Theory review for $(g-2)_{\mu}$: SM and BSM



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The situation after the Fermilab announcement



This talk: theory review see talk by B. Kiburg for the experimental result

The Standard Model prediction for $a_{\mu} = (g-2)_{\mu}/2$



• Dipole moments: definition

$$\mathcal{H} = -\boldsymbol{\mu}_{\ell} \cdot \boldsymbol{B} \qquad \boldsymbol{\mu}_{\ell} = -g_{\ell} \frac{e}{2m_{\ell}} \boldsymbol{S} \qquad \boldsymbol{a}_{\ell} = \frac{g_{\ell}-2}{2}$$

Overview of theory status (Standard Model)

$$a^{ ext{SM}}_{\mu} = a^{ ext{QED}}_{\mu} + a^{ ext{EW}}_{\mu} + a^{ ext{had}}_{\mu} \qquad \qquad a^{ ext{had}}_{\mu} = a^{ ext{HVP}}_{\mu} + a^{ ext{HLbL}}_{\mu}$$

Comments on possible BSM explanations

QED: mass-independent terms

$$\begin{aligned} \mathbf{a}_{\mu}^{\mathsf{OED}} &= \mathbf{A}_{1} + \mathbf{A}_{2} \left(\frac{m_{\mu}}{m_{e}} \right) + \mathbf{A}_{2} \left(\frac{m_{\mu}}{m_{\tau}} \right) + \mathbf{A}_{3} \left(\frac{m_{\mu}}{m_{e}}, \frac{m_{\mu}}{m_{\tau}} \right) \\ \mathbf{A}_{i} &= \sum_{j=1}^{\infty} \left(\frac{\alpha}{\pi} \right)^{j} \mathbf{A}_{i}^{(2j)} \end{aligned}$$

Mass-independent term A₁ universal

$$\begin{split} A_1^{(2)} &= 0.5 \\ A_1^{(4)} &= -0.328478965579193784582\ldots \\ A_1^{(6)} &= 1.181241456587200\ldots \\ A_1^{(8)} &= -1.912245764926445574\ldots \\ Laporta 2017 \\ A_1^{(10)} &= 6.737(159) \\ \end{split}$$

• 4.8 σ discrepancy between $A_1^{(10)}$ [no lepton loops] = 7.668(159) Aoyama, Kinoshita, Nio 2019 and $A_1^{(10)}$ [no lepton loops] = 6.793(90) Volkov 2019

QED: fine-structure constant



• With new Rb measurement LKB 2020, Nature 588 (2020) 61

 $\begin{aligned} a_e^{\text{SM}}[\text{Rb}] &= 1,159,652,180.25(1)_{5\text{-loop}}(1)_{\text{had}}(9)_{\alpha(\text{Rb})} \times 10^{-12} \\ a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Rb}] &= 0.48(30) \times 10^{-12} [1.6\sigma] \\ a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Cs}] &= -0.88(36) \times 10^{-12} [-2.5\sigma] \end{aligned}$

← see following talk by S. Guellati-Khélifa

• Large contribution from mass-dependent terms due to $\log m_{\mu}/m_e$ enhancement

$$\begin{split} A_2^{(12)} \Big(\frac{m_\mu}{m_e}\Big) &\sim A_2^{(6)} \Big(\frac{m_\mu}{m_e}; \text{LbL}\Big) \times \left\{\frac{2}{3}\log\left(\frac{m_\mu}{m_e}\right) - \frac{5}{9}\right\}^3 \times 10 \sim 5400\\ A_2^{(12)} \Big(\frac{m_\mu}{m_e}\Big) &\times \Big(\frac{\alpha}{\pi}\Big)^6 \sim 0.8 \times 10^{-12} \end{split}$$

 \hookrightarrow much bigger than uncertainty in mass-independent 5-loop coefficient

• With α from Cs measurement or a_e

$$\begin{split} & a_{\mu}^{\text{QED}}(\alpha(\text{Cs})) = 116\ 584\ 718.931(7)(17)(6)(100)(23)[104] \times 10^{-11} \\ & a_{\mu}^{\text{QED}}(\alpha(\textbf{a}_{e})) = 116\ 584\ 718.842(7)(17)(6)(100)(28)[106] \times 10^{-11} \end{split}$$

 \hookrightarrow dominant uncertainty from 6-loop QED, $(g-2)_e$ tension irrelevant for $(g-2)_{\mu}$

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Electroweak contribution to $(g-2)_{\mu}$

Electroweak contribution Gnendiger et al. 2013

 $a_{\mu}^{\text{EW}} = (194.8 - 41.2) \times 10^{-11} = 153.6(1.0) \times 10^{-11}$

- Remaining uncertainty dominated by q = u, d, s loops \hookrightarrow nonperturbative effects Czarnecki, Marciano, Vainshtein 2003
- Two-loop calculation recently revisited without asymptotic expansion Ishikawa, Nakazawa, Yasui 2019

$$a_{\mu}^{\mathsf{EW}} = 152.9(1.0) imes 10^{-11}$$

- 3-loop corrections?
 - 3-loop RG estimate accidentally cancels in scheme chosen by Gnendiger et al. 2013, with an error of 0.2 \times 10^{-11}
 - *α_s* corrections to *t*-loop should scale as

$$\left. a_{\mu}^{t ext{-loop}}
ight|_{ ext{2-loop}} imes rac{lpha_{s}}{\pi} \sim 0.3 imes 10^{-12}$$





Hadronic vacuum polarization: need hadronic two-point function

 $\Pi_{\mu\nu} = \langle 0 | T\{j_{\mu}j_{\nu}\} | 0 \rangle$

• Hadronic light-by-light scattering: need hadronic four-point function

 $\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_{\mu}j_{\nu}j_{\lambda}j_{\sigma}\}|0\rangle$

• In the following: status of the hadronic contributions

The Muon g - 2 Theory Initiative



- Formed in 2017, series of workshops since (last plenary one at the INT in 2019) https://indico.fnal.gov/event/21626/ https://muon-gm2-theory.illinois.edu/
- Map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental results
- Next meeting at KEK (virtual) June 28–July 2 https://www-conf.kek.jp/muong-2theory/

M. Hoferichter (Institute for Theoretical Physics)

Theory review for $(g - 2)_{\mu}$: SM and BSM

Higher-order hadronic effects



- Once $\Pi_{\mu\nu}$ and $\Pi_{\mu\nu\lambda\sigma}$ known, higher-order iterations determined
- Standard for NLO HVP Calmet et al. 1976
- NNLO HVP found to be relevant Kurz et al. 2014
- NLO HLbL already further suppressed Colangelo et al. 2014

Hadronic vacuum polarization from e^+e^- data

- General principles yield direct connection with experiment
 - Gauge invariance

$$\sum_{k,\mu}^{k,\mu} = -i(k^2 g^{\mu\nu} - k^{\mu} k^{\nu}) \Pi(k^2)$$

Analyticity

$$\Pi_{\rm ren} = \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi} \int_{4M_\pi^2}^{\infty} {\rm d}s \frac{{\rm Im}\,\Pi(s)}{s(s-k^2)}$$

Unitarity

$$\operatorname{Im}\Pi(s) = \frac{s}{4\pi\alpha}\sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons}) = \frac{\alpha}{3}R(s)$$

Master formula

$$a_{\mu}^{\mathsf{HVPLO}} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{s_{\mathsf{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R(s)$$



- Decades-long effort to measure e^+e^- cross sections
 - Up to about 2 GeV: sum of exclusive channels
 - Above: inclusive data + narrow resonances + pQCD
- Tensions in the data: most notably between KLOE and BaBar 2π data

Hadronic vacuum polarization from e^+e^- data: 2π channel



Relative difference between data sets and fit result

- Tension between KLOE and BaBar data:
 - Cross checks from analyticity and unitarity of the pion form factor
 - Affects the combination of data sets: different results depending on methodology
 - For white paper: adopt a conservative merging procedure that accounts for the 2π tension
- Our final recommendation: $a_{\mu}^{\text{HVPLO}}(e^+e^-) = 693.1(4.0) \times 10^{-10}$

Hadronic light-by-light scattering

- In the past: hadronic models, inspired by various QCD limits, but error estimates difficult
- Dispersive approach: use again analyticity, unitarity, crossing, and gauge invariance for data-driven approach Colangelo, MH, Procura, Stoffer 2014,...
- For simplest intermediate states: relation to $\pi^0 \rightarrow \gamma^* \gamma^*$ transition form factor and $\gamma^* \gamma^* \rightarrow \pi \pi$ partial waves







HLbL scattering: white paper

• Reference points:

$$\begin{split} \mathbf{a}_{\mu}^{\text{HLbL}}|_{\text{"Glasgow consensus" 2009}} &= 105(26) \times 10^{-11} \\ \mathbf{a}_{\mu}^{\text{HLbL}}|_{\text{Jegerlehner, Nyffeler 2009}} &= 116(39) \times 10^{-11} \end{split}$$

- Strategy in the white paper
 - Take well-controlled results for the low-energy contributions
 - Combine errors in quadrature
 - Take best guesses for medium-range and short-distance matching
 - Add these errors linearly, since errors hard to disentangle at the moment
- Recommended value

 a_{μ}^{HLbL} (phenomenology) = 92(19) \times 10⁻¹¹

• Lattice QCD: first complete calculation RBC/UKQCD 2019

 a_{μ}^{HLbL} (lattice, *uds*) = 79(35) × 10⁻¹¹

 \hookrightarrow can combine with phenomenological value

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Contribution	Section	Equation	Value ×10 ¹¹	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, udsc)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18-30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP $(e^+e^-, LO + NLO + NNLO)$	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2-8, 18-24, 31-36]
Difference: $\Delta a_{\mu} := a_{\mu}^{\exp} - a_{\mu}^{SM}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_{μ}^{SM} . After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

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• White paper average: a_{μ}^{HVPLO} (lattice) = 711.6(18.4) × 10⁻¹⁰

 \hookrightarrow large uncertainty, consistent with both e^+e^- data and "no new physics"

- Does not include a_{μ}^{HVPLO} (BMWc) = 707.5(5.5) × 10⁻¹⁰ 2002.12347, first lattice result at < 1% precision
 - \hookrightarrow 2.1 σ above e^+e^- , 1.6 σ below "no new physics"
- How to resolve this?
 - Scrutiny by other lattice collaborations ongoing
 - Need to know at which energies the changes to the e^+e^- cross section occur \rightarrow 2002.12347 points to low energies below 2 GeV
 - Would require changes to 2π cross section much bigger than the KLOE/BaBar tension
 - New 2π data: SND (just published), CMD3 (forthcoming), BaBar (reanalysis on larger data set), Belle II, BESIII
 - MUonE project: extract space-like HVP from µe scattering

BSM effect sizable

$$a_{\mu}^{ ext{exp}} - a_{\mu}^{ ext{SM}} = 251(59) imes 10^{-11} > a_{\mu}^{ ext{EW}}$$

- Requires some form of enhancement:
 - Chiral enhancement: chirality flip $\propto m_{\mu}^2$ in SM
 - \hookrightarrow enhancement by tan $\beta \sim 50$ in SUSY, $m_t/m_\mu \sim 1600$ in leptoquark models
 - Light BSM: axion-like particles, Z', $L_{\mu} L_{\tau}$, light scalars
- Connections to other recent hints for the violation of lepton flavor universality?
 - *B* anomalies: $b \rightarrow s\ell\ell$ ($R(K^{(*)}, P'_5, ...), b \rightarrow c\tau\nu$ ($R(D^{(*)})$)
 - First-row CKM unitarity, CMS dilepton data
 - Anomalous magnetic moment of the electron (?)

BSM: many possible models



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There are many more examples...

SUSY: MSSM, MRSSM

- MSugra...many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns

Two-Higgs doublet model

• Type I, II, Y, Type X(lepton-specific), flavour-aligned

Lepto-quarks, vector-like leptons

 $\bullet\,$ scenarios with muon-specific couplings to μ_L and μ_R

Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_{\mu} L_{\tau}$)





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Dominik Stöckinger Briefly		Briefly some general remarks, then general MSSM	14/26	- E	うく
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Conclusions

• QED and EW contribution well under control

Hadronic vacuum polarization

- Presently largest systematic uncertainty in $\pi\pi$ channel
- Comparison with lattice QCD just beginning
- New data: SND, CMD-3, BaBar, Belle II, BESIII

• Hadronic light-by-light scattering

- Use dispersion relations to remove model dependence as far as possible (π^0 and leading $\pi\pi$ effects done)
- Evaluation of subleading terms and comparison to lattice-QCD calculations in progress
- Current theory matches expected experimental precision after first E989 release, but need to go further!
- Plethora of BSM explanations, possible relation to other lepton-flavor-universality violating "anomalies"



QED: fine-structure constant

Input from atom interferometry

$$lpha^2 = rac{4\pi R_\infty}{c} imes rac{m_{
m atom}}{m_e} imes rac{\hbar}{m_{
m atom}}$$

With Rb measurement LKB 2011

$$\begin{aligned} a_{e}^{\text{exp}} &= 1,159,652,180.73(28) \times 10^{-12} \\ a_{e}^{\text{SM}} &= 1,159,652,182.03(1)_{5\text{-loop}}(1)_{\text{had}}(72)_{\alpha(\text{Rb})} \times 10^{-12} \\ a_{e}^{\text{exp}} &- a_{e}^{\text{SM}} &= -1.30(77) \times 10^{-12} [1.7\sigma] \end{aligned}$$

 $\hookrightarrow \alpha$ limiting factor, but more than an order of magnitude to go in theory

With Cs measurement Berkeley 2018, Science 360 (2018) 191

$$\begin{aligned} a_{e}^{\text{SM}} &= 1,159,652,181.61(1)_{5\text{-loop}}(1)_{\text{had}}(23)_{\alpha(\text{Cs})} \times 10^{-12} \\ a_{e}^{\text{exp}} &- a_{e}^{\text{SM}} = -0.88(36) \times 10^{-12} [2.5\sigma] \end{aligned}$$

 \hookrightarrow for the first time a_e^{\exp} limiting factor

Hadronic light-by-light scattering: data input



• Reconstruction of $\gamma^* \gamma^* \to \pi \pi, \pi^0$: combine experiment and theory constraints

• Need input on $\gamma^* \gamma^*$ matrix elements for as many states as possible

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Hadronic light-by-light scattering: pion pole



• Pion pole from data MH et al. 2018, Masjuan, Sánchez-Puerto 2017 and lattice QCD Gérardin et al. 2019

$$\begin{split} \left. a_{\mu}^{\pi^{0}\text{-pole}} \right|_{\text{dispersive}} &= 63.0^{+2.7}_{-2.1} \times 10^{-11} \\ \left. a_{\mu}^{\pi^{0}\text{-pole}} \right|_{\text{Canterbury}} &= 63.6(2.7) \times 10^{-11} \\ \left. a_{\mu}^{\pi^{0}\text{-pole}} \right|_{\text{lattice}+\text{PrimEx}} &= 62.3(2.3) \times 10^{-11} \\ \left. a_{\mu}^{\pi^{0}\text{-pole}} \right|_{\text{lattice}} &= 59.7(3.6) \times 10^{-11} \end{split}$$

 \hookrightarrow agree within uncertainties well below Fermilab goal

Singly-virtual results agree well with BESIII measurement

Theory review for $(g - 2)_{\mu}$: SM and BSM

HVP from *e*+*e*- data

$$\begin{aligned} \mathbf{a}_{\mu}^{\text{HVP,LO}} &= \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} \mathrm{d}s \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2} \sigma(e^+e^- \to \text{hadrons}(+\gamma))(s) \\ &= 6931(40) \times 10^{-11} \end{aligned}$$

- The "theory" prediction a_{μ}^{SM} is actually based on experiments (ISR, direct scan)
 - \hookrightarrow propagation of experimental uncertainties
- Uncertainty estimate includes:
 - different methodologies for the combination of data sets Davier et al. 2019, Keshavarzi et al. 2020
 - conservative estimate of systematic errors from tensions in the data
 - cross checks from analyticity/unitarity constraints Colangelo et al. 2018, Ananthanarayan et al.

2018, Davier et al. 2019, MH et al. 2019

full NLO radiative corrections Campanario et al. 2019

Cross checks from analyticity and unitarity



• For "simple" channels $e^+e^- \rightarrow 2\pi$, 3π can derive form of the cross section from general principles of QCD (analyticity, unitarity, crossing symmetry)

 \hookrightarrow strong cross check on the data sets (covering about 80% of HVP)

 Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

Cross checks from analyticity and unitarity

- In direct integration: local combination of data
 - \hookrightarrow local scale factor in case tensions arise
- $e^+e^- \rightarrow 2\pi$ determined by pion vector form factor F_{π}^V
- Unitarity for pion vector form factor

$$\operatorname{Im} F_{\pi}^{V}(s) = \theta(s - 4M_{\pi}^{2})F_{\pi}^{V}(s)e^{-i\delta_{1}(s)}\sin\delta_{1}(s)$$



- \hookrightarrow final-state theorem: phase of F_{π}^{V} equals $\pi\pi P$ -wave phase δ_1 Watson 1954
- Can derive a global fit function that depends on
 - Two values of δ_1 (elastic 2π intermediate states)
 - ω mass, width, and residue (3 π intermediate states)
 - Conformal polynomial (4π intermediate states)

HVP from e^+e^- data

$$a_{\mu}^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} imes 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error

 \hookrightarrow would get 4.2 $\sigma \rightarrow$ 4.8 σ when ignoring additional systematic error

- There was broad consensus to adopt conservative error estimates
 → merging procedure in WP20 covers tensions in the data and different methodologies for the combination of data sets
- Systematic effect dominated by [fit w/o KLOE fit w/o BaBar]/2



plots from Gülpers et al. 2018

Matches data-driven convention for leading-order HVP

 \hookrightarrow diagram (f) F without additional gluons is subtracted

Theory review for $(g - 2)_{\mu}$: SM and BSM

$\pi\pi$ contribution below 1 GeV



Assumption: suppose all changes occur in $\pi\pi$ channel below 1 GeV

$$\hookrightarrow a_{\mu}^{ ext{total}}[ext{WP20}] - a_{\mu}^{2\pi, <1 \, ext{GeV}}[ext{WP20}] = 197.7 imes 10^{-10}$$

Changing the $\pi\pi$ cross section below 1 GeV



- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
 - "Low-energy" scenario: $\pi\pi$ phase shifts
 - High-energy scenario: conformal polynomial
 - Combined scenario
 - \hookrightarrow 2. and 3. lead to uniform shift, 1. concentrated in ρ region

Correlations



Correlations with other observables:

- Pion charge radius $\langle r_{\pi}^2 \rangle$
 - \hookrightarrow significant change in scenarios 2. and 3.
 - \hookrightarrow can be tested in lattice QCD
- Hadronic running of α
- Space-like pion form factor





Window quantities



- Weight functions in Euclidean time proposed by RBC/UKQCD 2018
 - \hookrightarrow long-distance, intermediate, and short-distance window
- For intermediate window $a_{\mu}^{\text{int}}[\text{RBC/UKQCD}] = 231.9(1.5) \times 10^{-10}$ and $a_{\mu}^{\text{int}}[\text{BMWc}] = 236.7(1.4) \times 10^{-10}$ differ by 2.3σ
- Difference between BMWc and e^+e^- in intermediate window is 3.7 σ , but $\pi\pi$ channel below 1 GeV split 69 : 28 : 3, relevant changes above 1 GeV?
- Detailed study of windows key tool for comparison among lattice and with e⁺e⁻



- New data from SND experiment not yet included in WP20 number
 - \hookrightarrow lie between BaBar and KLOE
- More data to come from: CMD3, BESIII, BaBar, Belle II
- MUonE project: extract space-like HVP from μe scattering

Hadronic running of $\boldsymbol{\alpha}$

$$\Delta \alpha_{\rm had}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\rm thr}}^{\infty} {\rm d}s \frac{R_{\rm had}(s)}{s(M_Z^2 - s)}$$

• $\Delta \alpha_{had}^{(5)}(M_Z^2)$ enters as input in global electroweak fit

- \hookrightarrow integral weighted more strongly towards high energy
- Changes in $R_{had}(s)$ have to occur at low energies, $\lesssim 2 \text{ GeV}$ Crivellin et al. 2020, Keshavarzi et al. 2020, Malaescu et al. 2020
- This seems to happen for BMWc calculation (translated from the space-like), with only moderate increase of tensions in the electroweak fit ($\sim 1.8\sigma \rightarrow 2.4\sigma$)
 - \hookrightarrow need large changes in low-energy cross section

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on \ensuremath{BMWc}
$\Delta lpha_{ m had}^{(5)}(M_Z^2) imes 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

80

0

⊽ 0.5 γ 0.0

-0.5

+01 × 10 40 20

• Time-like formulation:

$$\Delta \alpha_{\rm had}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\rm thr}}^{\infty} {\rm d}s \frac{R_{\rm had}(s)}{s(M_Z^2 - s)}$$

• Space-like formulation:

Global EW fit

1

- Difference between HEPFit and GFitter implementation mainly treatment of M_W
- Pull goes into opposite direction



KNT18+rhad

lattice incl. bottom -