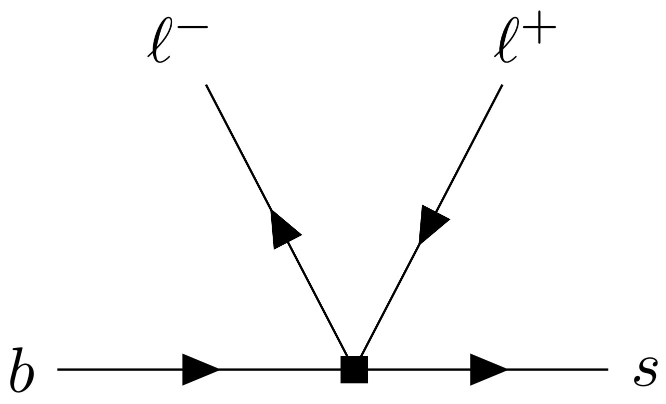
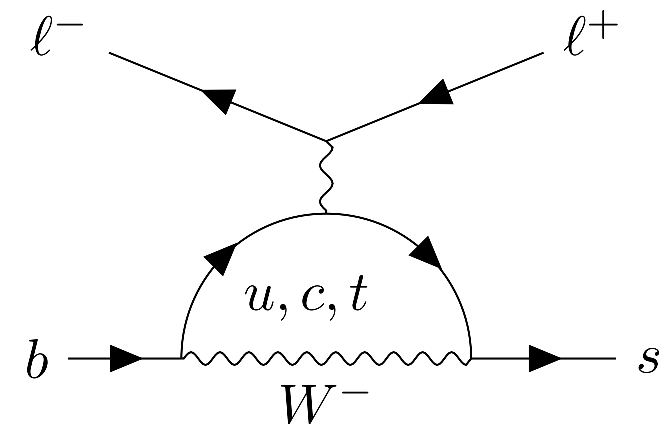




Inclusive $\bar{B} \to X_s \ell^+ \ell^-$ decay

Qin Qin (秦溱)

HUST (华中科技大学)

T. Huber, **QQ**, K. Vos, 1806.11521
T. Huber, T. Hurth, J. Jenkins, E. Lunghi, **QQ**, K. Vos, 1908.07507
T. Huber, T. Hurth, J. Jenkins, E. Lunghi, **QQ**, K. Vos, 2007.04191

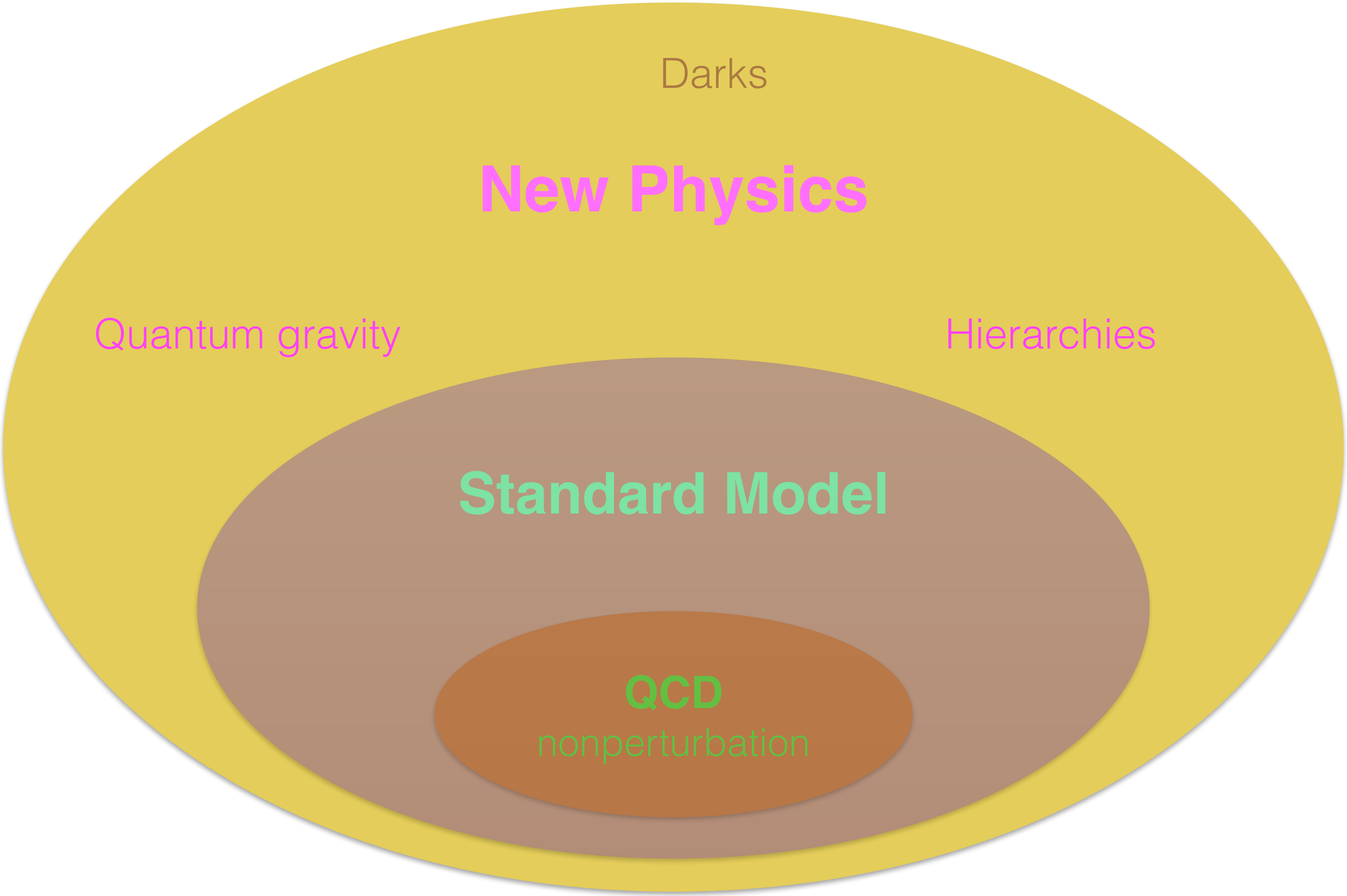
Conference on Flavor Physics and CP Violation (FPCP2021)
Shanghai, 7-11 June 2021

Contents

- Motivation
- Calculation —— within & beyond OPE
- Results —— SM predictions & NP effects

to judge B anomalies!

B decays are important!



Categories of B decays

leptonic, radiative or hadronic; tree or penguin; …

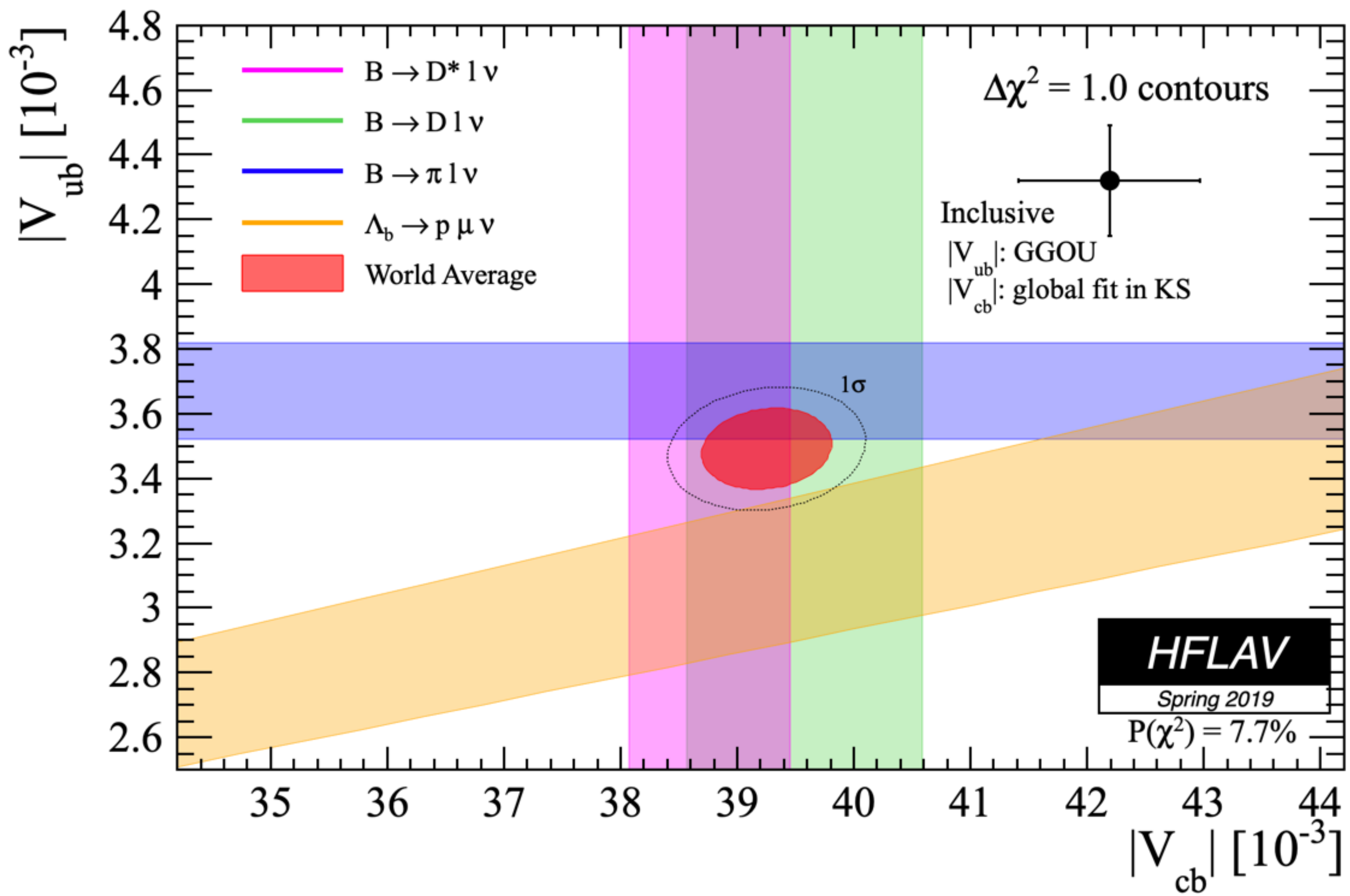
Inclusive	Exclusive
Cleaner theory ✓	
	Easier experiments ✓
	More channels ✓

Inclusive and exclusive B decays

- Play an important role by themselves
- **Complementary** to each other

Inclusive vs Exclusive

—— an example: inclusively and exclusively extracted **Vub & Vcb**



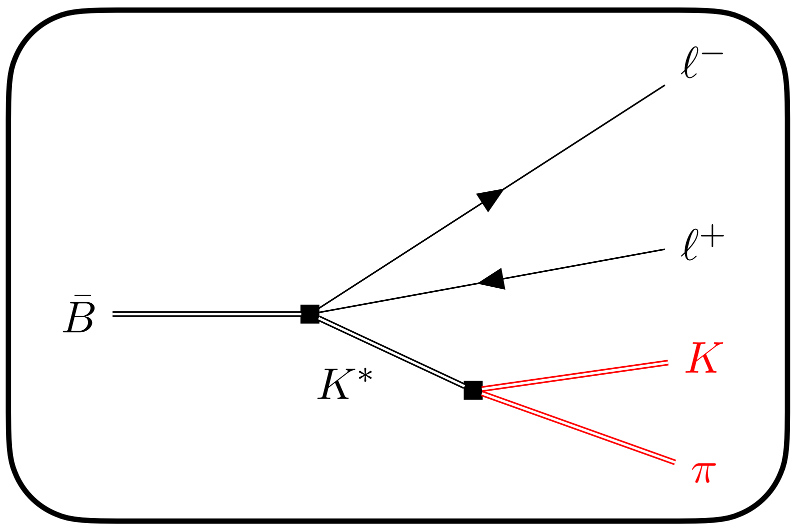
see also talk by De Cian et al

[HFLAV,1909.12524]

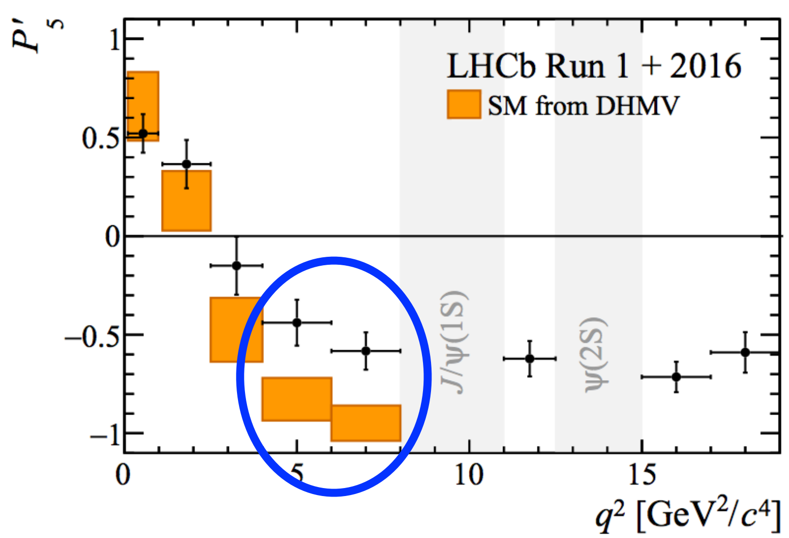
Inclusive vs Exclusive

—— $b \to s\ell\ell$ anomalies

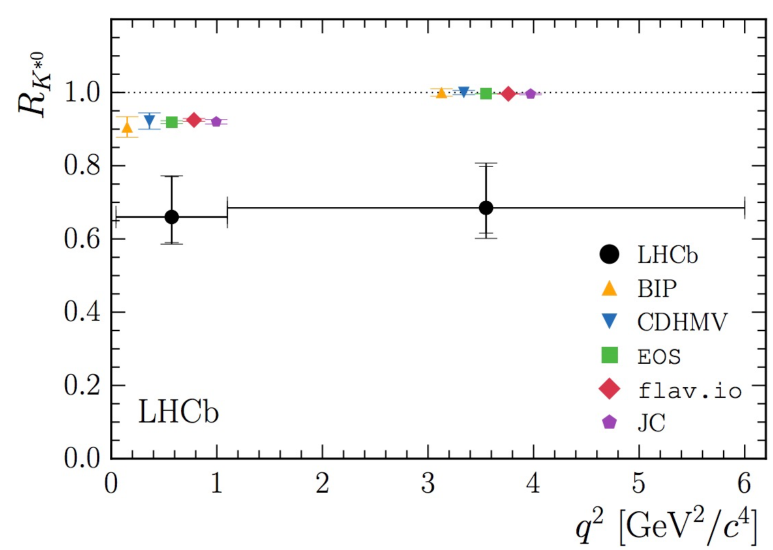
see also talks by Watanukl, Mannel, Materok et al



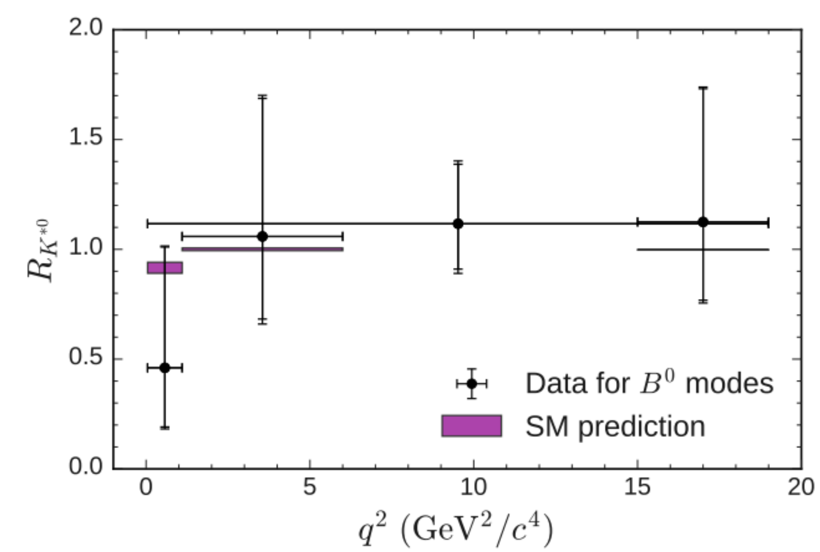
- P_5': an angular-distribution observable
- $R_{K^*} \equiv \dfrac{B(\bar{B} \to K^*\mu\mu)}{B(\bar{B} \to K^*ee)}$



[LHCb, 2003.04831]



[LHCb, 1705.05802]



[Belle, 1612.05014]

Inclusive vs Exclusive

—— $b \to s\ell\ell$ anomalies

Question: why don't we study the correponding inclusive channel $\bar{B} \to X_s \ell^+ \ell^-$ as a cross check?

Answer: lack of data for a precision study.

Good news ——Belle II

Inclusive vs Exclusive

—— $b \to s\ell\ell$ anomalies

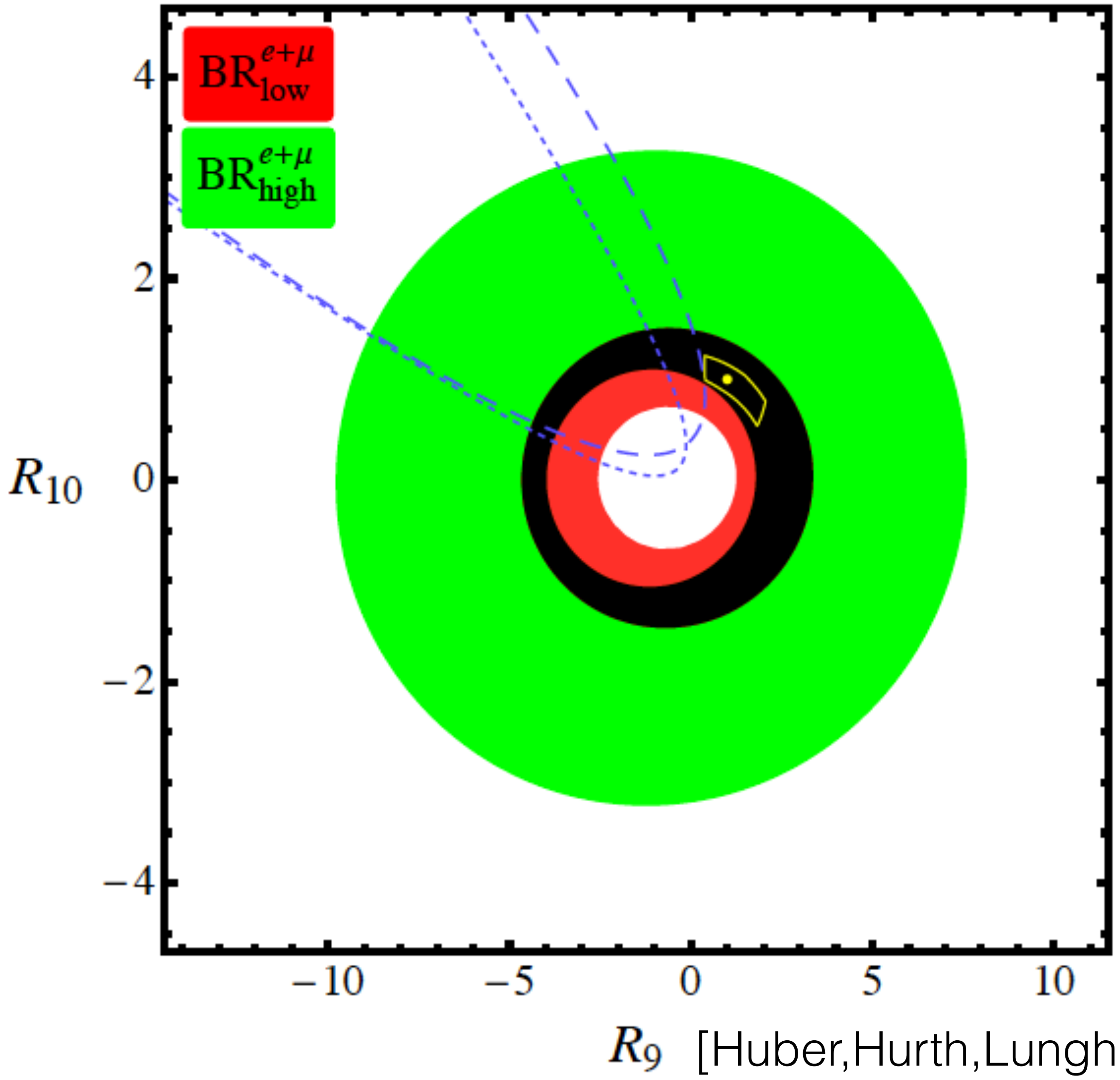
Belle II precision for $\bar{B} \to X_s \ell\ell$ (expectation):

Observables	Belle 0.71 ab^{-1}	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
$\mathrm{Br}(B \to X_s \ell^+\ell^-)$ ($[1.0, 3.5]$ GeV^2)	29%	13%	6.6%
$\mathrm{Br}(B \to X_s \ell^+\ell^-)$ ($[3.5, 6.0]$ GeV^2)	24%	11%	6.4%
$\mathrm{Br}(B \to X_s \ell^+\ell^-)$ (> 14.4 GeV^2)	23%	10%	4.7%
$A_{\mathrm{CP}}(B \to X_s \ell^+\ell^-)$ ($[1.0, 3.5]$ GeV^2)	26%	9.7 %	3.1 %
$A_{\mathrm{CP}}(B \to X_s \ell^+\ell^-)$ ($[3.5, 6.0]$ GeV^2)	21%	7.9 %	2.6 %
$A_{\mathrm{CP}}(B \to X_s \ell^+\ell^-)$ (> 14.4 GeV^2)	21%	8.1 %	2.6 %
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ ($[1.0, 3.5]$ GeV^2)	26%	9.7%	3.1%
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ ($[3.5, 6.0]$ GeV^2)	21%	7.9%	2.6%
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ (> 14.4 GeV^2)	19%	7.3%	2.4%
$\Delta_{\mathrm{CP}}(A_{\mathrm{FB}})$ ($[1.0, 3.5]$ GeV^2)	52%	19%	6.1%
$\Delta_{\mathrm{CP}}(A_{\mathrm{FB}})$ ($[3.5, 6.0]$ GeV^2)	42%	16%	5.2%
$\Delta_{\mathrm{CP}}(A_{\mathrm{FB}})$ (> 14.4 GeV^2)	38%	15%	4.8%

Better by one order!

[“The Belle II Physics Book”, by Huber, Ishikawa and Virto]

Inclusive vs Exclusive —— $b \to s\ell\ell$ anomalies



[Huber,Hurth,Lunghi,'15]

B factories (red, green)

⟶ **Belle II (yellow)**

To exclusive anomalies, the **inclusive** channel serves as

- **a strong cross check** (different theoretical framework)
- **a solid cross check** (systematic OPE)
- **a practical cross check** (Belle II)

Theoretical calculation

- Within OPE
  - Radiative corrections
  - Power corrections
- Beyond OPE (Long-distance)
  - Factorizable
  - Non-factorizable

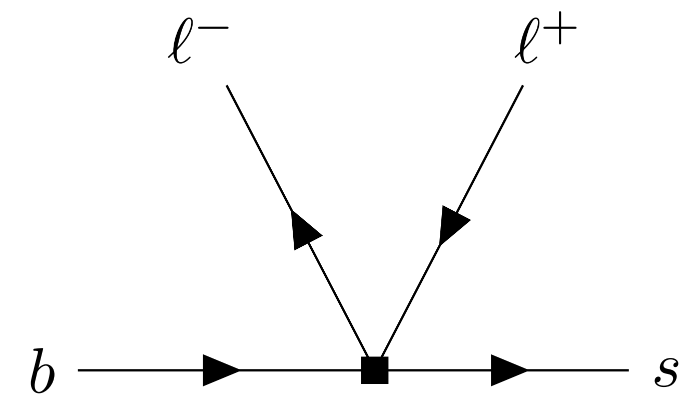
Theoretical calculation —— within OPE

Operator product expansion: [Falk,Luke,Savage,'93]

$$\sum_{X_s} \langle \bar{B} | \mathcal{J}_q^\dagger | X_s \rangle \langle X_s | \mathcal{J}_q | \bar{B} \rangle$$

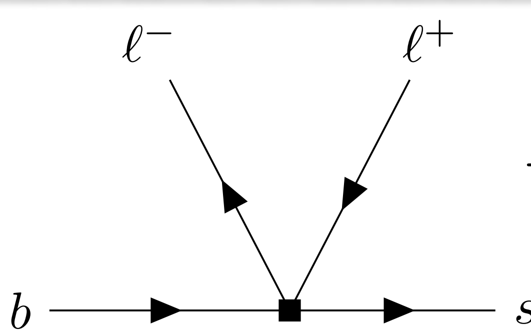
$$\propto \mathrm{Im} \int d^4 e^{-iq\cdot x} \langle \bar{B} | T\{\mathcal{J}_q^\dagger(x) \mathcal{J}_q(0)\} | \bar{B} \rangle$$

(OPE in HQET) $= \mathrm{Im}\ \langle \bar{B} | C_0 \mathcal{O}_0 + \frac{C_1}{m_b}\mathcal{O}_1 + \frac{C_2}{m_b^2}\mathcal{O}_2 + \cdots | \bar{B} \rangle$



$O_{9,10}$

leading power:
= quark-level process



$+\alpha_{s,e}$ corrections

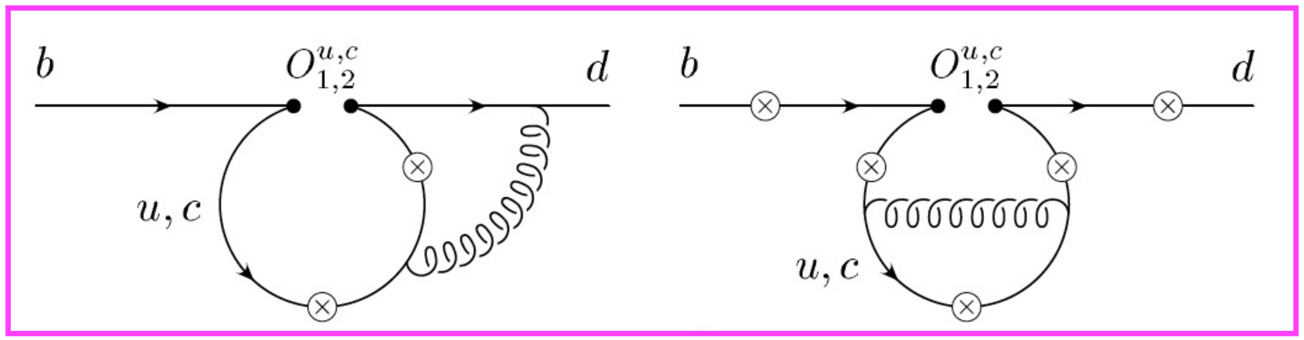
$1/m_b$ power corrections:

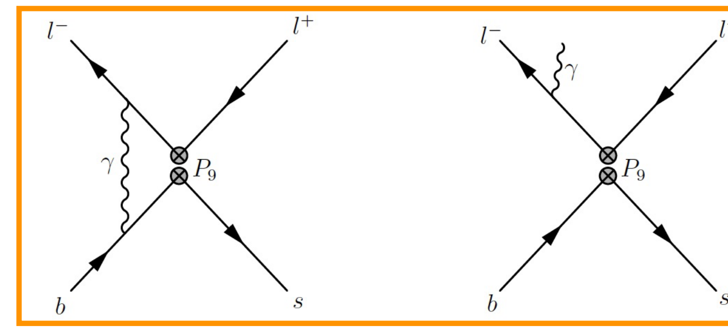
- coefficients **Ci** obtained by **matching**
- **matrix elements** extracted from **data**

Theoretical calculation —— within OPE

Perturbative $\alpha_{s,e}$ corrections include

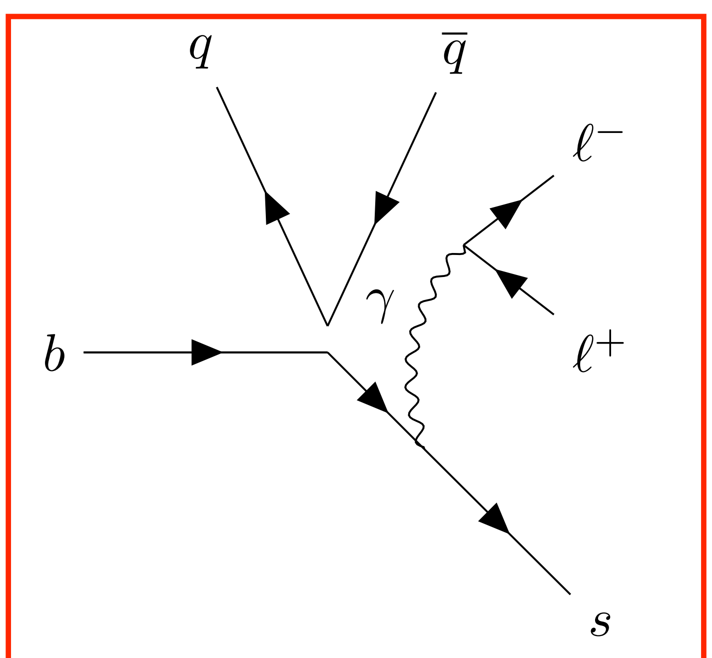
- NNLO QCD corrections [Misiak et al,'92,'99;Greub et al,'01,'02,'03,'04;Seidel,'04]
- log-enhanced NLO QED corrections [Huber et al,'05,'07]





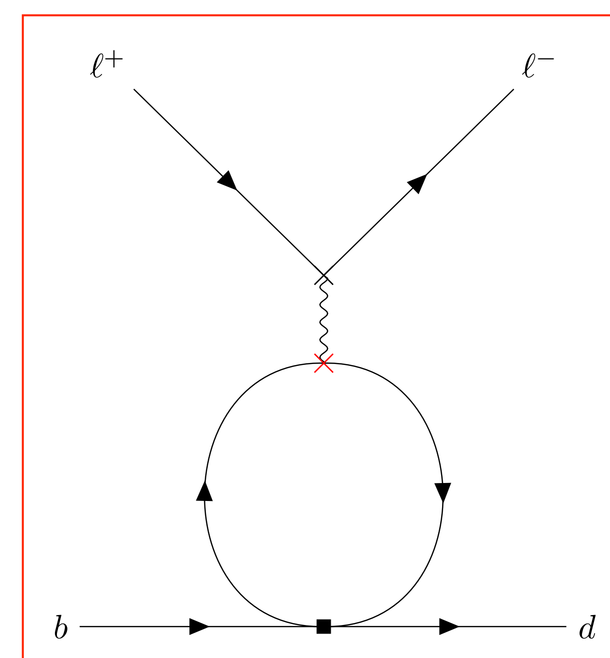
- **multi-particle contributions** as $b \to s\ell^+\ell^-\bar{q}q$

[T. Huber, **QQ**, K. Vos, 1806.11521]



Contribute from LO!

vesus



Theoretical calculation —— within OPE

Power corrections include [Ali,Hiller,Handoko,Morozumi,'97;Ligeti,Tackmann,'07]

$$\Gamma \propto C_0 + C_\pi \frac{\mu_\pi^2}{m_b^2} + C_G \frac{\mu_G^2}{m_b^2} + C_D \frac{\rho_D^3}{m_b^3} + C_{LS} \frac{\rho_{LS}^3}{m_b^3} + \sum_q C_q \frac{f_q}{m_b^3} + \ldots$$

$\mathcal{O}(1/m_b^2)$ $\mathcal{O}(1/m_b^3)$

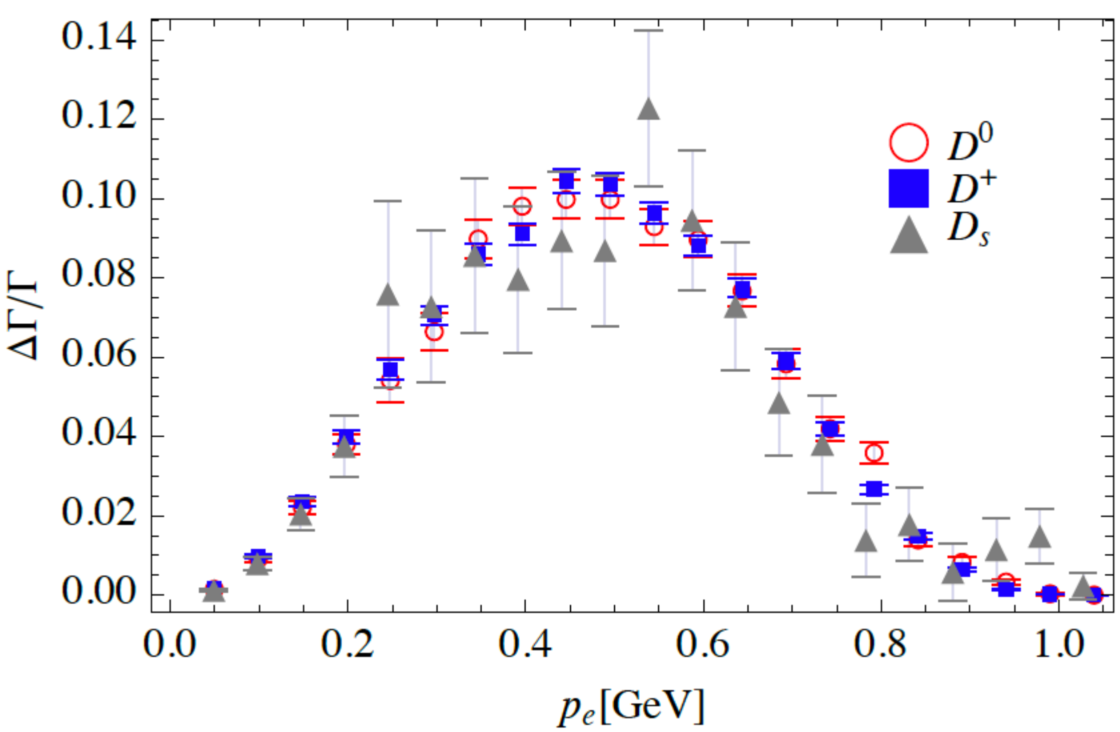
$$m_H = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_H}$$

$$\mu_\pi^2 \equiv \frac{1}{2m_H} \langle H_Q | \bar{Q}(iD)^2 Q | H_Q \rangle$$

$$\mu_G^2 \equiv \frac{1}{2m_H} \langle H_Q | \bar{Q} \frac{1}{2} \sigma \cdot G | H_Q \rangle$$

$$\ldots\ldots$$

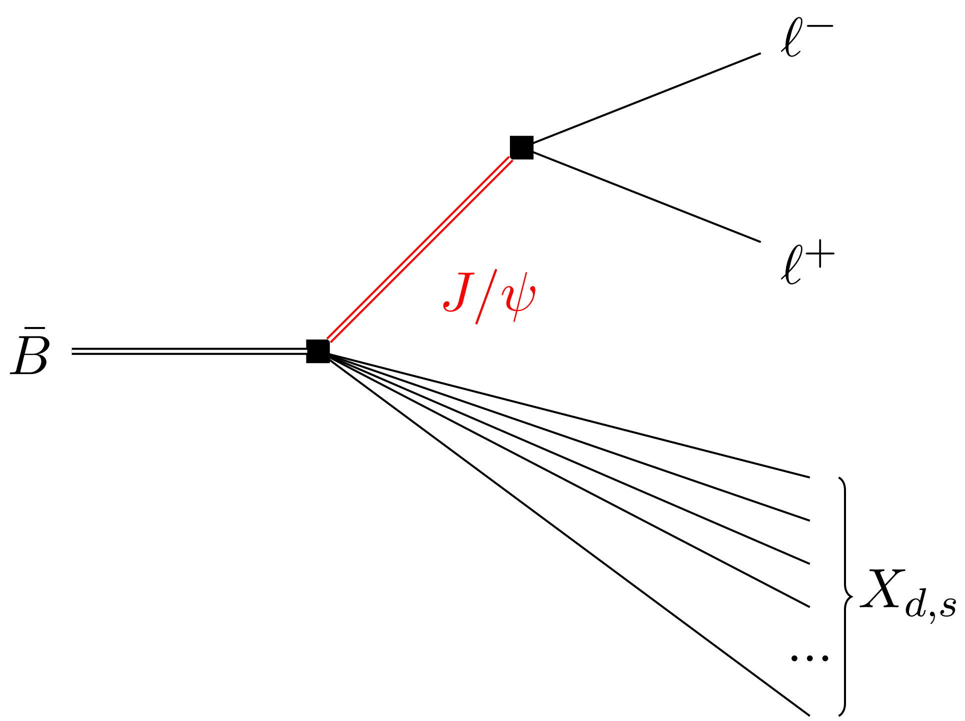
Heavy meson decay data:

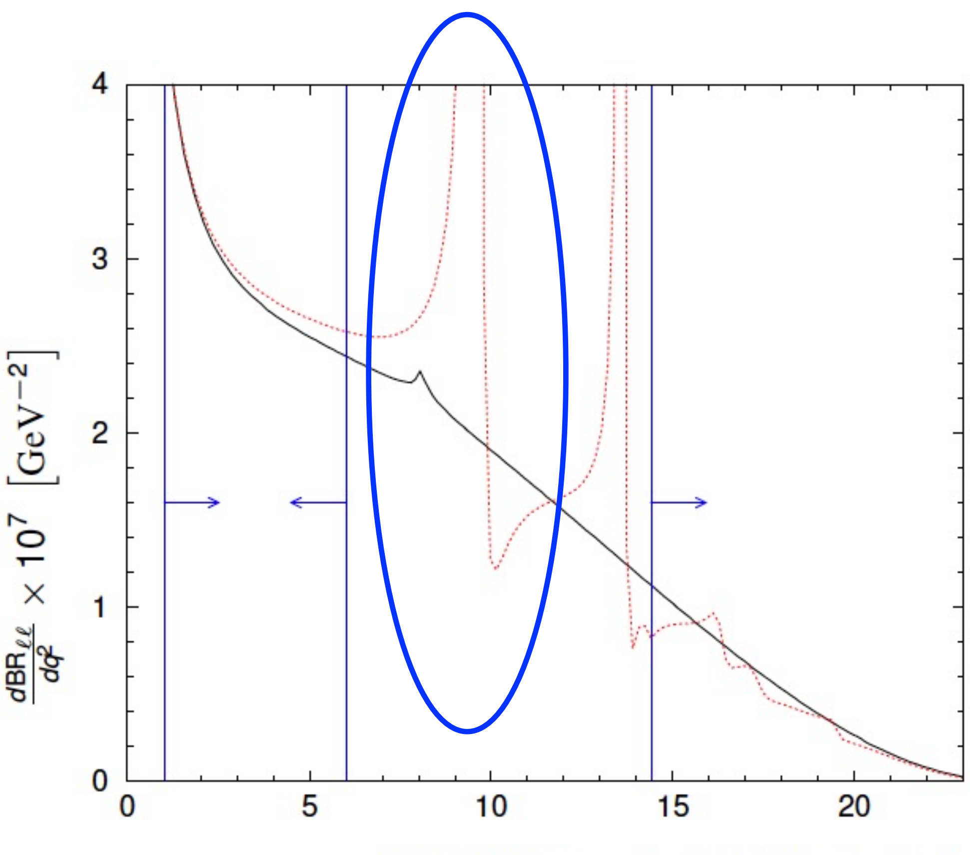


[CLEO,'09;Gambino,Kamenik,'10]

Theoretical calculation —— beyond OPE

Long-distance contributions induced by charmonium resonances and others





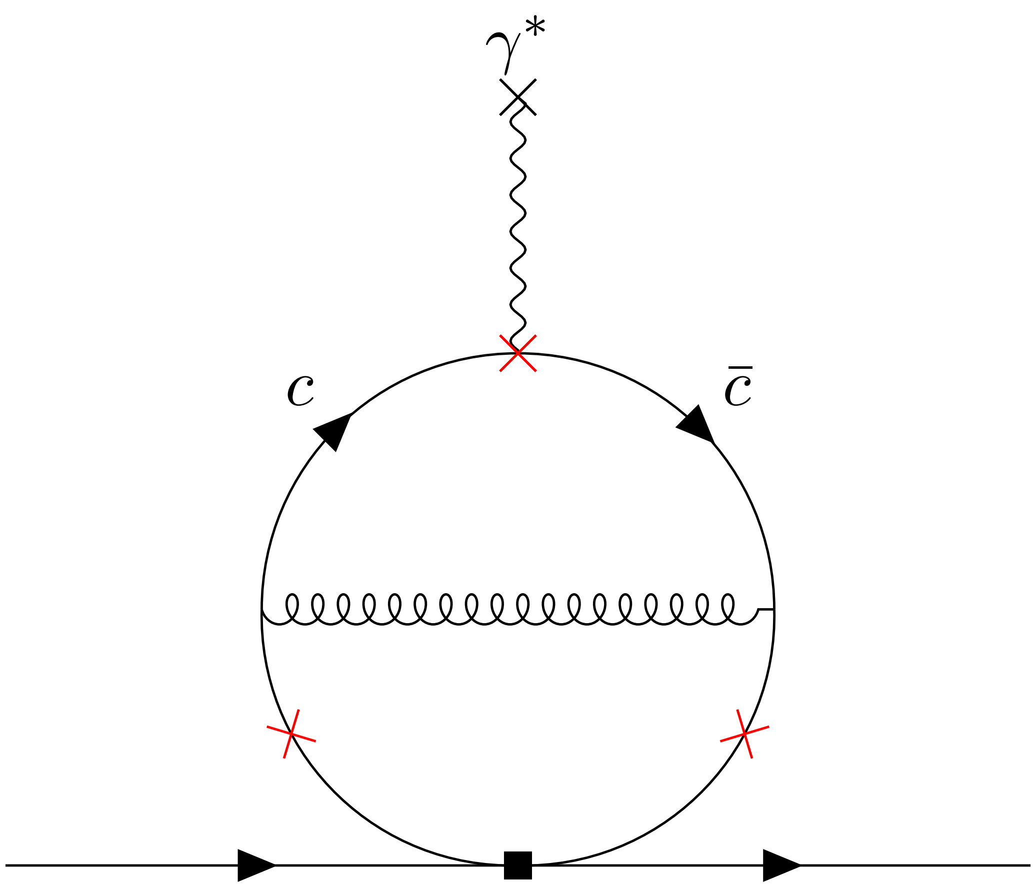
[Ghinculov,Hurth,Isidori,Yao]

Cut off the resonance region?

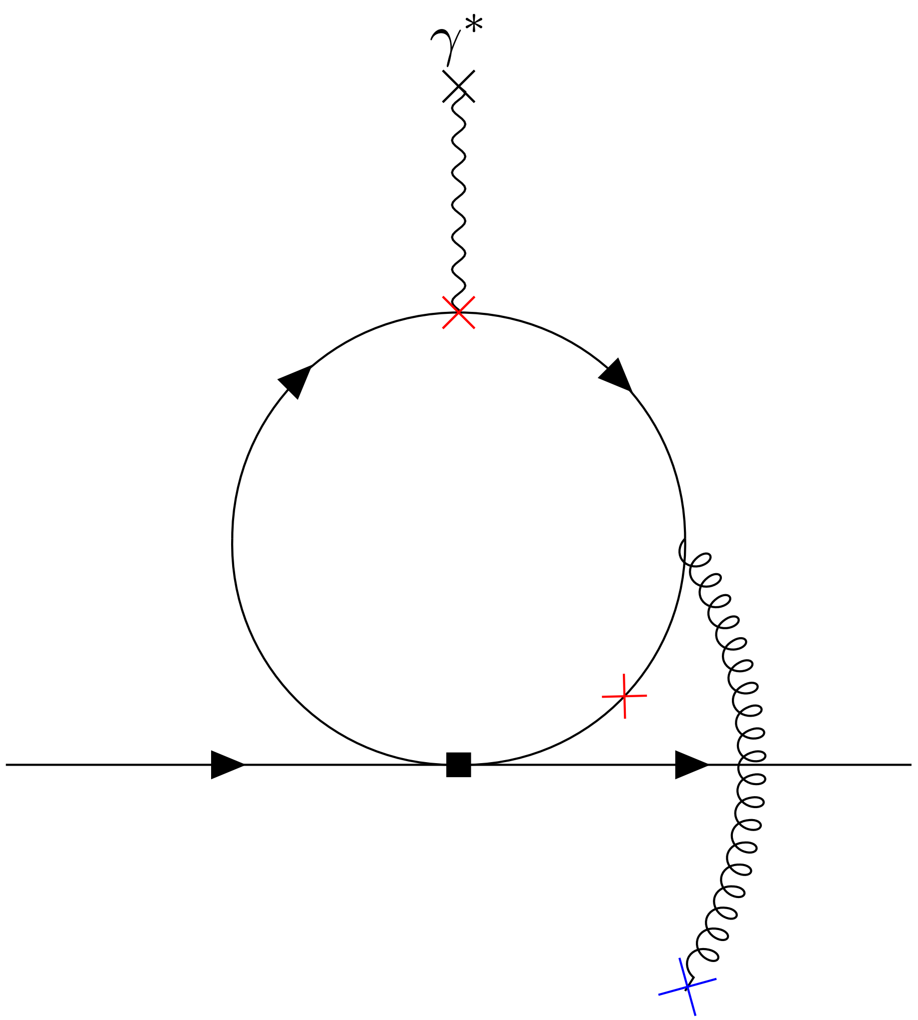
The tail effects are still considerable!

Theoretical calculation —— beyond OPE

Two kinds of long-distance contributions:



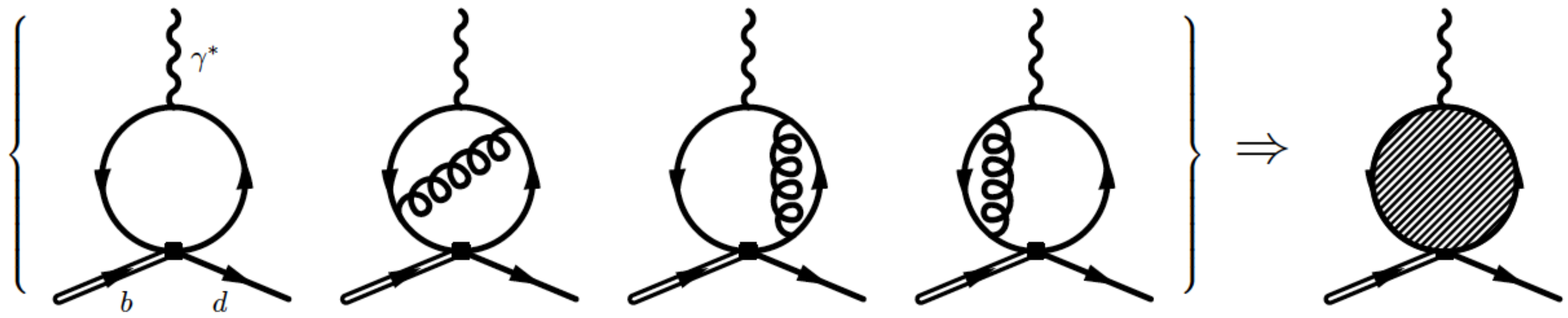
Factorizable
(color singlet)



Nonfactorizable
(color octet)

Long-distance effects (factorizable)

[T. Huber, T. Hurth, J. Jenkins, E. Lunghi, **QQ**, K. Vos,'19,'20]



- The current-current-operator matrix element is in general written as

$$\boxed{(q^\mu q^\nu - q^2 g^{\mu\nu}) h_q^{\mathrm{KS}}(q^2) = \frac{16\pi^2}{9Q_q}\, i \int d^4x\, e^{iqx} \langle 0|T J_q^\mu(0) J_{\mathrm{em}}^\nu(x)|0\rangle}$$

the Kruger-Sehgal function

[Kruger,Sehgal,96']

- Extract it from data —— **the R-ratio** & Tau decays.

[see Keshavarzi,Nomura,Teubner,'18]

Long-distance effects (factorizable)

[T. Huber, T. Hurth, J. Jenkins, E. Lunghi, **QQ**, K. Vos,'19,'20]

The R-ratio give the <u>imaginary part</u>

$$R_{\rm had}(s) = \frac{3s}{4\pi\alpha^2}\sigma_{\rm had}(s) = 12\pi {\rm Im}[\Pi_\gamma(s)] \approx \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^=e^- \to \mu^+\mu^-)}$$

$$\Pi_\gamma(q^2) \propto h_q^{KS}(q^2)$$

The real part is obtained by **<u>dispersion relation</u>**

$${\rm Re}[h_q^{\rm KS}(s)] = {\rm Re}[h_q^{\rm KS}(s_0)] + \frac{s-s_0}{\pi}\int_0^\infty dt\, \frac{{\rm Im}[h_q^{\rm KS}(t+i\epsilon)]}{(t-s_0)(t-s-i\epsilon)}.$$

perturbative input, $s_0 = -(5\ \text{GeV})^2$

two-loop match [de Boer,'17]

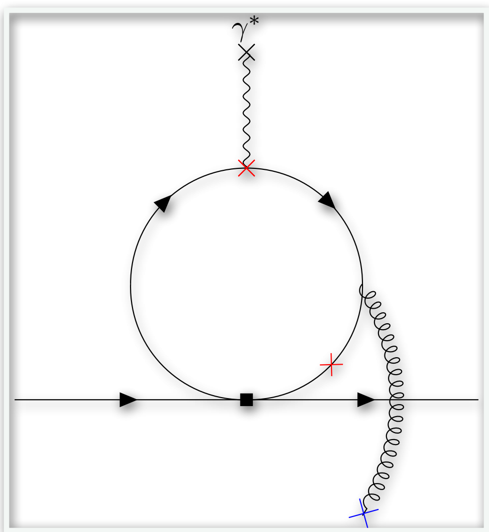
Phenomenologically, the decay rates are modified by

- ~ **5%** in the low-q^2 region
- ~ **15%** in the high-q^2 region

Long-distance effects (non-factorizable)

So-called resolved-photon contributions, operator matching

[See Voloshin,’96;Buchalla,Isidori,Rey,’97;Benzke,Hurth,Turczyk,’17]



$$\boxed{\frac{(l+q)_\alpha}{l\cdot q}\left[F^{\mu\alpha}\tilde{G}_{\mu\beta}+G^{\mu\alpha}\tilde{F}_{\mu\beta}\right]\bar{s}\Gamma_1\gamma^\beta\gamma_5\Gamma_2 b}$$

a **non-local** operator
(gluon momentum in the denominator)

Calculation of nonlocal operator matrix elements

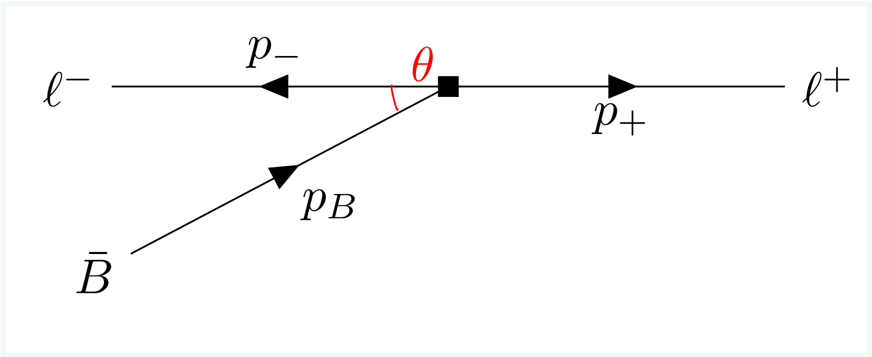
- seriously dependent on modeled shape functions of B meson
- phenomenologically, **~5%** uncertainty is assumed

Observables

$$\boxed{q^2 \equiv (p_{\ell^+} + p_{\ell^-})^2}$$

We consider the observables in two kinetic regions $q^2 \in [1,6]$ GeV^2 and $q^2 > 14.4$ GeV^2, to reduce the resonance effects, including

- The branching ratios in the low- and high-q^2 regions
- The angular-distribution observables in the low-q^2 region



$$\boxed{H_I^{\ell\ell}(q^2) = \int_{-1}^{+1} dz \, \frac{d^2\Gamma(\bar{B} \to X_s \ell\ell)}{dq^2 dz} W_I(z) \,,}$$

- The ratio between $\bar{B} \to X_s \ell^+ \ell^-$ and $B^0 \to X_u \ell \nu$ the high-q^2 region to reduce power-correction uncertainties

[Ligeti,Tackmann,'07]

$$\boxed{\mathcal{R}(s_0) = \int_{\hat{s}_0}^{1} \mathrm{d}\hat{s} \, \frac{\mathrm{d}\Gamma(\bar{B} \to X_d \ell^+ \ell^-)}{\mathrm{d}\hat{s}} \Big/ \int_{\hat{s}_0}^{1} \mathrm{d}\hat{s} \, \frac{\mathrm{d}\Gamma(B^0 \to X_u \ell \nu)}{\mathrm{d}\hat{s}}}$$

SM predictions

$$10^6 \cdot B[1,6]_{ee} = 1.78 \pm 0.13 = 1.78 \pm 0.08_{\text{scale}} \pm 0.09_{\text{resolved}} \pm \ldots$$

$$10^6 \cdot B[1,6]_{\mu\mu} = 1.73 \pm 0.13 = 1.78 \pm 0.08_{\text{scale}} \pm 0.09_{\text{resolved}} \pm \ldots$$

exp: $\boxed{\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)^{\text{exp}}_{\text{low}} = (1.58 \pm 0.37) \times 10^{-6}}$

$$10^7 \cdot B[>14.4]_{\mu\mu} = 2.38 \pm 0.87 = 2.38 \pm 0.27_{\text{scale}} \pm 0.79_{\text{power}} \pm \ldots$$

Lepton-universality observables

$$\boxed{R_{X_s} \equiv \frac{\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-)}{\mathcal{B}(\bar{B} \to X_s e^+ e^-)}}$$

$$R_{X_s}[1, 3.5] = 0.961 \pm 0.004 \ ,$$

$$R_{X_s}[3.5, 6] = 0.984 \pm 0.002 \ ,$$

$$R_{X_s}[1, 6] = 0.971 \pm 0.003 \ ,$$

$$R_{X_s}[> 14.4] = 1.17 \pm 0.08 \ .$$

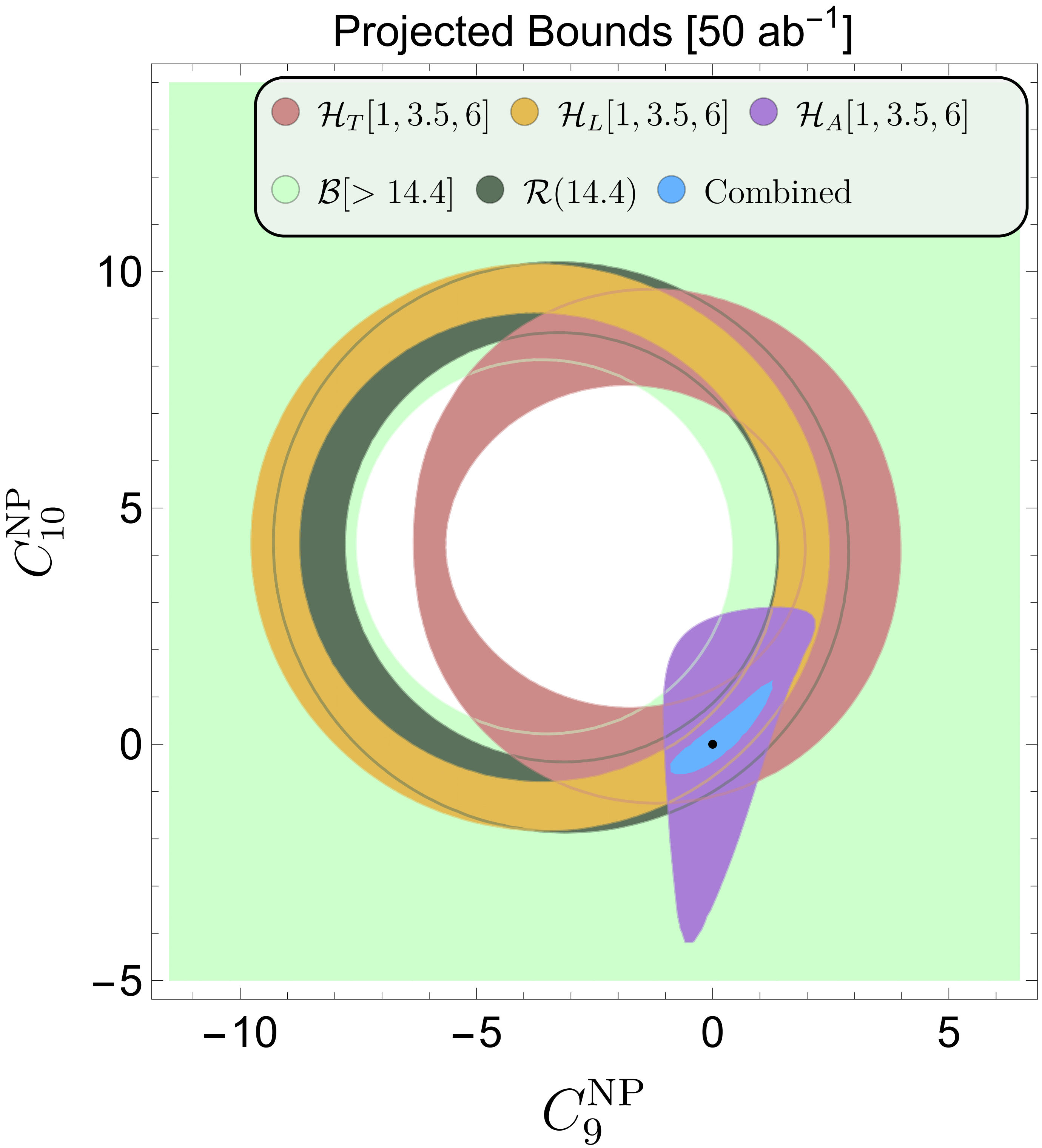
to be tested at Belle II!

New-physics analysis

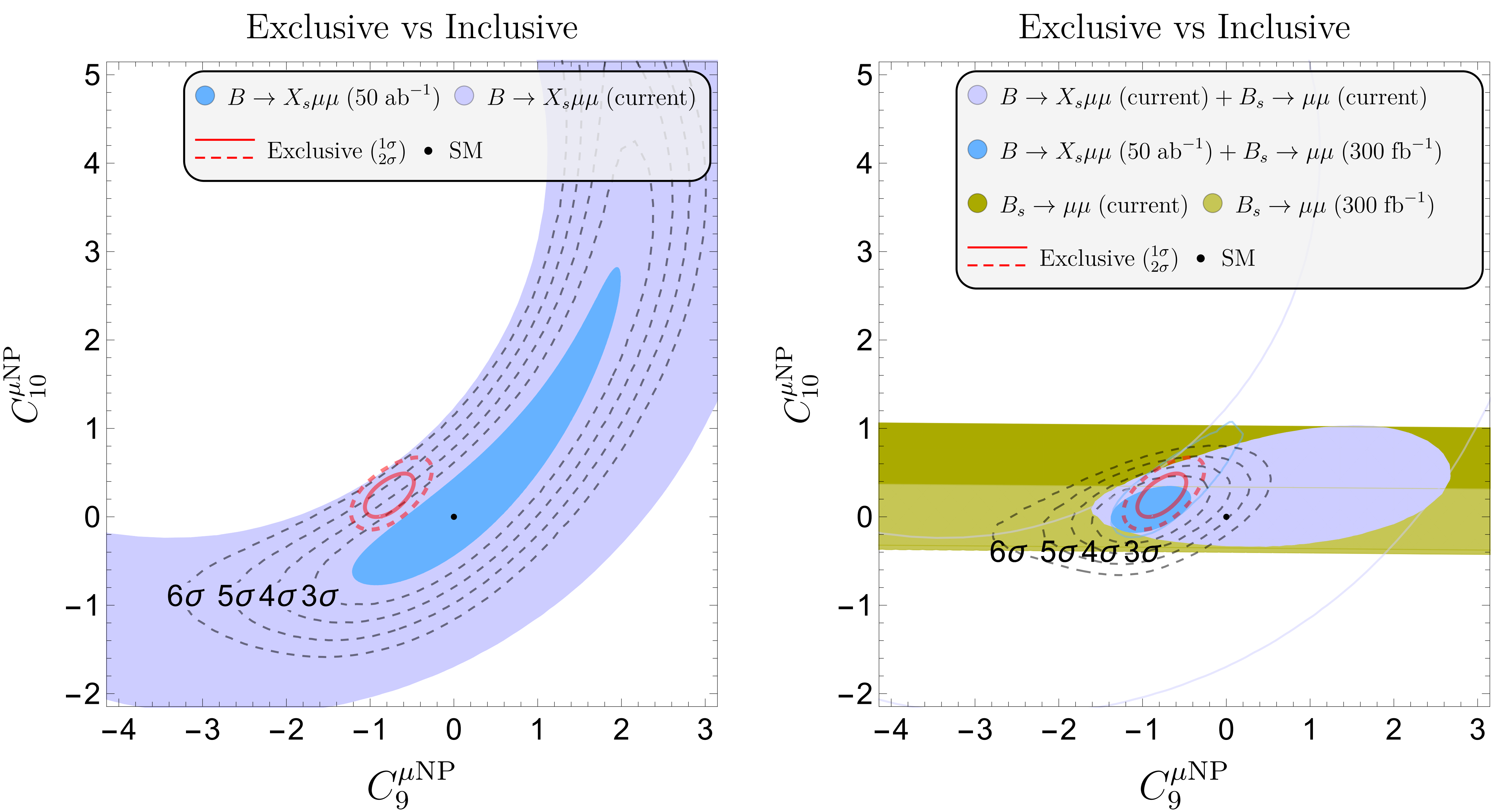
$$
\begin{aligned}
\mathcal{B}[1,6]_{ee} = \Big[& 0.288327\,|R_7|^{\,2} + 0.00384785\,|R_8|^{\,2} + 1.56513\,|R_9|^{\,2} \\
& + 11.3588\,|R_{10}|^{\,2} + 0.0232071\mathcal{I}\left(R_7 R_8^*\right) + 0.00507683\mathcal{I}\left(R_7 R_9^*\right) \\
& + 0.0266804\mathcal{I}\left(R_8 R_9^*\right) - 0.00054448\mathcal{I}\left(R_8 R_{10}^*\right) + 0.068984\mathcal{R}\left(R_7 R_8^*\right) \\
& - 0.89029\mathcal{R}\left(R_7 R_9^*\right) + 0.015688\mathcal{R}\left(R_7 R_{10}^*\right) - 0.101342\mathcal{R}\left(R_8 R_9^*\right) \\
& + 0.00188925\mathcal{R}\left(R_8 R_{10}^*\right) - 0.109829\mathcal{R}\left(R_9 R_{10}^*\right) + 0.0303878\mathcal{I}\left(R_7\right) \\
& + 0.00630353\mathcal{I}\left(R_8\right) + 0.047988\mathcal{I}\left(R_9\right) - 0.00298649\mathcal{I}\left(R_{10}\right) \\
& - 0.245038\mathcal{R}\left(R_7\right) - 0.0336275\mathcal{R}\left(R_8\right) + 3.44232\mathcal{R}\left(R_9\right) \\
& - 0.571287\mathcal{R}\left(R_{10}\right) + 3.00949 \Big] \times 10^{-7},
\end{aligned}
$$

$$
\boxed{R_{7,8} = \frac{C_{7,8}^{(00)\mathrm{eff}}(\mu_0)}{C_{7,8}^{(00)\mathrm{eff,SM}}(\mu_0)} \qquad \text{and} \qquad R_{9,10} = \frac{C_{9,10}^{(11)}(\mu_0)}{C_{9,10}^{(11)\mathrm{SM}}(\mu_0)}}
$$

New-physics analysis



New-physics analysis



Judgement on B anomalies at 5σ level

Summary

- We have systematically studied the inclusive $\bar{B} \to X_s \ell \ell$ decay, to the state of the art

- We make predictions for its br's, angular observables and observables to test lepton flavor universality

- Our results, together with the Belle II data, will be able to judge the anomalies from exclusive $b \to s \ell \ell$ decays

Thank you!

Backup

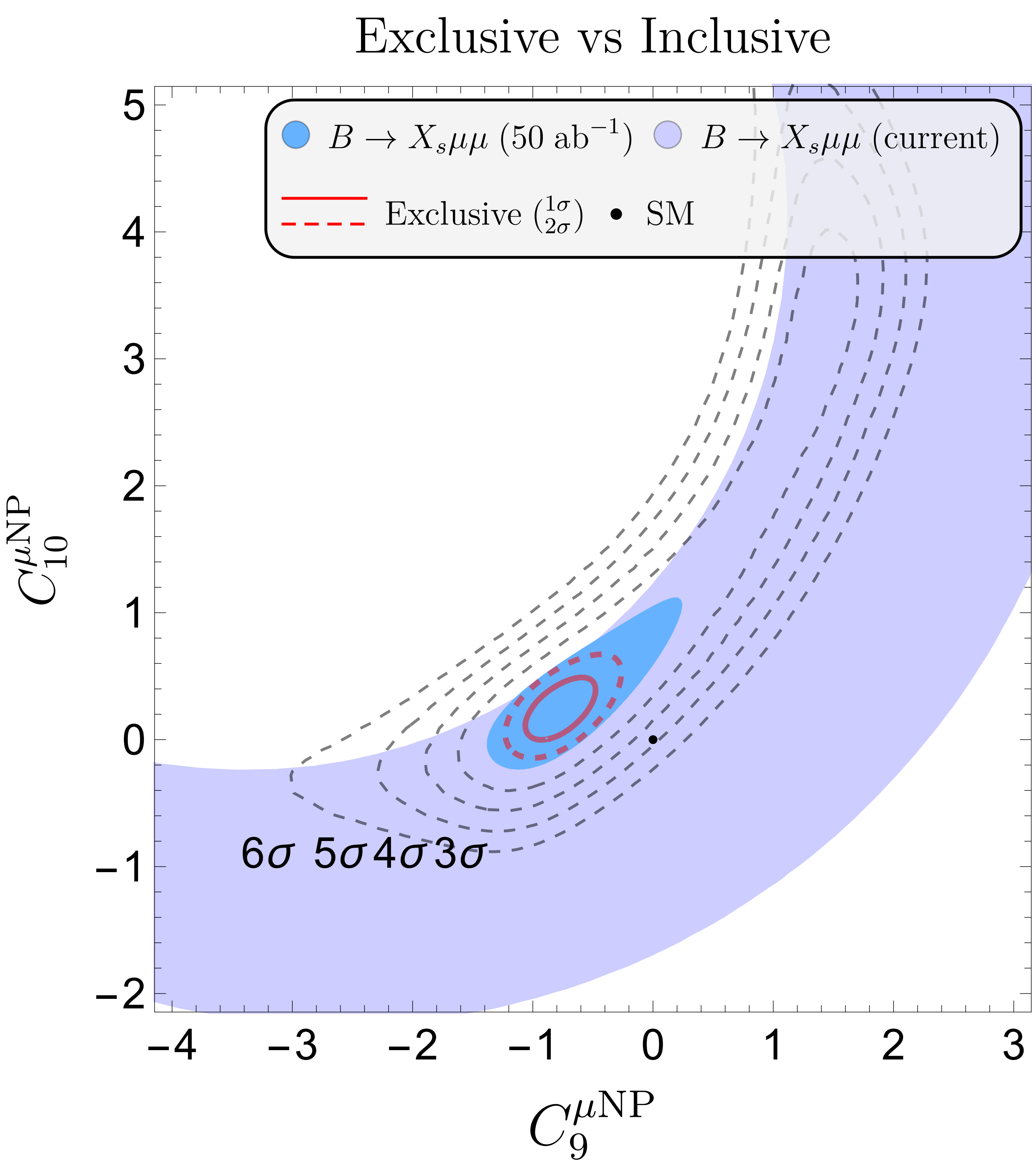
★ New physics analysis for $\boxed{\bar{B} \to X_s \ell^+ \ell^-}$

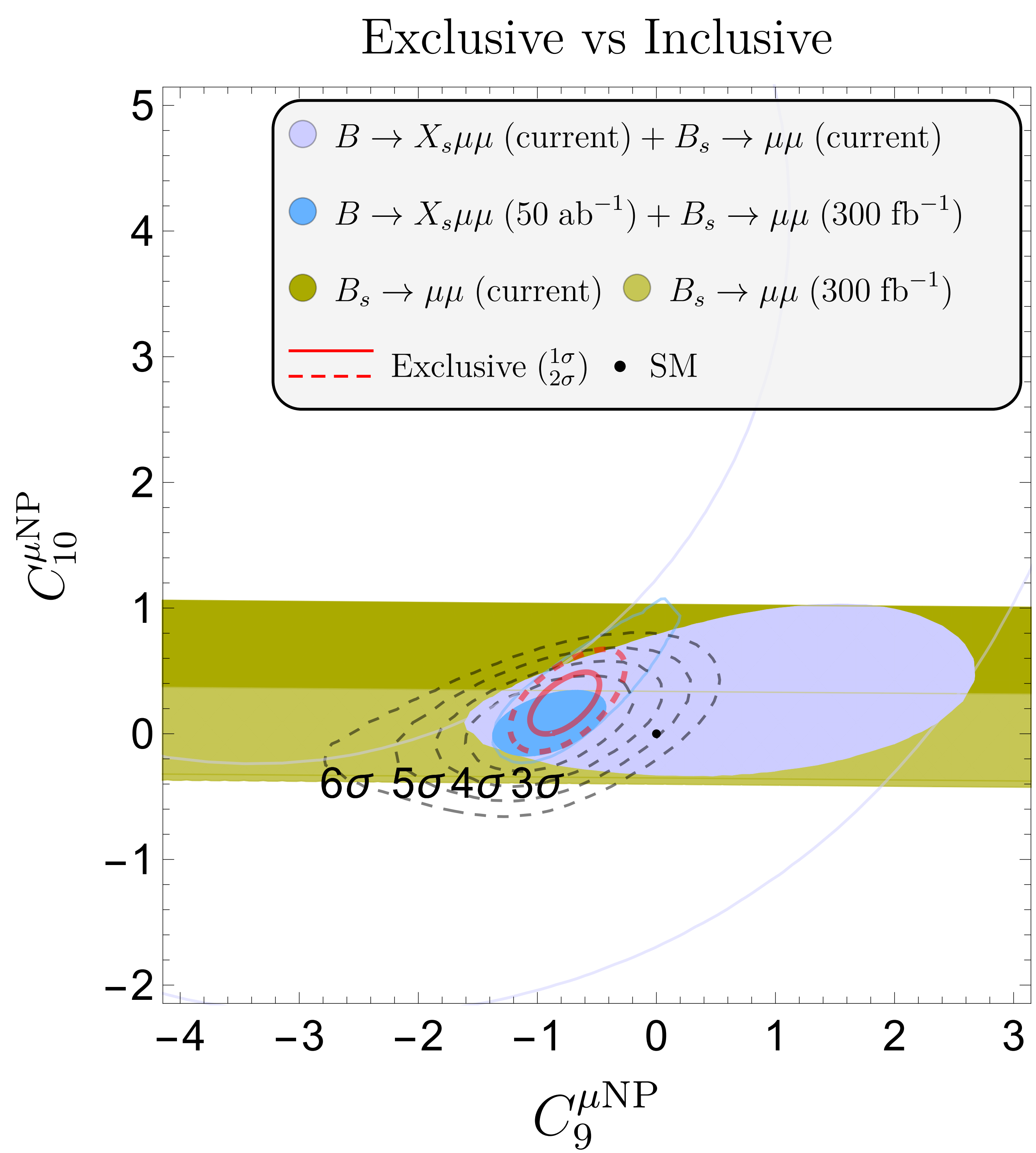
	$[1, 3.5]$	$[3.5, 6]$	$[1, 6]$	> 14.4
$\mathcal{B}$	3.1 %	2.6 %	2.0 %	2.6%
$\mathcal{H}_T$	24 %	15 %	13 %	-
$\mathcal{H}_L$	5.5 %	5.0 %	3.7 %	-
$\mathcal{H}_A$	40 %	33 %	- %	-
$\mathcal{H}_3$	240 %	140 %	120 %	-
$\mathcal{H}_4$	140 %	270 %	120 %	-

Table 4: Projected statistical uncertainties that we expect at Belle II with 50 ab^{-1} of integrated luminosity. The first row gives the considered q^2 bin in GeV^2. The total projected error is obtained by adding a 5.8(3.9)% systematic uncertainty to all low-q^2 (high-q^2) observables.

Phenomenology

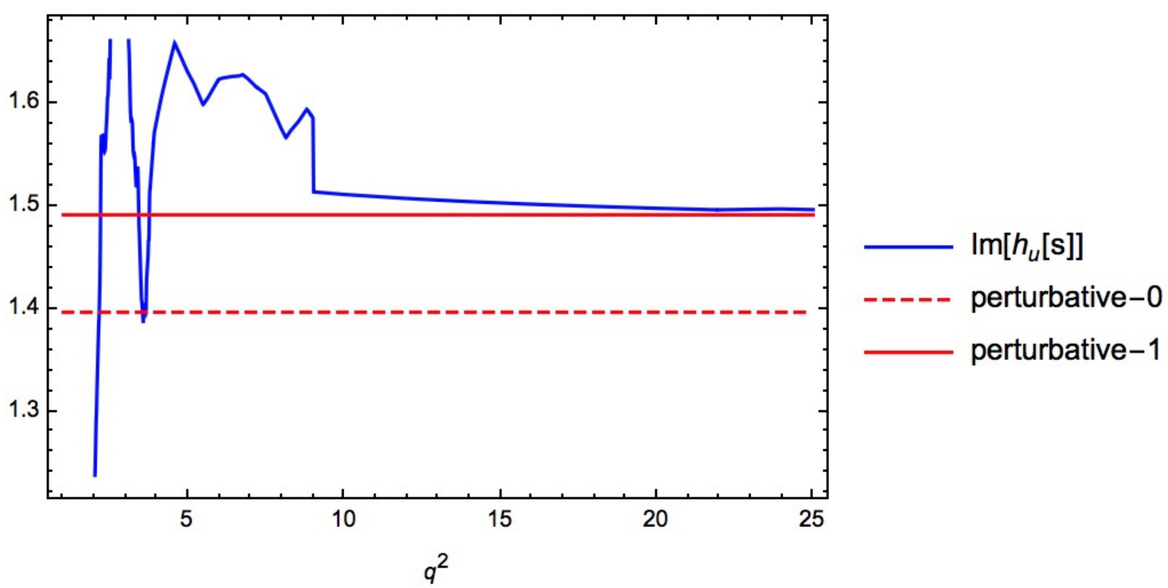
★ New physics analysis for $\bar{B} \to X_s \ell^+ \ell^-$

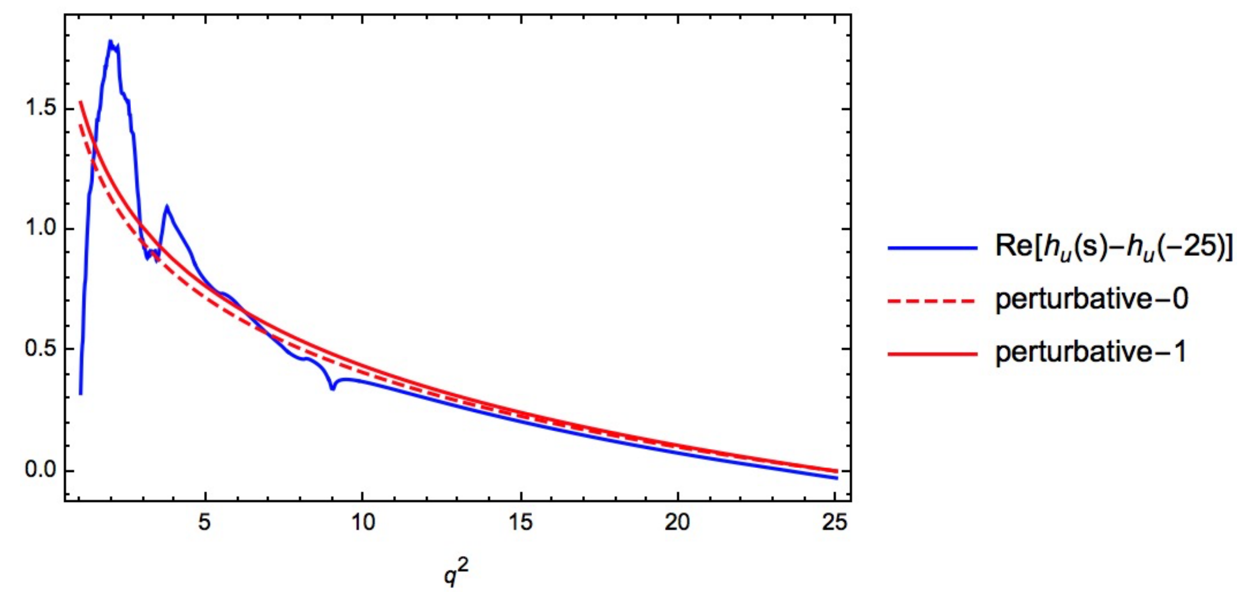




Long-distance effects (factorizable)

- Use latest data from **BESIII**, BaBar and ALEPH
  [see Keshavarzi,Nomura,Teubner,'18]
- The matching point is calculated at the two-loop level. [de Boer,'17]





Asymptotic behaviours of perturbative and Kruger-Sehgal functions

Phenomenologically, the decay rates are modified by

- ~ **5%** in the low-q^2 region
- ~ **15%** in the high-q^2 region

SM predictions

for branching ratios (high-q^2)

$$\mathcal{B}[>14.4]_{ee} = (2.04 \pm 0.28_{\rm scale} \pm 0.02_{m_t} \pm 0.03_{C,m_c} \pm 0.19_{m_b} \pm 0.002_{\rm CKM} \pm 0.03_{\rm BR_{sl}}$$
$$\pm 0.006_{\alpha_s} \pm 0.13_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \cdot 10^{-7} = (2.04 \pm 0.87) \cdot 10^{-7} \,,$$
$$\mathcal{B}[>14.4]_{\mu\mu} = (2.38 \pm 0.27_{\rm scale} \pm 0.03_{m_t} \pm 0.04_{C,m_c} \pm 0.21_{m_b} \pm 0.002_{\rm CKM} \pm 0.04_{\rm BR_{sl}}$$
$$\pm 0.006_{\alpha_s} \pm 0.12_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \cdot 10^{-7} = (2.38 \pm 0.87) \cdot 10^{-7} \,.$$

$$\mathcal{R}(14.4)_{ee} = (21.53 \pm 0.54_{\rm scale} \pm 0.25_{m_t} \pm 0.15_{C,m_c} \pm 0.09_{m_b} \pm 0.06_{\alpha_s} \pm 0.92_{\rm CKM}$$
$$\pm 0.11_{\lambda_2} \pm 1.38_{\rho_1} \pm 1.54_{f_{u,s}}) \times 10^{-4} = (21.53 \pm 2.35) \times 10^{-4} \,,$$
$$\mathcal{R}(14.4)_{\mu\mu} = (25.33 \pm 0.27_{\rm scale} \pm 0.29_{m_t} \pm 0.14_{C,m_c} \pm 0.03_{m_b} \pm 0.07_{\alpha_s} \pm 1.09_{\rm CKM}$$
$$\pm 0.04_{\lambda_2} \pm 0.83_{\rho_1} \pm 1.29_{f_{u,s}}) \times 10^{-4} = (25.33 \pm 1.93) \times 10^{-4} \,.$$

SM predictions

for angular observables

$$\overline{A}_{\rm FB}[1,3.5]_{ee} = (-7.28 \pm 0.67_{\rm scale} \pm 0.01_{m_t} \pm 0.11_{C,m_c} \pm 0.23_{m_b} \pm 0.19_{\alpha_s} \pm 0.04_{\lambda_2} \pm 0.51_{\rm resolved})\% = (-7.28 \pm 0.90)\%\,,$$

$$\overline{A}_{\rm FB}[3.5,6]_{ee} = (8.57 \pm 0.74_{\rm scale} \pm 0.01_{m_t} \pm 0.13_{C,m_c} \pm 0.37_{m_b} \pm 0.18_{\alpha_s} \pm 0.11_{\lambda_2} \pm 0.60_{\rm resolved})\% = (8.57 \pm 1.05)\%\,,$$

$$\overline{A}_{\rm FB}[1,6]_{ee} = (-0.18 \pm 0.79_{\rm scale} \pm 0.004_{m_t} \pm 0.13_{C,m_c} \pm 0.30_{m_b} \pm 0.20_{\alpha_s} \pm 0.02_{\lambda_2} \pm 0.01_{\rm resolved})\% = (-0.18 \pm 0.88)\%\,,$$

$$\mathcal{H}_T[1,3.5]_{ee} = (2.91 \pm 0.15_{\rm scale} \pm 0.03_{m_t} \pm 0.05_{C,m_c} \pm 0.02_{m_b} \pm 0.005_{\alpha_s} \pm 0.003_{\rm CKM} \pm 0.04_{\rm BR_{sl}} \pm 0.01_{\lambda_1} \pm 0.004_{\lambda_2} \pm 0.15_{\rm resolved}) \cdot 10^{-7} = (2.91 \pm 0.22) \cdot 10^{-7}$$

$$\mathcal{H}_L[1,6]_{ee} = (12.35 \pm 0.53_{\rm scale} \pm 0.13_{m_t} \pm 0.29_{C,m_c} \pm 0.14_{m_b} \pm 0.09_{\alpha_s} \pm 0.01_{\rm CKM} \pm 0.19_{\rm BR_{sl}} \pm 0.03_{\lambda_1} \pm 0.11_{\lambda_2} \pm 0.62_{\rm resolved}) \cdot 10^{-7} = (12.35 \pm 0.92) \cdot 10^{-7}$$