Lepton flavor universality: $\mathcal{R}(\mathcal{D}), \mathcal{R}(\mathcal{D}^*)$ measurements

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(On behalf of Belle collaboration,

Materials from LHCb and Belle II also included)

Conference on Flavor Physics and CP Violation
Fudan University, Shanghai, China
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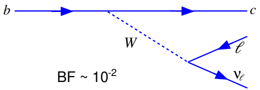
Lepton flavor universality (LFU)

- Equal coupling of the gauge bosons to the three lepton generations in Standard Model (SM): LFU

- The only difference in their interactions is caused by the difference in the mass

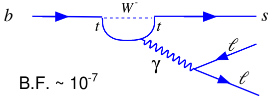
- In the recent years, several discrepancies from the SM have been reported in the $B-$decays related to the LFU ratio measurements

Decays : $B \to D^{(*)}\ell\nu_\ell$..
Variables : $\mathcal{R}(\mathcal{D}), \mathcal{R}(\mathcal{D}^*)$ ..



This talk

Decays : $B \to K^{(*)}\ell\ell$
Variables : $\mathcal{R}(\mathcal{K}), \mathcal{R}(\mathcal{K}^*)$..



See talk by Shun Watanuki

$\mathcal{R}(\mathcal{D}^{(*)})$ in semi**tau**onic B-decays

- Semi**tau**onic decays such as $B \to D^{*0}\tau\nu_\tau$ are more interesting, given the third generation of lepton family are involved in the transition

- Sensitive to New Physics (NP): Two Higgs doublets, leptoquarks, etc.

- The presence of NP impact the experimentally observed branching fractions that can be observed in LFU ratios:

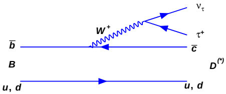
$$\mathcal{R}(\mathcal{D}^{(*)}) \equiv \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}\ell\nu_\ell)}$$

where, $\ell = e$ and μ

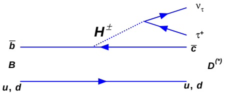
$$\mathcal{B}(B \to D^{(*)}\ell\nu_\ell) = \text{Avg of } e \text{ and } \mu \text{ modes}$$

- Beyond SM particles can also alter the kinematic distributions of final state: τ lepton/D^* meson polarization Belle PRD **97**, 012004 (2018)

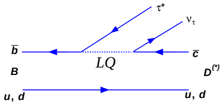
SM



2HDM[1]



Leptoquarks[2]



[1] Front. Phy. **80**, 1 (2000)

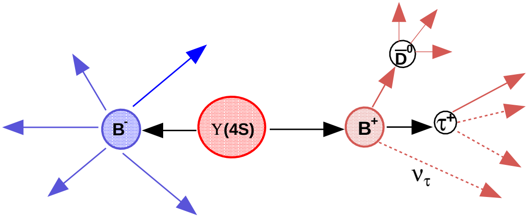
[2] Phys. Lett. B **191**, 442 (1987); 448, 320(E) (1999)

$B \to D^{(*)}\tau\nu$ reconstruction at B factories

- Challenging with the presence of multiple neutrinos in the final state

- Select the $\mathrm{B_{sig}}$ with
  $\rightarrow$ a $D^{(*)}$
  $\rightarrow$ a charged daughter of τ
  (hadronic/leptonic τ decays)

- Relies on reconstruction of accompanying B-meson ($\mathrm{B_{tag}}$) to provide the necessary level of background suppression



Tagging methods:

- Hadronic Tag:

$\rightarrow$Fully reconstructed in $B \to DX$ modes

$\rightarrow$Tagging efficiency $\sim 0.2\%$

$\rightarrow$Low background

- Semileptonic Tag:

$\rightarrow$Reconstruct $B \to D^{(*)}\ell\nu$

$\rightarrow$Tagging efficiency $\sim 0.5\%$

$\rightarrow$More background

- Inclusive Tag :

$\rightarrow$Reconstruct $\mathrm{B_{tag}}$ with all particles except signal side

$\rightarrow$Higher efficiency

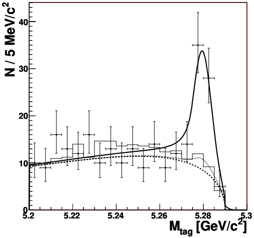
$\rightarrow$Need clean signal side final state

First observation of $B \to D^{*0}\tau\nu_\tau$

Belle, PRL**99**, 191807 (2007)

(Received 29 June 2007; published 9 November 2007)

We report an observation of the decay $B^0 \to D^{*-}\tau^+\nu_\tau$ in a data sample containing 535×10^6 $B\bar{B}$ pairs collected with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. We find a signal with a significance of 5.2σ and measure the branching fraction $\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau) = (2.02^{+0.40}_{-0.37}(\text{stat}) \pm 0.37(\text{syst}))\%$. This is the first observation of an exclusive B decay with a $b \to c\tau\nu_\tau$ transition.

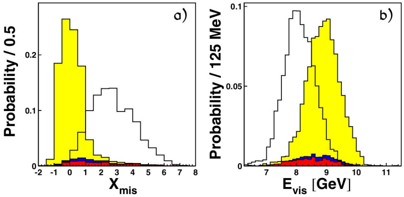


arXiv : 0708.4089

Flavor Physics and CP Violation Conference, Bled, 2007

Measurement of $B \to D^{*-}\tau^+\nu_\tau$ and $B \to h^{(*)}\nu\overline{\nu}$ Decays at Belle

K.-F. Chen
Department of Physics, National Taiwan University, Taipei



X$_{\text{mis}}$: variable is closely related to the missing mass in the $\mathrm{B_{sig}}$ decay

Visible energy $(\mathrm{E_{vis}})$: sum of the energies of all particles in the event

$\mathcal{R}(\mathcal{D})$ and $\mathcal{R}(\mathcal{D}^*)$ overview and status

$$\mathcal{R}(\mathcal{D}^{(*)}) \equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

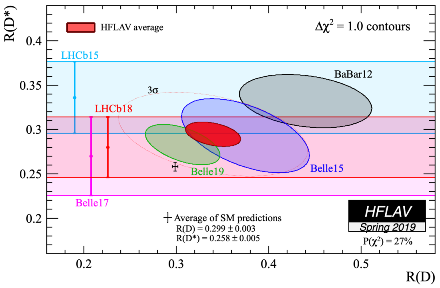
- Common systematics will cancel out
  - $\rightarrow$ Detection efficiency
  - $\rightarrow$ Theoretical uncertainty of form factor
  - $\rightarrow$ Uncertainty of $|V_{cb}|$
- Predictions are theoretically clean

The SM average:

$$\mathcal{R}(\mathcal{D}) = 0.299 \pm 0.003$$
$$\mathcal{R}(\mathcal{D}^*) = 0.258 \pm 0.005$$

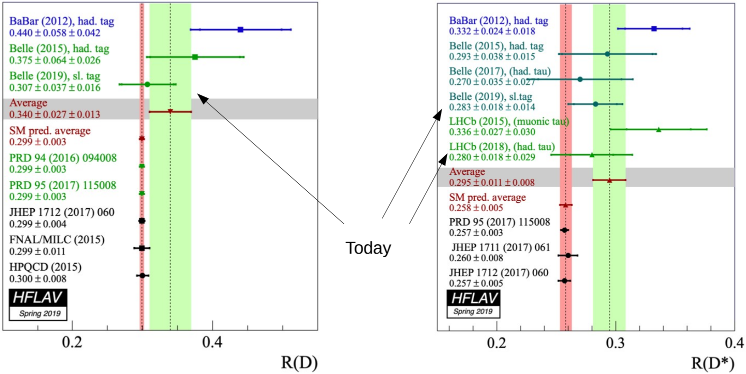
[Phys. Rev. D **94**, 094008 (2016)] [Phys. Rev. D **95**, 115008 (2017); **97**, 059902(E) (2018)] [JHEP 1711 061 (2017), JHEP 1712 060 (2017)]

Updated averages of the HFLAV:



- Latest Belle measurement brings the world average discrepancy down to 3.1σ form 3.8σ (HFAV 2018).

$\mathcal{R}(\mathcal{D})$ and $\mathcal{R}(\mathcal{D}^*)$ overview and status



Experimental avg:

$\boxed{\mathcal{R}(\mathcal{D}) = 0.340 \pm 0.027 \pm 0.013}$

Theory avg:

$\boxed{\mathcal{R}(\mathcal{D}) = 0.299 \pm 0.003}$

Experimental avg:

$\boxed{\mathcal{R}(\mathcal{D}^*) = 0.295 \pm 0.011 \pm 0.008}$

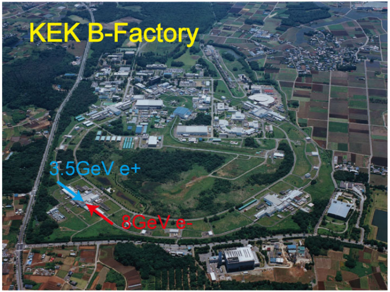
Theory avg:

$\boxed{\mathcal{R}(\mathcal{D}^*) = 0.258 \pm 0.005}$

Recent $\mathcal{R}(\mathcal{D}^{(*)})$ measurements in Belle with semileptonic tagging

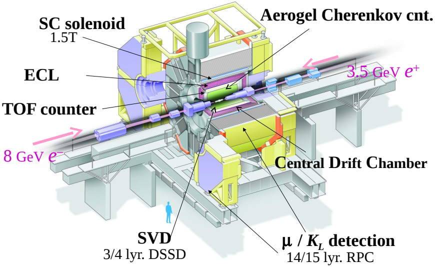
[PRL **124**, 161803 (2020)]

KEKB and Belle detector



- Asymmetric e^+e^- collider at the High Energy Accelerator Research Organization(KEK), Japan
- 8 GeV e^- collides to 3.5 GeV e^+ at $\Upsilon(4S)$ resonance

- Collected about 772 million $\mathrm{B\bar{B}}$ till 2010
- Result covered in this talk is based on the complete belle data set



Recent $\mathcal{R}(\mathcal{D}^{(*)})$ measurements in Belle with semileptonic tagging

Previous Belle semileptonic tag: [PRD **94** 072007 (2016)]

Measure $\mathcal{R}(\mathcal{D}^{*})$ with $B^0 \to D^{*-}\tau^+\nu$ decays

New semileptonic analysis: [PRL **124**, 161803 (2020)]

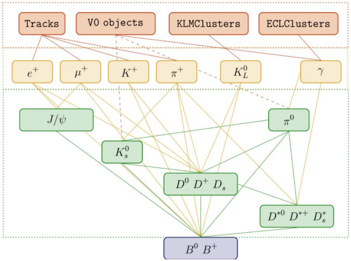
Using Full Event Interpretation (FEI) tool developed in Belle II software framework

$\rightarrow$ The $\mathrm{B_{tag}}$ is reconstructed using a hierarchical algorithm based on a multivariate analysis with Boosted-Decision Tree (BDT) in the $B \to D^{(*)}\ell\nu$ channel

$\rightarrow$ Better efficiency and enables to use more signal decay modes

$\rightarrow$ Simultaneous measurements of $\mathcal{R}(\mathcal{D})$ and $\mathcal{R}(\mathcal{D}^{*})$

Compt. Softw. Big. Sci. (2019) 3:6



Analysis overview

- Analysis with the Belle II software framework

D^0 channels	D^+ channels
$K^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+$
$K^- \pi^+ \pi^+ \pi^-$	$K_S^0 \pi^+ \pi^0$
$K^- \pi^+$	$K_S^0 \pi^+ \pi^+ \pi^-$
$K_S^0 \pi^+ \pi^-$	$K_S^0 \pi^+$
$K_S^0 \pi^0$	$K^- K^+ \pi^+$
$K^- K^+$	$K_S^0 K^+$
$\pi^- \pi^+$	
$K_S^0 K^+ K^-$	
~ 30% of total	~ 22% of total

D* channels
$D^{*+} \to D^0 \pi^+$
$D^{*+} \to D^+ \pi^0$
$D^{*0} \to D^0 \pi^0$

- 772×10^6 $B\bar{B}$ events
- 4 data samples: $D^+\ell^-$, $D^0\ell^-$, $D^{*+}\ell^-$, $D^{*0}\ell^-$

B-Signal: $B^{\pm/0} \to D^{(*)}\tau^-(\ell^-\bar{\nu}\nu)\nu$ $\leftarrow$ $\Upsilon(4S)$ $\rightarrow$ B-Tag: $\mathrm{B}^{\pm/0} \to D^{(*)}\ell^-\nu$

Signal mode

Use BDT hierarchical algorithm

$\mathrm{B}^{\pm/0} \to D^{(*)}\ell^-\nu$

Normalization mode

Where, $\ell = e$ and μ

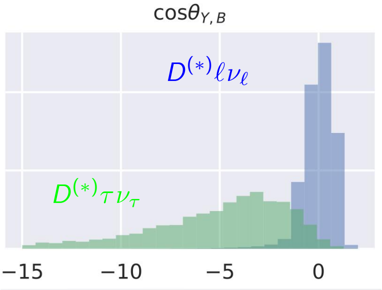
$\rightarrow$ $\mathrm{B_{tag}}$ and $\mathrm{B_{sig}}$ are required to be of opposite flavor

Event selection

- Well-reconstructed $\mathrm{B_{tag}}$ candidates are selected with a requirement on classifier output
- Veto on the events for $B_{tag} \to D^{(*)}\tau\nu_\tau$ are applied using a criterion on:

$$\boxed{cos\theta_{B,D^{(*)}\ell} = \frac{2E_{beam}E_{D^{(*)}\ell} - m_B^2 - m_{D^{(*)}\ell}^2}{2|p_B||p_{D^{(*)}\ell}|}}$$

cosine of the angle between the momentum of the B meson and $D^{(*)}\ell$ under the assumption that only one massless particle is not reconstructed



correctly reconstructed $\mathrm{B_{tag}}$:
$-1 < \cos\theta_{B,D^{(*)}\ell} < 1$

- Best candidates are selected in case of multiple $\mathrm{B_{tag}}$ (on highest tagging classifier output) and $\mathrm{B_{sig}}$ (on highest p-value of vertex fit of the charm daughter)

Key observables

1. $\mathbf{E}_{\mathrm{ECL}}$: Energy deposited in the Electromagnetic Calorimeter not associated with reconstructed particles

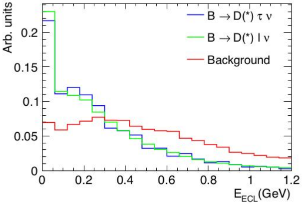
$\rightarrow$ $B \rightarrow D^{(*)}\tau\nu$ and $B \rightarrow D^{(*)}\ell\nu$ events peak near zero in $\mathbf{E}_{\mathrm{ECL}}$

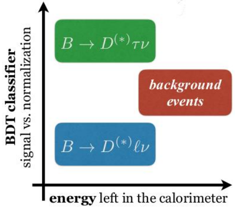
2. **class:** BDT classifier output (Based on the XGBoost package)

$\rightarrow$ To separate signal and normalization modes

$\rightarrow$ Input variables:

- Visible energy
- $m^2_{miss} = (E_{beam} - E_{D^{(*)}-E_\ell})^2 - p_{D^{(*)}} + p_\ell)^2$
- $cos\theta_{B,D^{(*)}\ell}$

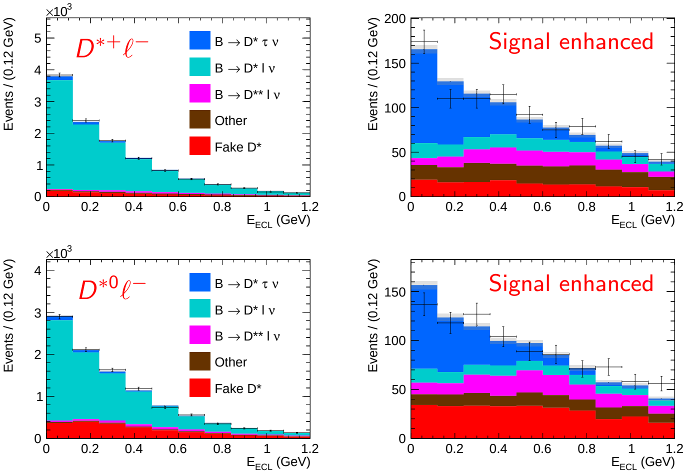


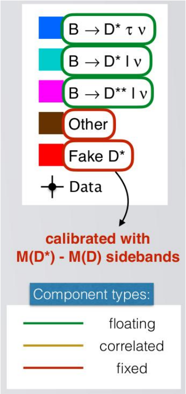


- The signal events peak around 1 and normalization events peak around 0 of the classifier output

Fit results: $D^*\ell$ sample

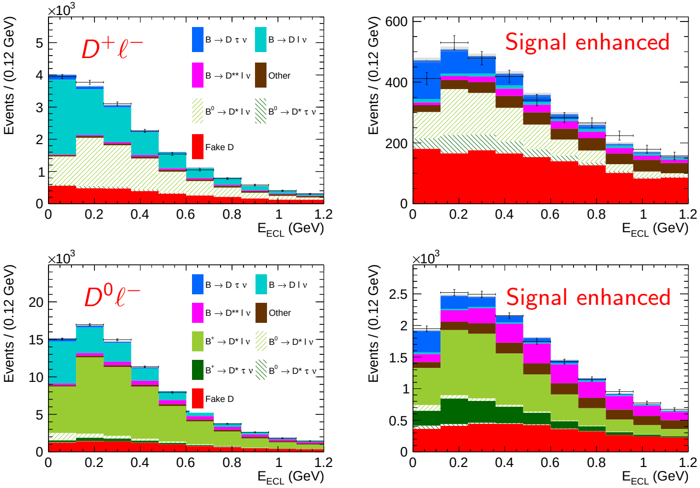
class > 0.9

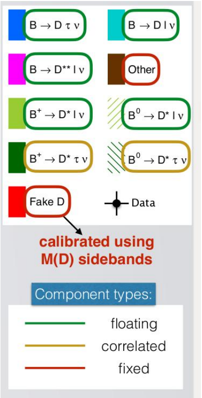




Fit results: $D\ell$ sample

class > 0.9





Results

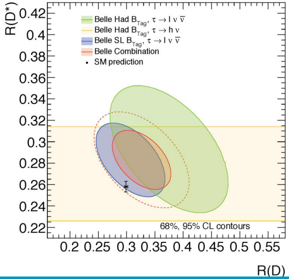
MC

$$\mathcal{R}(\mathcal{D}^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau^+\nu_\tau)}{\mathcal{B}(B \to D^{(*)}l^+\nu_l)} = \frac{1}{2\mathcal{B}(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau)} \cdot \frac{\epsilon_{\mathrm{sig}}}{\epsilon_{norm}} \cdot \frac{N_{\mathrm{sig}}}{N_{norm}}$$

$\boxed{\mathcal{R}(\mathcal{D}^*) = 0.283 \pm 0.018 \pm 0.014}$ Agrees with SM within 1.1σ

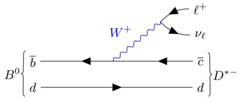
$\boxed{\mathcal{R}(\mathcal{D}) = 0.307 \pm 0.037 \pm 0.016}$ Agrees with SM within 0.2σ

- Most precise measurements of $\mathcal{R}(\mathcal{D}^{(*)})$ to date
- First $\mathcal{R}(\mathcal{D})$ measurement performed with a semileptonic tag
- Results compatible with SM expectation within 1.2σ
- $\mathcal{R}(\mathcal{D}) - \mathcal{R}(\mathcal{D}^*)$ Belle average is now within 1.6σ of the SM prediction



e/μ LFUV measurements in Belle

[Phys. Rev. D **100**, 052007]

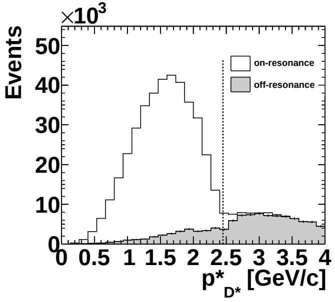


- $B^0 \to D^* \ell^+ \nu_\ell$ using un-tagged approach
- Measure $|V_{\mathrm{cb}}|$ using Belle $711~\mathrm{fb}^{-1}$
  $D^{*-} \to D^0 \pi_s$, $D^0 \to K\pi$ (vertex fit)
- LFU test by forming a ratio of the B.F. of modes with electrons and muons:

$$\frac{\mathcal{B}(B^0 \to D^* e^+ \nu_e)}{\mathcal{B}(B^0 \to D^* \mu^+ \nu_\mu)}$$

- Signal Selection:
  $|\cos\,\theta_{B,D^*\ell}| < 1$
  $|m_{D^0} - m_{D^0_{\mathrm{PDG}}}| < 14~\mathrm{MeV/c^2}$
  $144 < |m_{D^*} - m_{D^0}| < 147~\mathrm{MeV/c^2}$
  $p_e > 0.80~GeV/c$, $p_\mu > 0.85~GeV/c$
- e and μ modes are reconstructed separately:

$$\boxed{\cos\theta_{B,D^{(*)}\ell} = \frac{2E_{beam}E_{D^{(*)}\ell} - m_B^2 - m_{D^{(*)}\ell}^2}{2|p_B||p_{D^{(*)}\ell}|}}$$



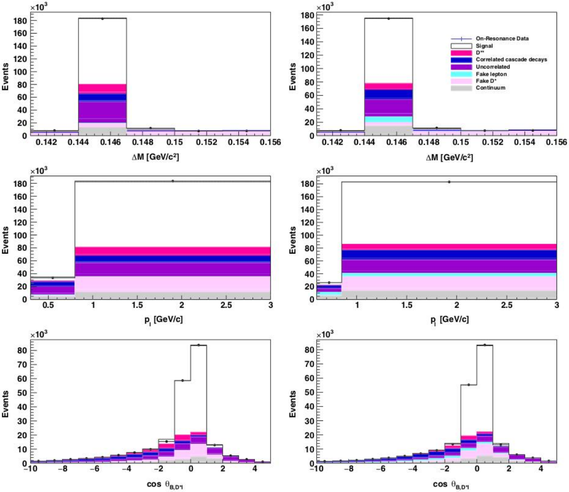
Suppress the continuum with $p^*_{D^*} > 2.45~\mathrm{GeV/c}$

Signal selection

3D fit to:

- $\rightarrow$ $\cos\theta_{B,D^*\ell}$
- $\rightarrow$ $\Delta M = m_{D^*} - m_{D^0}$
- $\rightarrow$ lepton momentum

	SVD2 (e)	SVD2 (μ)
Signal yield	88622	87060
Signal	81.00 ± 0.19	79.86 ± 0.20
Fake ℓ	0.10 ± 0.79	1.15 ± 0.38
Fake D^*	2.94 ± 0.01	2.81 ± 0.01
D^{**}	5.08 ± 0.14	3.62 ± 0.08
Signal corr.	1.42 ± 0.07	2.39 ± 0.14
Uncorrelated	4.96 ± 0.15	5.00 ± 0.24
Continuum	4.48 ± 0.38	5.16 ± 0.46



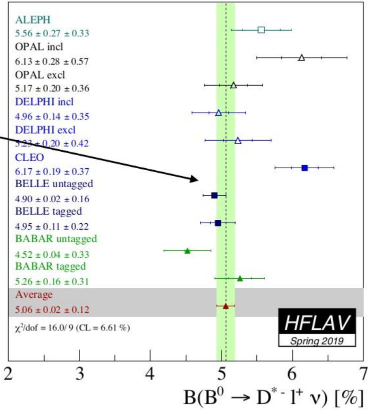
e/μ LFUV measurements in Belle

[Phys. Rev. D **100**, 052007]

- Extract BR from yield for e and μ and the total BR of the decay.

$$\mathcal{B} = \frac{N_{\text{signal}}}{N_{B^0} \times \epsilon \times \mathcal{B}(D^{*+} \to D^0\pi^+) \times \mathcal{B}(D^0 \to K^-\pi^+)}$$

$$\boxed{\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell) = (4.90 \pm 0.02 \pm 0.16)\%}$$



- First direct e/μ LFUV measurement - cancelling common systematics where only remaining are dominated by e and μ ID.

$$\boxed{\frac{\mathcal{B}(B^0 \to D^{*-}e^+\nu_e)}{\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)} = 1.01 \pm 0.01 \pm 0.03}$$

$\mathcal{R}(\mathcal{D}^*)$ measurement at LHCb

By measuring the $B^0 \to D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong τ decays

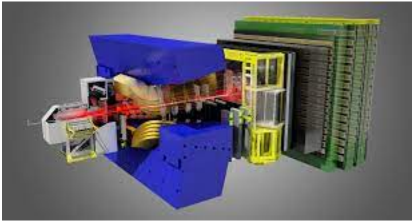
PRL **120**, 171802 (2018) PRD **97**, 072013 (2018)

LHCb

- pp collisions
- b quarks produced by gluon fusion $\rightarrow$ forward direction
- More B's, with a lot more background
- Boosted CM energy helps to reconstruct vertices

Run 1: 2011 – 12: 3 fb^{-1} @ 7 – 8 TeV

Run 2: 2015 – 18: 6 fb^{-1} @ 13 TeV



- LFUV in LHCb:

Leptonic [PRL **115** (2015) 111803]

$$\mathcal{R}(\mathcal{D}^*) = \frac{\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)}{\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)}$$

Hadronic [PRL **120**, 171802 (2018)]

$$\mathcal{K}(\mathcal{D}^*) = \frac{\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)}{\mathcal{B}(B^0 \to D^{*-}\pi^+\pi^-\pi^+)}$$

Hadronic $\mathcal{R}(\mathcal{D}^*)$ measurement at LHCb

- First determination of $\mathcal{R}(\mathcal{D}^*)$ using the three-prong τ decays

$$\mathcal{R}(\mathcal{D}^*) = \mathcal{K}(\mathcal{D}^*)_{\text{meas}} \times \left(\frac{\mathcal{B}(B^0 \to D^{*-}\pi^+\pi^-\pi^+)}{\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu_\mu})}\right)_{\text{external}}$$

where, $\mathcal{K}(\mathcal{D}^*) = \frac{\mathcal{B}(B^0 \to D^{*-}\tau^+\nu_\tau)}{\mathcal{B}(B^0 \to D^{*-}\pi^+\pi^-\pi^+)}$

- Signal and normalisation channels have the same final state, such that many systematics cancel in the ratio
- Data: 3 fb^{-1} @ $7-8$ TeV

$B^0 \to D^{*-}\tau^+\nu_\tau$ Signal

$\tau^+ \to \pi^+\pi^-\pi^+\bar{\nu_\tau}$

$\pi^+\pi^-\pi^+\pi^0\bar{\nu_\tau}$

$D^{*-} \to \bar{D}^0(\to K^+\pi^-)\pi^-$

$B^0 \to D^{*-}\pi^+\pi^-\pi^+$

$D^{*-} \to \bar{D}^0(\to K^+\pi^-)\pi^-$

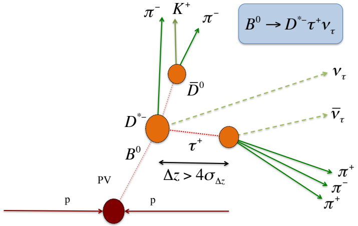
Normalization

PRL **120**, 171802 (2018)

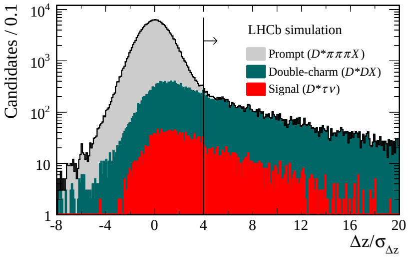
PRD **97**, 072013 (2018)

Analysis overview

- Use decay vertex topology to suppress the dominant background ($B \to D^* 3\pi(X)$ system)



- Suppress the background by 3 orders of magnitude and has an efficiency of 35% for the signal



- Other bkg: $B \to DD^{*-}(X)$ decays (suppressed by means of an MVA)

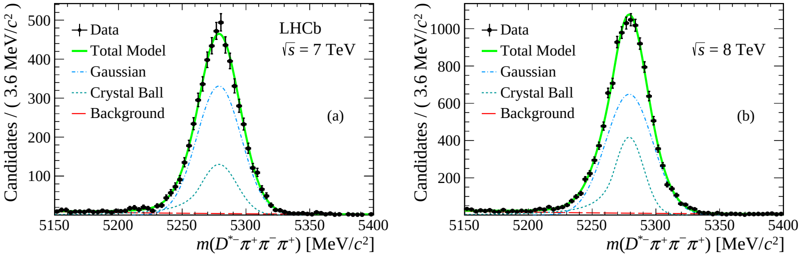
- $\bar{D}^0$, D^{*-}, and τ candidates are selected based on kinematic, geometric, and particle identification criteria

Signal extraction

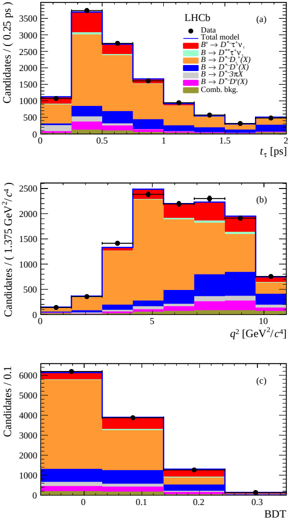
Normalization mode:

- 1D fit to $M_{D^{*}-3\pi}$ ((Gauss+CB)+Exp)

$$\boxed{\mathrm{N_{norm}} = 17660 \pm 158}$$



Signal extraction



Signal mode:

- 3D binned maximum likelihood fit to q^2, t_τ, and the BDT output

q^2 : squared invariant mass of the (τ, ν_τ) pair

t_τ: τ decay time

$$\boxed{\mathrm{N_{sig}} = 1296 \pm 86}$$

$\mathcal{R}(\mathcal{D}^*)$ results

PRL **120**, 171802 (2018) PRD **97**, 072013 (2018)

$$\mathcal{R}(\mathcal{D}^*) = \mathcal{K}(\mathcal{D}^*)_{\text{meas}} \times \left(\frac{\mathcal{B}(B^0 \to D^{*-}\pi^+\pi^-\pi^+)}{\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu_\mu})}\right)_{\text{external}}$$

$$\mathcal{K}(\mathcal{D}^*) \equiv \frac{\mathcal{B}(\mathcal{B}' \to \mathcal{D}^{*-}\tau^+\nu_\tau)}{\mathcal{B}(\mathcal{B}' \to \mathcal{D}^{*-}\pi^+\pi^-\pi^+)}$$

$$= \frac{\mathrm{N_{sig}}}{\mathrm{N_{norm}}} \cdot \frac{\epsilon_{\text{norm}}}{\epsilon_{\text{sig}}} \cdot \frac{1}{\mathcal{B}(\tau^+ \to 3\pi\bar{\nu_\tau}) + \mathcal{B}(\tau^+ \to 3\pi\pi^0\bar{\nu_\tau})}$$

$$\mathcal{K}(\mathcal{D}^*) = 1.97 \pm 0.13\ (\text{stat}) \pm 0.18\ (\text{syst})$$

$$\mathcal{R}(\mathcal{D}^*) = 0.291 \pm 0.019\ (\text{stat}) \pm 0.026\ (\text{syst}) \pm 0.013\ (\text{BR})$$

External inputs have been recently updated by HFLAV

$$\boxed{\mathcal{R}(\mathcal{D}^*) = 0.280 \pm 0.018\ (\text{stat}) \pm 0.029\ (\text{syst})}$$

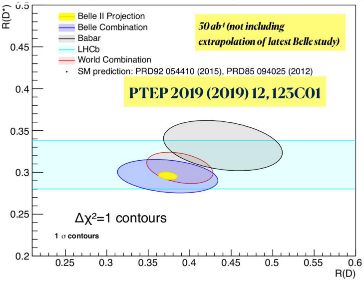
- Compatible with SM within 1σ

Future prospects

- A lot more data to be analysed from the upgraded experiments (LHCb and Belle II)

Expected precision at Belle II

Observable	5 ab^{-1}	50 ab^{-1}
R_D	$(\pm 6.0 \pm 3.9)\%$	$(\pm 2.0 \pm 2.5)\%$
R_{D^*}	$(\pm 3.0 \pm 2.5)\%$	$(\pm 1.0 \pm 2.0)\%$
$P_\tau(D^*)$	$\pm 0.18 \pm 0.08$	$\pm 0.06 \pm 0.04$



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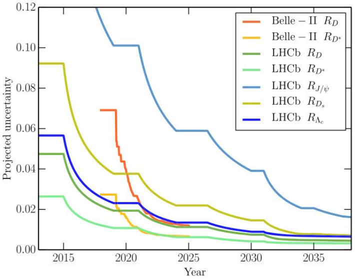


Prospects of various decays modes in the coming years

Talk on Friday: 'The status of Belle II and its Prospects' by M. Nayak

Summary

- Belle recently provided the most precise measurements of $\mathcal{R}(\mathcal{D}^{(*)})$

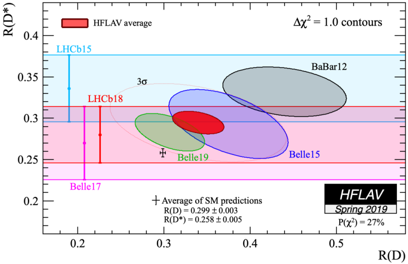
- After the updates from LHCb's hadronic $\mathcal{R}(\mathcal{D}^{*})$ and the new Belle semileptonic result, the new HFLAV averages are:

$$\boxed{\mathcal{R}(\mathcal{D}^{*}) = 0.295 \pm 0.011 \pm 0.008}$$

$$\boxed{\mathcal{R}(\mathcal{D}) = 0.340 \pm 0.027 \pm 0.013}$$

https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html

- $\mathcal{R}(\mathcal{D}^{(*)})$: Expecting $\mathcal{O}(5\%)$ precision (total uncertainty) with $5\ \mathrm{ab}^{-1}$ Belle II data



- Still 3.1σ difference with the SM!

Conclusion

• $\mathcal{R}(\mathcal{D}^{(*)})$ is a clean variable to search for NP

• A lot more data to be analysed, different decay channels to study, and new experiments on the way

• Upgraded experiments (LHCb and Belle II) are expected to finally confirm or rule out lepton flavor universality violation.

Thank you!

Extra slides

$B \to D^{(*)}\pi\ell\nu$ [Belle, arXiv:1803.064]

- Hadron tag
- $B^+ \to D^{(*)}\pi^0 l \nu$ (1.4k signal)
- $B^+ \to D^{(*)}\pi^+ l \nu$ (1.1k signal)
- Binned fit to $m^2_{miss/\nu}$ to D and D* simultaneously (B^+, B^0 separately)

  - $\mathcal{B}(B^+ \to D^-\pi^+\ell^+\nu) = [4.55 \pm 0.27\ (\text{stat.}) \pm 0.39\ (\text{syst.})] \times 10^{-3}$,
  - $\mathcal{B}(B^0 \to \bar{D}^0\pi^-\ell^+\nu) = [4.05 \pm 0.36\ (\text{stat.}) \pm 0.41\ (\text{syst.})] \times 10^{-3}$,
  - $\mathcal{B}(B^+ \to D^{*-}\pi^+\ell^+\nu) = [6.03 \pm 0.43\ (\text{stat.}) \pm 0.38\ (\text{syst.})] \times 10^{-3}$,
  - $\mathcal{B}(B^0 \to \bar{D}^{*0}\pi^-\ell^+\nu) = [6.46 \pm 0.53\ (\text{stat.}) \pm 0.52\ (\text{syst.})] \times 10^{-3}$.

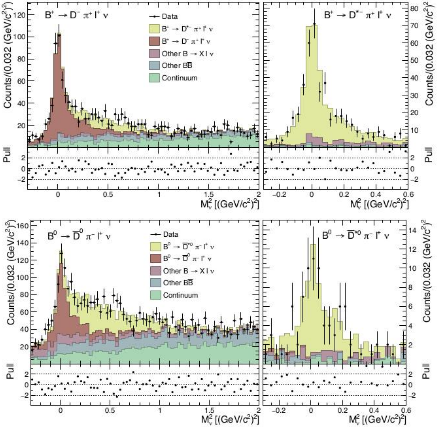


TABLE I. Systematic uncertainties contributing to the $\mathcal{R}(D^{(*)})$ results, together with their correlation.

Source	$\Delta\mathcal{R}(D)(\%)$	$\Delta\mathcal{R}(D^*)(\%)$	Correlation
D^{**} composition	0.76	1.41	−0.41
PDF shapes	4.39	2.25	−0.55
Feed-down factors	1.69	0.44	0.53
Efficiency factors	1.93	4.12	−0.57
Fake $D^{(*)}$ calibration	0.19	0.11	−0.76
B_{tag} calibration	0.07	0.05	−0.76
Lepton efficiency and fake rate	0.36	0.33	−0.83
Slow pion efficiency	0.08	0.08	−0.98
B decay form factors	0.55	0.28	−0.60
Luminosity, f^{+-}, f^{00}, and $\mathcal{B}(\Upsilon(4S))$	0.10	0.04	−0.58
$\mathcal{B}(B \to D^{(*)}\ell\nu)$	0.05	0.02	−0.69
$\mathcal{B}(D)$	0.35	0.13	−0.65
$\mathcal{B}(D^*)$	0.04	0.02	−0.51
$\mathcal{B}(\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau)$	0.15	0.14	−0.11
Total	5.21	4.94	−0.52

TABLE I. Relative systematic uncertainties on $\mathcal{R}(D^{*-})$.

Source	$\delta R(D^{*-})/R(D^{*-})$ [%]
Simulated sample size	4.7
Empty bins in templates	1.3
Signal decay model	1.8
$D^{**}\tau\nu$ and $D_s^{**}\tau\nu$ feeddowns	2.7
$D_s^+ \to 3\pi X$ decay model	2.5
$B \to D^{*-}D_s^+X$, $B \to D^{*-}D^+X$, $B \to D^{*-}D^0X$ backgrounds	3.9
Combinatorial background	0.7
$B \to D^{*-}3\pi X$ background	2.8
Efficiency ratio	3.9
Normalization channel efficiency (modeling of $B^0 \to D^{*-}3\pi$)	2.0
Total uncertainty	9.1

Event selection : Belle semileptonic

Particle	Variable type	Cut
$K^{\pm}$	PID	KID > 0.1
$e^{\pm}$	PID	eID > 0.3
	Kinematics	$p_{lab} > 0.200$ GeV/c^2
$\mu^{\pm}$	PID	muID > 0.5
all charged	Track parameter	$\|d_0\| < 2.0$ cm
	Track parameter	$\|z_0\| < 5.0$ cm
π^0	Invariant mass	$0.120 < M($ GeV/c^2 $) < 0.150$
	E_{γ}	$> 50/100/150$ MeV/c^2 (barrel/fwd/back)
	$cos\theta_{\gamma\gamma}$	> 0
	Kinematics	$p_{lab} > 0.200$ GeV/c^2
π^0_{slow}	Invariant mass	$\|(dM($ GeV/c^2 $)\| < 0.010$
	E_{γ}, high	> 50 MeV/c^2
	E_{γ}, low	> 20 MeV/c^2
K^0_S	Invariant mass	$0.483 < M($ GeV/c^2 $) < 0.513$
	nisKSFinder	$nb_{vlike} > 0.5$ and $nb_{nolam} > -0.4$

- The requirement on $\cos\theta_{B,D^{(*)}\ell} < 1$ and momentum of D^* in CM frame < 2.5 GeV/c
- No extra charged tracks, K^0_S and π^0
- A criteria on the invariant mass is applied to be close to nominal mass of π^0, K^0_S, D^0 and D^+

D^+	with π^0	InvM	-36 MeV/c^2 $< dM < 24$ MeV/c^2
D^+	without π^0	InvM	-15 MeV/c^2 $< dM < 15$ MeV/c^2
		Vertex fit	pValue > 0
D^0	with π^0	InvM	-45 MeV/c^2 $< dM < 30$ MeV/c^2
D^0	without π^0	InvM	-15 MeV/c^2 $< dM < 15$ MeV/c^2
		Vertex fit	pValue > 0
D^{*0}	$D^0\pi^0$	$\|dM\| < 2.0$ MeV/c^2	
D^{*+}	$D^0\pi^+$	$\|dM\| < 2.5$ MeV/c^2	
	$D^+\pi^0$	$\|dM\| < 2.0$ MeV/c^2	