Light-cone distribution amplitudes of pion from the electromagnetic form factor

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Overview

Background-Light cone (relativistic) wave function (LCDA)

Applying pQCD theory on hadron involved observations,
hadron is presented by LCDA, PDA, PDF

- $\phi_\pi(x,\mu) = 6x(1-x)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(x)$, Gegenbauer expansion.
- Asymptotic behaviour ($Q^2 \to \infty$): a_0^π is determined by f_π;
  Correlation: $a^\pi_{n\geqslant 2}(\mu)$ rely on the non-perturbative theory and lattice QCD.
- The first few a_n^π is of utmost importance to an accuracy description of pion.

$\phi_\pi(x,\mu) = 6x(1-x)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(x)$

- Definition in QCD

$$a_n^\pi(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots |0\rangle \,, \qquad (1)$$

- LQCD: $a_2^\pi(1GeV) = 0.334 \pm 0.129$ [RBC and UKQCD, 1011.5906]
  0.135 ± 0.032 [RQCD, 1903.08038]

  a_4^π is not available $\leftarrow$ the growing number of derivatives in $q\bar{q}$ operator.
  new technique [RQCD, 1709.04325, 1807.06671] .

- 2pSRs: $a_2^\pi(1GeV) = 0.19 \pm 0.06$ [Chernyak et.al., Phys.Rept. 112 (1984) 173]
  $0.26^{+0.21}_{-0.09}$ [Khodjamirian et.al., 0407226]
  0.28 ± 0.08 [Ball et.al., 0603063]

  $a_{n>2}^\pi$, model dependent, nonlocal vacuum condensate [Bakulev et.al., 0103119]

‡ QCD sum rules as an inverse problem, [Li et.al., 2006.16593]
quark-hadron duality $\rightarrow$ *Legendre expansion of spectral density*

Background-a_n^π

$\phi_\pi(x,\mu) = 6x(1-x)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(x)$

- $a_2^\pi(1GeV)$ obtained from LCSRs VS Data

$F_{B\to\pi}$: 0.19 ± 0.19 [Ball et.al., 0507076]
0.16 [Khodjamirian et al., 1103.2655], uncertainty from B meson.

$F_{\pi\gamma\gamma^*}$: 0.14, BABAR+CLEO data [Agaev et.al., 1012.4671]
0.10, Belle+CLEO data [Agaev et al., 1206.3968]

  * large uncertainty is obtained for the result of $a_{n>2}^\pi$.
  * discrepancy between different experiments result, especially at large Q^2.

F_π: 0.24 ± 0.17, Wilson Lab+NA7 data [Bebek et al., PRD17(1978)1693]
0.20 ± 0.03, Wilson Lab+JLab data [Agaev, 0509345]

  * large uncertainty is obtained for the result of $a_{n>2}^\pi$.
  * LCSRs is applicable for F_π when $|q^2| \in [1, 10]$ GeV^2, the measurement at JLab is $|q^2| \leqslant 2.5$ GeV^2, JLab 12 GeV (9 GeV^2).

Background-a_n^π

- Is there any observables which are theory clean and experiment precise ?
- The measurements of F_π at B factories provide an opportunity.

  BABAR: $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 [J. Lees et al., 1205.2228]
  Belle: $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2 [M. Fujikawa et al., 0805.3773]
  BESIII: $e^+e^-(\gamma) \to \pi^+\pi^-$, $0.6 \leqslant q^2 \leqslant 0.9\,\mathrm{GeV}^2$ [M. Ablikim et al., 1507.08188, 2009.05011]
- BABAR measurement in timelike region, LCSR study in spacelike region, dispersion relation (DR) to relate the FF in two different regions.

Dispersion relation (DR)

The standard DR

$$F_\pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im} F_\pi(s)}{s - q^2 - i\epsilon}\,, \quad q^2 < 0\,, \tag{2}$$

- integral over $\mathrm{Im} F_\pi(s)$ in $s > 4m_\pi^2$,
- the direct measurement at BABAR is $|F_\pi(s)|^2$,
- model dependence to parameterize $F_\pi(s)$ to reproduce $|F_\pi(s)|$, additional uncertainty.

The modulus representation of DR

$$\frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0 - q^2}} = \frac{1}{2\pi}\int_{s_0}^{\infty} \frac{ds \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}, \qquad q^2 < s_0, \tag{3}$$

$$\Leftrightarrow \qquad F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} \frac{ds \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right], \quad q^2 < s_0. \tag{4}$$

The modulus representation of DR

- obtained by introducing an auxiliary function $g_\pi(q^2) \equiv \frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0 - q^2}}$.
  [Geshkenbein, 9806418]
- derived by Cauchy theorem and Schwartz reflection principle for g_π.
- The only assumption: $F_\pi(q^2)$ is free from zeros in the complex q^2 plane. then $\ln F_\pi(q^2)$ does not diverge. [Leutwyler, 0212324] [Ananthanarayan et.al., 1102.3299]
- If $F_\pi(q^2)$ has zeros in the complex q^2 plane, deserves a separate analysis.
  [Dominguez, 0102190] [Ananthanarayan, et al., 0409222]
- $F_\pi(q^2)$ evaluated by the standard and modulus DR have a tiny difference, $\rightarrow$ the zeros of $F_\pi(q^2)$ are either absent or their influence is beyond our accuracy.

$F_\pi(|q^2|)$ from LCSRs

$$F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0-q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\ \ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}\right], \quad q^2 < s_0$$

LCSRs on $F_\pi(|q^2|)$: energetic pion [Bijnens et.al., 0206252] [Braun et.al., 9907495]

- soft dominated, one quark carries almost the whole momentum.
- asymptotic LCDA, asymptotic result of pQCD.
- $F_\pi(Q^2 = |q^2|)$ in terms of LCDA, Gegenbauer moments dependence.
- accuracy:

$$F_\pi^{(\mathrm{LCSR})}(Q^2) = F_\pi^{(\mathrm{as})}(Q^2) + \sum_{n=2,4,..} a_n(\mu_0)\, f_n(Q^2,\mu,\mu_0)\,, \tag{5}$$

$$F_\pi^{(\mathrm{as})}(Q^2) = F_\pi^{(\mathrm{tw2,as})}(Q^2) + F_\pi^{(\mathrm{tw4,\ LO})}(Q^2) + F_\pi^{(\mathrm{tw6,\ fact})}(Q^2) \tag{6}$$

the coefficient functions $f_n(Q^2,\mu,\mu_0)$ are the integral of Gegenbauer polynomials with the Borel exponent.

$F_\pi(s)$ from the BABAR measurement, $[4m_\pi^2, 2.95^2]$ GeV^2

$$F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0-q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\,\ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}\right], \quad q^2 < s_0$$

$$F_\pi^{data}(s) = \sum_{n=0,\cdots}^{N}\frac{c_n^\pi BW_n^{GS}(s)}{c_n^\pi}, \quad c_0^\pi \to \frac{1+c_\omega^\pi\,BW_n^{KS}(s)}{1+c_\omega^\pi}, \tag{7}$$

$$BW_{\rho_n}^{GS}(s) = \frac{m_n^2 + m_n\Gamma_n d(m_n)}{m_n^2 - s + f(s) - i\sqrt{s}\Gamma_n(s)}, \quad BW_\omega^{KS}(s) = \frac{m_\omega^2}{m_\omega^2 - s - i\,m_\omega\Gamma_\omega} \tag{8}$$

- Vector dominate model (VDM), with the modified Breit-Winger formula, Gounaris-Sakuria (GS) and Kühn-Santamaria (KS) representations,
  [Gounaris et.al.,, Phys.Rev.Lett. 21(1968)244] [Kühn et.al., Z.Phys.C48(1990)445]
- $N = 4\,\&\,\rho - \omega$ interaction, describes the BABAR data by 18 parameters.
  [J. Lees et al., 1205.2228]

High energy tail of $F_\pi(s)$, $[2.95^2, \infty)$ GeV^2

- DR is written in the integral of whole timelike region $[4m_\pi^2, \infty)$, the high energy tail should be considered, even though small.
- The application of $F_\pi^{data}(s)$ at high energy is not physical: resonances above $N = 4$ are not included, $F_\pi^{data}(s) \nrightarrow 1/s$ at adequate large s.
- The dual-resonance models and $N_c = \infty$ limit of QCD.

[Dominguez, 0102190] [Bruch et.al., 0409080]

$$F_\pi^{(tail)}(s) = F_\pi^{(dQCD)}(s) = \sum_{n=0}^{\infty} c_n \; BW_n(s) \,, \quad BW_n(s) = BW_n^{KS}(s), \tag{9}$$

$$c_n = \frac{(-1)^n \Gamma(\beta - 1/2)}{\alpha' m_n^2 \sqrt{\pi}\, \Gamma(n+1)\Gamma(\beta - 1 - n)} \,, \quad m_n^2 = m_\rho^2(1+2n) \,. \tag{10}$$

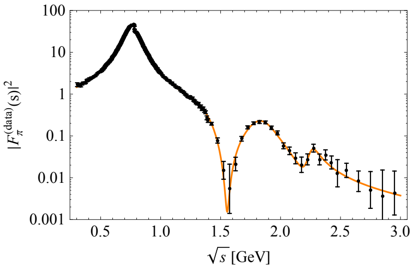
- $\alpha' = 1/2m_\rho^2$, $\Gamma_n = \gamma\, m_n$, $\gamma = 0.193$ is adjusted to the total width of $\rho(770)$.
- Matching at $|F_\pi^{(data)}(s_{max})| = |F_\pi^{(tail)}(s_{max})|$ indicates $\beta = 2.09 \pm 0.13$.

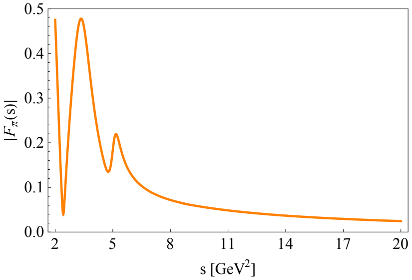
† reproduce $F_\pi^{(\mathrm{dQCD})}(0) = 1$ and $\lim_{s\to -\infty} F_\pi^{(\mathrm{dQCD})}(s) \sim 1/s^{\beta-1}$.

$F_\pi(s)$: $[4m_\pi^2, 2.95^2] + [2.95^2, \infty)$ GeV^2

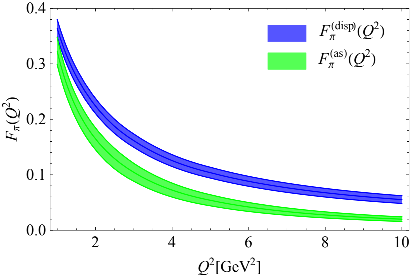
$$F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0-q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\ \ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}\right], \quad q^2 < s_0$$

$$|F_\pi(s)| = \Theta(s_{\rm max} - s)\,|F_\pi^{(\rm data)}(s)| + \Theta(s - s_{\rm max})\,|F_\pi^{(\rm tail)}(s)|\,. \tag{10}$$





$F_\pi(|q^2|)$ obtained from asymptotic LCSRs and DR



- $$F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0-q^2}}{2\pi}\int_{s_0}^{\infty}\frac{ds\ \ln|F_\pi(s)|^2}{s\sqrt{s-s_0}\,(s-q^2)}\right], \quad q^2 < s_0$$
- $$F_\pi^{(\text{LCSR})}(Q^2) = F_\pi^{(\text{as})}(Q^2) + \sum_{n=2,4,..} a_n(\mu_0)\, f_n(Q^2,\mu,\mu_0)\,,$$
  $$F_\pi^{(\text{as})}(Q^2) = F_\pi^{(\text{tw2,as})}(Q^2) + F_\pi^{(\text{tw4, LO})}(Q^2) + F_\pi^{(\text{tw6, fact})}(Q^2)$$
- the 2nd term in $F_\pi^{(\text{LCSR})}(Q^2)$ gives significant contribution
- the asymptotic LCDA of pion at leading twist is not enough

- χ^2 fit to reveal the more inner structures of pion meson,

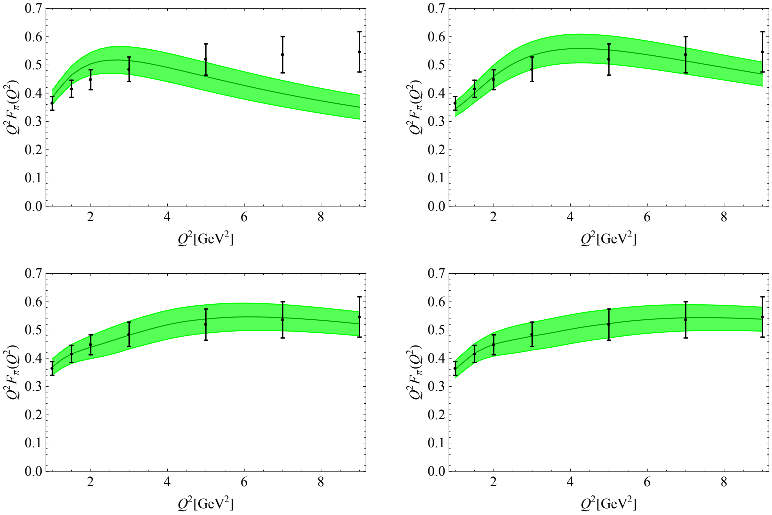
$$\chi^2 = \sum_{i=1}^{N_p} \frac{1}{\sigma_i^2} \left[\sum_{n=2,4,..}^{n_{\max}} a_n(\mu_0)\, f_n(Q_i^2, \mu_0) + F_\pi^{(\mathrm{as})}(Q_i^2) - F_\pi^{(\mathrm{disp})}(Q_i^2) \right]^2 . \qquad (11)$$

- $a_2^\pi(1\ GeV)$ obtained in our formalism,

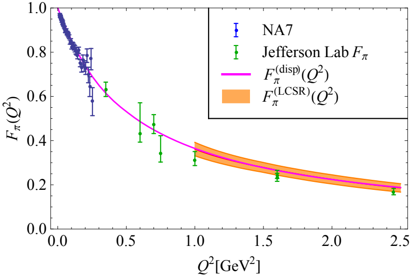
Model	$a_2(1\,\mathrm{GeV})$	$a_4(1\,\mathrm{GeV})$	$a_6(1\,\mathrm{GeV})$	$a_8(1\,\mathrm{GeV})$	$\chi^2_{\min}/\mathrm{ndf}$
$\{a_2\}$	0.302 ± 0.046				4.08
$\{a_2, a_4\}$	0.279 ± 0.047	0.189 ± 0.060			0.75
$\{a_2, a_4, a_6\}$	0.270 ± 0.047	0.179 ± 0.060	0.123 ± 0.086		0.073
$\{a_2, a_4, a_6, a_8\}$	0.269 ± 0.047	0.185 ± 0.062	0.141 ± 0.096	0.049 ± 0.116	0.013

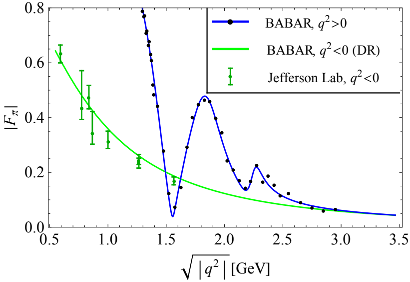
$\approx -15\%$ correlations are found between different moments.

Result of $a_n^\pi(1\ GeV)$



Performing with the direct measurement





- $|F_\pi(s)| \sim |F_\pi^{(disp)}(|q^2|)|$ at $|\sqrt{q^2}| \gtrsim 3\,GeV$,
- manifests analyticity of the modulus representation.

- Modulus representation of DR, LCSRs calculation + BABAR data,

  $a_2(1\,\mathrm{GeV}) = (0.22 - 0.33)\,, \quad a_4(1\,\mathrm{GeV}) = (0.12 - 0.25)\,.$

- Pion deviates from the purely asymptotic one,
  a_2^π is not enough, more inner structures.

† the role of form factor zeros.

† contribution from the high-energy tail

† model-independent study of F_π and $F_{\pi\gamma\gamma^*}$.

The End, Thanks.

Review of $F_\pi(Q^2)$ measurements, [K.K. Seth, arXiv:1401.7054[hep-ex]]

- Cyclotron Lab, $Q^2 = 0.176\mathrm{GeV}^2$ in electron production;
- NOVOSIBIRSK and ORSAY, $0.64 \leqslant Q^2 \leqslant 1.40\,\mathrm{GeV}^2$ & $1.35 \leqslant Q^2 \leqslant 2.38\,\mathrm{GeV}^2$, e^+e^- annihilation .
- CLEO, $Q^2 = 9.6, 13.48\,\mathrm{GeV}^2$.
- Belle, $4m_\pi^2 \leqslant Q^2 \leqslant 3.125\,\mathrm{GeV}^2$, $\tau^- \to \pi^-\pi^0\nu_\tau$ decay.
- BABAR, $4m_\pi^2 \leqslant Q^2 \leqslant 3.1^2\,\mathrm{GeV}^2$, $e^+e^- \to \pi^+\pi^-(\gamma)$.
- BESIII, $0.6 \leqslant Q^2 \leqslant 0.9\,\mathrm{GeV}^2$, $e^+e^-(\gamma) \to \pi^+\pi^-$.

Review of $F_\pi(q^2)$ measurements, [M.R. Whalley, et al., J.Phys. G 29 (2003) A1]

- Harvard & Cornell, $0.15 \leqslant |q^2| \leqslant 10\mathrm{GeV}^2$;
- DESY, $|q^2| = 0.35, 0.70\mathrm{GeV}^2$, electron production;
- Jefferon Lab F_π, $0.6 \leqslant |q^2| \leqslant 1.6\mathrm{GeV}^2, |q^2| = 2.45\mathrm{GeV}^2$.
  12 GeV^2 upgrade, to $9\,\mathrm{GeV}^2$.

Review of $F_\pi(q^2)$ theory

† Lattice QCD (LQCD): a few points of small Q^2

[HPQCD, UKQCD, MILC and Fermilab Lattice Collaborations 2004]
[QCDSF/UKQCD 2007; ETM 2009; JLQCD and TWQCD 2009; JLQCD 2016, et.al.,]
up to 1 GeV^2, [χQCD 2020]

† QCD-based approaches

DSE: 0-6 GeV^2; [Z.F Cui 2020, M-Y Chen 2018]
QCDSR: 1-2 GeV^2; [B.L Ioffe 1982]
LCSRs: 1-15 GeV^2; [P. Ball 1991, V. Braun 1994,2000, SC 2020]
pQCD: $\gtrsim$ 10 GeV^2; [Li 2001, Li 2012, SC 2014]

Light cone (relativistic) wave function (LCDA)

- DA: matrix element of nonlocal light-ray operator sandwiched between $\langle h|$ and $|0\rangle$.
- Physics: transparent in the infinite reference frame $P^3 \to \infty$ $(P_\perp = 0)$,
  hadron momentum $P = (P^+, P^-, P_\perp)$,
  x_i distribution of partons in a hadron at small b_i.
- Isochronous hadron Bethe-Salpeter wave function $\phi_{BS}(x_i, k_{\perp i}, \lambda_i)$,
  $\sum_{i=1}^n k_{\perp i} = 0, \quad \sum_{i=1}^n x_i = 1.$
- Light cone gauge $A^+ = A^0 + A^3 = 0$, physical partons,
  LCDA relates to the k_T integrals of ϕ_{BS},

$$\psi_n(x_i, \lambda_i) = \int^{|k_{\perp i}|<\mu} d^2 k_{\perp i}\, \phi_{BS}(x_i, k_{\perp i}, \lambda_i) \tag{12}$$

- ie., $|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$.
  for each component n, $\psi_n^\pi(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i|\pi\rangle$
- At large Q^2, $k_\perp$ can been neglected/integrated $\psi_n^\pi(x_i, \lambda_i)$,
  put out the spin and obtain the LCDA $\psi_n^\pi(x_i, Q)$.
- Corrections $\mathcal{O}(k_\perp^2/Q^2, m^2/Q^2, \alpha_s)$, scale dependence,
  RGE with the general solution in terms of Gegenbauer polynomials.