CP Violating Phase Sum Rule for CKM and PMNS Matrix

Jin Sun
TDLI & SJTU

work with Junxing Pan, Xiao-Dong Ma, Xiao-Gang He,
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Research Motivation

Lagrangian for quark and neutrino mixings

$$L = -\frac{g}{\sqrt{2}}\bar{U}_L \underline{V_{CKM}} \gamma^\mu D_L W_\mu^+ - \frac{g}{\sqrt{2}}\bar{E}_L \underline{V_{PMNS}} \gamma^\mu \nu_L W_\mu^- + H.C.$$

Siginify CPV

$$\delta_q^{KM} \simeq \pi/2 \qquad \delta_l^{KM} \simeq -\pi/2$$

$$\delta_q^{KM} + \delta_l^{KM} = 0,\ \delta_q^{KM} \simeq \pi/2$$

Accident? **Correlation?**

Spontaneous CPV Model

Relate these two Dirac phases

Parameterization

PDG standard parameterization

$$V_i = \begin{pmatrix} c^i_{12}c^i_{13} & s^i_{12}c^i_{13} & s^i_{13}e^{-i\delta_i} \\ -s^i_{12}c^i_{23} - c^i_{12}s^i_{23}s^i_{13}e^{i\delta_i} & c^i_{12}c^i_{23} - s^i_{12}s^i_{23}s^i_{13}e^{i\delta_i} & s^i_{23}c^i_{13} \\ s^i_{12}s^i_{23} - c^i_{12}c^i_{23}s^i_{13}e^{i\delta_i} & -c^i_{12}s^i_{23} - s^i_{12}c^i_{23}s^i_{13}e^{i\delta_i} & c^i_{23}c^i_{13} \end{pmatrix}$$

KM parameterization

$$V_i = \begin{pmatrix} c^i_1 & -s^i_1c^i_3 & -s^i_1s^i_3 \\ s^i_1c^i_2 & c^i_1c^i_2c^i_3 - s^i_2s^i_3e^{i\delta_i} & c^i_1c^i_2s^i_3 + s^i_2c^i_3e^{i\delta_i} \\ s^i_1s^i_2 & c^i_1s^i_2c^i_3 + c^i_2s^i_3e^{i\delta_i} & c^i_1s^i_2s^i_3 - c^i_2c^i_3e^{i\delta_i} \end{pmatrix} \qquad i = q/l$$

Angles and phases are parameterization dependent.

Decompose KM

$$V_i = \begin{pmatrix} c_1^i & -s_1^i c_3^i & -s_1^i s_3^i \\ s_1^i c_2^i & c_1^i c_2^i c_3^i - s_2^i s_3^i e^{i\delta_i} & c_1^i c_2^i s_3^i + s_2^i c_3^i e^{i\delta_i} \\ s_1^i s_2^i & c_1^i s_2^i c_3^i + c_2^i s_3^i e^{i\delta_i} & c_1^i s_2^i s_3^i - c_2^i c_3^i e^{i\delta_i} \end{pmatrix}$$

$$V_{i1}^\dagger \cdot V_{i2} = 0$$

$$V_i = V_{i1} + e^{i\delta_i} V_{i2}$$

$$V_{i1} = \begin{pmatrix} c_1^i & -s_1^i c_3^i & -s_1^i s_3^i \\ s_1^i c_2^i & c_1^i c_2^i c_3^i & c_1^i c_2^i s_3^i \\ s_1^i s_2^i & c_1^i s_2^i c_3^i & c_1^i s_2^i s_3^i \end{pmatrix} \qquad V_{i2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s_2^i s_3^i & s_2^i c_3^i \\ 0 & c_2^i s_3^i & -c_2^i c_3^i \end{pmatrix}$$

Convert angle and phase

PDG	KM
Quark mixing Utfit	**Quark mixing**
$\delta^q_{\rm PDG}/\pi = 0.3717$	$\delta^q_{\rm KM}/\pi = 0.4950$
Lepton mixing Nufit, T2K	**Lepton mixing**
NH $s^l_{23} = 0.7503$	**NH** $s^l_2 = 0.7894$
$\delta_{\rm PDG}/\pi = -0.6016$	$\delta^l_{\rm KM}/\pi = -0.5757$
IH $s^l_{23} = 0.7517$	**IH** $s^l_2 = 0.7202$
$\delta_{\rm PDG}/\pi = -0.4393$	$\delta^l_{\rm KM}/\pi = -0.4275$

see backup for details

$$\theta^l_{23} = \theta^l_2 = \pi/4 \qquad \delta^l_{\rm KM/PDG} = -\pi/2 \qquad \delta^q_{\rm KM} = \pi/2$$

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Multi-Higgs Model

Gauge Group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{PQ}$

$$Q_L : (3, 2, 1/6)(0),\ U_R : (3, 1, 2/3)(+1),\ D_R : (3, 1, -1/3)(+1)$$

$$L_L : (1, 2, -1/2)(0),\ E_R : (1, 1, -1)(+1),\ \nu_R : (1, 1, 0)(+1)$$

$$\phi_{1,2} : (1, 2, -1/2)(+1),\ \phi_3 : (1, 2, -1/2)(-1),\ \tilde{S} : (1, 1, 0)(+2)$$

Spontaneous CPV with PQ Symmetry

$$\phi_i = e^{i\theta_i} H_i = e^{i\theta_i}((v_i + R_i + iA_i)/\sqrt{2}, h_i^-)^T,\ \tilde{S} = e^{i\theta_s} S = e^{i\theta_s}(v_s + R_s + iA_s)/\sqrt{2}$$

Why 3 doublets and 1 singlet?

$$L_Y^L = -\bar{L}_L Y_\nu \phi_3 \nu_R - \bar{L}_L (Y_{e1} \tilde{\phi}_1 + Y_{e2} \tilde{\phi}_2) E_R - \frac{1}{2} \bar{\nu}_R^c Y_s \tilde{S}^\dagger \nu_R$$

Higgs Potential

$$V = -m_1^2 H_1^\dagger H_1 - m_2^2 H_2^\dagger H_2 - m_3^2 H_3^\dagger H_3 - m_{12}^2 \left(H_1^\dagger H_2 e^{i(\theta_2-\theta_1)} + \text{h.c.} \right) - m_s^2 S^\dagger S$$
$$+ \lambda_1 \left(H_1^\dagger H_1 \right)^2 + \lambda_2 \left(H_2^\dagger H_2 \right)^2 + \lambda_t \left(H_3^\dagger H_3 \right)^2 + \lambda_s \left(S^\dagger S \right)^2$$
$$+ \lambda_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + \lambda_3' \left(H_1^\dagger H_1 \right) \left(H_3^\dagger H_3 \right) + \lambda_3'' \left(H_2^\dagger H_2 \right) \left(H_3^\dagger H_3 \right)$$
$$+ \lambda_4 \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) + \lambda_4' \left(H_1^\dagger H_3 \right) \left(H_3^\dagger H_1 \right) + \lambda_4'' \left(H_2^\dagger H_3 \right) \left(H_3^\dagger H_2 \right)$$
$$+ \frac{1}{2}\lambda_5 \left(\left(H_1^\dagger H_2 \right)^2 e^{i2(\theta_2-\theta_1)} + \text{h.c.} \right) + \lambda_6 \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 e^{i(\theta_2-\theta_1)} + \text{h.c.} \right)$$
$$+ \lambda_7 \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 e^{i(\theta_2-\theta_1)} + \text{h.c.} \right) + \lambda_8 \left(H_3^\dagger H_3 \right) \left(H_1^\dagger H_2 e^{i(\theta_2-\theta_1)} + \text{h.c.} \right)$$
$$+ \lambda_9 \left(H_3^\dagger H_2 \right) \left(H_1^\dagger H_3 \right) e^{i(\theta_2-\theta_1)} + \lambda_{10} \left(H_3^\dagger H_1 \right) \left(H_2^\dagger H_3 \right) e^{i(\theta_1-\theta_2)}$$
$$+ f_1 H_1^\dagger H_1 S^\dagger S + f_2 H_2^\dagger H_2 S^\dagger S + f_3 H_3^\dagger H_3 S^\dagger S + d_{12} \left(H_1^\dagger H_2 e^{i(\theta_2-\theta_1)} + H_2^\dagger H_1 e^{i(\theta_1-\theta_2)} \right) S^\dagger S$$
$$+ f_{13} \left(H_1^\dagger H_3 S e^{i(\theta_3+\theta_s-\theta_1)} + h.c. \right) + f_{23} \left(H_2^\dagger H_3 S e^{i(\theta_3+\theta_s-\theta_2)} + \text{h.c.} \right)$$

Annotations (right column, per line):

$H_i^\dagger H_i$

$(H_i^\dagger H_i)^2$

$(H_i^\dagger H_i)(H_j^\dagger H_j)$

$(H_i^\dagger H_j)(H_j^\dagger H_i)$

$(H_1^\dagger H_2)(H_1^\dagger H_2)$

$(H_1^\dagger H_2)(H_i^\dagger H_i)$

$(H_3^\dagger H_i)(H_3^\dagger H_j)$

$(H_i^\dagger H_i)(S^\dagger S)$

$(H_i^\dagger H_3)S$

Only CPV Source

3 phases $\theta_1 - \theta_2$ $\quad \theta_3 + \theta_s - \theta_1$ $\quad \theta_3 + \theta_s - \theta_2$

Only 2 $\delta_{sp} = \theta_1 - \theta_2$ $\quad \delta_s = \theta_3 + \theta_s - \theta_1$ $\quad \delta_s + \delta_{sp}$

Minimal condition of potential

$$f_{13} v_1 v_3 v_s \sin \delta_s + f_{23} v_2 v_3 v_s \sin (\delta_s + \delta_{sp}) = 0$$

$$\tan \delta_s = - \frac{f_{23} v_2 \sin \delta_{sp}}{f_{13} v_{21} + f_{23} v_2 \cos \delta_{sp}}$$

the only source of spontaneous CPV —— δ_{sp}

Maximal CPV $\delta_{sp} = \pi/2$

Absorb phases

Yukawa coupling

$$L_Y = -\bar{L}_L Y_\nu \phi_3 \nu_R - \bar{L}_L (Y_{e1}\tilde{\phi}_1 + Y_{e2}\tilde{\phi}_2) E_R - \frac{1}{2}\bar{\nu}_R^c Y_s \tilde{S}^\dagger \nu_R$$ **Lepton**

$$-\bar{Q}_L Y_u \phi_3 U_R - \bar{Q}_L (Y_{d1}\tilde{\phi}_1 + Y_{d2}\tilde{\phi}_2) D_R + H.C.$$ **Quark**

Absorb phases to redefine fermion fields

$$\theta_3, -\theta_s/2, -(\theta_3+\theta_s/2), -\theta_1, -(\theta_1+\theta_3+\theta_s/2) \rightarrow U_R, \nu_R, L_L, D_R, E_R$$

$$\delta_{sp} = \theta_1 - \theta_2$$

$$L_m = -\overline{D}_L M_d D_R - \overline{U}_L M_u U_R - \overline{E}_L M_e E_R - \overline{L}_L M_D \nu_R - \frac{1}{2}\overline{\nu_R^c} M_R \nu_R$$

$$M_d = M_{d1} + M_{d2} e^{i\delta_{sp}}\ ,\ M_e = M_{e1} + M_{e2} e^{i\delta_{sp}}\ ,\ M_{ai} = Y_{ai}\frac{v_i}{\sqrt{2}}\ ;$$

$$M_u = Y_u \frac{v_3}{\sqrt{2}}\ ,\ M_D = Y_\nu \frac{v_3}{\sqrt{2}}\ ,\ M_R = Y_s \frac{v_s}{\sqrt{2}},\ M_\nu = -M_D M_R^{-1} M_D^T$$

Relate Dirac phases to CP phase

$$M_d = V_L^{d\dagger} \hat{M}_d V_R^d, \;\; M_e = V_L^{e\dagger} \hat{M}_e V_R^e \;\rightarrow\; V_q = V_L^{d\dagger}, \;\; V_l = V_L^e$$

Solution?

$$V_i^{KM} = V_{i1}^{KM} + e^{i\delta_i} V_{i2}^{KM}$$

$$M_{di} = V_{qi} \hat{M}_d V_R^d = \tilde{M}_{di} V_R^d \qquad M_{ei} = V_{li}^\dagger \hat{M}_e V_R^e = \tilde{M}_{ei} V_R^e$$

Absorb V_R into fermion field

$$V_q \hat{M}_d = (\mathrm{Re}V_q + i\mathrm{Im}V_q)\hat{M}_d = (\tilde{M}_{d1} + \tilde{M}_{d2} e^{i\delta_{sp}})$$

$$V_l^\dagger \hat{M}_e = (\mathrm{Re}V_l^\dagger + i\mathrm{Im}V_l^\dagger)\hat{M}_e = (\tilde{M}_{e1} + \tilde{M}_{e2} e^{i\delta_{sp}})$$

$$\delta_{\mathrm{KM}}^q = \delta_{sp}$$

$$\delta_{\mathrm{KM}}^l = -\delta_{sp}$$

$$\delta_{\mathrm{KM}}^q + \delta_{\mathrm{KM}}^l = 0$$

Only CPV source

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Physical fields

Additional Higgs bosons → **New Interaction**

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_s \end{pmatrix} = \begin{pmatrix} v_2/v_{12} & -v_1 v_3 v_s/v_{12}N_a & v_1/v & v_1 v_3^2/vN_a \\ -v_1/v_{12} & -v_2 v_3 v_s/v_{12}N_a & v_2/v & v_2 v_3^2/vN_a \\ 0 & v_{12}v_s/N_a & v_3/v & -v_{12}^2 v_3/vN_a \\ 0 & v_{12}v_3/N_a & 0 & vv_s/N_a \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ h_z \\ a \end{pmatrix}$$

$$\begin{pmatrix} h_1^- \\ h_2^- \\ h_3^- \end{pmatrix} = \begin{pmatrix} v_2/v_{12} & v_1 v_3/vv_{12} & v_1/v \\ -v_1/v_{12} & v_2 v_3/vv_{12} & v_2/v \\ 0 & -v_{12}/v & v_3/v \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \\ h_w \end{pmatrix}$$

Axion field

Physical DoF

$$v^2 = v_1^2 + v_2^2 + v_3^2,\ v_{12}^2 = v_1^2 + v_2^2,\ N_a^2 = (v_{12}^2 v_3^2 + v_s^2 v^2) \simeq v_s^2 v^2$$

Same rotation $(R_1, R_2, R_3, R_s)^T \rightarrow (H_1^0, H_2^0, H_3^0, H_4^0)^T$

arXiv:0705.0399, 1010.5204

Mass Eigenstates

Yukawa coupling

$$L_Y = -\bar{L}_L Y_\nu \tilde{\phi}_3 \nu_R - \bar{L}_L (Y_{e1}\tilde{\phi}_1 + Y_{e2}\tilde{\phi}_2) E_R - \frac{1}{2}\bar{\nu}_R^c Y_s \tilde{S}^\dagger \nu_R$$
$$-\bar{Q}_L Y_u \phi_3 U_R - \bar{Q}_L (Y_{d1}\tilde{\phi}_1 + Y_{d2}\tilde{\phi}_2) D_R + H.C.$$

$$M_{ai} = Y_{ai}\frac{v_i}{\sqrt{2}}$$

$$M_d = M_{d1} + M_{d2} e^{i\delta_{sp}} \qquad V_i^{KM} = V_{i1}^{KM} + e^{i\delta_i} V_{i2}^{KM}$$

Neutral Higgs

$$L_Y = -\bar{U}_L \frac{M_u}{v} U_R \left[\frac{v_{12} v v_s}{v_3 N_a}(H_2^0 + i a_2) + H_3^0 - \frac{v_{12}^2}{N_a}(H_4^0 + i a)\right]$$
$$-\left(\bar{D}_L \frac{M_d}{v} D_R + \bar{E}_L \frac{M_e}{v} E_R\right)\left[\frac{v_2 v}{v_1 v_{12}}(H_1^0 - i a_1) - \frac{v_3 v v_s}{v_{12} N_a}(H_2^0 - i a_2) + H_3^0 + \frac{v_3^2}{N_a}(H_4^0 - i a)\right]$$
$$+\left(\bar{D}_L V_{q2}^\dagger V_{q2}\frac{M_d}{v} D_R + \bar{E}_L V_{l2} V_{l2}^\dagger \frac{M_e}{v} E_R\right)\frac{v v_{12}}{v_1 v_2}(H_1^0 - i a_1) + H.C..$$

FCNC

Numerical Analysis

$$\begin{cases} M_u = Y_u \frac{v_3}{\sqrt{2}} \\ M_d = M_{d1} + M_{d2} e^{i\delta_{sp}} \end{cases}$$

→ **top quark mass**

down-type quark mass

Couplings around 1 to not upset perturbative calculation

$$v_3 \sim v, \; v_{12}/v_3 \sim m_b/m_t, \; v_2 \sim v_1$$

↓

$$v_2/(v_1 v_{12}) \sim (1/v)(m_t/m_b), \; v_{12}/(v_1 v_2) \sim (2/v)(m_t/m_b)$$

$v_2 \neq v_1$ → **even stronger constraint**

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Flavor Changing Neutral Current (FCNC)

$$V_{i2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s_2^i s_3^i & s_2^i c_3^i \\ 0 & c_2^i s_3^i & -c_2^i c_3^i \end{pmatrix}$$

→ **FCNC between 2 and 3**

↓

$\tau \to \mu\gamma$

$\tau \to \mu\mu\bar{\mu}$

$(g-2)_\mu$

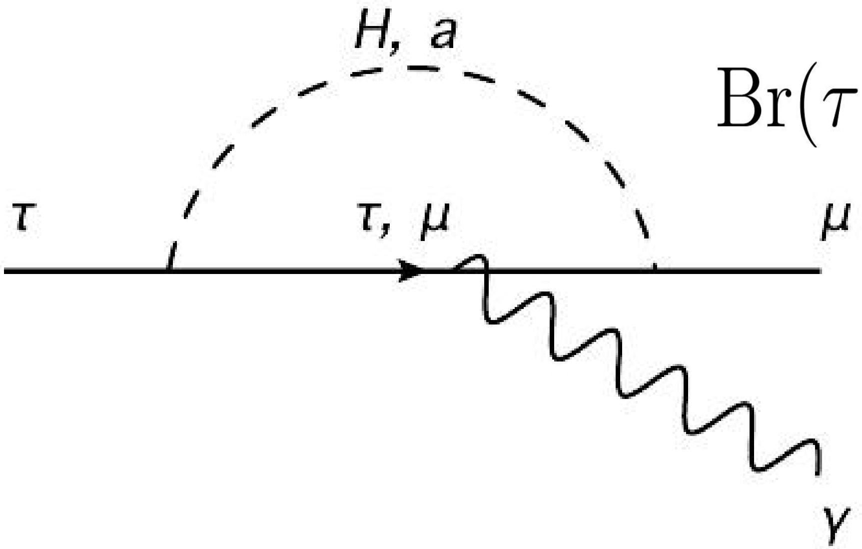
$B_s - \bar{B}_s$

Constrain ←

New scalar mass scale

Radiative Decay

$$\tau \to \mu\gamma$$



$$\mathrm{Br}(\tau \to \mu\gamma)^{\mathrm{exp}} < 4.4 \times 10^{-8},\ \ 90\%\mathrm{CL}$$

M. Tanabashi et al. PDG2018

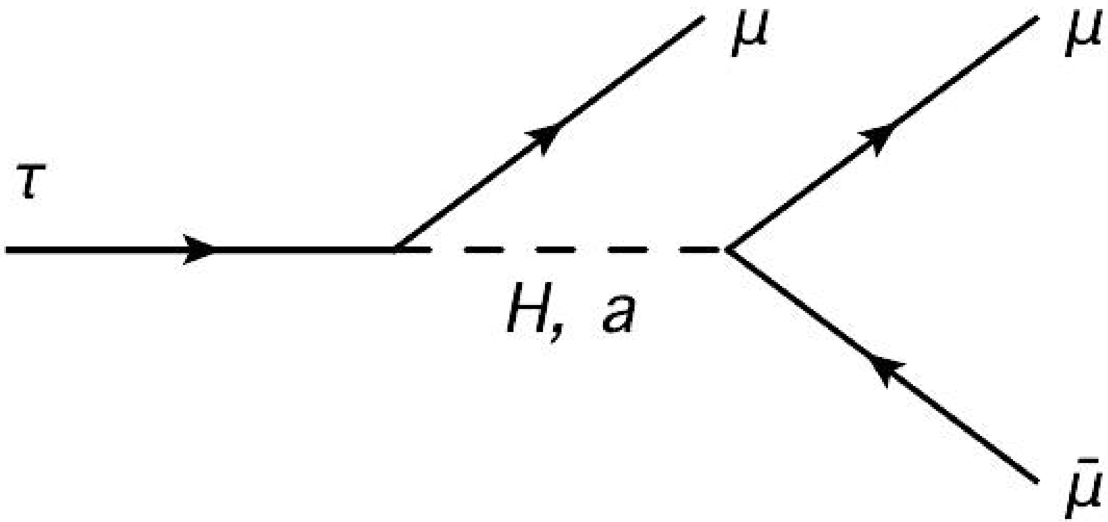
$$m_{a1}, m_{H1}$$

$$\mathrm{Br}(\tau \to \mu\gamma) = t_\tau \frac{\alpha_{\mathrm{em}} m_\tau}{64\pi^4} \frac{(s_2^l c_2^l)^2}{16} \frac{m_\tau^4}{v_1^4} \left(1 - (c_2^l)^2 \frac{v_{12}^2}{v_2^2}\right)^2$$

$$\times \left[\frac{m_\tau^2}{m_{a_1}^2}\left(\ln\frac{m_\tau^2}{m_{a_1}^2} + \frac{5}{3}\right) - \frac{m_\tau^2}{m_{H_1}^2}\left(\ln\frac{m_\tau^2}{m_{H_1}^2} + \frac{4}{3}\right)\right]^2$$

Tri-lepton Decay

$$\tau \to \mu\mu\bar{\mu}$$



$$\mathrm{Br}(\tau \to \mu\mu\bar{\mu})^{\mathrm{exp}} < 2.1 \times 10^{-8},\ 90\%\mathrm{CL}$$

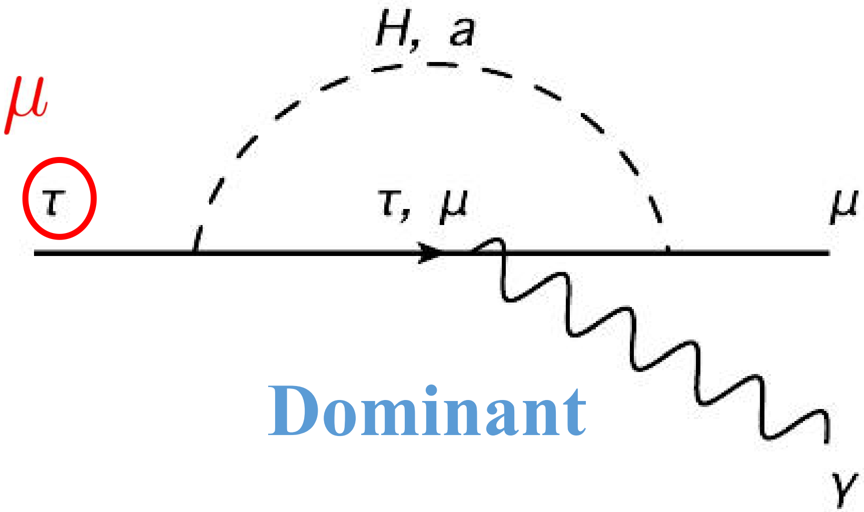
Formula in Backup

$$m_{a1}, m_{H1}$$

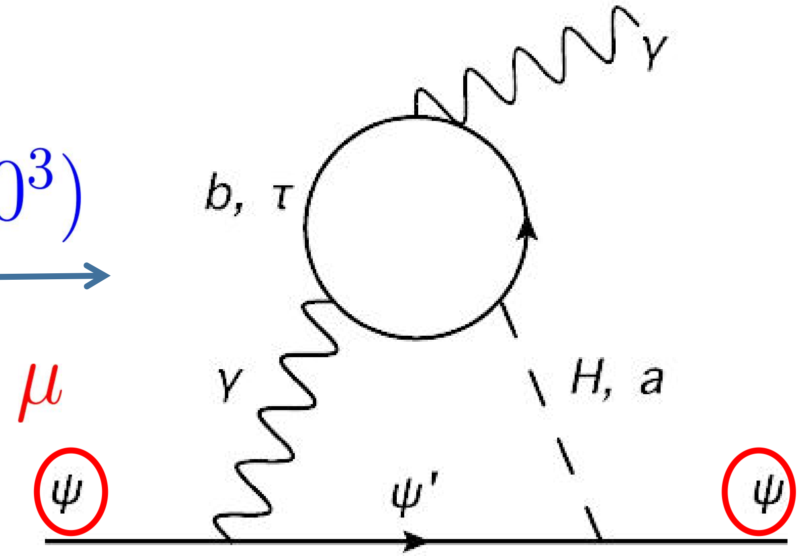
M. Tanabashi et al. PDG2018

Anomalous Magnetic Moment

$$(g-2)_\mu$$



$O(10^3)$ →



$$\Delta a_\mu = (28.02 \pm 7.37) \times 10^{-10} (3.8\sigma)$$

arXiv:1911.00367

$$\Delta a_\mu = (251 \pm 59) \times 10^{-11} (4.2\sigma)$$

Phys. Rev. Lett. 126 (2021) 141801

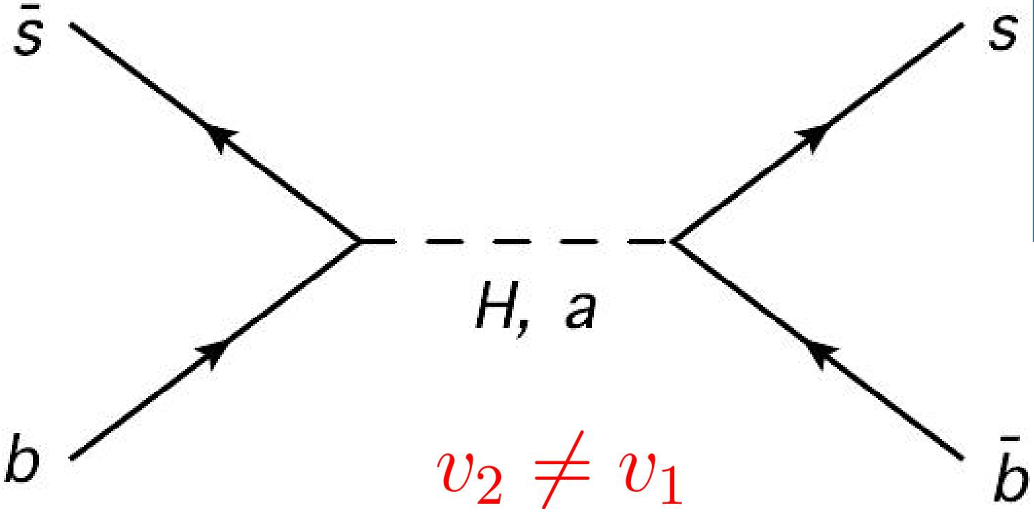
↓

$$m_{a1}, m_{H1}$$

Formula in Backup

B_S Meson Mixing

$$B_s - \bar{B}_s$$



$v_2 \neq v_1$

$$\Delta M_{B_s}^{\text{exp}} = (17.757 \pm 0.021)\text{ps}^{-1}$$

$$\Delta M_{B_s}^{SM} = (17.25 \pm 0.85)\text{ps}^{-1}$$

UTfit

$$m_{a1}, m_{H1}$$

$B_S = 0.835$

$$\Delta M_{B_s}^{NP} = \frac{1}{2}(s_3^q c_3^q)^2 \left(\frac{vv_{12}}{v_1 v_2}\right)^2 \left\{ \frac{5}{12}\left(\frac{1}{m_{H_1}^2} - \frac{1}{m_{a_1}^2}\right) \frac{m_s^2+m_b^2}{v^2} \frac{m_{B_s}^2}{(m_s+m_b)^2} B_S - \left(\frac{1}{m_{H_1}^2} + \frac{1}{m_{a_1}^2}\right) \frac{m_s m_b}{v^2} \left[\frac{m_{B_s}^2}{(m_s+m_b)^2} B_S + \frac{1}{6} B_V\right] \right\} f_{B_s}^2 m_{B_s}$$

$f_{B_s} = 227.2\text{MeV}$

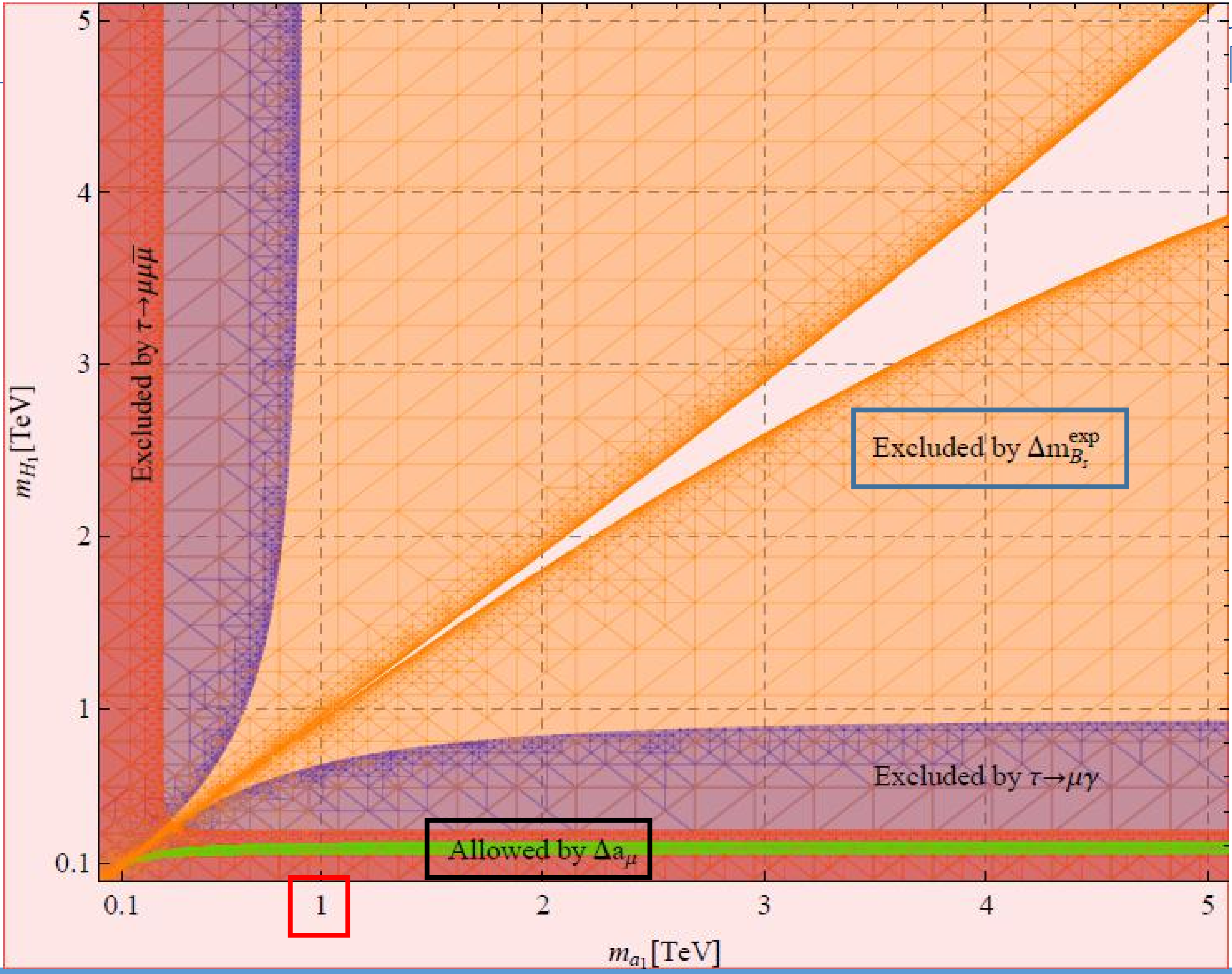
$B_V = 0.849$

arXiv:1909.11087

$$\langle \bar{B}_s | (\bar{b}\gamma_\mu P_L s)(\bar{b}\gamma^\mu P_L s) | B_s \rangle = (2/3) f_{B_s}^2 m_{B_s}^2 B_V$$

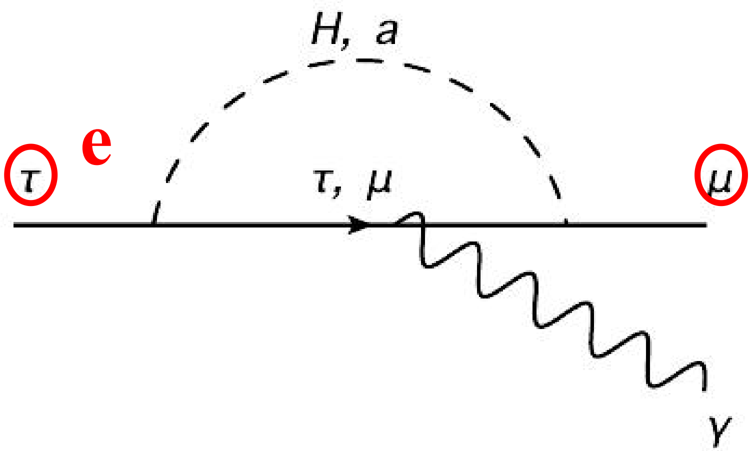
$$\langle \bar{B}_s | (\bar{b} P_R s)(\bar{b} P_R s) | B_s \rangle = -(5/12) \frac{f_{B_s}^2 m_{B_s}^4}{(m_s+m_b)^2} B_S$$

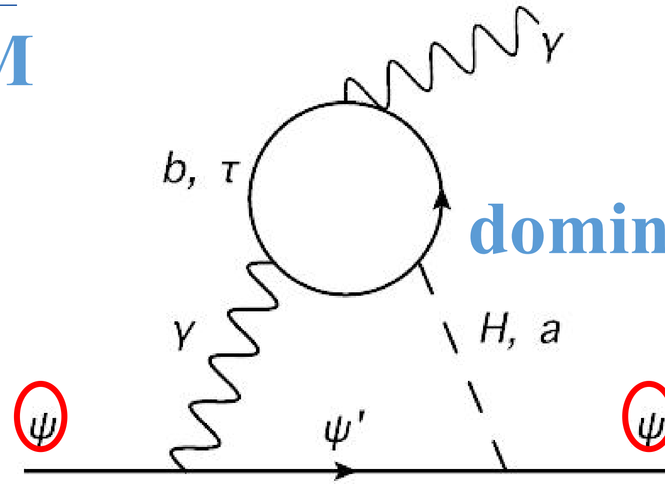
Constraints for FCNC



EDM (eEDM, nEDM)

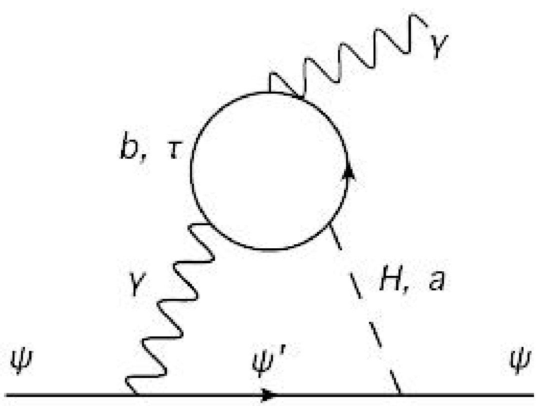
eEDM



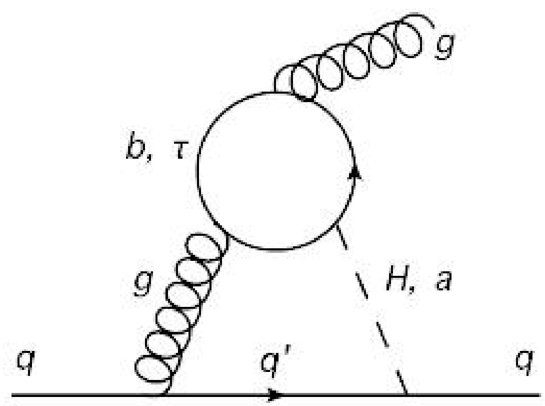


dominant

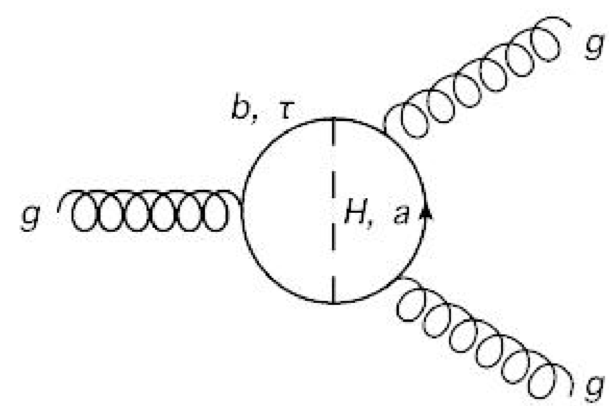
nEDM



Barr-Zee

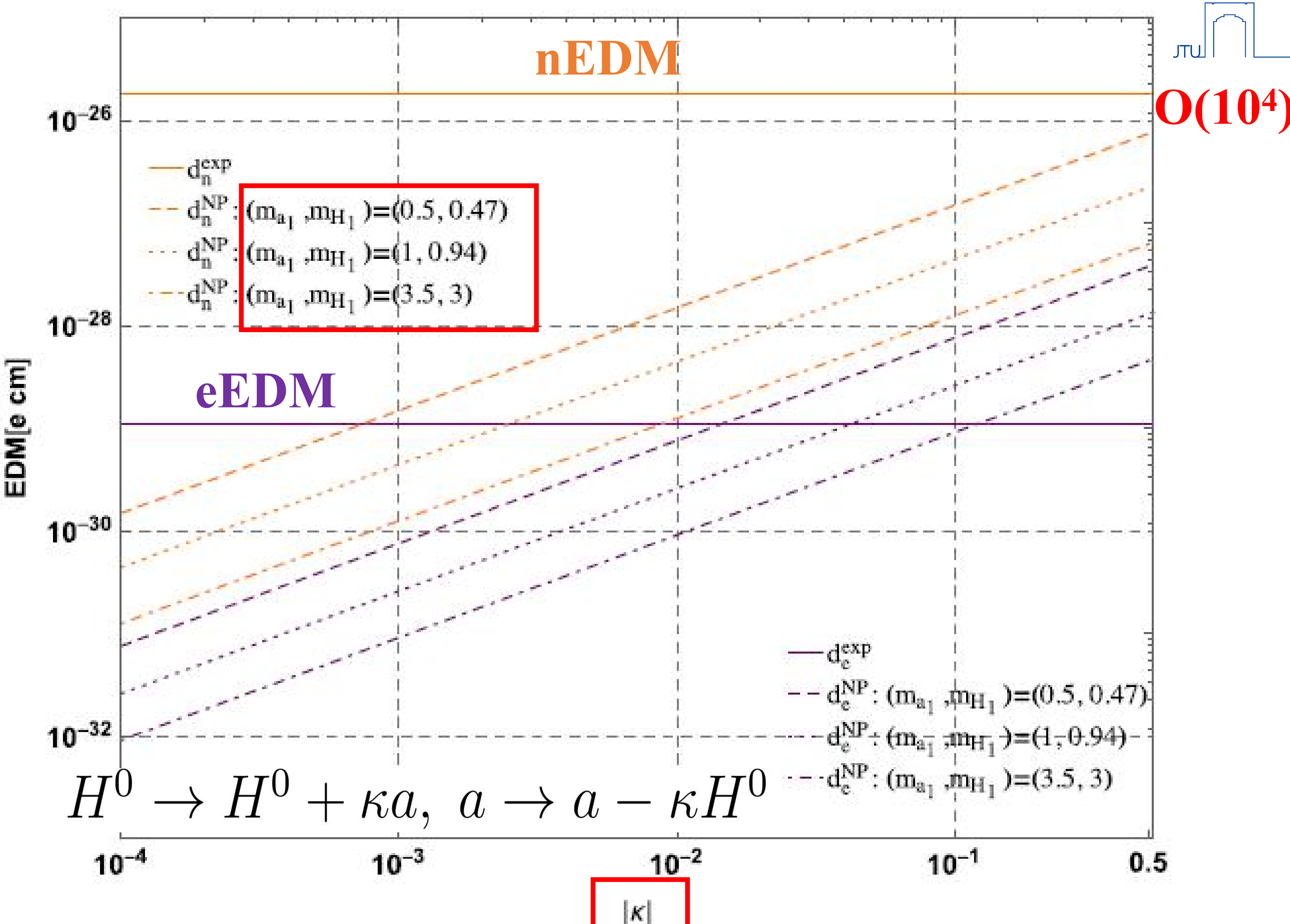


color EDM



Weinberg Operator

EDM


nEDM
O(10^4)
eEDM
d_n^{exp}
d_n^{NP}: $(m_{a_1}, m_{H_1})=(0.5, 0.47)$
d_n^{NP}: $(m_{a_1}, m_{H_1})=(1, 0.94)$
d_n^{NP}: $(m_{a_1}, m_{H_1})=(3.5, 3)$
d_e^{exp}
d_e^{NP}: $(m_{a_1}, m_{H_1})=(0.5, 0.47)$
d_e^{NP}: $(m_{a_1}, m_{H_1})=(1, 0.94)$
d_e^{NP}: $(m_{a_1}, m_{H_1})=(3.5, 3)$
EDM[e cm]
10^{-26}
10^{-28}
10^{-30}
10^{-32}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
0.5
$|\kappa|$
$H^0 \to H^0 + \kappa a,\ a \to a - \kappa H^0$

Conclusion

1. **In KM parameterization, sum rule** $\delta_q + \delta_l = 0$

2. **This sum rule can be accommodated in model with spontaneous CP violation.**

3. $B_s - \bar{B}_s$ **provides very strong constraints on the new neutral scalar mass scales and the model allow (e, n)EDM to reach current upper bounds.**

4. **This model can be tested by future experiments.**

Thanks!

Backup

PDG

$s^q_{12} = 0.22500, (3\sigma : 0.22400, 0.22600)$
$s^q_{23} = 0.04220, (3\sigma : 0.04141, 0.04259)$
$s^q_{13} = 0.003675, (3\sigma : 0.003580, 0.003770)$
$\delta^q_{\rm PDG}/\pi = 0.3717, (3\sigma : 0.3606, 0.3828)$

Utfit

NH/IH

$s^l_{12} = 0.5568, (3\sigma : 0.5244, 0.5916)$
$s^l_{23} = 0.7503, (3\sigma : 0.6580, 0.7804)$
$s^l_{13} = 0.1496, (3\sigma : 0.1430, 0.1560)$
$\delta_{\rm PDG}/\pi = -0.6016, (3\sigma : -1.085, -0.0095)$

$s^l_{12} = 0.5568, (3\sigma : 0.5244, 0.5916)$
$s^l_{23} = 0.7517, (3\sigma : 0.6603, 0.7810)$
$s^l_{13} = 0.1503, (3\sigma : 0.1437, 0.1567)$
$\delta_{\rm PDG}/\pi = -0.4393, (3\sigma : -0.8085, -0.1019)$

Nufit, T2K.

KM

$s^q_1 = 0.2250, (3\sigma : 0.2240, 0.2260)$
$s^q_2 = 0.03863, (3\sigma : 0.03751, 0.03974)$
$s^q_3 = 0.01633, (3\sigma : 0.01584, 0.01683)$
$\delta^q_{\rm KM}/\pi = 0.4950, (3\sigma : 0.4780, 0.5120)$

NH/IH

$s^l_1 = 0.5705, (3\sigma : 0.5383, 0.6048)$
$s^l_2 = 0.7894, (3\sigma : 0.4530, 0.9101)$
$s^l_3 = 0.2622, (3\sigma : 0.2372, 0.2885)$
$\delta^l_{\rm KM}/\pi = -0.5757, (3\sigma : -1, -0.0094)$

$s^l_1 = 0.5706, (3\sigma : 0.5385, 0.6050)$
$s^l_2 = 0.7202, (3\sigma : 0.4677, 0.8882)$
$s^l_3 = 0.2634, (3\sigma : 0.2383, 0.2897)$
$\delta^l_{\rm KM}/\pi = -0.4275, (3\sigma : -0.8603, -0.1004)$

Formula

$$
\begin{aligned}
\Gamma(\tau\to 3\mu) = {} & \frac{\alpha_{\rm em}^2 m_\tau}{3{\cdot}2^8\pi^5}\frac{s_2^2c_2^2}{16}\frac{m_\tau^4}{v_1^4}\left[1-\left(c_2^l\right)^2\frac{v_{12}^2}{v_2^2}\right]^2\left[\frac{m_\tau^2}{m_{a_1}^2}\left(\ln\frac{m_\tau^2}{m_{a_1}^2}+\frac{5}{3}\right)-\frac{m_\tau^2}{m_{H_1}^2}\left(\ln\frac{m_\tau^2}{m_{H_1}^2}+\frac{4}{3}\right)\right]^2\left(4\ln\frac{m_\tau^2}{m_\mu^2}-11\right) \\
& -\frac{\alpha_{\rm em} m_\tau}{3{\cdot}2^8\pi^4}\frac{s_2^2c_2^2}{4}\frac{m_\tau^2m_\mu^2}{v_1^4}\left[1-\left(s_2^l\right)^2\frac{v_{12}^2}{v_2^2}\right]\left(\frac{m_\tau^2}{m_{a_1}^2}+\frac{m_\tau^2}{m_{H_1}^2}\right) * \left\{\frac{1}{6}\left[1-\left(s_2^l\right)^2\frac{v_{12}^2}{v_2^2}\right]\left(\frac{m_\tau^2}{m_{a_1}^2}+\frac{m_\tau^2}{m_{H_1}^2}\right)\right. \\
& \left. +\left[1-\left(c_2^l\right)^2\frac{v_{12}^2}{v_2^2}\right]\left[\frac{m_\tau^2}{m_{a_1}^2}\left(2\ln\frac{m_\tau^2}{m_{a_1}^2}+\frac{7}{2}\right)-\frac{m_\tau^2}{m_{H_1}^2}\left(2\ln\frac{m_\tau^2}{m_{H_1}^2}+\frac{5}{2}\right)\right]\right. \\
& +\frac{m_\tau s_2^2c_2^2}{3(16\pi)^3}\frac{m_\tau^2m_\mu^2}{v_1^4}\left[1-\left(s_2^l\right)^2\frac{v_{12}^2}{v_2^2}\right]^2\left[2\left(\frac{m_\tau^2}{m_{a_1}^2}+\frac{m_\tau^2}{m_{H_1}^2}\right)^2+\left(\frac{m_\tau^2}{m_{a_1}^2}-\frac{m_\tau^2}{m_{H_1}^2}\right)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
\Delta a_\mu = {} & \frac{1}{24\pi^2}\frac{m_\mu^4 v_2^2}{v_1^2 v_{12}^2}\left(1-s_2^2\frac{v_{12}^2}{v_2^2}\right)^2\left[\frac{1}{m_{H_1}^2}+\frac{1}{m_{a_1}^2}-\frac{3}{m_{H_1}^2}\left(\ln\frac{m_\mu^2}{m_{H_1}^2}+\frac{3}{2}\right)+\frac{3}{m_{a_1}^2}\left(\ln\frac{m_\mu^2}{m_{a_1}^2}+\frac{3}{2}\right)\right] \\
& +\frac{s_2^2c_2^2}{48\pi^2}\frac{m_\mu^4 v_{12}^2}{v_1^2v_2^2}\left[\left(1+\frac{m_\tau^2}{m_\mu^2}\right)\left(\frac{1}{m_{H_1}^2}+\frac{1}{m_{a_1}^2}\right)-\frac{m_\tau^2}{m_\mu^2}\left(\frac{6}{m_{\vec{H}_1}^2}\left(\ln\frac{m_\tau^2}{m_{H_1}^2}+\frac{3}{2}\right)-\frac{6}{m_{a_1}^2}\left(\ln\frac{m_\tau^2}{m_{a_1}^2}+\frac{3}{2}\right)\right)\right]
\end{aligned}
$$

Electric Dipole Moment (EDM)

CPV in Higgs potential

$$(H_1^\dagger H_2)^2 e^{-i2\delta_{sp}},\ (H_1^\dagger H_1)(H_1^\dagger H_2)e^{-i\delta_{sp}},\ (H_2^\dagger H_2)(H_1^\dagger H_2)e^{-i\delta_{sp}}$$

$$(H_3^\dagger H_3)(H_1^\dagger H_2)e^{-i\delta_{sp}},\ (H_3^\dagger H_2)(H_1^\dagger H_3)e^{-i\delta_{sp}}$$

$$H^0_{1,2,3} - a_{1,2}$$

Parameterizing mixing

$$H_i^0 \to H_i^0 + \kappa_{ij} a_j,\ a_i \to a_i - \kappa_{ji} H_j^0,\ \kappa_{ij} = m_{ij}^2/(m_{a_j}^2 - m_{H_i}^2)$$

EDM

EDM

$$d_e^{1L} = -\frac{e\kappa}{8\pi^2}\frac{m_e v_2^2}{v_1^2 v_{12}^2}\left[\frac{m_e^2}{m_{H_1}^2}\left(\ln\frac{m_e^2}{m_{H_1}^2}+\frac{3}{2}\right)-\frac{m_e^2}{m_{a_1}^2}\left(\ln\frac{m_e^2}{m_{a_1}^2}+\frac{3}{2}\right)\right]$$

$$d_e^{2L} = \frac{e\alpha_{\rm em}}{96\pi^3}m_e\left[G(m_b,e)+3G(m_\tau,e)\right]$$

$$G(m,e) = 2\frac{\kappa v_2^2}{v_1^2 v_{12}^2}\left(1-c_m^2\frac{v_{12}^2}{v_2^2}\right)\left[f\left(\frac{m^2}{m_{a_1}^2}\right)+g\left(\frac{m^2}{m_{a_1}^2}\right)-f\left(\frac{m^2}{m_{H_1}^2}\right)-g\left(\frac{m^2}{m_{H_1}^2}\right)\right]$$

$$f(z) = \frac{z}{2}\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z} \qquad g(z) = \frac{z}{2}\int_0^1 \mathrm{d}x\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z}$$

$$d_n \approx \eta_d\left(\frac{4}{3}d_d-\frac{1}{3}d_u\right)_\Lambda + e\eta_f\left(\frac{4}{9}d_d+\frac{2}{9}f_u\right)_\Lambda + ef_\pi\xi C_W$$

$$d_d^{2L} = \frac{e\alpha_{\rm em}}{288\pi^3}m_d\left[G(m_b,d)+3G(m_\tau,d)\right], \quad f_d^{2L} = -\frac{\alpha_s}{64\pi^3}m_d G(m_b,d)$$

$$C_W^{2L} = -\frac{1}{4\pi}\frac{\kappa v_2^2}{}\mathrm{v}_1^2 v_{12}^2\left(1-(c_3^q)^2\frac{v_{12}^2}{v_2^2}\right)^2\left[h\left(\frac{m_b^2}{m_{a_1}^2}\right)-h\left(\frac{m_b^2}{m_{H_1}^2}\right)\right]$$

$$h(z) = \frac{z^2}{2}\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}u\frac{u^3x^3(1-x)}{[zx(1-ux)+(1-u)(1-x)]^2}$$

$$\eta_d \approx 0.166, \eta_f \approx 0.0117, \xi \approx 1.2\times 10^{-4}$$

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