Exotic hadrons in lattice QCD

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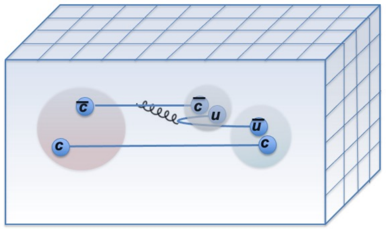
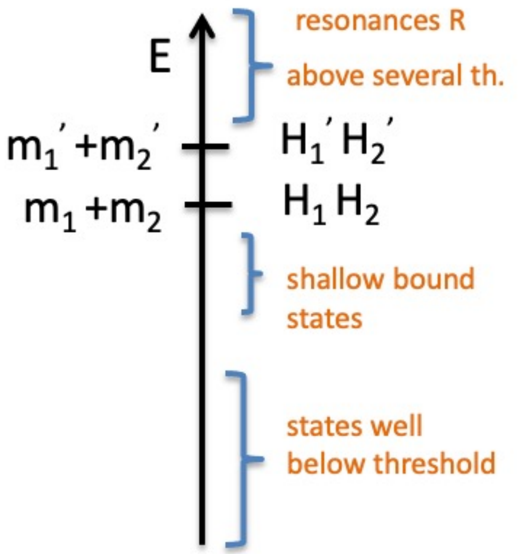
Challenge: all experimentally discovered exotic hadrons are strongly decaying resonances !

most of them decay via several decay channels $R \to H_1 H_2,\ H_1' H_2', \dots$





The status of hadrons in lattice QCD

States:

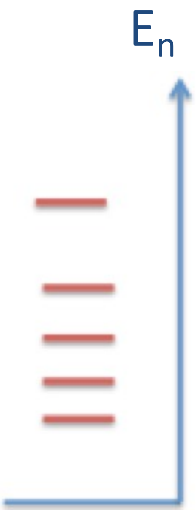
- well below threshold “straighforward”
- above 1 threshold or just below it “mostly done”
- above 2 or 3 threshold “challenging, some of them studied”
- above more than 3 thresholds “very challenging, mostly unexplored”

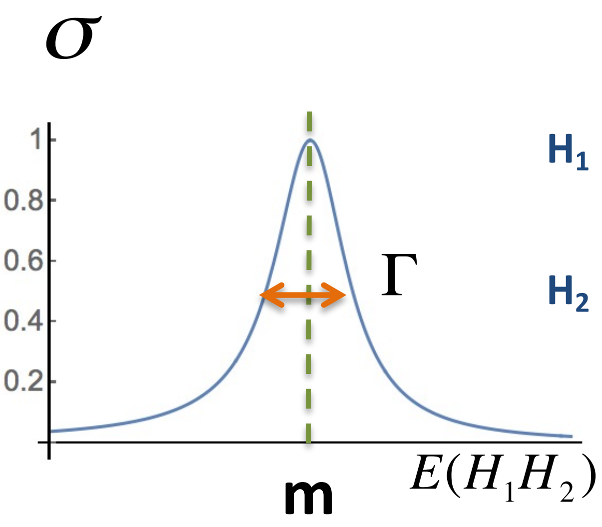
➢ P_c, Z_c, Z_b, *X(6900),... above many threshold:* that is why lattice has not concluded yet on their nature

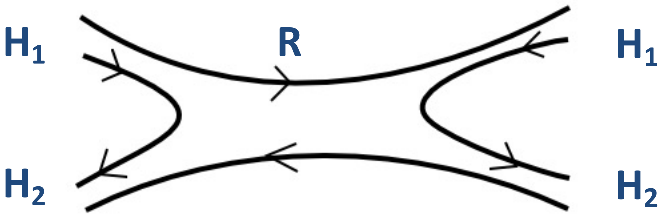
➢ still, lattice contributed conclusions on certain interesting states .. to be presented in this talk

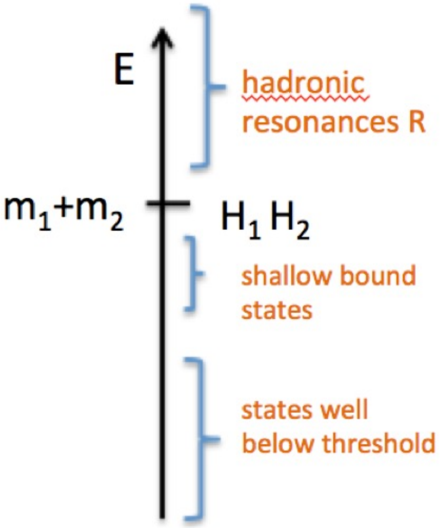
All experimentally discovered exotic hadrons are strongly decaying resonances !

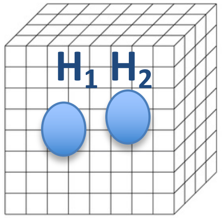
Resonances and shallow bound states from one-channel scattering











energy of eigenstate

scattering matrix for real E

E → $T(E)$

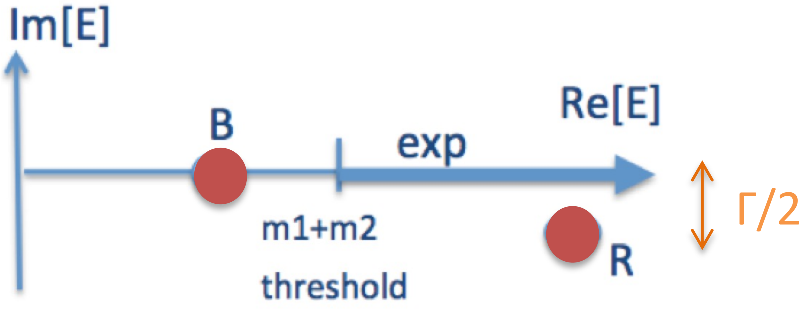
analytic relation:
Luscher 1991

$$\sigma(E) \propto |T(E)|^2$$

continuation to complex E

$$T_B(E) \propto \frac{1}{E^2 - m_B^2} \qquad T_R(E) = \frac{-m_R\Gamma}{E^2 - m_R^2 + i m_R \Gamma}$$

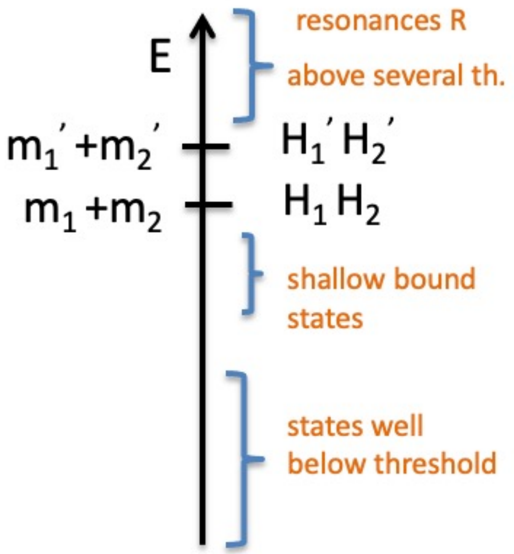
$$T_B(E = m_B) = \infty$$



location of poles in complex E plane

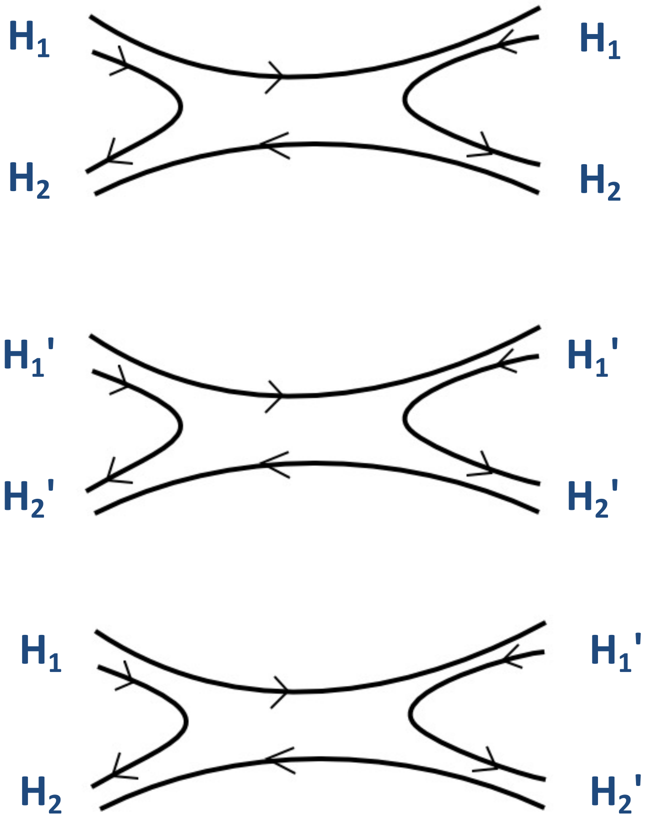
$E=E_{cm}$

Resonances from coupled-channel scattering : challenging



$$R \to H_1H_2,\ H_1'H_2',\ldots$$

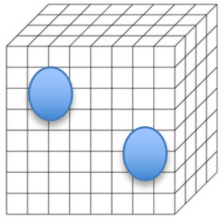
$$\text{channel } a: \quad H_1H_2$$
$$\text{channel } b: \quad H_1'H_2'$$



$$T(E) = \left[\begin{array}{cc} \text{a -> a} & \text{a -> b} \\ T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \\ \text{b->a} & \text{b -> b} \end{array}\right] \leftarrow E_n$$

generalization of. Luscher's method

- Challenging to get enough accurate E_n to determine the whole matrix T
- HadSpec extracted T for several channels

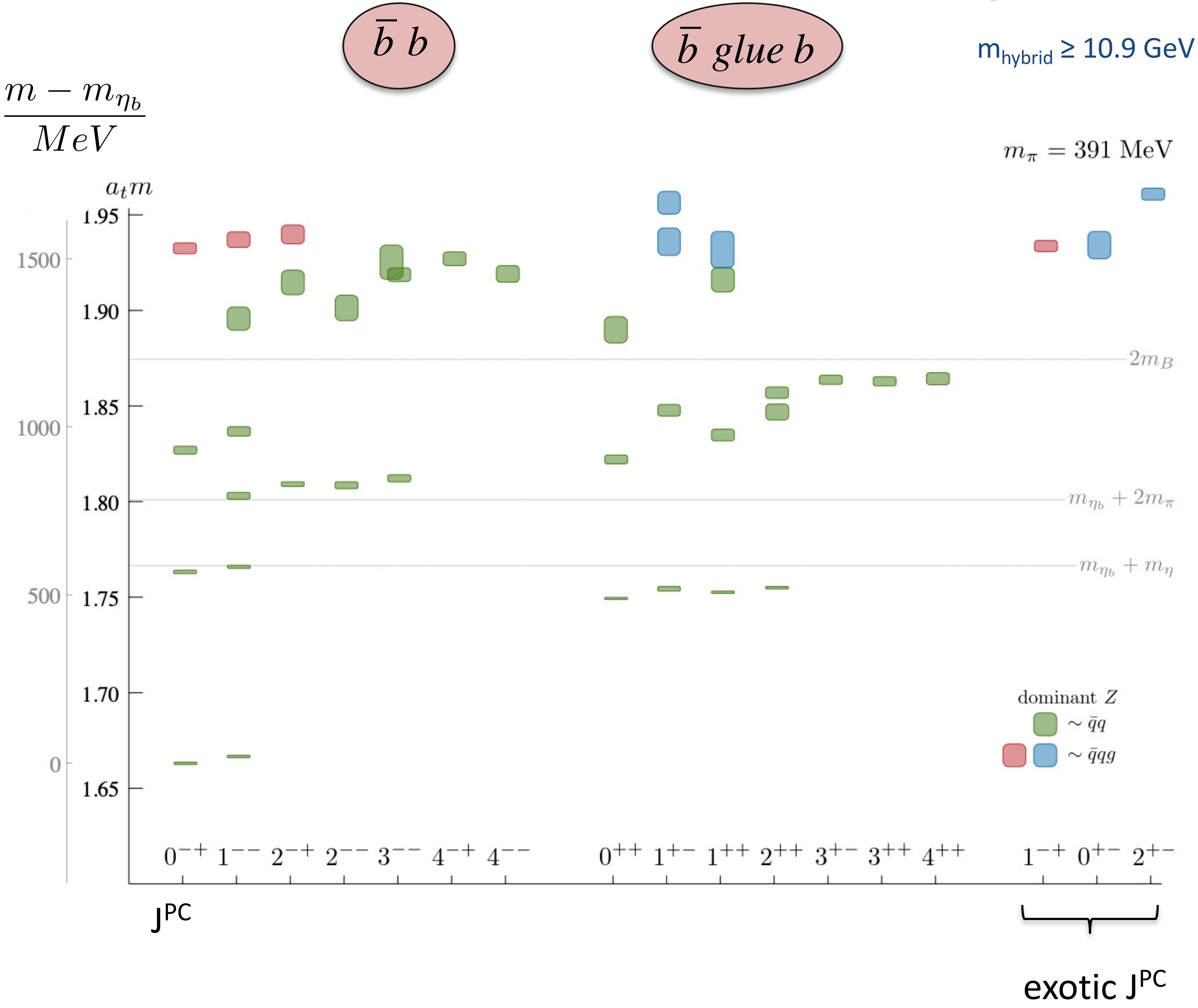


Mesonic sector

$$\bar{q}q \qquad \underbrace{\bar{q}q\bar{q}q \qquad \bar{q}Gq}_{\text{"exotic"}}$$

*

Excited bottomonia, bottomonium hybrids



$m_{hybrid} \geq 10.9$ GeV

Ryan & Wilson (HadSpec) 2008.02656

relativistic b-quark:

main challenge a m_b errors

states above $B\underline{B}$ threshold

treated as strongly stable

EFT+lattice prediction of hybrids [Brambilla et al, 1805.07713, PRD 2019]

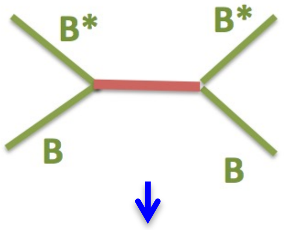
discretization effects in hybrid static potentials for SU(2) quenched theory:

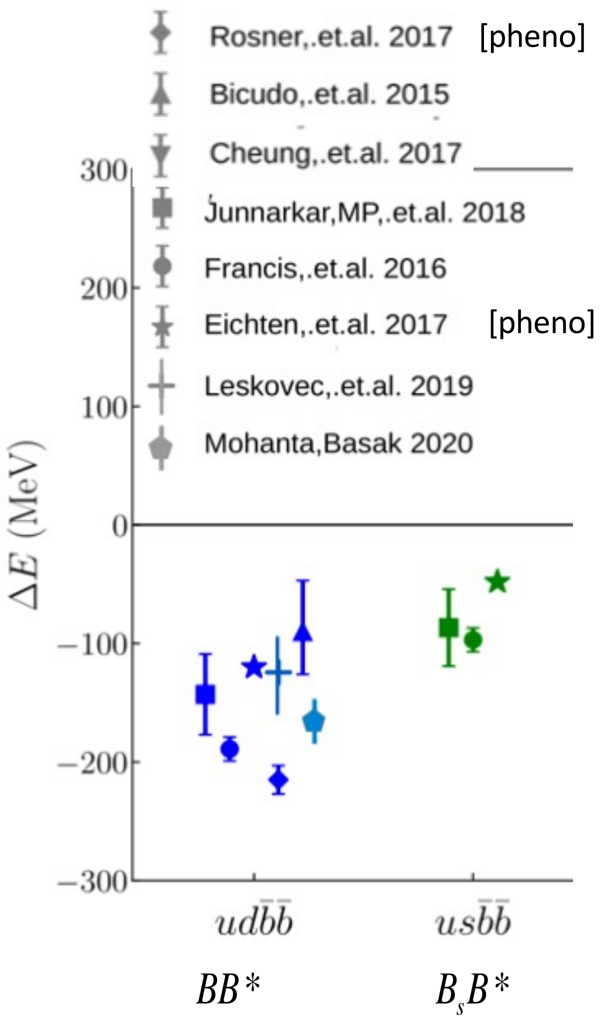
Riehl, Wagner, 2008.1221

Strongly stable doubly bottom tetraquarks

$\bar{b}\bar{b}ud \quad (J^P = 1^+, I = 0)$

$\bar{b}\bar{b}us \quad (J^P = 1^+)$





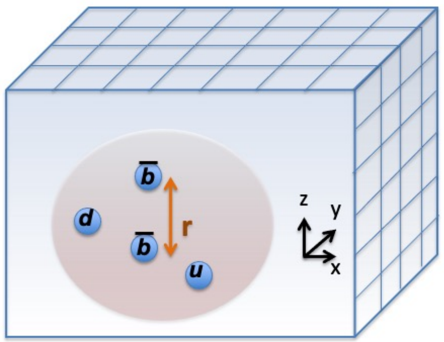
taken from Padmanath @ Charm2021

Hudspith et al 2006.14294

also reliably confirms those

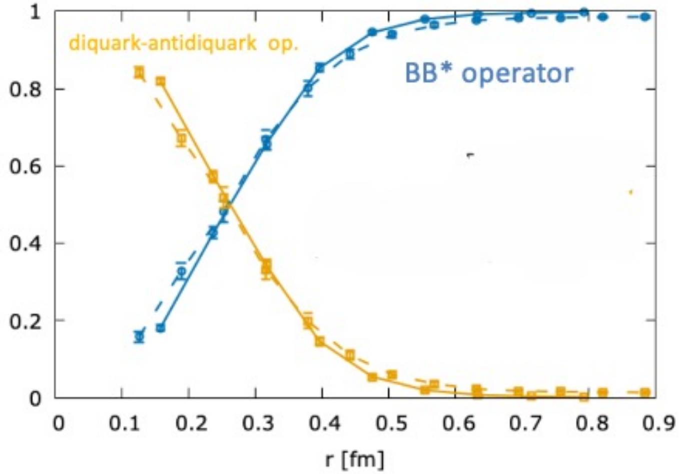
Exp: udbb and usbb not (yet) found

and it will be challenging to find them

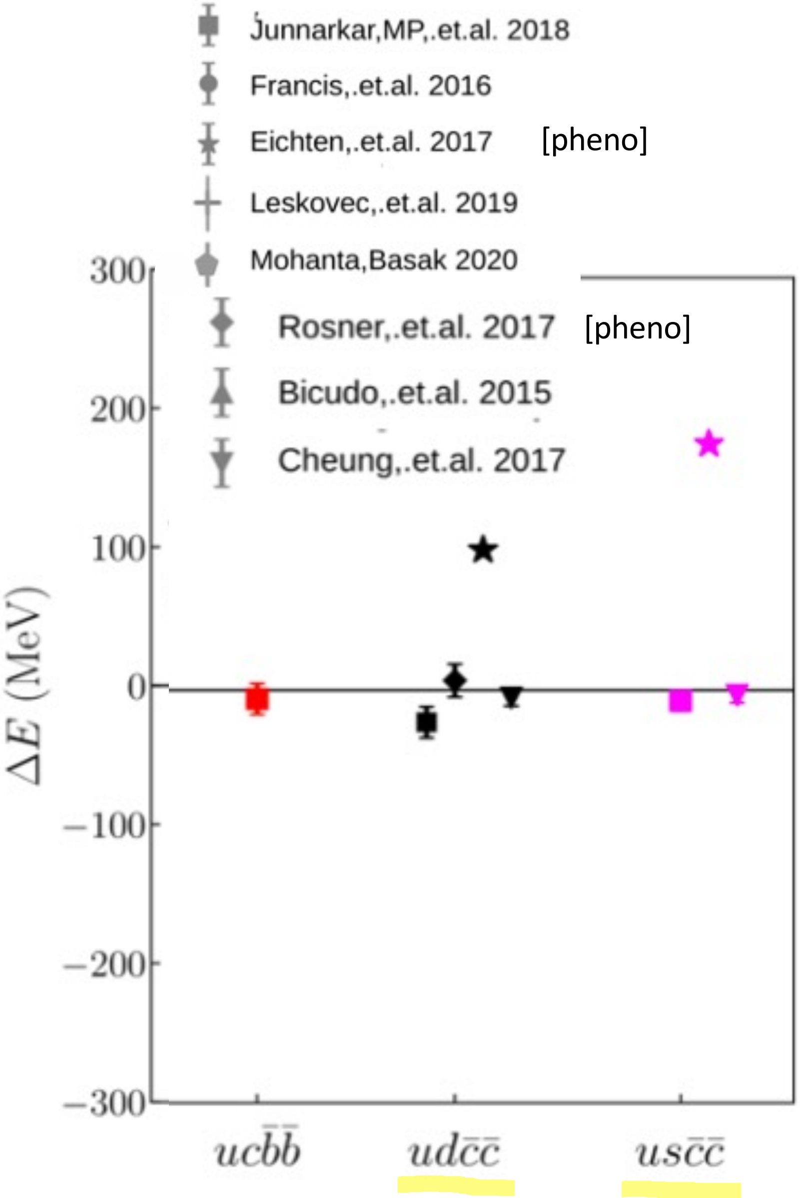


Bicudo et al 2101.00723

|eigenvector compon. for the bbud|^2



No bound state with sizable binding found in other channels (listed below)



taken from Padmanath @ Charm2021

Hudspith et al 2006.14294 (lattice)

no bound state found with

binding energy larger than O(20 MeV) in channels

(significantly constrains pheno models!)

$$ud\bar{c}\bar{b}\ ,\ ud\bar{s}\bar{b}\ ,\ ud\bar{s}\bar{c}:\ I=0,\ J^P=1^+,\ 0^+$$

$$us\bar{c}\bar{b}\ :\ I=\tfrac{1}{2},\ J^P=1^+,\ 0^+$$

$$uc\bar{b}\bar{b}\ ,\ \bar{s}\bar{c}bb:\ I=\tfrac{1}{2},\ J^P=1^+$$

Strongly stable $\underline{bb}du$

theory: straightforward

exp: challenging x

$\bar{b}\bar{b}du$

Resonance Z_b^+

challenging

discovered

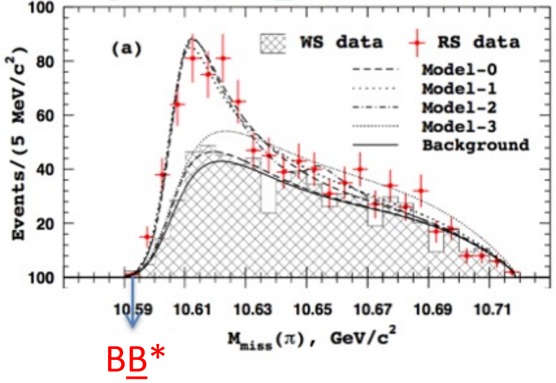
Belle 2011, 2014

$\bar{b}\,b\,\bar{d}\,u$

decays that make $\underline{b}b\underline{d}u$ channel much more challenging than $\underline{bb}du$

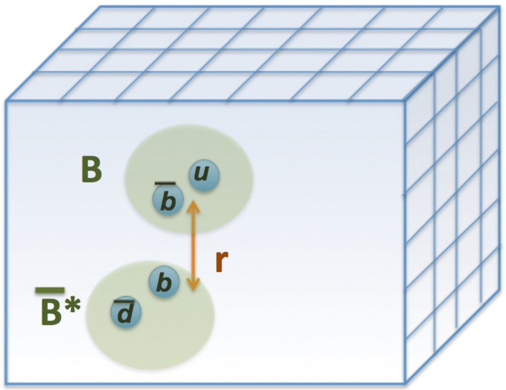
$$\bar{b}\,b\,\bar{d}\,u \rightarrow \underbrace{\bar{b}b}_{\Upsilon} + \underbrace{\bar{d}u}_{\pi}$$

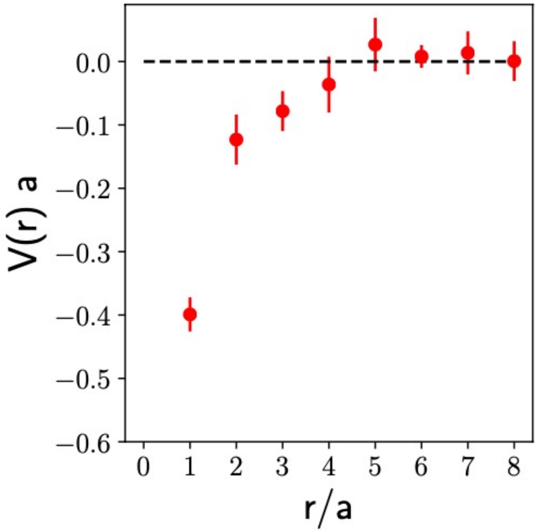
$Z_b(10610)$ in $B\underline{B}^*$ decay mode



first study: Bicudo et al, proceedings : Lat16: 1602.07621

recent study: S.P., H. Bahtiyar, J. Petkovic: 1912.02656v4

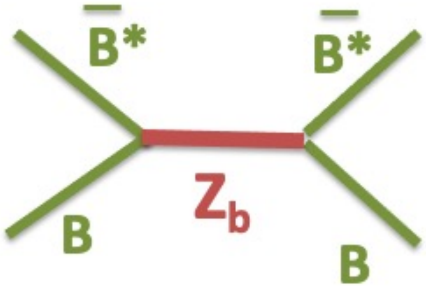




$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \mathrm{MeV}$$

all parametrizations of V lead to a bound state

the binding energy depends on the parametrization



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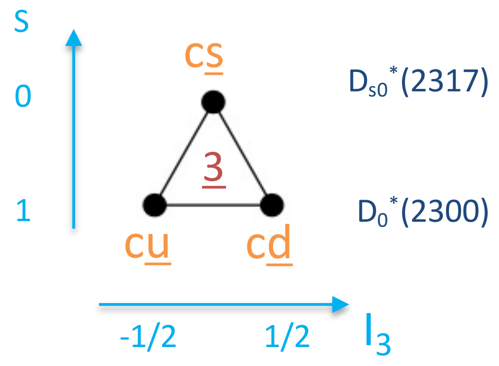
New paradigm for heavy-light mesons

$c\bar{q}$ $c\bar{q}\ q\bar{q}$

Example: single-charm mesons with $J^P=0^+$

Quark model expectation:



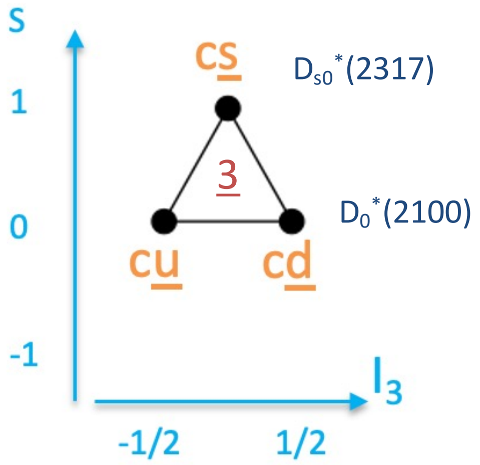
Many puzzles: $m[D_0^*(2300)] \approx m[D_{s0}^*(2317)]$??

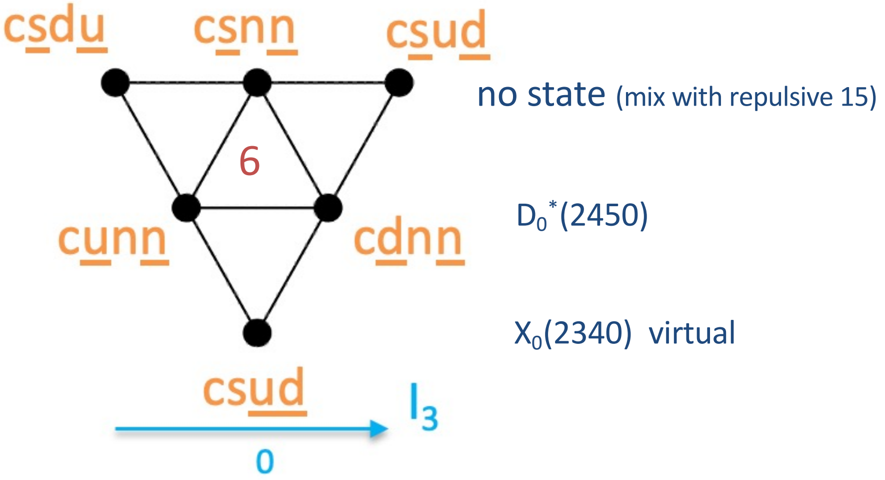
New paradigm: several states owe existence to scattering of $(c\underline{q})$ and $(q\underline{q})$

$SU(3)_F \quad \underline{3} \otimes 8 = \underline{3} \oplus 6 \oplus 15$

New paradigm & ChPT:
Du et al, 1712.07957
Albaladejo et al, 1610.06727
Lutz et al, 0371332

Lattice with Nf=2+1:
S=1
Mohler et al, 1308.3175
Lang et al, 1403.8103
Cox et al (RQCD), 1706.01247
S=1,-1
Cheung et al (HadSpec), 2008.06432
S=0
Moir et al (HadSpec), 1607.07093
Gayer et al (HadSpec) 2102.04973





no state (mix with repulsive 15)

$D_0^*(2450)$

$X_0(2340)$ virtual

meson masses at m_q^{phy} from 1712.07957

New paradigm for heavy-light mesons, cont'd

$c\bar{q}$ $c\bar{q}q\bar{q}$

Example: single-charm mesons with $J^P=0^+$

Lattice results seem to support this paradigm, more results on heavier S=0 and S=-1 state(s) are welcome

DK

S=1 & I=0: $D_{s0}^*(2317)$: lighter than quark model prediction due to DK threshold Mohler et al 1308.3175 ; Lang et al 1403.8103; RQCD 1706.01247; HadSpec 2008.06432

S=0: $m_1 = 2105^{+6}_{-8}$ MeV $\frac{1}{2}\Gamma_1 = 102^{+10}_{-12}$ MeV $m_2 = 2451^{+36}_{-26}$ MeV $\frac{1}{2}\Gamma_2 = 134^{+7}_{-8}$ MeV

reanalysis of HadSpec 1607.07093 ($m_\pi \approx 390$ MeV)
by Albaladejo et al 1610.06727 and chirally extrapolated

$D\pi - D\eta - D_s\bar{K}$

$m_1 = 2189 \pm 72$ MeV $\frac{1}{2}\Gamma_1 = 205 \pm 49$ MeV 2: not explored

HadSpec 2102.04973
($m_\pi \approx 240$ MeV)

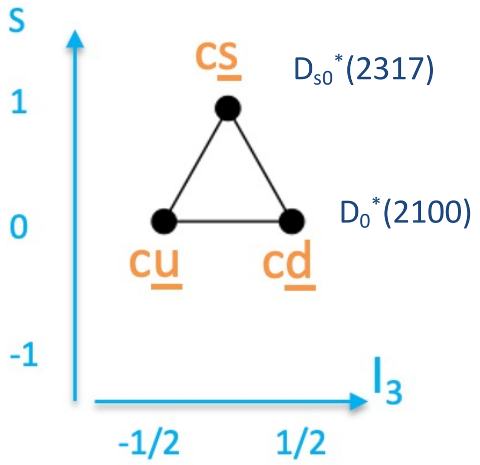
S=-1 & I=0 : $m = 2170 \pm 140\ MeV$

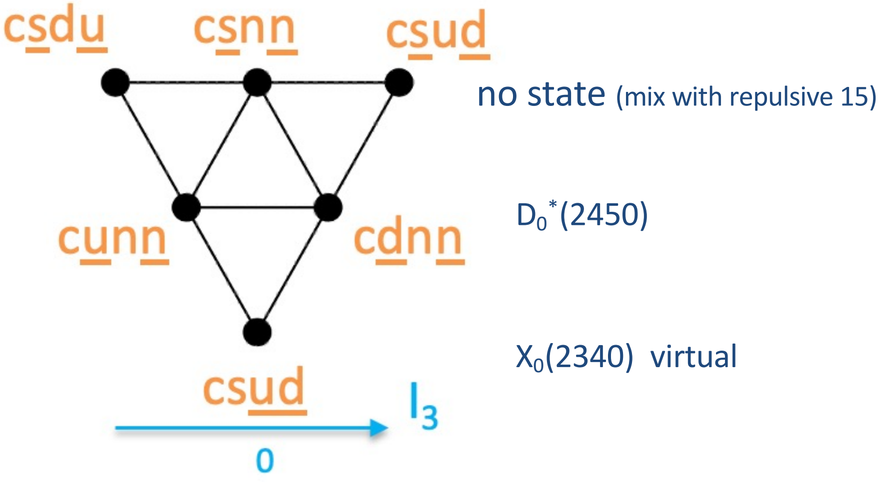
$cs\underline{ud}$ suggestion of a virtual bound state ~0.2 GeV below $D\underline{K}$

HadSpec 2008.06432
($m_\pi \approx 240$ MeV)

partner of $X_0(2900)=cs\underline{ud}$?
LHCb 2009,00025
challenging for lattice, not (yet) done
(high, above several thresholds)

$D\bar{K}$

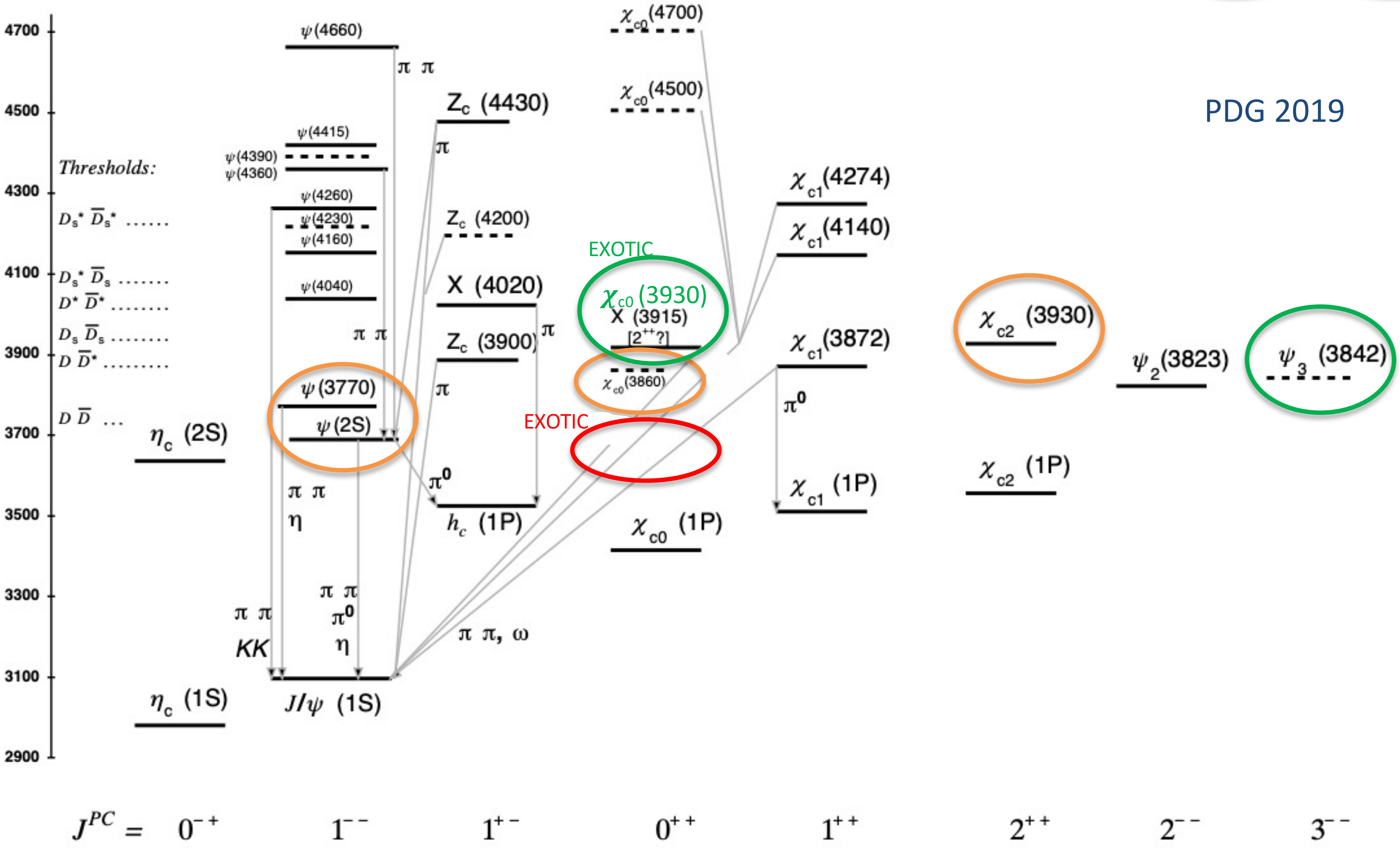




meson masses at m_q^{phy} from 1712.07957

Charmonium resonances with isospin 0

$\bar{c}c$ $\bar{c}q\bar{q}c$ $\bar{c}s\bar{s}c$



PDG 2019

lattice study

S.P et al 2011.02541, J=0,2

Piemonte et al 1905.03506: J=1,3

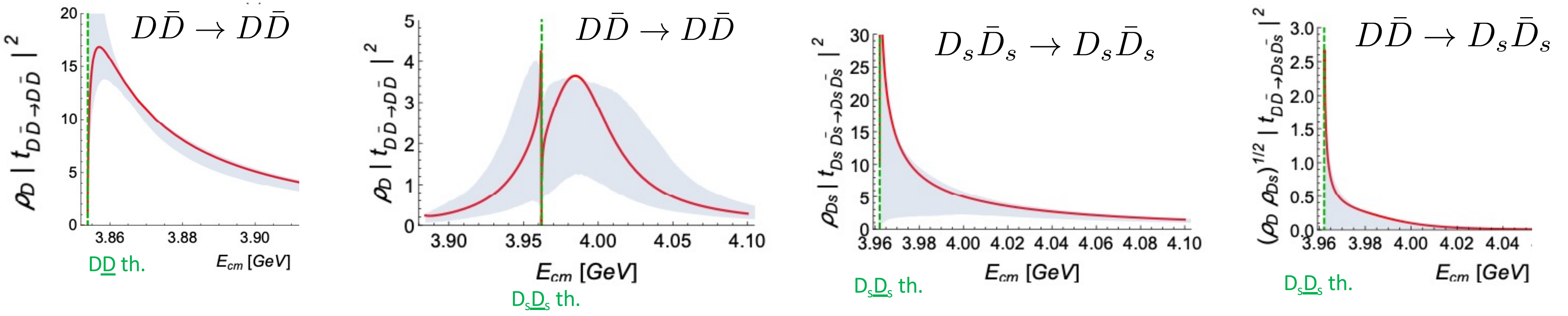
postdicted

predicted, exp discovered

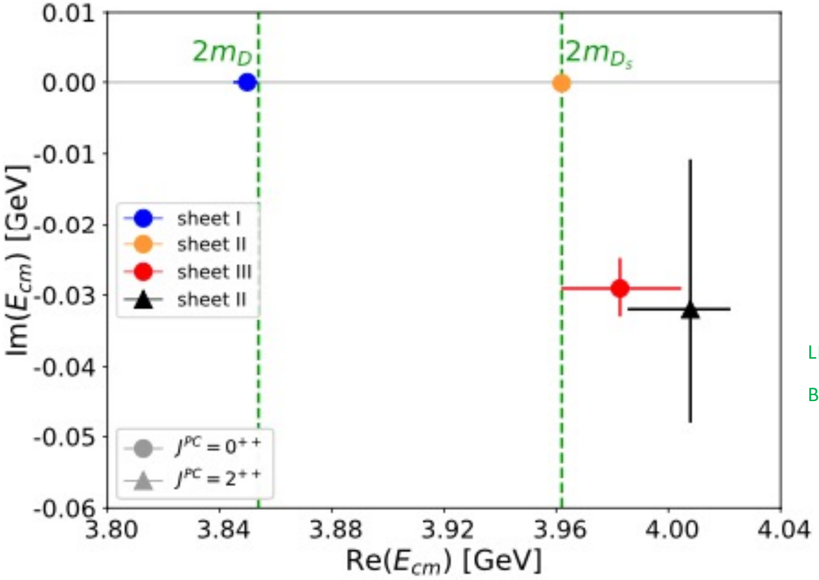
predicted, exp not (yet) discovered

Charmonium–like resonances: exotic candidates with $J^{PC}=0^{++}$, I=0

S-wave (L=0, $J^{PC}=0^{++}$)



Positions of poles in complex energy plane



- broad resonance coupling mostly to DD : conventional

  in rough agreement with X(3860) [Belle 2017]

- state near D_sD_s threshold coupling mostly to D_sD_s : exotic

lat : $m - 2m_{D_s} = -0.2\,^{+0.16}_{-4.9}$ MeV , $g = 0.10\,^{+0.21}_{-0.03}$ GeV $\quad \Gamma = g^2 p_D/E_{cm}^2$

LHCb 2020 $\chi_{c0}(3930): \; m - 2m_{D_s} = -12.9 \pm 1.6$ MeV , $\Gamma = 17 \pm 5$ MeV , $g = 0.67 \pm 0.10$ GeV

Babar 2008 $X(3915): \; m - 2m_{D_s} = -18.3 \pm 1.9$ MeV , $\Gamma = 20 \pm 5$ MeV , $g = 0.72 \pm 0.10$ GeV

pheno: Lebed, Polosa et al., 1602.08421
2005.07100, Chet et al. 1706.097231

- state near DD threshold : exotic

  not yet claimed by experiment

  X(3700) proposed by many pheno works

  DD partner of DD*=X(3872)

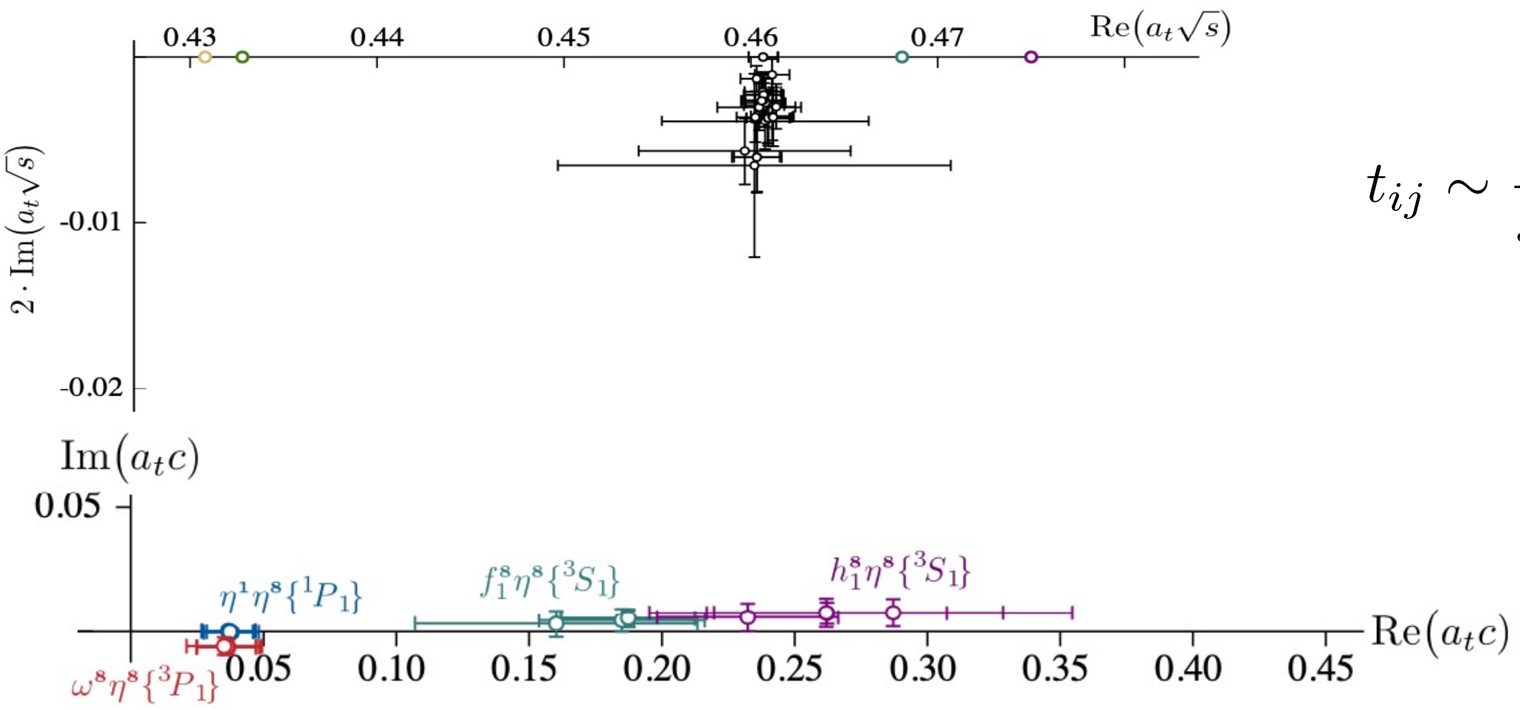
pheno: Oset et al 0612179, 0712.1758,
1211.1862, 2004.05204,...
Baru et al, PLB**763** 20 (2016) 20
Nieves et al, PRD86 (2012) 056004

near pole $\quad t_{ij} \sim \dfrac{c_i\; c_j}{(E^p_{cm})^2 - E^2_{cm}}$

light hybrid meson π_1

$J^{PC} = 1^{-+}$

$\bar{d}Gu$



$$t_{ij} \sim \frac{c_i\ c_j}{s_0 - s}$$

Woss et al. (HadSpec)

2009.10034

$m_u = m_d = m_s$, $m_\pi \approx 700\ MeV$

pheno

analysis

$\rho\ \pi$ $\eta'\ \pi$ $f_1\ \pi$ $b_1\ \pi$

dominant coupling

physical world

resemblence to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Non-existence of strongly stable fully beautiful tetraquark

$\bar{b}b\bar{b}b$

Lattice QCD: No indication for strongly stable state (below threshold) with

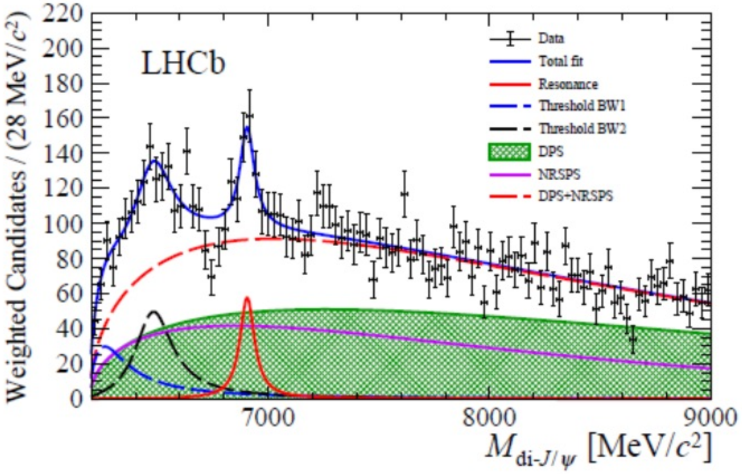
J^{PC} = 0^{++} , 1^{+-} , 2^{++}

threshold $\eta_b\eta_b$ $\eta_b\Upsilon$ $\Upsilon\Upsilon$

[Hughes, Eichten, Davies, HPQCD, 1710.03236, PRD 2018]

Existence of fully charming tetraquark resonances (high above threshold)

$\bar{c}c\bar{c}c$



LHCb 2006.16957

X(6900)

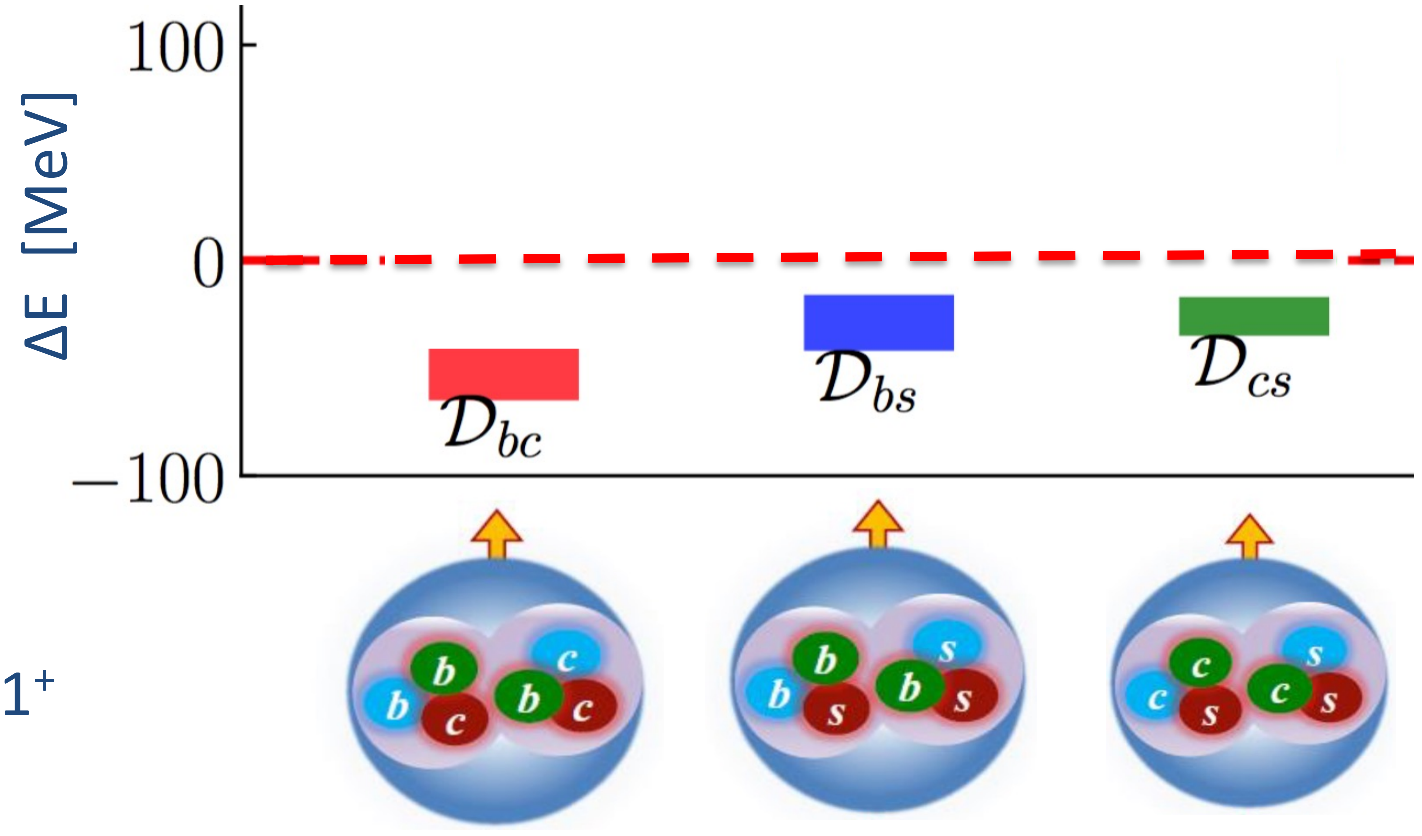
challenging for lattice, not (yet) done
(high, above several thresholds)

di-baryons

$$qqq\ qqq$$

Strongly and EM stable di-baryons

lattice QCD: Junnarkar, Mathur, [1906.06054, PRL 2019]



$J^P=1^+$

deuterium-like

strong decay threshold

$\Omega^{J=3/2}_{ccc}\Omega^{J=3/2}_{bbb}$ $\Omega^{J=3/2}_{sss}\Omega^{J=3/2}_{bbb}$ $\Omega^{J=1/2}_{css}\Omega^{J=1/2}_{ccs}$

Most charming di-baryon

$ccc\ ccc$

$\Omega_{ccc}\Omega_{ccc}$

- HALQCD method, $m_q \approx m_q^{phy}$, Lyu et al. (HALQCD) 2102.00181

  QCD: shallow bound state found: B $\approx 6\ MeV$

  QCD + Coulomb repulsion: no bound state

H-dibaryon

$uds\ uds$

$\Lambda\Lambda,\ \Sigma\Sigma,\ N\Xi$

- HALQCD method, $m_q \approx m_q^{phy}$, Sasaki et al. (HALQCD) 1912.08630

  only small attraction in channel $\Lambda\Lambda$ found, not enough to form the bound state

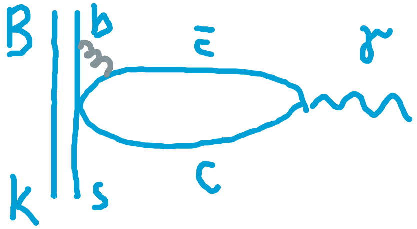
- Luscher's method, $m_u = m_d = m_s$, $m_\pi \approx 420\ MeV$, extrapolation a→ 0 Green et al (Mainz) 2103.01054

  shallow bound state found which is sensitive to the lattice spacing

$$B_H = 3.97 \pm 1.16 \pm 0.86\ \mathrm{MeV}$$

Unrelated to exotic hadrons, but relevant to relation of FCP & lattice: JLQCD, 1901.08784

Lattice study of B->K l^+l^- beyond factorization, incorporating long-distance effects from $\bar{c}c$



Conclusions concerning recent studies of exotic hadrons from lattice

predictions (exotic)	exp
$\chi_{c0}(3930)=\bar{c}s\bar{s}c$	yes
bottomonium hybrids	no
doubly bottom tetraquarks	yes or no?
D0*(2100) & D0* (2450)	no
virtual st. below $D\bar{K}$	no
$D\bar{D}$ threshold state	yes or no?

ruled out a number of bound states

postdictions (exotic):

Z_b^+

$D_s^0(2317)$

$\pi_1(1564)$

postdictions (conventional): many

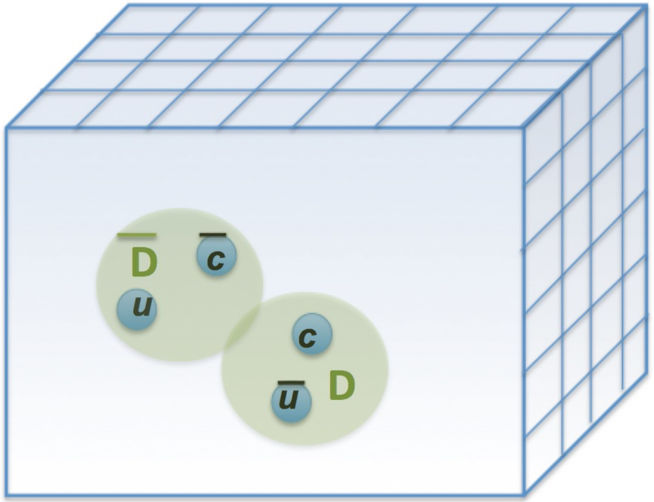
numerous bottomonia

resonant charmonia: X(3860), $\psi(3770)$, $\psi_3(3842)$, $\chi_{c2}(3930)$

....

Backup

Lattice QCD



$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G^\mu_a T^a) q - m_q \bar{q} q$$

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$

discretized finite Eucledian

space-time

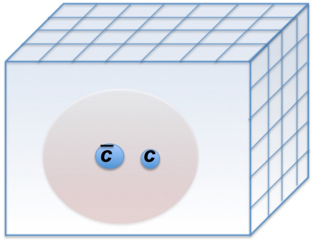
Determine energies of eigenstates E_n and overlaps

$$\text{charmonium}: \quad J^{PC}: \quad \bar{c}\Gamma c\,, \quad (\bar{c}\Gamma_1 u)(\bar{u}\Gamma_2 c) = D\bar{D}\,, \qquad [\bar{c}\Gamma_3 \bar{u}][c\Gamma_4 u]$$

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t}\ \langle n | \mathcal{O}_j^+ | 0 \rangle$$

overlap

energy of
eigenstate |n>



$$J^{PC} = 1^{--}: \quad E_1(\vec{p} = 0) = m_{J/\psi}$$

$c\underline{c}$ and $b\underline{b}$ annihilation omitted for all result in this talk.

Then hadrons below $\underline{D}D$ or $\underline{B}B$ are strongly stable

Charmonium resonances

S.P et al 2011.02541, J=0,2
Piemonte et al 1905.03506: J=1,3