Kaon decays from lattice QCD

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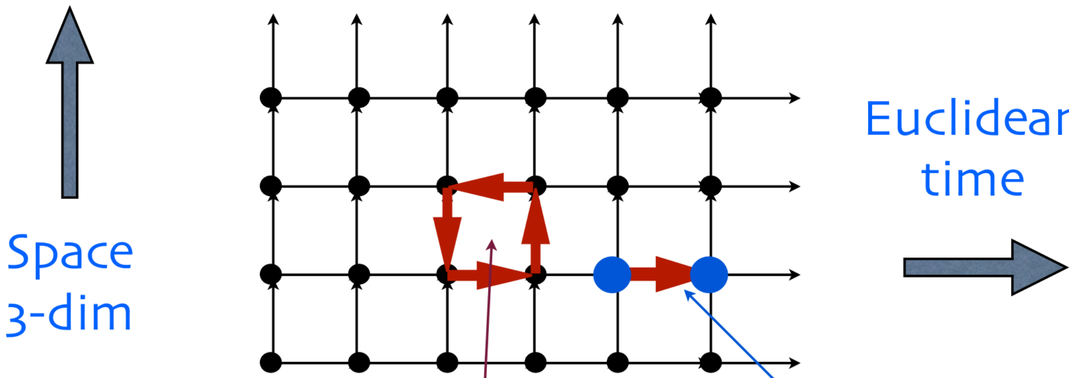
Outline

- **Introduction**
- $K \to \pi\pi$ and CP violation
- $K \to \ell\nu$, $K \to \pi\ell\nu$ and $|V_{us}|$
- Rare kaon decays
  - $K \to \pi\nu\bar{\nu}$
  - $K \to \pi\ell^+\ell^-$
  - $K \to \mu^+\mu^-$
  - $K \to \ell\nu\ell'^+\ell'^-$
- Conclusion and outlook

Lattice QCD: action



$$S_E^{\mathrm{latt}} = -\sum_{\Box} \frac{6}{g^2} \mathrm{Re}\, \mathrm{tr}_N (U_{\Box,\mu\nu}) - \sum_q \overline{q}(D_\mu^{\mathrm{lat}} \gamma_\mu + a m_q) q$$

Wilson gauge action   Lattice fermion action

Figure credit: Stephen R. Sharpe.

Operator quantum expectation value:

$$\begin{aligned}\langle \mathcal{O}(U,q,\bar{q})\rangle &= \frac{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\mathrm{lat}}} \mathcal{O}(U,q,\bar{q})}{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\mathrm{lat}}}} \\ &= \frac{\int [\mathcal{D}U] e^{-S_{\mathrm{gauge}}^{\mathrm{lat}}} \prod_q \det\left(D_\mu^{\mathrm{lat}}\gamma_\mu + am_q\right) \tilde{\mathcal{O}}(U)}{\int [\mathcal{D}U] e^{-S_{\mathrm{gauge}}^{\mathrm{lat}}} \prod_q \det\left(D_\mu^{\mathrm{lat}}\gamma_\mu + am_q\right)}\end{aligned}$$

Monte Carlo:

- The integration is performed for all the link variables, U. Dimension is $L^3 \times T \times 4 \times 8$.
- Sample points in this large dimensional configuration space with the following distribution:

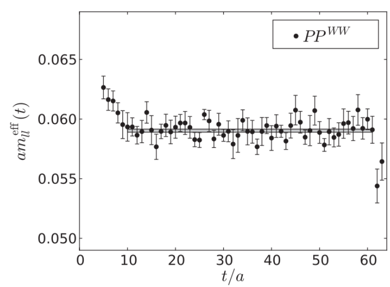
$$e^{-S_{\mathrm{gauge}}^{\mathrm{lat}}(U)} \prod_q \det\left(D_\mu^{\mathrm{lat}}(U)\gamma_\mu + am_q\right)$$

- Sampled points are referred to as the “configurations”, $U^{(k)}$.

$$\langle \mathcal{O}(U,q,\bar{q})\rangle = \frac{1}{N_{\mathrm{conf}}} \sum_{k=1}^{N_{\mathrm{conf}}} \tilde{\mathcal{O}}(U^{(k)})$$

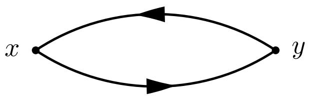
- $N_{\mathrm{conf}} \sim 100$. Large lattice size, *i.e.* higher integration dimension, require fewer N_{conf}.

64I $a^{-1} = 2.359$GeV $am_\pi = 0.059$ RBC-UKQCD



Correlation function:

$$
\begin{aligned}
C(t) &= \langle \pi^-(\vec{x},t) \sum_{\vec{y}} \pi^+(\vec{y},0) \rangle \\
&= \langle \bar{d}(\vec{x},t) i\gamma_5 u(\vec{x},t) \sum_{\vec{y}} \bar{u}(\vec{y},0) i\gamma_5 d(\vec{y},0) \rangle \\
&= \frac{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\text{lat}}} \bar{d}(\vec{x},t) i\gamma_5 u(\vec{x},t) \sum_{\vec{y}} \bar{u}(\vec{y},0) i\gamma_5 d(\vec{y},0)}{\int [\mathcal{D}U] \prod_q [\mathcal{D}q_q][\mathcal{D}\bar{q}_q] e^{-S_E^{\text{lat}}}} \\
&= \frac{\int [\mathcal{D}U] e^{-S_{\text{glue}}^{\text{lat}}} \prod_q \det\left(D_\mu^{\text{lat}}\gamma_\mu + am_q\right) \times \sum_{\vec{y}} \text{Tr}\left[\left(D_\mu^{\text{lat}}\gamma_\mu + am_u\right)^{-1}_{(\vec{x},t;\vec{y},0)} \gamma_5 \left(D_\mu^{\text{lat}}\gamma_\mu + am_d\right)^{-1}_{(\vec{y},0;\vec{x},t)} \gamma_5\right]}{\int [\mathcal{D}U] e^{-S_{\text{glue}}^{\text{lat}}} \prod_q \det\left(D_\mu^{\text{lat}}\gamma_\mu + am_q\right)} \\
&\propto e^{-m_\pi t}
\end{aligned}
$$



$$m_\pi^{\text{eff}}(t) = ln\left(\frac{C(t)}{C(t+1)}\right)$$

Lattice QCD: parameters

- How many parameters?

$$g \qquad am_l \qquad am_s$$

  isospin symmetric ($m_u = m_d = m_l$) and three flavor u, d, s theory.

- We are supposed to take $a \to 0$ limit, how?

$$g \to 0$$

  For different g, as long as it is small, the lattice calculation is describe the same physics, just with different a.

$$a = a(g) \approx a_0 \exp\left(-\frac{1}{11 - \frac{2}{3}N_f}\frac{8\pi^2}{g^2}\right)$$

  This is the renormalization equation. Since a decrease very fast when g decrease, g is not very small in realistic lattice QCD calculations.

- We need two physical inputs

$$m_\pi / m_\Omega \qquad m_K / m_\Omega$$

  to determine the remaining parameters am_l, am_s.

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$K \to \pi\pi$ and CP violation

- Two *neutral* kaons with definite CP quantum number:

$$
\begin{aligned}
|K_+\rangle &= |K^0\rangle + |\bar{K}^0\rangle \qquad \text{(CP even, decays to } \pi\pi\text{)} \\
|K_-\rangle &= |K^0\rangle - |\bar{K}^0\rangle \qquad \text{(CP odd, decays to } \pi\pi\pi\text{)}
\end{aligned}
$$

- $K_L (\approx K_-) \to \pi\pi$ observed in experiments indicate CP violation! Two sources:

- Indirect CP violation: the decaying states are ($\epsilon = \bar{\epsilon} + i\mathrm{Im}A_0/\mathrm{Re}A_0$):

$$
\begin{aligned}
|K_S\rangle &= \frac{|K_+\rangle + \bar{\epsilon}|K_-\rangle}{\sqrt{1+|\bar{\epsilon}|^2}} \\
|K_L\rangle &= \frac{|K_-\rangle + \bar{\epsilon}|K_+\rangle}{\sqrt{1+|\bar{\epsilon}|^2}}
\end{aligned}
$$

- Direct CP violation due to $K_- \to \pi\pi$, characterized by ϵ'

$$
\begin{aligned}
\eta_{+-} &= \frac{\langle \pi^+\pi^- | H_w | K_L\rangle}{\langle \pi^+\pi^- | H_w | K_S\rangle} = \epsilon + \epsilon' \\
\eta_{00} &= \frac{\langle \pi^0\pi^0 | H_w | K_L\rangle}{\langle \pi^0\pi^0 | H_w | K_S\rangle} = \epsilon - 2\epsilon'
\end{aligned}
$$

This direct CP violation is a highly suppressed $\mathcal{O}(10^{-6})$ effect in the Standard Model, making it a quantity which is especially sensitive to the effects of new physics in general, and new sources of CP violation in particular.

- Indirect CP violation characterized by ϵ. Experimental value: $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$
- Direct CP violation characterized by ϵ'.

  Combined measurement from KTeV (Fermilab 2011) and NA48 (CERN 2002):

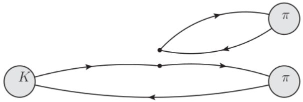
  $$\epsilon'/\epsilon \approx \mathrm{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}$$

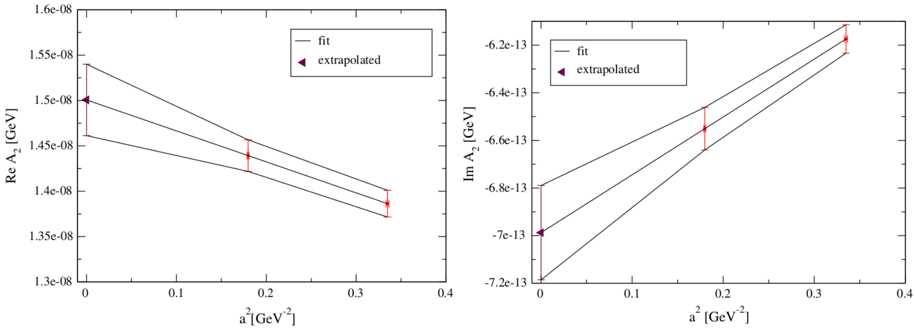
  Non-perturbative QCD inputs is needed to obtain the Standard Model prediction!
- Lattice calculation is needed for the following two processes to determine A_0 and A_2:

$$\begin{aligned} \langle \pi\pi(I=2)|H_w|K^0\rangle &= A_2 e^{i\delta_2} \\ \langle \pi\pi(I=0)|H_w|K^0\rangle &= A_0 e^{i\delta_2} \end{aligned}$$

$$\epsilon' = \frac{i e^{\delta_2 - \delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)$$

- $\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2}$
  [PRD 91, 074502 (2015)] by the RBC-UKQCD collaborations.





- Calculation is at physical pion mass with domain wall fermion.
- Dominate source of error is from the perturbatively calculated Wilson coefficients for the low energy 4-quark operators. (12%, not included in the plot)
- $E_{\pi\pi} \to \delta_2 = -11.6(2.5)_{\text{stat}}(1.2)^\circ_{\text{sys}}$

$K \to \pi\pi$ and CP violation

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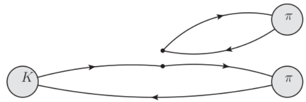
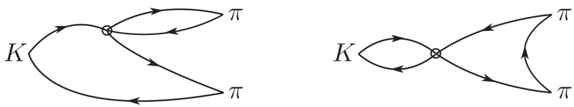
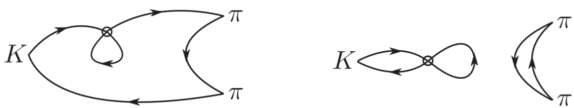
  [PRD 91, 074502 (2015)] by the RBC-UKQCD collaborations.



- $\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_2}$
  [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations.
  Chris Kelly (BNL), Tianle Wang (Columbia University, Norman Christ's current graduate student).



(a) *type1* (b) *type2*



(c) *type3* (d) *type4*

- Calculation much more difficult due to the $I = 0$ $\pi\pi$ final state.

- $\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_2}$
  [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations.
- Calculation direct at physical pion mass.
- G-parity boundary condition (and appropriate lattice size $L = 4.6$ fm) is used to ensure the ground state $\pi\pi$ system has the same energy as the kaon.
- All-to-All propagator technique are used to enhance the statistics efficiently.
- Use RI/MOM and step scaling up to 4 GeV to match with perturbatively obtained Wilson Coefficients.

Bare matrix elements

i	Q'_i [GeV^3]	Q_i [GeV^3]
1	0.143(93)	−0.119(32)
2	−0.147(24)	0.261(27)
3	0.233(23)	0.023(74)
4	...	0.403(72)
5	−0.723(91)	−0.723(91)
6	−2.211(144)	−2.211(144)
7	1.876(52)	1.876(52)
8	5.679(107)	5.679(107)
9	...	−0.190(39)
10	...	0.190(35)

Contribution to A_0

i	Re(A_0)		Im(A_0)	
	$(\not{q},\not{q})$ ($\times 10^{-7}$ GeV)	(γ^μ,γ^μ) ($\times 10^{-7}$ GeV)	$(\not{q},\not{q})$ ($\times 10^{-11}$ GeV)	(γ^μ,γ^μ) ($\times 10^{-11}$ GeV)
1	0.383(77)	0.335(64)	0	0
2	2.89(30)	2.81(28)	0	0
3	0.0081(58)	0.0050(42)	0.20(14)	0.12(10)
4	0.081(23)	0.088(17)	1.24(35)	1.34(27)
5	0.0380(68)	0.0339(53)	0.552(99)	0.492(77)
6	−0.410(28)	−0.398(27)	−8.78(60)	−8.54(57)
7	0.001863(56)	0.001900(56)	0.02491(75)	0.02540(75)
8	−0.00726(14)	−0.00708(13)	−0.2111(40)	−0.2060(39)
9	$-8.7(1.5)\times 10^{-5}$	$-8.5(1.4)\times 10^{-5}$	−0.133(22)	−0.128(21)
10	$2.37(38)\times 10^{-4}$	$2.13(32)\times 10^{-4}$	−0.0304(49)	−0.0273(41)
Total	2.99(32)	2.86(31)	−7.15(66)	−6.93(64)

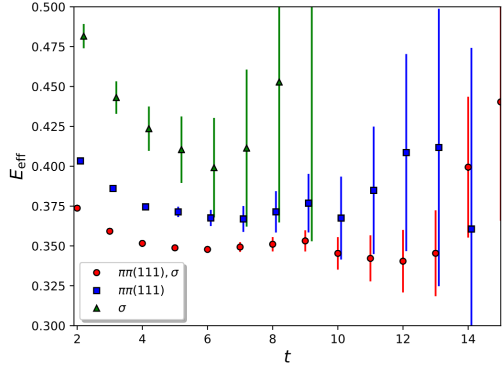
- $\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_2}$
  [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations.

- Major improvements over the previous RBC-UKQCD 2015 $\Delta I = 1/2$ calculation
  - Increase statistics from 216 to 741 configurations.
  - Additional interpolation operators for the $\pi\pi$ state.

  Lead to a more precise $\pi\pi$ ground state on the lattice.

$$E_0 = 0.3479(11) \leftarrow 0.3606(74)$$

- $E_0 \to \delta_0 = 32.3(1.0)_{\text{stat}}(1.8)^{\circ}_{\text{sys}}$
- The new σ operator is extremely effective.
- Extensive checks are made to ensure the true ground state is obtained.
- Consistent with dispersive value $\delta_0 = 35.9^{\circ}$.



$K \to \pi\pi$ and CP violation

- $\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_2}$
  [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations.

- Systematic error estimation of the calculation:

Error source	Value
Excited state	...
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

Error source	Value	
	$\mathrm{Re}(A_0)$	$\mathrm{Im}(A_0)$
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

- Two leading source of uncertainties (both 12%) are
  - Finite lattice spacing: only one lattice spacing @ $a^{-1} = 1.378$ GeV
  - Wilson coefficients: match 4 flavor theory to 3 flavor theory perturbatively.

- QED and strong isospin breaking correction can be important due to large A_0/A_2. ChPT calculation of these effects available.
  [JHEP. 02 (2020) 032] V. Cirigliano, H. Gisbert, A. Pich and A. Rodríguez-Sánchez.

- [PRD 102, 054509 (2020)] by the RBC-UKQCD collaborations.
- Final result $\mathrm{Re}(\epsilon'/\epsilon) = 2.17(26)_{\text{stat}}(62)_{\text{sys}}(50)_{\text{isospin}} \times 10^{-3}$
  Or, include the ChPT evaluation of the QED and strong isospin breaking effects: 1.67×10^{-3}. Recall the experimental value is $1.66(23) \times 10^{-3}$.
- Good agreement at this precision. RBC-UKQCD efforts to reduce the error:
  - Repeat the calculation with finer lattices.
  - Non-perturbative 3- to 4-flavor operator matching. Masaaki Tomii.
    M. Tomii, Proc. Sci., LATTICE2018 (2019) 216.
  - Periodic boundary condition $K \to \pi\pi$. Masaaki Tomii and Daniel Hoying.
- Developing method to study the QED and strong isospin breaking effects on the lattice
  - N. Christ and X. Feng, EPJ Web Conf. 175, 13016 (2018)
  - Y. Cai and Z. Davoudi, Proc. Sci., LATTICE2018 (2018) 280
- [PRD 98 (2018) 11, 114512] N. Ishizuka, K. I. Ishikawa, A. Ukawa and T. Yoshié
  Independent calculation with $m_\pi = 260$ MeV.

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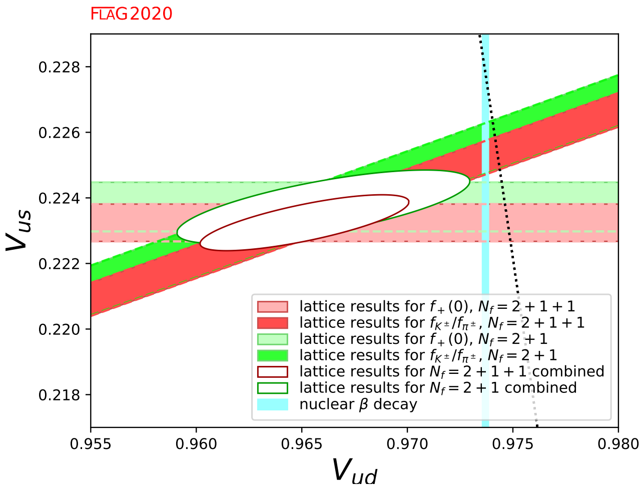
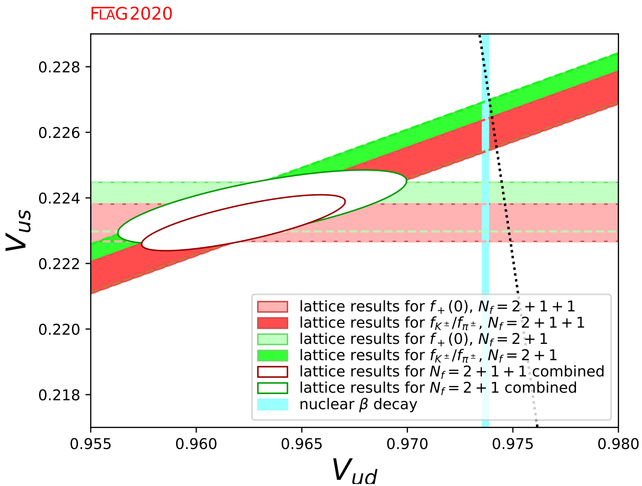


- Dashed line is the CKM matrix first row unitary constraint.
- All the bands and line should cross the same point. There are visible tensions in the plot.
- QED and strong isospin breaking corrections from ChPT calculations.

$K \to \ell\nu$, $K \to \pi\ell\nu$ and $|V_{us}|$



- Dashed line is the CKM matrix first row unitary constraint.
- All the bands and line should cross the same point. There are visible tensions in the plot.
- QED and strong isospin breaking corrections for $f_{K^\pm}/f_{\pi^\pm}$ from lattice QCD.
  [PRD 100 (2019) 034514] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo

- [PRD 100 (2019) 034514] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo
- Main result:

$$\Gamma(\pi^{\pm} \to \mu^{\pm}\nu_{\ell}[\gamma]) = (1.0153 \pm 0.0019)\Gamma^{(0)}(\pi^{\pm} \to \mu^{\pm}\nu_{\ell}),$$

$$\Gamma(K^{\pm} \to \mu^{\pm}\nu_{\ell}[\gamma]) = (1.0024 \pm 0.0010)\Gamma^{(0)}(K^{\pm} \to \mu^{\pm}\nu_{\ell}),$$

- Calculation is performed on several lattices with different pion masses, physical sizes, lattice spacings.
- Final result is obtained by extrapolating to the physical point.
- QED effects are included in lattice simulation in finite volume.
  $\Rightarrow \mathcal{O}(1/L)$ universal finite volume effects.
  $\Rightarrow \mathcal{O}(1/L^2)$ structure dependent finite volume effects.
  Subtract the $\mathcal{O}(1/L)$ effects and fit the remaining $\mathcal{O}(1/L^2)$ effects.

- [PRD 100 (2019) 034514] M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C.T. Sachrajda, F. Sanfilippo, S. Simula, N. Tantalo

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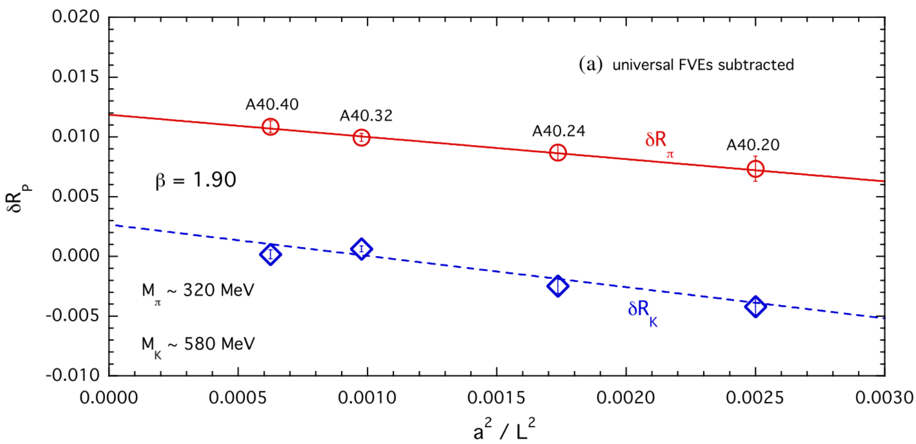

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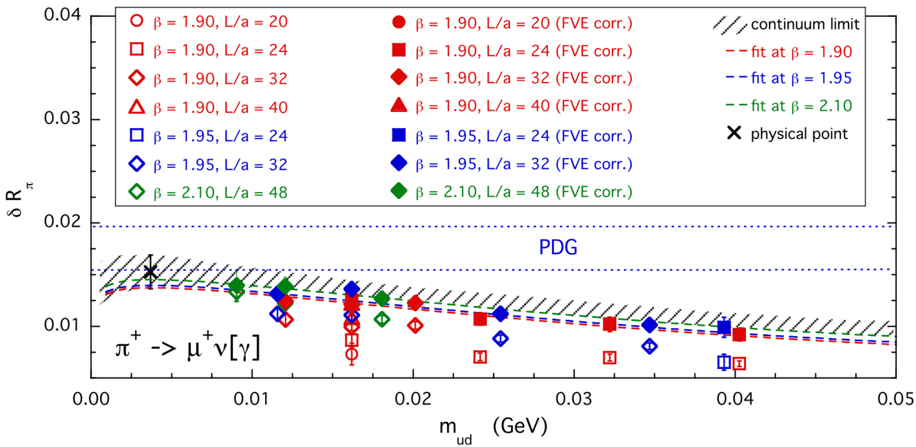

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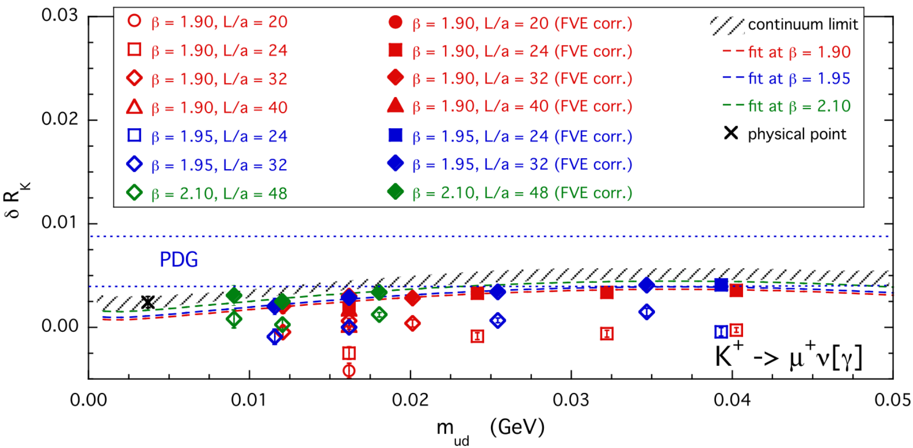
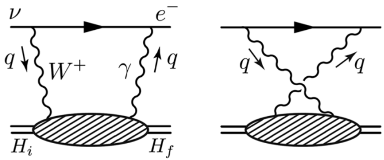


- [PRL. 124 (2020) 19, 192002] X. Feng, M. Gorchtein, L. C. Jin, P. X. Ma and C. Y. Seng
  - The γW-box contribution to $\pi^+ \to \pi^0 e^+ \nu$ is calculated on the lattice.



    - ∗ All the non-hadronic part of the diagram is analytically calculated in infinite volume. Finite volume effects is exponentially suppressed by the volume.
    - ∗ Calculation is directly performed at physical pion mass with domain wall fermion.
  - In the dispersive framework, this is the only hadronic structure dependent part.
  - The obtained QED correction is $\delta = 0.0332(1)_{\gamma W}(3)_{\text{higher order QED}}$.
  - In comparison, ChPT gives $\delta = 0.0334(10)_{\text{LEC}}(3)_{\text{higher order QED}}$

- [arXiv:2102.12048] P. X. Ma, X. Feng, M. Gorchtein, L. C. Jin and C. Y. Seng
  - Data of the lattice calculation above re-analyzed to obtain the ChPT LECs.
  - $\mathcal{O}(e^2 p^4)$ uncertainties from ChPT remain.

$$
\begin{aligned}
\delta^e_{K^0} &= 0.99(19)_{e^2p^4}(11)_{\text{LEC}} &\to\quad & 1.00(19)\\
\delta^\mu_{K^0} &= 1.40(19)_{e^2p^4}(11)_{\text{LEC}} &\to\quad & 1.41(19)\\
\delta^e_{K^\pm} &= 0.10(19)_{e^2p^4}(16)_{\text{LEC}} &\to\quad & -0.01(19)\\
\delta^\mu_{K^\pm} &= 0.02(19)_{e^2p^4}(16)_{\text{LEC}} &\to\quad & -0.09(19).
\end{aligned}
$$

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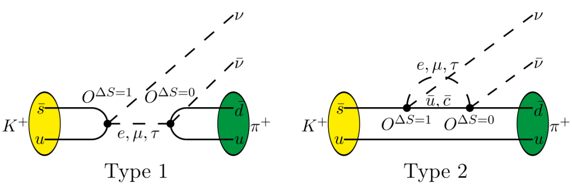
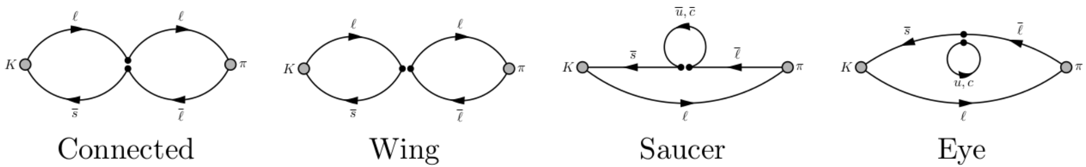
- [PRD 100 (2019) 11, 114506] by the RBC-UKQCD collaborations Xu Feng (Peking University).
- The golden modes: an ideal process in which to search for signs of new physics.
  - NA62 at CERN: $K^+ \to \pi^+\nu\bar{\nu}$ (aim at 10% accuracy)
  - KOTO at J-PARC: $K_L \to \pi^0\nu\bar{\nu}$ (long distance contributions negligible)
- The long distance ($\mathcal{O}(1/m_c)$) contribution is estimated to be about 5% to 10% in $K^+ \to \pi^+\nu\bar{\nu}$.
- Two pioneer lattice calculations @ $m_\pi = 430$ MeV and $m_\pi = 170$ MeV and lighter charm quark mass due to coarse lattice spacing.

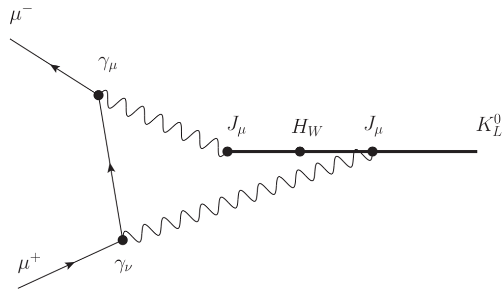


W-W diagram

- [PRD 94, 114516 (2016), J. Phys.: Conf. Ser. 1526 012015] by the RBC-UKQCD collaborations
  Antonin Portelli, Fionn Ó. HÓgáin (University of Edinburgh)
- Similar to $K \to \pi \nu \bar{\nu}$:
  - NA62 at CERN: $K^+ \to \pi^+ \ell^+ \ell^-$
  - Prospective experiments planned at LHCb to study the: $K_S \to \pi^0 \ell^+ \ell^-$
  - Pioneer lattice calculations @ $m_\pi = 430$ MeV
  - Calculation at physical pion mass under way.



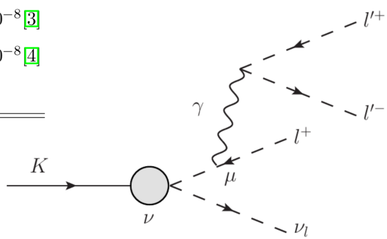
- [PoS LATTICE2019 (2020) 128, PoS LATTICE2019 (2020) 097] by the RBC-UKQCD collaborations
  Yidi Zhao (Columbia University, Norman Christ's current graduate student)
- Branching fraction is accurately measured: $\mathrm{BR}(K_L \to \mu^+\mu^-) = 6.84 \pm 0.11) \times 10^{-9}$.
- Two mechanism of comparable sizes for the decay:
  - One-loop, second-order weak process, involving exchange of two weak bosons.
  - $\mathcal{O}(\alpha_{\mathrm{EM}}^2 G_F)$ process shown below.
- First step calculation $\pi \to e^+e^-$ successful.
- Second step calculation $K_L \to \gamma\gamma$ in progress.
- Final goal: lattice calculation of $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$



Rare kaon decays: $K \to \ell\nu\ell'^+\ell'^-$

- [arXiv:2103.11331] X. Y. Tuo, X. Feng, L. C. Jin and T. Wang
- Good test of the lattice calculation for the kaon form factors also needed for the QED corrections to the kaon leptonic decay (photon can be emitted from kaon).
- Techniques are developed to treat the four (non-interacting) particle final state.
- Infinite volume reconstruction method [PRD 100, 094509 (2019)] X. Feng and L. Jin used to control the finite volume effects (no power-law suppressed finite volume error). Method will be used to calculate the full QED corrections to kaon leptonic decay.
- $m_\pi = 352$ MeV used in this calculation. Physical pion mass calculation underway.

Channels	m_{ee} cuts	Lattice ($m_\pi = 352$ MeV)	ChPT[5]	experiments
$\mathrm{Br}[K \to e\nu_e e^+e^-]$	140 MeV	$3.29(35) \times 10^{-8}$	3.39×10^{-8}	$2.91(23) \times 10^{-8}$[3]
$\mathrm{Br}[K \to \mu\nu_\mu e^+e^-]$	140 MeV	$11.08(39) \times 10^{-8}$	8.51×10^{-8}	$7.93(33) \times 10^{-8}$[3]
$\mathrm{Br}[K \to e\nu_e \mu^+\mu^-]$	——	$0.94(8) \times 10^{-8}$	1.12×10^{-8}	$1.72(45) \times 10^{-8}$[4]
$\mathrm{Br}[K \to \mu\nu_\mu \mu^+\mu^-]$	——	$1.52(7) \times 10^{-8}$	1.35×10^{-8}	——



Outline

- Introduction
- $K \to \pi\pi$ and CP violation
- $K \to \ell\nu$, $K \to \pi\ell\nu$ and $|V_{us}|$
- Rare kaon decays
  - $K \to \pi\nu\bar{\nu}$
  - $K \to \pi\ell^+\ell^-$
  - $K \to \mu^+\mu^-$
  - $K \to \ell\nu\ell'^+\ell'^-$
- **Conclusion and outlook**

Conclusion and outlook

- Accuracy of lattice QCD calculation is improving steadily.
- More hadronic processes are becoming accessible to lattice QCD calculations.
- Many lattice calculations are currently performed with physical parameters (pion/kaon masses).
- QED corrections in lattice QCD calculations are becoming important.

Thank You!