Effective Theories of Black Hole Dynamics

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All Things EFT

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EFT, or the Search for a Separation of Scales

EFT

- A scheme to systematically parametrize our ignorance about short-distance dynamics

- An approach to simplify the long-distance physics of a known complex theory

EFT

In this talk

- An approach to simplify the long-distance physics of a known complex theory

General Relativity

Not *low-energy EFT*,

but *long-wavelength EFT*

Quantum EFT: $length \sim \frac{\hbar}{E}$

Gravitational classical EFT: $length \sim GE$

(assume relativistic invce)

EFT & separation of scales

Integrate out (solve) short-distance dynamics

to obtain coefficients/functions of long-distance state in a systematic expansion

Can be classical or quantum

Long-wavelength states can differ

Fluctuations around:

- vacuum
- thermal state
- finite-energy localized object (soliton, **black hole**)

Black holes and Black branes

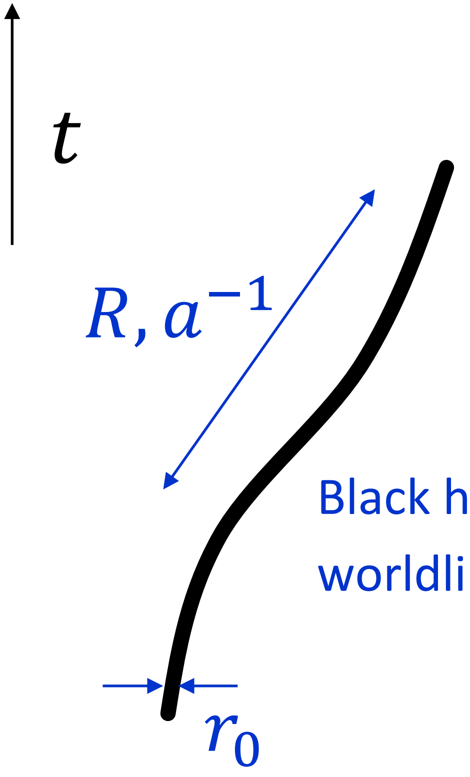
Two problems amenable to EFT:

Motion in spacetime

Horizon fluctuations

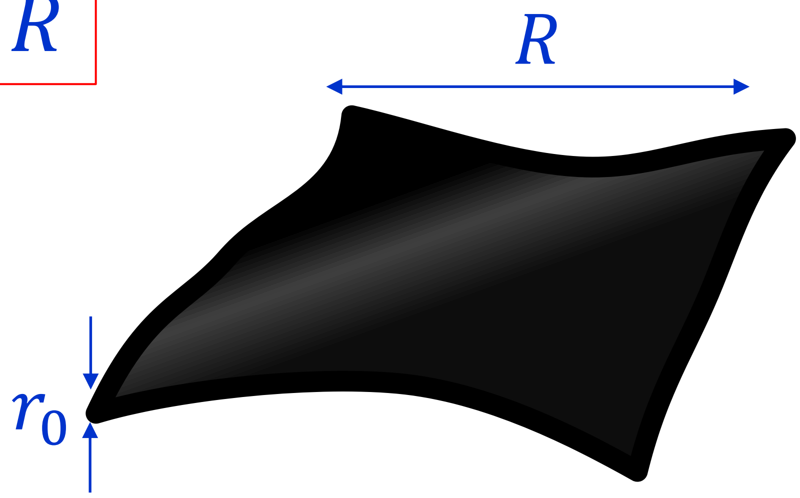
Motion in spacetime

Worldline/worldvolume dynamics



Black hole worldline

$$\boxed{r_0 \ll R}$$

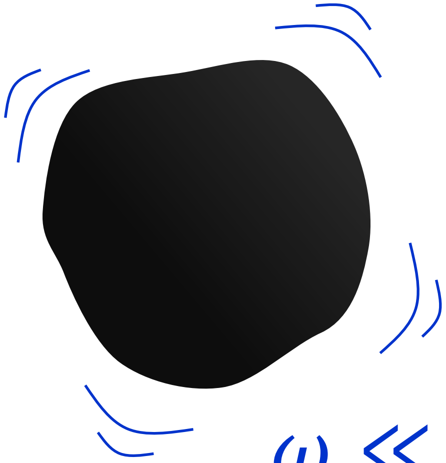


Black brane

Horizon fluctuations

Quasinormal ringing & Stability

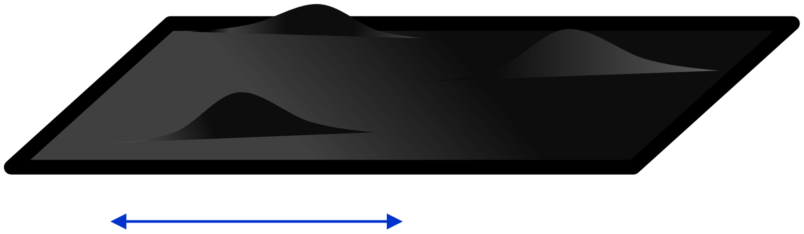
Non-linearly



$\omega \ll \kappa \sim 1/r_0$

surface gravity

(?)



$\lambda \gg r_0$

BHs and Black branes are dissipative

absorption, qnm ringdown, tidal distortion

Effective eqns of mo and stress tensor

rather than effective action

Symmetry and conservation

$$\nabla_\mu T^{\mu\nu} = 0$$

$T_{\mu\nu}$: gradient expansion

worldline/worldvolume covariance

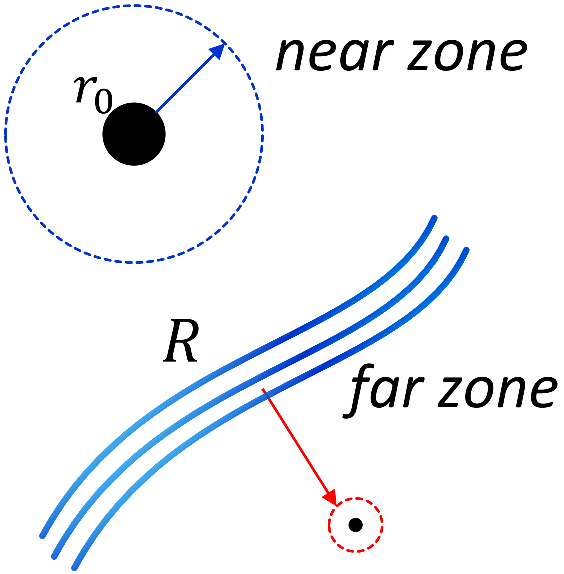
Goldberger talk@AllEFT
M Levi ReptProgPhys 2018
Poisson et al LivRevRel 2011

BH worldline theory

bh

background

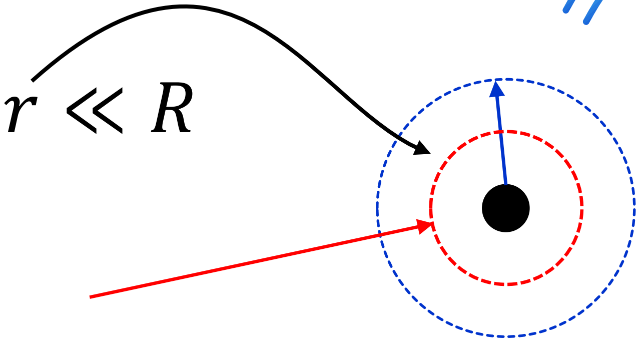
$$\boxed{r_0 \ll R}$$



$$ds^2_{(near)} = ds^2(Schw) + O(r/R)$$

$$ds^2_{(far)} = ds^2(bckg) + O(r_0/r)$$

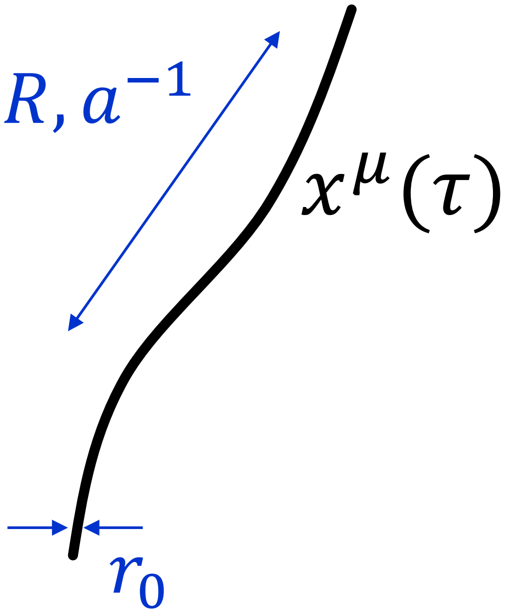
overlap: $r_0 \ll r \ll R$



In *far*, $r \gg r_0$: BH → worldline $x^\mu(\tau)$

$$T^{\mu\nu} = c_1(x(\tau))\ \dot{x}^\mu(\tau)\dot{x}^\nu(\tau) + \cdots$$

(add tetrad for spin)



$$T_{\mu\nu} = c_1(x(\tau))\, u_\mu(\tau) u_\nu(\tau) + \cdots$$

(minimal coupling)

$$u^\mu = \dot{x}^\mu(\tau)$$

Symmetries

General coord invce

Worldline reparametrization

Internal Lorentz invariance

$$T^{\mu\nu} = c_1\big(x(\tau)\big)\, u_\mu(\tau) u_\nu(\tau)$$

Effective eom

$$\nabla_\mu T^{\mu\nu} = 0$$

ν *parallel* to worldline $\Rightarrow \partial_\tau c_1 = 0 \Rightarrow c_1 = m$

ν *orthogonal* to worldline $\Rightarrow m a^\mu(\tau) = 0$

m is matched to BH mass in overlap zone

Coefficients in effective stress tensor obtained by integrating out short distance physics

BH worldline theory

Derivation from Einstein’s equations

- match asymptotics: perturb BH with generic weak asymptotic field (*far* field sources)
- radial (Gauss-Codazzi) constraint → eff eqs

BH worldline theory

EFT coefficients can also be obtained diagramatically

Classical Feynman diagrams with on-shell external legs, no virtuality in loops

At lowest orders both methods are similarly simple

RE+Harmark+Niarchos+Obers 2009
Camps+RE+Haddad 2010,2012

Black Brane Worldvolume Theory

$$x^{\mu}(\tau, \sigma^{i})$$

- Derivatives *orthogonal* to worldvolume:
  extrinsic curvature $K_{\mu\nu}{}^{n}$

orthogonal

worldvolume

- Derivatives *parallel* to worldvolume:
  worldvolume velocities u^{i}

worldvol Lorentz broken in black brane
not purely tensional brane, can support wv velocities

Symmetries

General coord invce

Worldvolume reparametrization

Worldvolume translations + rotations

$$T^{\mu\nu} = c_1(\tau, \sigma^i) u^\mu u^\nu + c_2(\tau, \sigma^i) \underset{\uparrow}{\gamma^{\mu\nu}}$$

wv induced metric

$$T^{\mu\nu} = c_1(\tau, \sigma^i) u^\mu \, u^\nu + c_2(\tau, \sigma^i) \gamma^{\mu\nu}$$

Effective eom

$$\nabla_\mu T^{\mu\nu} = 0$$

Orthogonal to wv ⇒ $T^{\mu\nu} {K_{\mu\nu}}^n = 0$

$$T^{\mu\nu} = c_1(\tau, \sigma^i) u^\mu \, u^\nu + c_2(\tau, \sigma^i) \gamma^{\mu\nu}$$

Effective eom

$$\nabla_\mu T^{\mu\nu} = 0$$

Parallel to wv ⇒ Relativistic fluid equations

with $c_1 = \varepsilon + P \qquad c_2 = P$

$\varepsilon(\tau, \sigma^i)$, $P(\tau, \sigma^i)$: effective *energy density* & *pressure*

Coefficients in effective stress tensor ε, P from integrating out short-distance physics

Match *near* to *far* at $r_0 \ll r \ll R$

$\Rightarrow$ Effective equation of state of black p-brane

$$P = -\frac{\varepsilon}{D-p-2}$$

Extrinsic dynamics EFT

Elasticity theory

Extrinsic dynamics EFT

Elastic membrane

Example: Dirac-Nambu-Goto pure-tension brane

$$T^{\mu\nu} = -T\gamma^{\mu\nu} \;\Rightarrow\; \gamma^{\mu\nu} K_{\mu\nu}{}^{n} = 0$$

minimal (extremal) surface

Blackfolds are elastic, but not minimal surfaces

$$T^{\mu\nu} = (\varepsilon + P) u^{\mu}\, u^{\nu} + P\gamma^{\mu\nu}$$

Extrinsic dynamics EFT

Elastic solid membrane

Bent brane: Young modulus

Armas + Camps,Gath,Harmark,Obers

Intrinsic dynamics EFT

Hydrodynamics

Fluid on the brane

$$T_{\mu\nu} = {T_{\mu\nu}}^{(0)} + {T_{\mu\nu}}^{(1)} + \cdots$$

zero gradient

perfect fluid

one gradient

viscous fluid

Black Branes as Fluids

Fluid/gravity correspondence

A classical limit of AdS/CFT

Bhattacharyya+Hubeny+Minwalla+Rangamani 2007

Black Branes as Fluids

Fluid/gravity correspondence

Universality of hydrodynamics as EFT

Long-wavelength fluctuations of *thermal state of CFT*:
hydrodynamics

Long-wavelength fluctuations of *black brane in AdS*:
hydrodynamics

Bhattacharyya+Hubeny+Minwalla+Rangamani 2007

Fluid/gravity correspondence

can be derived from classical gravity in AdS
without assuming AdS/CFT

Uniform black brane:
constant temperature & velocity

Allow them to vary along brane, correcting the metric to satisfy Einstein’s eqs

Solve Einstein: integrate radial direction
Obtain eq of state, transport coeffs

Black branes as fluids: highlights

- AdS branes: Viscosity of “quark-gluon-like” plasma *Policastro+Son+Starinets+Kovtun*
- AF branes: Gregory-Laflamme instability analytically solved as hydro instability
- Correlated stability conjecture *Gubser+Mitra* reduces to

thermodynamic instability ⇒ hydrodynamic instability

Limitations

Effective description of horizon fluctuations requires *long horizon* to support long wavelengths

$$\lambda \gg |\partial_r|^{-1}$$

needed for integrating radial dependence

$|\partial_r|^{-1}$: surface gravity, temperature

$$\lambda \gg \frac{1}{\kappa}, \frac{1}{T}$$

acceleration length, thermal length

For Schw or Kerr (or non-ultraspinning bhs)

Horizon length ~ Thermal length

$$r_0 \sim |\partial_r|^{-1},\ T^{-1},\ \kappa^{-1}$$

$$\lambda \simeq r_0$$

$$\omega \cancel{\ll} \kappa \sim 1/r_0$$

Can’t have long wavelengths or low frequencies on horizon

No EFT for black hole fluctuations

In more detail

$$T^{-1} = \frac{4\pi}{D-3} r_0$$

$\Longrightarrow \quad T^{-1} \sim r_0$: only scale in system

$\Longrightarrow \quad \nexists$ separation of scales, $\nexists$ EFT ☹

In more detail – try again

$$T^{-1} = \frac{4\pi}{D-3} r_0$$

$$D \gg 1$$

$\Longrightarrow \quad T^{-1} \sim r_0/D \ll r_0$: two scales

$\Longrightarrow$ $\exists$ separation of scales, $\exists$ EFT ☺

Asnin+Gorbonos+Hadar+Kol+Levi 2007

RE+Suzuki+Tanabe 2015

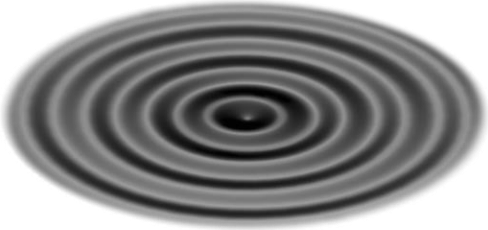
RE+Herzog RevModPhys *2020*

Study BH fluctuations in an expansion in 1/D

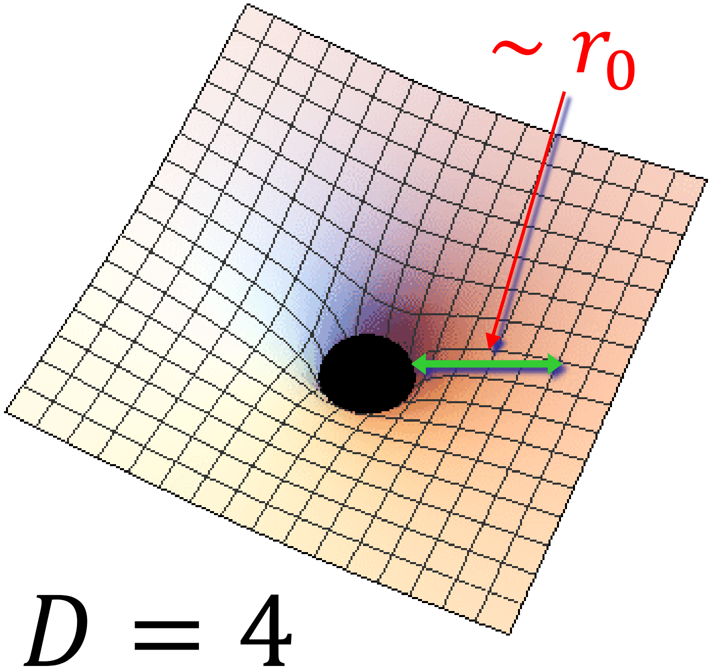
What kind of EFT?

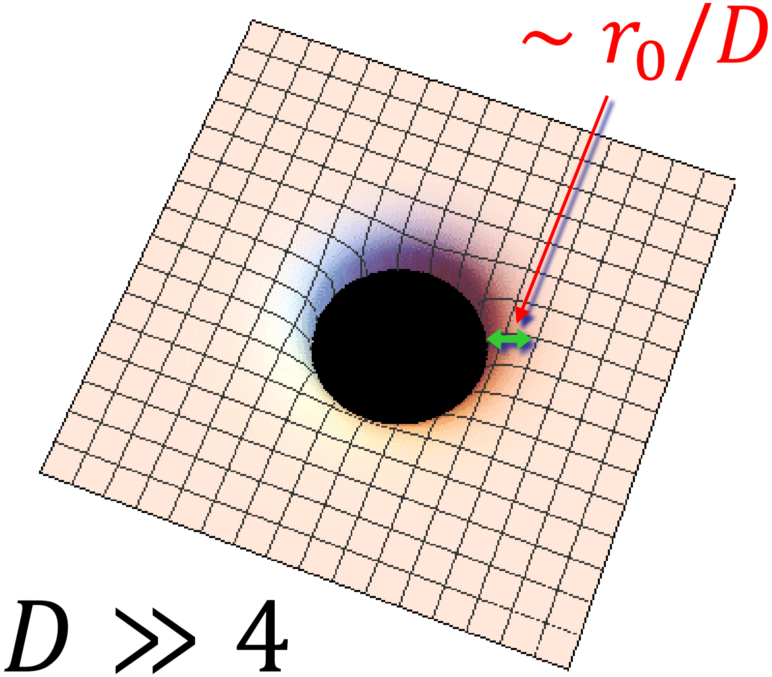
Elasticity theory or Hydrodynamic theory?

Ripples on a pond, or wrinkles on a membrane?



Not *or*, but *and*

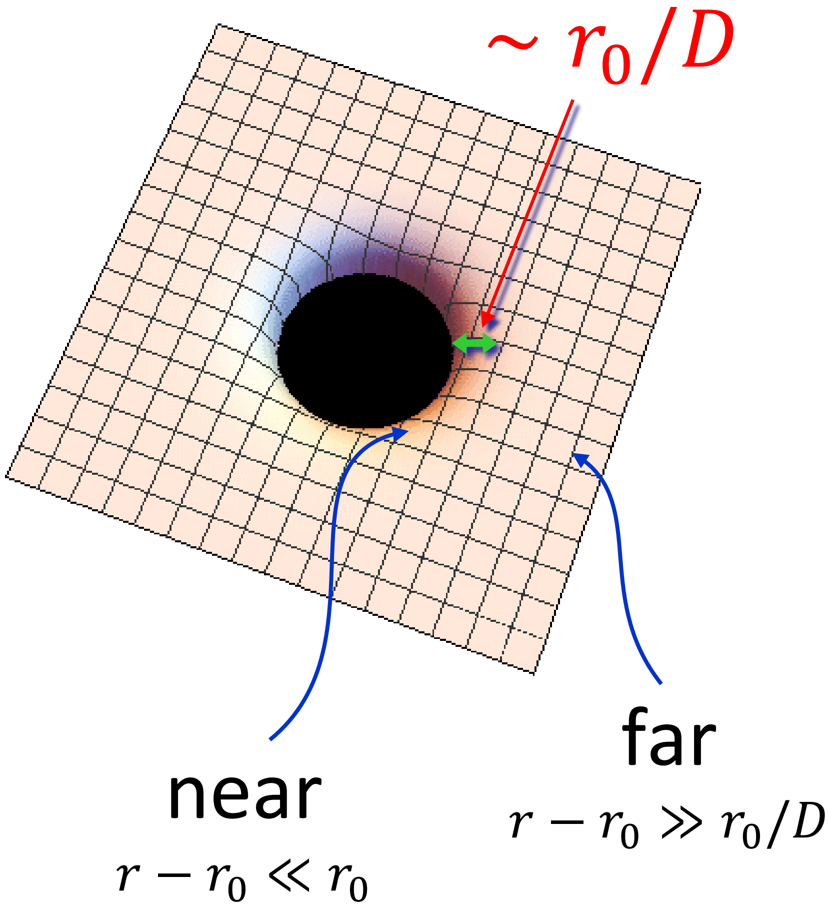

$\sim r_0$
$D = 4$


$\sim r_0/D$
$D \gg 4$

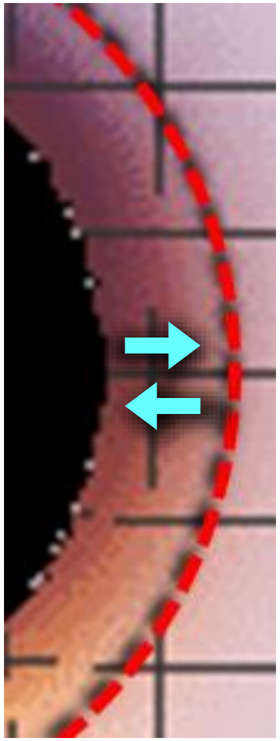
So near, so far…

Near and *far* defined independently of any background length scale

Just $r_0 \gg r_0/D$ intrinsic black hole scales



Slowest fluctuations of BH



$$\omega \sim \frac{1}{r_0} \ll \frac{D}{r_0}$$

Almost static in near-horizon region

Decoupled from far zone

∃ parametrically slow fluctuations

Find EFT for non-linear slow fluctuations

Integrate radial direction away from horizon

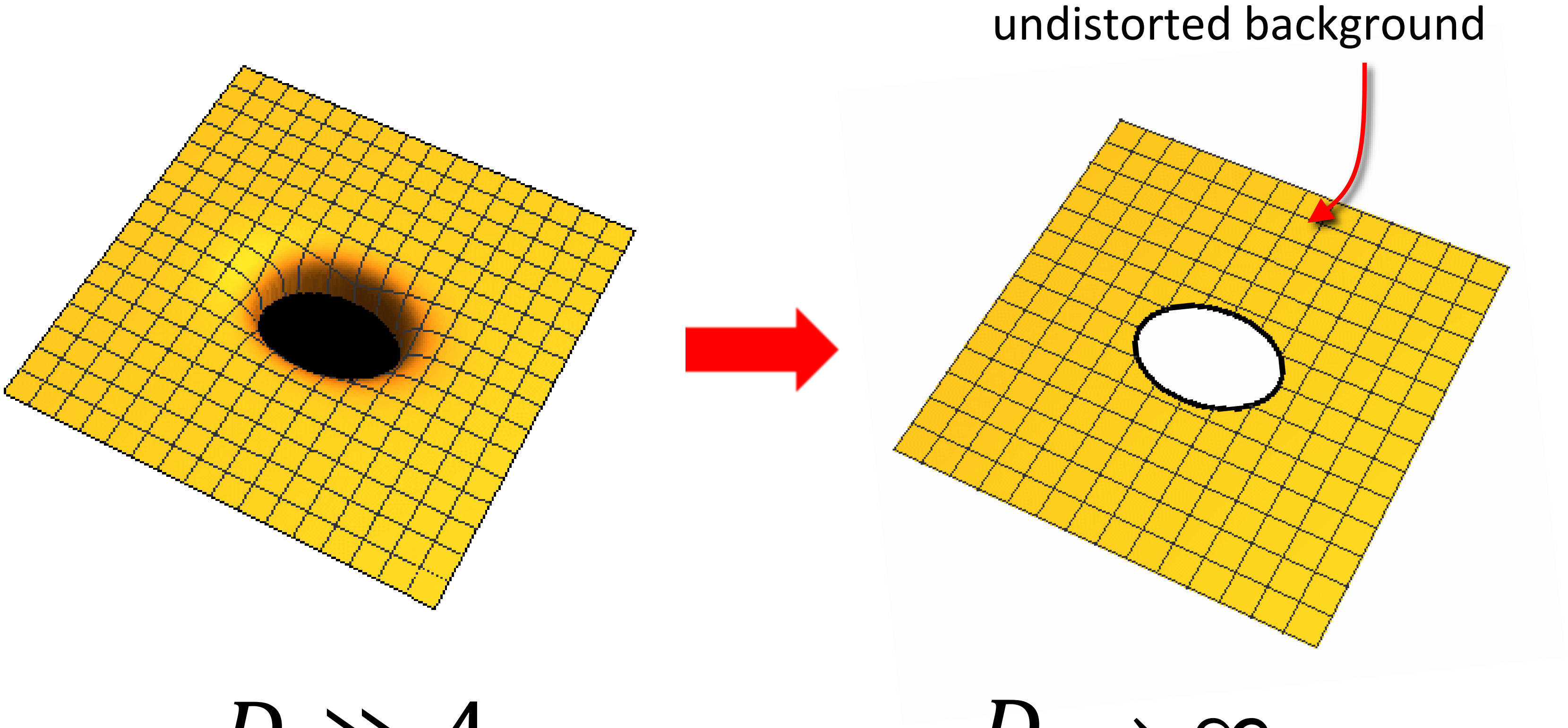
ie, integrate out near-horizon dynamics

Gradient hierarchy

$\perp$ Horizon: $\partial_r \sim D$

$\parallel$ Horizon: $\partial_t , \partial_z \sim 1$ $(\text{or} \sim \sqrt{D}\,)$

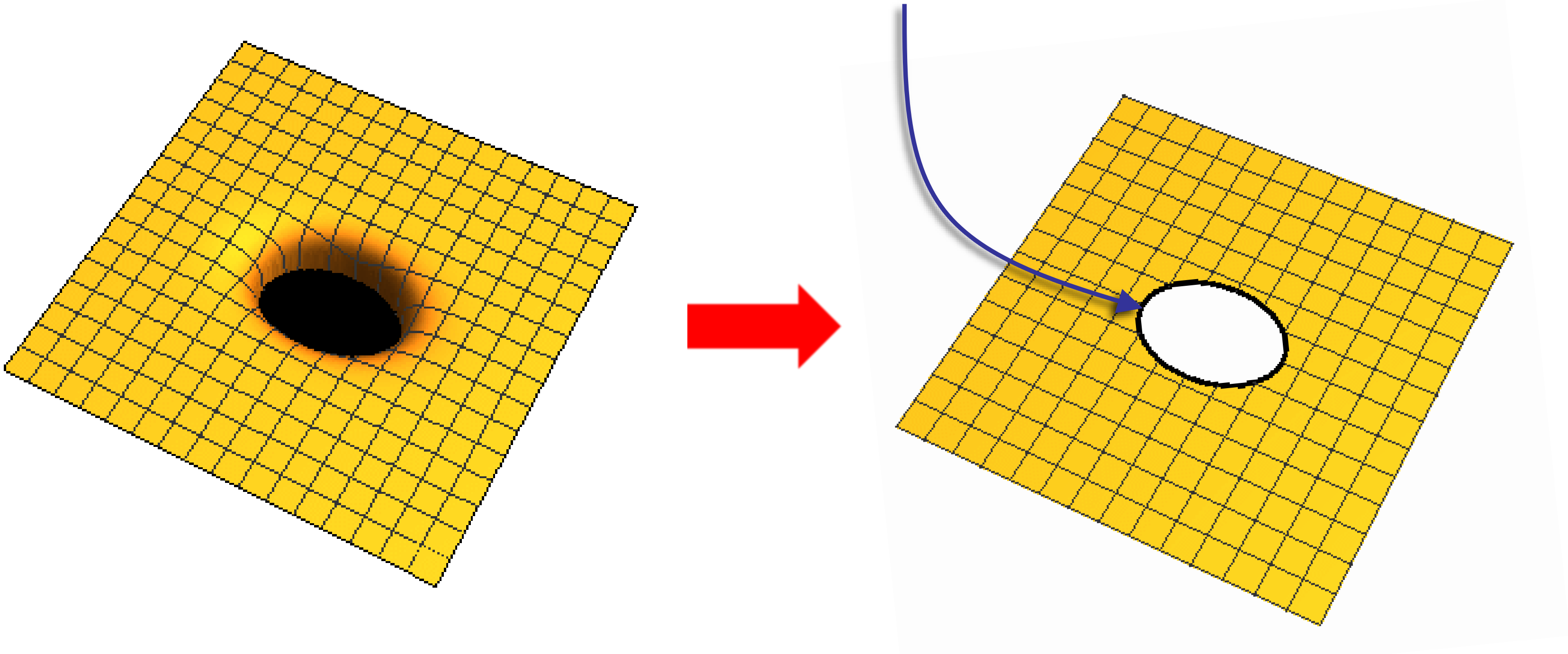
Replace BH → ‘effective membrane’



$D \gg 4$

$D \to \infty$

What's the dynamics* of this membrane?



$D \gg 4$ $\qquad$ $D \to \infty$

*what shapes? what evolution?

Effective fields

- Horizon shape (embedding into background)

$$X(\sigma)$$

w/ finite number of non-trivial horizon directions σ

- Horizon velocity

$$u(\sigma)$$

Effective equations for $X(\sigma), u(\sigma)$

Most general & elegant formulation by

Bhattacharyya+Minwalla et al 2015

$$\left(\frac{\nabla^2 u}{\mathcal{K}} - \frac{\nabla \mathcal{K}}{\mathcal{K}} + u \cdot K - (u \cdot \nabla) u\right) \cdot \mathcal{P} = 0$$

$$\mathcal{K} = \eta^{AB} K_{AB}$$

$$\nabla \cdot u = 0\,, \qquad n \cdot u = 0$$

$$\mathcal{P}_{AB} = \eta_{AB} - n_A n_B + u_A u_B$$

n, K_{AB}: normal & **extrinsic curvature** of membrane

u: **velocity** field on membrane

Simplifies
–conceptually and technically–
in two important cases:

1. Stationary black holes

2. Black branes, AdS or AF

RE+Shiromizu+Suzuki+Tanabe+Tanaka 2015

Stationary solution

Soap-bubble equation

$$K = 2\gamma\kappa$$

K = trace **extrinsic curvature** of membrane

γ = **redshift** on membrane

κ = **surface gravity**

Effective equations for fluctuating black brane

RE+Suzuki+Tanabe 2015

Effective fields:

$m(t,\sigma^i)$: mass and area density of black brane

$v_i(t,\sigma^j)$: velocity along brane

Effective equations for fluctuating black brane

$$\partial_t m + \partial_i (m v^i) = 0$$

$$\partial_t (m v_i) + \partial^j \left(\pm m\, \delta_{ij} + m v_i v_j - 2\, m\, \partial_{(i} v_{j)} - m\, \partial^2_{ij} \ln m \right) = 0$$

pressure viscosity higher transport

Hydrodynamics truncates exactly

Large D explains unexpected success of hydro?

Effective equations for fluctuating black brane

$$\partial_t m + \partial_i (m v^i) = 0$$

$$\partial_t (m v_i) + \partial^j \left(\pm m\, \delta_{ij} + m v_i v_j - 2\, m\, \partial_{(i} v_{j)} - m\, \partial^2_{ij} \mathrm{lnm} \right) = 0$$

Can be rewritten as Soap Bubble eqn $K = 2\gamma\kappa$

with $m(t, \sigma^i)$ local bubble radius

(recall $\ell \sim GE$!)

BH “membrane paradigm”

Damour

Thorne et al

Not an EFT

Clever & suggestive way of writing boundary conditions for the gravitational field on a null hypersurface

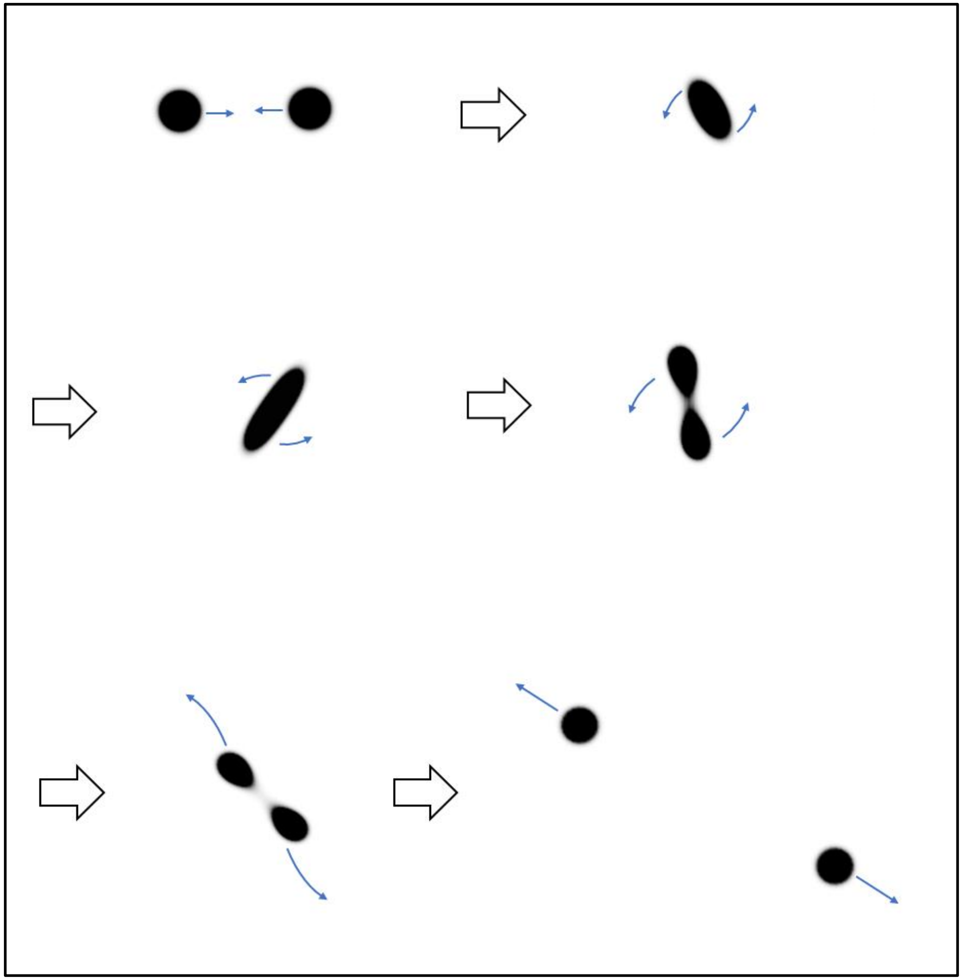
No separation of scales

No integration of any short-scale degrees of freedom

Highlight of large D effective theory:

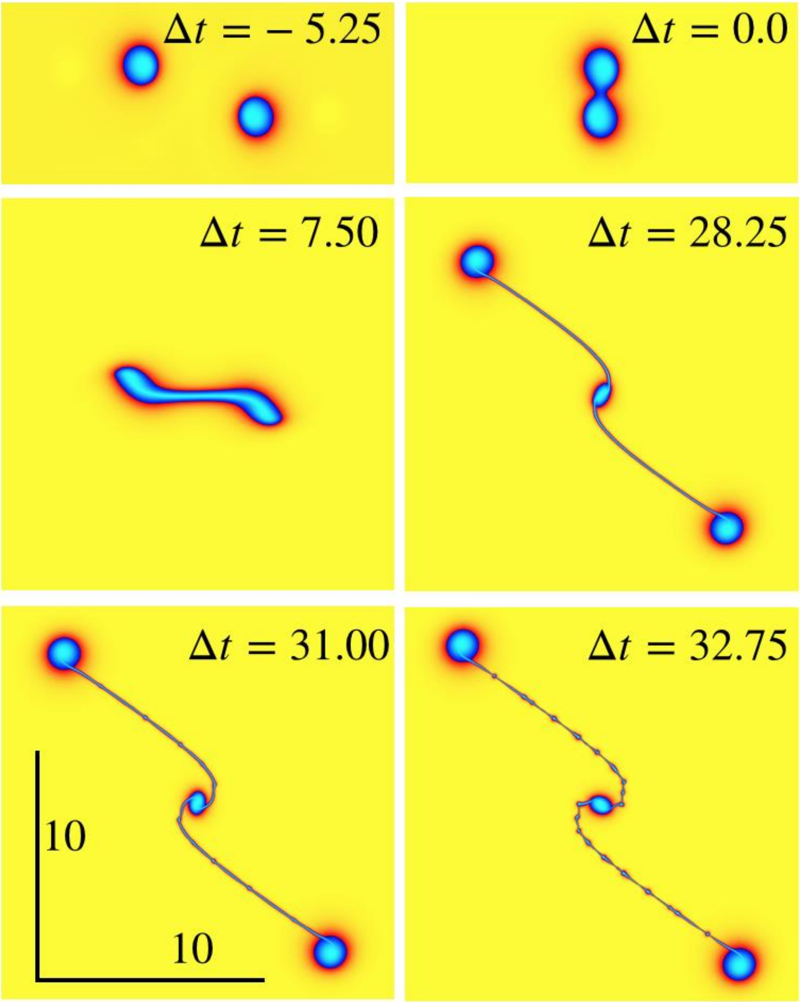
Black hole collision in higher D w/ cosmic censorship violation

● ●



Large-D EFT

Andrade+RE+Licht+Luna 2019



D=6 NumGR

Andrade+Figueras+Sperhake 2020

Final remarks

- EFT philosophy and concepts can be very usefully applied to classical theories

  Separation of scales, integrating short-distance dynamics, matching conditions, expansion in derivatives…

Final remarks

- Diagrammatic techniques, renormalization group?

  So far of little use beyond BH worldline theory

- Matched asymptotics works well enough

Final remarks

- Universal EFTs: hydrodynamics, elasticity apply also to black holes

  Different EFTs for different dynamical regimes

Final remarks

EFTs very efficient for isolating and reformulating slow non-linear dynamics of horizons

Thank you

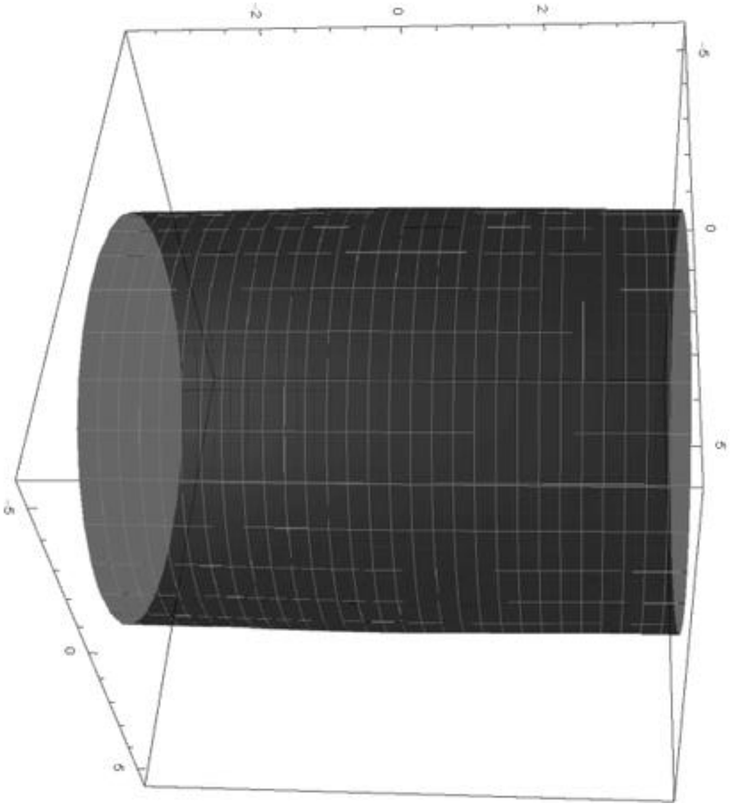
Brane blobology

Very effective theory of black holes as blobs on a brane

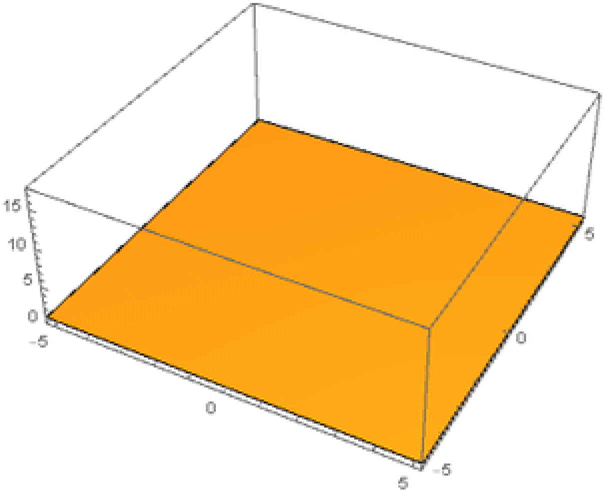
Black hole as blob on a string or brane

Black string & black brane instability *Gregory-Laflamme 1993*

Evolved w/ large-D effective theory

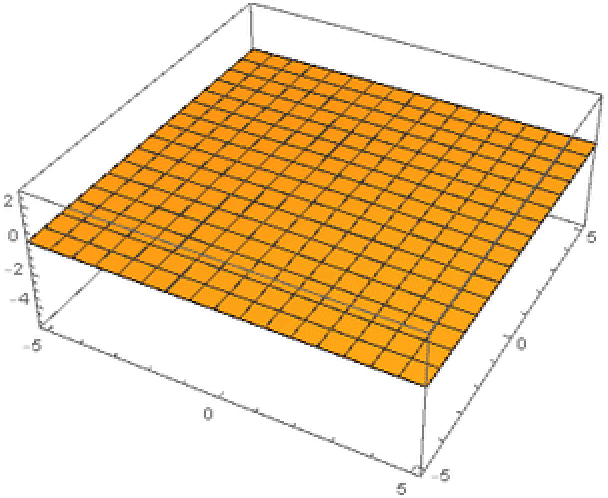


“Black hole blob” in a black membrane



Area density

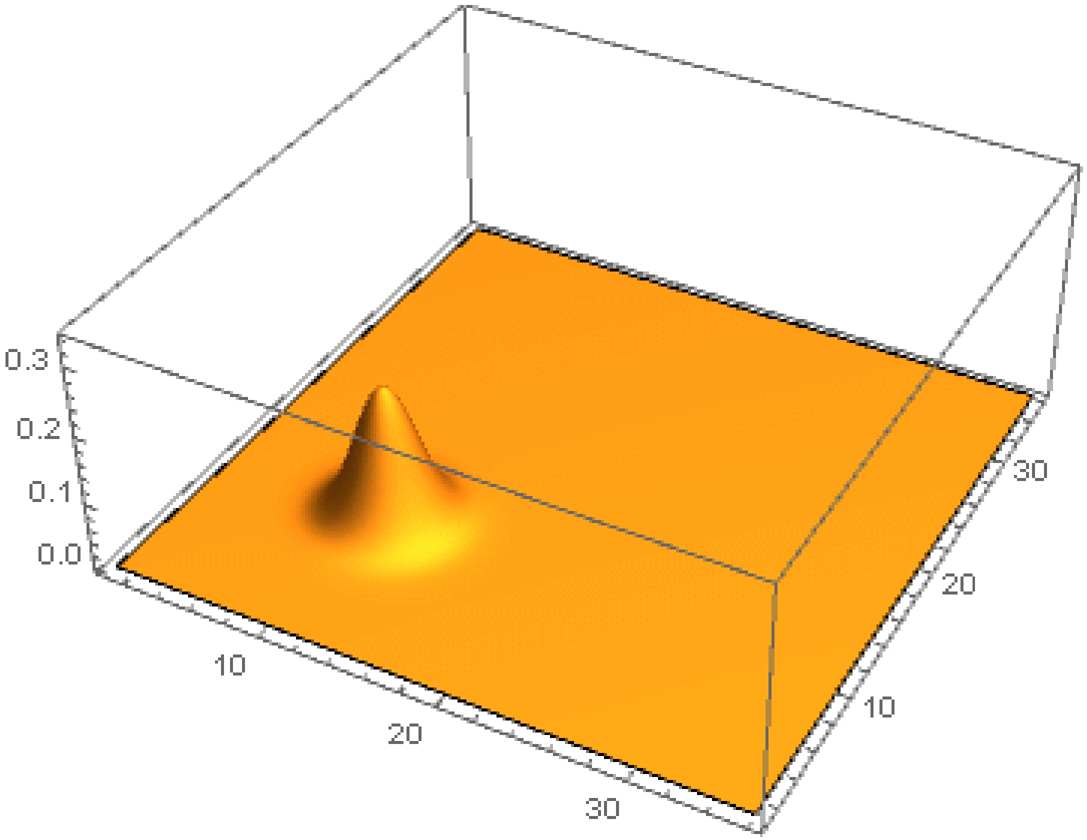
m

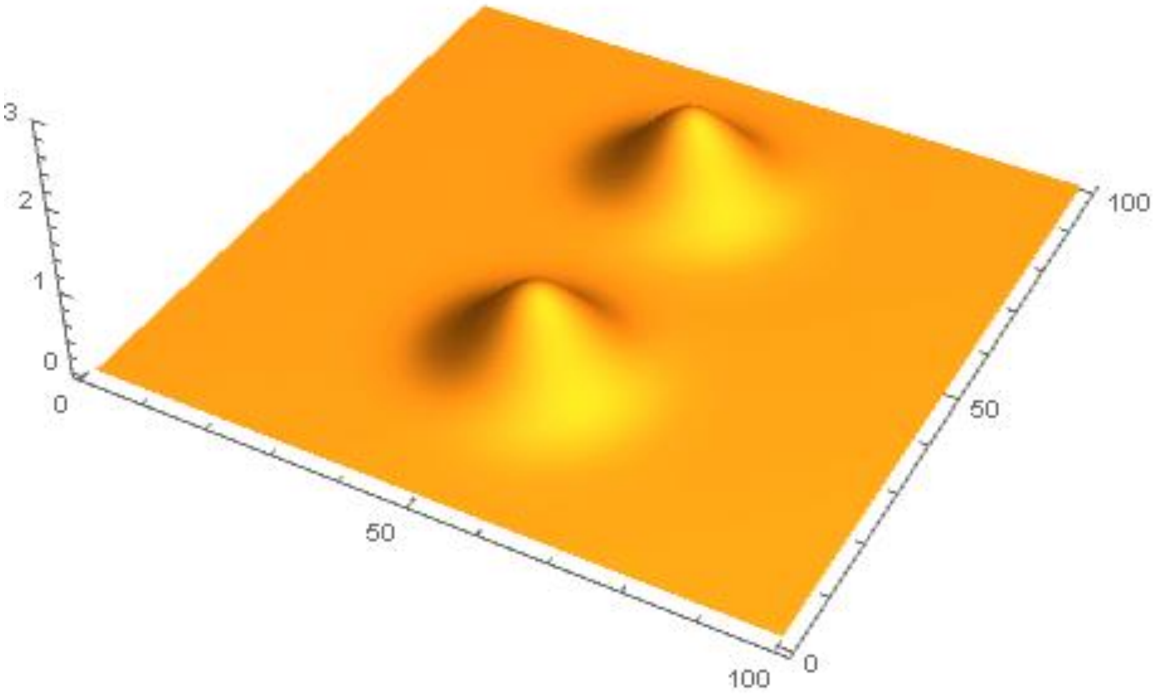


Area radius

$r_H = \ln m$

Moving black hole




3
2
1
0
0
50
100
0
50
100

Collisions of black hole blobs

Brane acts as a “regulator”:
continuous horizon

BHs never really merge nor split:
smooth evolution