

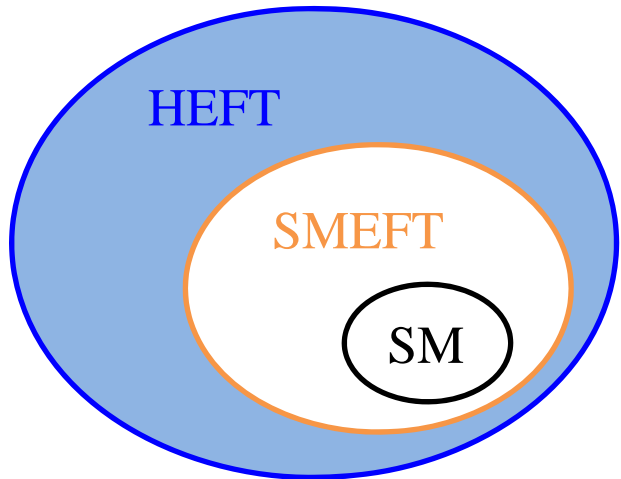
# What is HEFT?

All Things EFT, Nov 4, 2020

Xiaochuan Lu

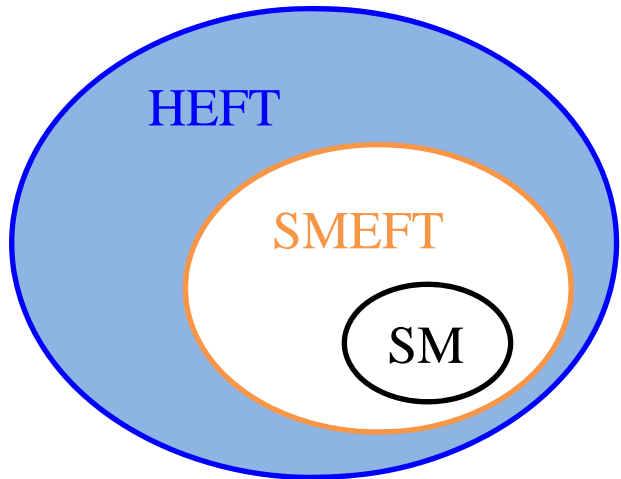
University of Oregon

arXiv: 2008.08597,  
with Timothy Cohen, Nathaniel Craig, and Dave Sutherland



$H$   $SU(2)_W$  doublet  
 $\left\{ \begin{array}{l} h \\ U \equiv e^{i\pi^a t^a / v} \end{array} \right.$  physical Higgs  
 Goldstones

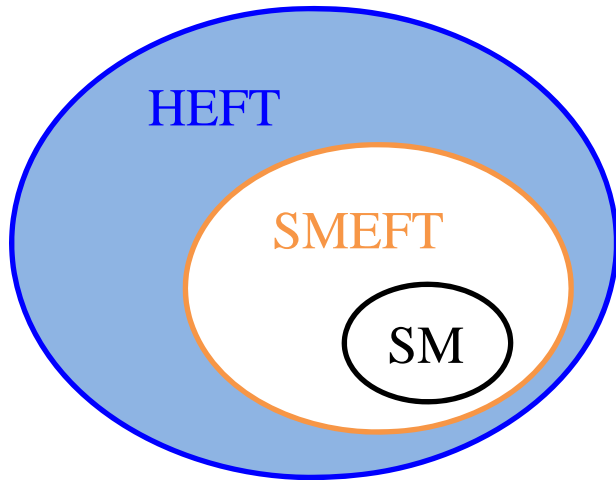
$$\mathcal{L}_{\text{SMEFT}}(H) = \mathcal{L}_{\text{SM}} + \frac{C_H}{\Lambda^2} |H|^6 + \frac{C_{H\Box}}{\Lambda^2} |H|^2 \partial^2 |H|^2 + \frac{C_R}{\Lambda^2} |H|^2 |DH|^2 + \dots$$



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 H & SU(2)_W \text{ doublet} \\
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$$\mathcal{L}_{\text{HEFT}}(h, U) = \frac{1}{2} (\partial h)^2 - V(h) + \frac{1}{2} [vF(h)]^2 \text{tr} \left[ -\frac{1}{2} (U^\dagger D_\mu U)(U^\dagger D^\mu U) \right] + \dots$$



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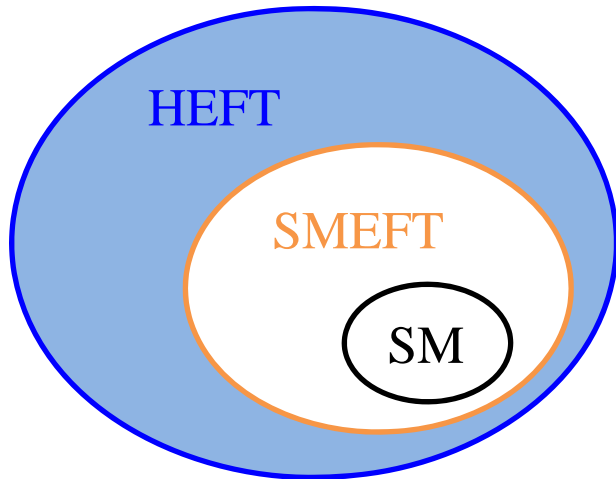
physical Higgs

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Linear

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# Outline

- HEFT  $\Leftrightarrow$  non-analyticities
- Field redefinitions: **physical** vs **unphysical** non-analyticities
  - Geometric criteria
- What UV Physics lead to HEFT?
  - Integrating out extra electroweak breaking
  - Integrating out “massless” states

SMEFT  $\Rightarrow$  HEFT

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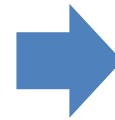
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Non-analyticity

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$$\begin{cases} \text{SMEFT: } (x, y) \\ \text{HEFT: } (r, \theta) \end{cases} \Rightarrow r = \sqrt{x^2 + y^2}$$



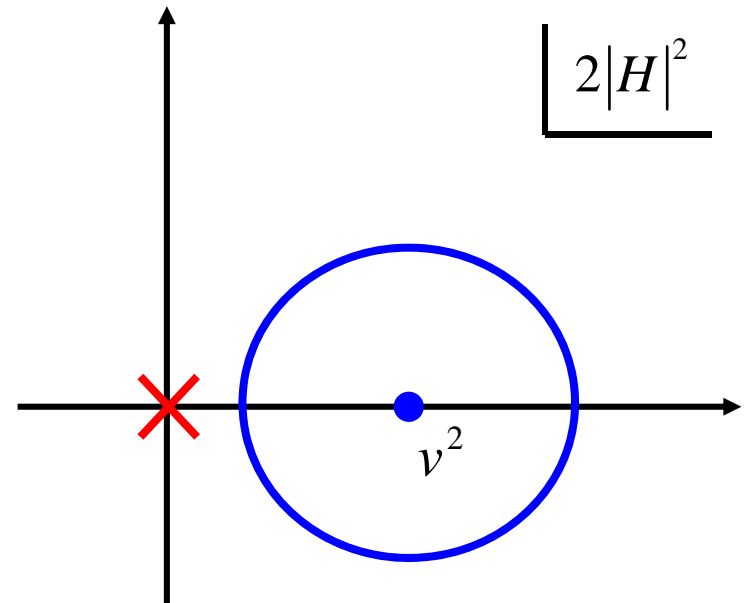
## Non-analyticity is an all-order effect

$$\begin{aligned}v + h &= \sqrt{2|H|^2} = \sqrt{v^2 + (2|H|^2 - v^2)} \\ &= v + \frac{1}{2v} \left( \underline{2|H|^2 - v^2} \right) + \dots\end{aligned}$$

$$2vh + h^2$$

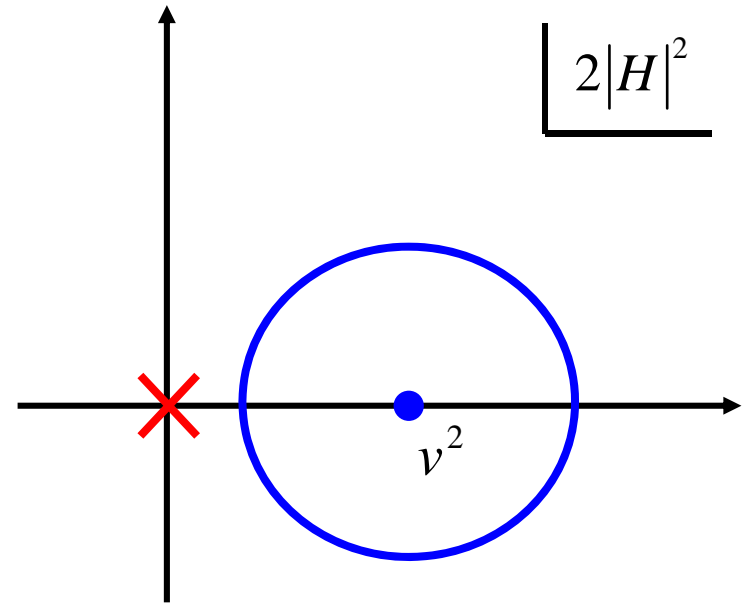
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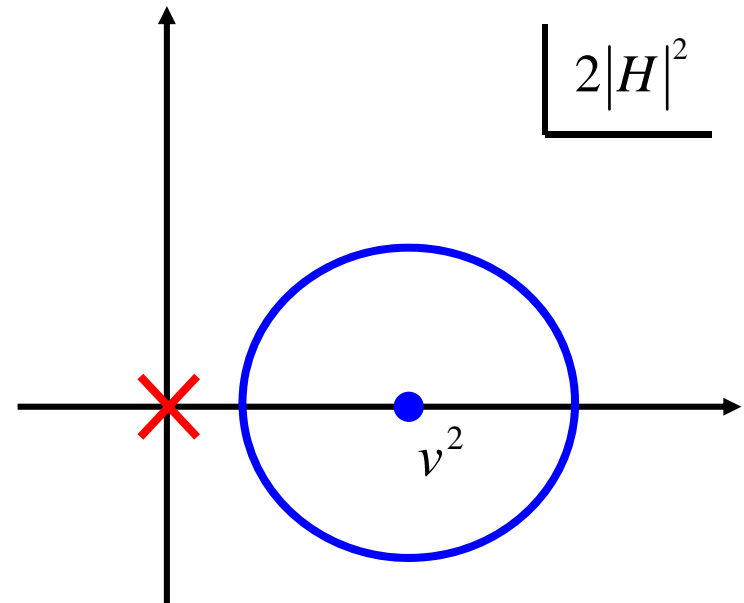
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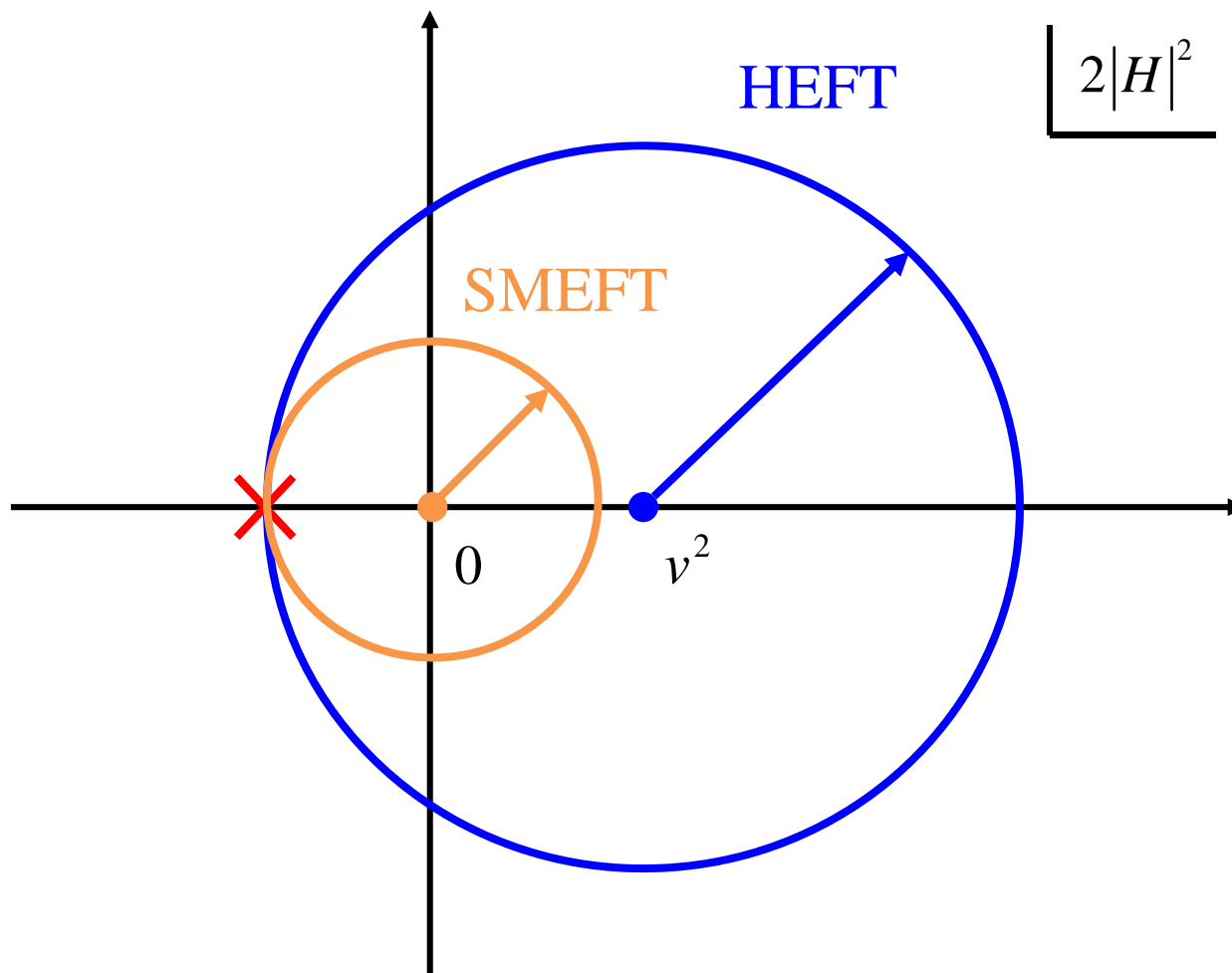
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Need to know to all powers



## Adam Falkowski and Riccardo Rattazzi: (arXiv: 1902.05936)

UV cut-off. Our distinction between analytic and non-analytic lagrangians coincides with the distinction, in use in the Higgs EFT community, between linear (so-called SMEFT) and non-linear (so-called HEFT) effective theory, or equivalently between  $h$  being or not being part of a  $SU(2)_W$  doublet. We however believe our classification is more adequate and enlightening from a physical point of view.

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SMEFT

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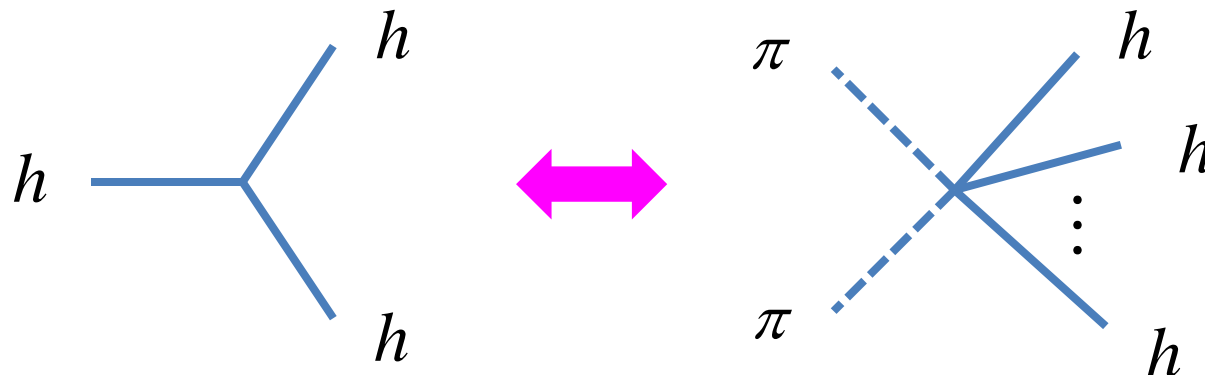
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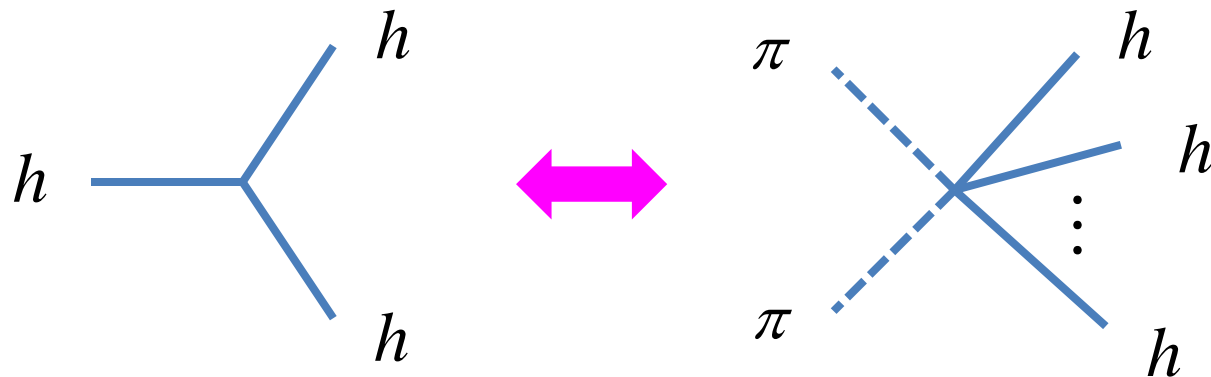




$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_1 + i\pi_2 \\ v + h + i\pi_3 \end{pmatrix} \quad \Sigma = \begin{pmatrix} H_2^* & H_1 \\ -H_1^* & H_2 \end{pmatrix} = \frac{1}{\sqrt{2}} (v + h) U$$

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One can convert the SMEFT Lagrangian to HEFT form using Eq. (2.11) to switch from Cartesian and polar coordinates. One can attempt to convert from HEFT to SMEFT form using

$$\frac{\phi}{(\phi \cdot \phi)^{1/2}} = n \quad (2.30)$$

with  $(\phi \cdot \phi)^{1/2}$  some function of  $h$ . This substitution gives a Lagrangian  $L(\phi)$  that need not be analytic in  $\phi$ . However, if there is an  $O(4)$  fixed point, then there is a suitable change of variables such that the resulting Lagrangian is analytic in  $\phi$ .

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{\text{Kinetic}} = \frac{1}{2} g_{ab}(\phi) (\partial_\mu \phi_a) (\partial^\mu \phi_b)$$

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Geometric invariants

$$ds^2 \equiv g_{ab}(\phi) d\phi_a d\phi_b$$

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = (v+h)\vec{n} \quad , \quad \vec{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ \sqrt{1-n_1^2-n_2^2-n_3^2} \end{pmatrix} \quad , \quad \vec{n} \cdot \vec{n} = 1$$

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2} [K(h)]^2 (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

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SM:

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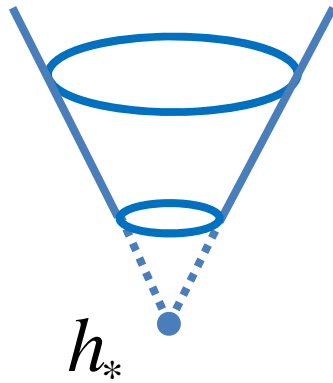
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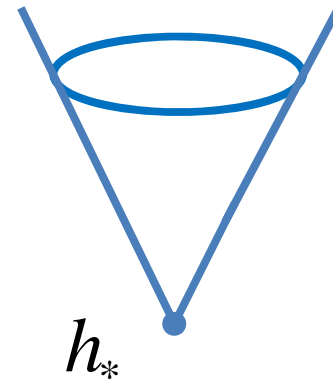
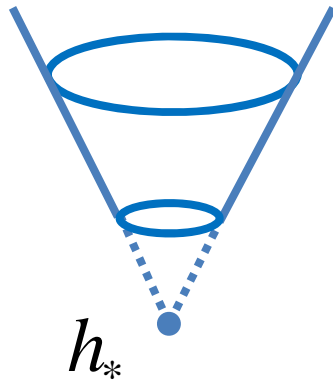
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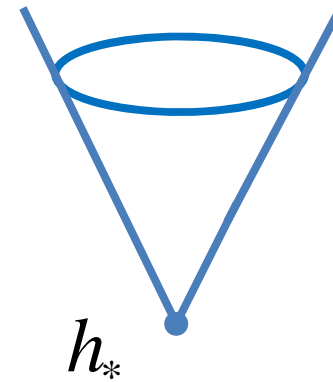
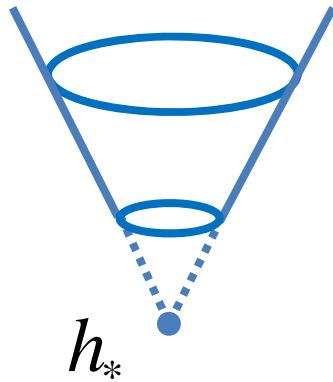
AJM (arXiv: 1605.03602)

$\exists h_*$  such that  $F(h_*) = 0$



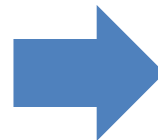
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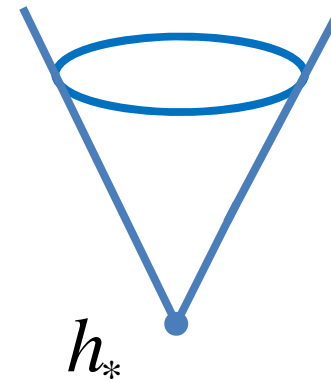
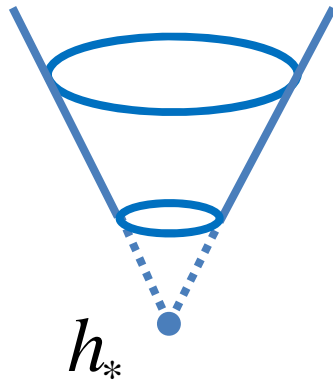


Our work (arXiv: 2008.08597)

$$F = 0$$

$$R, \nabla^2 R, \nabla^4 R, \dots < \infty$$

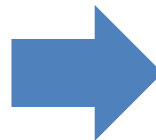
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Leading Order  
Criterion



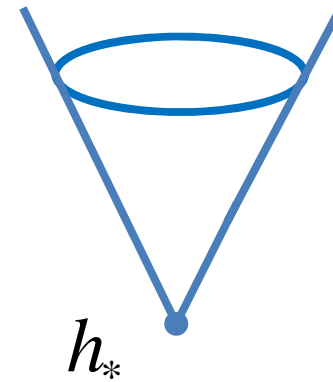
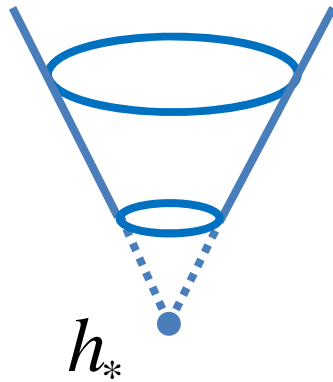
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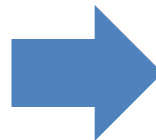
# UV theories that will generate HEFT?



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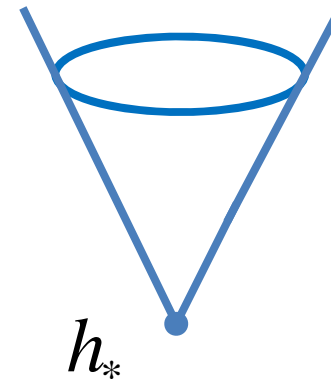
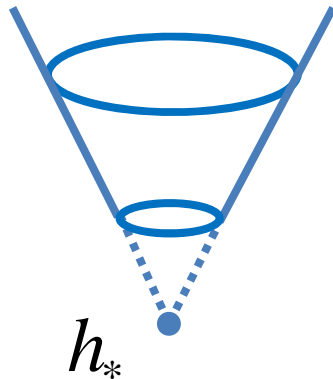
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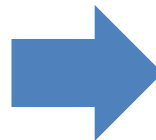
Extra electroweak  
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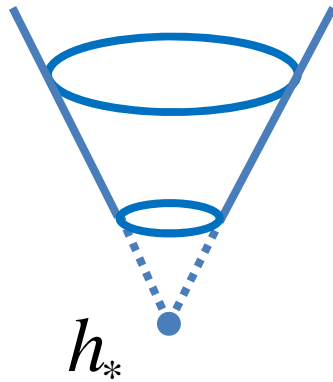
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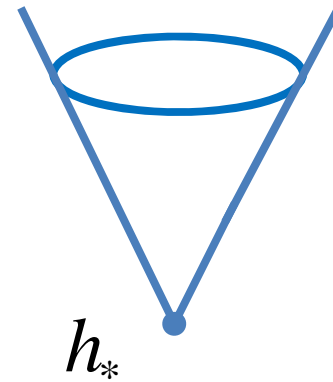


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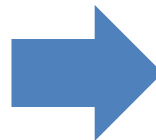
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## Insight from tree-level matching

$$\mathcal{L}_{\text{UV}}[\phi, \Phi] \Rightarrow \mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_{\text{UV}}[\phi, \Phi_c[\phi]] \quad , \quad \left. \frac{\delta S_{\text{UV}}}{\delta \Phi} \right|_{\Phi_c[\phi]} = 0$$

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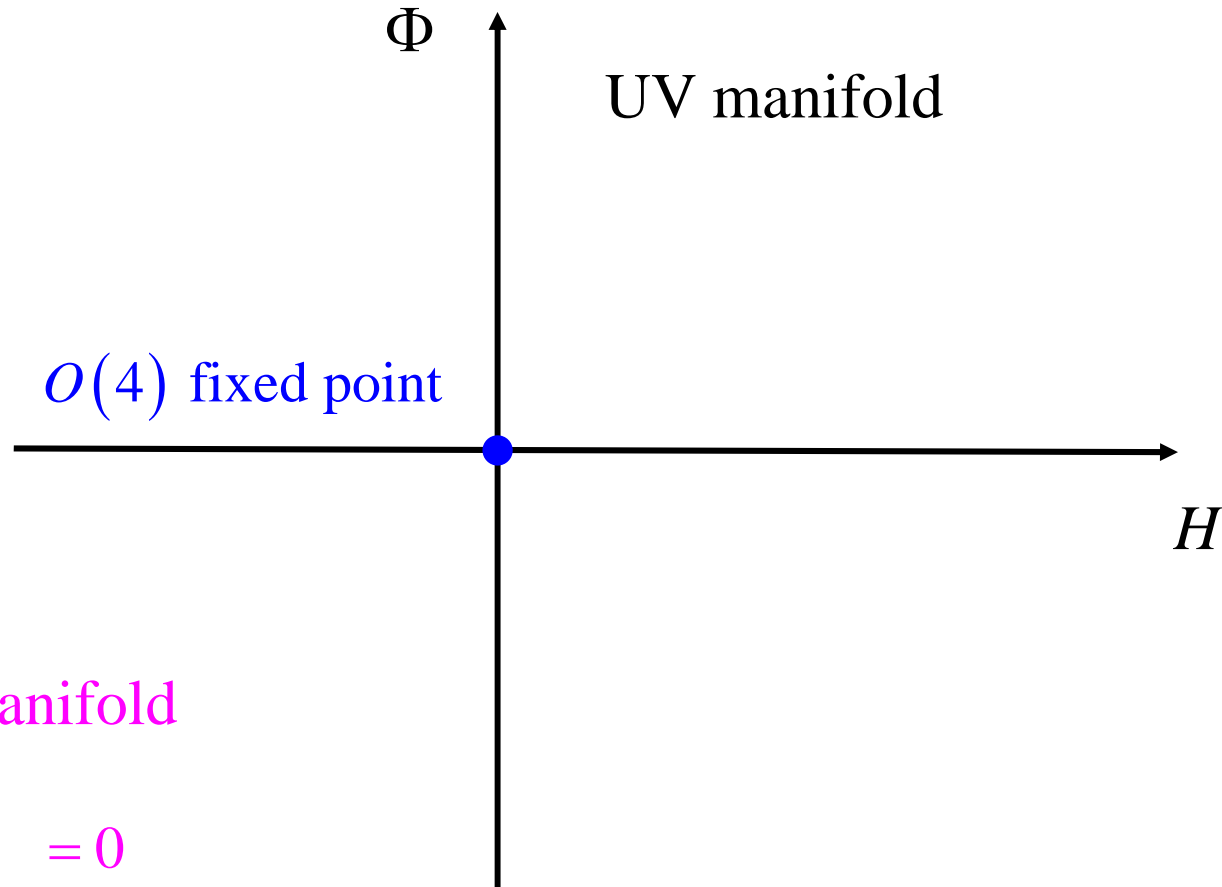
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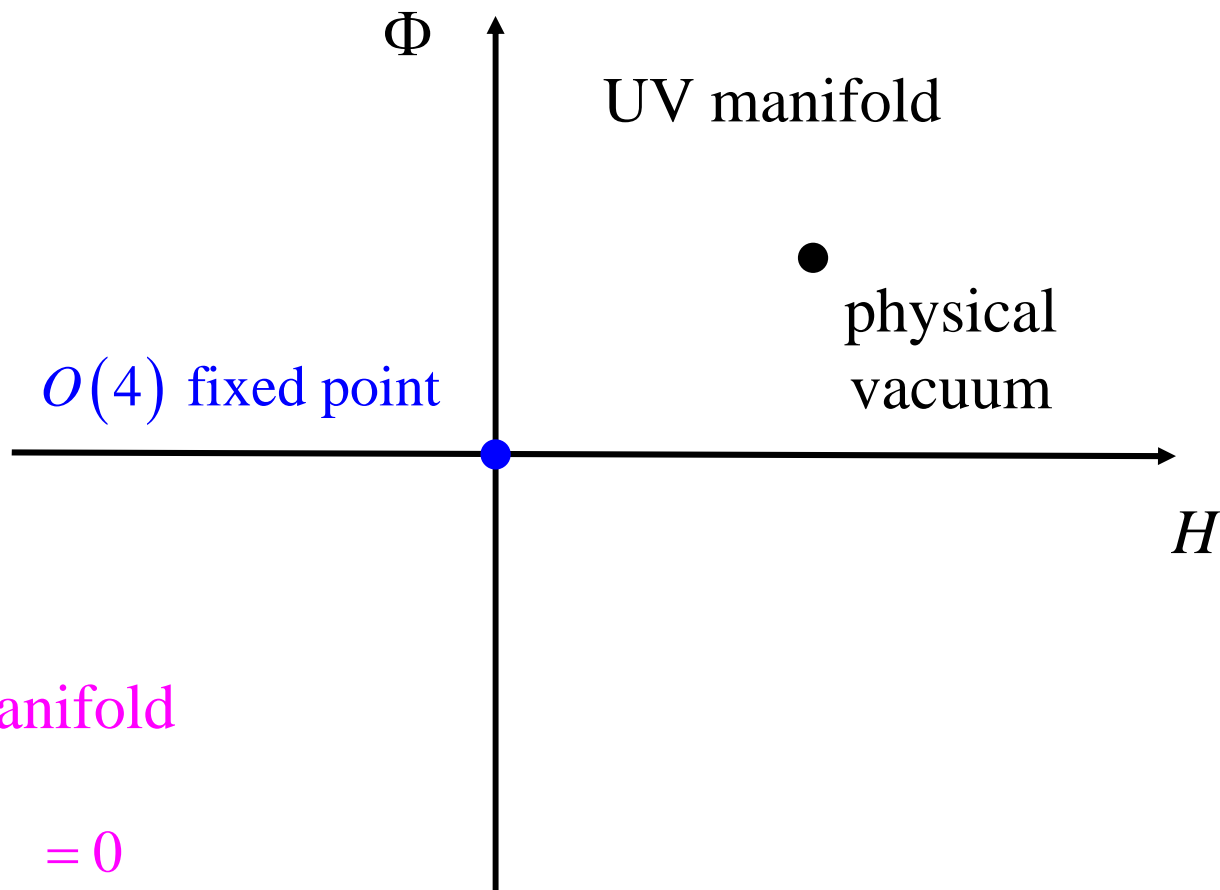
# Insight from tree-level matching



EFT submanifold

$$\left. \frac{\partial V_{\text{UV}}}{\partial \Phi} \right|_{\Phi_c^{(0)}(\phi)} = 0$$

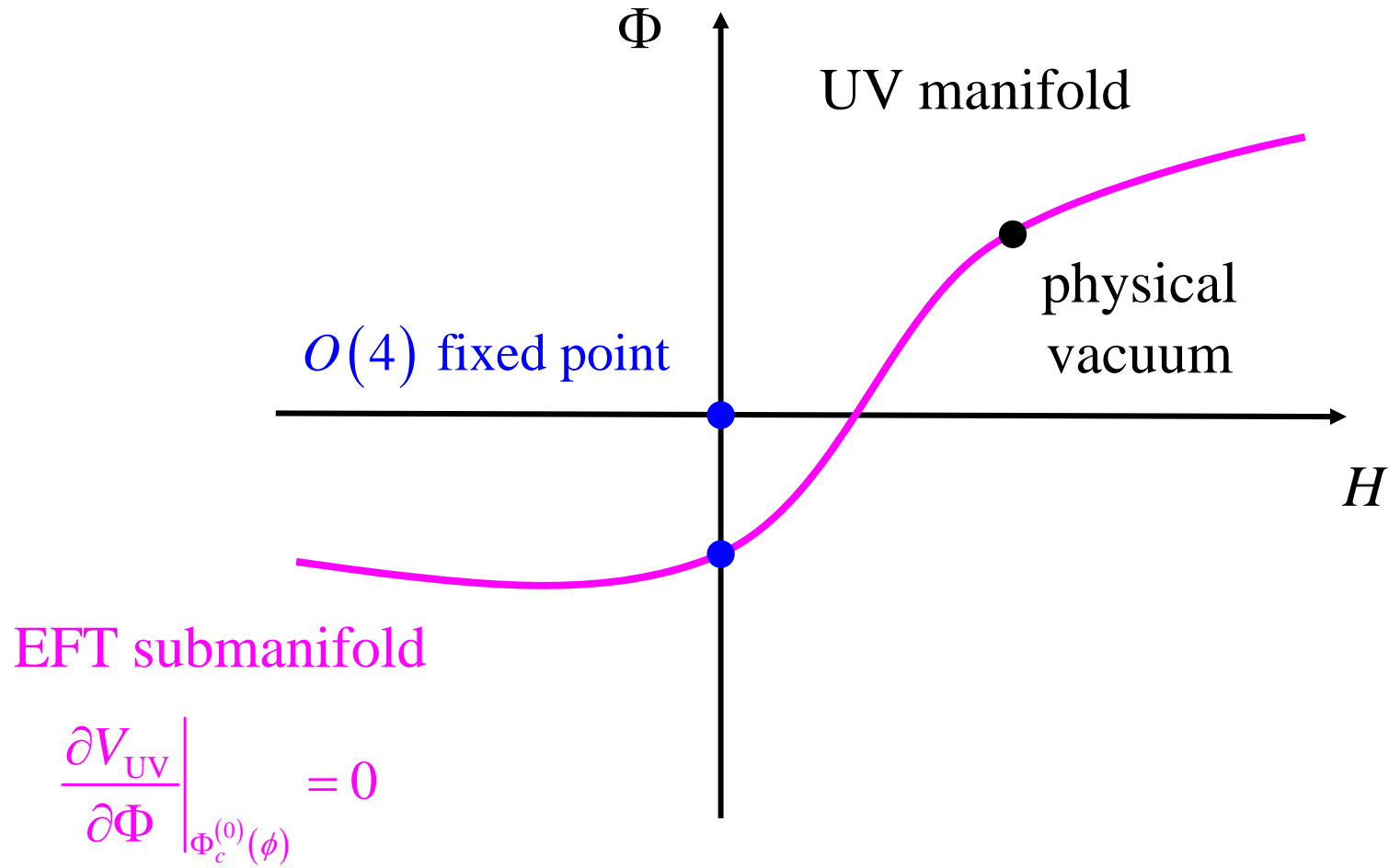
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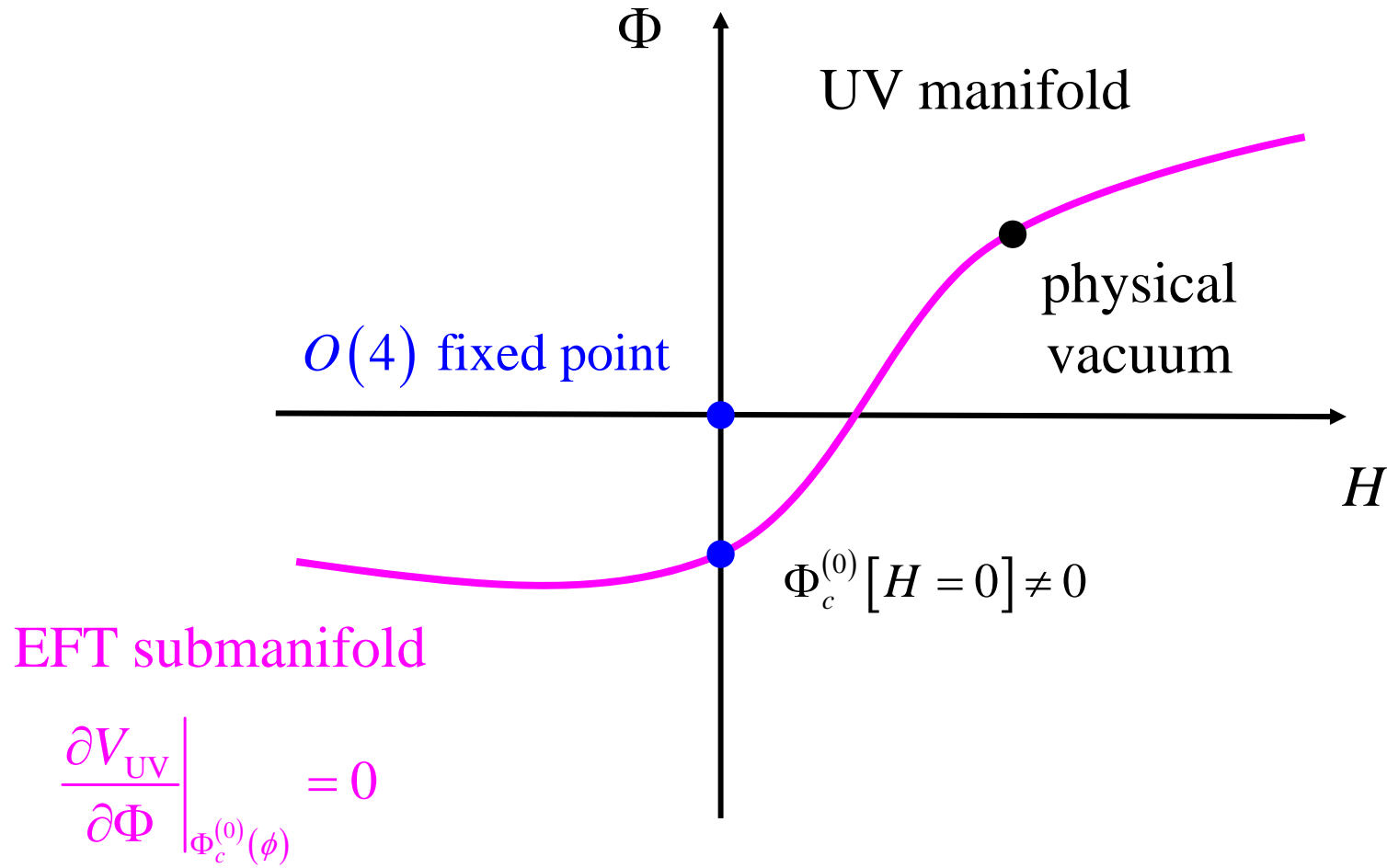
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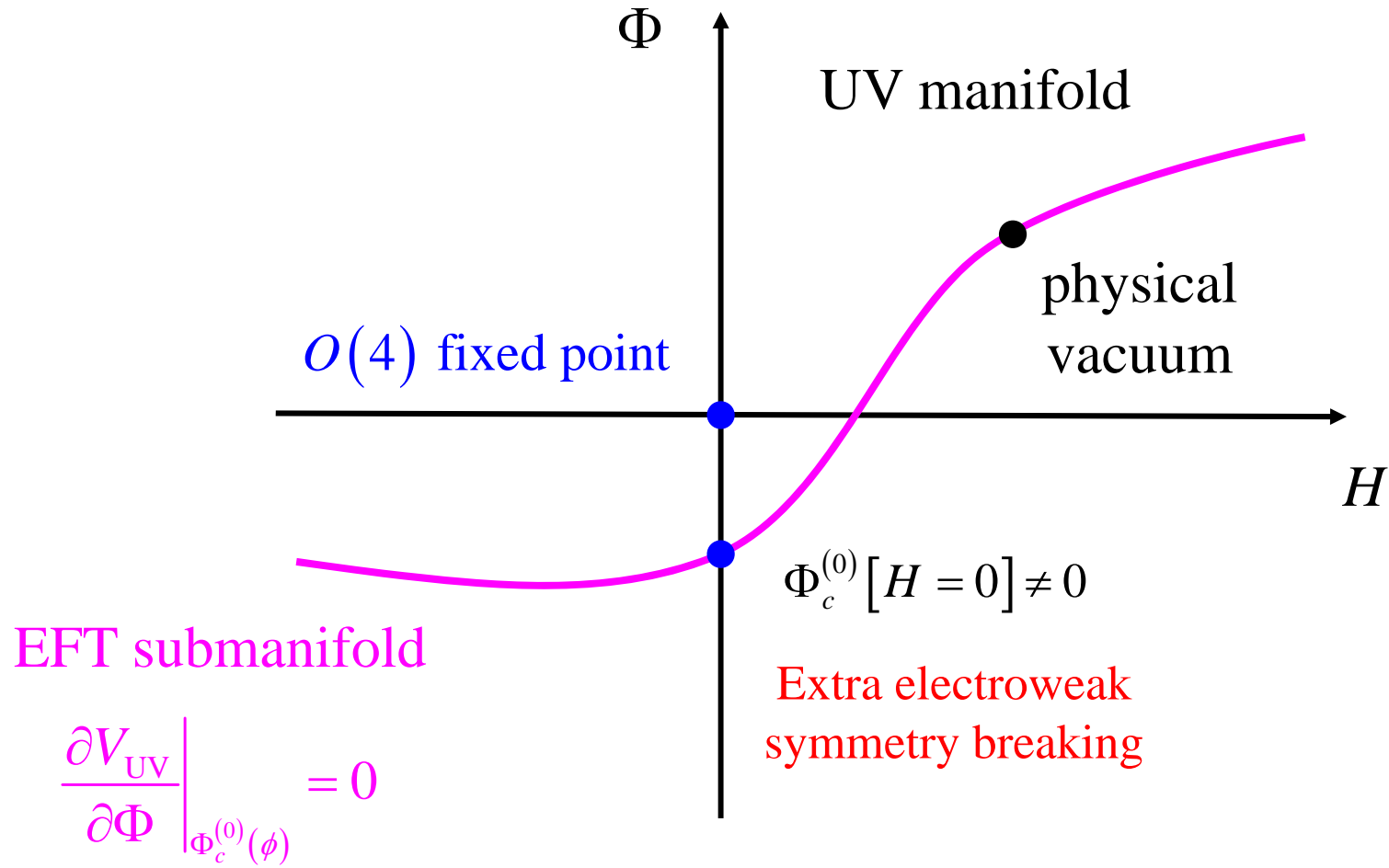
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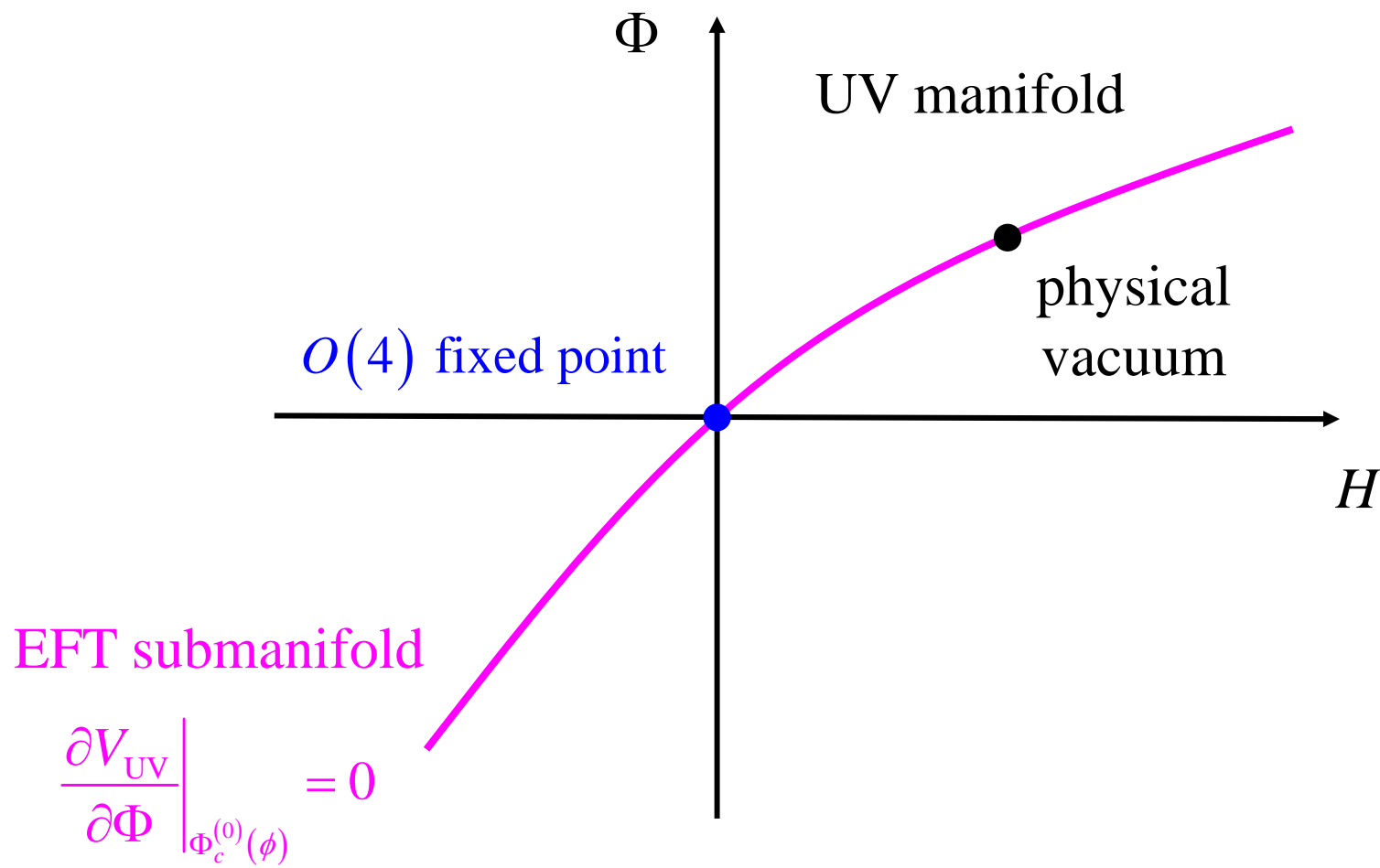
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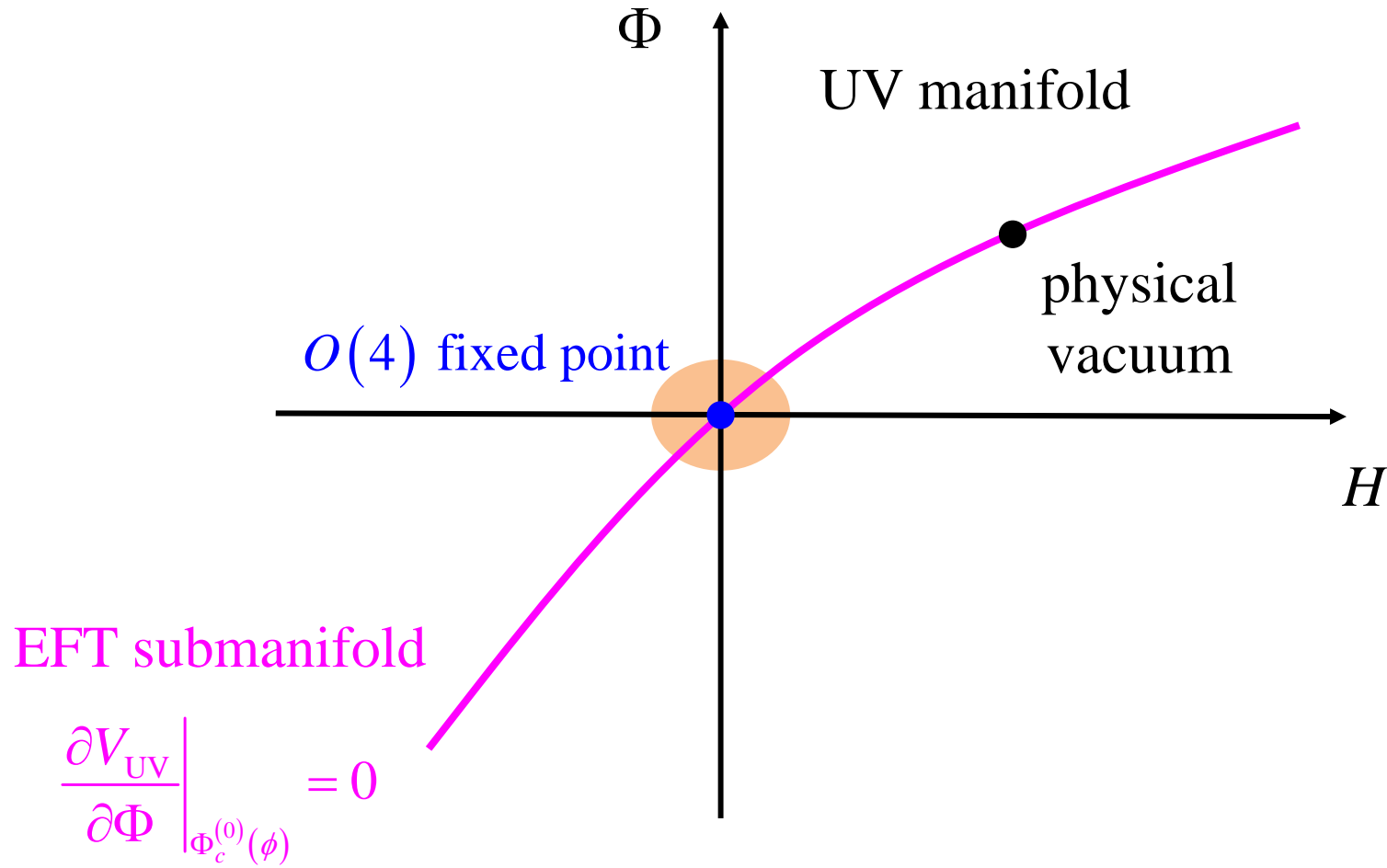


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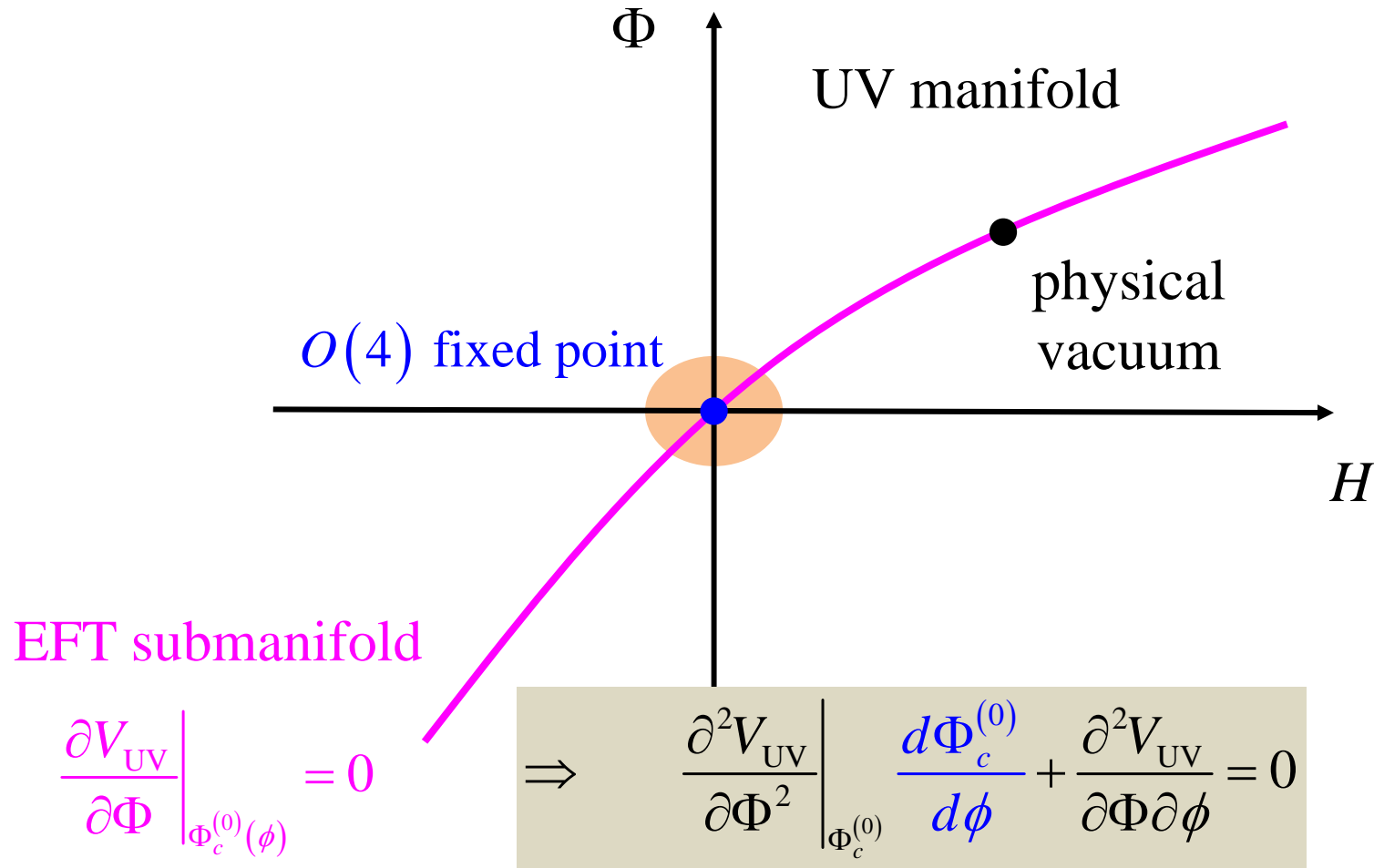




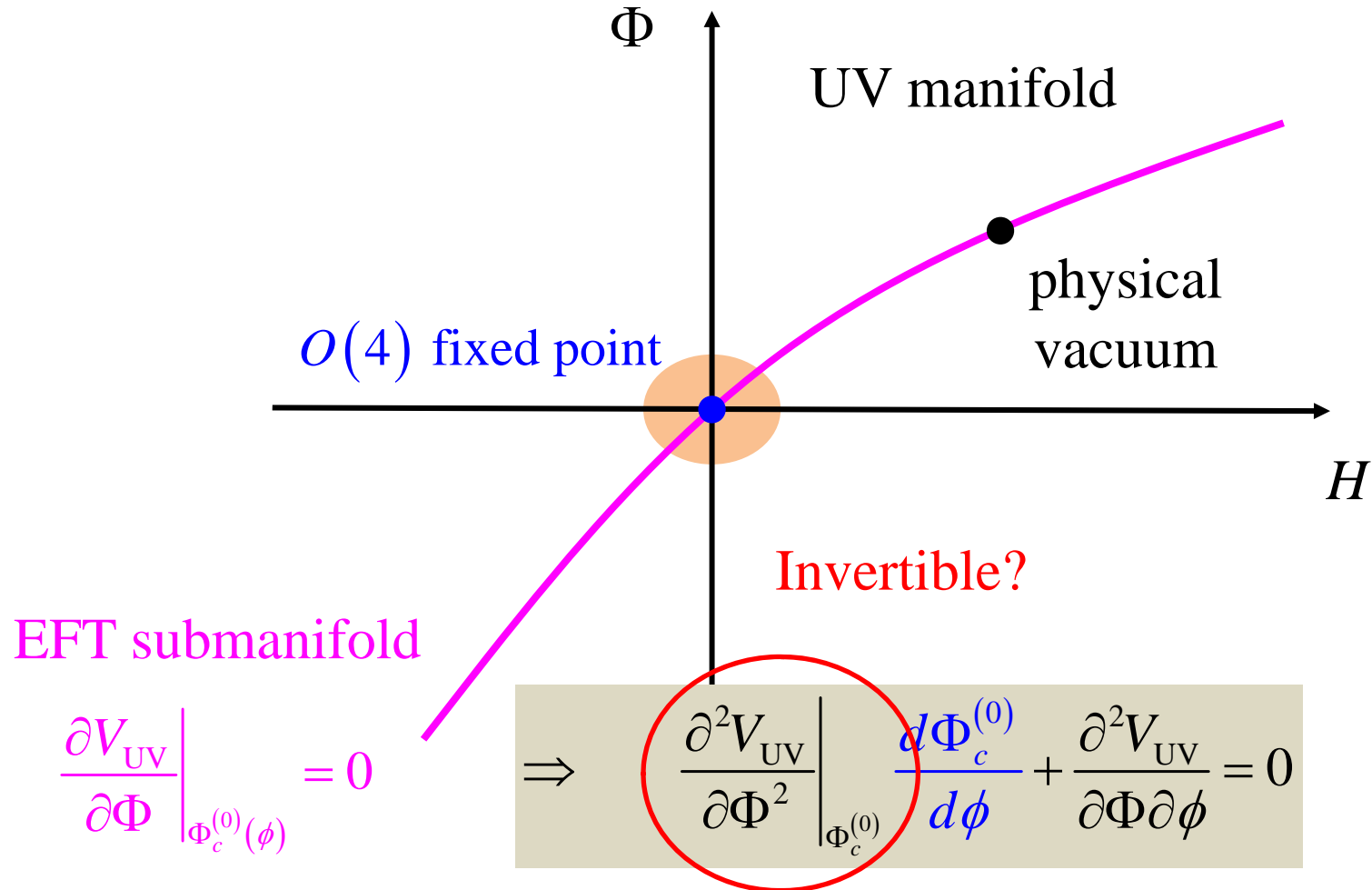
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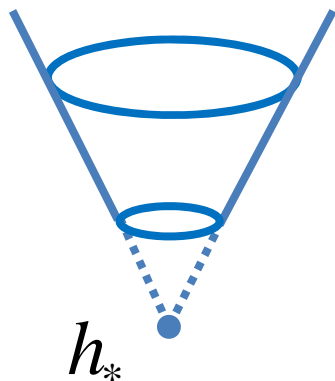


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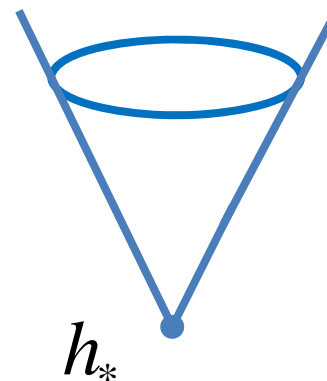
## UV theories that will generate HEFT?

Extra electroweak  
symmetry breaking



$$F(h) \neq 0$$

Mass fully from electroweak  
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$$F(h_*) = 0$$

$$R(h_*) \text{ ill defined}$$

## Matching to **all-order in fields**

Coleman-Weinberg:  $\mathcal{L}_{\text{UV}}[\phi, \Phi] \supset -\frac{1}{2}\Phi[\partial^2 + M^2 + U(\phi)]\Phi$

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$$\begin{aligned}\mathcal{L}_{\text{EFT}}[\phi] &= \frac{i}{2} \ln \det(\partial^2 + M^2 + U) = \frac{i}{2} \text{Tr} \ln(\partial^2 + M^2 + U) \\ &= \frac{i}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \ln(-p^2 + M^2 + U) + \frac{1}{6(p^2 - M^2 - U)^3} (\partial U)^2 \right] \\ &= \frac{1}{2} \int d^4x \frac{1}{16\pi^2} \text{tr} \left[ \frac{1}{2} (M^2 + U)^2 \left( \ln \frac{\mu^2}{M^2 + U} + \frac{3}{2} \right) + \frac{1}{12} \frac{1}{M^2 + U} (\partial U)^2 + \dots \right]\end{aligned}$$

## Matching to **all-order in fields**

**Coleman-Weinberg:**  $\mathcal{L}_{\text{UV}}[\phi, \Phi] \supset -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U(\phi)\right]\Phi$

$$\begin{aligned}\mathcal{L}_{\text{EFT}}[\phi] &= \frac{i}{2} \ln \det(\partial^2 + M^2 + U) = \frac{i}{2} \text{Tr} \ln(\partial^2 + M^2 + U) \\ &= \frac{i}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \ln(-p^2 + M^2 + U) + \frac{1}{6(p^2 - M^2 - U)^3} (\partial U)^2 \right] \\ &= \frac{1}{2} \int d^4x \frac{1}{16\pi^2} \text{tr} \left[ \frac{1}{2} (M^2 + U)^2 \left( \ln \frac{\mu^2}{M^2 + U} + \frac{3}{2} \right) + \frac{1}{12} \frac{1}{M^2 + U} (\partial U)^2 + \dots \right]\end{aligned}$$

If  $[U, \partial_\mu U] \neq 0$ : see App. D in 2008.08597

## A heavy singlet at one-loop level

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} - \frac{1}{2} S \left( \partial^2 + M^2 + \kappa |H|^2 \right) S \quad , \quad m_S^2 = M^2 + \frac{1}{2} \kappa v^2$$

$M^2 > 0$  , SMEFT       $M^2 = 0$  , HEFT



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$$\mathcal{L}_{\text{UV}} = |\partial H_A|^2 + |\partial H_B|^2 - V$$

$$V = m_A^2 |H_A|^2 + m_B^2 |H_B|^2 + \lambda_A |H_A|^4 + \lambda_B |H_B|^4 \\ + 2\kappa |H_A|^2 |H_B|^2 + (\mu H_A^* H_B^2 + h.c.)$$

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# Summary

- A geometric criterion to tell a HEFT Lagrangian from SMEFT
- An understanding of what UV theories would generate HEFT
- A functional method to match to all order in *fields* (not *derivatives* yet)
- A few UV examples show that the leading order criterion works