



## $\left|\mathrm{V}_{\mathrm{ub}}\right|$ Measurements over Time



# $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$ measurements at LHCb and Belle (II) 

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Michel De Cian, EPFL<br>on behalf of the LHCb and Belle (II) collaborations

## in the main roles:

## The Belle (II) experiment at the $e^{+} e^{-}$SuperKEK B accelerator.


and: The LHCb experiment at the $p p$ accelerator LHC.

## Measuring $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$ at LHCb and Belle (II)

- Two main ways to measure $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$
- Exclusive:
- Focus on a single final state, e.g. $B^{+} \rightarrow \pi^{+} \mu^{-} \bar{\nu}_{\mu}$ or $B^{0} \rightarrow D^{*-} \mu^{+} \nu_{\mu}$.
- Challenges: Possibly small signal yield, knowledge of hadronic form factors.
- Inclusive:
- Consider "all" final states, e.g. $B^{+} \rightarrow X_{c} \mu^{-} \bar{\nu}_{\mu}$ or $B^{0} \rightarrow X_{u} \mu^{+} \nu_{\mu}$.
- Challenges: Background contamination, shape functions, ...

Table: Matrix of success

|  | Belle (II) | LHCb |
| :---: | :---: | :---: |
| exclusive | $\left\|V_{c b}\right\|$ ) | $\left\|V_{c b}\right\|$ ) |
|  | $\left\|V_{u b}\right\|$ ) | $\left\|V_{u b}\right\|$ ) |
| inclusive | $\left\|V_{c b}\right\|$ ) | $\left.\left\|V_{c b}\right\|\right)^{*}$ |
|  | $\left\|V_{u b}\right\|$ ) | $\left.\left\|V_{u b}\right\|\right)^{*}$ |



## Exclusive measurements - | $V_{c b} \mid$

- $\frac{d^{4} \Gamma\left(B \rightarrow D^{* 0} \mu \nu\right)}{d w d \Omega}=\frac{3 m_{B}^{3} m_{D^{* 0}}^{2} G_{F}^{2}}{16(4 \pi)^{4}} \eta_{E W}^{2}\left|V_{c b}\right|^{2}|\mathcal{A}(w, \Omega)|^{2}, w=\frac{m_{B}^{2}+m_{D^{*}}^{2}-q^{2}}{2 m_{B} m_{D^{* 0}}}$
- Helicity amplitudes in $\mathcal{A}(w, \Omega)$ depend on 3 form factors: $h_{A_{1}}(w), R_{1}(w), R_{2}(w)$
- External input: $\eta_{E W}=1.0066$

CLN parametrisation $\rightarrow 4$ free parameters: $\rho^{2}, h_{A_{1}}, R_{1}(1), R_{2}(1)$ [Nucl. Phys. B530, 153(1998)]

$$
\begin{aligned}
h_{A_{1}}(w) & =h_{A_{1}}(1)\left(1-8 \rho^{2} z+\left(53 \rho^{2}-15\right) z^{2}-\left(231 \rho^{2}-91\right) z^{3}\right) \\
R_{1}(w) & =R_{1}(1)-0.12(w-1)+0.05(w-1)^{2} \\
R_{2}(w) & =R_{2}(1)-0.11(w-1)-0.06(w-1)^{2}
\end{aligned}
$$

BGL parametrisation $\rightarrow$ Converging series [PRL 74, 4603(1995)]

$$
\begin{array}{rlr}
f(z) & =\frac{1}{P_{1^{+}}(z) \phi_{f}(z)} \sum_{n=0}^{\infty} b_{n} z^{n} & z=\frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}} \\
g(z) & =\frac{1}{P_{1^{-}}(z) \phi_{g}(z)} \sum_{n=0}^{\infty} a_{n} z^{n} & \\
\mathcal{F}_{1}(z) & =\frac{1}{P_{1^{+}}(z) \phi_{\mathcal{F}_{1}}(z)} \sum_{n=0}^{\infty} c_{n} z^{n}
\end{array}
$$

## Exclusive measurements - | $V_{u b} \mid$

- $\frac{d \Gamma(B \rightarrow \pi \ell \nu)}{d q^{2}}=\frac{G_{F}^{2}\left|V_{u b}\right|^{2}}{14 \pi^{3}}\left|p_{\pi}\right|^{3}\left|f_{+}\left(q^{2}\right)\right|^{2}$
- Less data available than for exclusive $\left|V_{c b}\right|$ measurements: Use theoretical input for form factors (and use different parametrisations), often a mix of LQCD (high $q^{2}$ ) and LCSR (low $q^{2}$ ).
- Different $b \rightarrow u$ transitions have different FF uncertainties and experimental challenges.


- Not discussing purely leptonic decays


## Inclusive measurements of $\left|V_{u b}\right|$ and $\left|V_{c b}\right|$

$\left|V_{c b}\right|$

- $\Gamma=\frac{G_{F}^{2} m_{b}^{5}}{192 \pi^{3}}\left|V_{c b}\right|^{2}\left(1+\frac{c_{5}(\mu)\left\langle O_{5}(\mu)\right\rangle}{m_{b}^{2}}+\frac{c_{6}(\mu)\left\langle O_{6}(\mu)\right\rangle}{m_{b}^{3}}+\mathcal{O}\left(\frac{1}{m_{b}^{4}}\right)\right)$
- HQE with non-perturbative coefficients, determined with moments of the lepton energy / $q^{2}$ / the hadronic mass of $B \rightarrow X_{c} \ell \nu$
$\left|V_{u b}\right|$
- Dominated by feed-down from $X_{c}$ : Select region in lepton energy / $m_{u}$ inaccessible to $B \rightarrow X_{c} \ell \nu$ decays.
- Use "shape-functions" to describe dynamics of the $b$ quark inside the hadron: Different models used with different uncertainties in the perturbative and non-perturbative parameters.



## Inclusive $\left|V_{c b}\right|$ using higher-order terms

- Use a novel strategy to measure $\left|V_{c b}\right|$ with higher-order terms: Exploit parametrization invariance to reduce number of non-perturbative elements in the expansion. [JHEP 02 (2019) 177]
- Holds only for some observables, e.g. $\left\langle q^{2}\right\rangle$ (but not for $\left\langle M_{X}\right\rangle$
- Reduces \# parameters at order 4 from $13 \rightarrow 8$.
- $\therefore$ Measure $\left\langle q^{2}\right\rangle^{1 \ldots 4}$ with Belle data
- Use hadronic tagging to fully reconstruct tag-side kinematics, using Full Reconstruction (FR).




Inclusive $\left|V_{c b}\right|$ using higher-order terms


- Using simulation, correct for:
- Difference between true and reconstructed $\left\langle q^{2}\right\rangle$

- Biases from the calibration procedure
- Biases from the reconstruction.
- as a function of the $q_{\text {cut }}^{2}$ value.
- Good agreement between electron and muon results
$\rightarrow$ Combine for the moments determination.


## Inclusive $\left|V_{c b}\right|$ using higher-order terms

In collaboration with M. Fael, K. Olschewsky, K. Vos



- $\left|V_{c b}\right|=(41.7 \pm 1.2) \cdot 10^{-3}$ (preliminary)


## Towards inclusive $\left|V_{c b}\right|$ with Belle II





- Similar strategy as in the Belle measurement, but much smaller dataset: Measure $\left\langle M_{X}^{1 \ldots}{ }^{1}\right\rangle$.
- Use hadronic tagging with Full Event Interpretation (FEI), reconstruct a high-momentum lepton and use the rest of the event to construct $M_{X}$.
- Similar correction procedure as for the Belle measurement.


## Towards inclusive $\left|V_{c b}\right|$ with Belle II






Results in agreement with earlier measurements but not competitive yet

## Exclusive $\left|V_{c b}\right|$ using $B_{s}^{0}$ decays

LHCb Simulation



- Use the decay $B_{s}^{0} \rightarrow D_{s}^{(*)-} \mu^{+} \nu_{\mu}, D_{s}^{-} \rightarrow K^{+} K^{-} \pi^{-}$to measure $\left|V_{c b}\right|$ exclusively.
- Normalize to $B^{0} \rightarrow D^{(*)-} \mu^{+} \nu_{\mu}, D^{-} \rightarrow K^{+} K^{-} \pi^{-}$
- $B_{s}^{0}$ mesons abundantly produced at LHCb .
- Use the corrected mass variable $m_{\text {corr }}=\sqrt{m_{v i s}^{2}+p_{\mathrm{T}}^{v i s}}{ }^{2}+p_{\mathrm{T}_{v i s}}$ to distinguish signal and background
- and the correlation between $p_{\mathrm{T}_{r e l}}$ and $\mathrm{w}, \cos \theta_{D}$ and $\cos \theta_{\mu}$ to determine the angular shape.


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## Exclusive $\left|V_{c b}\right|$ using $B_{s}^{0}$ decays





- Clean signal and normalization samples
- $D_{s}^{*+} \rightarrow D_{s}^{+} \gamma / \pi^{0}$ : Neutral objects with low $p_{\mathrm{T}}$ have a low reconstruction efficiency in LHCb, so only $D_{s}^{+}$is reconstructed.
- Similar for $D^{*-} \rightarrow D^{-} \gamma / \pi^{0}$


## Exclusive $\left|V_{c b}\right|$ using $B_{s}^{0}$ decays




- Perform fit using both, CLN and BGL, parametrisation.
- Leading to:
- $\left|V_{c b}\right|_{\text {CLN }}=(40.8 \pm 0.6 \pm 0.9 \pm 1.1) \cdot 10^{-3}$
- $\left|V_{c b}\right|_{\mathrm{BGL}}=(41.7 \pm 0.8 \pm 0.9 \pm 1.1) \cdot 10^{-3}$
- Updated results wrt to [Phys. Rev. D101 (2020) 072004] due to new value of $f_{s} / f_{d}$ from


## Exclusive $\left|V_{c b}\right|$ using $B_{s}^{0}$ decays



ALEPH [PLB 395, 373 (1997)] [PRL 82, 3746 (1999)] Belle [PRD 93, 032006 (2016)] BaBar [PRD 79, 012002 (2009)] BaBar PRL 104, 011802 (2010) LEPH [PLB 395, 373 (1997) OPAL [PLB 482 , 15 (2000)] OPAL [PB 482, 50 (2000)] DELPHI [PLB 510, 55 (2001)] DELPHI [EPJ C33, 213 (2004)] BaBar [PRD 77, 032002 (2008)] 012002 (2009) Belle [PRD 100, 052007 (2019)] BaBar [PRL 123, 091801 (2019) LHCb [PRD 101, 072004 (2020)

- Exclusive average (HFLAV 2019)

Inclusive average (HFLAV 2019)
$\begin{array}{lllllllll}5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45\end{array} \begin{array}{lll}50 \\ & & \end{array}$

- Used the latest average of $f_{s} / f_{d}$ from [LHCb-PAPER-2020-046] for the LHCb numbers.


## Measuring the shape of the $B_{s}^{0} \rightarrow D_{s}^{*-} \mu^{+} \nu_{\mu}$ decay rate

## LHCD HCG





- Determine the FFs of $B_{s}^{0} \rightarrow D_{s}^{*-} \mu^{+} \nu_{\mu}$ by measuring the shape of its differential decay rate.
- Fully reconstruct $D_{s}^{*-}$ with (low- $p_{\mathrm{T}}$ ) photon from $D_{s}^{*-} \rightarrow D_{s}^{-} \gamma$
- Use $m_{\text {corr }}$ to separate signal from background, determine $q^{2}$ (and $w$ ) using kinematical constraints.

Measuring the shape of the $B_{s}^{0} \rightarrow D_{s}^{*-} \mu^{+} \nu_{\mu}$ decay rate



- Provide $w$ data for phenomenological analysis for the first time.
- Good agreement with FFs from [Phys. Rev. D101 (2020) 072004]
- and with recent LQCD calculation from HPQCD [arXiv:2105.11433]

$$
\left|V_{u b}\right|
$$

## Partial $\mathcal{B}$ and inclusive $\left|V_{u b}\right|$

- The dilemma of inclusive $\left|V_{u b}\right|$ :

- Want to reduce contributions from $B \rightarrow X_{c} \ell \nu$ as much as possible $\rightarrow$ Only consider small phase space.
- Precision of shape functions decreases for small phase space $\rightarrow$ Increase phase space.
- "Resolve" dilemma by enlarging the regions where $\Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$ is measured and suppress $B \rightarrow X_{c} \ell \nu$ background using machine learning.
- Use hadronic tagging with FR to constrain the kinematics of $B_{s i g}$
- Use $M_{\text {miss }}^{2}, D^{*}$ veto variables, presence of kaons, $B_{\text {sig }}$ vertex fit quality and the total net charge of the event to distinguish $B \rightarrow X_{c} \ell \nu$ from $B \rightarrow X_{u} \ell \nu$.




before BDT




after BDT


## Partial $\mathcal{B}$ and inclusive $\left|V_{u b}\right|$

$$
\begin{aligned}
& \left|V_{u b}\right|(\mathrm{BLNP})=\left(4.05 \pm 0.09_{-0.22}^{+0.21+0.18}\right) \times 10^{-3}, \\
& \left|V_{u b}\right|(\mathrm{DGE})=\left(4.16 \pm 0.09_{-0.22-0.12}^{+0.21+0.11}\right) \times 10^{-3}, \\
& \left|V_{u b}\right|(\mathrm{GGOU})=\left(4.15 \pm 0.09_{-0.22}^{+0.21+0.09}\right) \times 10^{-3}, \\
& \left|V_{u b}\right|(\mathrm{ADFR})=\left(4.05 \pm 0.09_{-0.22}^{+0.21} \pm 0.18\right) \times 10^{-3} .
\end{aligned}
$$

- $\left|V_{u b}\right|=\sqrt{\frac{\Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)}{\tau_{B} \Delta \Gamma\left(B \rightarrow X_{u} \ell \nu\right)}}$
- Use 4 different theoretical predictions for $\Delta \Gamma\left(B \rightarrow X_{u} \ell \nu\right)$
- Use a 2D fit in $M_{X}$ and $q^{2}$ in the most inclusive sample with $E_{\ell}^{B}>1 \mathrm{GeV} / c$ to determine $\left|V_{u b}\right|$ with the 4 different predictions.


- Average of the most precise ones leads to: $\left|V_{u b}\right|=(4.10 \pm 0.09 \pm 0.22 \pm 0.15) \cdot 10^{-3}$

Partial $\mathcal{B}$ and inclusive $\left|V_{u b}\right|$


- Obtained result shows a discrepancy with respect to $\left|V_{u b}\right|$ exclusive from $B \rightarrow \pi \ell \nu(1.3 \sigma)$ and from CKM constraints (1.6 $\sigma$ ).

First Belle II results on $B \rightarrow X_{u} \ell \nu$



- Rediscover $B \rightarrow X_{u} \ell \nu$ close to the kinematic endpoint of $p_{\ell}^{*}$
- And $B^{0} \rightarrow \pi^{-} \ell \nu$ using FEI.


## Exclusive $\left|V_{u b}\right|$ from $B_{s}^{0}$ mesons




- Use $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$ decays to measure $\left|V_{u b}\right|$ : First observation of $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$.
- Measure $\frac{\left|V_{u b}\right|}{\left|V_{c b}\right|}=R_{F F} \cdot \frac{\mathcal{B}\left(B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}\right)}{\mathcal{B}\left(B_{s}^{0} \rightarrow D_{s}^{-} \mu^{+} \nu_{\mu}\right)}$, use $m_{\text {corr }}$ to discriminate signal and background.
- Divide $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$ in two bins of $q^{2}$ with equal number of signal events.


## Exclusive $\left|V_{u b}\right|$ from $B_{s}^{0}$ mesons




- Two different FF predictions for $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$ used to extract $\left|V_{u b}\right|$ :
- Low $q^{2}$ : LCSR based on [JHEP 08 112]
- High $q^{2}$ : LQCD based on [Phys. Rev. D100, 034501]
- Provide two values of $\left|V_{u b}\right|$. Differential rate will help understanding the $B_{s}^{0} \rightarrow K^{-} \mu^{+} \nu_{\mu}$ decay better.


## Exclusive $\left|V_{u b}\right|$ from $B_{s}^{0}$ mesons



| Uncertainty | All $q^{2}$ | low $q^{2}$ | high $q^{2}$ |
| :--- | :---: | :---: | :---: |
| Tracking | 2.0 | 2.0 | 2.0 |
| Trigger | 1.4 | 1.2 | 1.6 |
| Particle identification | 1.0 | 1.0 | 1.0 |
| $\sigma\left(m_{\text {corr }}\right)$ | 0.5 | 0.5 | 0.5 |
| Isolation | 0.2 | 0.2 | 0.2 |
| Charged BDT | 0.6 | 0.6 | 0.6 |
| Neutral BDT | 1.1 | 1.1 | 1.1 |
| $q^{2}$ migration | - | 2.0 | 2.0 |
| Efficiency | 1.2 | 1.6 | 1.6 |
| Fit template | ${ }_{-2.9}^{+2.3}$ | ${ }_{-2.4}^{+1.8}$ | ${ }_{-3.4}^{+3.0}$ |
| Total | ${ }_{-4.3}^{+4.0}$ | ${ }_{-4.5}^{+4.3}$ | ${ }_{-5.3}^{+5.0}$ |

- Measurement (in individual $q^{2}$ bins) is systematically limited, many are connected with limited size of simulation sample.
- More $q^{2}$ bins will allow for a more precise measurement using the full LHCb data set.


## Conclusions

- After 20 years of precision measurements, $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$ are still an exciting research topic with unresolved puzzles.
- Many recent measurements by Belle (II) and LHCb add precision, but did not resolve all mysteries.
- Belle II and LHCb will continue to show a "collaborative competition".
- Exciting new results expected for the future: More exploitation of the Belle data set, new results by Belle II in the following months, start of the LHCb upgrade, results using $B_{c}^{+}$mesons, $B^{+} \rightarrow \tau^{+} / \mu^{+} \nu$, etc.



## Exclusive $\left|V_{u b}\right|$ from $\Lambda_{b}^{0}$ baryons



- First measurement of $\left|V_{u b}\right|$ (relative to $\left|V_{c b}\right|$ ) at LHCb
- Use the corrected mass variable to separate signal and background.

$$
\text { - } m_{\text {corr }}=\sqrt{m_{v i s}^{2}+p_{\mathrm{T}}^{2}}+p_{\mathrm{T}}
$$

- Yields about $15^{\prime} 000$ signal events.
- Measure $\left|V_{u b}\right|=R_{F F} \cdot \frac{\mathcal{B}\left(\Lambda_{b}^{0} \rightarrow p \mu^{-} \bar{\nu}_{\mu}\right)}{\mathcal{B}\left(\Lambda_{b}^{0} \rightarrow \Lambda_{c}^{+} \mu^{-} \bar{\nu}_{\mu}\right)} \cdot\left|V_{c b}\right|$ in high $-q^{2}$ region, using $R_{F F}$ from [Phys Rev D. 92 (2015) 034503]
- $\left|V_{u b}\right|=(3.27 \pm 0.15 \pm 0.16 \pm 0.06) \cdot 10^{-3}$. Most precise exclusive result.


## Exclusive $\left|V_{c b}\right|$ using $B_{s}^{0}$ decays




- Clean signal and normalization samples
- $D_{s}^{*+} \rightarrow D_{s}^{+} \gamma / \pi^{0}$ : Neutral objects with low $p_{\mathrm{T}}$ have a low reconstruction efficiency in LHCb, so only $D_{s}^{+}$is reconstructed.
- Similar for $D^{*-} \rightarrow D^{-} \gamma / \pi^{0}$


## $B \rightarrow D^{(*)} \ell \nu$ at Belle II

Belle II Preliminary $\int \mathcal{L d t}=34.6 \mathrm{fb}^{-}$


Belle II preliminary $\int \mathcal{L} d t=34.6 \mathrm{fb}^{-1}$



Belle II preliminary $\int \mathcal{L} d t=34.6 \mathrm{fb}^{-1}$

- Reconstruct $B \rightarrow D^{*} \ell \nu$ using hadronic tagging and FEI or $B \rightarrow D^{(*)} \ell \nu$ without tagging.
- Determine $\mathcal{B}\left(B^{0} \rightarrow D^{*-} \ell^{+} \nu\right)=(4.51 \pm 0.41 \pm 0.27 \pm 0.45) \%$ with FEI
- and $\mathcal{B}\left(B^{0} \rightarrow D^{*-} \ell^{+} \nu\right)=(4.60 \pm 0.05 \pm 0.17 \pm 0.45) \%$ without tagging.


## $\left|V_{u b}\right|$ from $B^{+} \rightarrow \rho / \omega \ell \nu$



- Large discrepancy between values extracted from $B^{+} \rightarrow \rho / \omega \ell \nu$ and $B \rightarrow \pi \ell \nu$

