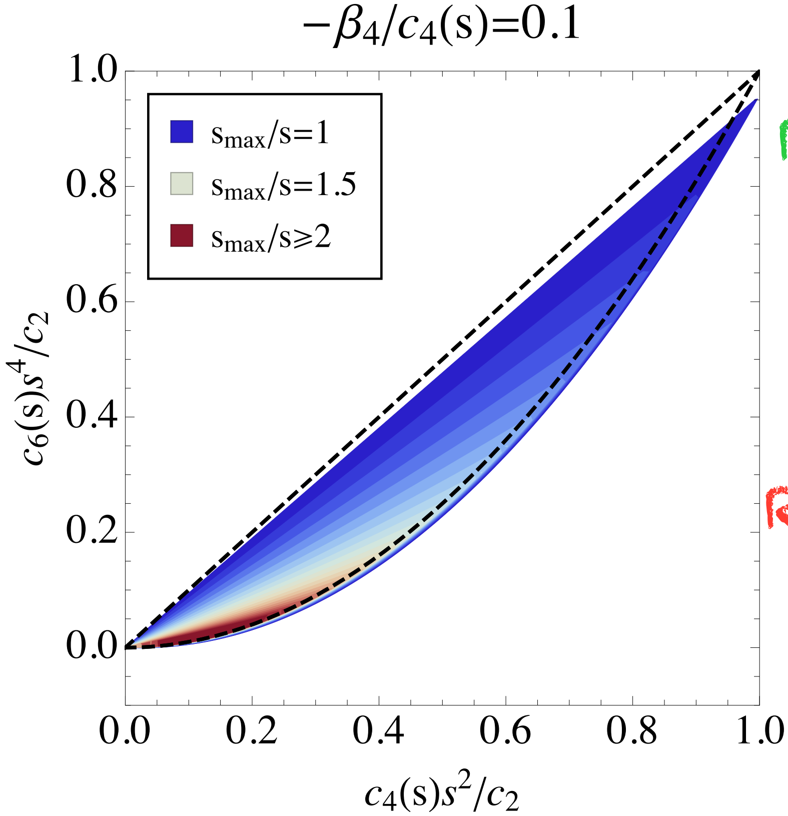

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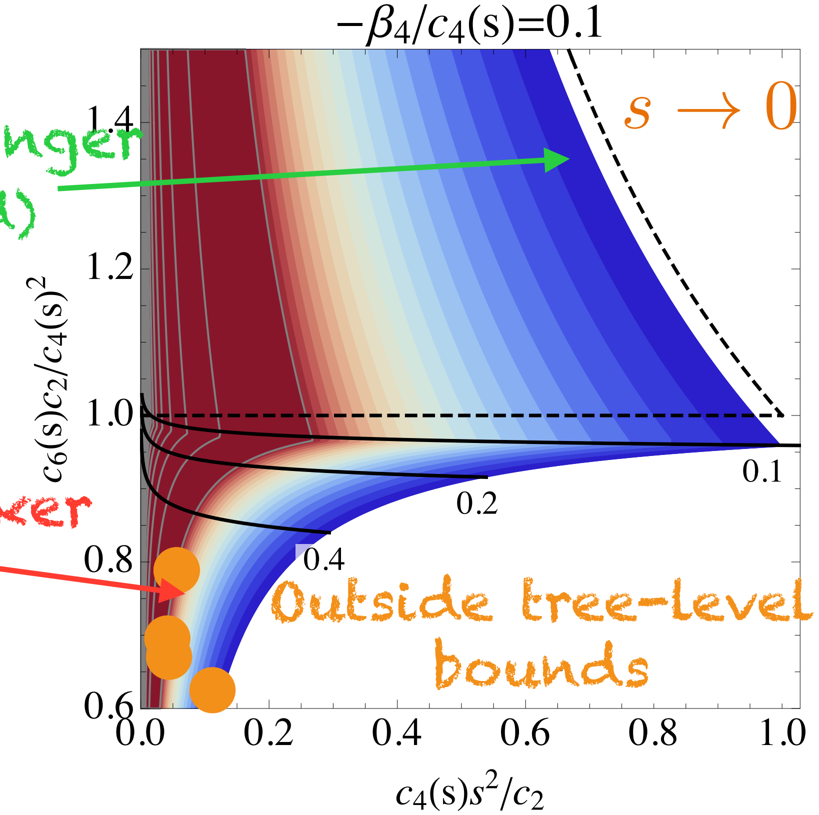

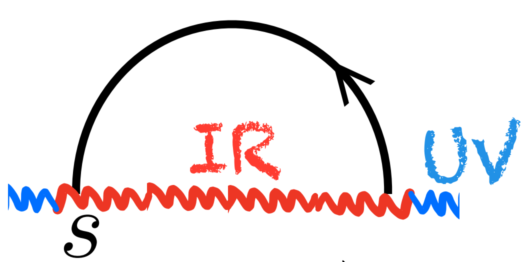

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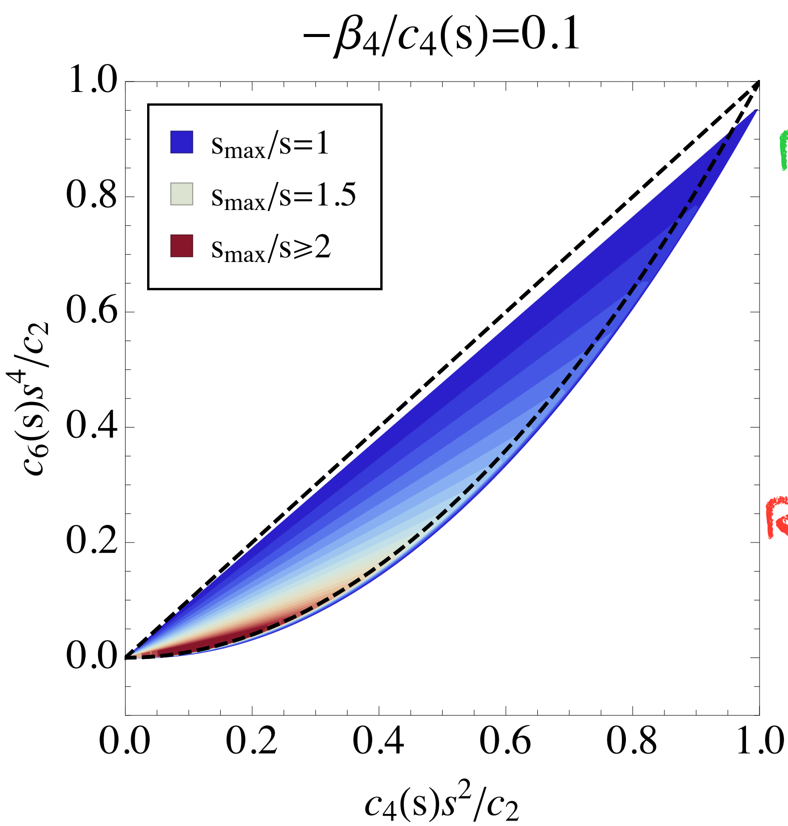

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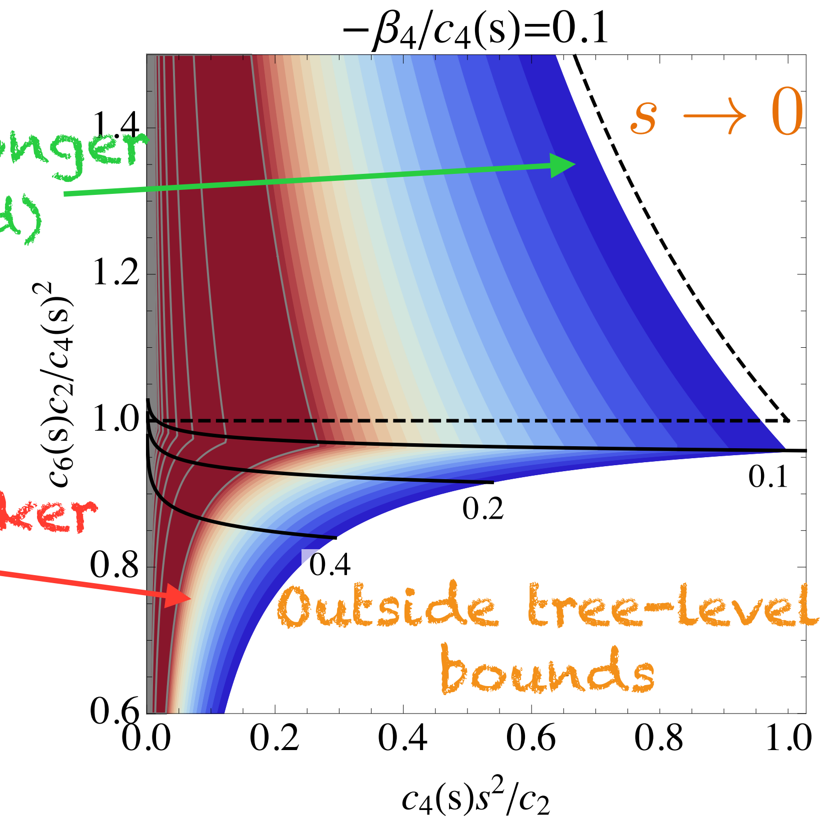

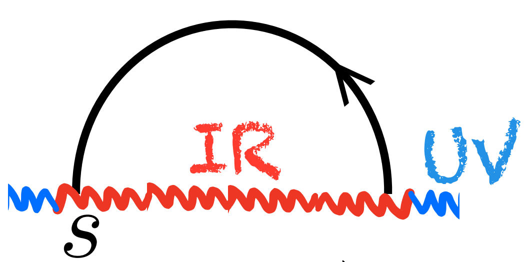

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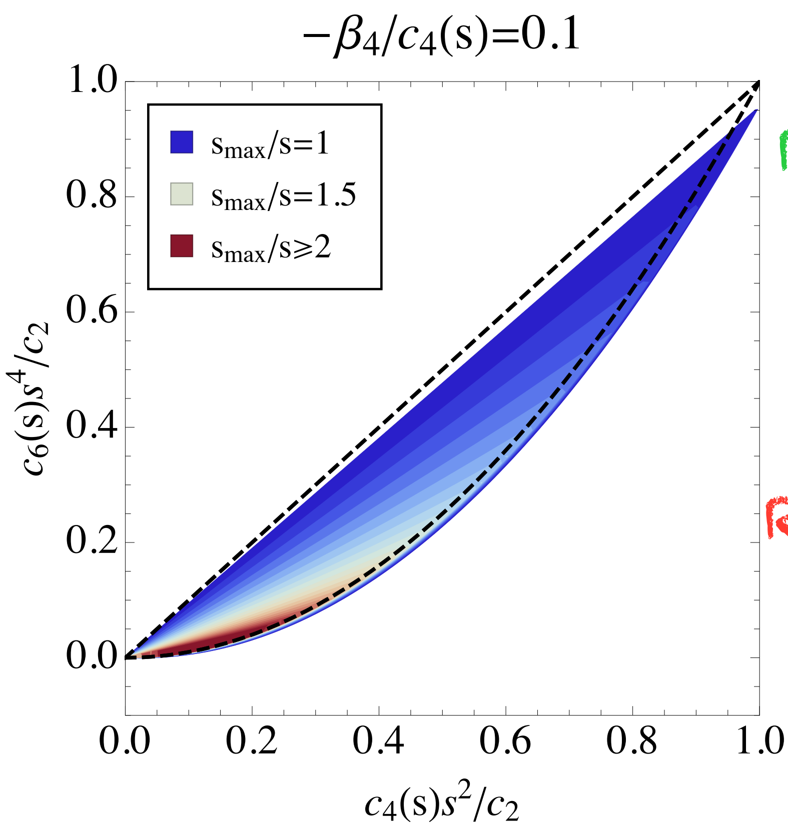

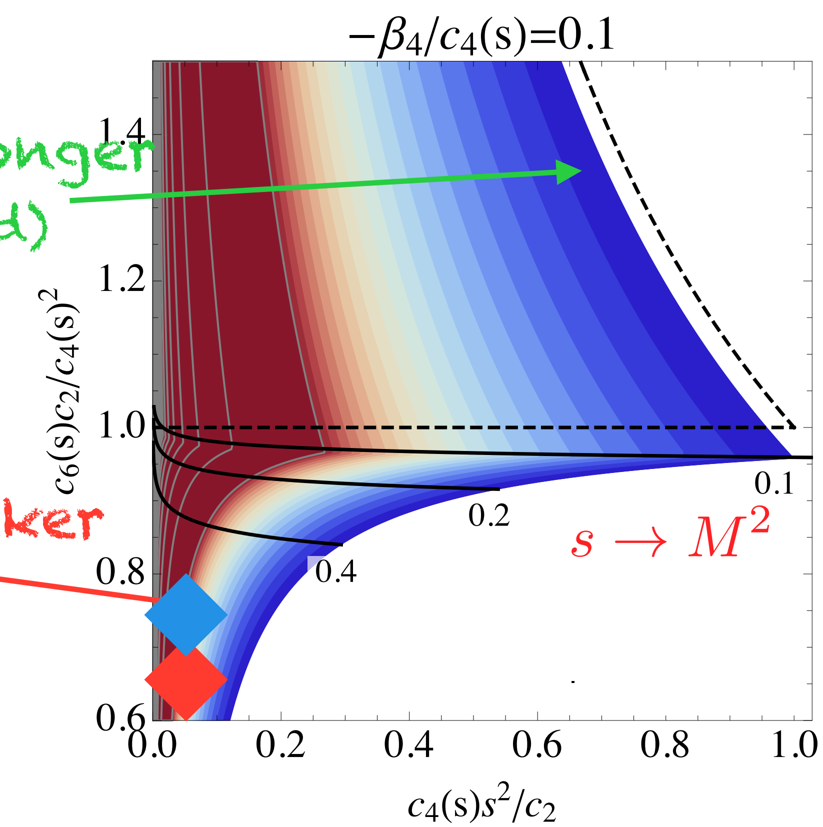

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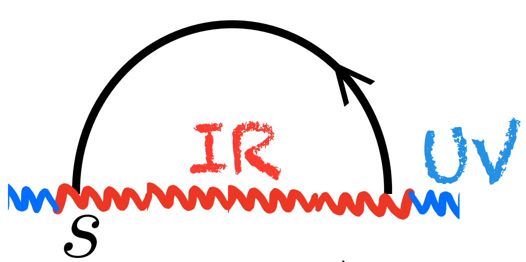

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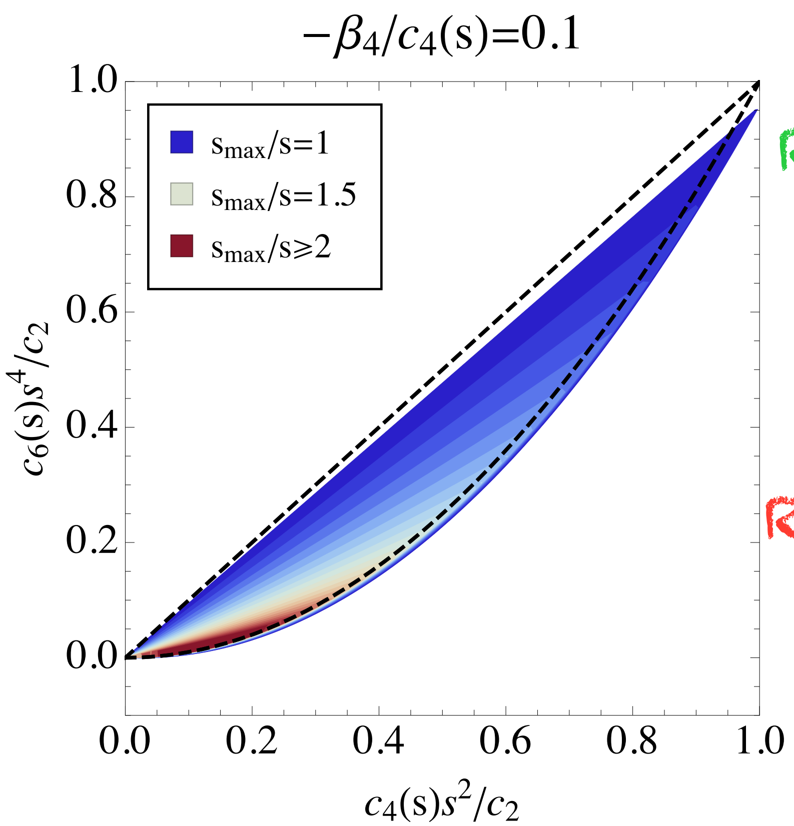

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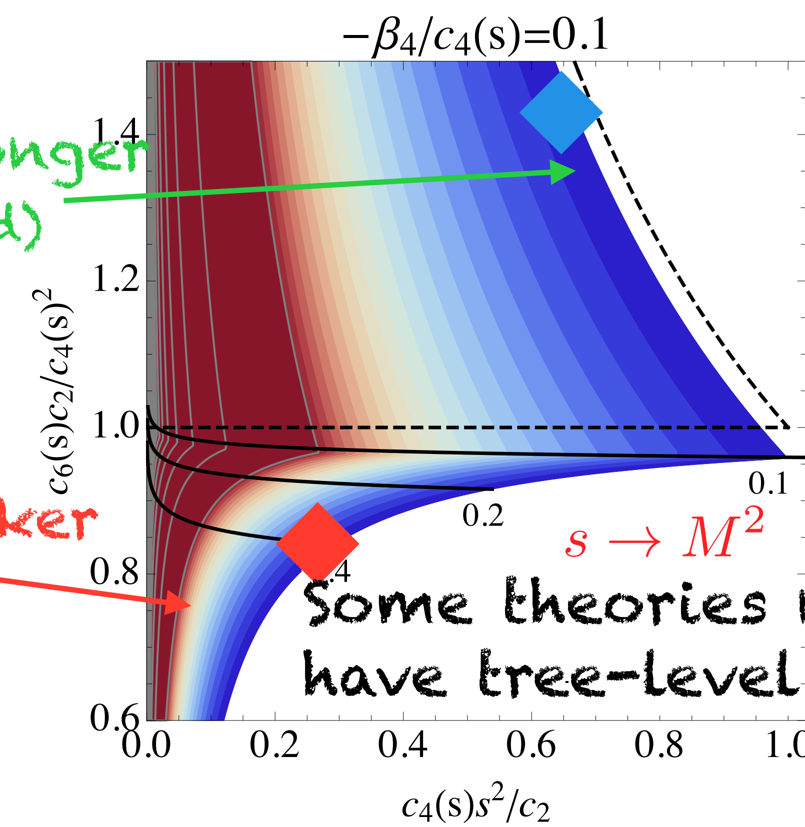
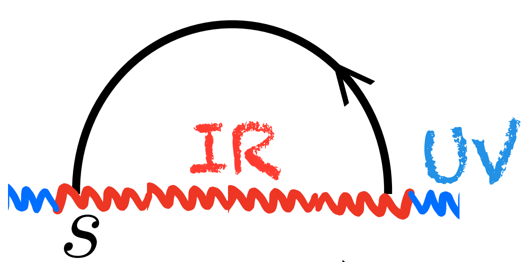


Some theories never have tree-level regime



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Collinear Divergences

Bellazzini et al, to appear

Large scale separation: $|t| \leq 4m^2 \ll s$

Amplitude analytic

Large EFT validity range

$$A(s,t) = \cdots + c_{2,2}(s)s^2t^2 + \frac{c_2^2}{16\pi^2}s^2t^2\log\frac{s}{m^2} + \cdots$$

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powers of t

powers of s

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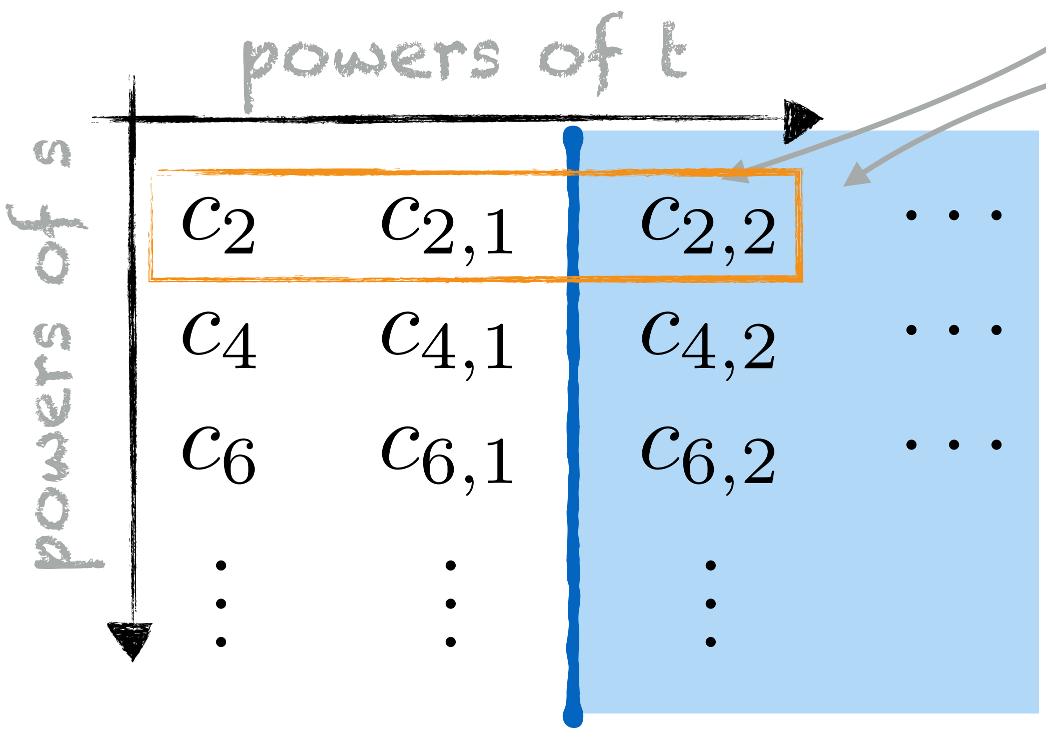

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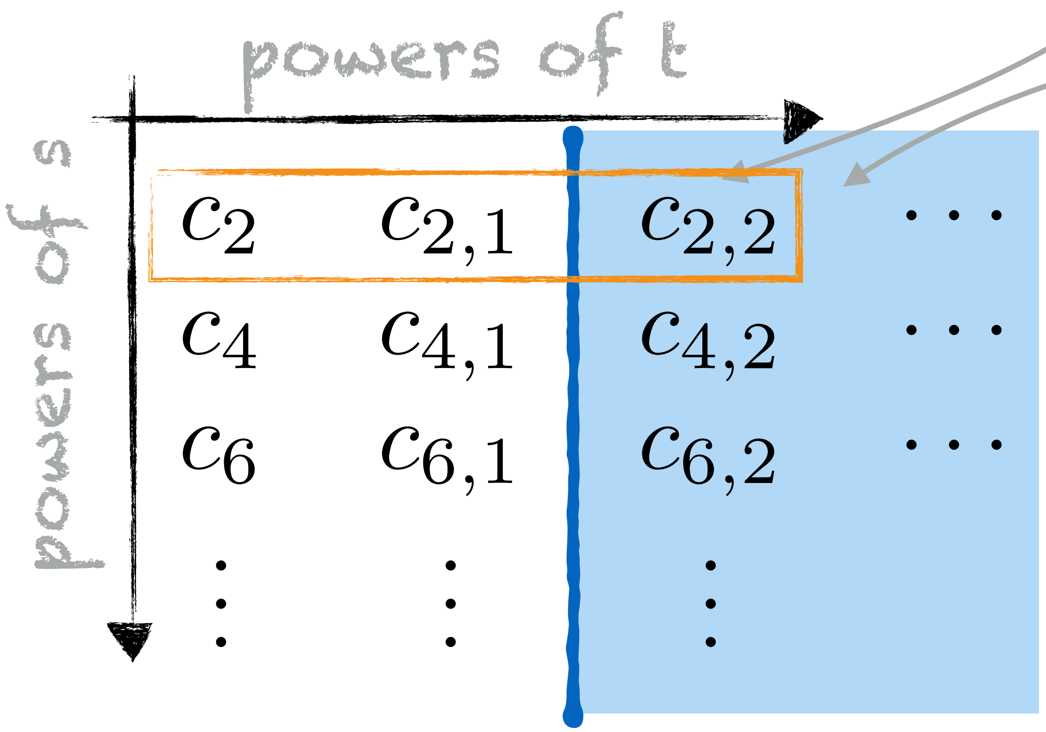
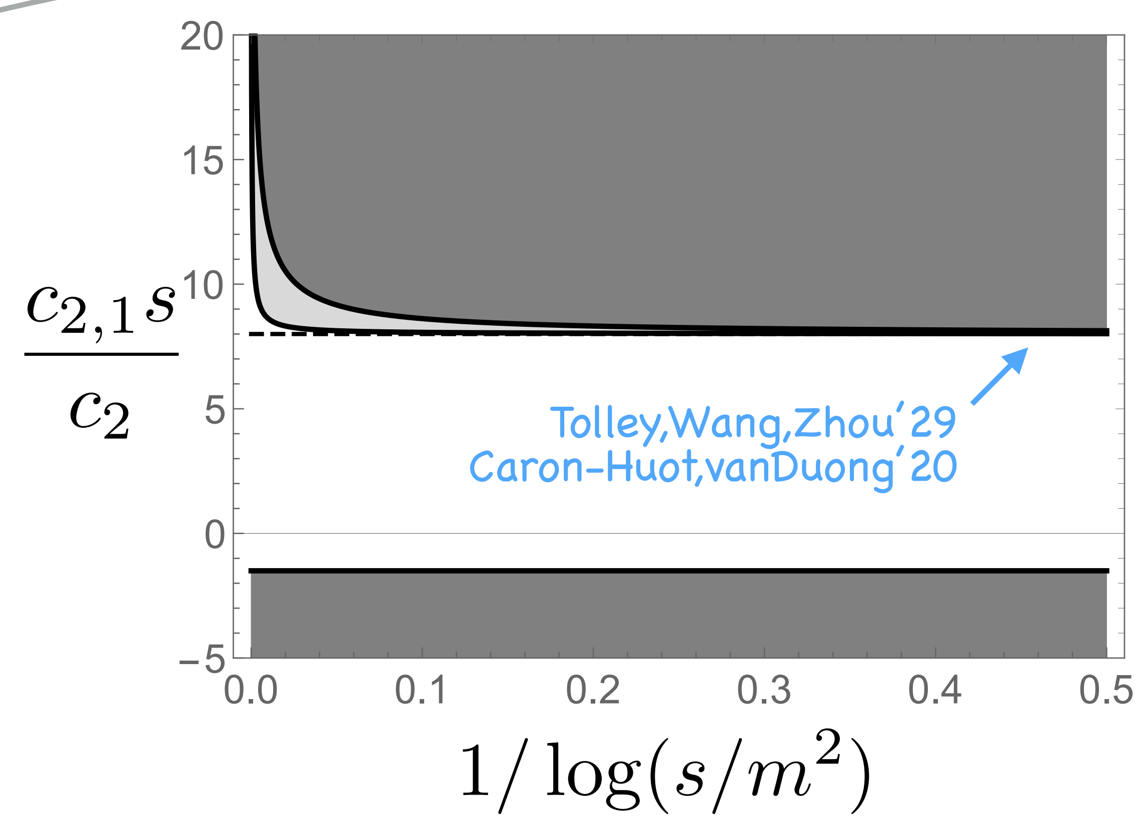
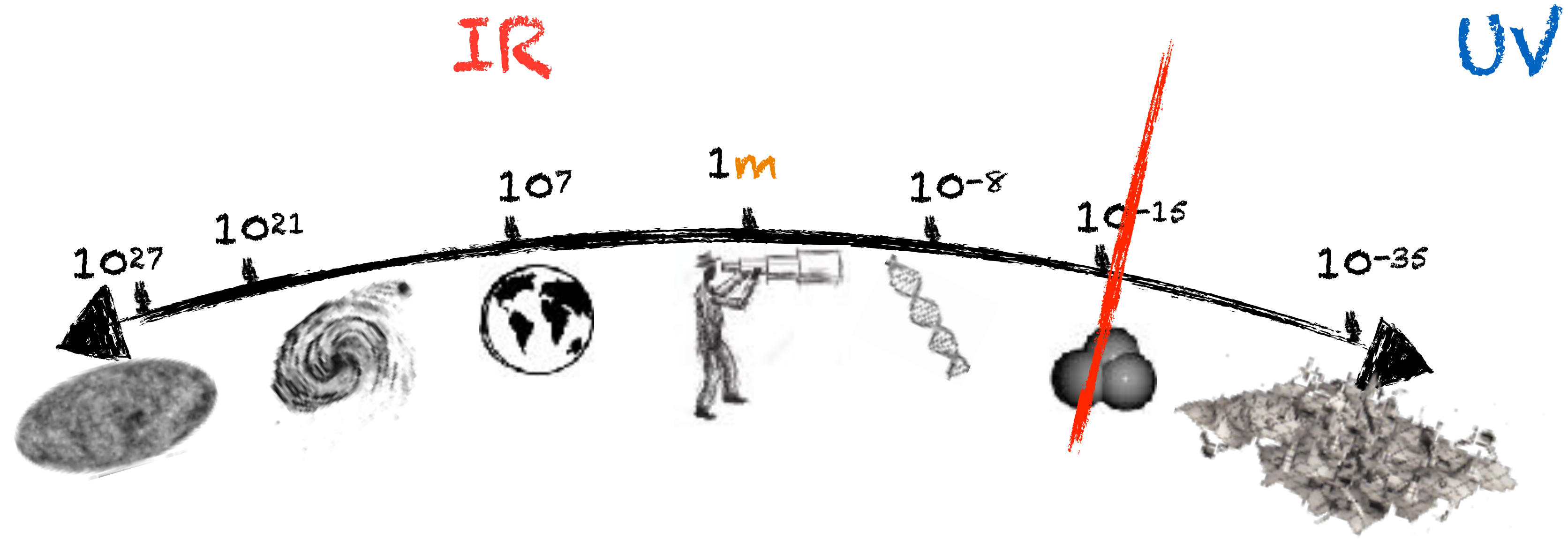




Bounds in t always affected: disappear for m->0

Effect small for finite mass

Summary



Constrained EFTs ← **moments** ← Causality, Unitarity, Lorentz invariance, Locality

- Only 3(2) coefficients can dominate
- ~~Supersoftness~~
- $m_{HS} > 1/L_{HS}$

IR running important

powers of t (→), powers of s (↓)

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Polygons vs Polynomials

Arkani-Hamed, Huang[2], 2020

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Positivity from geometry

Different "functional" approach

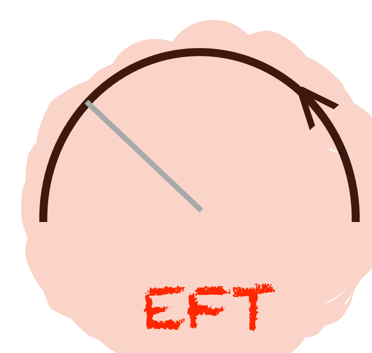
Forward Bounds for infinite arcs same

$$1 - x/2 - x^2/8 + \cdots = q(x) = \sqrt{1-x}$$

- Focus on "Optimal" bounds for finite many arcs, (both forward and at finite-t)
- Two-sided bounds

Residues

Arcs



- Suitable for EFT cutoff estimate
- Ideal for running

Galileon Bounds

Galileons have tree-amplitudes Nicolis,Rattazzi,Trincherini'09

$$A = -c_{2,1} stu = c_{2,1} s^2 t + c_{2,1} s t^2 .$$

Vanish in the forward limit, but enter at loop in β_6

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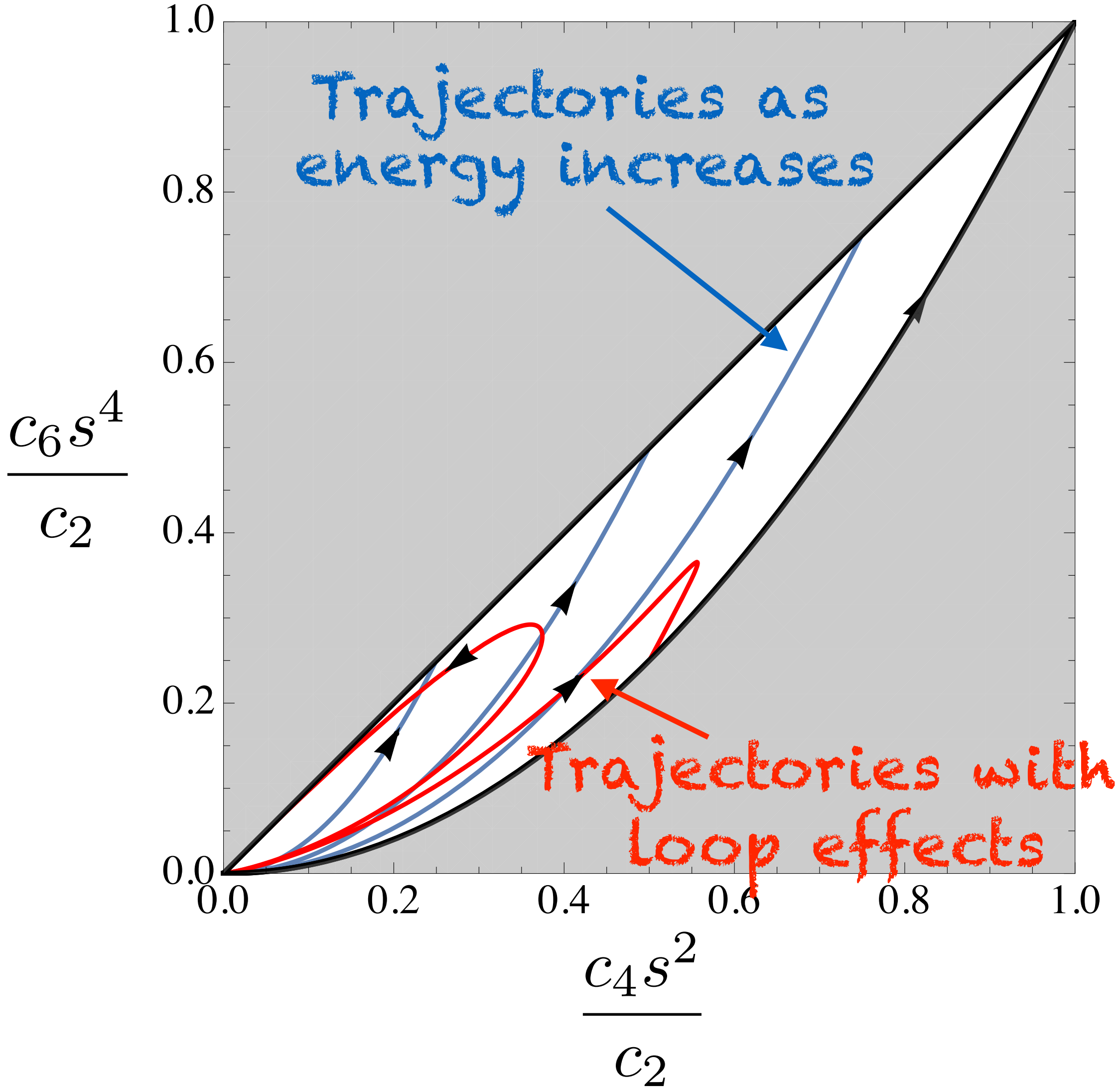

$$c_{2,1}^2 < 16\pi^2 c_4 s^3$$

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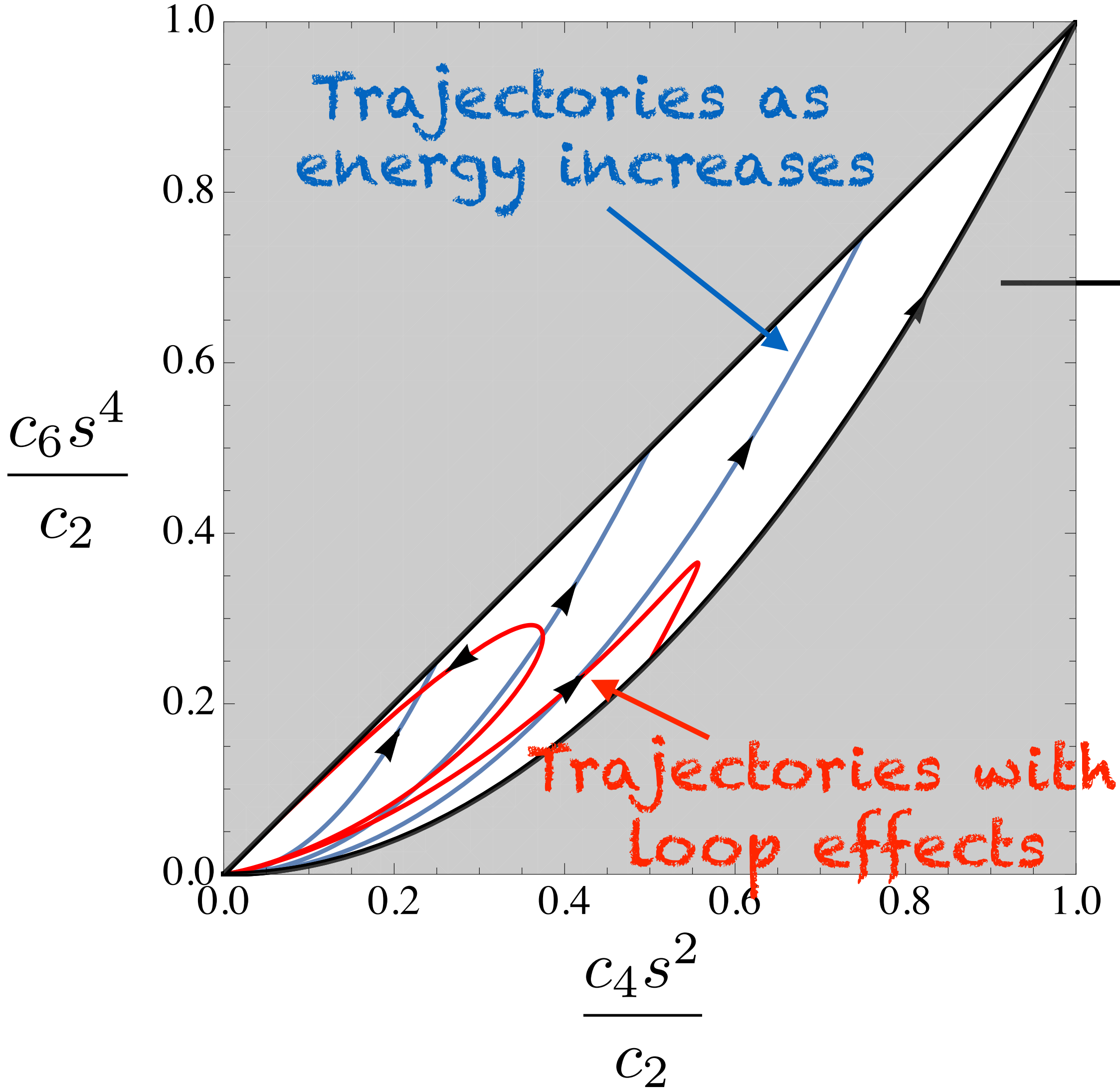

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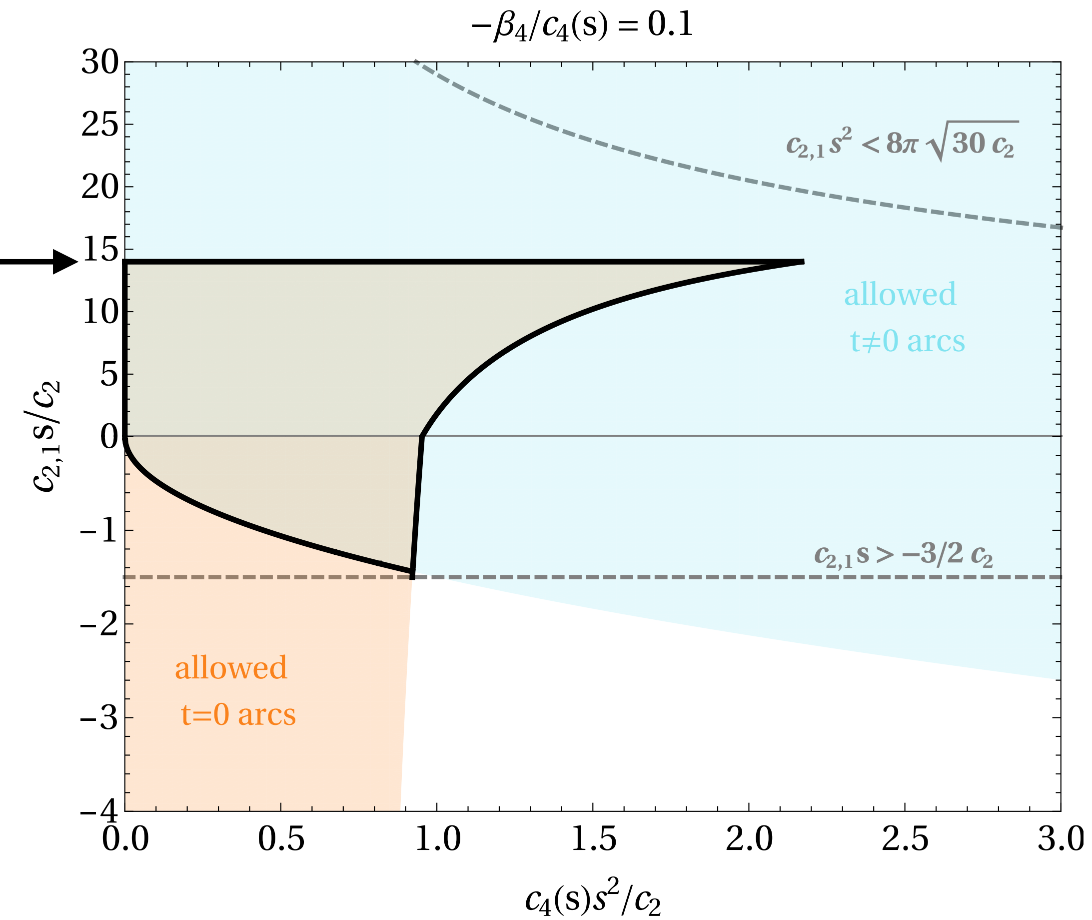

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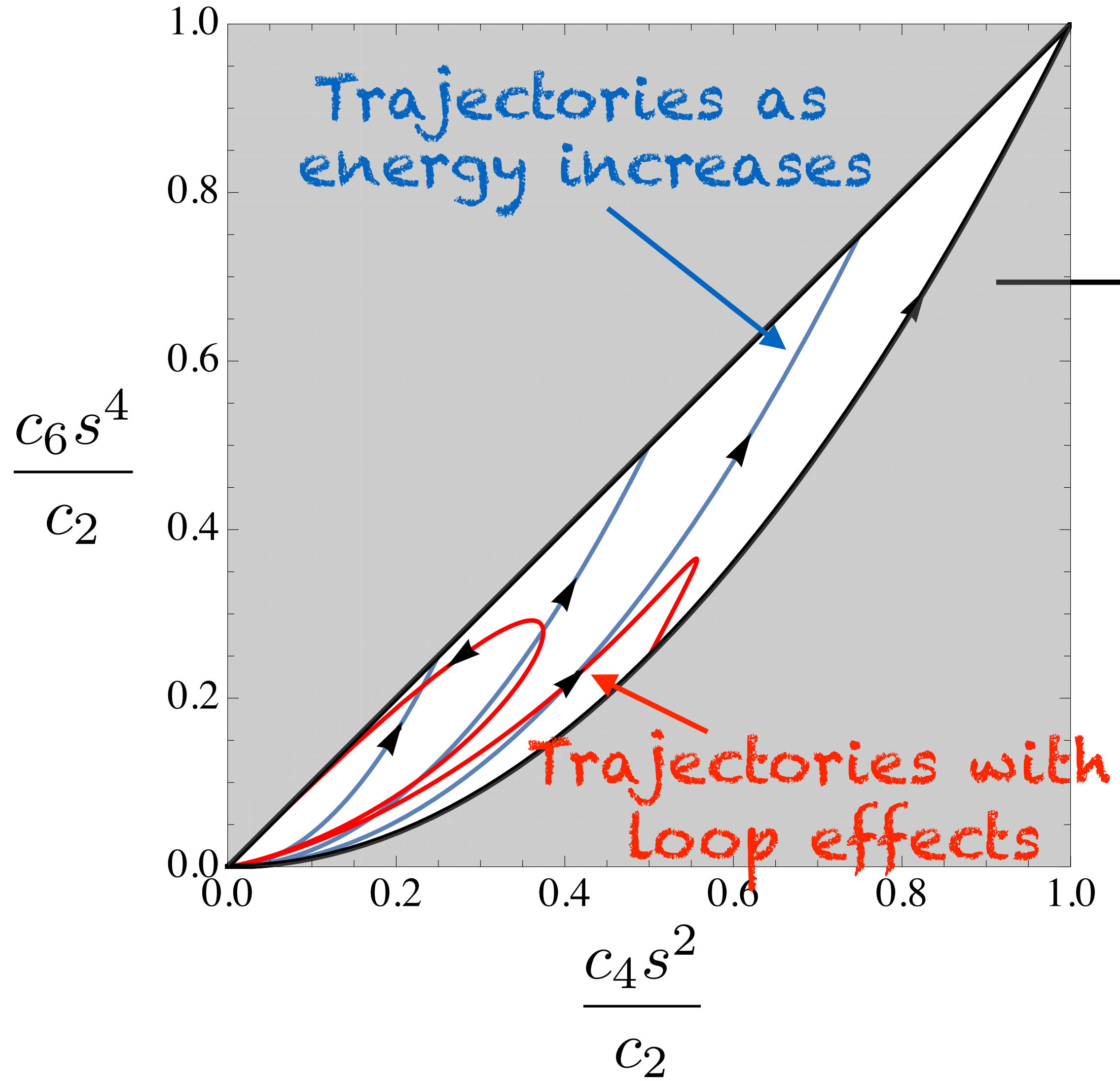

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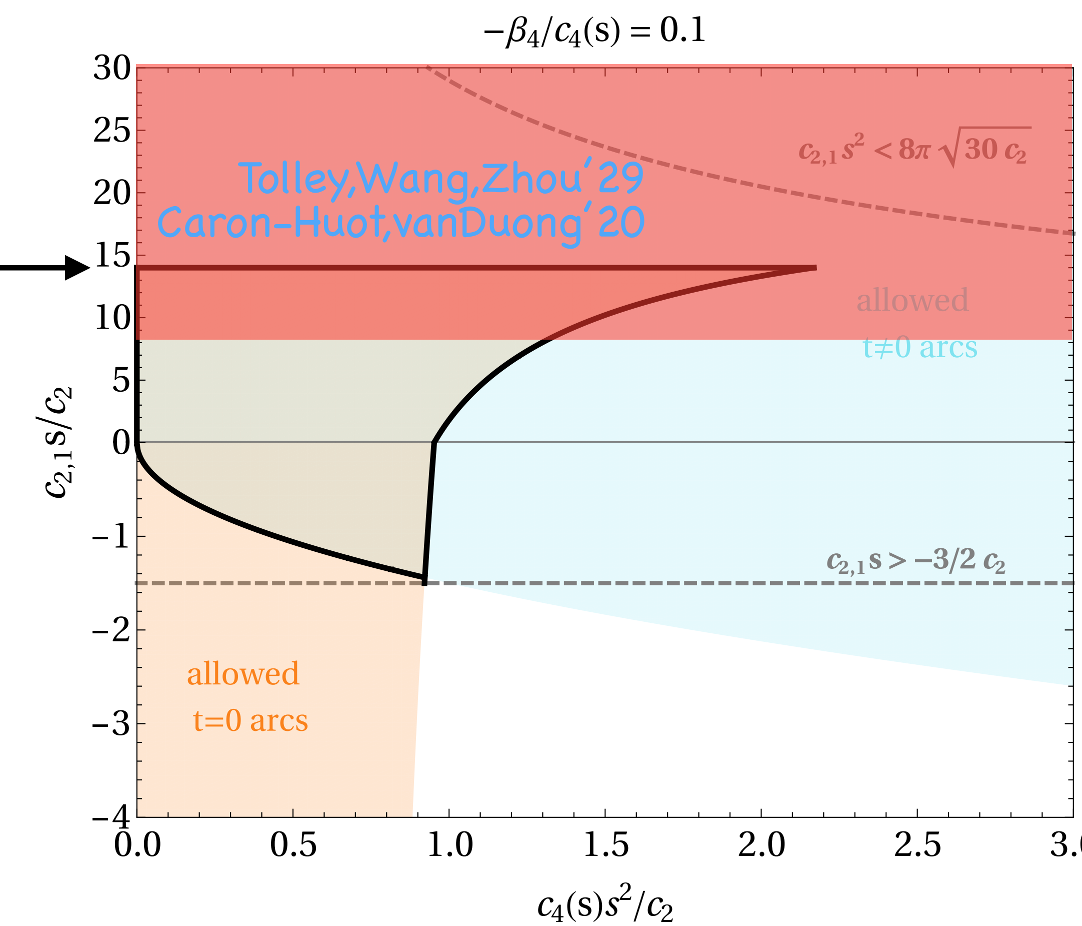

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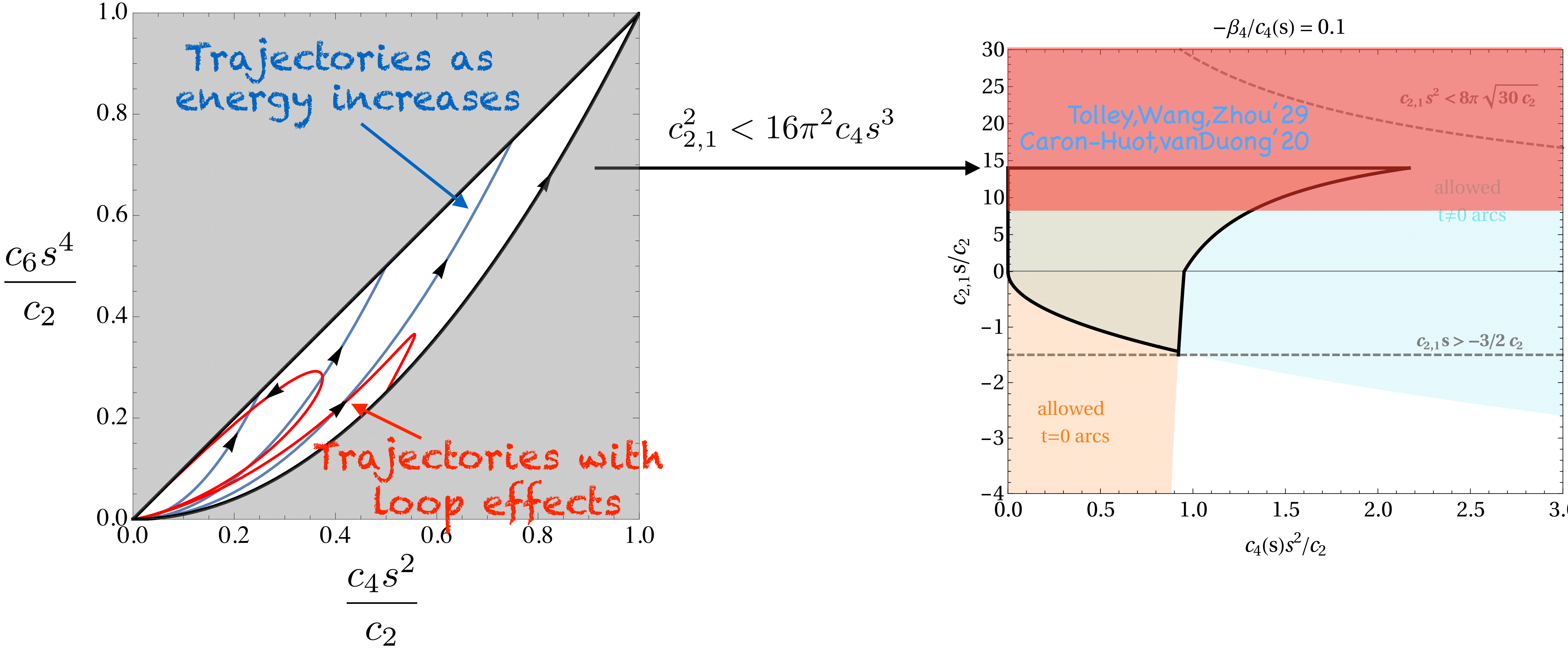

- Upper bound crucial for massive gravity Bellazzini, FR, Serra, Sgarlata '17(PRL)