Functional Prescription for STrEAMlined EFT Matching

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Based on 2011.02484, 2012.07851 (w/ Tim Cohen, Xiaochuan Lu)

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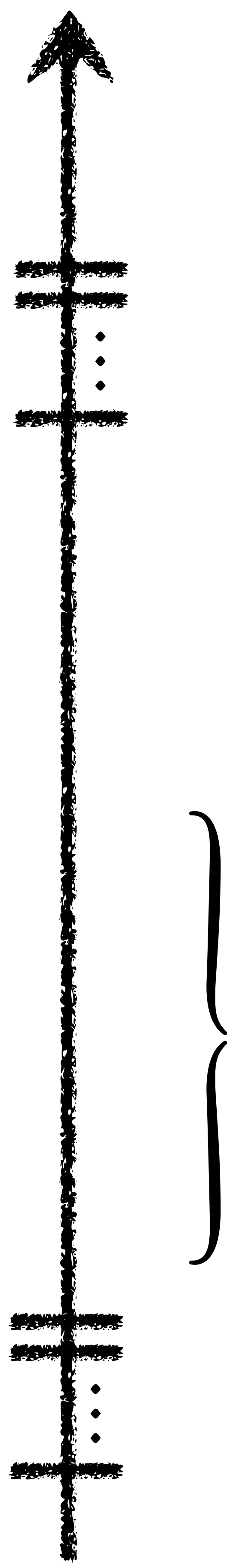
EFT matching



m_Φ heavy fields (e.g. BSM states $\sim$ TeV)

energy scales of experimental interest (e.g. LHC energies)

m_ϕ light fields (e.g. SM particles)

$$\boxed{\mathscr{L}_{\text{UV}}[\Phi, \phi]}$$

match & run (resum $\log \frac{m_\Phi}{m_\phi}$)

$$\boxed{\mathscr{L}_{\text{EFT}}[\phi]}$$

Many pheno applications (e.g. SMEFT).

Highly desirable to have efficient algorithms for EFT matching.

Outline

- What is functional matching, and what is new?
- The prescription.
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

A toy model

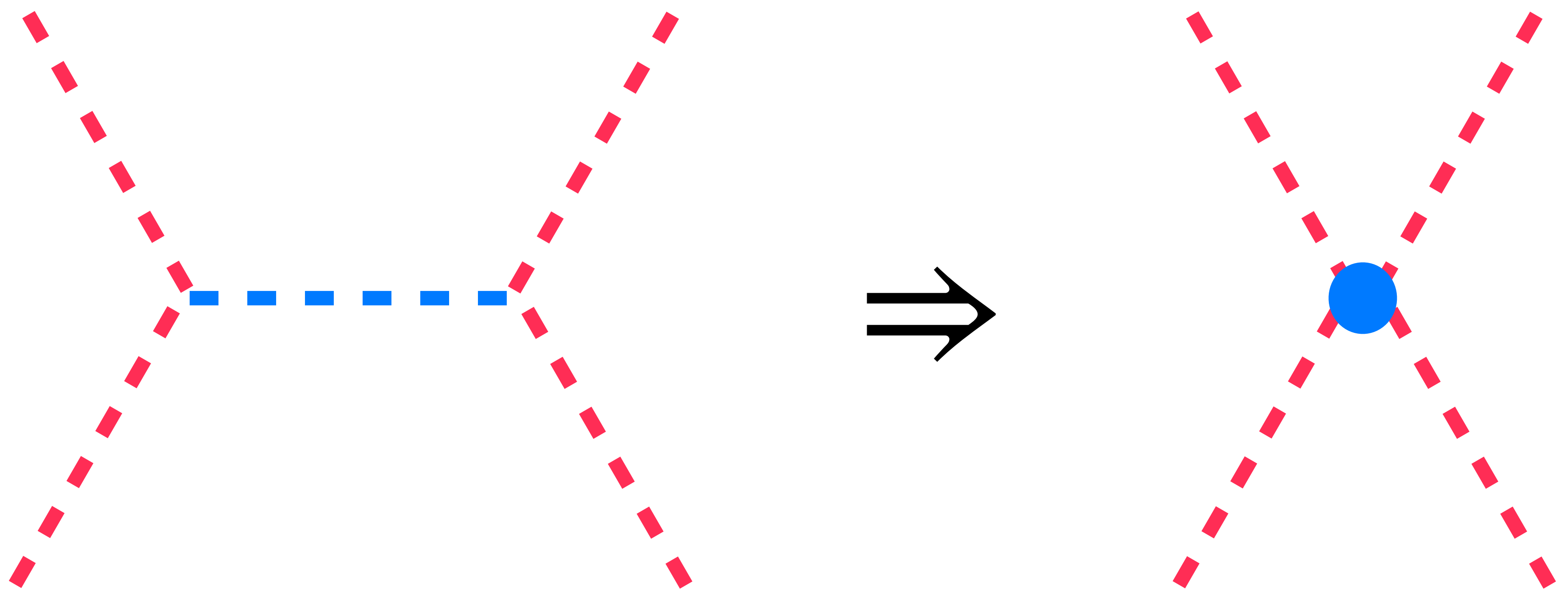
- $\mathscr{L}_{\mathrm{UV}} = \underbrace{\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2}_{\text{heavy scalar}} + \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{light scalar}} + \underbrace{\frac{1}{2}\kappa\,\Phi\,\phi^2}_{\text{interaction}}$

- $\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + ??$

A toy model

- $$\mathscr{L}_{\mathrm{UV}} = \underbrace{\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2}_{\text{heavy scalar}} + \underbrace{\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{light scalar}} + \underbrace{\frac{1}{2}\kappa\,\Phi\,\phi^2}_{\text{interaction}}$$

- $$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \boxed{\frac{1}{4!}c_1\,\phi^4 + \frac{1}{4}c_2\,\phi^2\partial^2\phi^2 + \ldots}$$



A toy model

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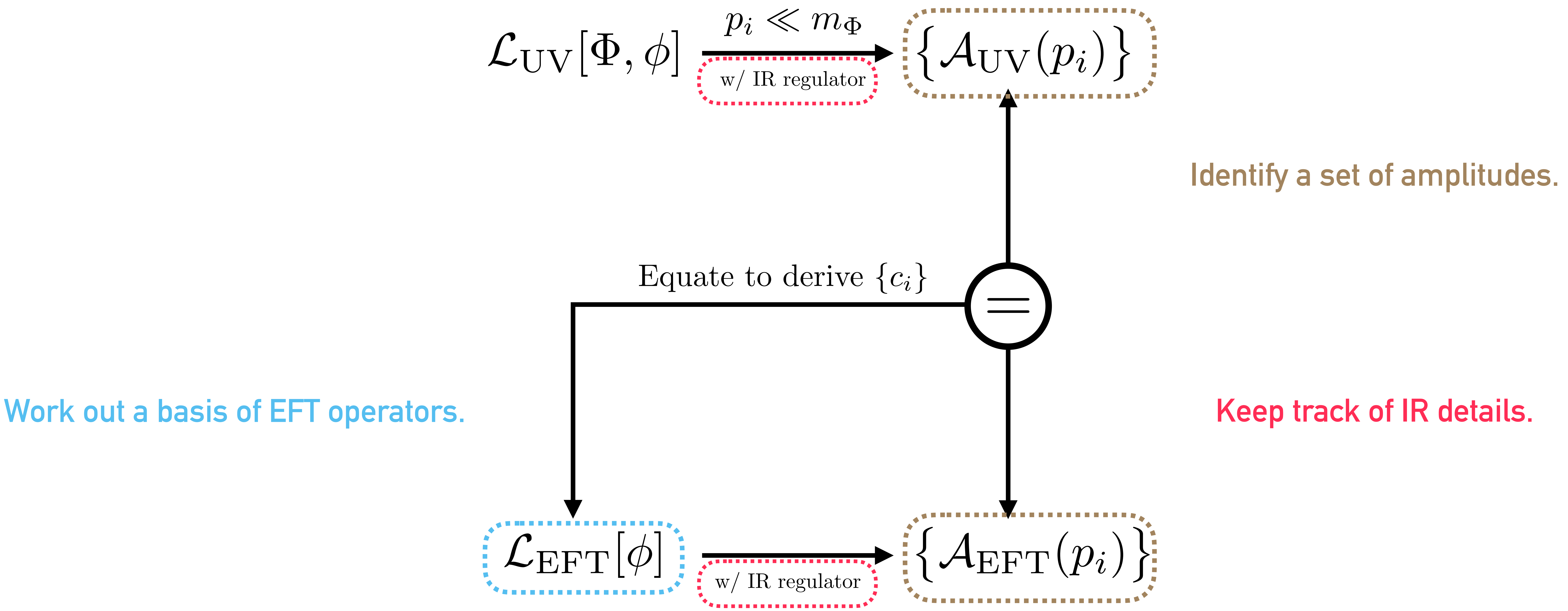
- $$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \boxed{\frac{1}{4!}c_1\,\phi^4 + \frac{1}{4}c_2\,\phi^2\partial^2\phi^2 + \ldots}$$

$$\mathscr{A}^{(\mathrm{tree})}_{\mathrm{UV}}(\phi\phi \to \phi\phi) = -\kappa^2\left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right) = \frac{3\kappa^2}{M^2} + \frac{\kappa^2}{M^4}(s+t+u) + \ldots$$

$$\mathscr{A}^{(\mathrm{tree})}_{\mathrm{EFT}}(\phi\phi \to \phi\phi) = c_1 - c_2\,(s+t+u) + \ldots$$

$$\Rightarrow \quad c_1^{(\mathrm{tree})} = \frac{3\kappa^2}{M^2}, \quad c_2^{(\mathrm{tree})} = -\frac{\kappa^2}{2M^4}.$$

What we have just done is "amplitude matching"



➤ Becomes cumbersome with more (higher-spin) fields, at higher operator dimensions, and higher loop orders.

A (familiar) more efficient approach

$$\mathcal{L}_{\mathrm{UV}}[\Phi, \phi]$$

$\downarrow$ Equation of motion (EOM): $\dfrac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \Phi} = 0 \, .$

$$\Phi_{\mathrm{c}}[\phi]$$

(EOM solution)

$\downarrow$ Set $\Phi = \Phi_{\mathrm{c}}[\phi]$

$$\mathcal{L}_{\mathrm{EFT}}[\phi]$$

A (familiar) more efficient approach

$\mathcal{L}_{\rm UV}[\Phi, \phi]$

$$\mathscr{L}_{\rm UV} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\kappa\,\Phi\,\phi^2$$

Equation of motion (EOM): $\dfrac{\delta \mathscr{L}_{\rm UV}}{\delta \Phi} = 0\,.$ $\quad\Rightarrow\ \left(\partial^2 + M^2\right)\Phi = \frac{1}{2}\kappa\,\phi^2$

$\Phi_{\rm c}[\phi]$ (EOM solution)

$$\Rightarrow\ \Phi_{\rm c}[\phi] = \frac{1}{2}\kappa\,\frac{1}{\partial^2 + M^2}\,\phi^2 = \frac{\kappa}{2M^2}\left(\phi^2 - \frac{1}{M^2}\partial^2\phi^2 + \ldots\right)$$

Set $\Phi = \Phi_{\rm c}[\phi]$

$\mathcal{L}_{\rm EFT}[\phi]$

$$\mathscr{L}^{\rm (tree)}_{\rm EFT} = \mathscr{L}_{\rm UV}\big[\Phi_{\rm c}[\phi]\,,\,\phi\big] = \frac{3\kappa^2}{M^2}\,\frac{1}{4!}\,\phi^4 - \frac{\kappa^2}{2M^4}\,\frac{1}{4}\,\phi^2\partial^2\phi^2 + \ldots$$

➤ This is nothing but functional matching at tree level.

Functional matching

- Match generating functionals of amplitudes, rather than amplitudes themselves.
- Path integral:

$$Z[J] = e^{-i E[J]} = \int D\varphi \, e^{i\int d^dx\,(\mathscr{L}[\varphi] + J\varphi)}$$

Generating functional of connected correlation functions: $\langle \varphi^n \rangle_{\text{conn}} = (-i)^{n+1} \dfrac{\delta^n E}{\delta J^n}$.

- Simpler to work with its Legendre transform $\Rightarrow$ 1PI effective action:

$$\Gamma\big[\langle\varphi\rangle\big] = -E[J] - \int d^dx\, J\,\langle\varphi\rangle$$

where $\langle\varphi\rangle$ and J are related by $\langle\varphi\rangle = -\dfrac{\delta E}{\delta J}$, $J = -\dfrac{\delta\Gamma}{\delta\langle\varphi\rangle}$

Generating functional of 1PI correlation functions: $\langle \varphi^n \rangle_{\text{1PI}} = i \dfrac{\delta^n \Gamma}{\delta\langle\varphi\rangle^n}$.

Wish to match between UV theory and EFT.

Matching 1PI effective actions

tree level matching

one-loop matching

$$\Gamma_{\mathrm{UV,L}}[\phi] = \boxed{\int d^dx\, \mathscr{L}_{\mathrm{UV}}\big[\Phi_c[\phi],\phi\big]} + \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) + \dots$$

1LPI

$$\Gamma_{\mathrm{EFT}}[\phi] = \int d^dx\, \mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}[\phi] + \int d^dx\, \mathscr{L}^{(1-\mathrm{loop})}_{\mathrm{EFT}}[\phi] + \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}}{\delta\phi^2}\right) + \dots$$

one-loop-generated operators used in tree graphs

tree-level-generated operators used in one-loop graphs

$$\Rightarrow \int d^dx\, \mathscr{L}^{(1-\mathrm{loop})}_{\mathrm{EFT}}[\phi] = \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) - \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2\mathscr{L}^{(\mathrm{tree})}_{\mathrm{EFT}}}{\delta\phi^2}\right)$$

$\varphi = \{\,\Phi\,,\,\phi\,\}$

Functional matching at one loop

$$\int d^dx\, \mathscr{L}_{\rm EFT}^{(1-{\rm loop})}[\phi] \;=\; \frac{i}{2}\, \log {\rm Sdet}\left(- \frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right) \;-\; \boxed{\frac{i}{2}\, \log {\rm Sdet}\left(- \frac{\delta^2 \mathscr{L}_{\rm EFT}^{(\rm tree)}}{\delta\phi^2} \right)}$$

- If $-\dfrac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\varphi^2} = \begin{array}{cc} \begin{array}{cc} \Phi & \phi \end{array} & \\ \begin{pmatrix} & 0 \\ 0 & \end{pmatrix} & \begin{array}{l} \Phi \\ \phi \end{array} \end{array}$ $\quad$ $\Rightarrow$ **heavy loops** $\quad$ $\Rightarrow$ **light loops (also present in the EFT)**

- Just compute heavy fields' contributions to the functional superdeterminant.

B. Henning, X. Lu, H. Murayama, 1412.1837.

Functional matching at one loop

$$\int d^d x\, \mathcal{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})}[\phi] = \frac{i}{2} \log \mathrm{Sdet}\left(- \frac{\delta^2 \mathcal{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right) - \boxed{\frac{i}{2} \log \mathrm{Sdet}\left(- \frac{\delta^2 \mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}}{\delta \phi^2} \right)}$$

➤ If $-\dfrac{\delta^2 \mathcal{L}_{\mathrm{UV}}}{\delta \varphi^2} = \begin{array}{c} \begin{array}{cc} \Phi & \phi \end{array} \\ \begin{pmatrix} \blacksquare & ? \\ ? & \blacksquare \end{pmatrix} \end{array} \begin{array}{l} \Phi \quad \Rightarrow \text{heavy loops} \\ \phi \quad \Rightarrow \text{light loops (also present in the EFT)} \end{array}$

➤ Just compute heavy fields' contributions to the functional superdeterminant.

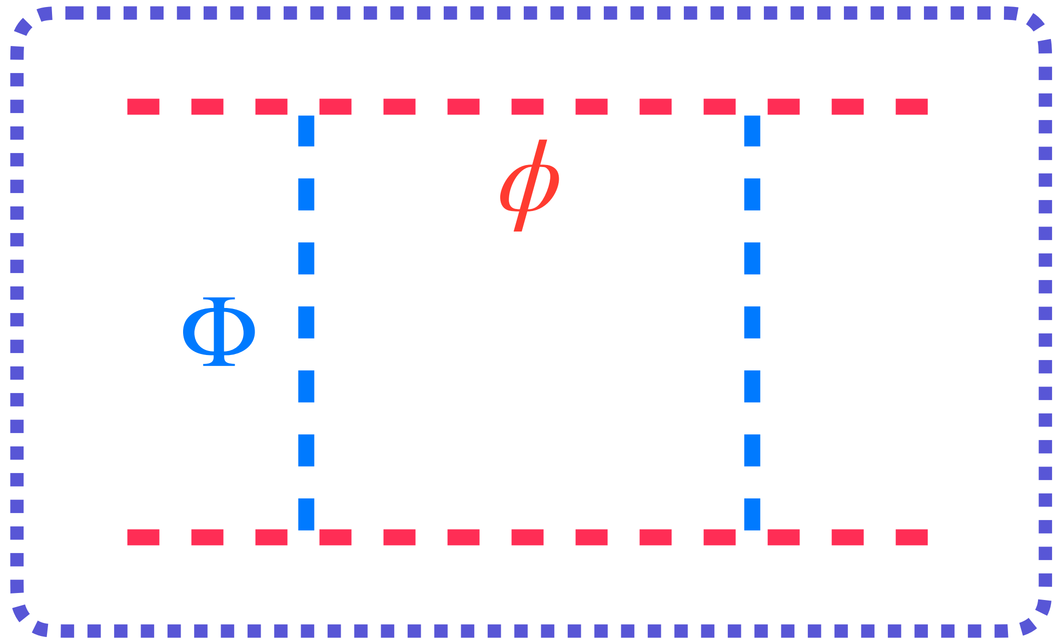
➤ More work is needed to include mixed heavy-light contributions.

B. Henning, X. Lu, H. Murayama, 1604.01019. S. A. R. Ellis, J. Quevillon, T. You, ZZ, 1604.02445. J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia, 1607.02142. ZZ, 1610.00710.

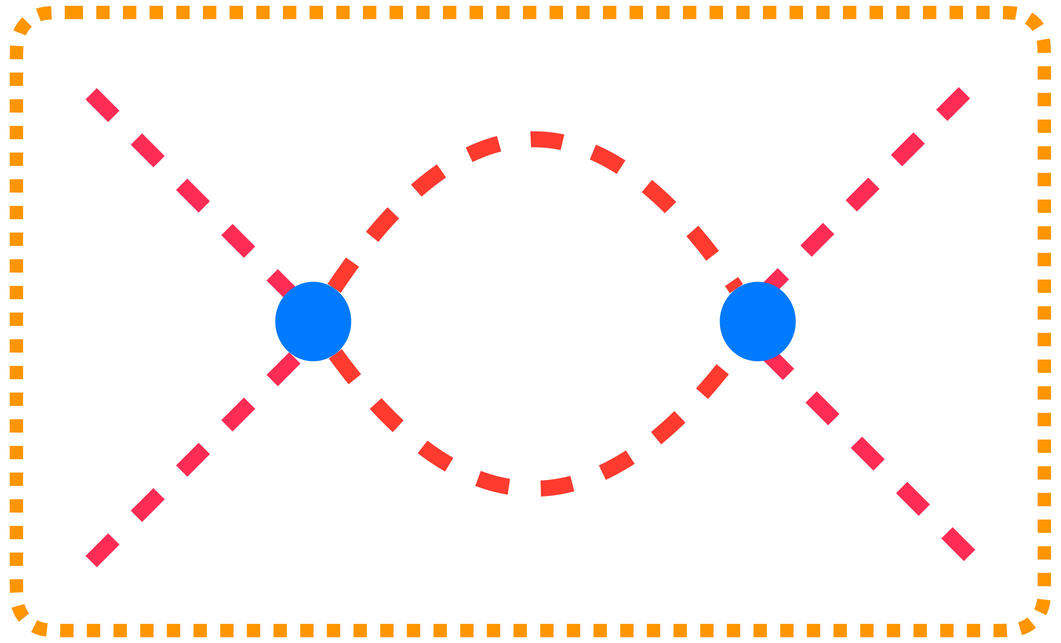
Functional matching at one loop

$$\int d^d x\, \mathscr{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})}[\phi] = \boxed{\frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)} - \boxed{\frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\mathrm{EFT}}^{(\mathrm{tree})}}{\delta \phi^2}\right)}$$

➤ A closer look at the second term (tree-generated operators used in 1-loop graphs).



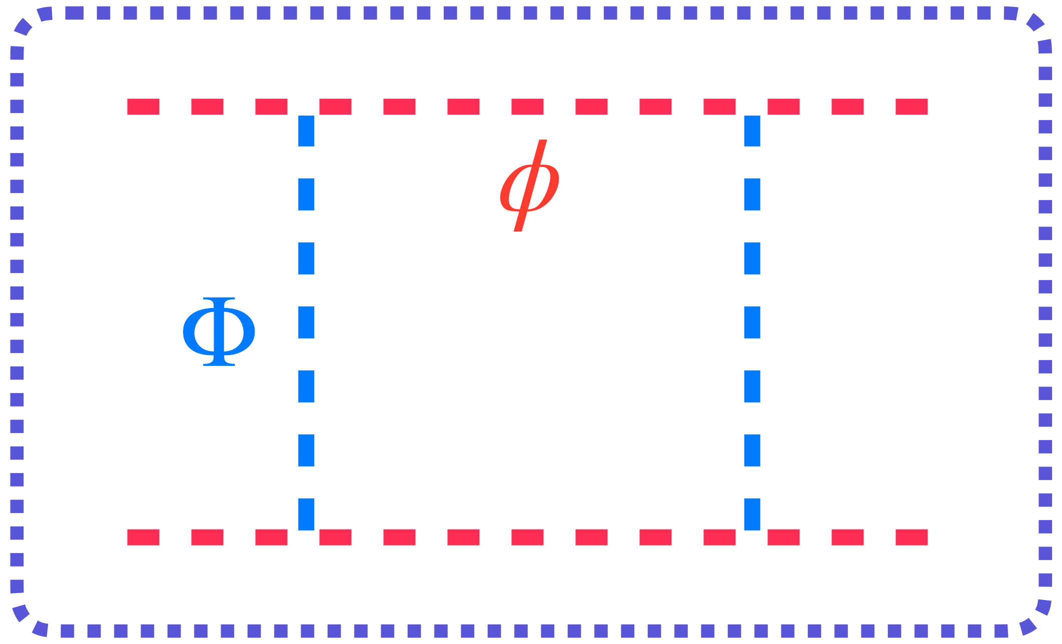
$\Rightarrow$



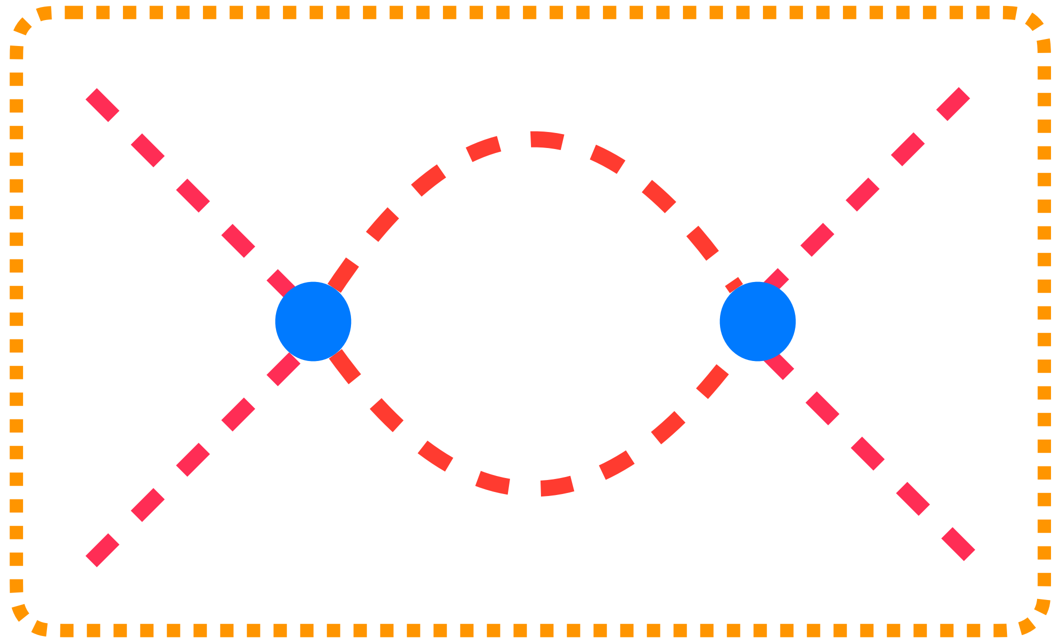
Functional matching at one loop

$$\int d^d x\, \mathscr{L}_{\rm EFT}^{(1-\rm loop)}[\phi] = \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right) - \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\rm EFT}^{(\rm tree)}}{\delta \phi^2} \right)$$

➤ A closer look at the second term (tree-generated operators used in 1-loop graphs).



$\neq$



Difference goes into $\mathscr{L}_{\rm EFT}^{(1-\rm loop)}[\phi]$.

Functional matching at one loop

$$\int d^dx\, \mathscr{L}^{(1-\text{loop})}_{\text{EFT}}[\phi] = \frac{i}{2}\log \text{Sdet}\left(-\frac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right) - \frac{i}{2}\log\text{Sdet}\left(-\frac{\delta^2\mathscr{L}^{(\text{tree})}_{\text{EFT}}}{\delta\phi^2}\right)$$

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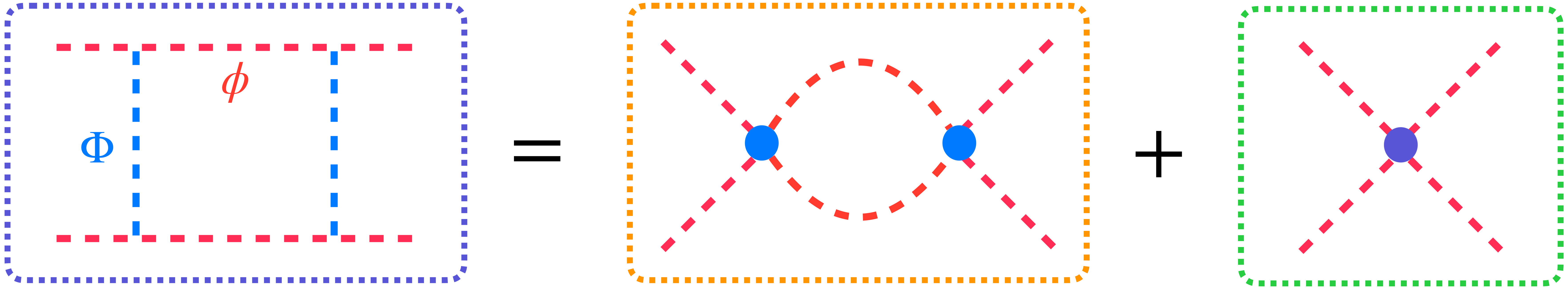

➤ This is nothing but the **method of regions**.

M. Beneke, V. A. Smirnov, hep-ph/9711391.
V. A. Smirnov's book (2002). B. Jantzen, 1111.2589.

➤ $\displaystyle\int d^dq = \int d^dq\,\bigg|_{\text{soft}} + \int d^dq\,\bigg|_{\text{hard}}$. Expand then integrate over full q space using DimReg.

$q \sim m_\phi$ $\Rightarrow$ heavy propagator shrinks to a point.

$q \sim m_\Phi$ $\Rightarrow$ entire loop shrinks to a point $\Rightarrow$ local operators in the EFT.

Functional matching at one loop

➤ Central formula:

$$\boxed{\int d^d x\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] \;=\; \frac{i}{2}\,\log \text{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_{\text{c}}[\phi]} \right)\bigg|_{\text{hard}}}$$

B. Henning, X. Lu, H. Murayama, 1604.01019. S. A. R. Ellis, J. Quevillon, T. You, ZZ, 1604.02445.
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Functional matching at one loop

$$\boxed{\int d^dx\, \mathscr{L}_{\rm EFT}^{(1-\rm loop)}[\phi] \;=\; \frac{i}{2}\log \mathrm{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]}\right)\bigg|_{\rm hard}}$$

- What makes the functional approach powerful is an elegant procedure to compute this superdeterminant, which crucially relies upon:
  - **Method of regions.**
    - Just need hard region, no IR details. ☺
  - **Covariant derivative expansion (CDE).**
    - Work with D_μ , $\phi \Rightarrow$ gauge-invariant operators directly, no momentum-space Feynman rules. ☺
    - Extension of Coleman-Weinberg including derivatives.

M. K. Gaillard, Nucl. Phys. B 268 (1986) 669.
O. Cheyette, Nucl. Phys. B 297 (1988) 183.
B. Henning, X. Lu, H. Murayama, 1412.1837.

Functional matching at one loop

$$\int d^dx\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] \;=\; \frac{i}{2}\, \log \text{Sdet}\left(-\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} \right)\bigg|_{\text{hard}}$$

- What makes the functional approach powerful is an elegant procedure to compute this superdeterminant, which crucially relies upon:
  - **Method of regions.**
  - **Covariant derivative expansion (CDE).**
- Quite some freedom on how to put these ingredients together + CDE can be tedious. ☹
  - Motivated new development: streamlined prescription + automated CDE calculation.

T. Cohen, X. Lu, ZZ, 2011.02484.

T. Cohen, X. Lu, ZZ, 2012.07851.

see also: J. Fuentes-Martin, M. König, J. Pagès, A. E. Thomsen, F. Wilsch, 2012.08506.

Outline

- What is functional matching, and what is new?

- The prescription.

- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.

- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

Two types of supertraces

$$
\begin{aligned}
\int d^d x\, \mathscr{L}_{\text{EFT}}^{(1-\text{loop})}[\phi] &= \frac{i}{2} \log \text{Sdet}\left(- \frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \varphi^2} \bigg|_{\Phi=\Phi_c[\phi]} \right) \bigg|_{\text{hard}} \\
&= \frac{i}{2} \text{STr} \log\left(- \frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \varphi^2} \bigg|_{\Phi=\Phi_c[\phi]} \right) \bigg|_{\text{hard}} \\
&= \frac{i}{2} \text{STr} \log \boxed{\left(\mathbf{K} - \mathbf{X} \right)} \bigg|_{\text{hard}} \\
&= \frac{i}{2} \text{STr} \log \mathbf{K} \bigg|_{\text{hard}} + \frac{i}{2} \text{STr} \log\left(\mathbf{1} - \mathbf{K}^{-1} \mathbf{X} \right) \bigg|_{\text{hard}} \\
&= \frac{i}{2} \boxed{\text{STr} \log \mathbf{K} \bigg|_{\text{hard}}} - \frac{i}{2} \sum_{n=1}^{\infty} \boxed{\frac{1}{n} \text{STr}\left[\left(\mathbf{K}^{-1} \mathbf{X} \right)^n \right] \bigg|_{\text{hard}}}
\end{aligned}
$$

generic form of the quadratic action:

inverse propagator ($\mathbf{K}$) – interaction ($\mathbf{X}$)

"log-type" "power-type"

K, X matrices in relativistic theories

- $\mathbf{K}$ is block-diagonal with elements:

$$K_i = \begin{cases} P^2 - m_i^2 & \text{(spin-0)} \\ \gamma^\mu P_\mu - m_i & \text{(spin-1/2)} \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & \text{(spin-1, Feynman gauge } \xi = 1\text{)} \end{cases}$$

where $P_\mu = iD_\mu$ is the hermitian covariant derivative.

- $\mathbf{X}$ admits a derivative expansion:

$$\mathbf{X}\left[\phi\,,\,P_\mu\right] \;=\; U[\phi] \;+\; \left(P_\mu\,\mathbf{Z}^\mu[\phi] + \overline{\mathbf{Z}}^\mu\,P_\mu\right) \;+\; \text{higher-derivative interactions}$$

K, X matrices in relativistic theories

➤ **K** is block-diagonal with elements:

$$K_i = \begin{cases} \boxed{P^2} - m_i^2 & \text{(spin-0)} \\ \gamma^\mu \boxed{P_\mu} - m_i & \text{(spin-1/2)} \\ -\eta^{\mu\nu}\left(\boxed{P^2} - m_i^2\right) & \text{(spin-1, Feynman gauge } \xi = 1) \end{cases}$$

where $P_\mu = iD_\mu$ is the hermitian covariant derivative.

➤ **X** admits a derivative expansion:

$$\mathbf{X}\left[\phi\,,\,P_\mu\right] \;=\; U[\phi] \;+\; \left(\boxed{P_\mu}\mathbf{Z}^\mu[\phi] + \overline{\mathbf{Z}}^\mu \boxed{P_\mu}\right) \;+\; \text{higher-derivative interactions}$$

These P_μ's are "open" covariant derivatives (act openly to the right),

as opposed to "closed" covariant derivatives [enclosed by "()"], e.g. $(P_\mu\phi) = i\,(D_\mu\phi) \equiv [P_\mu, \phi]$ (act just on ϕ).

Log-type supertraces

$$\frac{i}{2}\,\mathrm{STr}\log \mathbf{K}\Big|_{\text{hard}} = \frac{i}{2}\sum_i \mathrm{STr}\log K_i\Big|_{\text{hard}}$$

- Heavy fields: full STr = hard + $\cancelto{0\ (\text{scaleless})}{\text{soft}}$.
- Light fields: full STr = $\cancelto{0}{\text{hard}}$ + soft.

$$\Rightarrow \quad \frac{i}{2}\,\mathrm{STr}\log \mathbf{K}\Big|_{\text{hard}} = \frac{i}{2}\sum_{i\in\{\Phi\}} \mathrm{STr}\log K_i$$

(Add up contributions from all heavy fields Φ)

Log-type supertraces

- Results are universal. Using CDE (later in the talk) to carry out the **functional part** of the supertraces, we obtain:

$$\frac{i}{2}\,\mathrm{STr}\log\left(P^2-m^2\right)=\int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\Bigg\{-\left(\frac{2}{\bar{\epsilon}}-\log\frac{m^2}{\mu^2}\right)\frac{1}{24}\,F_{\mu\nu}F^{\mu\nu}$$

$$+\frac{1}{m^2}\left[-\frac{1}{120}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)-\frac{1}{180}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\},$$

$$\frac{i}{2}\,\mathrm{STr}\log\left(\not{P}-m\right)=\int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\Bigg\{-\left(\frac{2}{\bar{\epsilon}}-\log\frac{m^2}{\mu^2}\right)\frac{1}{24}\,F_{\mu\nu}F^{\mu\nu}$$

$$+\frac{1}{m^2}\left[-\frac{1}{60}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)+\frac{1}{360}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\}.$$

Remaining trace is over field components.

Log-type supertraces

- Results are universal. Using CDE (later in the talk) to carry out the **functional part** of the supertraces, we obtain:

$$\frac{i}{2}\,\mathrm{STr}\log\left(P^2-m^2\right)=\int \mathrm{d}^dx\,\frac{1}{16\pi^2}\boxed{\mathrm{tr}}\Bigg\{-\left(\frac{2}{\bar\epsilon}-\log\frac{m^2}{\mu^2}\right)\frac{1}{24}F_{\mu\nu}F^{\mu\nu}$$
$$+\frac{1}{m^2}\left[-\frac{1}{120}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)-\frac{1}{180}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\},$$

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$$+\frac{1}{m^2}\left[-\frac{1}{60}\left(D^\mu F_{\mu\nu}\right)\left(D_\rho F^{\rho\nu}\right)+\frac{1}{360}\,i\,F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\right]+\ldots\Bigg\}.$$

Remaining trace is over field components.

Log-type supertraces

Integrate out a heavy ...	Operator coefficients $\times\, 16\pi^2$		
	$\mathrm{tr}_G\big(F_{\mu\nu}F^{\mu\nu}\big)$	$\mathrm{tr}_G\big[(D^\mu F_{\mu\nu})\,(D_\rho F^{\rho\nu})\big]$	$\mathrm{tr}_G\big(iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\big)$
real scalar	$\frac{1}{24}\,\log\frac{m^2}{\mu^2}$	$-\frac{1}{120}\,\frac{1}{m^2}$	$-\frac{1}{180}\,\frac{1}{m^2}$
complex scalar	$\frac{1}{12}\,\log\frac{m^2}{\mu^2}$	$-\frac{1}{60}\,\frac{1}{m^2}$	$-\frac{1}{90}\,\frac{1}{m^2}$
Majorana fermion	$\frac{1}{6}\,\log\frac{m^2}{\mu^2}$	$-\frac{1}{15}\,\frac{1}{m^2}$	$\frac{1}{90}\,\frac{1}{m^2}$
Dirac fermion	$\frac{1}{3}\,\log\frac{m^2}{\mu^2}$	$-\frac{2}{15}\,\frac{1}{m^2}$	$\frac{1}{45}\,\frac{1}{m^2}$
real vector	$\frac{1}{6}\left(\log\frac{m^2}{\mu^2}+\frac{1}{2}\right)$	$-\frac{1}{30}\,\frac{1}{m^2}$	$-\frac{1}{45}\,\frac{1}{m^2}$
ghost	$-\frac{1}{12}\,\log\frac{m^2}{\mu^2}$	$\frac{1}{60}\,\frac{1}{m^2}$	$\frac{1}{90}\,\frac{1}{m^2}$

Table 1. Universal results for log-type supertraces up to dimension six.

trace over gauge indices:

$$\mathrm{tr}_G\big(F_{\mu\nu}F^{\mu\nu}\big) = C_\Phi\, g^2\, G^a_{\mu\nu} G^{a\mu\nu}\,,$$

$$\mathrm{tr}_G\big[(D^\mu F_{\mu\nu})\,(D_\rho F^{\rho\nu})\big] = C_\Phi\, g^2 \big(D^\mu G^a_{\mu\nu}\big)^2\,,$$

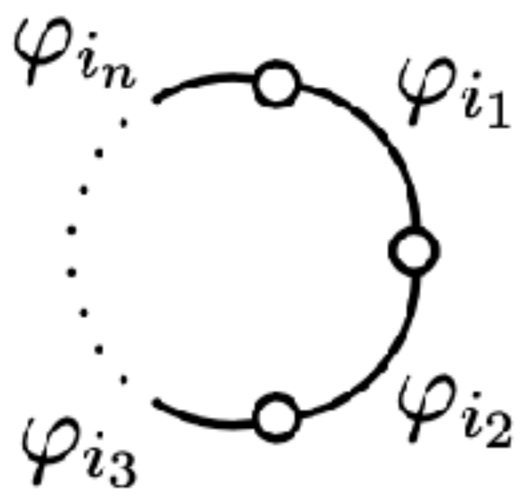
$$\mathrm{tr}_G\big(iF_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu\big) = -C_\Phi\,\frac{1}{2}\,g^3 f^{abc} G^{a\nu}_{\mu} G^{b\rho}_{\nu} G^{c\mu}_{\rho}\,,$$

where C_Φ is defined by $\mathrm{tr}_G\big(T^a_\Phi\, T^b_\Phi\big) = C_\Phi\, \delta^{ab}$.

Power-type supertraces

$$-\frac{i}{2}\frac{1}{n}\,\mathrm{STr}\left[\left(\mathbf{K}^{-1}\mathbf{X}\right)^n\right]\Big|_{\text{hard}} = -\frac{i}{2}\frac{1}{n}\sum_{i_1,\ldots,i_n}\mathrm{STr}\left(\boxed{\frac{1}{K_{i_1}}}\boxed{X_{i_1 i_2}}\boxed{\frac{1}{K_{i_2}}}\boxed{X_{i_2 i_3}}\cdots\boxed{\frac{1}{K_{i_n}}}\boxed{X_{i_n i_1}}\right)\Big|_{\text{hard}}$$

- This has the structure of a 1-loop graph (propagators & vertices).



$$\equiv -\frac{i}{2}\boxed{\frac{1}{r}}\mathrm{STr}\left[\frac{1}{K_{i_1}}X_{i_1 i_2}\frac{1}{K_{i_2}}X_{i_2 i_3}\cdots\frac{1}{K_{i_n}}X_{i_n i_1}\right]\Big|_{\text{hard}}$$

symmetry factor (graph has $\mathbb{Z}_r$ symmetry under rotation)

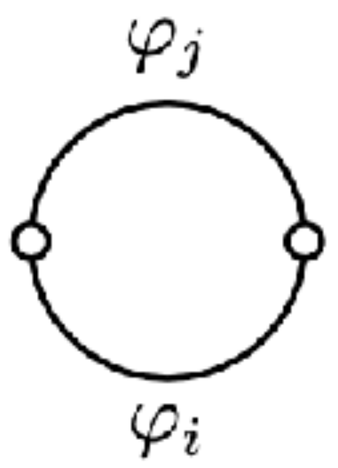
- Enumerate distinct graphs (just 1 topology).
- Finite # of graphs with $\dim X_{i_1 i_2} + \dim X_{i_2 i_3} + \cdots + \dim X_{i_n i_1} \leq$ desired operator dim in EFT.

Power-type supertraces

- Results depend on specific forms of X_{ij} .
  - Example:

Suppose: $K_i = P^2 - M^2$ (heavy), $K_j = P^2$ (massless),

$$X_{ij} = U_1 + \bar{Z}^\mu P_\mu \ , \ \ X_{ji} = U_2 + P_\mu Z^\mu .$$



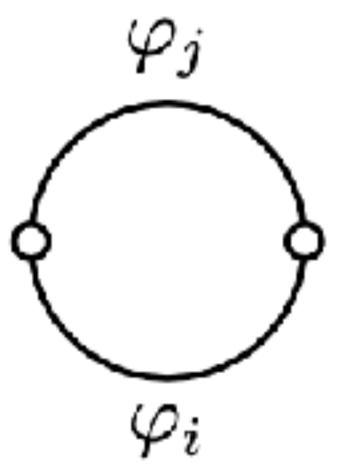
$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{K_i}X_{ij}\,\frac{1}{K_j}X_{ji}\right)\bigg|_{\text{hard}}$$

$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}U_2\right)\bigg|_{\text{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}P_\mu Z^\mu\right)\bigg|_{\text{hard}}$$

$$- \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}U_2\right)\bigg|_{\text{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}P_\nu Z^\nu\right)\bigg|_{\text{hard}} .$$

Power-type supertraces

- Results depend on specific forms of X_{ij} .
  - Example:



$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{K_i}X_{ij}\,\frac{1}{K_j}X_{ji}\right)\Big|_{\mathrm{hard}}$$

$$= -\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}U_2\right)\Big|_{\mathrm{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}P_\mu Z^\mu\right)\Big|_{\mathrm{hard}}$$

$$- \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}U_2\right)\Big|_{\mathrm{hard}} - \frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}P_\nu Z^\nu\right)\Big|_{\mathrm{hard}}$$

2) substitute in explicit expressions of U_1, U_2, Z^μ, $\overline{Z}^\mu$ that are derived from $\mathscr{L}_{\mathrm{UV}}$.

1) carry out the functional part of the supertraces with CDE.

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}U_2\right)\Big|_{\mathrm{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{2}\left(1-\log\frac{M^2}{\mu^2}\right)U_1U_2 + \frac{1}{4M^2}\,(D^\mu U_1)\,(D_\mu U_2) + \ldots\right\},$$

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}U_1\,\frac{1}{P^2}P_\mu Z^\mu\right)\Big|_{\mathrm{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{4}\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)i\,U_1\,(D_\mu Z^\mu) + \ldots\right\},$$

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}U_2\right)\Big|_{\mathrm{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{-\frac{1}{4}\left(\frac{1}{2}-\log\frac{M^2}{\mu^2}\right)i\,(D_\mu\bar{Z}^\mu)\,U_2 + \ldots\right\},$$

$$-\frac{i}{2}\,\mathrm{STr}\left(\frac{1}{P^2-M^2}\bar{Z}^\mu P_\mu\,\frac{1}{P^2}P_\nu Z^\nu\right)\Big|_{\mathrm{hard}}$$
$$= \int \mathrm{d}^d x\,\frac{1}{16\pi^2}\,\mathrm{tr}\left\{\frac{1}{8}M^2\left(\frac{3}{2}-\log\frac{M^2}{\mu^2}\right)\bar{Z}^\mu Z_\mu\right.$$
$$-\frac{1}{8}\,i\,\bar{Z}^\mu F_{\mu\nu}Z^\nu - \frac{1}{24}\left(\frac{5}{6}-\log\frac{M^2}{\mu^2}\right)(D^\mu\bar{Z}^\nu)\,(D_\mu Z_\nu)$$
$$\left.+\frac{1}{12}\left(\frac{1}{3}-\log\frac{M^2}{\mu^2}\right)\left[(D_\mu\bar{Z}^\mu)(D_\nu Z^\nu) + (D_\nu\bar{Z}^\mu)(D_\mu Z^\nu)\right] + \ldots\right\}.$$

Summary of the prescription

$$\mathcal{L}_{\mathrm{UV}}[\Phi, \phi]$$

$\boxed{\mathrm{EOM}}$ $\qquad \Phi_{\mathrm{c}}[\phi]$

$$\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi]$$

Summary of the prescription

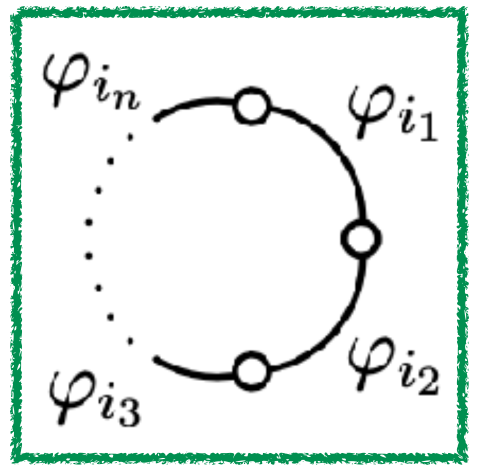
$$
\begin{tikzcd}
 & & \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \arrow[dl] \arrow[dr] & \\
\boxed{\text{EOM}} & \Phi_{\mathrm{c}}[\phi] \arrow[rr] \arrow[d] & & \boldsymbol{K}, \boldsymbol{X} \\
 & \mathcal{L}^{(\mathrm{tree})}_{\mathrm{EFT}}[\phi] & &
\end{tikzcd}
$$

$$\boxed{-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_{\mathrm{c}}[\phi]} = \mathbf{K} - \mathbf{X}}$$

Summary of the prescription

$$\begin{tikzcd}[column sep=small]
 & \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \arrow[dl] \arrow[dr] & \\
\boxed{\text{EOM}} \quad \Phi_{\mathrm{c}}[\phi] \arrow[rr] \arrow[dd] & & \boldsymbol{K}, \boldsymbol{X} \arrow[d, "\text{Enumerate}"] \\
 & & \text{Functional supertraces} \\
\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi] & &
\end{tikzcd}$$

$$\boxed{-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\bigg|_{\Phi=\Phi_{\mathrm{c}}[\phi]} = \mathbf{K} - \mathbf{X}}$$

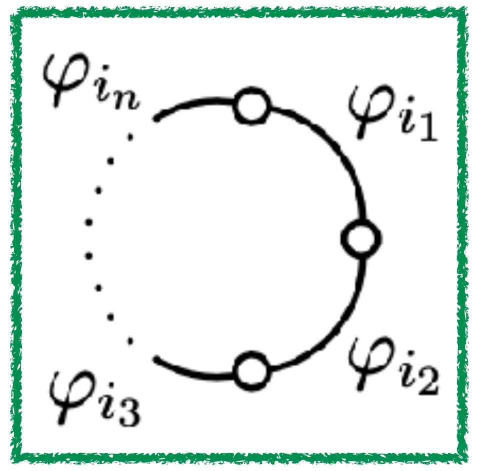


Summary of the prescription

$$\begin{tikzcd}
 & \mathcal{L}_{\mathrm{UV}}[\Phi, \phi] \arrow[dl] \arrow[dr] & \\
\Phi_{\mathrm{c}}[\phi] \arrow[rr] \arrow[dd] & & \boldsymbol{K}, \boldsymbol{X} \arrow[d, "\text{Enumerate}"] \\
 & & \text{Functional supertraces} \arrow[d, "\text{Evaluate}"] \\
\mathcal{L}_{\mathrm{EFT}}^{(\mathrm{tree})}[\phi] & & \mathcal{L}_{\mathrm{EFT}}^{(1\text{-loop})}[\phi]
\end{tikzcd}$$

$\boxed{\text{EOM}}$

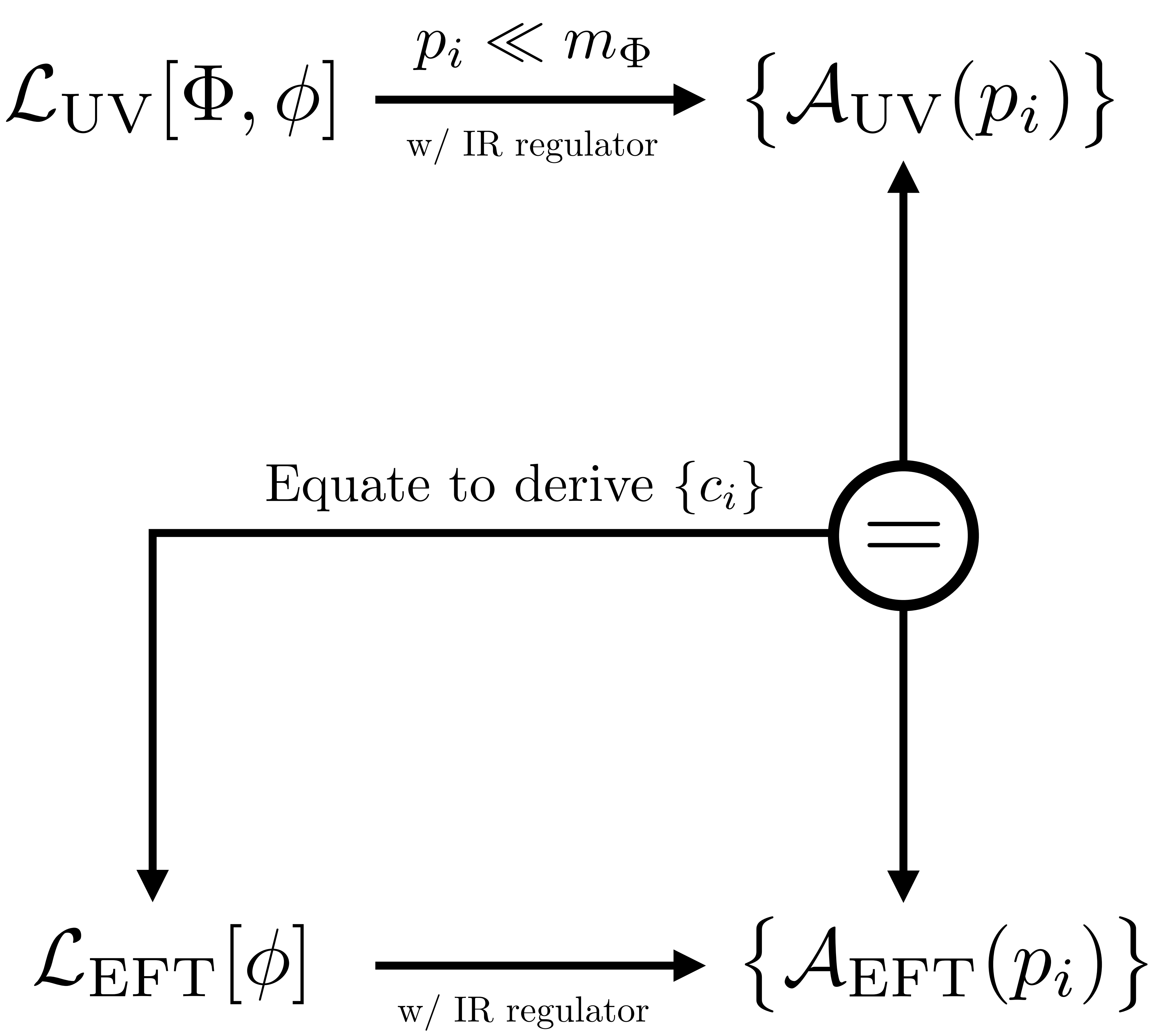
$$\boxed{-\left.\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \varphi^2}\right|_{\Phi=\Phi_{\mathrm{c}}[\phi]} = \mathbf{K} - \mathbf{X}}$$



$\boxed{\text{(automated) CDE + matrix algebra}}$

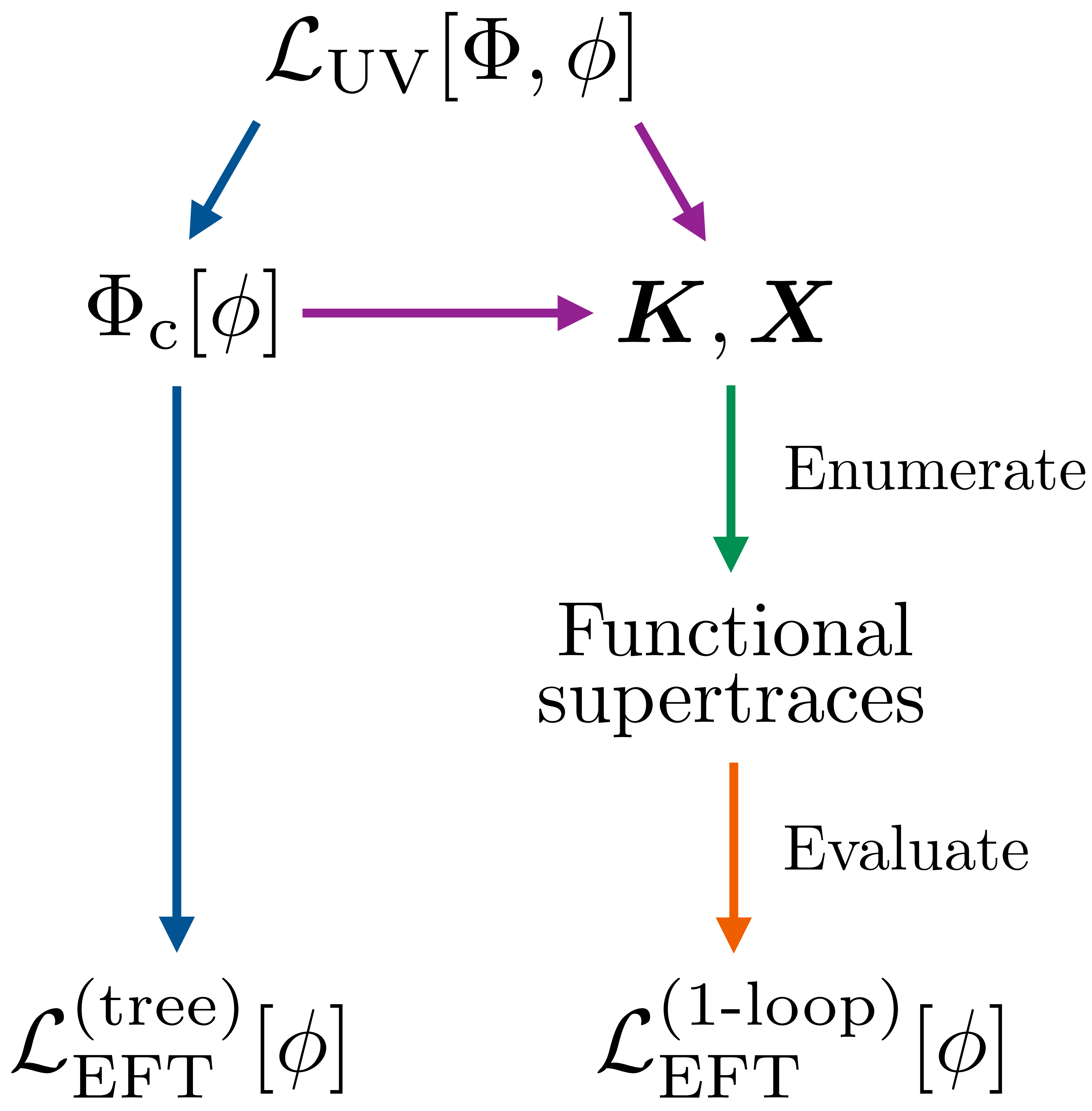
Amplitude matching

(with Feynman diagrams)



Functional matching

(our prescription)



No prior determination of operator basis.
No (amplitude) calculations within EFT.
No keeping track of IR details.

Outline

- ➤ What is functional matching, and what is new? ✅
- ➤ The prescription. ✅
- ➤ Example: matching the singlet scalar extended SM onto SMEFT up to dim-6.
- ➤ CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

SM + heavy singlet scalar S

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2S^2 - A|H|^2S - \frac{1}{2}\kappa|H|^2S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

S. A. R. Ellis, J. Quevillon, T. You, ZZ, 1706.07765 (bosonic sector, functional matching)
M. Jiang, N. Craig, Y. Li, D. Sutherland, 1811.08878 (full dim-6, functional + amplitude matching)
U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini, A. Weiler, 2003.05936 (full dim-6, amplitude matching)

Step 1: tree-level matching

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2S^2 - A|H|^2S - \frac{1}{2}\kappa|H|^2S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Heavy field's EOM:

$$\frac{\delta S_{\text{UV}}}{\delta S} = -A|H|^2 + \left(P^2 - M^2 - \kappa|H|^2\right)S - \frac{1}{2}\mu_S S^2 - \frac{1}{3!}\lambda_S S^3 = 0\,.$$

- Solve order by order:

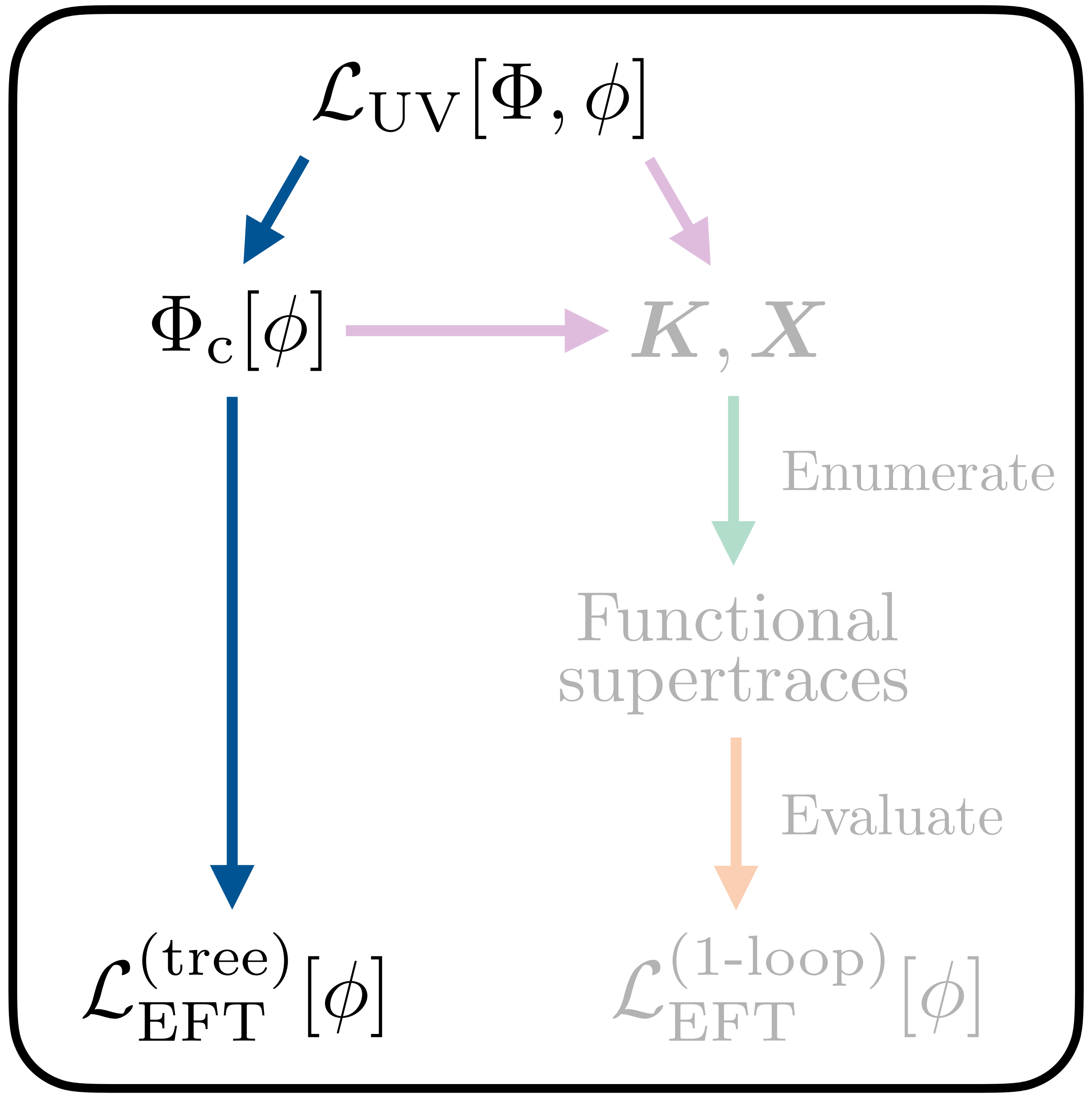
$$S_{\text{c}} = S_{\text{c}}^{(2)} + S_{\text{c}}^{(4)} + S_{\text{c}}^{(6)} + \dots$$

$$S_{\text{c}}^{(2)} = -\tfrac{A}{M^2}|H|^2\,,$$

$$S_{\text{c}}^{(4)} = \tfrac{A}{M^4}\left[\left(\partial^2|H|^2\right) + \left(\kappa - \tfrac{\mu_S A}{2M^2}\right)|H|^4\right],$$

$$S_{\text{c}}^{(6)} = -\tfrac{A}{M^6}\left\{\left(\kappa - \tfrac{\mu_S A}{M^2}\right)|H|^2\left(\partial^2|H|^2\right) + \left[\left(\kappa - \tfrac{\mu_S A}{M^2}\right)\left(\kappa - \tfrac{\mu_S A}{2M^2}\right) - \tfrac{\lambda_S A^2}{6M^2}\right]|H|^6 + \partial^2\left[\left(\partial^2|H|^2\right) + \left(\kappa - \tfrac{\mu_S A}{2M^2}\right)|H|^4\right]\right\}.$$

$$\Rightarrow \quad \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{SM}} + \frac{A^2}{2M^2}|H|^4 - \frac{A^2}{2M^4}|H|^2\left(\partial^2|H|^2\right) - \frac{A^2}{2M^4}\left(\kappa - \frac{\mu_S A}{3M^2}\right)|H|^6\,.$$

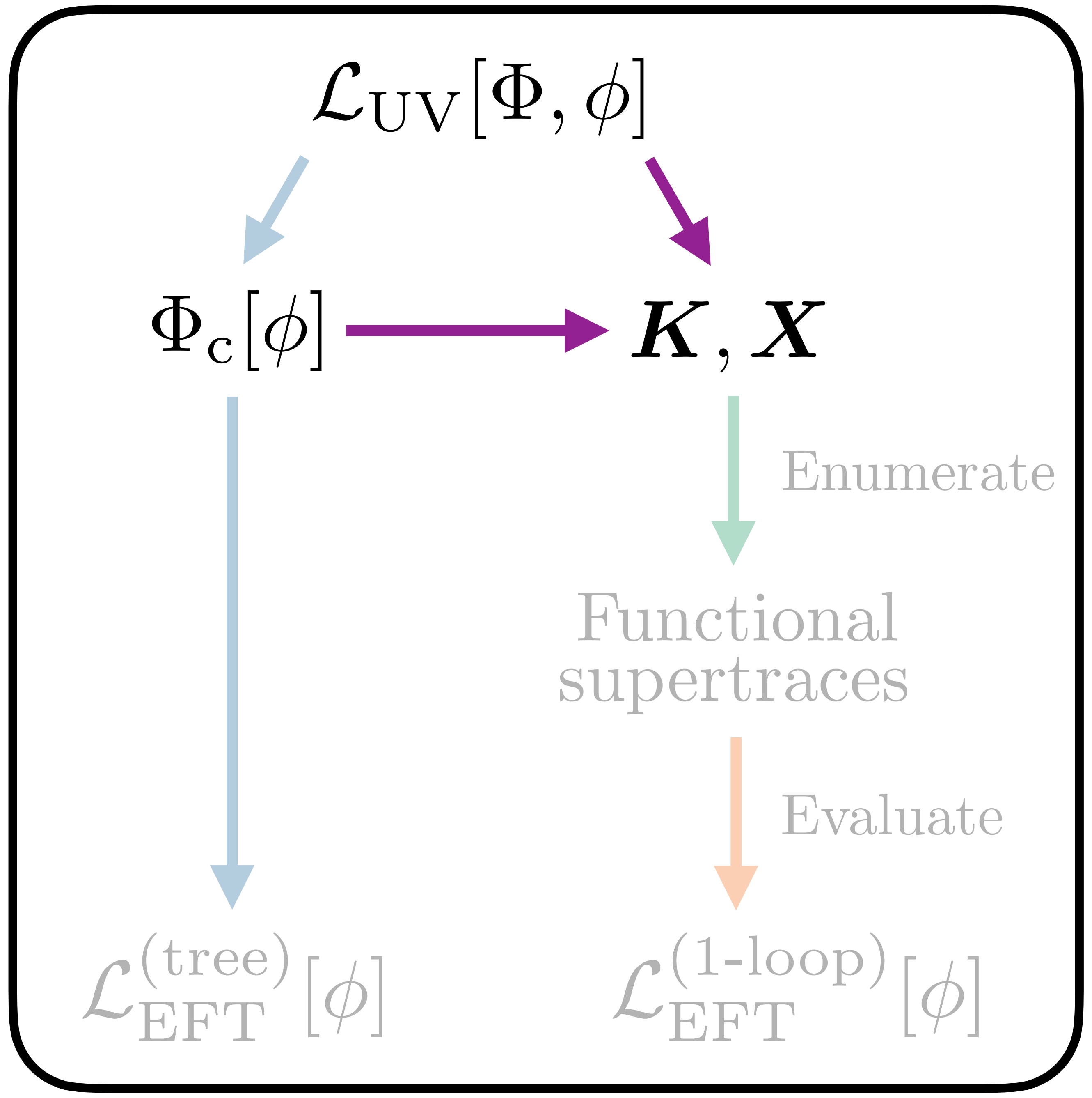


Step 2: derive K and X matrices

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa |H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Recall:

$$-\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\varphi^2} = \mathbf{K} - \mathbf{X} \qquad K_i = \begin{cases} P^2 - m_i^2 & \text{(spin-0)} \\ \gamma^\mu P_\mu - m_i & \text{(spin-1/2)} \\ -\eta^{\mu\nu}\left(P^2 - m_i^2\right) & \text{(spin-1, Feynman gauge } \xi = 1\text{)} \end{cases}$$



- Field content:

$$\varphi_i \in \{\varphi_S\,,\quad \varphi_H\,,\quad \varphi_q\,,\quad \varphi_u\,,\quad \varphi_d\,,\quad \varphi_l\,,\quad \varphi_e\,,\quad \varphi_G\,,\quad \varphi_W\,,\quad \varphi_B\}\,,$$

$$\boxed{\bar\varphi_i \in \{\bar\varphi_S\,,\quad \bar\varphi_H\,,\quad \bar\varphi_q\,,\quad \bar\varphi_u\,,\quad \bar\varphi_d\,,\quad \bar\varphi_l\,,\quad \bar\varphi_e\,,\quad \bar\varphi_G\,,\quad \bar\varphi_W\,,\quad \bar\varphi_B\}}\,,$$

where conjugate fields defined s.t. **K** has the assumed block-diagonal form

$$\varphi_S = S\,,\qquad \varphi_H = \begin{pmatrix} H \\ H^* \end{pmatrix}\,,\qquad \boxed{\varphi_f = \begin{pmatrix} f \\ f^c \end{pmatrix}}\,,\qquad \varphi_V = V\,,$$

$$\bar\varphi_S = S\,,\qquad \bar\varphi_H = \begin{pmatrix} H^\dagger & H^T \end{pmatrix}\,,\qquad \bar\varphi_f = \begin{pmatrix} \bar f & \bar f^c \end{pmatrix}\,,\qquad \bar\varphi_V = V\,,$$

with $f = q, u, d, l, e,$ and $V = G, W, B.$

Dirac spinors including auxiliary wrong-chirality fields (denoted with prime):

$$q = \begin{pmatrix} q_a \\ q'^{\dagger\dot a} \end{pmatrix}\,,\qquad q^c = \begin{pmatrix} q'_a \\ q^{\dagger\dot a} \end{pmatrix}$$

$$u = \begin{pmatrix} u'_a \\ u^{\dagger\dot a} \end{pmatrix}\,,\qquad u^c = \begin{pmatrix} u_a \\ u'^{\dagger\dot a} \end{pmatrix}$$

Step 2: derive K and X matrices

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Example: SS block.

$$\delta^2 \mathcal{L}_{\rm UV} \supset \delta^2 \left[\frac{1}{2}\, S\,(P^2 - M^2)\, S - \frac{1}{2}\,\kappa\,|H|^2 S^2 - \frac{1}{3!}\,\mu_S\, S^3 - \frac{1}{4!}\,\lambda_S\, S^4\right]$$

$$\supset \delta S\, \underbrace{(P^2 - M^2)}_{K_S}\, \delta S - \delta S\, \underbrace{\left(\kappa\,|H|^2 + \mu_S\, S + \frac{1}{2}\,\lambda_S\, S^2\right)}_{X_{SS} = U_{SS}\ \text{(non-derivative interactions)}}\, \delta S\,.$$

Step 2: derive K and X matrices

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

➤ Example: *SH* and *HS* blocks.

$$\delta^2\mathcal{L}_{\rm UV} \supset \delta^2\left[-A\,|H|^2 S - \frac{1}{2}\,\kappa\,|H|^2 S^2\right]$$

$$\supset -2\,(\delta H^\dagger H + H^\dagger \delta H)\,(A+\kappa\,S)\,\delta S$$

$$= -(\delta H^\dagger H + H^T \delta H^* + H^\dagger \delta H + \delta H^T H^*)\,(A+\kappa\,S)\,\delta S$$

$$= -\delta S\boxed{\begin{pmatrix}(A+\kappa\,S)\,H^\dagger & (A+\kappa\,S)\,H^T\end{pmatrix}}\begin{pmatrix}\delta H \\ \delta H^*\end{pmatrix}$$

$X_{SH} = U_{SH}$

$$-\begin{pmatrix}\delta H^\dagger & \delta H^T\end{pmatrix}\boxed{\begin{pmatrix}(A+\kappa\,S)\,H \\ (A+\kappa\,S)\,H^*\end{pmatrix}}\delta S\,.$$

$X_{HS} = U_{HS}$

Step 2: derive K and X matrices

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2 S^2 - A|H|^2 S - \frac{1}{2}\kappa|H|^2 S^2 - \frac{1}{3!}\mu_S S^3 - \frac{1}{4!}\lambda_S S^4$$

- Example: HW and WH blocks. (The only derivative interactions in the SM are these and HB, BH.)

$$\begin{aligned}
\delta^2\mathcal{L}_{\rm UV} &\supset \delta^2\left(|D_\mu H|^2\right) \\
&= \left[\delta^2(D_\mu H)^\dagger\right](D^\mu H) + (D_\mu H^\dagger)\left[\delta^2(D^\mu H)\right] + 2\left[\delta\,(D_\mu H)^\dagger\right]\left[\delta\,(D^\mu H)\right] \\
&\supset i g_2\, \delta W^I_\mu\, \delta H^\dagger \sigma^I\, (D^\mu H) - i g_2\, (D_\mu H)^\dagger \sigma^I\, \delta H\, \delta B^\mu \\
&\quad - i g_2\, (D_\mu \delta H)^\dagger \sigma^I\, H\, \delta W^I_\mu + i g_2\, \delta W^I_\mu\, H^\dagger \sigma^I\, (D^\mu \delta H) \\
&= -\frac{g_2}{2}\, \delta W^I_\mu \left[-i\, \delta H^\dagger \sigma^I\, (D^\mu H) - i\, (D^\mu H)^T \sigma^{I*}\, \delta H^*\right. \\
&\qquad + i\, (D_\mu H)^\dagger \sigma^I\, \delta H + i\, \delta H^T \sigma^{I*}\, (D_\mu H)^* \\
&\qquad + i\, (D_\mu \delta H)^\dagger \sigma^I\, H + i\, H^T \sigma^{I*}\, (D_\mu \delta H)^* \\
&\qquad \left. - i\, H^\dagger \sigma^I\, (D^\mu \delta H) - i\, (D^\mu \delta H)^T \sigma^{I*}\, H^*\right] \\
&\overset{\rm IBP}{=} -\begin{pmatrix}\delta H^\dagger & \delta H^T\end{pmatrix} \boxed{\left[\begin{pmatrix} -\frac{i g_2}{2}\, \sigma^J\, (D^\nu H) \\ \frac{i g_2}{2}\, \sigma^{J*}\, (D^\nu H)^* \end{pmatrix} + i D_\rho \begin{pmatrix} -\eta^{\rho\nu}\, \frac{g_2}{2}\, \sigma^J\, H \\ \eta^{\rho\nu}\, \frac{g_2}{2}\, \sigma^{J*}\, H^* \end{pmatrix}\right]} \delta W^J_\nu \\
&\quad - \delta W^I_\mu \boxed{\left[\begin{pmatrix} \frac{i g_2}{2}\, (D^\mu H)^\dagger \sigma^I & -\frac{i g_2}{2}\, (D^\mu H)^T \sigma^{I*} \end{pmatrix} + \begin{pmatrix} -\eta^{\rho\mu}\, \frac{g_2}{2}\, H^\dagger \sigma^I & \eta^{\rho\mu}\, \frac{g_2}{2}\, H^T \sigma^{I*} \end{pmatrix} i D_\rho\right]} \begin{pmatrix} \delta H \\ \delta H^* \end{pmatrix},
\end{aligned}$$

$$X_{HW} = U_{HW} + P_\rho Z^\rho_{HW}$$

$$X_{WH} = U_{WH} + \bar{Z}^\rho_{WH} P_\rho$$

Step 2: derive K and X matrices

➤ Full results:

Scalar sector entries

$$U_{SS} = \kappa\,|H|^2 + \mu_S\,S_c + \frac{1}{2}\lambda_S\,S_c^2\,. \tag{B.5}$$

$$U_{SH} = (A+\kappa\,S_c)\begin{pmatrix} H^\dagger & H^T\end{pmatrix}\,,\quad U_{HS} = (A+\kappa\,S_c)\begin{pmatrix} H \\ H^*\end{pmatrix}\,. \tag{B.6}$$

$$U_{HH} = \left(A\,S_c + \frac{1}{2}\kappa\,S_c^2\right)\begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1}\end{pmatrix} + \lambda_H\begin{pmatrix} |H|^2\mathbb{1} + H H^\dagger & H H^T \\ H^* H^\dagger & |H|^2\mathbb{1} + H^* H^T\end{pmatrix}. \tag{B.7}$$

Fermion-fermion entries

$$U_{qu} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_u\,\frac{1+\gamma^5}{2}\,\widetilde{H} & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2}\,\widetilde{H}^* \end{pmatrix},\quad U_{uq} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2}\,\widetilde{H}^\dagger & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2}\,\widetilde{H}^T \end{pmatrix}. \tag{B.8}$$

$$U_{qd} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_d\,\frac{1+\gamma^5}{2}\,H & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2}\,H^* \end{pmatrix},\quad U_{dq} = \begin{pmatrix} \mathbb{1}\,\boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2}\,H^\dagger & 0 \\ 0 & \mathbb{1}\,\boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2}\,H^T \end{pmatrix}. \tag{B.9}$$

$$U_{le} = \begin{pmatrix} \boldsymbol{y}_e\,\frac{1+\gamma^5}{2}\,H & 0 \\ 0 & \boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2}\,H^* \end{pmatrix},\qquad U_{el} = \begin{pmatrix} \boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2}\,H^\dagger & 0 \\ 0 & \boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2}\,H^T \end{pmatrix}. \tag{B.10}$$

Vector-vector entries

$$U_{GG}^{\mu A,\,\nu B} = 2\,g_3\,f^{ABC}G^{C\mu\nu}\,. \tag{B.11}$$

$$U_{WW}^{\mu I,\,\nu J} = 2\,g_2\,\epsilon^{IJK}W^{K\mu\nu} - \frac{g_2^2}{2}\,\eta^{\mu\nu}\,\delta^{IJ}\,|H|^2\,. \tag{B.12}$$

$$U_{BB}^{\mu,\,\nu} = -\frac{g_1^2}{2}\,\eta^{\mu\nu}\,|H|^2\,. \tag{B.13}$$

$$U_{WB}^{\mu I,\,\nu} = -\frac{g_1 g_2}{2}\,\eta^{\mu\nu}\,H^\dagger\sigma^I H\,,\qquad U_{BW}^{\mu,\,\nu J} = -\frac{g_1 g_2}{2}\,\eta^{\mu\nu}\,H^\dagger\sigma^J H\,. \tag{B.14}$$

Higgs-fermion entries

$$U_{Hq} = \begin{pmatrix} \mathbb{1}\,\bar{d}\,\boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2} & -\boldsymbol{\epsilon}\,\bar{u}^c\,\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2} \\ -\boldsymbol{\epsilon}\,\bar{u}\,\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2} & \mathbb{1}\,\bar{d}^c\,\boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2} \end{pmatrix},\quad U_{qH} = \begin{pmatrix} \boldsymbol{y}_d\,\frac{1+\gamma^5}{2}\,d\,\mathbb{1} & \boldsymbol{y}_u\,\frac{1+\gamma^5}{2}\,u\,\boldsymbol{\epsilon} \\ \boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2}\,u^c\,\boldsymbol{\epsilon} & \boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2}\,d^c\,\mathbb{1} \end{pmatrix}. \tag{B.15}$$

$$U_{Hu} = \begin{pmatrix} (\bar{q}\,\boldsymbol{\epsilon})_{\bar\alpha}\,\boldsymbol{y}_u\,\frac{1+\gamma^5}{2} & 0 \\ 0 & (\bar{q}^c\,\boldsymbol{\epsilon})_\alpha\,\boldsymbol{y}_u^*\,\frac{1-\gamma^5}{2} \end{pmatrix},\quad U_{uH} = \begin{pmatrix} -\boldsymbol{y}_u^\dagger\,\frac{1-\gamma^5}{2}\,(\boldsymbol{\epsilon}\,q)_\beta & 0 \\ 0 & -\boldsymbol{y}_u^T\,\frac{1+\gamma^5}{2}\,(\boldsymbol{\epsilon}\,q^c)_{\bar\beta} \end{pmatrix}. \tag{B.16}$$

$$U_{Hd} = \begin{pmatrix} 0 & \bar{q}^c_{\bar\alpha}\,\boldsymbol{y}_d^*\,\frac{1-\gamma^5}{2} \\ \bar{q}_\alpha\,\boldsymbol{y}_d\,\frac{1+\gamma^5}{2} & 0 \end{pmatrix},\qquad U_{dH} = \begin{pmatrix} 0 & \boldsymbol{y}_d^\dagger\,\frac{1-\gamma^5}{2}\,q_\beta \\ \boldsymbol{y}_d^T\,\frac{1+\gamma^5}{2}\,q^c_{\bar\beta} & 0 \end{pmatrix}. \tag{B.17}$$

$$U_{Hl} = \begin{pmatrix} \mathbb{1}\,\bar{e}\,\boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2} & 0 \\ 0 & \mathbb{1}\,\bar{e}^c\,\boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2} \end{pmatrix},\qquad U_{lH} = \begin{pmatrix} \boldsymbol{y}_e\,\frac{1+\gamma^5}{2}\,e\,\mathbb{1} & 0 \\ 0 & \boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2}\,e^c\,\mathbb{1} \end{pmatrix}. \tag{B.18}$$

$$U_{He} = \begin{pmatrix} 0 & \bar{l}^c_{\bar\alpha}\,\boldsymbol{y}_e^*\,\frac{1-\gamma^5}{2} \\ \bar{l}_\alpha\,\boldsymbol{y}_e\,\frac{1+\gamma^5}{2} & 0 \end{pmatrix},\qquad U_{eH} = \begin{pmatrix} 0 & \boldsymbol{y}_e^\dagger\,\frac{1-\gamma^5}{2}\,l_\beta \\ \boldsymbol{y}_e^T\,\frac{1+\gamma^5}{2}\,l^c_{\bar\beta} & 0 \end{pmatrix}. \tag{B.19}$$

Higgs-vector entries

$$U_{HW}^{\nu J} = \frac{i g_2}{2}\begin{pmatrix} -\sigma^J(D^\nu H) \\ \sigma^{J*}(D^\nu H)^* \end{pmatrix},\quad U_{WH}^{\mu I} = \frac{i g_2}{2}\begin{pmatrix} (D^\mu H)^\dagger\,\sigma^I & -(D^\mu H)^T\,\sigma^{I*}\end{pmatrix}. \tag{B.20}$$

$$Z_{HW}^{\rho\nu J} = \eta^{\rho\nu}\,\frac{g_2}{2}\begin{pmatrix} -\sigma^J H \\ \sigma^{J*}H^* \end{pmatrix},\qquad \bar{Z}_{WH}^{\rho\mu I} = \eta^{\rho\mu}\,\frac{g_2}{2}\begin{pmatrix} -H^\dagger\sigma^I & H^T\sigma^{I*}\end{pmatrix}. \tag{B.21}$$

$$U_{HB}^{\nu} = \frac{i g_1}{2}\begin{pmatrix} -D^\nu H \\ (D^\nu H)^* \end{pmatrix},\qquad U_{BH}^{\mu} = \frac{i g_1}{2}\begin{pmatrix} (D^\mu H)^\dagger & -(D^\mu H)^T\end{pmatrix}. \tag{B.22}$$

$$Z_{HB}^{\rho\nu} = \eta^{\rho\nu}\,\frac{g_1}{2}\begin{pmatrix} -H \\ H^* \end{pmatrix},\qquad \bar{Z}_{BH}^{\rho\mu} = \eta^{\rho\mu}\,\frac{g_1}{2}\begin{pmatrix} -H^\dagger & H^T\end{pmatrix}. \tag{B.23}$$

Fermion-vector entries

$$U_{qG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B q \\ \gamma^\nu\lambda^{B*}q^c \end{pmatrix},\qquad U_{Gq}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{q}\,\gamma^\mu\lambda^A & \bar{q}^c\,\gamma^\mu\lambda^{A*}\end{pmatrix}, \tag{B.24}$$

$$U_{uG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B u \\ \gamma^\nu\lambda^{B*}u^c \end{pmatrix},\qquad U_{Gu}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{u}\,\gamma^\mu\lambda^A & \bar{u}^c\,\gamma^\mu\lambda^{A*}\end{pmatrix}, \tag{B.25}$$

$$U_{dG}^{\nu B} = \frac{g_3}{2}\begin{pmatrix} -\gamma^\nu\lambda^B d \\ \gamma^\nu\lambda^{B*}d^c \end{pmatrix},\qquad U_{Gd}^{\mu A} = \frac{g_3}{2}\begin{pmatrix} -\bar{d}\,\gamma^\mu\lambda^A & \bar{d}^c\,\gamma^\mu\lambda^{A*}\end{pmatrix}, \tag{B.26}$$

$$U_{qW}^{\nu J} = \frac{g_2}{2}\begin{pmatrix} -\gamma^\nu\sigma^J q \\ \gamma^\nu\sigma^{J*}q^c \end{pmatrix},\qquad U_{Wq}^{\mu I} = \frac{g_2}{2}\begin{pmatrix} -\bar{q}\,\gamma^\mu\sigma^I & \bar{q}^c\,\gamma^\mu\sigma^{I*}\end{pmatrix}, \tag{B.27}$$

$$U_{lW}^{\nu J} = \frac{g_2}{2}\begin{pmatrix} -\gamma^\nu\sigma^J l \\ \gamma^\nu\sigma^{J*}l^c \end{pmatrix},\qquad U_{Wl}^{\mu I} = \frac{g_2}{2}\begin{pmatrix} -\bar{l}\,\gamma^\mu\sigma^I & \bar{l}^c\,\gamma^\mu\sigma^{I*}\end{pmatrix}, \tag{B.28}$$

$$U_{qB}^{\nu} = \frac{g_1}{6}\begin{pmatrix} -\gamma^\nu q \\ \gamma^\nu q^c \end{pmatrix},\qquad U_{Bq}^{\mu} = \frac{g_1}{6}\begin{pmatrix} -\bar{q}\,\gamma^\mu & \bar{q}^c\,\gamma^\mu\end{pmatrix}, \tag{B.29}$$

$$U_{uB}^{\nu} = \frac{2g_1}{3}\begin{pmatrix} -\gamma^\nu u \\ \gamma^\nu u^c \end{pmatrix},\qquad U_{Bu}^{\mu} = \frac{2g_1}{3}\begin{pmatrix} -\bar{u}\,\gamma^\mu & \bar{u}^c\,\gamma^\mu\end{pmatrix}, \tag{B.30}$$

$$U_{dB}^{\nu} = -\frac{g_1}{3}\begin{pmatrix} -\gamma^\nu d \\ \gamma^\nu d^c \end{pmatrix},\qquad U_{Bd}^{\mu} = -\frac{g_1}{3}\begin{pmatrix} -\bar{d}\,\gamma^\mu & \bar{d}^c\,\gamma^\mu\end{pmatrix}, \tag{B.31}$$

$$U_{lB}^{\nu} = -\frac{g_1}{2}\begin{pmatrix} -\gamma^\nu l \\ \gamma^\nu l^c \end{pmatrix},\qquad U_{Bl}^{\mu} = -\frac{g_1}{2}\begin{pmatrix} -\bar{l}\,\gamma^\mu & \bar{l}^c\,\gamma^\mu\end{pmatrix}, \tag{B.32}$$

$$U_{eB}^{\nu} = -g_1\begin{pmatrix} -\gamma^\nu e \\ \gamma^\nu e^c \end{pmatrix},\qquad U_{Be}^{\mu} = -g_1\begin{pmatrix} -\bar{e}\,\gamma^\mu & \bar{e}^c\,\gamma^\mu\end{pmatrix}. \tag{B.33}$$

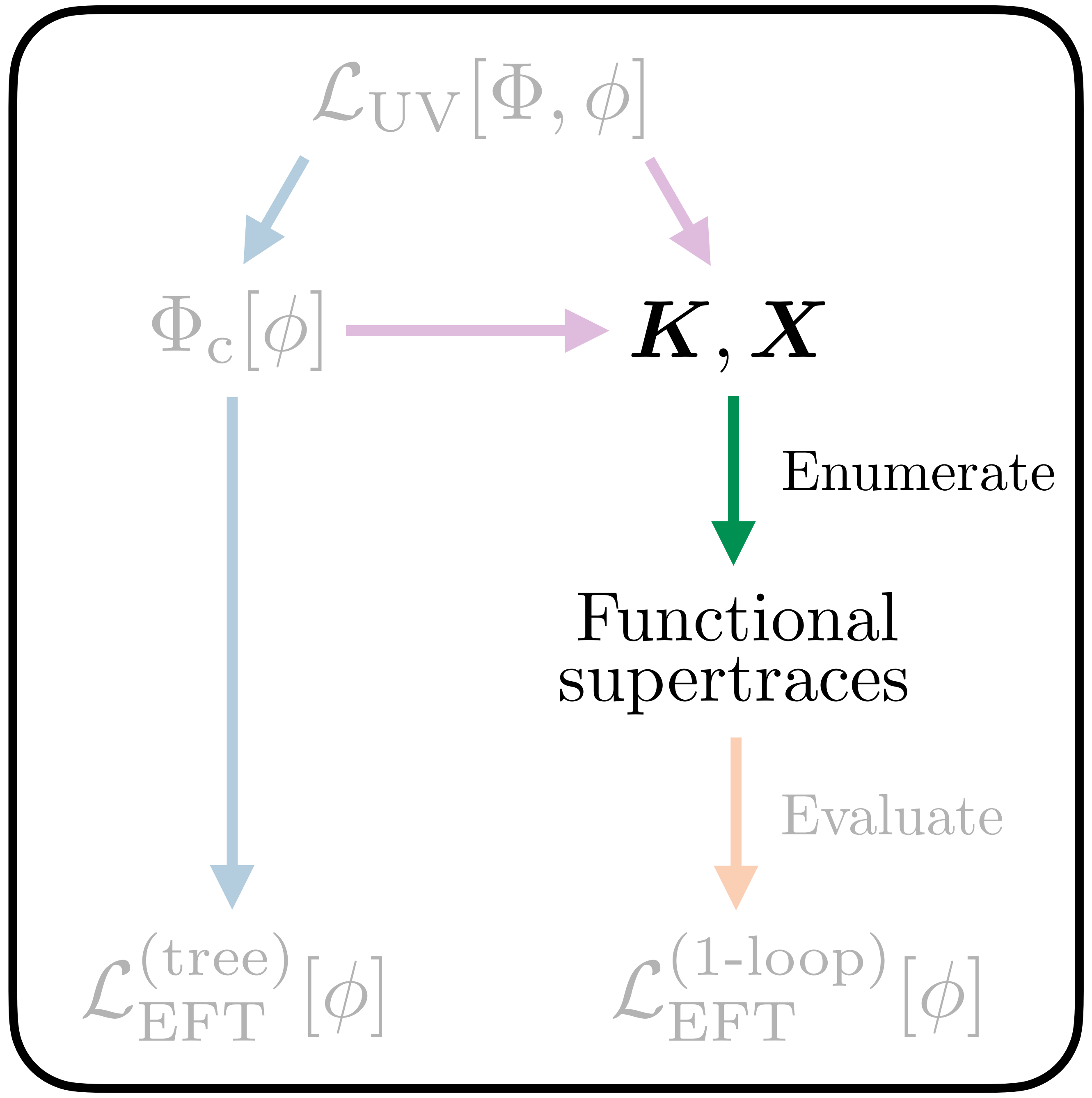
➤ A bit tedious, but the SM part is done once and for all.

Step 3: enumerate supertraces

➤ No log-type supertraces (S is a gauge singlet).

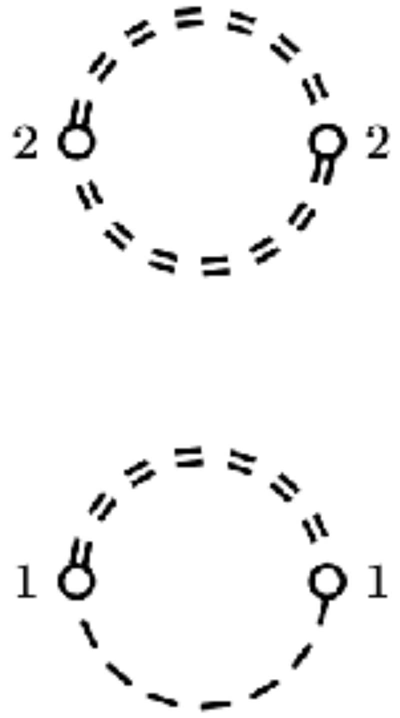
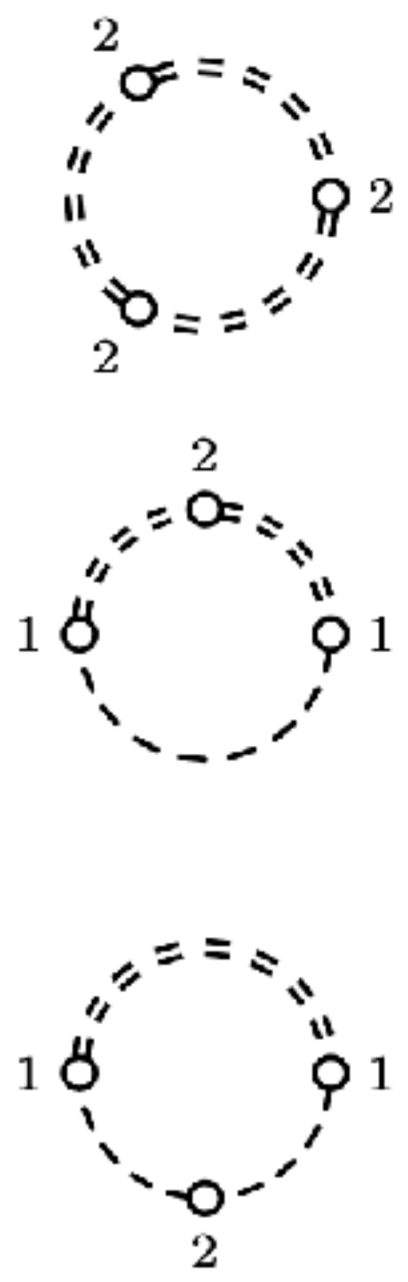
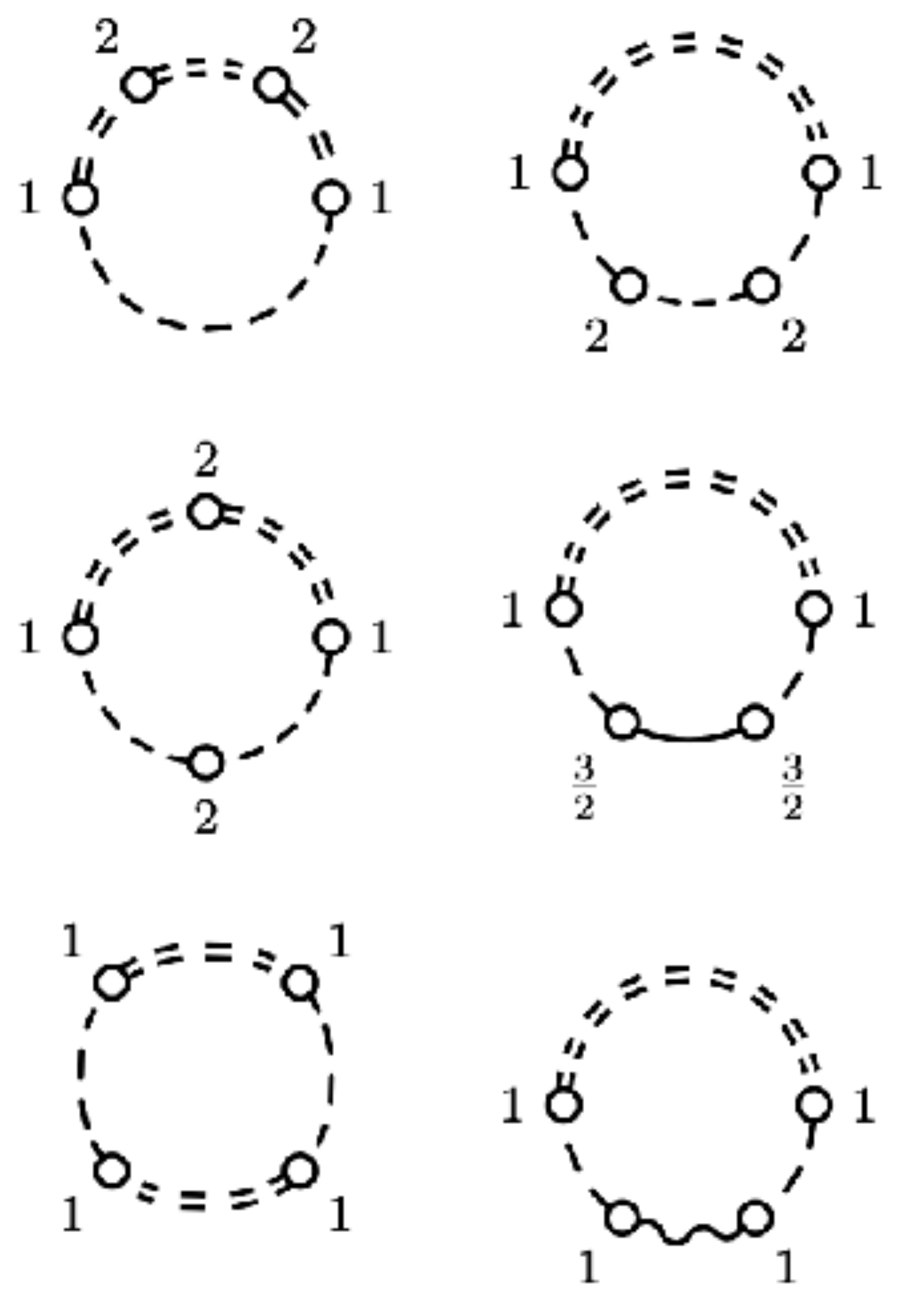
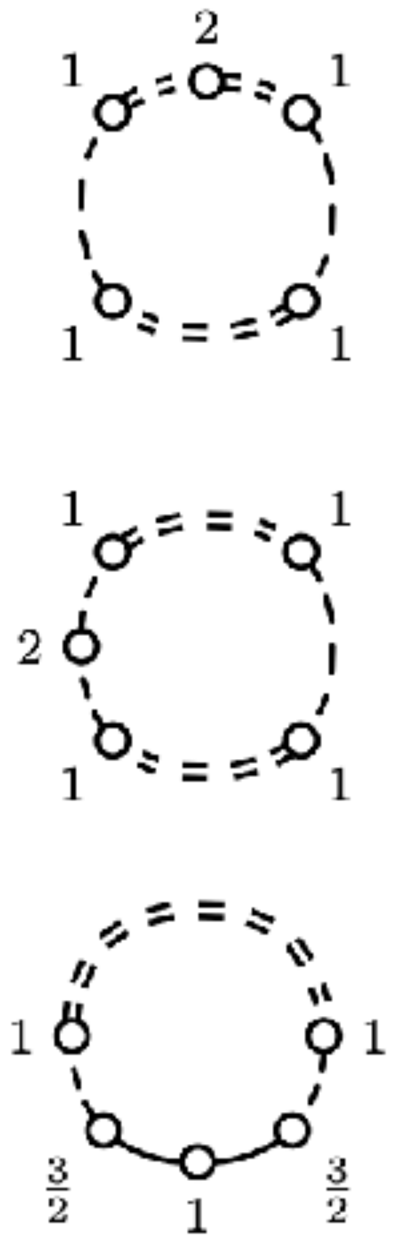
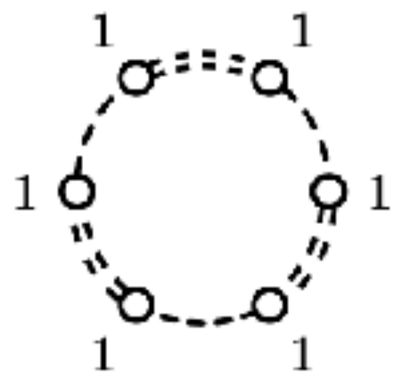
➤ Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.

$$
\dim(\boldsymbol{X}) \geq
\begin{array}{cc}
 & \begin{array}{cccccccccc} S & H & q & u & d & l & e & G & W & B \end{array} \\
\begin{array}{c} S \\ H \\ q \\ u \\ d \\ l \\ e \\ G \\ W \\ B \end{array} &
\begin{pmatrix}
2 & 1 & & & & & & & & \\
1 & 2 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & 1 & 1 \\
 & \frac{3}{2} & & 1 & 1 & & & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} \\
 & \frac{3}{2} & 1 & & & & & \frac{3}{2} & & \frac{3}{2} \\
 & \frac{3}{2} & 1 & & & & & \frac{3}{2} & & \frac{3}{2} \\
 & \frac{3}{2} & & & & & 1 & & \frac{3}{2} & \frac{3}{2} \\
 & \frac{3}{2} & & & & 1 & & & & \frac{3}{2} \\
 & & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & & 2 & & \\
 & 1 & \frac{3}{2} & & & \frac{3}{2} & & & 2 & 2 \\
 & 1 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & & 2 & 2
\end{pmatrix}
\end{array} .
$$



Step 3: enumerate supertraces

- No log-type supertraces (S is a gauge singlet).
- Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.

# of propagators	1	2	3	4	5	6
						

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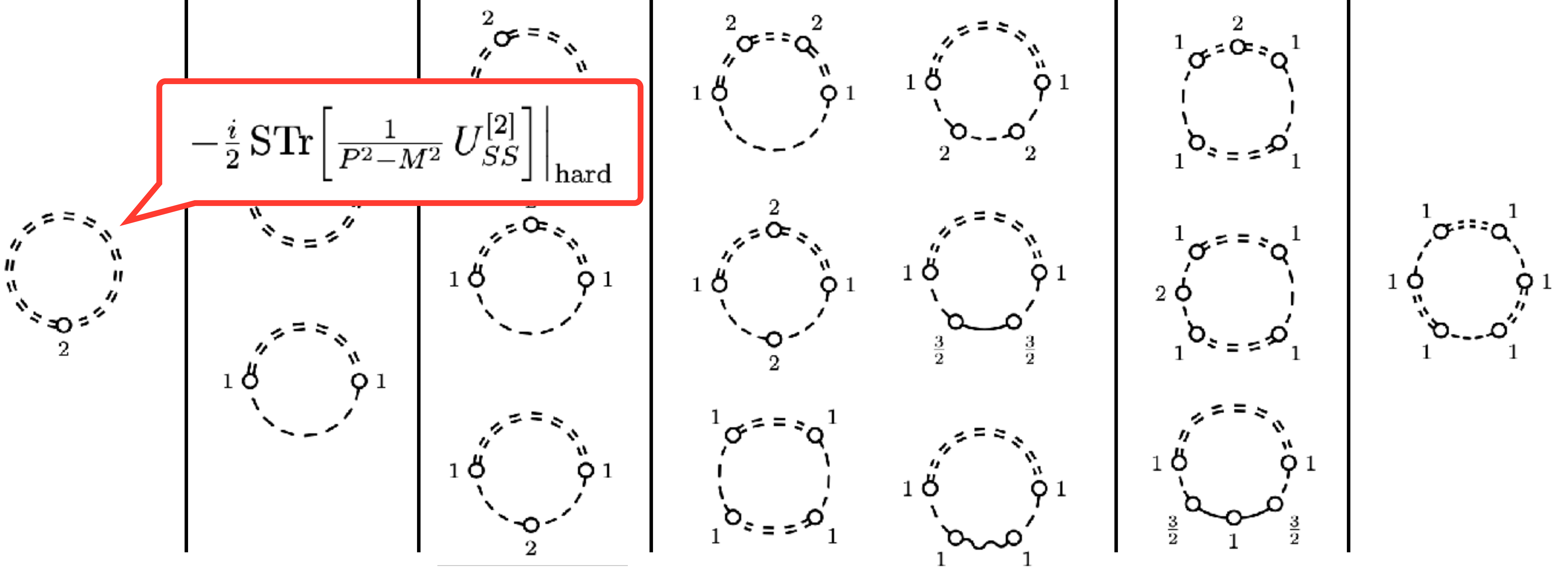

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[2]}_{SS}\right]\Big|_{\text{hard}}$$

Step 3: enumerate supertraces

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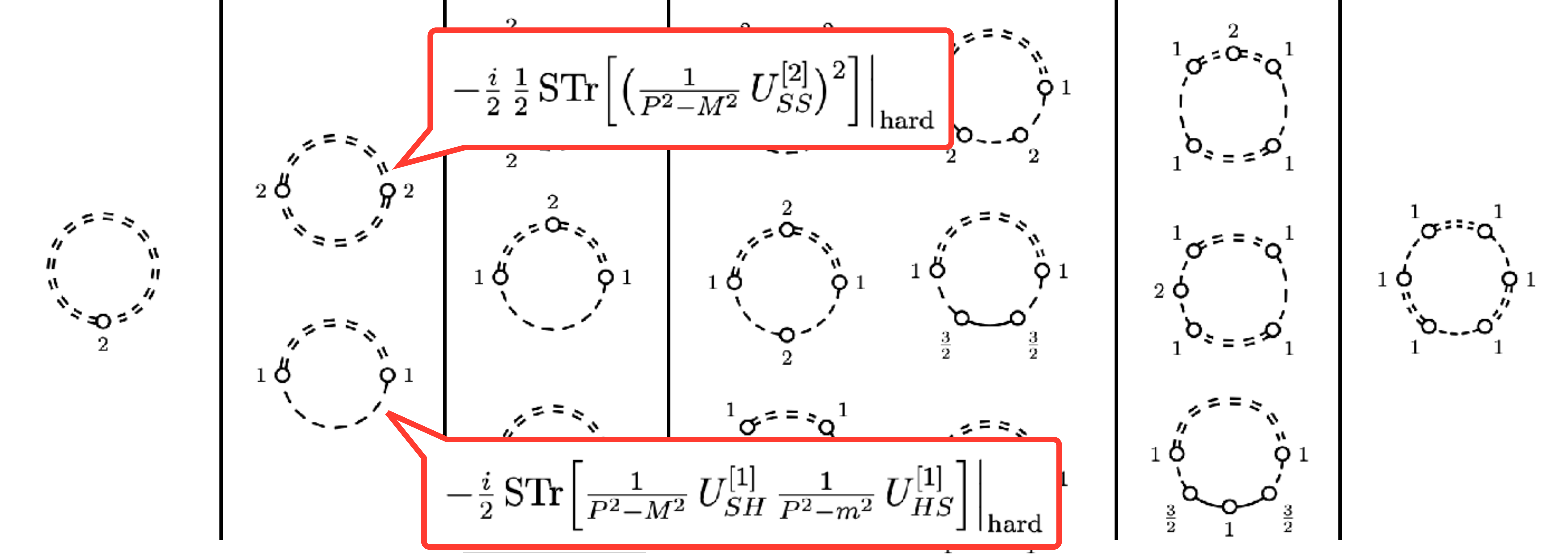
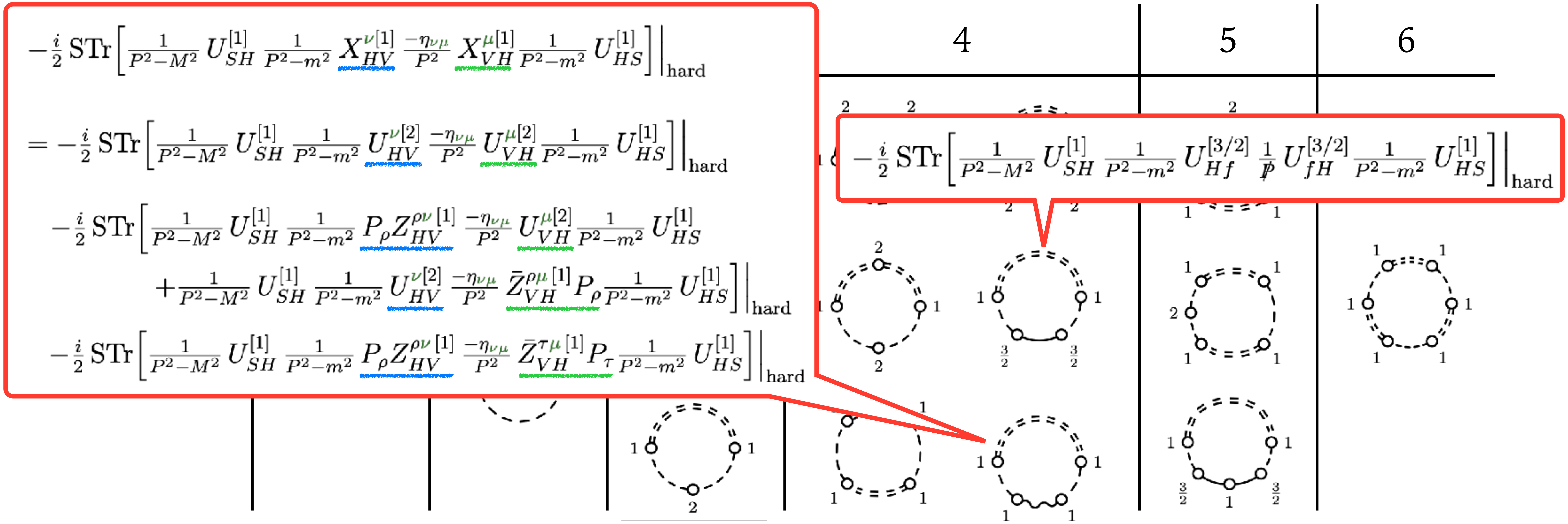


$$-\frac{i}{2}\,\frac{1}{2}\,\mathrm{STr}\left[\left(\frac{1}{P^2-M^2}\,U^{[2]}_{SS}\right)^2\right]\Big|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\,U^{[1]}_{SH}\,\frac{1}{P^2-m^2}\,U^{[1]}_{HS}\right]\Big|_{\text{hard}}$$

Step 3: enumerate supertraces

- ➤ No log-type supertraces (S is a gauge singlet).
- ➤ Enumerate power-type supertraces with $\sum \dim(X_{ij}) \leq 6$.



$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}\underline{X^{\nu[1]}_{HV}}\frac{-\eta_{\nu\mu}}{P^2}\underline{X^{\mu[1]}_{VH}}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

$$= -\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}\underline{U^{\nu[2]}_{HV}}\frac{-\eta_{\nu\mu}}{P^2}\underline{U^{\mu[2]}_{VH}}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}\underline{P_\rho Z^{\rho\nu[1]}_{HV}}\frac{-\eta_{\nu\mu}}{P^2}\underline{U^{\mu[2]}_{VH}}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right.$$

$$\left.+\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}\underline{U^{\nu[2]}_{HV}}\frac{-\eta_{\nu\mu}}{P^2}\underline{\bar{Z}^{\rho\mu[1]}_{VH}P_\rho}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}\underline{P_\rho Z^{\rho\nu[1]}_{HV}}\frac{-\eta_{\nu\mu}}{P^2}\underline{\bar{Z}^{\tau\mu[1]}_{VH}P_\tau}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

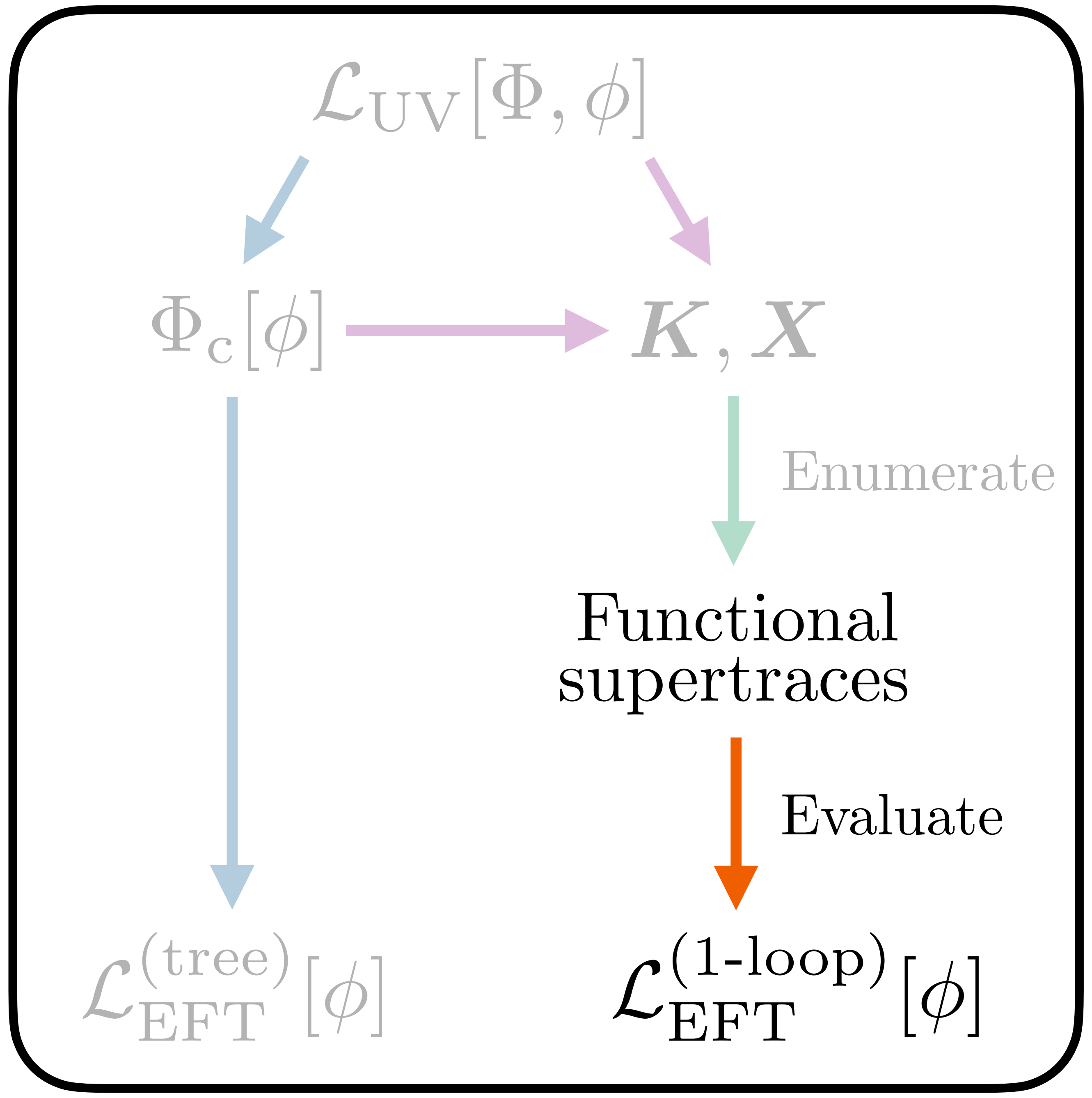
$$-\frac{i}{2}\,\mathrm{STr}\left[\frac{1}{P^2-M^2}U^{[1]}_{SH}\frac{1}{P^2-m^2}U^{[3/2]}_{Hf}\frac{1}{\not{P}}U^{[3/2]}_{fH}\frac{1}{P^2-m^2}U^{[1]}_{HS}\right]\Bigg|_{\text{hard}}$$

Step 4: evaluate supertraces

- First, use CDE to evaluate STr with generic X_{ij} .
- Then, substitute in $X_{ij}[\phi\,,P_\mu]$ and perform matrix algebra.

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\boxed{U_1^{[2]}}\right]\Big|_{\mathrm{hard}}$$ generic functional of ϕ with operator dimension ≥ 2

$$= \int \mathrm{d}^d x\, \frac{1}{16\pi^2}\,\mathrm{tr}\left[M^2\left(1-\log\frac{M^2}{\mu^2}\right)\boxed{U_1} + \frac{1}{12M^2}\,F_{\mu\nu}F^{\mu\nu}\boxed{U_1}\right].$$



- (*) If desired, post-process the results into a non-redundant operator basis.

Step 4: evaluate supertraces

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$$-i\,\mathrm{STr}\left[\frac{1}{P^2-M^2}\boxed{U_1^{[2]}}\right]\Big|_{\mathrm{hard}}$$

generic functional of ϕ with operator dimension ≥ 2

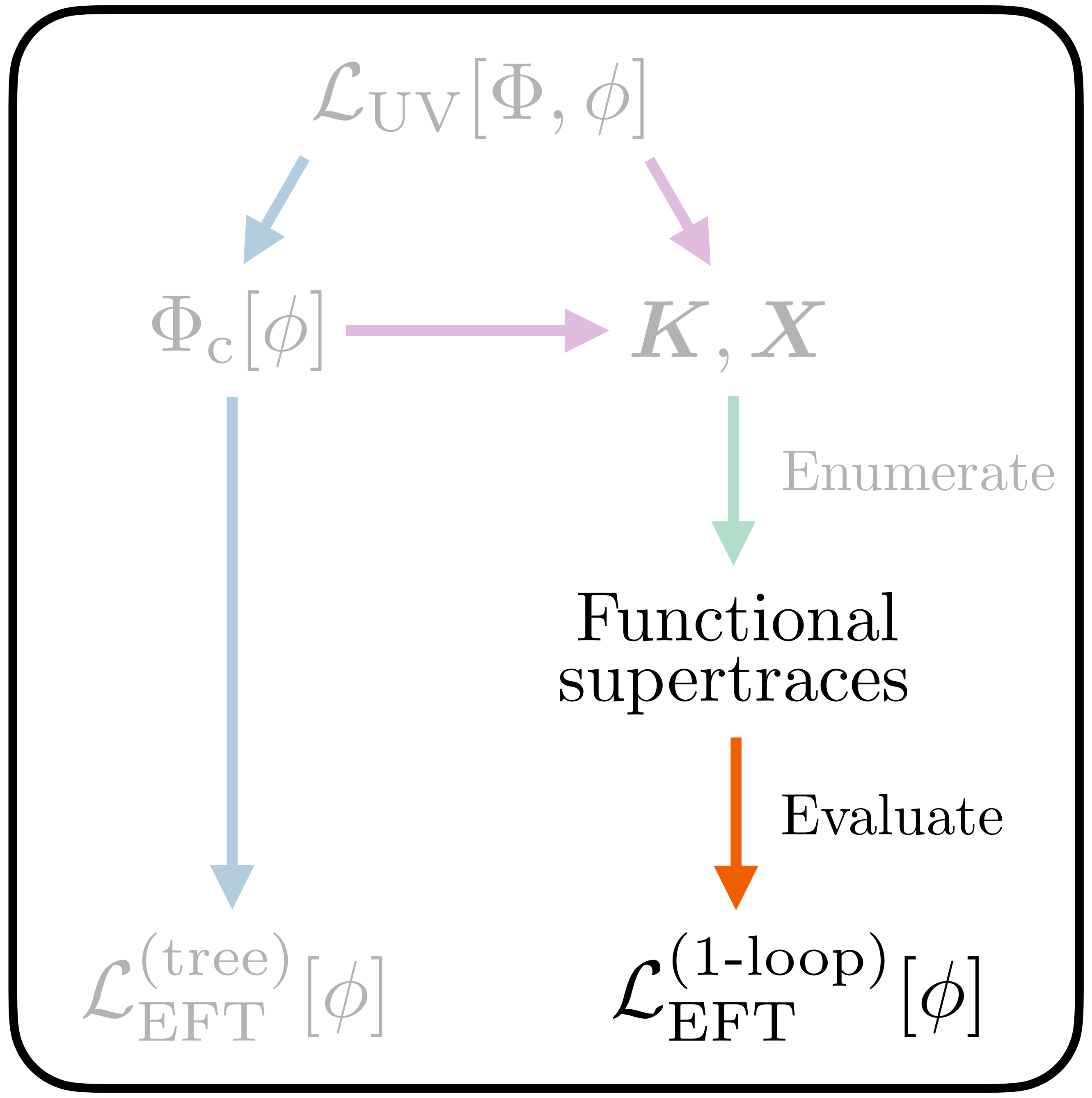
$$= \int \mathrm{d}^d x\, \frac{1}{16\pi^2}\,\mathrm{tr}\left[M^2\left(1-\log\frac{M^2}{\mu^2}\right)\boxed{U_1} + \frac{1}{12M^2}\,F_{\mu\nu}F^{\mu\nu}\boxed{U_1}\right].$$

$$U_{SS} = \kappa\,|H|^2 + \mu_S\,S_c + \frac{1}{2}\,\lambda_S\,S_c^2$$

0

$$\Rightarrow\ \mathscr{L}_{\mathrm{EFT}}^{(1-\mathrm{loop})} \supset \frac{1}{16\pi^2}\frac{1}{2}\left(1-\log\frac{M^2}{\mu^2}\right)\Bigg\{(\kappa M^2-\mu_S A)|H|^2 + \left[\frac{\lambda_S A^2}{2M^2}+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{2M^2}\right)\right]|H|^4$$
$$-\frac{1}{M^2}\left[\frac{\lambda_S A^2}{M^2}\left(\kappa-\frac{2\mu_S A}{3M^2}\right)+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{M^2}\right)\left(\kappa-\frac{\mu_S A}{2M^2}\right)\right]|H|^6$$
$$-\frac{1}{M^2}\left[\frac{\lambda_S A^2}{M^2}+\frac{\mu_S A}{M^2}\left(\kappa-\frac{\mu_S A}{M^2}\right)\right]|H|^2(\partial^2|H|^2)\Bigg\},$$

- (*) If desired, post-process the results into a non-redundant operator basis.



Step 4: evaluate supertraces

- Full results in agreement with existing results obtained using amplitude matching.
- Nontrivial test of our functional prescription.

Operator	Coefficient $\times\, 16\pi^2$
$\lvert H\rvert^2$	$\left[\frac{1}{2}(\kappa M^2 - \mu_S A) + A^2\left(1 + \frac{m^2}{M^2} + \frac{m^4}{M^4}\right)\right]\left(1 - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^4$	$\frac{\kappa^2}{4}\left(-\log\frac{M^2}{\mu^2}\right) + \frac{\mu_S A}{M^2}\left(\frac{\kappa}{2} - \frac{\mu_S A}{4M^2} + \frac{A^2}{M^2}\right)$ $+\frac{A^2}{M^2}\left[\left(\frac{\lambda_S}{4} + 3\lambda_H\right)\left(1 - \log\frac{M^2}{\mu^2}\right) - 2\left(\kappa + \frac{A^2}{M^2}\right)\left(\frac{3}{2} - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{m^2}{M^2}\frac{A^2}{M^2}\left[6\lambda_H\left(1 - \log\frac{M^2}{\mu^2}\right) - 3\left(\kappa + \frac{2A^2}{M^2}\right)\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right) + \frac{\mu_S A}{M^2}\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$
$\lvert D_\mu H\rvert^2$	$\frac{A^2}{2M^2} + \frac{A^2 m^2}{M^4}\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$

Table 2. Corrections to renormalizable operators.

Operator	Coefficient $\times\, 16\pi^2$
$\lvert H\rvert^6$	$\frac{1}{M^2}\left(-\frac{\kappa^3}{12} - \frac{\kappa^2\mu_S A}{4M^2} + \frac{\kappa\mu_S^2 A^2}{2M^4} - \frac{\lambda_S A^4}{2M^4} - \frac{\mu_S^3 A^3}{6M^6} + \frac{\mu_S^2 A^4}{M^6}\right)$ $+\frac{\kappa A^2}{M^4}\left[3\kappa\left(\frac{11}{6} - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_S}{4}\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{9\lambda_H A^2}{M^4}\left[-\kappa\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right) + \lambda_H\left(1 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{\mu_S A^3}{M^6}\left[-\kappa\left(5 - \log\frac{M^2}{\mu^2}\right) + \frac{\lambda_S}{12}\left(4 - \log\frac{M^2}{\mu^2}\right) + 3\lambda_H\left(2 - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{A^4}{M^6}\left[\frac{21\kappa}{2}\left(\frac{37}{21} - \log\frac{M^2}{\mu^2}\right) - 18\lambda_H\left(\frac{4}{3} - \log\frac{M^2}{\mu^2}\right)\right]$ $-\frac{7\mu_S A^5}{2M^8}\left(\frac{15}{7} - \log\frac{M^2}{\mu^2}\right) + \frac{9A^6}{M^8}\left(\frac{43}{27} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\partial^2\lvert H\rvert^2)$	$-\frac{\kappa^2}{24M^2} - \frac{5\kappa\mu_S A}{12M^4}$ $+\frac{A^2}{M^4}\left[2\kappa\left(\frac{17}{12} - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_S}{2}\left(1 - \log\frac{M^2}{\mu^2}\right) - \frac{\lambda_H}{2}\left(\frac{9}{2} - \log\frac{M^2}{\mu^2}\right)\right]$ $+\frac{11\mu_S^2 A^2}{24M^6} - \frac{4\mu_S A^3}{3M^6} + \frac{3A^4}{2M^6}\left(\frac{20}{9} - \log\frac{M^2}{\mu^2}\right) - \frac{3g_2^2 A^2}{8M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\lvert D_\mu H\rvert^2$	$\frac{A^2}{M^4}\left[\left(\lambda_H - \frac{A^2}{M^2}\right)\left(\frac{9}{2} - \log\frac{M^2}{\mu^2}\right) - \frac{3\kappa}{2} + \frac{\mu_S A}{2M^2}\right] - \frac{3g_2^2 A^2}{2M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\frac{1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)^2$	$\frac{3g_1^2 A^2}{4M^4}\left(\frac{5}{6} - \log\frac{M^2}{\mu^2}\right)$
$\lvert D^2 H\rvert^2$	$\frac{A^2}{6M^4}$

Operator	Coefficient $\times\, 16\pi^2$
$\frac{ig_2}{2}(H^\dagger\sigma^I \overleftrightarrow{D}^\mu H)(D^\nu W^I_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3} - \log\frac{M^2}{\mu^2}\right)$
$\frac{ig_1}{2}(H^\dagger \overleftrightarrow{D}^\mu H)(\partial^\nu B_{\mu\nu})$	$-\frac{A^2}{6M^4}\left(\frac{7}{3} - \log\frac{M^2}{\mu^2}\right)$
$ig_2(D^\mu H)^\dagger\sigma^I(D^\nu H)\, W^I_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$ig_1(D^\mu H)^\dagger(D^\nu H)\, B_{\mu\nu}$	$-\frac{A^2}{12M^4}$
$\lvert H\rvert^2 W^I_{\mu\nu}W^{I\mu\nu}$	$\frac{g_2^2 A^2}{16M^4}$
$\lvert H\rvert^2 B_{\mu\nu}B^{\mu\nu}$	$\frac{g_1^2 A^2}{16M^4}$
$H^\dagger\sigma^I H W^I_{\mu\nu}B^{\mu\nu}$	$\frac{g_1 g_2 A^2}{8M^4}$

Table 3. Dimension six bosonic operators.

Operator	Coefficient $\times\, 16\pi^2$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar{q}\sigma^I\gamma^\mu q)$	$\frac{A^2}{8M^4}(y_u y_u^\dagger + y_d y_d^\dagger)\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$	$-\frac{A^2}{8M^4}(y_u y_u^\dagger - y_d y_d^\dagger)\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$	$\frac{A^2}{4M^4}\, y_u^\dagger y_u\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$	$-\frac{A^2}{4M^4}\, y_d^\dagger y_d\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I i\overleftrightarrow{D}_\mu H)(\bar{l}\sigma^I\gamma^\mu l)$	$\frac{A^2}{8M^4}\, y_e y_e^\dagger\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l)$	$\frac{A^2}{8M^4} y_e y_e^\dagger\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$	$-\frac{A^2}{4M^4}\, y_e^\dagger y_e\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(\widetilde{H}^\dagger i(D_\mu H))(\bar{u}\gamma^\mu d)$ (+h.c.)	$-\frac{A^2}{2M^4}\, y_u^\dagger y_d\left(\frac{5}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar{q}\,\sigma^I i\overleftrightarrow{\slashed{D}}\, q)$	$-\frac{A^2}{8M^4}(y_u y_u^\dagger - y_d y_d^\dagger)\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{q}\, i\overleftrightarrow{\slashed{D}}\, q)$	$\frac{A^2}{8M^4}(y_u y_u^\dagger + y_d y_d^\dagger)\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{u}\, i\overleftrightarrow{\slashed{D}}\, u)$	$\frac{A^2}{4M^4}\, y_u^\dagger y_u\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{d}\, i\overleftrightarrow{\slashed{D}}\, d)$	$\frac{A^2}{4M^4}\, y_d^\dagger y_d\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$(H^\dagger\sigma^I H)(\bar{l}\,\sigma^I i\overleftrightarrow{\slashed{D}}\, l)$	$\frac{A^2}{8M^4}\, y_e y_e^\dagger\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{l}\, i\overleftrightarrow{\slashed{D}}\, l)$	$\frac{A^2}{8M^4}\, y_e y_e^\dagger\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2(\bar{e}\, i\overleftrightarrow{\slashed{D}}\, e)$	$\frac{A^2}{4M^4}\, y_e^\dagger y_e\left(\frac{1}{2} - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\, u\, \widetilde{H}$ (+h.c.)	$\frac{A^2}{M^4}\, y_u y_u^\dagger y_u\left(1 - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{q}\, d\, H$ (+h.c.)	$\frac{A^2}{M^4}\, y_d y_d^\dagger y_d\left(1 - \log\frac{M^2}{\mu^2}\right)$
$\lvert H\rvert^2\,\bar{l}\, e\, H$ (+h.c.)	$\frac{A^2}{M^4}\, y_e y_e^\dagger y_e\left(1 - \log\frac{M^2}{\mu^2}\right)$

Table 4. Dimension six operators with fermions.

Outline

- What is functional matching, and what is new? ✅
- The prescription. ✅
- Example: matching the singlet scalar extended SM onto SMEFT up to dim-6. ✅
- CDE (Covariant Derivative Expansion) & STrEAM (SuperTrace Evaluation Automated for Matching).

Evaluating functional supertraces with CDE

- The goal is to compute the two types of functional supertraces:

$$\int \mathrm{d}^d x\, \mathcal{L}_{\mathrm{EFT}}^{(\text{1-loop})}[\phi] = \frac{i}{2}\,\mathrm{STr}\log \boldsymbol{K}\Big|_{\mathrm{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\Big[\big(\boldsymbol{K}^{-1}\boldsymbol{X}\big)^n\Big]\Big|_{\mathrm{hard}}\,.$$

- Log-type can be converted to power-type by differentiation:

$$\frac{\partial}{\partial m_\Phi^2}\Big[i\,\mathrm{STr}\log\big(P^2 - m_\Phi^2\big)\Big] = -i\,\mathrm{STr}\left[\frac{1}{P^2 - m_\Phi^2}\right] \qquad \frac{\partial}{\partial m_\Phi}\Big[i\,\mathrm{STr}\log\big(\not{P} - m_\Phi\big)\Big] = -i\,\mathrm{STr}\left[\frac{1}{\not{P} - m_\Phi}\right]$$

- So we only need to deal with supertraces of the form:

$$-\,i\,\mathrm{STr}\Big[f\big(P_\mu\,,\,\{U_k\}\big)\Big]\Big|_{\mathrm{hard}} \qquad f = \Big[\;\cdots\;\big(P_{\mu_1}\ldots P_{\mu_n}\big)\big(\Delta_i \text{ or } \Lambda_i\big)\big(P_{\nu_1}\ldots P_{\nu_m}\big)\,U_k\;\cdots\;\Big]$$

where $\quad \Delta_i \equiv \dfrac{1}{P^2 - m_i^2}\,, \qquad \Lambda_i \equiv \dfrac{1}{\not{P} - m_i}\,.$

Evaluating functional supertraces with CDE

- First, address the "super" part: $-\,i\,\mathrm{STr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] = \boxed{\pm}\Big\{-\,i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big\}$

determined by the first propagator block

- Next, the "functional" part.
  - Conveniently, work in the quantum mechanics language:

$$
\begin{aligned}
-i\,\mathrm{Tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big] &= -i\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\Big\langle q\Big|\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big|q\Big\rangle \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\langle q|x\rangle\,\Big\langle x\Big|\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\Big|q\Big\rangle \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,e^{iq\cdot x}\,\mathrm{tr}\Big[f\big(P_\mu,\{U_k\}\big)\Big]\,e^{-iq\cdot x}\,. \\
&= -i\int\mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\Big[f\big(P_\mu-q_\mu,\{U_k\}\big)\Big]\,.
\end{aligned}
$$

$$
e^{iq\cdot x}\,P_\mu\,e^{-iq\cdot x} = P_\mu + q_\mu\,,
$$
$$
e^{iq\cdot x}\,U_k\,e^{-iq\cdot x} = U_k\,;
$$

Note: $P_\mu(\hat{x},\hat{q}) = \hat{q}_\mu + g_a\,G^a_\mu(\hat{x})\,T^a$

Evaluating functional supertraces with CDE

$$-\,i\,\mathrm{Tr}\left[f\big(P_\mu, \{U_k\}\big)\right] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\;\mathrm{tr}\left[f\big(P_\mu - q_\mu, \{U_k\}\big)\right].$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\big(P_\mu - q_\mu, \{U_k\}\big)$ in powers of *P* and *U*.
    - Produce non-gauge-invariant terms (e.g. when a $P_\mu(\hat{x},\hat{q}) = \hat{q}_\mu + g_a\, G^a_\mu(\hat{x})\, T^a$ appears on the rightmost).
    - All such terms cancel in the final result, as they must.
    - Simplifying trick: use gauge invariance as a constraint to reduce the number of terms to compute.

B. Henning, X. Lu, H. Murayama, 1604.01019.
J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia, 1607.02142.
ZZ, 1610.00710.

Evaluating functional supertraces with CDE

$$-i\,\mathrm{Tr}\left[f\big(P_\mu,\{U_k\}\big)\right] = -i\int \mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[f\big(P_\mu-q_\mu,\{U_k\}\big)\right].$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\big(P_\mu-q_\mu,\{U_k\}\big)$ in powers of P and U.
  - "Original CDE."
    - First apply a transformation: $-i\,\mathrm{Tr}\left[f\big(P_\mu,\{U_k\}\big)\right] = -i\int \mathrm{d}^d x\int\frac{\mathrm{d}^d q}{(2\pi)^d}\,e^{P\cdot\frac{\partial}{\partial q}}\,\mathrm{tr}\left[f\big(P_\mu-q_\mu,\{U_k\}\big)\right]e^{-P\cdot\frac{\partial}{\partial q}}$
    - This makes all covariant derivatives "closed."

$$P_\mu^{\mathrm{CDE}} \equiv e^{P\cdot\frac{\partial}{\partial q}}\,(P_\mu-q_\mu)\,e^{-P\cdot\frac{\partial}{\partial q}} = -q_\mu + G_{\mu\nu}^{\mathrm{CDE}}\partial^\nu\,,\qquad G_{\mu\nu}^{\mathrm{CDE}} \equiv -i\sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}\,(P_{\alpha_1}\cdots P_{\alpha_n}F_{\mu\nu})\,\partial^{\alpha_1}\cdots\partial^{\alpha_n}$$

$$U_k^{\mathrm{CDE}} \equiv e^{P\cdot\frac{\partial}{\partial q}}\,U_k\,e^{-P\cdot\frac{\partial}{\partial q}} = \sum_{n=0}^{\infty}\frac{1}{n!}\,(P_{\alpha_1}\cdots P_{\alpha_n}U_k)\,\partial^{\alpha_1}\cdots\partial^{\alpha_n}\qquad \partial^\alpha \equiv \frac{\partial}{\partial q_\alpha}$$

- Automatically guarantees gauge-invariance, at the cost of an additional sum + q differentiation.

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O. Cheyette, Nucl. Phys. B 297 (1988) 183.
B. Henning, X. Lu, H. Murayama, 1412.1837.

Evaluating functional supertraces with CDE

$$-i\,\mathrm{Tr}\left[f\left(P_\mu,\{U_k\}\right)\right] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\mathrm{tr}\left[f\left(P_\mu - q_\mu,\{U_k\}\right)\right].$$

- Different ways to proceed.
  - "Simplified CDE."
    - Directly expand $f\left(P_\mu - q_\mu,\{U_k\}\right)$ in powers of P and U.
  - "Original CDE."
    - First apply a transformation: $-i\,\mathrm{Tr}\left[f\left(P_\mu,\{U_k\}\right)\right] = -i\int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d}\, e^{P\cdot\frac{\partial}{\partial q}}\,\mathrm{tr}\left[f\left(P_\mu - q_\mu,\{U_k\}\right)\right] e^{-P\cdot\frac{\partial}{\partial q}}$
- We chose to automate the original CDE.
  - Result is a series of terms involving loop integrals of the form $-i\int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{(q^2)^r}{\left(q^2-m_1^2\right)^{n_1}\cdots\left(q^2-m_k^2\right)^{n_k}}$

    $\times$ gauge-invariant operators built from $(P_{\alpha_1}\cdots P_{\alpha_n}F_{\mu\nu})$ and $(P_{\alpha_1}\cdots P_{\alpha_n}U_k)$.

STrEAM (SuperTrace Evaluation Automated for Matching)

T. Cohen, X. Lu, ZZ, 2012.07851.

A Simple Example

In[•]:= `SuperTrace[6, {Δ1, U1}, Udimlist → {2}, display → True]`

$$-i\mathrm{STr}\left[\frac{1}{P^2 - m_1^2}U_1\right]\Big|_{\mathrm{hard}} = \int d^4x\ \frac{1}{16\pi^2}\ \mathrm{tr}\{$$

$$-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2 \qquad (U_1) \qquad (\text{dim-}2)$$

$$\frac{1}{12\,m_1^2} \qquad (F_{\mu1,\mu2})\,(F_{\mu1,\mu2})\,(U_1) \qquad (\text{dim-}6)$$

$$\}$$

Out[•]= $\left\{\left\{\left\{\left\{-\left(-1+\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]\right)m_1^2\right\}\right\},\ \{\{U_1\}\},\ 2\right\},\ \left\{\left\{\left\{\frac{1}{12\,m_1^2}\right\}\right\},\ \left\{\{F_{\mu1,\mu2}\},\ \{F_{\mu1,\mu2}\},\ \{U_1\}\right\},\ 6\right\}\right\}$

STrEAM (SuperTrace Evaluation Automated for Matching)

T. Cohen, X. Lu, ZZ, 2012.07851.

Additional Examples

```
In[•]:= SuperTrace[6, {Δ1}, display → True];
```

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}\right]\Big|_{\mathrm{hard}} = \int d^4x\ \frac{1}{16\pi^2}\ \mathrm{tr}\{$$

$$\frac{1}{12\,m_1^2}\quad (F_{\mu_1,\mu_2})(F_{\mu_1,\mu_2})\qquad (\text{dim-4})$$

$$\frac{i}{90\,m_1^4}\quad (F_{\mu_1,\mu_2})(F_{\mu_1,\mu_3})(F_{\mu_2,\mu_3})\qquad (\text{dim-6})$$

$$\frac{1}{60\,m_1^4}\quad (P_{\mu_1}P_{\mu_2}F_{\mu_2,\mu_3})(F_{\mu_1,\mu_3})\qquad (\text{dim-6})$$

$$\}$$

```
In[•]:= SuperTrace[6, {Δ1}, NoγinU → True, display → True];
```

$$-i\,\mathrm{STr}\left[\frac{1}{\mathrm{Pslash}-m_1}\right]\Big|_{\mathrm{hard}} = \int d^4x\ \frac{1}{16\pi^2}\ \mathrm{tr}\{$$

$$\frac{1}{6\,m_1}\quad (F_{\mu_1,\mu_2})(F_{\mu_1,\mu_2})\qquad (\text{dim-4})$$

$$-\frac{i}{90\,m_1^3}\quad (F_{\mu_1,\mu_2})(F_{\mu_1,\mu_3})(F_{\mu_2,\mu_3})\qquad (\text{dim-6})$$

$$\frac{1}{15\,m_1^3}\quad (P_{\mu_1}P_{\mu_2}F_{\mu_2,\mu_3})(F_{\mu_1,\mu_3})\qquad (\text{dim-6})$$

$$\}$$

reproduce log-type supertraces upon integration over m :

$$\frac{\partial}{\partial m_\Phi^2}\left[i\,\mathrm{STr}\log\left(P^2-m_\Phi^2\right)\right] = -i\,\mathrm{STr}\left[\frac{1}{P^2-m_\Phi^2}\right] \qquad \frac{\partial}{\partial m_\Phi}\left[i\,\mathrm{STr}\log\left(\not{P}-m_\Phi\right)\right] = -i\,\mathrm{STr}\left[\frac{1}{\not{P}-m_\Phi}\right]$$

STrEAM (SuperTrace Evaluation Automated for Matching)

T. Cohen, X. Lu, ZZ, 2012.07851.

Additional Examples

```
In[•]:= SuperTrace[6, {Δ1, U1, Δ2, Pν, Zν, Δ0, U3, Δ2, U4}, Udimlist → {1, 1, 2, 1}, display → True];
```

$$-i\,\mathrm{STr}\left[\frac{1}{P^2-m_1^2}U_1\frac{1}{P^2-m_2^2}P_\nu Z_\nu\frac{1}{P^2}U_3\frac{1}{P^2-m_2^2}U_4\right]\Big|_{\mathrm{hard}} = \int d^4x\,\frac{1}{16\pi^2}\,\mathrm{tr}\{$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (U_1)\,(Z_{\mu_1})\,(P_{\mu_1}U_3)\,(U_4)\qquad (\mathrm{dim}-6)$$

$$\frac{3-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{2\,m_1^4}\quad (U_1)\,(P_{\mu_1}Z_{\mu_1})\,(U_3)\,(U_4)\qquad (\mathrm{dim}-6)$$

$$\frac{5-2\,\mathrm{Log}\left[\frac{m_1^2}{\mu^2}\right]}{4\,m_1^4}\quad (P_{\mu_1}U_1)\,(Z_{\mu_1})\,(U_3)\,(U_4)\qquad (\mathrm{dim}-6)$$

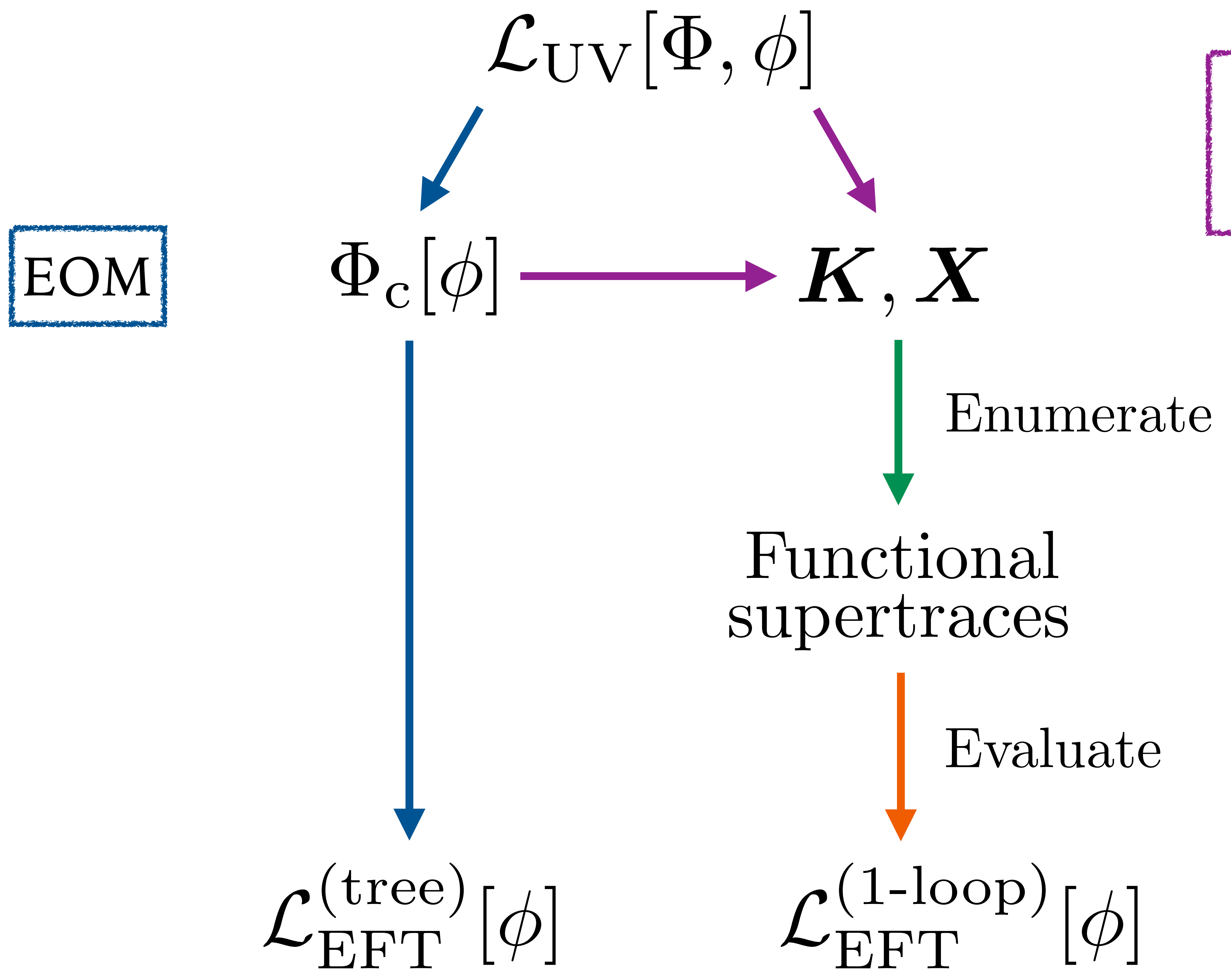
$\}$

derivative interaction

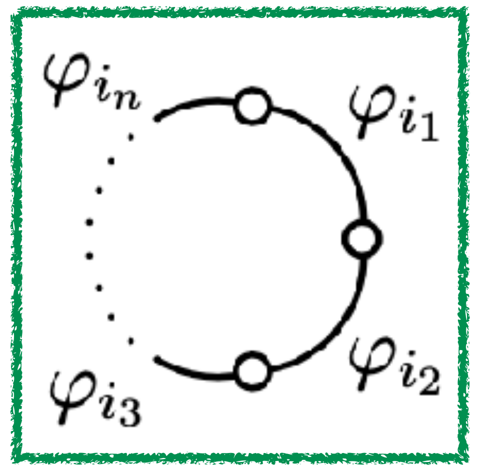
By default:
1 = heavy mass
2, 3, … = light masses
0 = zero mass
Otherwise the user should specify which masses are heavy.

Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.



$$\boxed{-\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta\varphi^2}\bigg|_{\Phi=\Phi_c[\phi]} = \mathbf{K} - \mathbf{X}}$$



(automated) CDE + matrix algebra

Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.

- Many pheno applications.

B. Henning, X. Lu, H. Murayama, 1404.1058, 1412.1837, 1604.01019.
C.-W. Chiang, R. Huo, 1505.06334.
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Summary & outlook

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- Many pheno applications.
- Beyond relativistic EFT?

T. Cohen, M. Freytsis, X. Lu, 1912.08814 (HQET).

Summary & outlook

- We have devised a prescription for EFT matching up to one loop that is functional (pun intended).
- STrEAM automates the most tedious part (CDE) in this STrEAMlined prescription.

- Many pheno applications.
- Beyond relativistic EFT?
- Beyond one loop?
- Beyond matching: also efficient RG calculation.
- New insights on other aspects of EFTs?
- ??

THANK YOU