Tow-body hadronic B decays at NNLO in QCDF

Xin-Qiang Li

Central China Normal University

In collaboration with G. Bell, M. Beneke, T. Huber, and S. Krankl

Based on JHEP 04 (2020), JHEP 09 (2016) 112, PLB 750 (2015) 348, NPB 832 (2010) 109

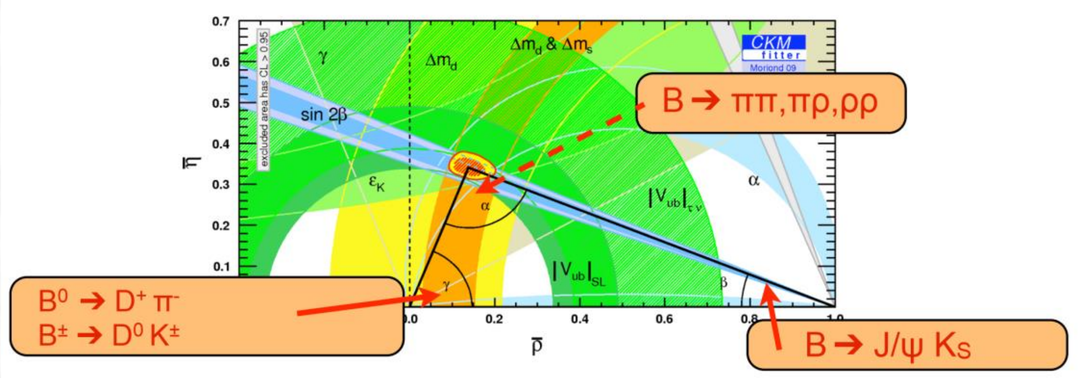
FPCP 2021, Shanghai, 2021/06/08

Outline

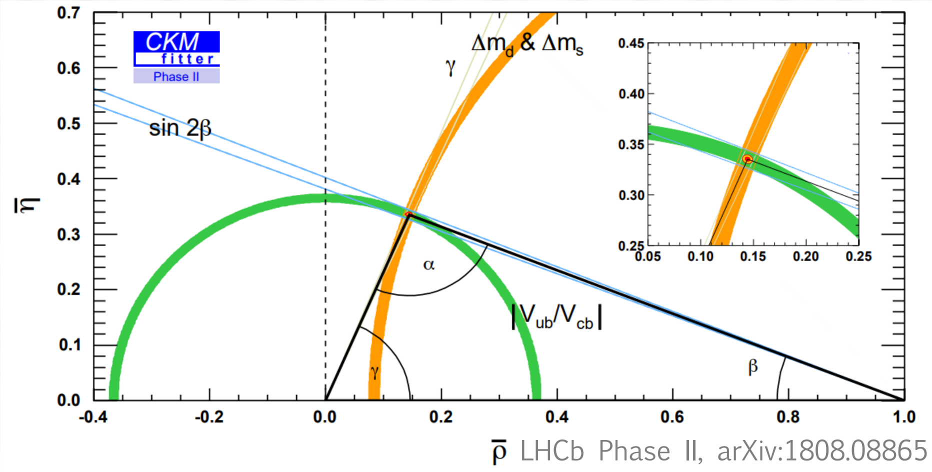
- □ Introduction to QCDF approach
- □ Status of NNLO calculations within QCDF:
  - ➢ Tree-dominated decay modes
  - ➢ Penguin-dominated decay modes:
  - ➢ Class-I $\bar{B}_q^0 \to D_q^{(*)+} L^-$ decays
- □ Summary

Why hadronic B decays

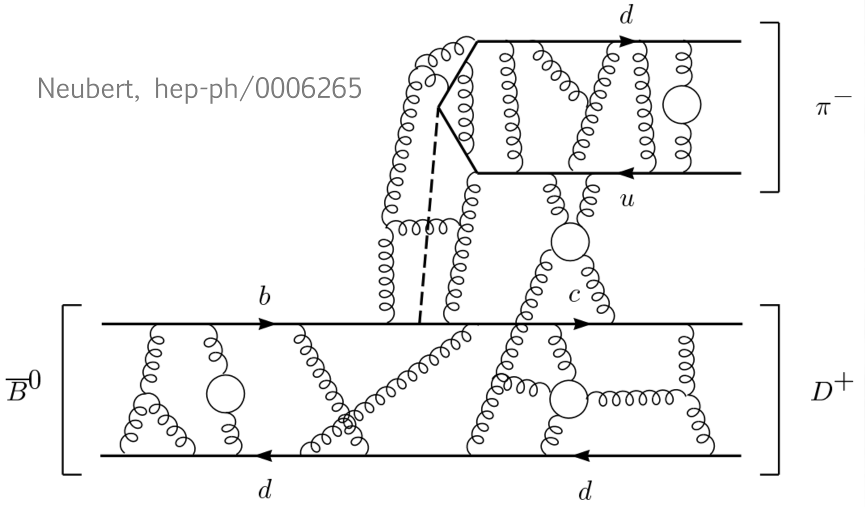
- direct access to the CKM parameters, especially to the **three angles of UT**.



- Thanks to exp. progress, precision era ahead!



- further insight into strong-interaction effects involved in these hadronic decays.



- From the theory side, we need also keep up with the same precision from experiment.

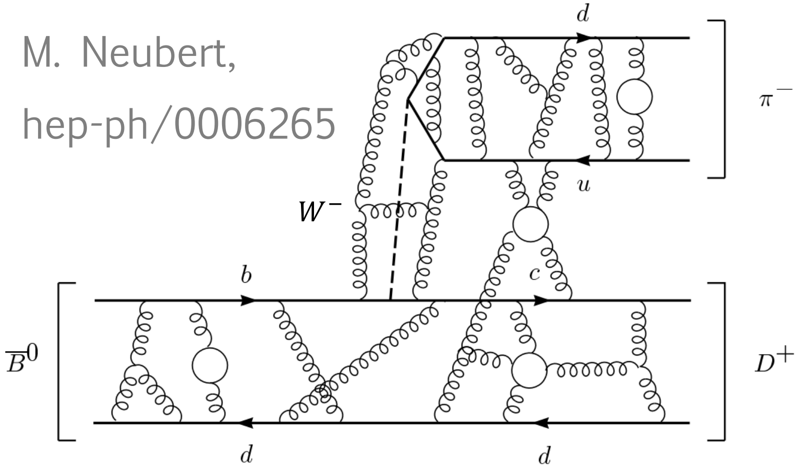
→ ***very difficult but necessary!***

Effective Hamiltonian for hadronic B decays

- **For hadronic decays:** simplicity of weak interactions overshadowed by complex QCD effects!

M. Neubert,

hep-ph/0006265



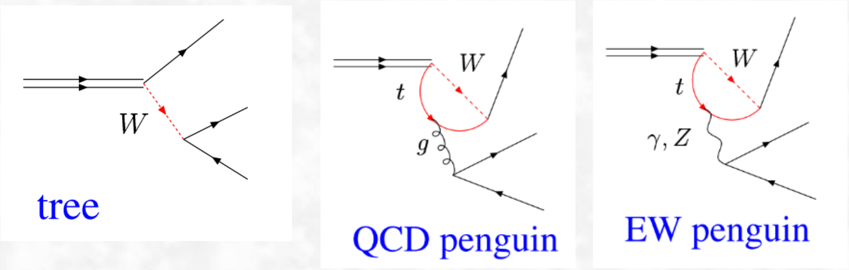
multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{QCD} \sim 1$ GeV

- **Starting point $\mathcal{H}_{\text{eff}} = -\mathcal{L}_{\text{eff}}$:** obtained after integrating out the heavy d.o.f. $(m_{W,Z,t} \gg m_b)$;

  [Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \Big(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\text{pen}} C_i \mathcal{O}_{i,\text{pen}} \Big)$$



- **Wilson coefficients C_i:** all physics above m_b; perturbatively calculable, and NNLL program now complete; [Gorbahn, Haisch '04]

Hadronic matrix elements

- **Decay amplitude for a given decay mode:**

$$\mathcal{A}(\bar{B} \to f) = \sum_i \left[\lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED} \right]_i$$

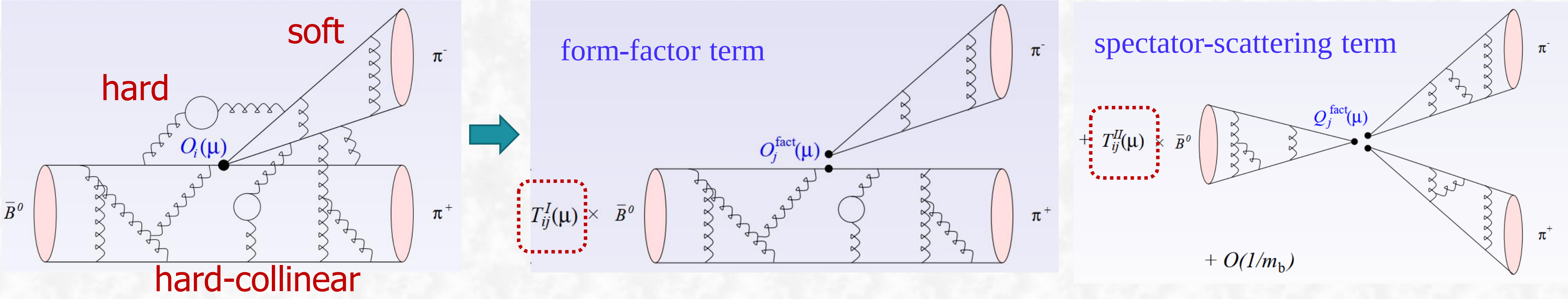
- $\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle$: depend on the spin and parity of $M_{1,2}$; also involve complicated QCD effects.

⟶ *A quite difficult, multi-scale, strong-interaction problem!*

- **Different methods for $\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle$:**

  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · · [Keum, Li, Sanda, Lü, Yang '00; Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · · [Zeppenfeld, '81; London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

- **QCDF:** systematic framework to all orders in α_s, but limited by $1/m_b$ corrections. [BBNS '99-'03]

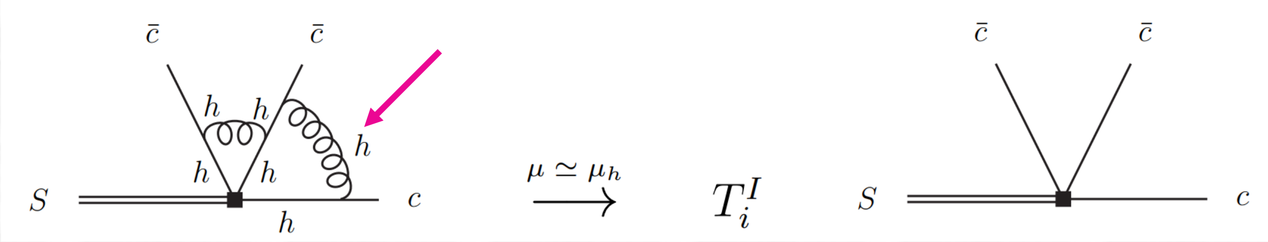


Soft-collinear factorization from SCET

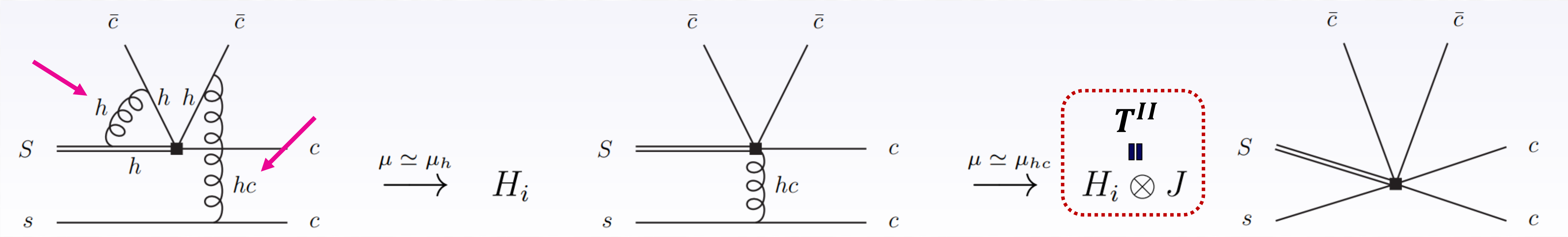
- **SCET diagrams reproduce precisely QCD diagrams in collinear & soft momentum regions**

→ **QCD - SCET = short-distance coefficients T^I & T^{II}**

- **For hard kernel T^I:** one-step matching, QCD → $\mathrm{SCET_I(hc, c, s)}$!



- **For hard kernel T^{II}:** two-step matching, QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!

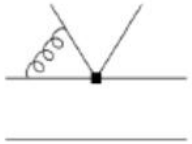
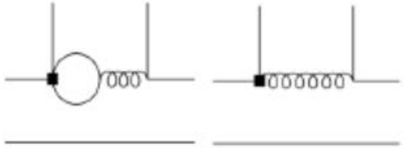
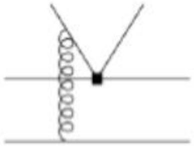
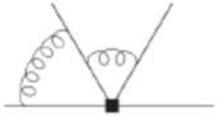
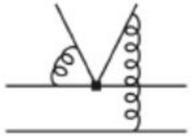
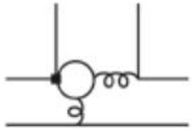


- **SCET result exactly the same as QCDF, but more apparent & efficient;** [Beneke, 1501.07374]

Status of NNLO calculations of T^I & T^{II}

- For each Q_i insertion, both **tree** & **penguin** topologies, and contribute to both T^I & T^{II}.

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$		$T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$		$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11, Bell Beneke, Huber, Li '15 Bell, Beneke, Huber, Li '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

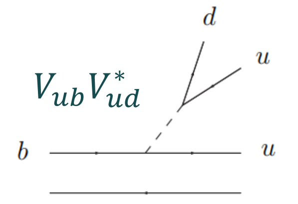
Tree-dominated B decays

- $B \to \pi\pi$ **decays:** mediated by $b \to u\bar{u}d$ transitions;

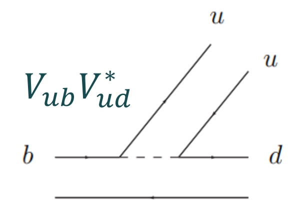
$$\sqrt{2}\,\langle \pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle = \lambda_u\left[\alpha_1(\pi\pi) + \alpha_2(\pi\pi)\right]\,A_{\pi\pi}$$

$$\langle \pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \left\{\lambda_u\left[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)\right] + \lambda_c\,\alpha_4^c(\pi\pi)\right\}\,A_{\pi\pi}$$

$$-\,\langle \pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \left\{\lambda_u\left[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)\right] - \lambda_c\,\alpha_4^c(\pi\pi)\right\}\,A_{\pi\pi}$$

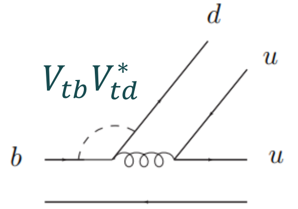


colour-allowed tree α_1



colour-suppressed tree α_2

Tree-dominated!



QCD penguins α_4

$$\lambda_u = V_{ub}V_{ud}^* \sim \mathcal{O}(\lambda^3),\quad \lambda_c = V_{cb}V_{cd}^* \sim \mathcal{O}(\lambda^3)$$ ➡ α_4 loop-suppressed vs $\alpha_{1,2}$

- α_2 **at NLO:** large cancellation between one-loop vertex correction and LO term;

$$a_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\rm NLO} + \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.123]_{\rm LOsp} + [0.072]_{\rm tw3}\right\}$$

$$r_{\rm sp} = \frac{9 f_{M_1}\hat{f}_B}{m_b f_+^{B\pi}(0)\,\lambda_B}$$

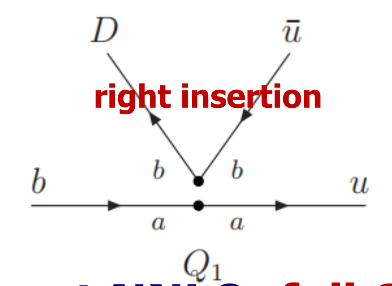
➡ **NNLO is in effect NLO for α_2; large effect still possible!**

Hard-kernel T^I at NNLO

□ QCD → SCETI matching calculation:

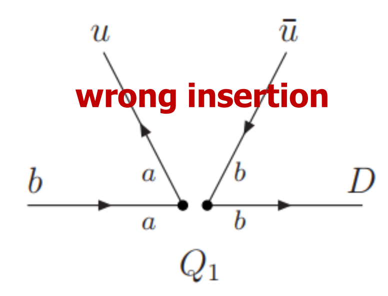
- For "right insertion":

$$\langle Q_i \rangle = T_i \, \langle O_{\rm QCD} \rangle + \sum_{a>1} H_{ia} \langle O_a \rangle$$



- For "wrong insertion":

$$\langle Q_i \rangle = \widetilde{T}_i \, \langle O_{\rm QCD} \rangle + \widetilde{H}_{i1} \langle \widetilde{O}_1 - O_1 \rangle + \sum_{a>1} \widetilde{H}_{ia} \langle \widetilde{O}_a \rangle$$



□ Master formula for T^I: right insertion

$$\begin{aligned}
T_i^{(0)} &= A_{i1}^{(0)} \,, \\
T_i^{(1)} &= A_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, A_{j1}^{(0)} \,, \\
T_i^{(2)} &= A_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} \, A_{j1}^{(1)} + Z_{ij}^{(2)} \, A_{j1}^{(0)} + Z_\alpha^{(1)} \, A_{i1}^{(1)\rm nf} + (-i) \, \delta m^{(1)} \, A_{i1}'^{(1)\rm nf} \\
&\quad - T_i^{(1)} [C_{FF}^{(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)}] - \sum_{b>1} H_{ib}^{(1)} \, Y_{b1}^{(1)} \,.
\end{aligned}$$

□ Master formula for T^I: wrong insertion

$$\begin{aligned}
\widetilde{T}_i^{(0)} &= \widetilde{A}_{i1}^{(0)} \,, \\
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \, \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \, \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} \,, \\
\widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \, \widetilde{A}_{j1}^{(0)} + Z_\alpha^{(1)} \, \widetilde{A}_{i1}^{(1)\rm nf} \\
&\quad + (-i) \, \delta m^{(1)} \, \widetilde{A}_{i1}'^{(1)\rm nf} + Z_{ext}^{(1)} \, [\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(0)}] \\
&\quad - \widetilde{T}_i^{(1)} [C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)} \, \widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2)\rm f} - A_{21}^{(2)\rm f} \, \widetilde{A}_{i1}^{(0)}] + (-i) \, \delta m^{(1)} \, [\widetilde{A}_{i1}'^{(1)\rm f} - A_{21}'^{(1)\rm f} \, \widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_\alpha^{(1)} + Z_{ext}^{(1)}) \, [\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \, \widetilde{A}_{i1}^{(0)}] \\
&\quad - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}] \, \widetilde{A}_{i1}^{(0)} \\
&\quad - (C_{FF}^{(1)} - \xi_{45}^{(1)}) \, [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \, \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \, \widetilde{A}_{i1}^{(0)} \,.
\end{aligned}$$

□ On-shell matrix elements at NNLO: full QCD side

$$\begin{aligned}
\langle Q_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \Big[A_{ia}^{(1)} + Z_{ext}^{(1)} \, A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \Big] \\
& + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} \, A_{ia}^{(1)} + Z_{ext}^{(2)} \, A_{ia}^{(0)} \\
& + Z_{ext}^{(1)} \, Z_{ij}^{(1)} A_{ja}^{(0)} + Z_\alpha^{(1)} A_{ia}^{(1)} + (-i) \, \delta m^{(1)} \, A_{ia}'^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \, \langle O_a \rangle^{(0)}
\end{aligned}$$

□ On-shell matrix elements at NNLO: SCET side

$$\begin{aligned}
\langle O_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \Big[M_{ab}^{(1)} + Y_{ext}^{(1)} \, \delta_{ab} + Y_{ab}^{(1)} \Big] + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M_{ab}^{(2)} + Y_{ac}^{(1)} M_{cb}^{(1)} \\
& + Y_{ab}^{(2)} + Y_{ext}^{(1)} \, M_{ab}^{(1)} + Y_{ext}^{(2)} \, \delta_{ab} + Y_{ext}^{(1)} \, Y_{ab}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \, \langle O_b \rangle^{(0)}
\end{aligned}$$

Final results for $\alpha_{1,2}$

$$\phi_M(u) = 6u(1-u)\left[1 + \sum_{n=1}^{\infty} a_n^M C_n^{(3/2)}(2u-1)\right],$$

- **Tree amplitudes $\alpha_{1,2}$ up to NNLO:**

$$\alpha_i(M_1 M_2) = \sum_j C_j V_{ij}^{(0)} + \sum_{l\geqslant 1}\left(\frac{\alpha_s}{4\pi}\right)^l \left[\frac{C_F}{2N_c}\sum_j C_j V_{ij}^{(l)} + P_i^{(l)}\right] + \cdots$$

$$V_{1j}^{(0)} = \int_0^1 du\, T_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{1j}^{(l)} = \int_0^1 du\, T_j^{(l)}(u)\phi_M(u),$$

$$V_{2j}^{(0)} = \int_0^1 du\, \widetilde{T}_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{2j}^{(l)} = \int_0^1 du\, \widetilde{T}_j^{(l)}(u)\phi_M(u).$$

- **Numerical results including the NNLO corrections:**

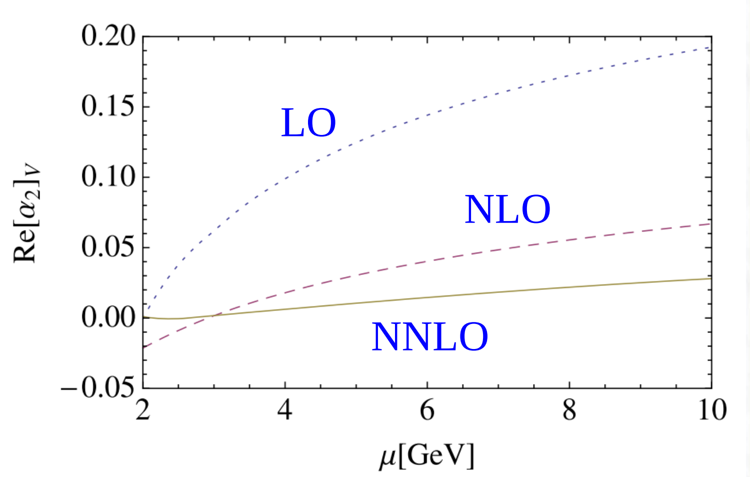
T^I

$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\, i]_{\rm NLO} + [0.026 + 0.028\, i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027 i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i
\end{aligned}$$

T^{II}

$$\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\, i]_{\rm NLO} - [0.031 + 0.050\, i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.049 + 0.051 i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i
\end{aligned}$$

individual NNLO corrections both significant, but cancelled between the vertex and the spectator term!



Branching ratios

$$-\langle \pi^0\pi^0|\mathcal{H}_{eff}|\bar{B}^0\rangle = \{\lambda_u[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)] - \lambda_c\,\alpha_4^c(\pi\pi)\}\, A_{\pi\pi}$$

	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43\,^{+0.06}_{-0.06}\,^{+1.45}_{-0.84}$	$(\star)$	$5.82\,^{+0.07}_{-0.06}\,^{+1.42}_{-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}_d^0 \to \pi^+\pi^-$	$7.37\,^{+0.86}_{-0.69}\,^{+1.22}_{-0.97}$	$(\star)$	$5.70\,^{+0.70}_{-0.55}\,^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}_d^0 \to \pi^0\pi^0$	$0.33\,^{+0.11}_{-0.08}\,^{+0.42}_{-0.17}$		$0.63\,^{+0.12}_{-0.10}\,^{+0.64}_{-0.42}$		1.55 ± 0.19
$B^- \to \pi^-\rho^0$	$8.68\,^{+0.42}_{-0.41}\,^{+2.71}_{-1.56}$	$(\star\star)$	$9.84\,^{+0.41}_{-0.40}\,^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38\,^{+0.90}_{-0.77}\,^{+2.18}_{-1.41}$	$(\star)$	$12.13\,^{+0.85}_{-0.73}\,^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80\,^{+0.62}_{-0.56}\,^{+1.76}_{-2.10}$	$(\star)$	$13.76\,^{+0.49}_{-0.44}\,^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28\,^{+0.39}_{-0.39}\,^{+1.37}_{-1.42}$	$(\star\star)$	$8.14\,^{+0.34}_{-0.33}\,^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08\,^{+0.27}_{-0.19}\,^{+3.82}_{-3.50}$	$(\dagger)$	$21.90\,^{+0.20}_{-0.12}\,^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52\,^{+0.04}_{-0.03}\,^{+1.11}_{-0.43}$		$1.49\,^{+0.07}_{-0.07}\,^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho_L^-\rho_L^0$	$18.42^{+0.23}_{-0.21}\,^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}\,^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}_d^0 \to \rho_L^+\rho_L^-$	$25.98^{+0.85}_{-0.77}\,^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}\,^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}_d^0 \to \rho_L^0\rho_L^0$	$0.39^{+0.03}_{-0.03}\,^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}\,^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

- **1st error: from CKM but without Vub;**
- **2nd error: all other hadronic parameters;**
- **Brackets: form factor error not included;**
- **Good agreement with all data, except $\pi^0\pi^0$;**

$(1.59 \pm 0.26) \times 10^{-6}$

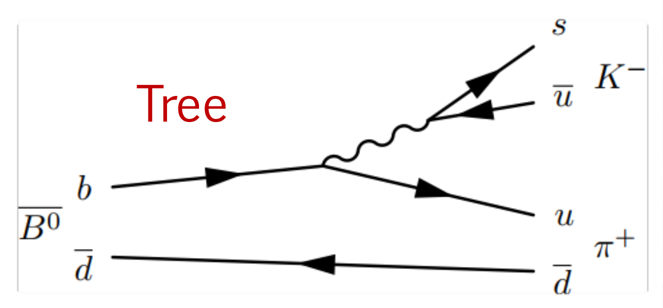
Penguin-dominated B decays

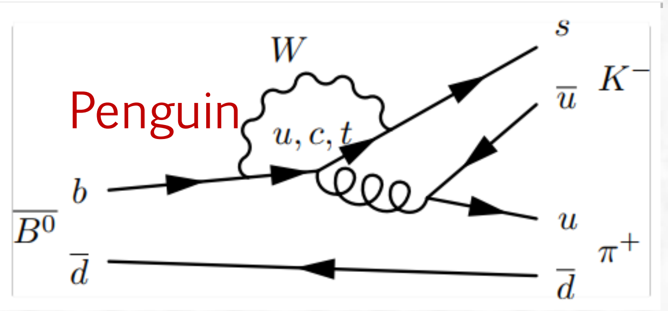
- $B \to \pi K$ **decays:** mediated by $b \to sq\bar{q}$ transitions;

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\bar{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\big],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

$$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4)$$
$$\lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$$

→ Penguin-dominated!





- **To predict accurately the direct CPV, we need calculate both tree & penguin to NNLO;**

- **Driven by the exp. data;** $\Delta A_{CP}(\pi K)$ puzzle

$$\Delta A_{CP} = A_{CP}(\pi^0K^-) - A_{CP}(\pi^+K^-)$$
$$= (11.5 \pm 1.4)\%$$ **differs from 0 by ~8σ**

Decay	BR($\times 10^{-6}$)	A_{CP}	S_{CP}
$B^+ \to \pi^+K^0$	23.79 ± 0.75	-0.017 ± 0.016	
$B^+ \to \pi^0K^+$	12.94 ± 0.52	0.025 ± 0.016	
$B_d^0 \to \pi^-K^+$	19.57 ± 0.53	-0.084 ± 0.004	
$B_d^0 \to \pi^0K^0$	9.93 ± 0.49	-0.01 ± 0.10	0.57 ± 0.17

→ ***How about the situation @ NNLO?***

Penguin topologies with various insertions

□ Effective Hamiltonian including penguin operators: [BBL '96; CMM '98]

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\, (\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\, (\bar{D}_L \gamma_\mu p_L),$$

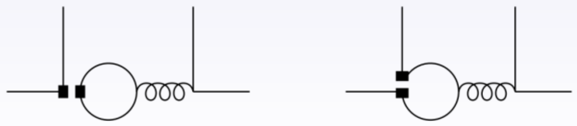
current-current operators

$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^A q)\,.$$

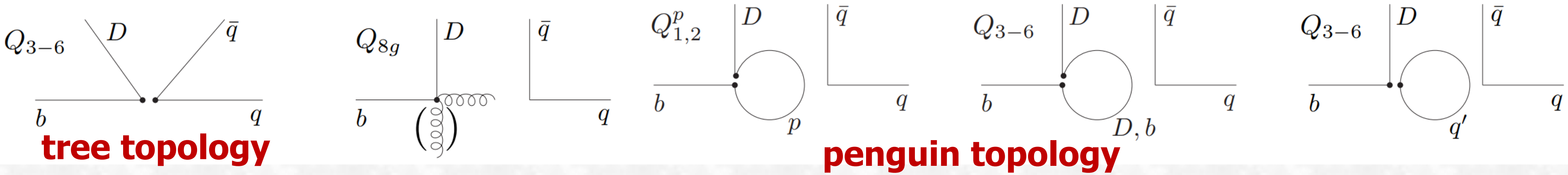
QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b,$$

chromo-magnetic dipole operators



□ Various types of operator insertions:

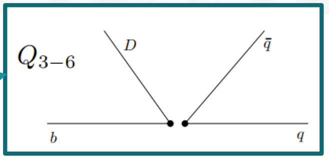


(i) Dirac structure of Q_i, (ii) color structure of Q_i, (iii) types of contraction, and (iv) quark mass in the fermion loop;

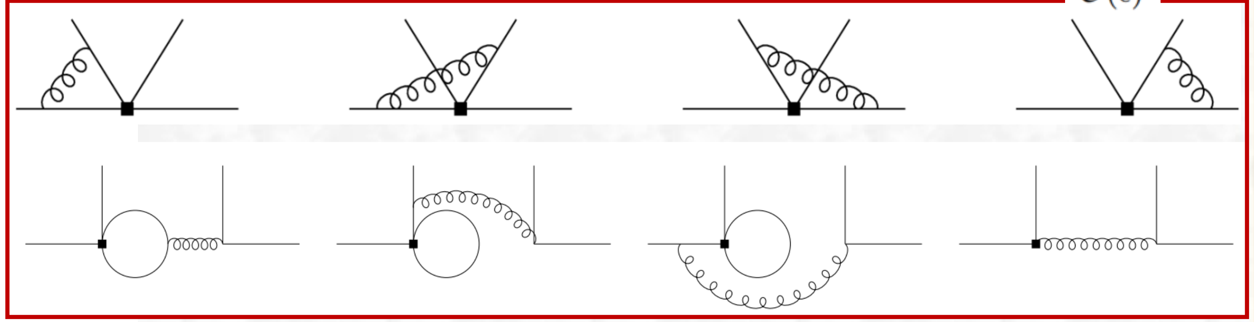
T^I up to NNLO

- **Master formulae for T^I:**

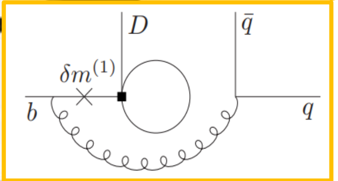
$$\frac{1}{2}\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)}\,,$$

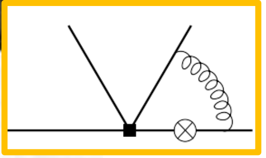


$$\frac{1}{2}\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{\sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(1)}}_{\mathcal{O}(\epsilon)}$$

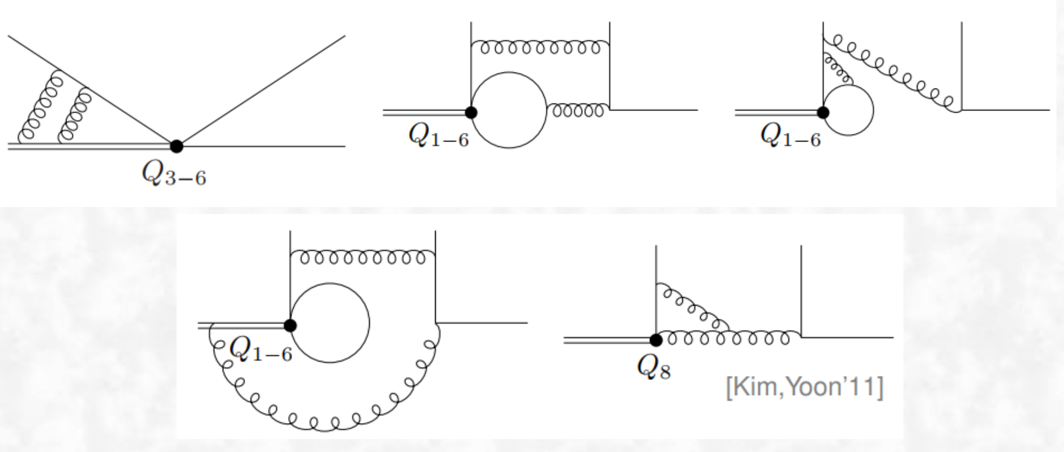


$$\begin{aligned}
\frac{1}{2}\widetilde{T}_i^{(2)} = {} & \widetilde{A}_{i1}^{(2)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
& + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}} + Z_{\mathrm{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A_{31}^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A_{31}^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& + (Z_{\alpha}^{(1)} + Z_{\mathrm{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}.
\end{aligned}$$

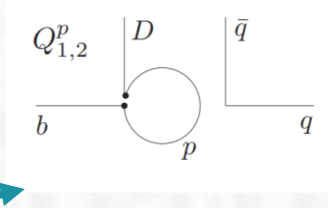




about 100 Feynman diagrams



Final results for a_4^p

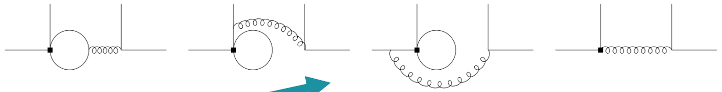


□ Final numerical results:

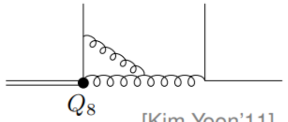
$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.49 - 1.32i]_{\mathrm{P}_1} - [0.32 + 0.71i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33 + 0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}}$$
$$+ \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} - [0.01 - 0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\}$$
$$= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,$$

- spectator-scattering has only a small effect.

$$T^{II} = (H_V^{II} + H_P^{II}) * J$$



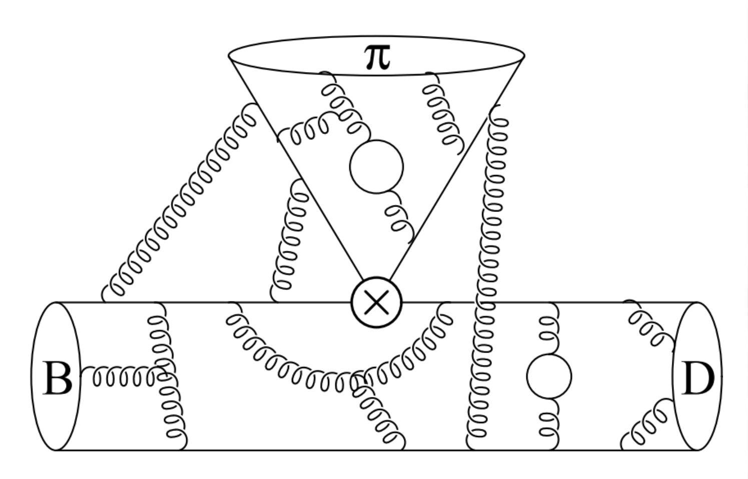
$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.05 - 0.62i]_{\mathrm{P}_1} - [0.77 + 0.50i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33 + 0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}}$$
$$+ \left[\frac{r_{\mathrm{sp}}}{0.434}\right]\left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} + [0.01 + 0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\}$$
$$= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.$$



[Kim,Yoon'11]

- **NNLO real part constitutes a (10 - 15)% correction relative to LO.**
- **NNLO imaginary part represents a -27% correction for a_4^u and reaches -54% for a_4^c.**
- **strong cancellation between NNLO correction from $Q_{1,2}^p$ and from $Q_{3-6,8g}$ observed!**

$B_q^0 \to D_q^{(*)-} L^+$ decays



- **At quark-level:** mediated by $b \to c\bar{u}d(s)$ transitions;

  all four flavors different from each other, no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- **For class-I decays:** QCDF formula much simpler;

  [Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du \, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

i) only color-allowed tree topology a_1;

ii) spectator & annihilation power-suppressed;

iii) annihilation absent in $B_{d(s)}^0 \to D_{d(s)}^- K(\pi)^+$ etal;

iv) they are theoretically simpler and cleaner!

- **Hard kernel T:** both NLO and NNLO results known;

  [Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Calculation of T

☐ Matching QCD onto SCET_I: [Huber, Kränkl, Li '16]

m_c is also heavy, keep m_c/m_b fixed as $m_b \to \infty$, thus needing two sets of SCET operator basis.

$$\langle \mathcal{Q}_i \rangle = \hat{T}_i \langle \mathcal{Q}^{\rm QCD} \rangle + \hat{T}_i' \langle \mathcal{Q}'^{\rm QCD} \rangle + \sum_{a>1} \left[H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle \right]$$

physical operators and factorizes into FF*LCDA.

$$\mathcal{O}_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) h_v \,,$$
$$\mathcal{O}_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v \,,$$
$$\mathcal{O}_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v$$

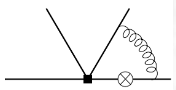
$$\mathcal{O}'_1 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) h_v \,,$$
$$\mathcal{O}'_2 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} h_v \,,$$
$$\mathcal{O}'_3 = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} \gamma_{\perp,\gamma} \gamma_{\perp,\delta} h_v$$

evanescent operators and must be renormalized to zero.

☐ Renormalized on-shell QCD amplitudes:

$$\begin{aligned}\langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\ & + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \\ & + (-i)\delta m_b^{(1)} A_{ia}^{*(1)} + (-i)\delta m_c^{(1)} A_{ia}^{**(1)} + Z_\alpha^{(1)} A_{ia}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\ & + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)} \,.\end{aligned}$$

on QCD side



☐ Renormalized on-shell SCET amplitudes:

$$\begin{aligned}\langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] \\ & + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \\ & + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \Big] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} \,,\end{aligned}$$

on SCET side

☐ Master formulas for hard kernels:

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

$$\hat{T}_i^{(0)} = A_{i1}^{(0)}$$
$$\hat{T}_i^{(1)} = A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)}$$
$$\begin{aligned}\hat{T}_i^{(2)} = & A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \left[C_{FF}^{\rm D(1)} + Y_{11}^{(1)} - Z_{\rm ext}^{(1)} \right] \\ & - C_{FF}^{\rm ND(1)} \hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b\neq 1} H_{ib}^{(1)} Y_{b1}^{(1)} \,.\end{aligned}$$

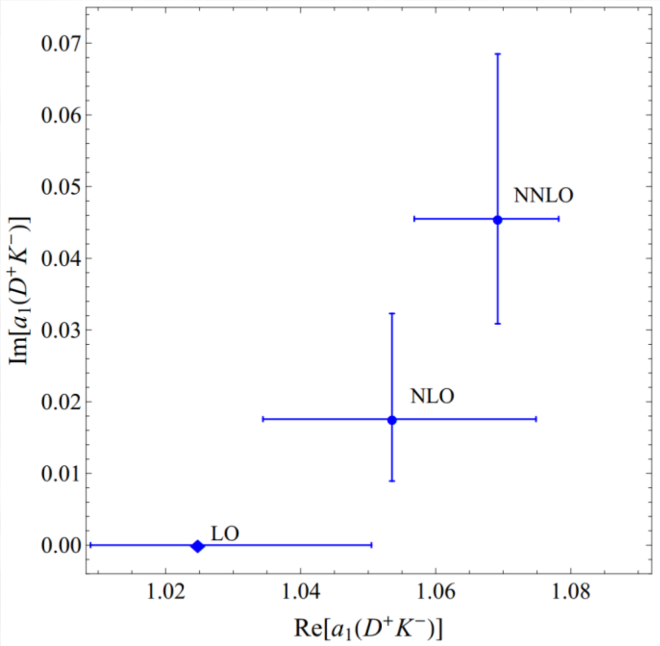
Decay amplitudes for $B_q^0 \to D_q^- L^+$

- **Color-allowed tree amplitude:**

$$a_1(D^+L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu),$$
$$a_1(D^{*+}L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) - \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu),$$

- **Numerical result:**

$$a_1(D^+K^-) = 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO} = (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,,$$



- **both NLO and NNLO add always constructively to LO result!**
- **NNLO corrections quite small in real (2%), but rather large in imaginary part (60%).**

- **For different decay modes:** *quasi-universal*, with a small process dependence from *non-fact. correction*.

$$a_1(D^+K^-) = (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,,$$
$$a_1(D^+\pi^-) = (1.072^{+0.011}_{-0.013}) + (0.043^{+0.022}_{-0.014})i\,,$$
$$a_1(D^{*+}K^-) = (1.068^{+0.010}_{-0.012}) + (0.034^{+0.017}_{-0.011})i\,,$$
$$a_1(D^{*+}\pi^-) = (1.071^{+0.012}_{-0.013}) + (0.032^{+0.016}_{-0.010})i\,.$$

Absolute branching ratios for $B_q^0 \to D_q^- L^+$

$$\mathcal{A}(\bar{B}^0_{(s)} \to D^+_{(s)} P^-) = i \frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq}\, a_1(D^+_{(s)} P^-)\, f_P\, F_0^{B_{(s)} \to D_{(s)}}(m_P^2)\, (m^2_{B_{(s)}} - m^2_{D^+_{(s)}}),$$

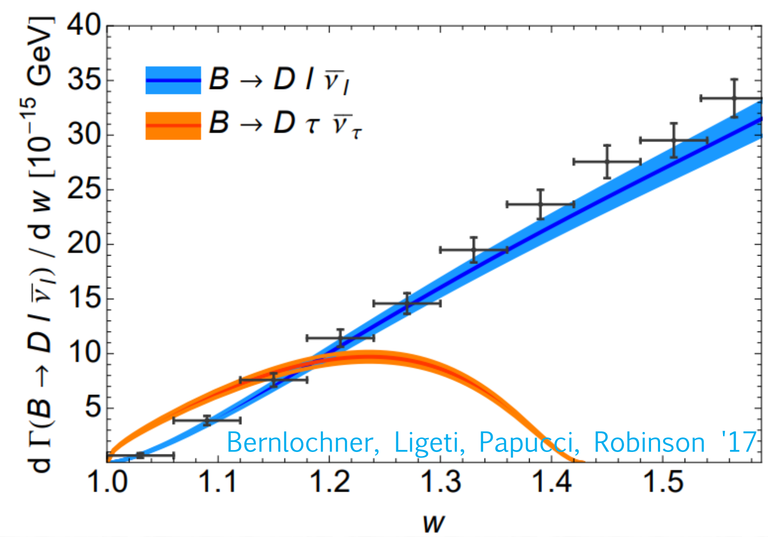
$$\mathcal{A}(\bar{B}^0_{(s)} \to D^{*+}_{(s)} P^-) = -i \frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq}\, a_1(D^{*+}_{(s)} P^-)\, f_P\, A_0^{B_{(s)} \to D^*_{(s)}}(m_P^2)\, 2m_{D^{*+}_{(s)}}\, (\epsilon^* \cdot p),$$

$$\mathcal{A}(\bar{B}^0_{(s)} \to D^+_{(s)} V^-) = -i \frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq}\, a_1(D^+_{(s)} V^-)\, f_V\, F_+^{B_{(s)} \to D_{(s)}}(m_V^2)\, 2m_V\, (\eta^* \cdot p),$$

□ $B \to D^{(*)}$ transition form factors:

Precision results available based on LQCD & LCSR calculations, together with data on $B_q^0 \to D_q^- l^+ \nu$;

[Bernlochner, Ligeti, Papucci, Robinson '17; Bordone, Gubernari, Jung, van Dyk '19]



□ Updated predictions vs data:

[Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

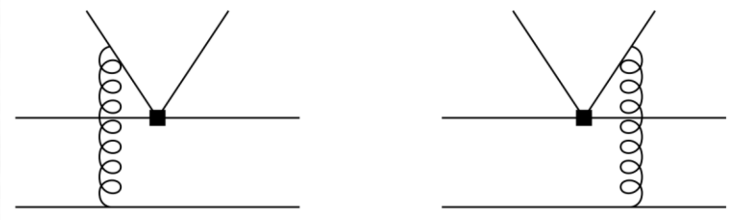
$|V_{cb}|$ and $B_{d,s} \to D^{(*)}_{d,s}$ form factors

Decay mode	LO	NLO	NNLO	Ref. [36]	Exp. [7, 8]
$\bar{B}^0 \to D^+\pi^-$	4.07	$4.32^{+0.23}_{-0.42}$	$4.43^{+0.20}_{-0.41}$	$3.93^{+0.43}_{-0.42}$	2.65 ± 0.15
$\bar{B}^0 \to D^{*+}\pi^-$	3.65	$3.88^{+0.27}_{-0.41}$	$4.00^{+0.25}_{-0.41}$	$3.45^{+0.53}_{-0.50}$	2.58 ± 0.13
$\bar{B}^0 \to D^+\rho^-$	10.63	$11.28^{+0.84}_{-1.23}$	$11.59^{+0.79}_{-1.21}$	$10.42^{+1.24}_{-1.20}$	7.6 ± 1.2
$\bar{B}^0 \to D^{*+}\rho^-$	9.99	$10.61^{+1.35}_{-1.56}$	$10.93^{+1.35}_{-1.57}$	$9.24^{+0.72}_{-0.71}$	6.0 ± 0.8
$\bar{B}^0 \to D^+K^-$	3.09	$3.28^{+0.16}_{-0.31}$	$3.38^{+0.13}_{-0.30}$	$3.01^{+0.32}_{-0.31}$	2.19 ± 0.13
$\bar{B}^0 \to D^{*+}K^-$	2.75	$2.92^{+0.19}_{-0.30}$	$3.02^{+0.18}_{-0.30}$	$2.59^{+0.39}_{-0.37}$	2.04 ± 0.47
$\bar{B}^0 \to D^+K^{*-}$	5.33	$5.65^{+0.47}_{-0.64}$	$5.78^{+0.44}_{-0.63}$	$5.25^{+0.65}_{-0.63}$	4.6 ± 0.8
$\bar{B}^0_s \to D^+_s\pi^-$	4.10	$4.35^{+0.24}_{-0.43}$	$4.47^{+0.21}_{-0.42}$	$4.39^{+1.36}_{-1.19}$	3.03 ± 0.25
$\bar{B}^0_s \to D^+_sK^-$	3.12	$3.32^{+0.17}_{-0.32}$	$3.42^{+0.14}_{-0.31}$	$3.34^{+1.04}_{-0.90}$	1.92 ± 0.22

Power corrections

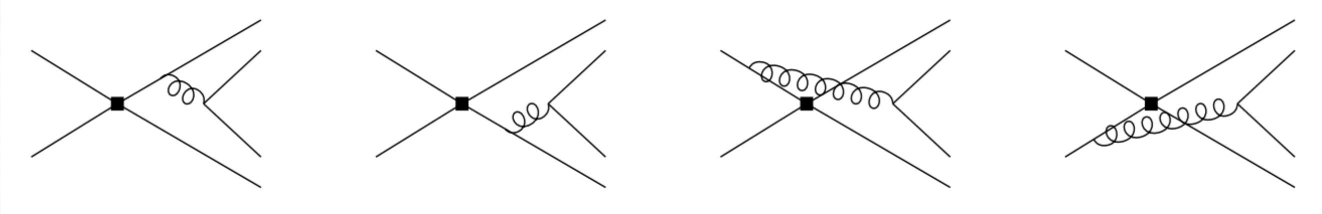
$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

- **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]
  - Non-factorizable spectator interactions;

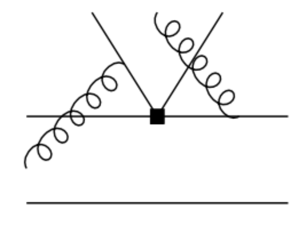


$\frac{\Lambda_{QCD}}{m_b}$

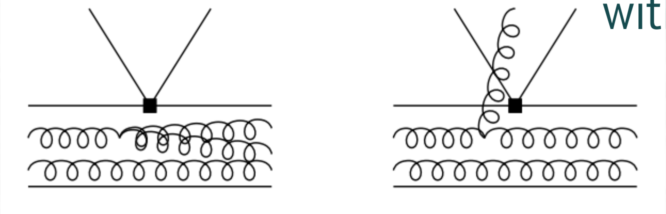
  - Annihilation topologies;



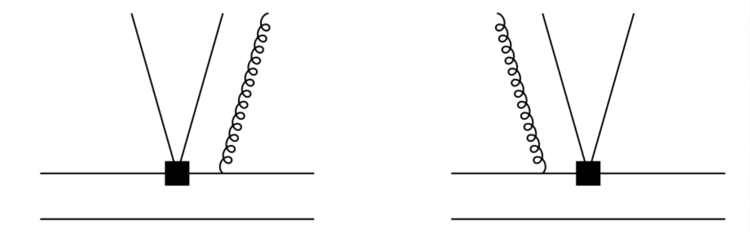
  - Non-leading Fock-state contributions;



included into FFs.



estimated with LCSR



$\left(\frac{\Lambda_{QCD}}{m_b}\right)^2$

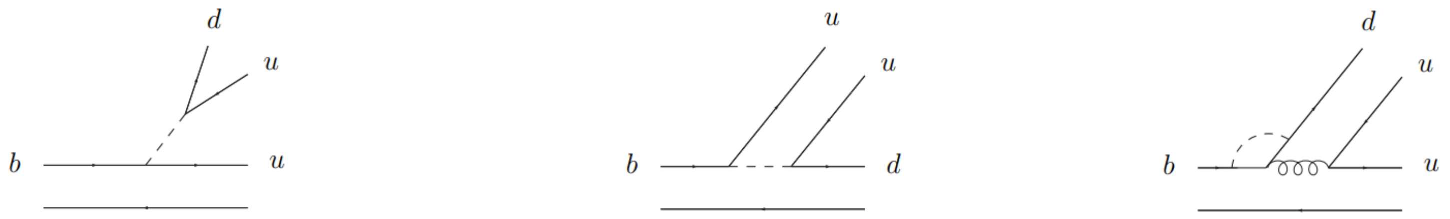
- **Scaling of the leading-power contribution:** [BBNS '01]

$$\mathcal{A}(\bar{B}_d \to D^+ \pi^-) \sim G_F m_b^2 F^{B \to D}(0) f_\pi \sim G_F m_b^2 \Lambda_{QCD}$$

  - $\propto \frac{C_1}{a_1} \simeq -\frac{1}{3}$, all are *ESTIMATED* to be power-suppressed; not *chirality-enhanced* due to (V-A)(V-A) structure
  - Current exp. data could not be easily explained within the SM, at least within the QCDF/SCET framework.

Summary

- NNLO calculation at LP in QCDF complete; ***soft-collinear factorization established!***



colour-allowed tree α_1 colour-suppressed tree α_2 QCD penguins α_4

- Individual contributions sizeable, but cancel with each other; → **NNLO shift small!**

$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO}$$
$$- \left[\frac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.034 + 0.027i]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\}$$

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1} - [0.32 + 0.71i]_{P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{P_2,\,Q_{3-6,8}}$$

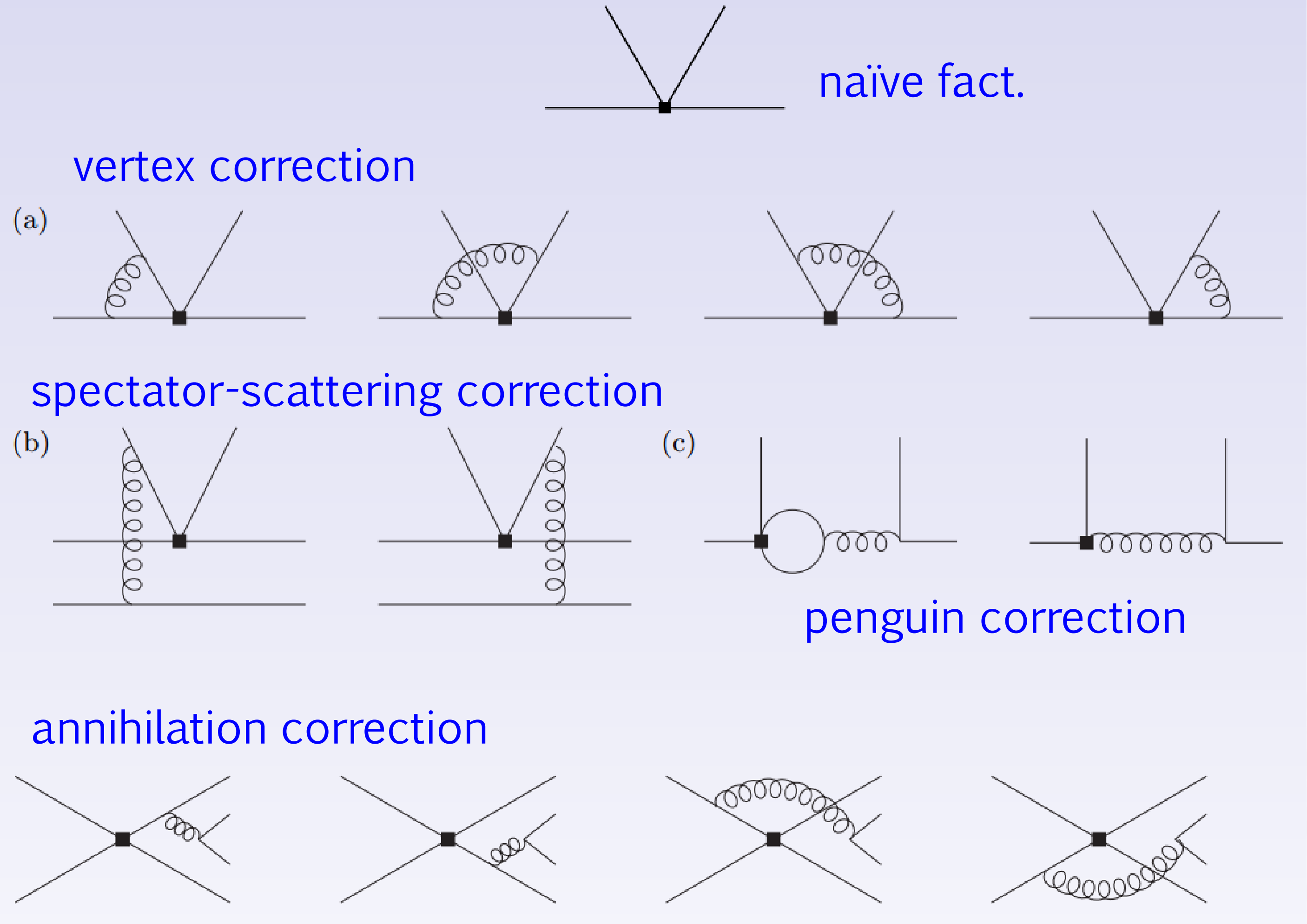
- Confronted with the current data, some puzzles remain; ***how about the NLP corrections?***

Thank You for your attention!

Back-up

Phenomenological analyses based on NLO

◻ **Hard kernels at NLO.**



◻ **complete sets of final states:**

- $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
- $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
- $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
- $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

QCDF: very successful but also with some issues!

Why higher orders in pert. & power corr.s?

- QCDF formulae:

$$\langle M_1 M_2 | Q | \bar{B} \rangle = T^{\rm I}(\mu_{\rm h}) * \phi_\pi(\mu_{\rm h})\, f_+^{B\pi}(0) + \overbrace{H^{\rm II}(\mu_{\rm h}) * U_\parallel(\mu_{\rm h}, \mu_{\rm hc}) * J(\mu_{\rm hc})}^{T^{\rm II}} * \phi_\pi(\mu_{\rm h}) * \phi_\pi(\mu_{\rm hc}) * \phi_{B+}(\mu_{\rm hc})$$

- Factorization of power correction generally broken, due to endpoint divergence; how to?

- How important the higher-order pert. corr.? Fact. theorem is still established for them?

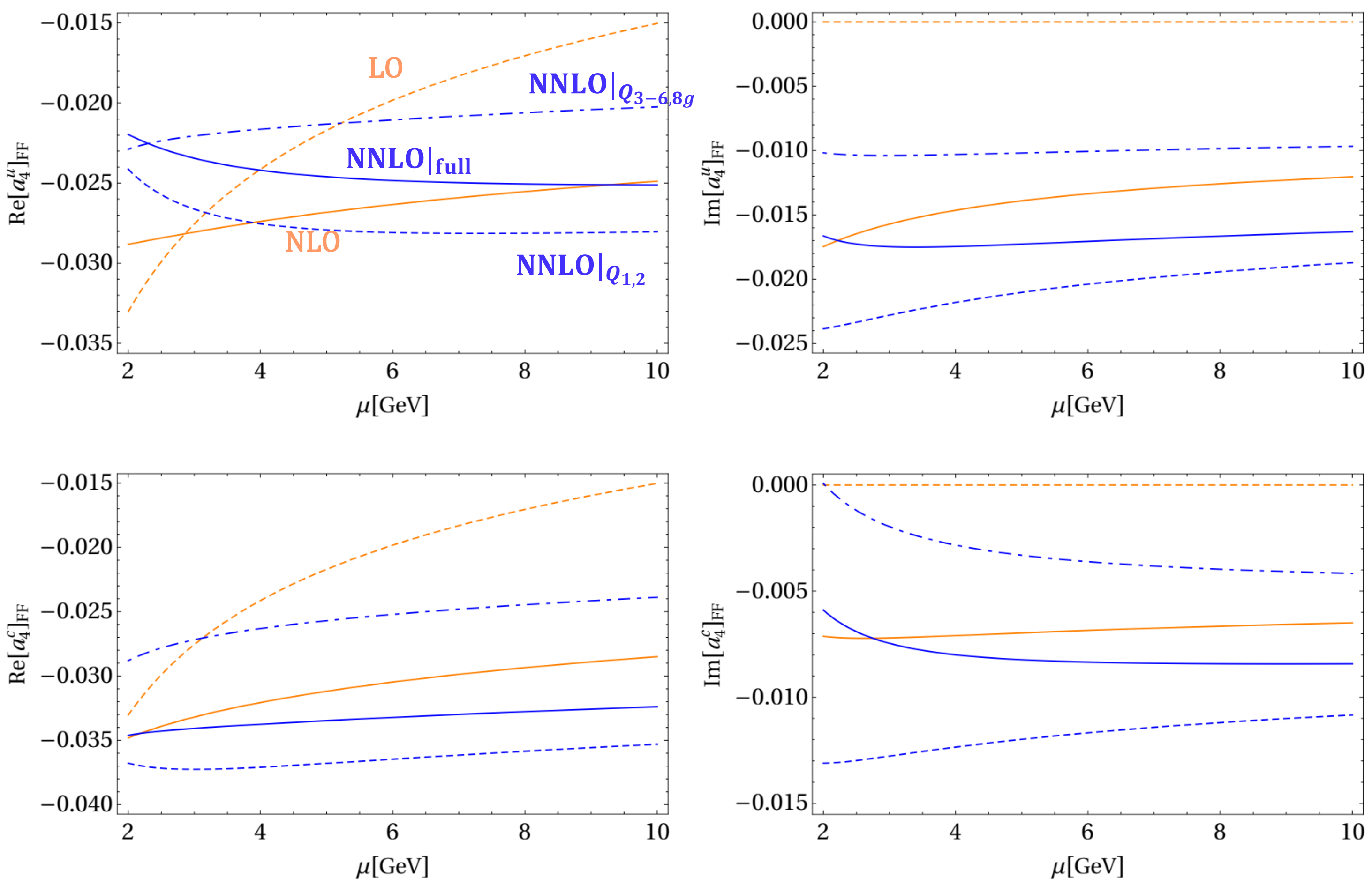
- As strong phase starts at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for A_{CP}?

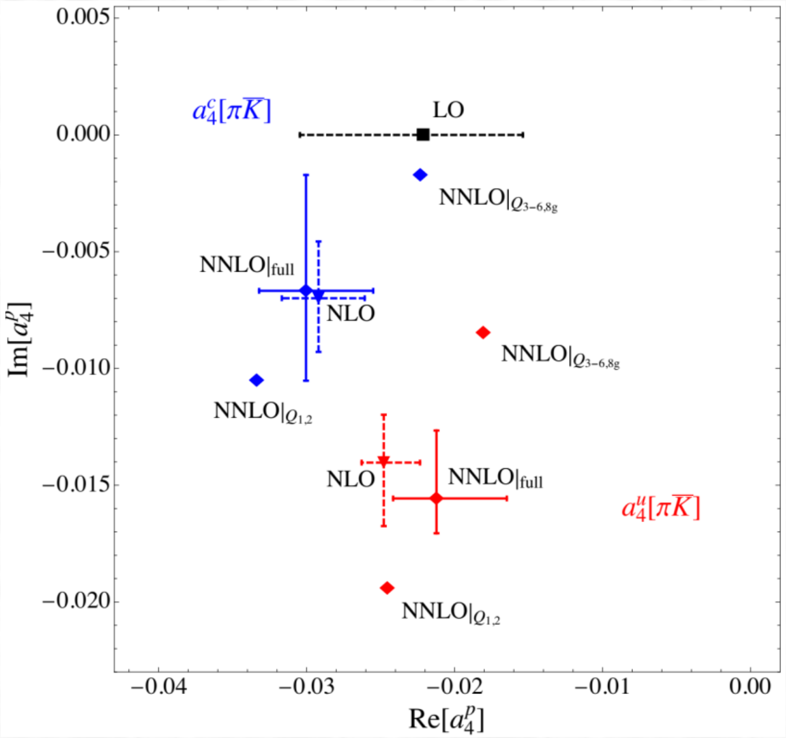
- Data driven: could not account for some data, such as large $Br(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$;

We need go beyond the LO in pert. and power corrections!

Scale dependence of a_4^p

- strong cancellation between $Q_{1,2}^p$ and; $Q_{3-6,8g}$;
- Scale dependence of a_4^p: only form-factor term;





- Theoretical uncertainty is larger at NNLO than at NLO.

- Scale dependence negligible, especially for μ > 4 GeV.