Heavy-heavy and heavy-light form factors from lattice QCD

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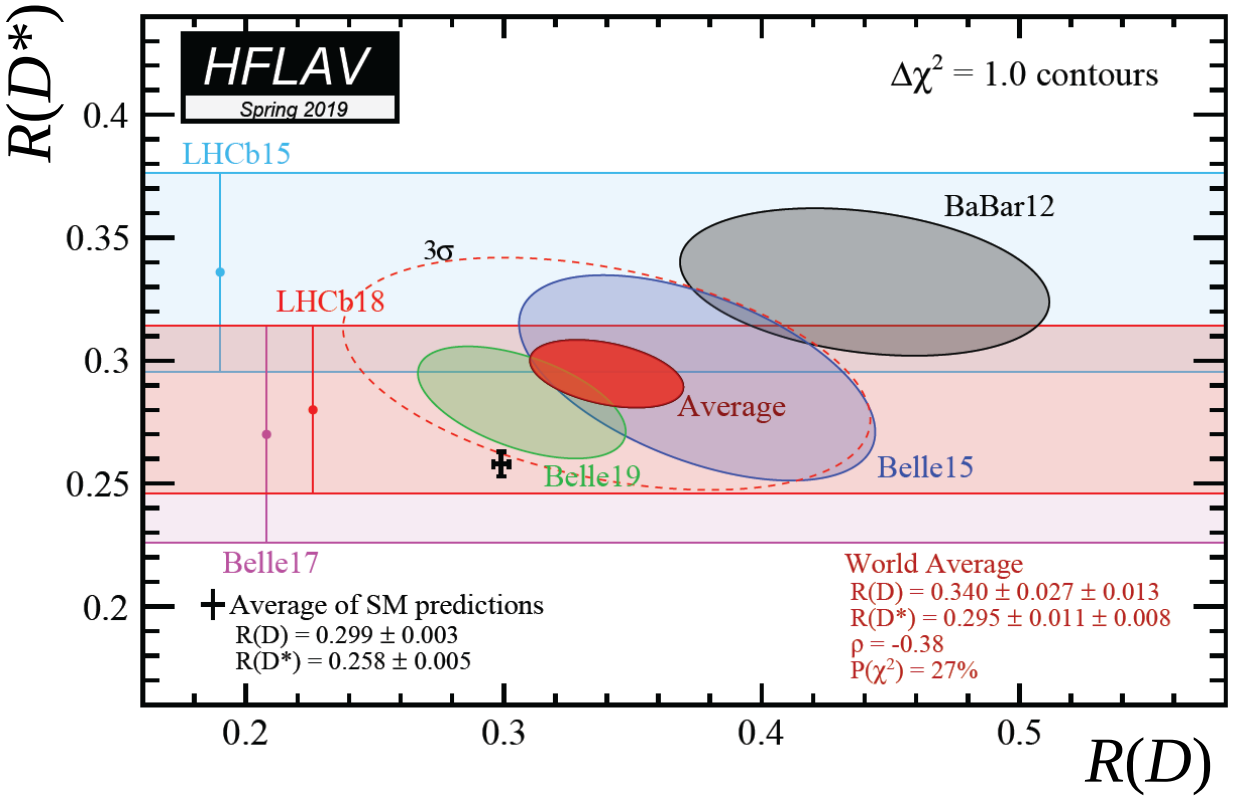
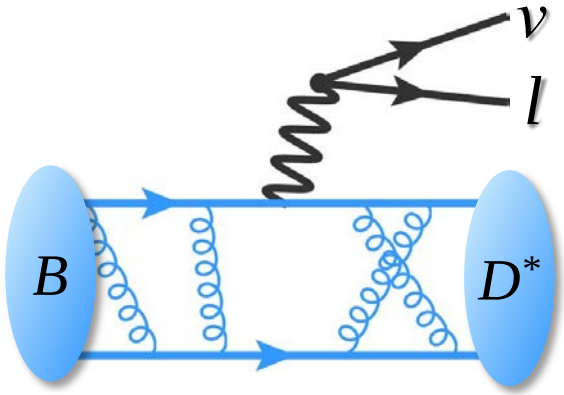
FPCP 2021 @ Shanghai, 10 June 2021

semileptonic decays

good probe of new physics (NP)



$$R\left(D^{(*)}\right) = \frac{\Gamma\left(B \to D^{(*)}\tau\nu\right)}{\Gamma\left(B \to D^{(*)}\{e,\mu\}\nu\right)}$$

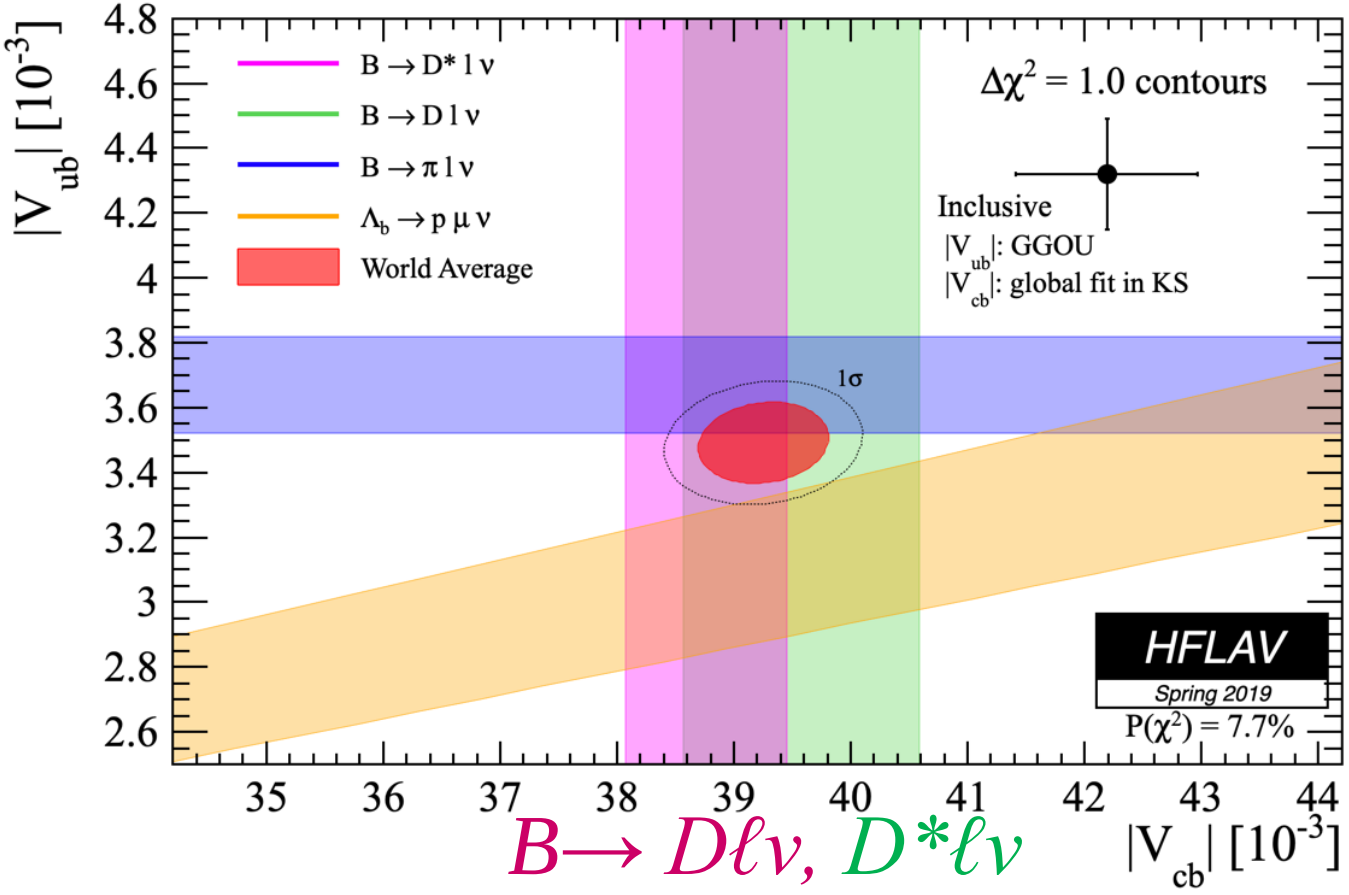


form factors (FFs) : non-perturbative QCD effects

precision search for NP @ Belle II, LHCb, ...

long-standing problem

tension in CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$



inclusive decay

= sum up over final state hadron(s)

$B \to X_u \ell \nu$

$= \pi\ell\nu + \rho\ell\nu + \ldots$

exclusive decay

$B \to \pi\ell\nu$

to be revolved for precision new physics search

lattice QCD

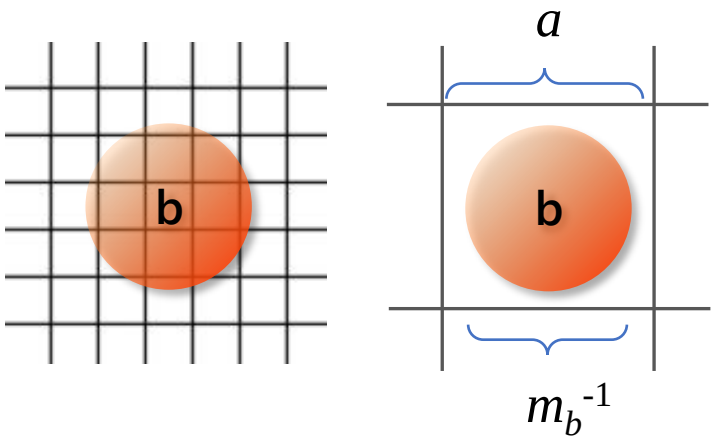
1st-principle calculation of form factors

fully realistic simulation for kaon and charm physics

bottom quarks on the lattice

- lattice spacing $a << m_b^{-1}$
- simulation cost $\propto a^{-7}$

$\Rightarrow$ difficult to simulate physical $m_{b,phys}$



practical strategies

- relativistic approach: simulate QCD w/ m_b's $\lesssim m_{b,\mathrm{phys}}$
  - ☺ direct simulation of QCD
  - ☹ extrapolate to $m_{b,\mathrm{phys}}$
- effective field theory (EFT) approach: use EFT action and operators
  - ☺ simulate $m_{b,\mathrm{phys}}$
  - ☹ need matching to QCD

$\Rightarrow$ independent calculations w/ different approaches

outline

heavy hadron form factors from lattice QCD

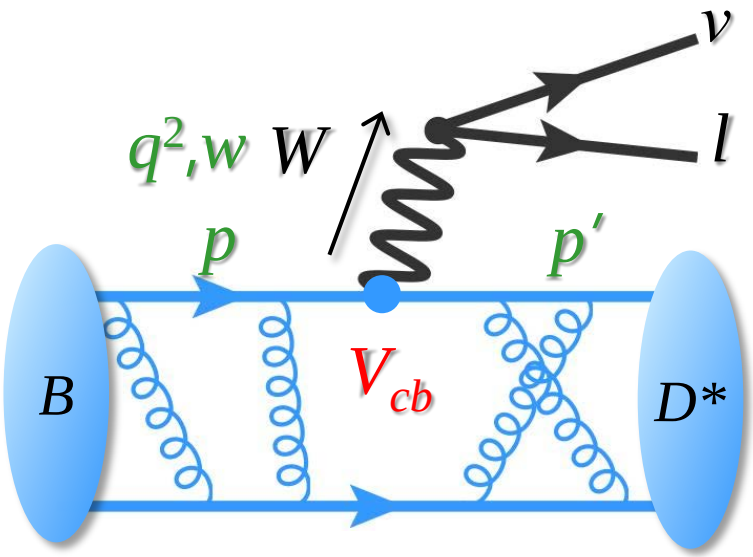
- $b \to c\ell\nu$
  - $B \to D^{*}\ell\nu$ at non-zero recoils
  - $B_s \to D_s^{*}\ell\nu$, $B_c \to J/\psi\ell\nu$ decays
- $b \to u\ell\nu$
  - current status and on-going calculations for $|V_{ub}|$
- other decays
  - D and baryon decays

not comprehensive review (see, Gamiz @ FPCP20)

subjectively - selected recent progress

1. $b \to c \ell \nu$ decays

hadronic matrix elements



- momentum recoil

$q^2 = (p - p')^2$

$w = vv' \geq 1$ (zero recoil, $\mathbf{p}=\mathbf{p}'=\mathbf{0}$)

$v = p/M_B,\ v'=p'/M_{D^*}$

$$\langle D^*(v',\varepsilon')|V^\mu|\bar{B}(v)\rangle = i h_V(w)\,\epsilon^{\mu\nu\alpha\beta}\,\varepsilon'^*_\nu v'_\alpha v_\beta$$

$$\langle D^*(v',\varepsilon')|A^\mu|\bar{B}(v)\rangle = h_{A_1}(w)\,(w+1)\,\varepsilon'^{*\mu} - \left[h_{A_2}(w)\,v^\mu + h_{A_3}(w)\,v'^\mu\right]\varepsilon^*\cdot v$$

lattice QCD : non-perturbatively calculate form factors from 1st principles

$B \to D^* \ell \nu$ decay

- $|V_{cb}|$ tension w/ inclusive decay $B \to X_c \ell \nu$
- new physics is unlikely source? *e.g.* Crivellin-Pokorski 1407.1320
- uncertainty of conventional determination not fully understood?

  $\Leftrightarrow$ lattice calculation was available for only single FF h_{A1} at w=1

- z-parameter expansion to parametrize the w dependence of FFs
  - Boyd-Grinstein-Lebed (BGL) : model-independent based on analyticity

$$h_X(w) = \frac{1}{P(w)\phi(w)} \sum_{n=0}^{N} a_n z^n, \quad z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} - \sqrt{2}}$$

  $\Leftrightarrow$ other information deduce from expr't data

- Fermilab/MILC published 1st calculation of all SM FFs at $w \neq 1$ (!)
- JLQCD calculation w/ different set up is on-going

$B \to D^* \ell \nu$ decay

simulation set up

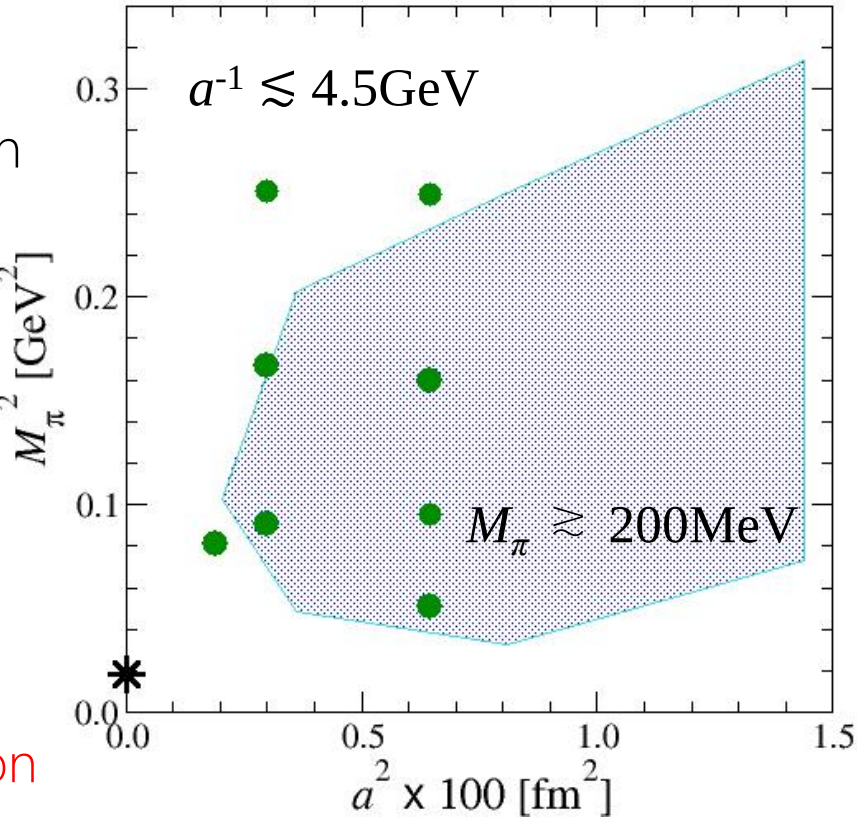
Fermilab/MILC 2105.14019

- c and b quarks in EFT approach
  - Fermilab interpretation of Wilson action
  - directly simulate physical $m_{b,\mathrm{phys}}$
  - need matching of action and op.s

JLQCD on-going

- relativistic apporach
  - chiral symmetric action for all quarks
  - no matching, automatic renormalization
  - simulate $m_b \leq 0.7 a^{-1}$, extrapolate to $m_{b,\mathrm{phys}}$

parameter region



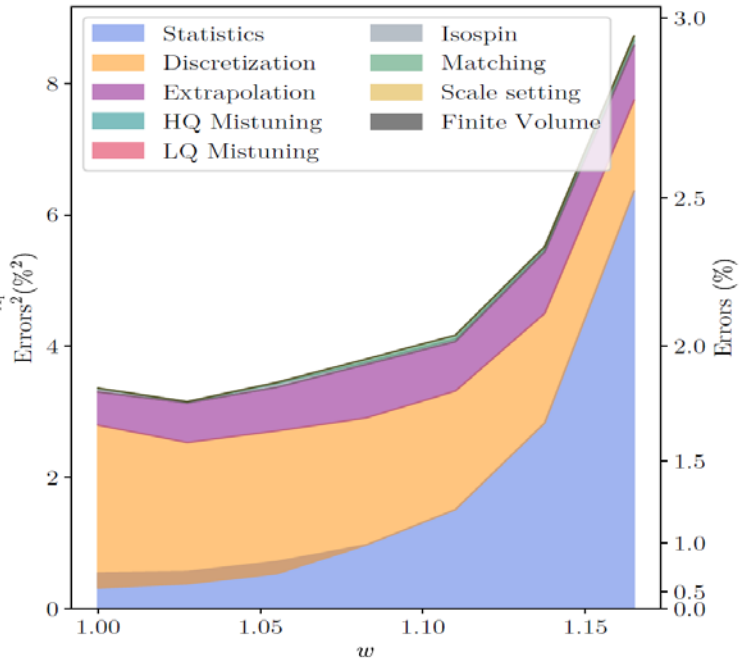
independent calc.s w/ very different systematics ⇒ firm prediction

$B \to D^* \ell \nu$ decay

uncertaties : h_{A1}

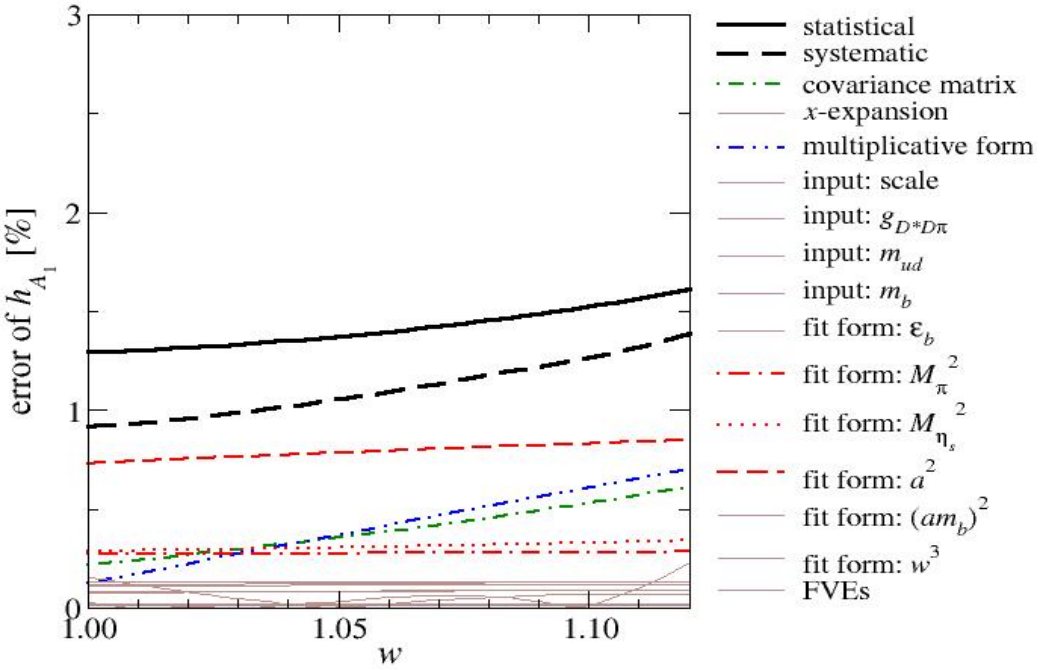
Fermilab/MILC

figure of accumulated error



JLQCD

figure of individual error



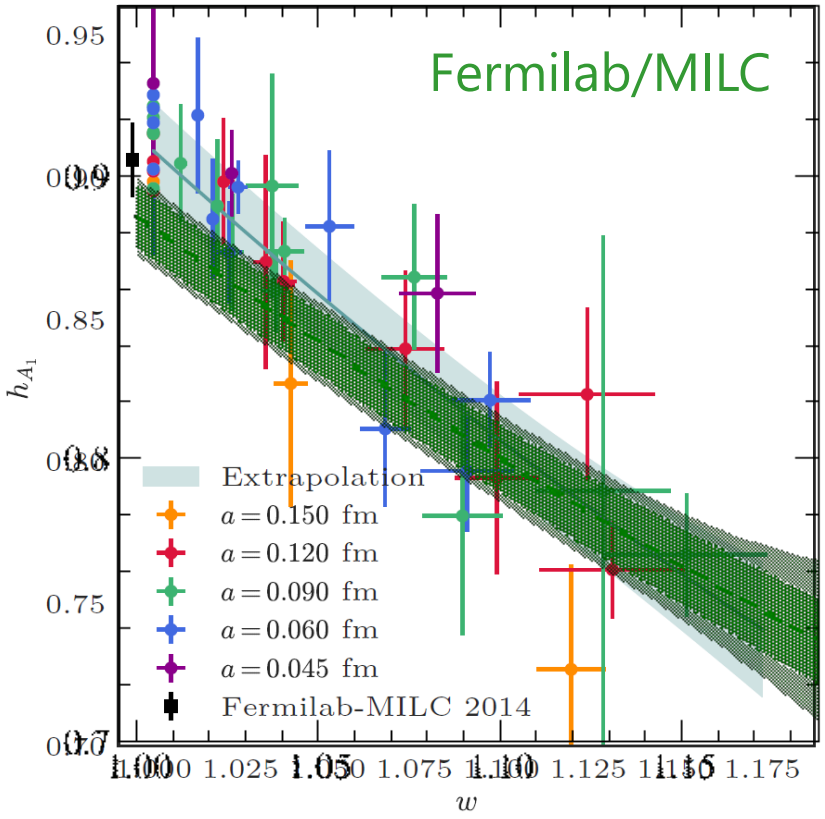
- largest errors : statistical and discretization [but 1% level for h_{A1}]

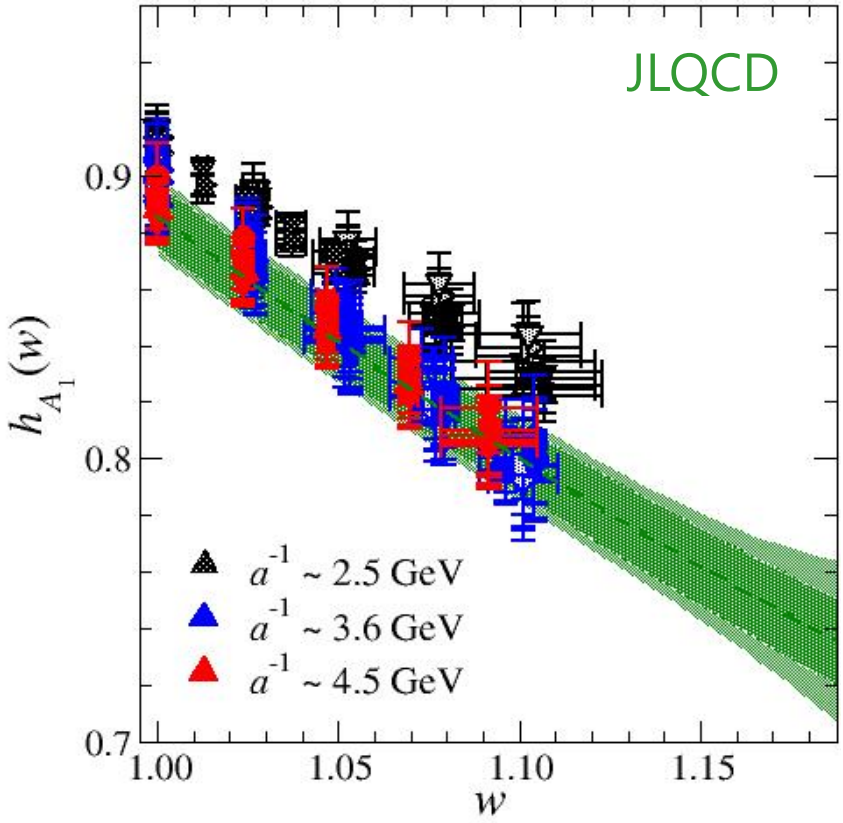
  chiral extrapolation ⇔ small m_q dependence, suppressed log.

- other FFs : larger and more dominant statistical error

$B \to D^* \ell \nu$ decay

FF comparison : h_{A1}

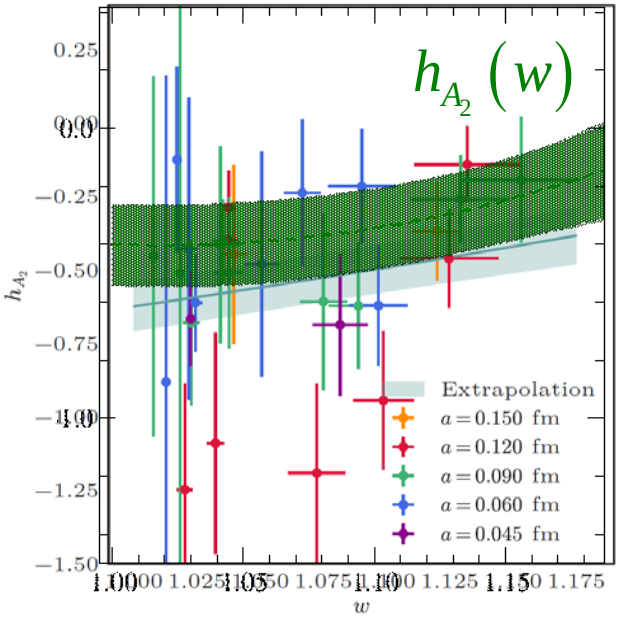


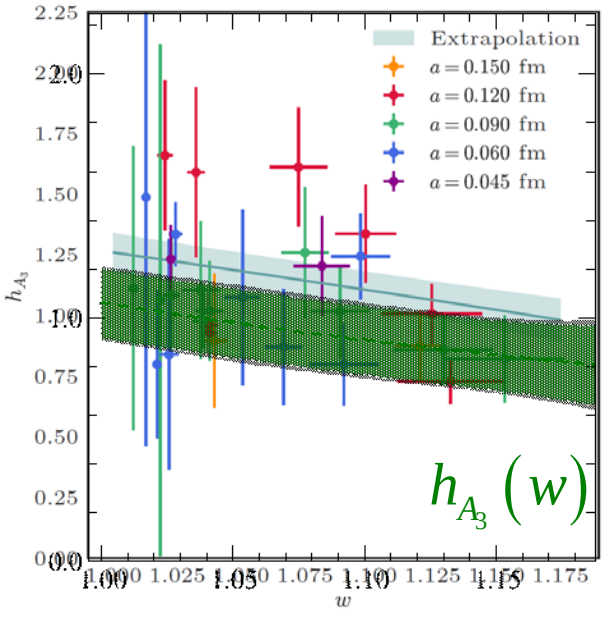


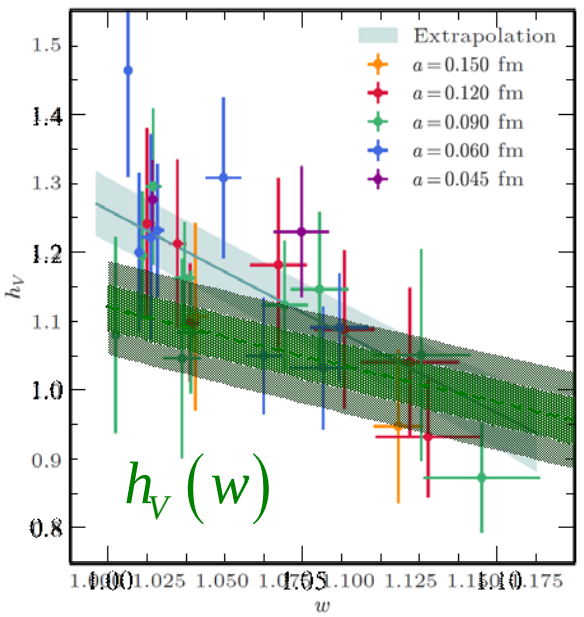
- reasonable consistency in spite of very different systematics
- JLQCD: slightly narrower w, only slightly gentle slope ??

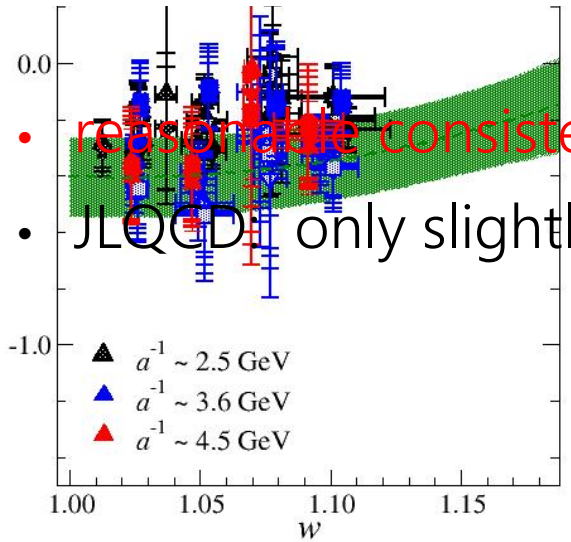
$B \to D^* \ell \nu$ decay

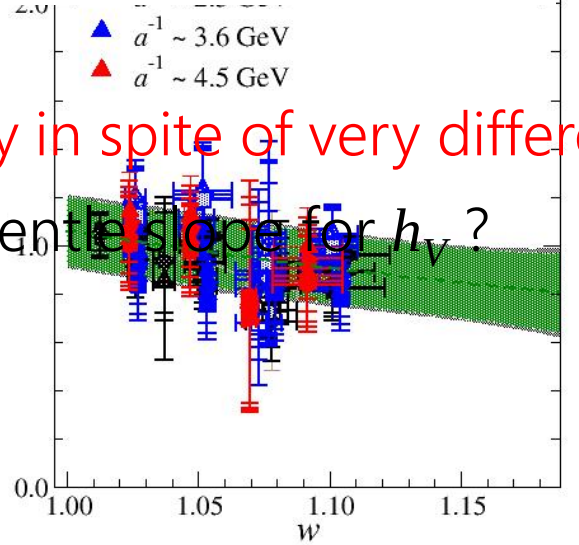
FF comparison

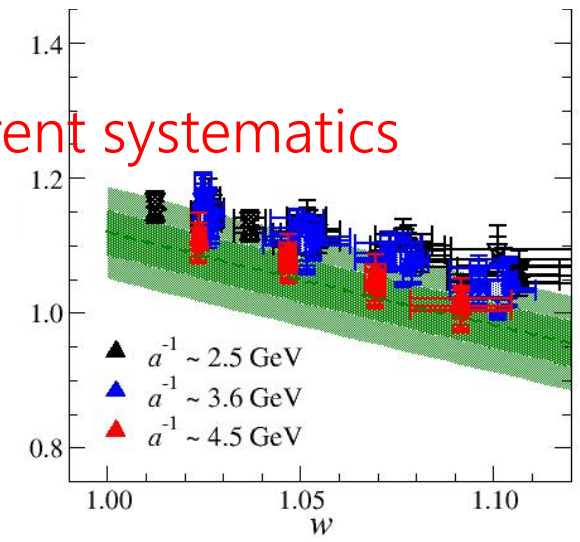










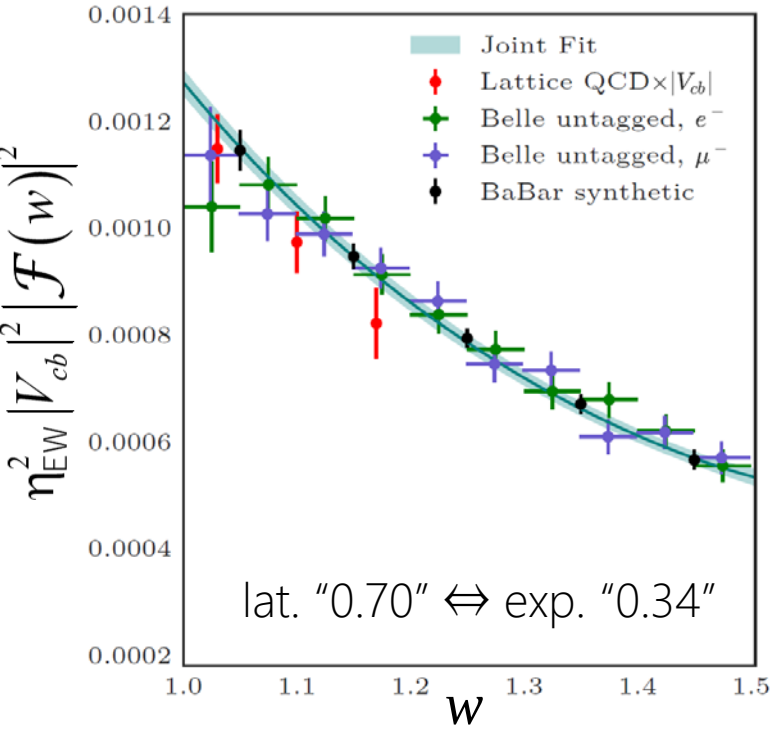


- reasonable consistency in spite of very different systematics
- JLQCD: only slightly gentle slope for h_V ?

$B \to D^* \ell \nu$ decay

Fermilab/MILC: impact on the SM test

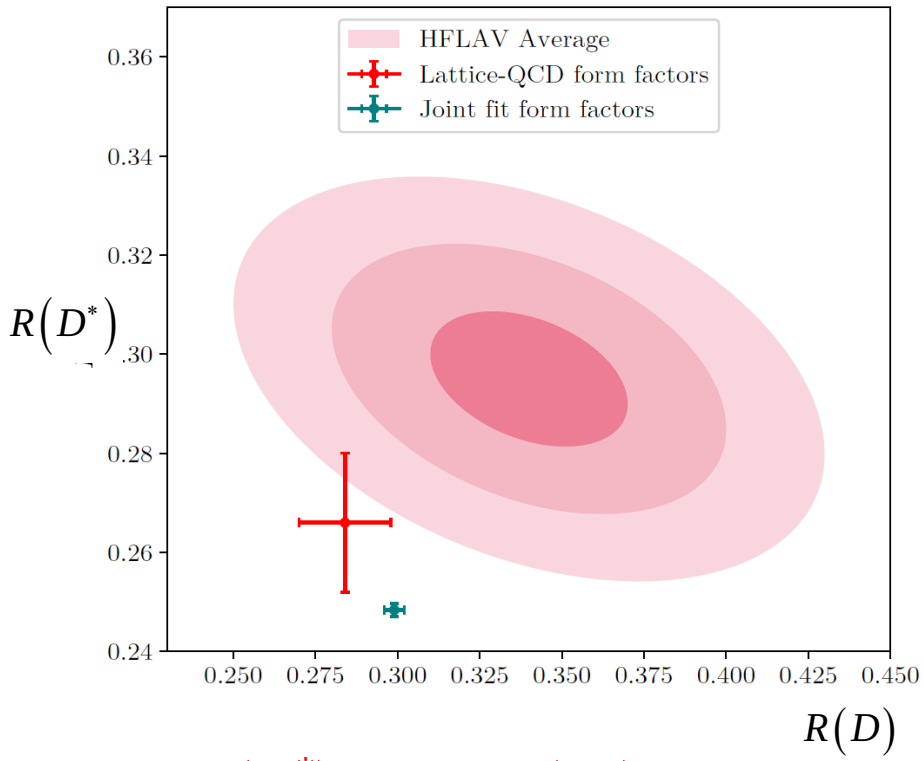
simultaneous fit to lat.+exp. data



$|V_{cb}| = 38.57(0.78) \times 10^3$

- consistent w/ previous exclusive calc.
- slight tension in slope b/w lat. & exp.??

$R(D^{(*)})$ from lattice QCD, and + exp.



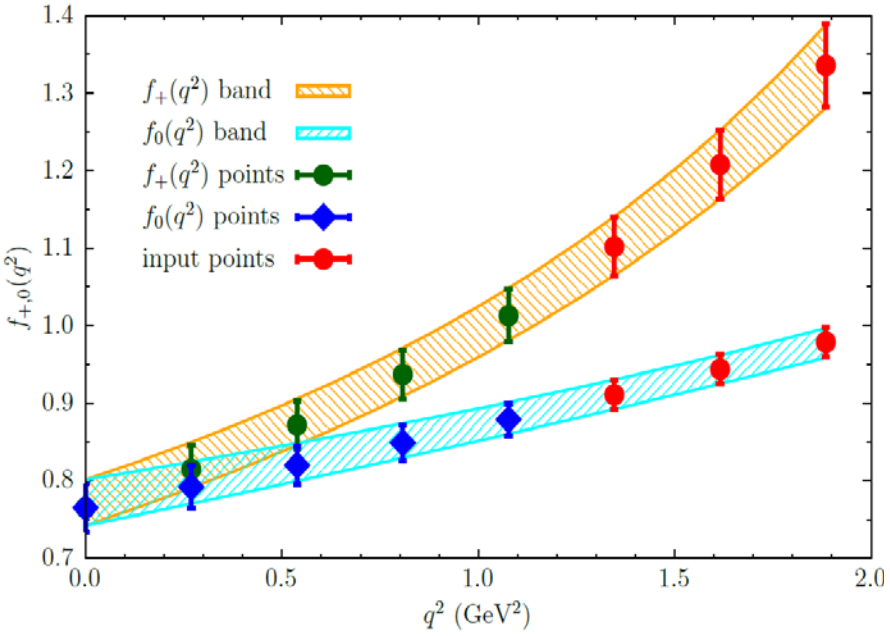
$R(D^*) = 0.266(14)$

- purely theoretical estimate
- *c.f.* w/ exp. 0.2484(13)

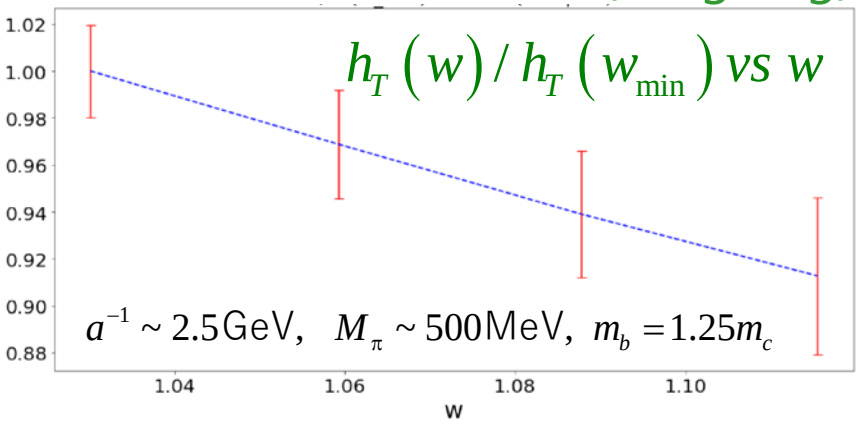
$B \to D^* \ell \nu$ decay

- $|V_{cb}|$ tension still remains
  - more independent lattice studies
  - more data from Belle II, LHCb
- strong isospin correction ~ small
- lattice are limited to small w
  - Di Carlo et al., 2105.02497
  - Martinelli et al., 2105.07851
    "susceptibilities" in z-parameter expansion
- FFs beyond SM
  - BSM interpretation of "hints"

Di Carlo et al. '21 for $D \to K \ell \nu$



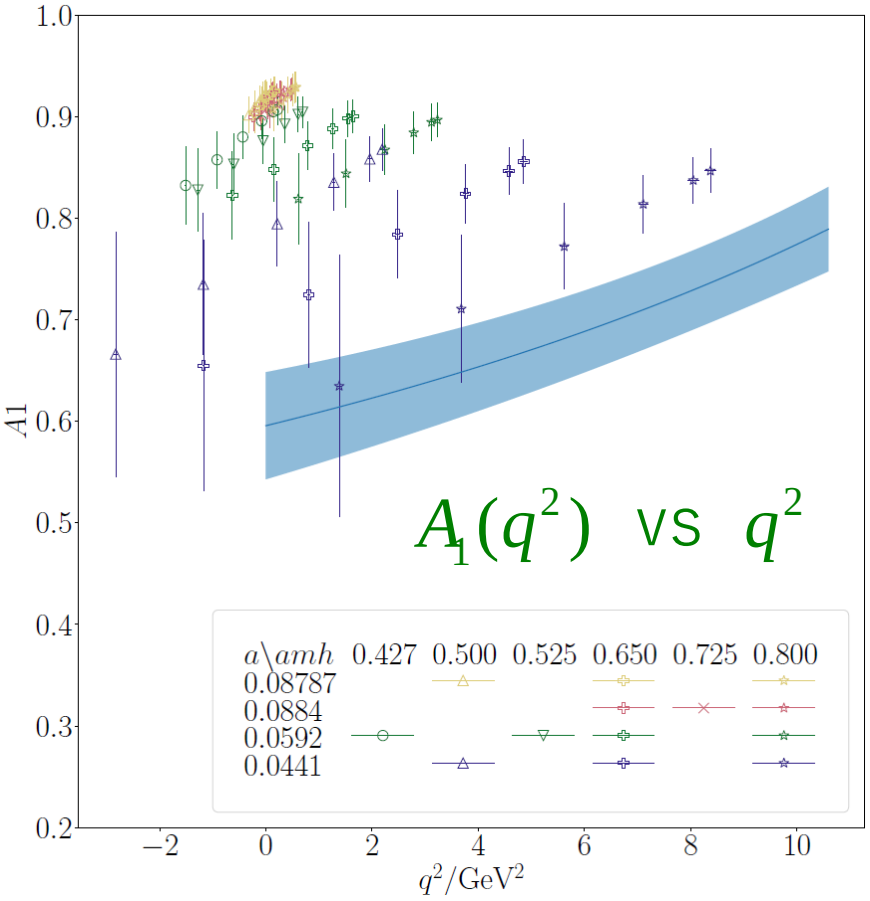
Faur, TK, Kou for JLQCD (on-going)



$B_s \to D_s^* \ell\nu$ decay

HPQCD arXiv:2105.11433

- $B \to D^* \ell\nu$ w/ s spectator quark
  - "stable" D_s^* w/ tiny width
  - no ud prop. : inexpensive
  - 1 vector, 3 axial vector FFs
- relativistic HISQ for b, c quarks
  - no matching
  - w/ $am_b \leq 0.8 \Rightarrow m_{b,phys}$
- fit to modified z-expansion



$$F(q^2) = \frac{1}{P(q^2)} \sum_{n=0}^{3} a_n z^n \mathcal{N}_n \qquad z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

+ coefficients in $1/M_{\eta b'}$ $m_{ud'}$ $m_{s'}$ a, $am_{c'}$ am_b

$B_s \to D_s^* \ell \nu$ decay

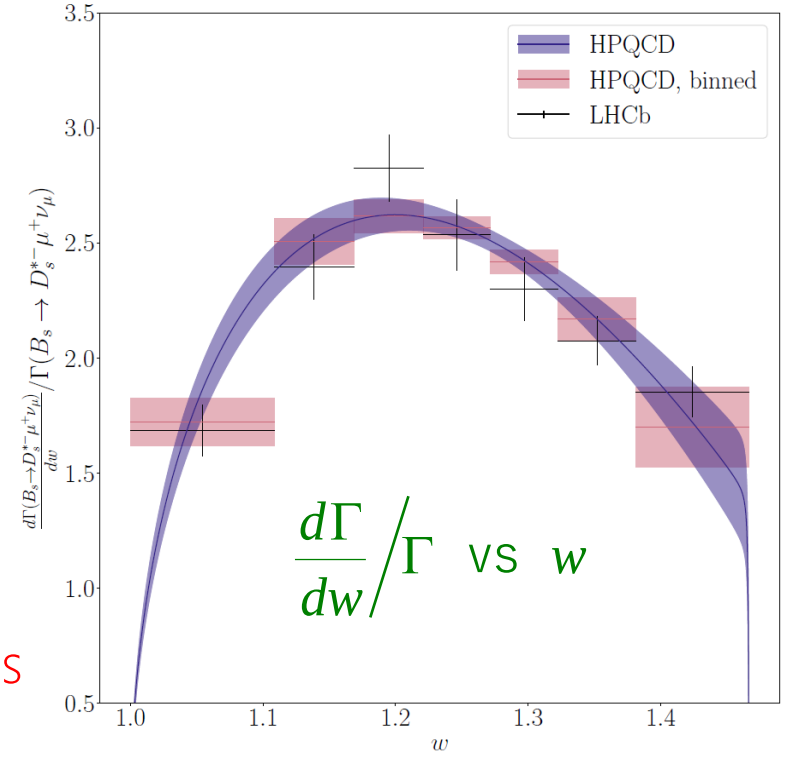
HPQCD arXiv:2105.11433

- shape of differential decay rate
  - good agreement w/ LHCb

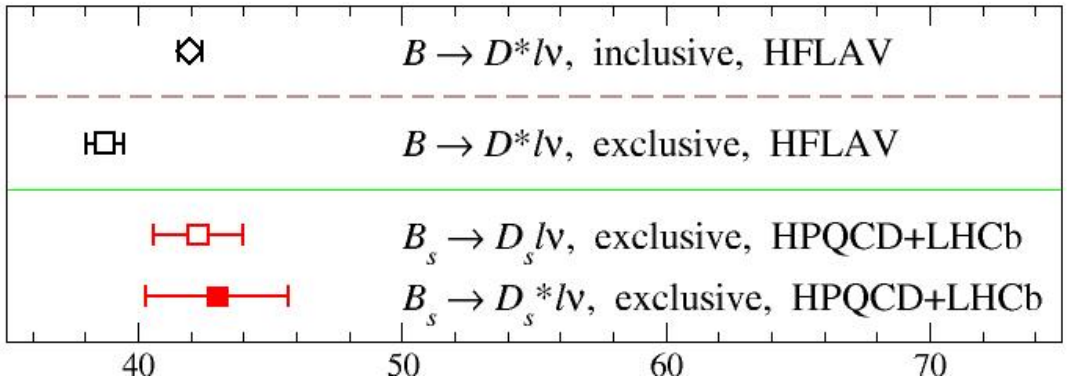
- $|V_{cb}|$ from simultaneous fit

  $|V_{cb}| = 43.0(2.1)_{\rm lat}(1.7)_{\rm exp}(0.4)_{\rm EM} \times 10^{-3}$

  - consistent w/ incl. & excl. $B \to D^* \ell \nu$
  - need improvements in th. and exp.t sides



$|V_{cb}| \times 10^3$

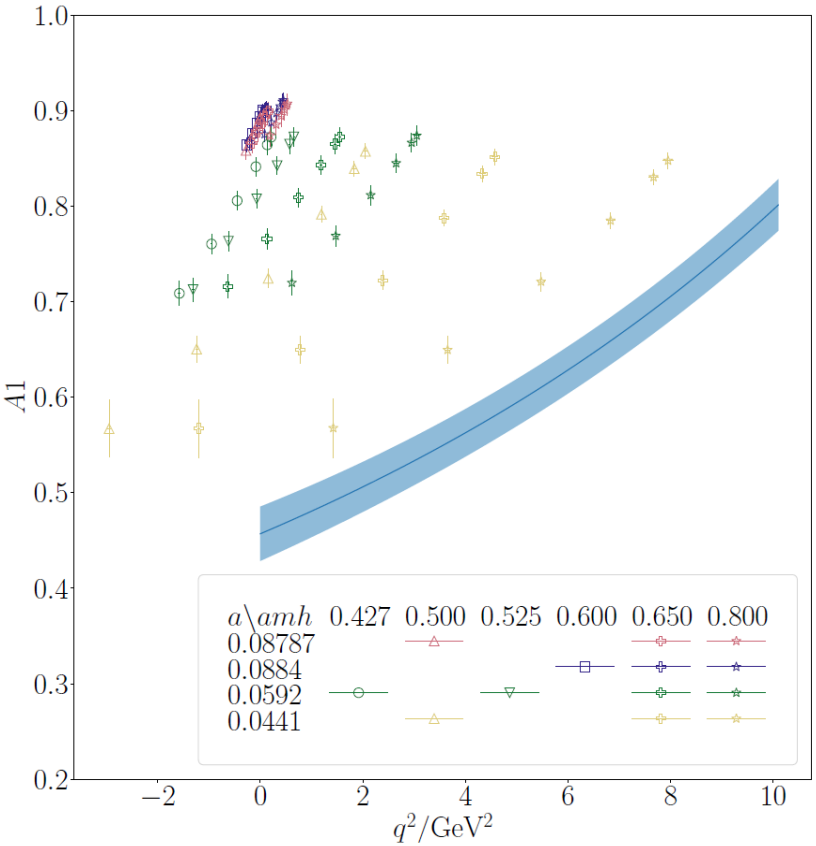


$B_c \to J/\psi \ell \nu$ decay

HPQCD 2007.06957(PRD), 2007.06956(PRL)

- $B_s \to D_s^* \ell \nu$ w/ c spectator quark
  - new physics in $R(J/\psi)$?
  - "stable" J/ψ, no ud propagator
  - 1 vector, 3 axial vector FFs
- set up similar to $B_s \to D_s^* \ell \nu$
  - relativistic HISQ for b, c quarks
    + no matching
    + extrapolation from $am_b \leq 0.8$
  - fit w/ modified z-expansion
    + coefficients in m_q's and a

$A_1(q^2)$ vs q^2



$B_c \to J/\psi^* \ell \nu$ decay

HPQCD 2007.06957(PRD), 2007.06956(PRL)

- 7% uncertainty in $(d\Gamma/dq^2)/|V_{cb}|^2$

  stat 3.6%, $a \neq 0$ 3.6%

  m_b 2.4%, $m_{ud,s}$ 3.4%

- 1.4% accuracy for $R(J/\psi)$

  $R(J/\psi) = 0.2582(38)$

  stat 1.1%, a ≠ 0 0.8%

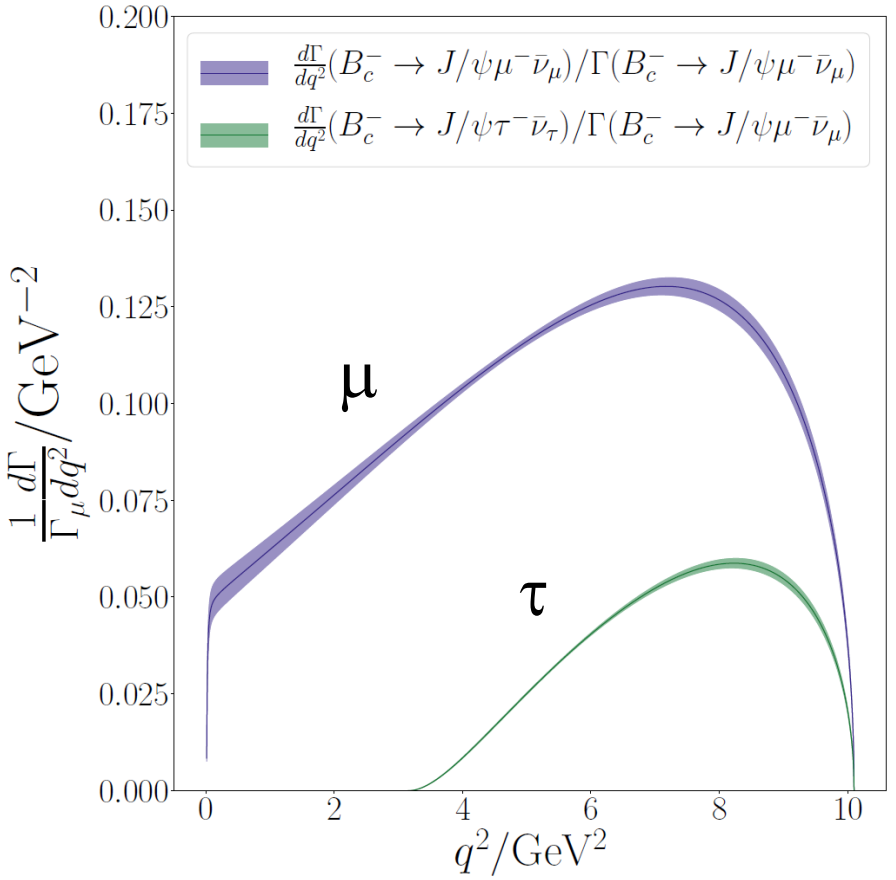
  m_b 0.6%

- 1.8 σ consistent w/ LHCb '18

  $R(J/\psi) = 0.71(18)_{\text{stat}}(17)_{\text{sys}}$

  ⇒ ΔR ∼ 0.07 / Run 3, 0.02 / Upgrade II

$$\frac{d\Gamma}{dq^2}\Big/\Gamma \quad \text{vs} \quad q^2$$

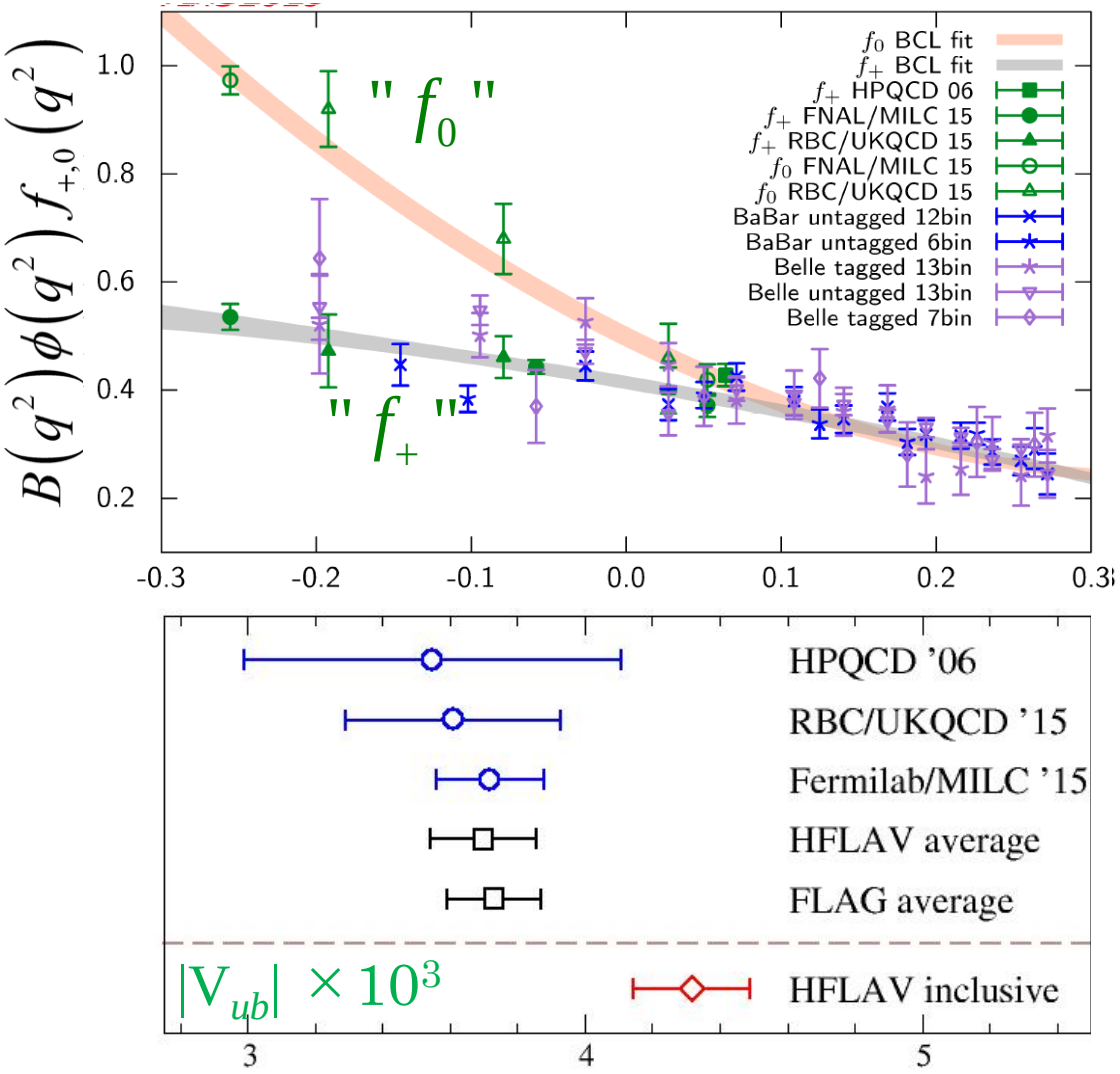


2. $b \to u \ell \nu$ decays

$B \to \pi \ell \nu$ decay

good publihed studies

Flavour Lattice Averaging Group (FLAG) '19



HPQCD '06

- NRQCD b
- $M_\pi \gtrsim 400$MeV
- stat. + matching error

RBC/UKQCD '15

- "relativistic heavy quark" b
- $M_\pi \gtrsim 270$MeV
- stat. + $a \Rightarrow 0$/chiral extrap.

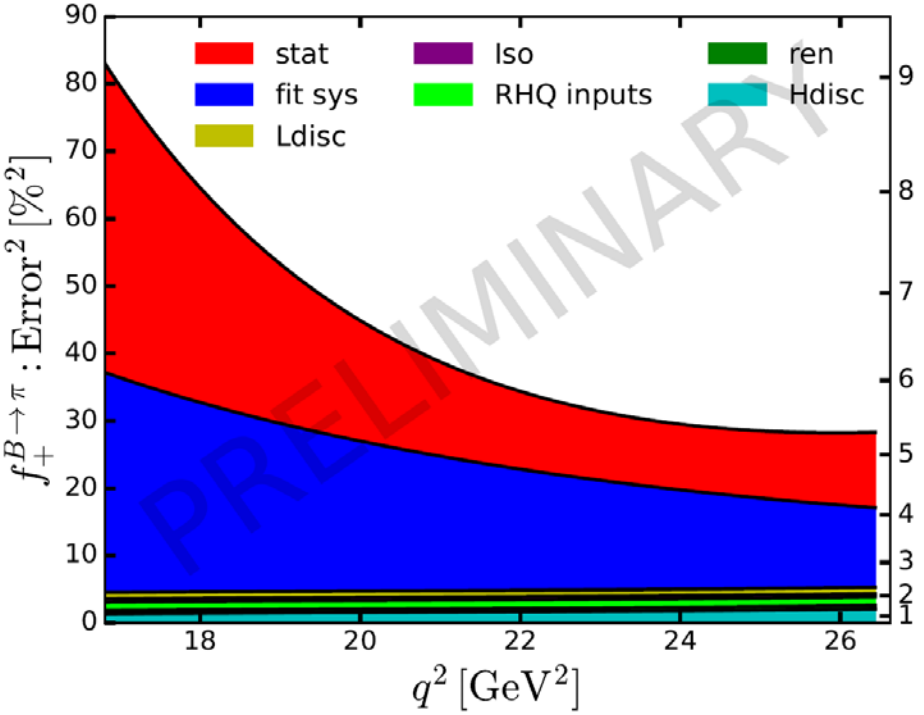
Fermilab/MILC '15

- "Fermilab interpretation" b
- $M_\pi \gtrsim 165$MeV, $a^{-1} \lesssim 4.5$GeV

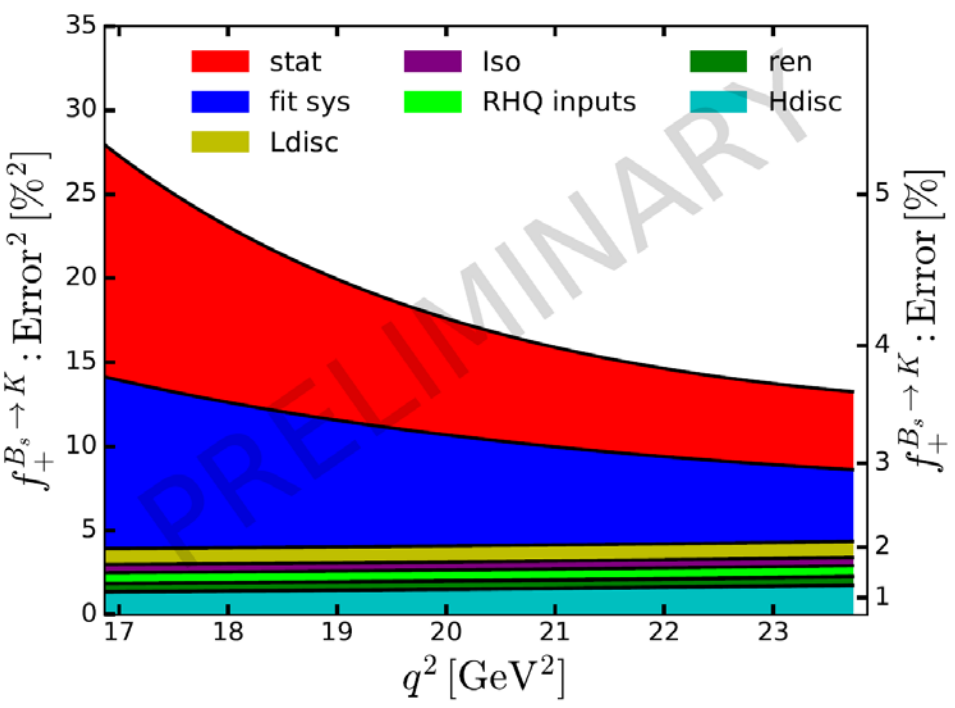
$B \to \pi \ell \nu$ and $B_s \to K \ell \nu$ decay

on-going study : RBC/UKQCD

$B \to \pi \ell \nu$



$B_s \to K \ell \nu$



- a finer lattice
- improved method to extract FFs
- ~~gauge ensemble @ physical M_π~~

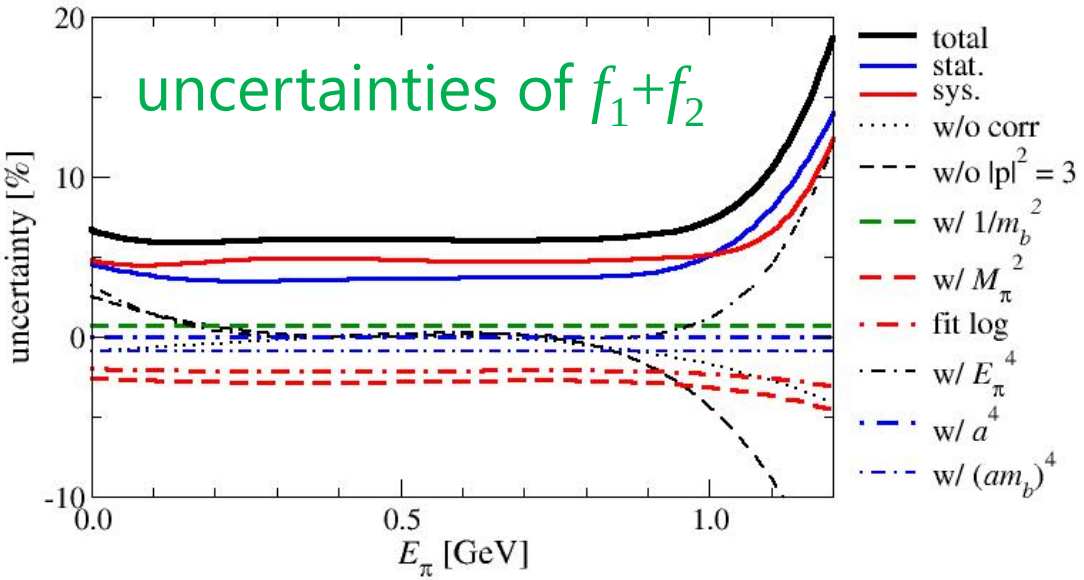
- (much) better accuracy
- measured by LHCb! 2012.05143

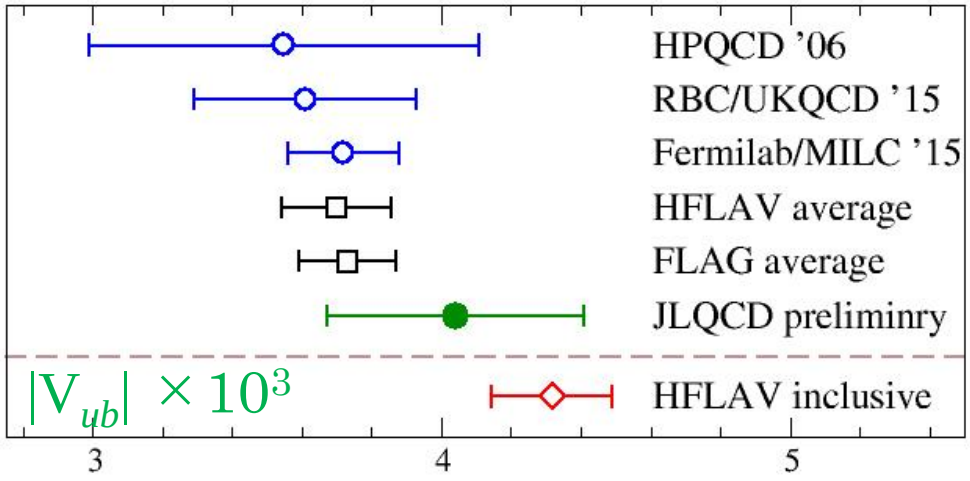
$B \to \pi \ell \nu$ decay

on-going study : JLQCD

- relativistic approach
  - chiral symmetric b
  - no $O(a)$ errors
  - simpler renormalization
- after FPCP 20
  - estimate of systematic uncertainties
- 9% accuracy
  - statistics
  - chiral extrapolation

  ⇒ Fugaku computer



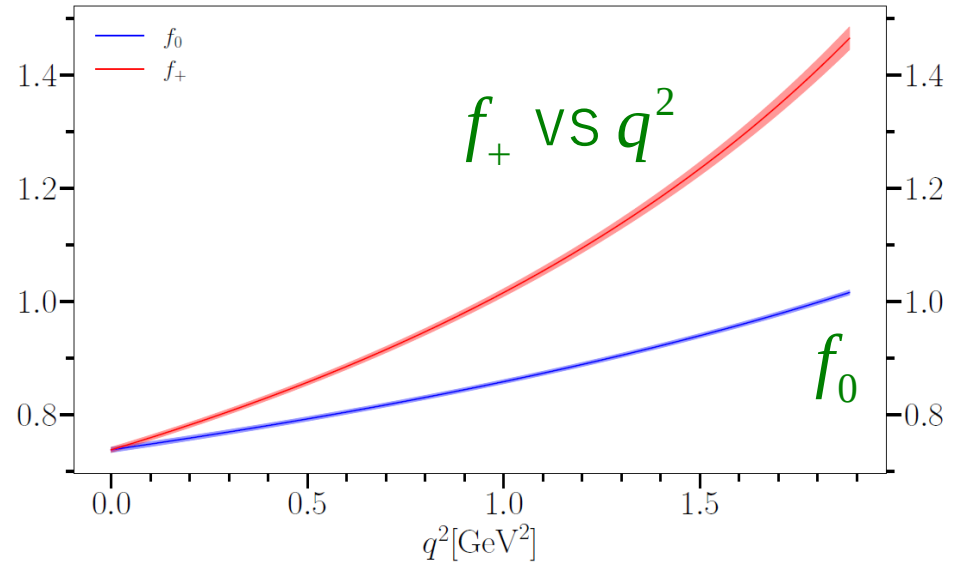


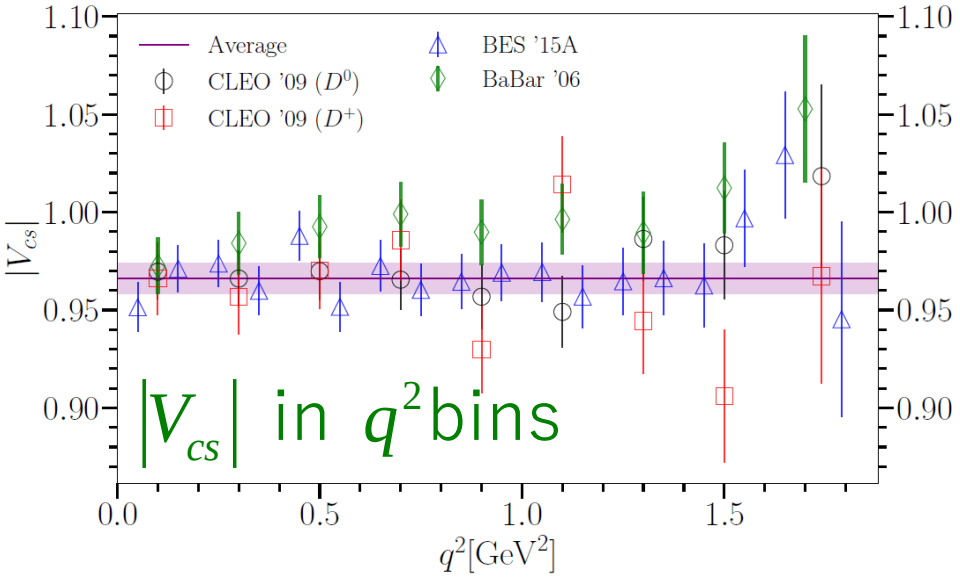
3. other decays

$D \to K \ell \nu$ decay

$|V_{cs}|$ and CKM unitarity in the 2nd row

HPQCD arXiv:2104.09883





- large range of a, physical M_π, full q^2 range $\Rightarrow$ precise data of f_+ and f_0
  (use CLEO, BES, BaBar data with covariance matrix available)

$|V_{cs}| = 0.994(15)$ (FLAG '19) $\Rightarrow$ $0.966\ (5)_{\text{lat}}(4)_{\text{exp}}(4)_{\text{EM+EW}}\ [8]_{\text{ttl}}$

$|V_{cs}|_{Ds \to \ell\nu} = 0.991(2)_{\text{lat}}(13)_{\text{exp}}$, $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.983(2)_{\text{Vcd}}(16)_{\text{Vcs}}$

baryon decays

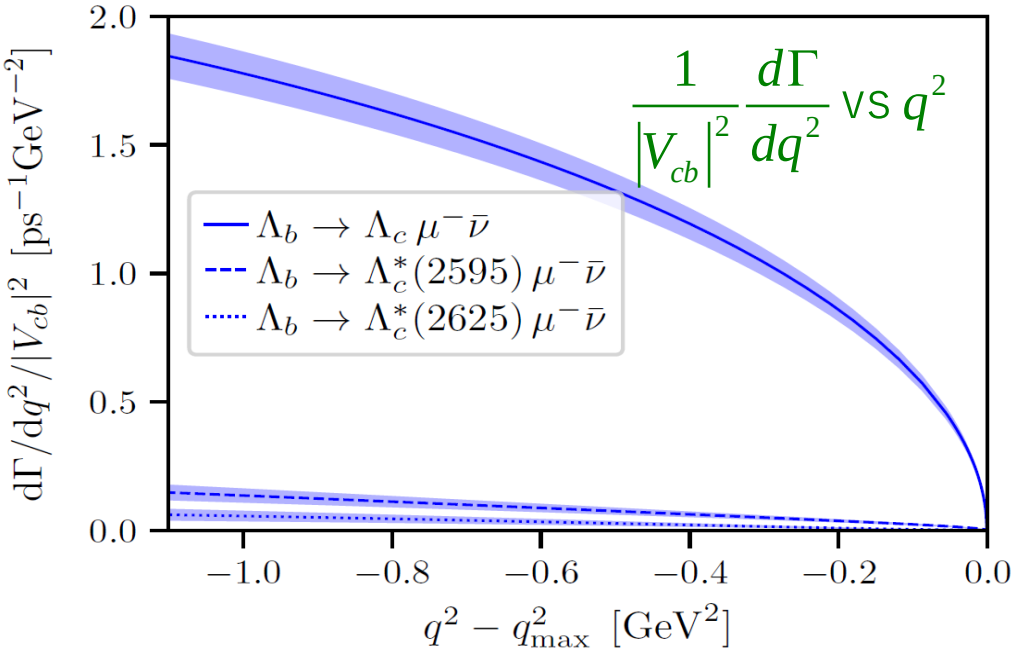
$b \to u\ell\nu$: $\Lambda_b \to p\ell\nu$ $\quad$ $c \to s\ell\nu$: $\Lambda_c \to \Lambda\ell\nu$

$b \to c\ell\nu$: $\Lambda_b \to \Lambda_c\ell\nu$, $\Lambda_b \to \Lambda_c^*(2596)\ell\nu$, $\Lambda_b \to \Lambda_c^*(2625)\ell\nu$ [new]

$b \to s\ell\ell$: $\Lambda_b \to \Lambda\ell\ell$, $\Lambda_b \to \Lambda^*(1520)\ell\ell$ [new]

Meinel-Rendon 2103.0875

- EFT approach
- calculate all SM FF
- predict $d\Gamma/dq^2$, asymmetry, angular observables



- only 2 a^{-1},s ≤ 2.5 GeV, 3 M_π 's ≥ 300 MeV, $M_{\pi,val}L$ ~ 3.1 not large

  ⇒ independent calculation(s) are highly welcome

Summary

heavy-heavy and heavy-light FFs from lattice QCD

- good progress for CKM elements and NP search
  - CKM : $B \to D^* \ell\nu$ @ $w \neq 1$, $B_s \to D_s \ell\nu$, $B \to \pi\ell\nu$, $D \to K\ell\nu$, ...
  - NP: $B_c \to J/\psi \ell\nu$
- $|V_{cb}|$ and $|V_{ub}|$ tensions still remain
  - more thorough study of exclusive decays
  - lattice study of inclusive decays [Gamibino-Hashimoto '20]
- more independent studies are highly welcome
- studies of BSM FFs