Resummation of Super-Leading Logarithms

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圈积分及相空间积分计算系列讲座 (online)

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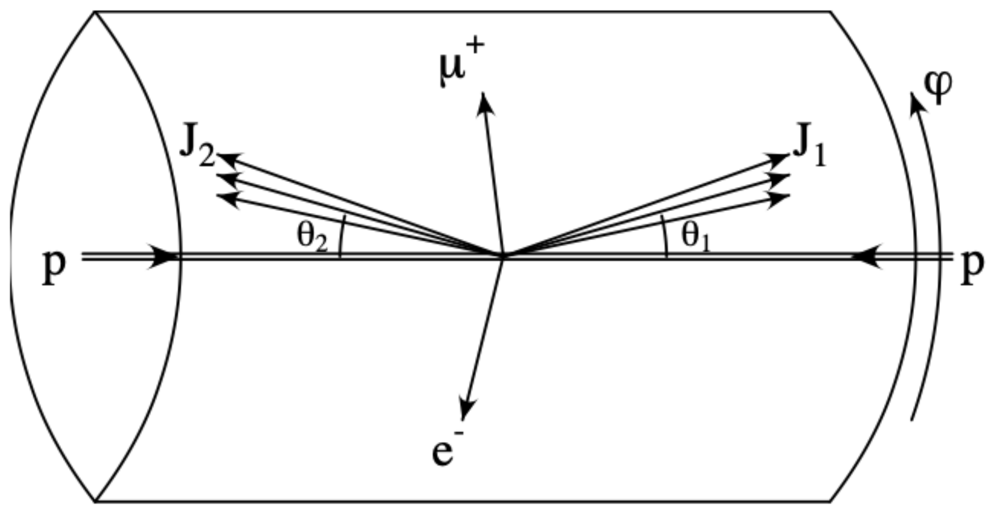
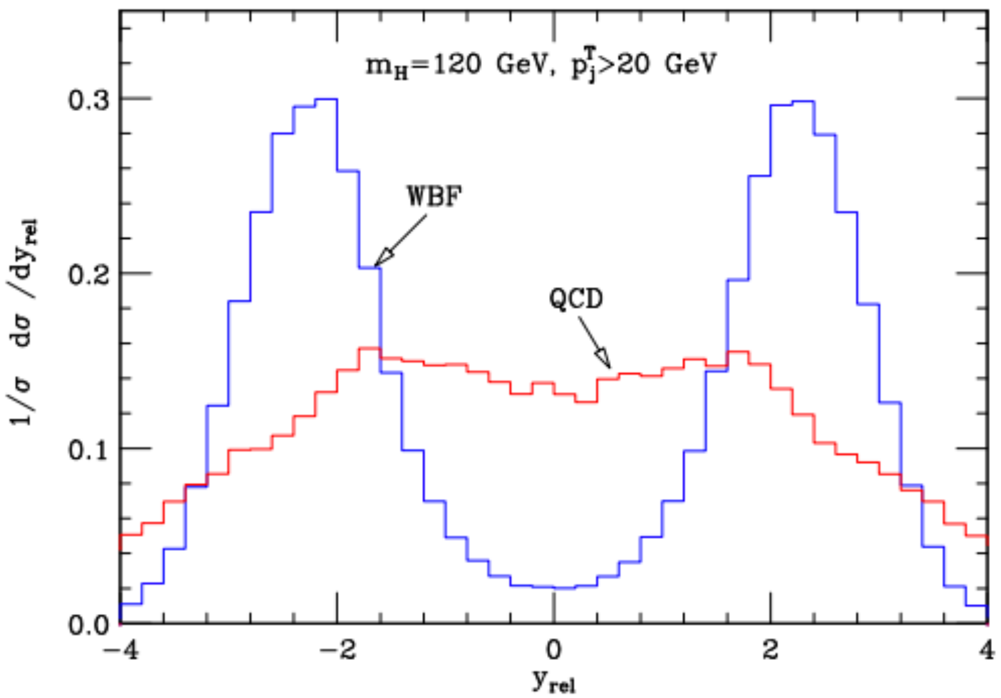
Central jet veto in Higgs production via VBF



VBF signature:

- **Energetic jets in the forward and backward directions**
- **Large rapidity separation and large invariant mass of two tagged jets**
- **Little radiation in the central-rapidity region**

- **Major QCD backgrounds: t-channel color octet exchange**
- **Central jet veto can suppresses QCD background**
- **Central jet veto: no extra jets between tagging jets**



Del Duca, Frizzo, Maltoni '05

Jet veto & QCD resummation

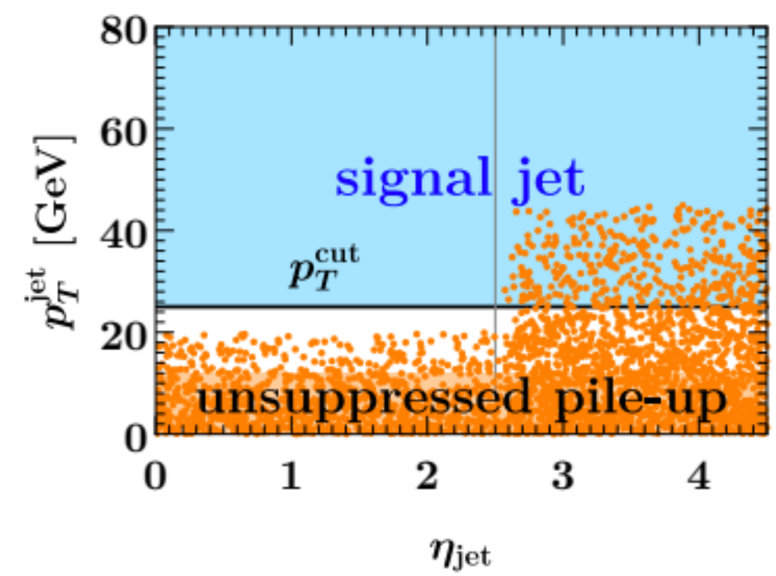
- Due to existence of a small scale p_T^{veto}, the fixed order calculations are unreliable
- QCD resummation is necessary, the large log should be resumed to all order
- Standard jet veto resummation for gg->H processes
  - Rapidity cut independent

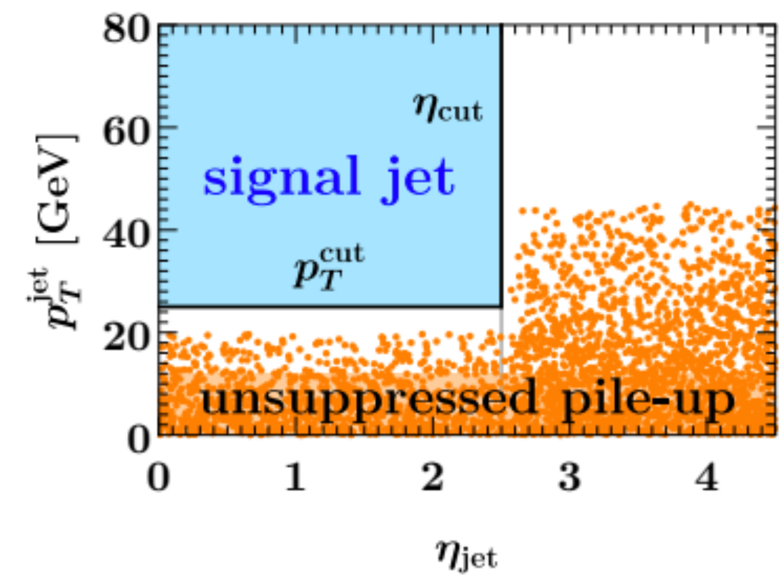
    Banfi, Monni, Salam, Zanderighi '12;

    Becher, Neubert, Rothen '12, '13;

    Stewart, Tackmann, Walsh, Zuberi '12, '13
  - Rapidity cut dependent

    Michel, Pietrulewicz, Tackmann '18
  - Nonfactorizable jet veto in VBF: Superleading Logs
    - **Four-loop** Forshaw, Kyrieleis, Seymour '06
    - **Five-loop** Keates, Seymour '09
    - **All-order** Becher, Neubert, DYS '21





Courtesy of Johannes Michel

Jet veto & QCD resummation

- **Due to existence of a small scale p_T^{veto}, the fixed order calculations are unreliable**
- **QCD resummation is necessary, the large log should be resumed to all order**
- **Standard jet veto resummation for gg->H processes**
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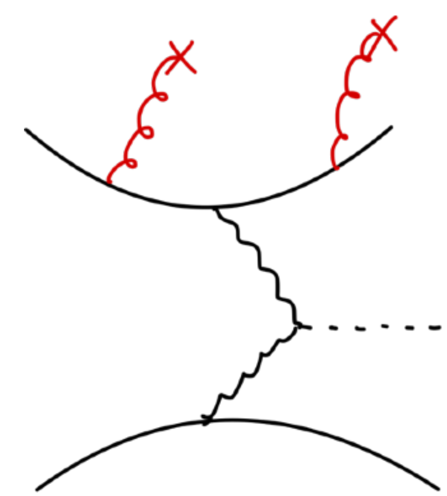

  - **Rapidity cut dependent**

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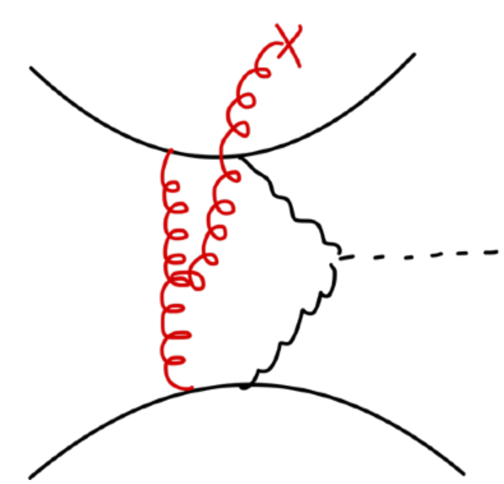
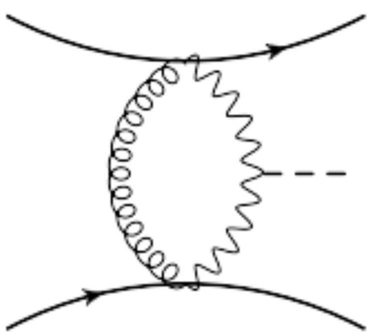
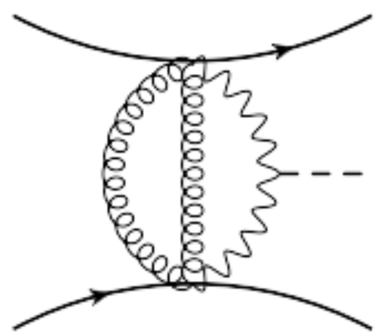


Nonfactorizable QCD effects in Higgs production via VBF

Liu, Melnikov, Penin ‘19





$$\mathcal{M}^{(2)} = -\frac{\tilde{\alpha}_s^2}{2!}\chi^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4)\mathcal{M}^{(0)}$$

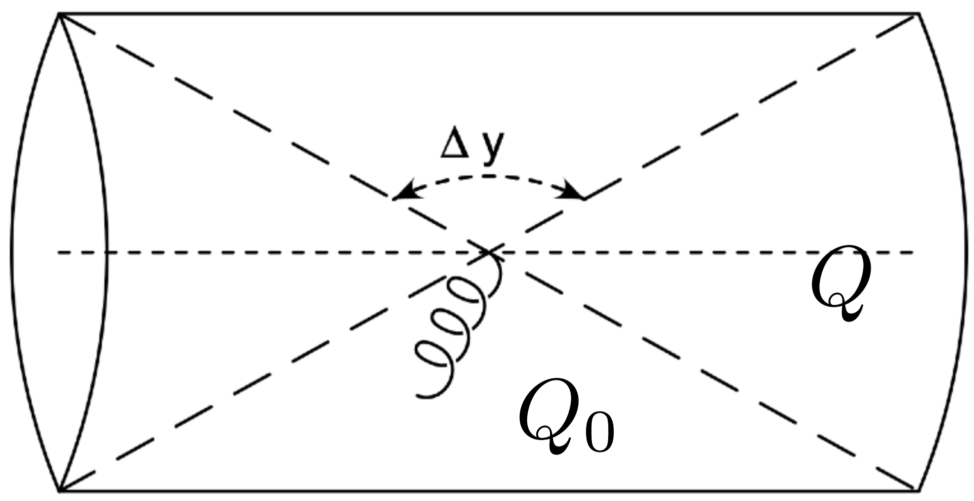
with $$\chi^{(2)}(\boldsymbol{q}_3, \boldsymbol{q}_4) = \frac{1}{\pi^2}\int \left(\prod_{i=1}^{2} \frac{\mathrm{d}^2\boldsymbol{k}_i}{\boldsymbol{k}_i^2 + \lambda^2}\right) \times \frac{\boldsymbol{q}_3^2 + M_V^2}{(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{q}_3)^2 + M_V^2} \frac{\boldsymbol{q}_4^2 + M_V^2}{(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{q}_4)^2 + M_V^2}$$

nonfactorizable correction: $$\Delta_{\mathrm{NF}} = \frac{\sigma_{\mathrm{VBF}}^{\mathrm{NNLO,NF}}}{\sigma_{\mathrm{VBF}}^{\mathrm{LO}}} \times 100\% = -0.39\%$$

- **the nonfactorizable correction is comparable to the NNNLO QCD factorizable corrections**
- **appear for the first time at NNLO, scale dependence is large**

See also Gaunt ’14, Schwartz, Yan & Zhu `17 `18 …

Central jet veto at the LHC

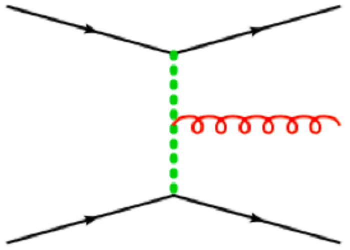


leading logs:

$$e^+e^-, ep: \quad \alpha_s^n \ln^n\left(\frac{Q}{Q_0}\right)$$

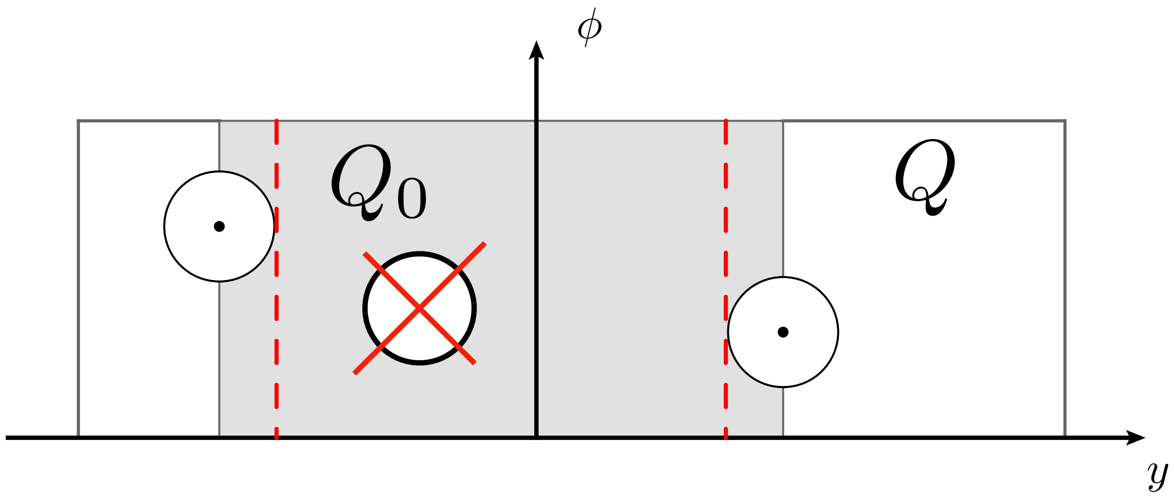
$$pp: \quad \cdots \quad + \alpha_s^3 (i\pi)^2 \ln^3\left(\frac{Q}{Q_0}\right) \times \alpha_s^n \ln^{2n}\left(\frac{Q}{Q_0}\right)$$

- Such events was originally suggested on the basis of color flow considerations in QCD Bjorken '93
- Global Logs resummation is first done by Oderda & Sterman '98
- Forshaw, Kyrieleis, Seymour '06 have analyzed the effect of Glauber phases in non-global observables directly in QCD
    - Non-zero contributions starting at 3 loops
    - Collinear logarithms starting at 4 loops: Super-leading logs



wide angle soft gluon emission developing a sensitivity to emission at small angles

Central jet veto at the LHC



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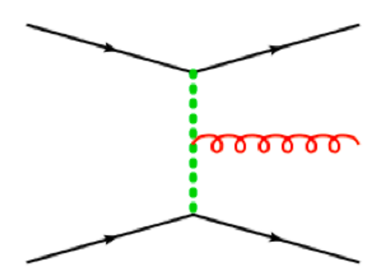
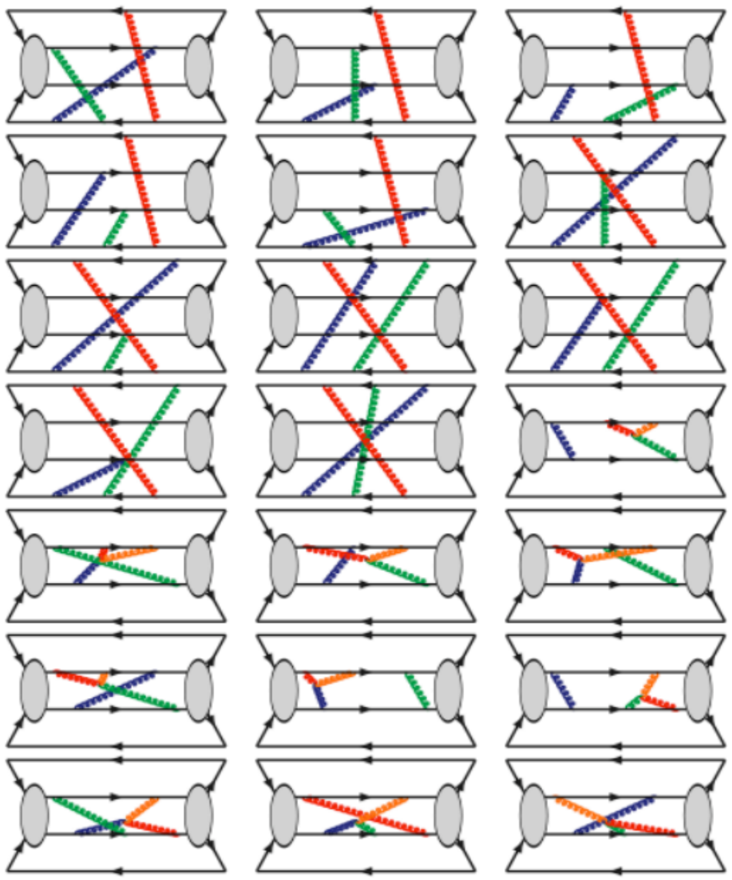


wide angle soft gluon emission developing a sensitivity to emission at small angles

Fixed order calculation

- Gluons are added in all possible ways to trace diagrams and colour factors calculated using COLOUR
- Diagrams are then cut in all ways consistent with strong ordering
- At fourth order there are 10,529 diagrams and 1,746,272 after cutting.
- SLL terms are confirmed at fourth order and computed for the first time at 5^{th} order

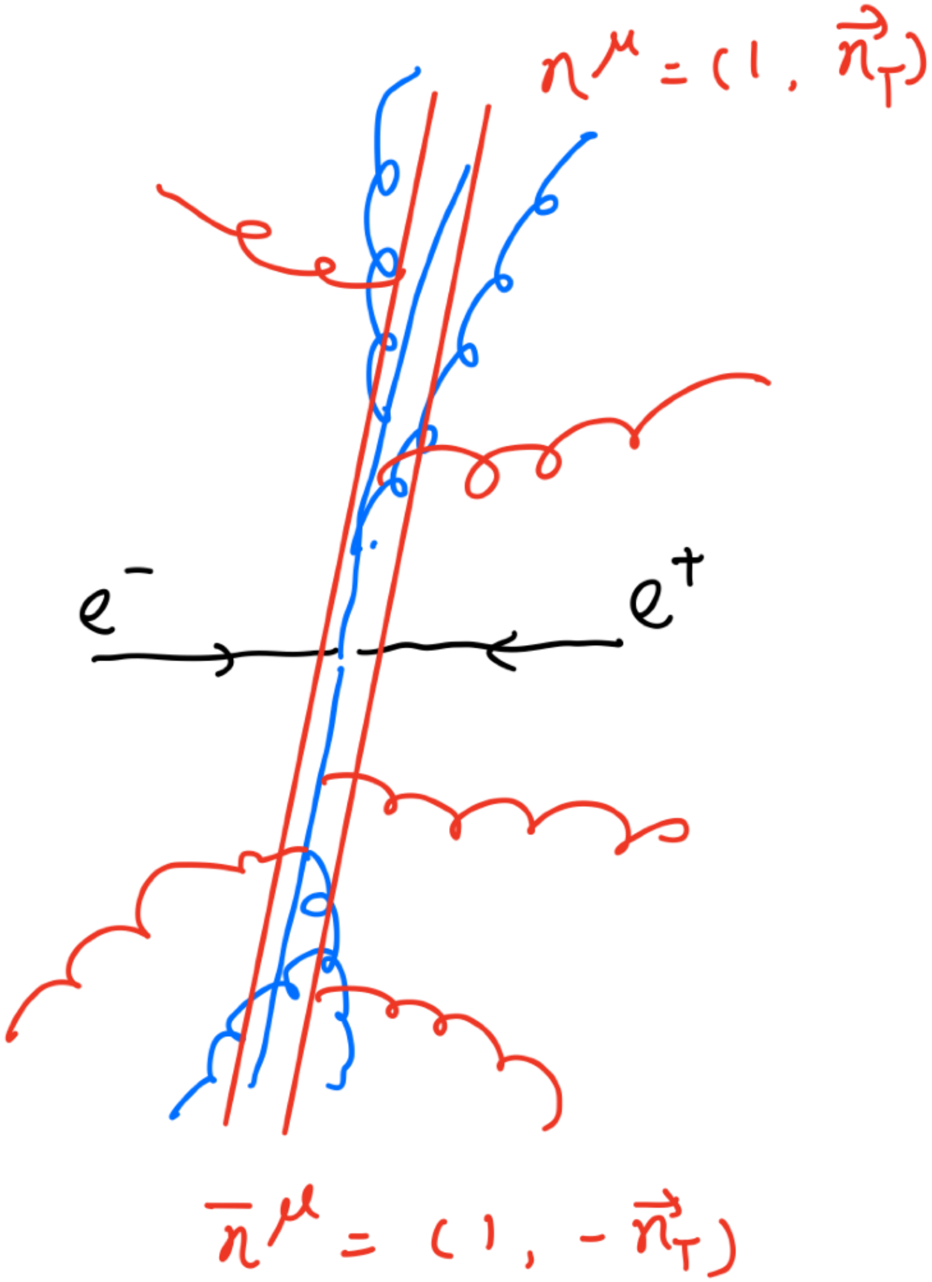


Keates and Seymour
arXiv:0902.0477 [hep-ph]

Simone Marzani's slide

Factorization in global event shapes

E.g. Thrust $T \sim 1$



$$\frac{d\sigma}{dT} = H \cdot J \otimes S$$

Soft radiation does not resolve individual energetic patrons. Sensitive only to direction and total charge of the jets

$$S \sim \sum_{X_s} \left| \langle X_s | \boldsymbol{S}(n) \boldsymbol{S}(\bar{n}) \left| 0 \right\rangle \right|^2$$

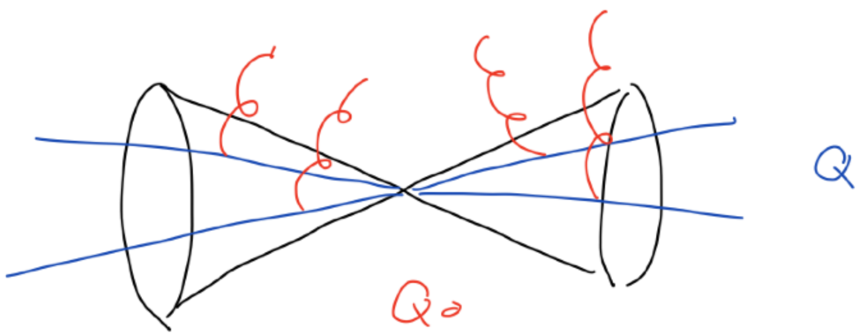
Simple structure -> N^{3}LL resummation

An effective field theory for jet processes

Becher, Neubert, Rothen, DYS ’15

EFT contains two modes:

$$\begin{aligned} \text{hard:} \quad & p_h \sim \quad Q\,(1,1,1) \\ \text{soft:} \quad & p_s \sim Q\beta\,(1,1,1) \end{aligned}$$



1. no collinear singularity, only single logs

2. method of region to verify at two-loop level

Hard parton described by collinear field $\Phi_i \in \{\chi_i, \bar{\chi}_i, \mathcal{A}^{\mu}_{i\perp}\}$

gauge invariant: $\chi_i(0) = W_i^\dagger(\bar{n}_i)\,\frac{\not{n}_i\bar{\not{n}}_i}{4}\,\psi_i(0)$

Perform decoupling transformation: $\Phi_i = \boldsymbol{S}_i(n_i)\,\Phi_i^{(0)}$ $\quad \boldsymbol{S}_i(n_i) = \mathbf{P}\exp\left(ig_s\int_0^\infty ds\, n_i\cdot A_s^a(sn_i)\,\boldsymbol{T}_i^a\right)$

Evaluates the matrix element of the operator with one collinear particle

$$\langle 0|\,\chi_j^{(0)}(0)\,|p_i\rangle = \delta_{ij}\,u(p_i)$$

Factorization for gap between jets in e+e-

(Becher, Neubert, Rothen, DYS, '15 PRL, '16 JHEP; Caron-Huot '15 JHEP)

Hard function
m hard partons along
fixed directions {n_1, …, n_m}
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

$$\sigma(Q, Q_\Omega) \sim \sum_{m=2}^{\infty} \prod_{i=1}^{m} \int \frac{d\Omega(\vec{n}_i)}{4\pi} \mathrm{Tr_c}\left[\mathcal{H}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q, \mu)\mathcal{S}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q_\Omega, \mu)\right]$$

Factorization for gap between jets in e+e-

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m hard partons along
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Soft function
squared amplitude
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Color Trace

Hard scale

Soft scale

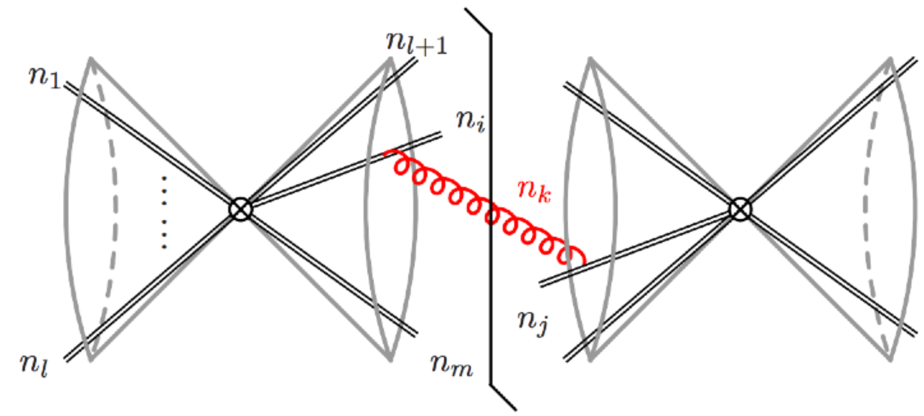
$$\sigma(Q, Q_\Omega) \sim \sum_{m=2}^{\infty} \prod_{i=1}^{m} \int \frac{d\Omega(\vec{n}_i)}{4\pi} \mathrm{Tr_c}\left[\mathcal{H}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q, \mu)\mathcal{S}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q_\Omega, \mu)\right]$$

# of jet not fixed

Integrate the angles for hard partons

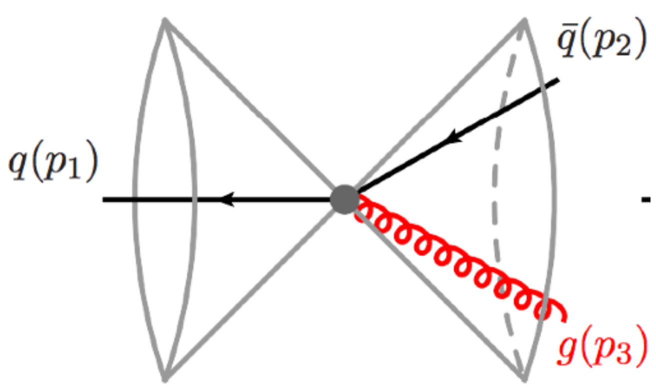
Renormalization

Bare hard and soft function suffer from divergence.



$$\boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q\beta, \delta, \epsilon) = \mathbf{1} + \frac{\alpha_s}{2\pi\epsilon}\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \int \frac{d\Omega(n_k)}{4\pi} W_{ij}^k\, \Theta_{\text{out}}^{n\bar{n}}(n_k)$$

$$W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k\, n_j \cdot n_k}$$



$p_1 > p_2 > p_3$

$$\boldsymbol{\mathcal{H}}_3^{(1)} = \frac{1}{2Q^2(2\pi)^{2-4\epsilon}} \prod_{i=1}^{3} \int dE_i\, E_i^{1-2\epsilon}\, |\mathcal{M}_3(\{p_1, p_2, p_3\})\rangle\langle\mathcal{M}_3(\{p_1, p_2, p_3\})|$$
$$\times\, \delta(Q - E_1 - E_2 - E_3)\, \delta^{(3-2\epsilon)}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)\; \Theta_{\text{in}}^{n\bar{n}}(\{p_1, p_2, p_3\})$$

$$\boldsymbol{\mathcal{H}}_{3,\text{I}}^{(1)}(u, v, Q, \delta, \epsilon) = C_F\, 4^\epsilon\, C_\epsilon\, e^{-2\epsilon L_h}\, \delta^{-2\epsilon}\, u^{-1-2\epsilon}\, v^{-1-\epsilon}\, h_3^{\text{I}}(u, v, \delta, \epsilon)\, \mathbf{1}$$

$$u \sim \theta_2, \quad v \sim \theta_3$$

See also Banfi, Dreyer, Monni '21

Renormalization

UV poles inside hard function removed by renormalizing the hard function as

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \delta, \epsilon) = \sum_{l=2}^{m} \boldsymbol{\mathcal{H}}_l(\{\underline{n}\}, Q, \delta, \mu)\, \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu)$$

1. **obtain the bare hard function from on-shell matching. The IR poles are in one-to-one correspondence to UV div, since the EFT loop-integrals are scaleless.**
2. **We can understand the UV div. of hard function from the structure of the IR div. in the real and virtual diagrams**
3. **lower multiplicity virtual diagrams are needed to cancel the div. of real emission diagrams**

$$\boldsymbol{\mathcal{H}}_3^{(1)}(Q, \mu) = \boldsymbol{\mathcal{H}}_3^{(1)}(Q, \epsilon) - Z_{23}^{(1)}(Q, \epsilon, \mu) \boldsymbol{\mathcal{H}}_2^{(0)}(Q, \mu)$$

the renormalization matrix must have the form:

$$\boldsymbol{Z}^H(\{\underline{n}\}, Q, \delta, \epsilon, \mu) \sim \begin{array}{c} \begin{array}{ccccc} 2 & 3 & 4 & 5 & \dots \end{array} \\ \begin{pmatrix} 1 & \alpha_s & \alpha_s^2 & \alpha_s^3 & \dots \\ 0 & 1 & \alpha_s & \alpha_s^2 & \dots \\ 0 & 0 & 1 & \alpha_s & \dots \\ 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array} \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ \dots \end{array}$$

At each higher order in perturbation theory, more off-diagonal contributions fill in

By consistency, the matrix Z^H must render the soft functions finite

$$\boldsymbol{S}_l(\{\underline{n}\}, Q\beta, \delta, \mu) = \sum_{m=l}^{\infty} \boldsymbol{Z}^H_{lm}(\{\underline{n}\}, Q, \delta, \epsilon, \mu) \,\hat{\otimes}\, \boldsymbol{S}_m(\{\underline{n}\}, Q\beta, \delta, \epsilon)$$

Higher multiplicity soft functions are needed to absorb the div. of matrix elements with fewer Wilson lines

$$S_2(\mu) = Z^H_{22} S_2(\varepsilon) + Z^H_{23} \hat{\otimes} S_3(\varepsilon) + Z^H_{24} \hat{\otimes} 1$$

Verify at two-loop order !!!

Leading Log Resummation

$$\sigma^{\rm LL}(Q,Q_0) = \sum_{m=2}^{\infty} \left\langle \boldsymbol{\mathcal{H}}_2(\{n_1,n_2\},Q,\mu_h) \otimes \boldsymbol{U}_{2m}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1}\right\rangle$$

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h) = \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma}^H(\{\underline{n}\},\mu)\right]$$

One-loop anomalous dim. : $\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

$$\boldsymbol{V}_m = 2\sum_{(ij)} (\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}) \int \frac{d\Omega(n_k)}{4\pi} W_{ij}^k,$$

$$\boldsymbol{R}_m = -4\sum_{(ij)} \boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R} W_{ij}^{m+1}\,\Theta_{\rm in}(n_{m+1}),$$

dipole: $W_{ij}^k = \dfrac{n_i\cdot n_j}{(n_i\cdot n_k)(n_j\cdot n_k)}$

virtual: $\boldsymbol{V}_m\mathcal{H}_m \sim \sum_a \boldsymbol{T}_i^a\cdot\boldsymbol{T}_j^a |\mathcal{M}_m\rangle\langle\mathcal{M}_m| + |\mathcal{M}_m\rangle\langle\mathcal{M}_m| \sum_a \boldsymbol{T}_i^a\cdot\boldsymbol{T}_j^a$

real: $\boldsymbol{R}_m\mathcal{H}_m \sim \boldsymbol{T}_i^a |\mathcal{M}_m\rangle\langle\mathcal{M}_m|\, \boldsymbol{T}_j^a$

RG equation: $\dfrac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}$ $\qquad t = \displaystyle\int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\frac{\alpha}{4\pi}$

Formal solution

$$\sigma(Q,Q_0)=\sum_{l=2}^{\infty}\langle\boldsymbol{\mathcal{H}}_l(\{\underline{n}'\},Q,\mu_h)\otimes\sum_{m\geq l}^{\infty}\boldsymbol{U}_{lm}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu_s)\rangle$$

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h)=\mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma}(\{\underline{n}\},\mu)\right]$$

A way to generate (super)-leading logs order-by-order

- Use lowest order $\mathcal{H}_2 = \sigma_0$ and $\mathcal{S}_m = \mathbf{1}$
- Expand

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h)=\mathbf{1}+\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma}+\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\int_{\mu}^{\mu_h}\frac{d\mu'}{\mu'}\boldsymbol{\Gamma}\boldsymbol{\Gamma}+\ldots$$

Resummation of Super-Leading Logs

arXiv:2107.01212 Becher, Neubert & DYS

Collinear singularities and SLLs at hadron colliders

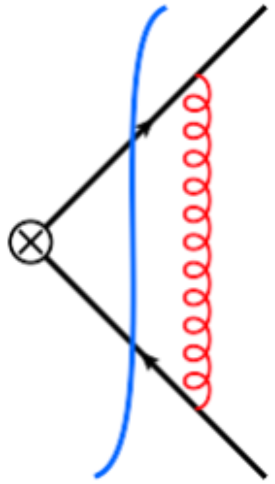
One-loop anomalous dimension:

$$\Gamma^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$$

$$\boldsymbol{V}_m = -2 \sum_{(ij)} \int \frac{d\Omega(n_k)}{4\pi} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \, W_{ij}^k \left[\Theta_{\text{in}}^{n\bar{n}}(k) + \Theta_{\text{out}}^{n\bar{n}}(k)\right]$$
$$+ 2\,i\pi \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \, \Pi_{ij} \,,$$
$$\boldsymbol{R}_m = 4 \sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \, W_{ij}^{m+1} \, \Theta_{\text{in}}(n_{m+1}) \,.$$

$\Pi_{ij} = 1$ **if both incoming or outgoing**



Individually R_m and V_m contain singularities when emitted gluon k gets collinear to parsons i or j.

- **Expect cancellation in inclusive soft observables such as gaps between jets at lepton colliders**
- **Glauber phases spoil this cancellation: soft+collinear double logs! "Super-leading logs"**

Simplification of the imaginary part

Imaginary part of the anomalous dimension:

For e+e-:

$$\sum_{i\neq j} \mathbf{T}_i \cdot \mathbf{T}_j = -\sum_i \mathbf{T}_i^2 = -\sum_i C_i$$

For pp:

$$\begin{aligned}\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Pi_{ij} &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + \sum_{i=3}^{m} \boldsymbol{T}_i \cdot (-\boldsymbol{T}_1 - \boldsymbol{T}_2 - \boldsymbol{T}_i) \\ &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2) - \sum_{i=3}^{m} C_i \\ &= 4\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + C_1 + C_2 - \sum_{i=3}^{m} C_i\end{aligned}$$

$$\sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R})\, \Pi_{ij} = 4\,(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R})$$

Extracting the collinear singularities: $\overline{W}_{ij}^{k} = \dfrac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k} - \dfrac{\delta(n_k - n_i)}{n_i \cdot n_k} - \dfrac{\delta(n_k - n_j)}{n_j \cdot n_k}$

The one-loop anomalous dimension is

$$\boldsymbol{V}_m = \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \, \ln \frac{\mu^2}{\hat{s}}$$

$$\boldsymbol{R}_m = \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \, \ln \frac{\mu^2}{\hat{s}} \,,$$

with

$$\overline{\boldsymbol{V}}_m = 2 \sum_{(ij)} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \int \frac{d\Omega\,(n_k)}{4\pi} \bar{W}_{ij}^{k}$$

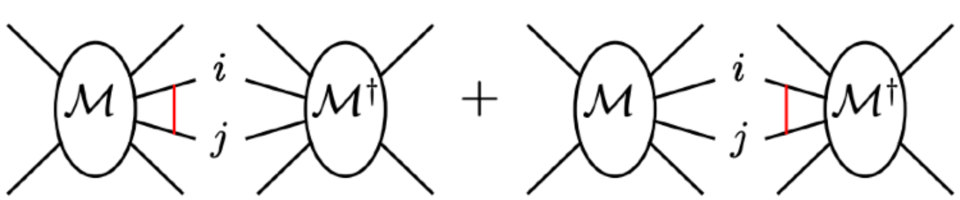
$$\boldsymbol{V}_i^c = 4 C_i \boldsymbol{1}$$

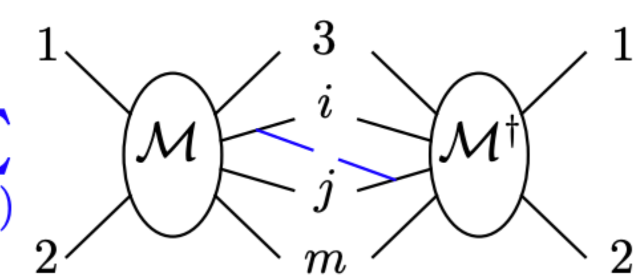
$$\boldsymbol{V}^G = -8 i \pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

$$\overline{\boldsymbol{R}}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} \bar{W}_{ij}^{m+1} \Theta_{\text{hard}}\left(n_{m+1}\right)$$

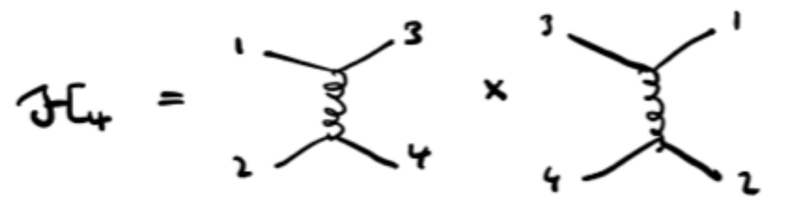
$$\boldsymbol{R}_i^c = -4 \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \delta\left(n_k - n_i\right)$$

$$\boxed{\boldsymbol{\mathcal{H}}_m \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R} = \boldsymbol{T}_i^a \, \boldsymbol{\mathcal{H}}_m \, \boldsymbol{T}_j^{\tilde{a}}}$$

$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{V}}_m = \sum_{(ij)}$ 

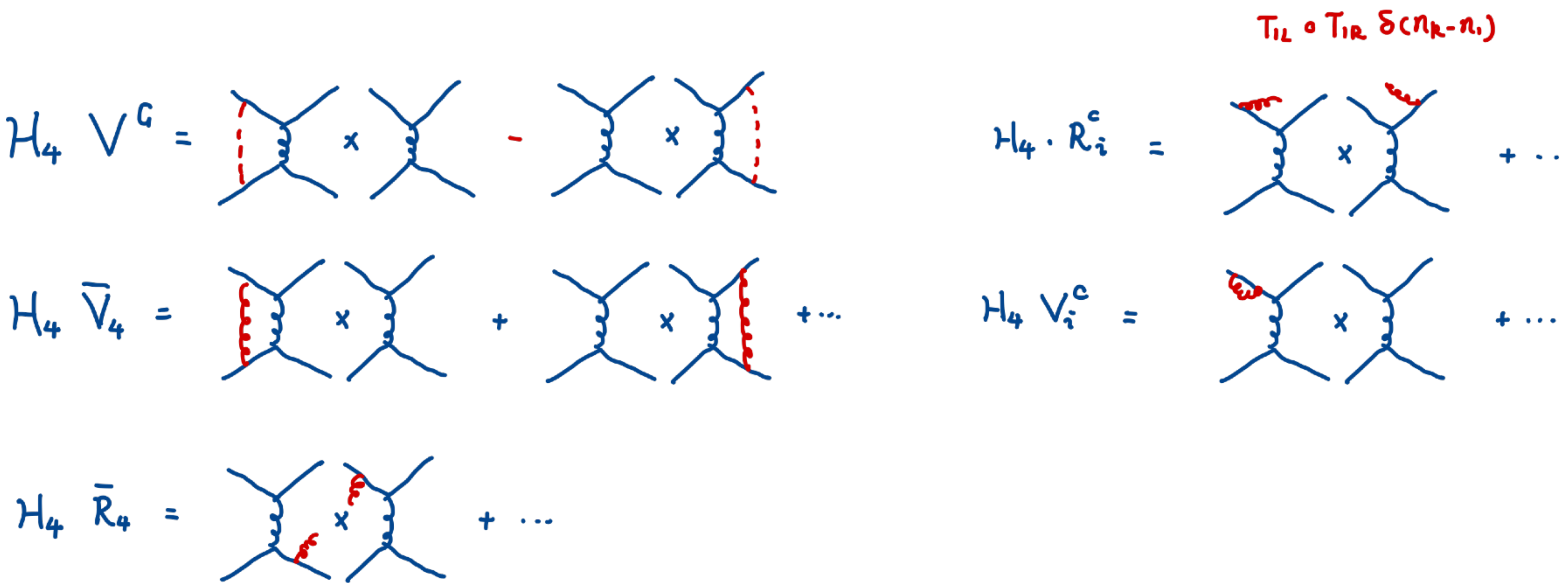
$\boldsymbol{\mathcal{H}}_m \overline{\boldsymbol{R}}_m = \sum_{(ij)}$ 

Hard function for octet exchange:



$$\sim t^{a}_{\alpha_3\alpha_1} t^{a}_{\alpha_4\alpha_2} t^{b}_{\beta_1\beta_3} t^{b}_{\beta_2\beta_4} \sigma_0$$

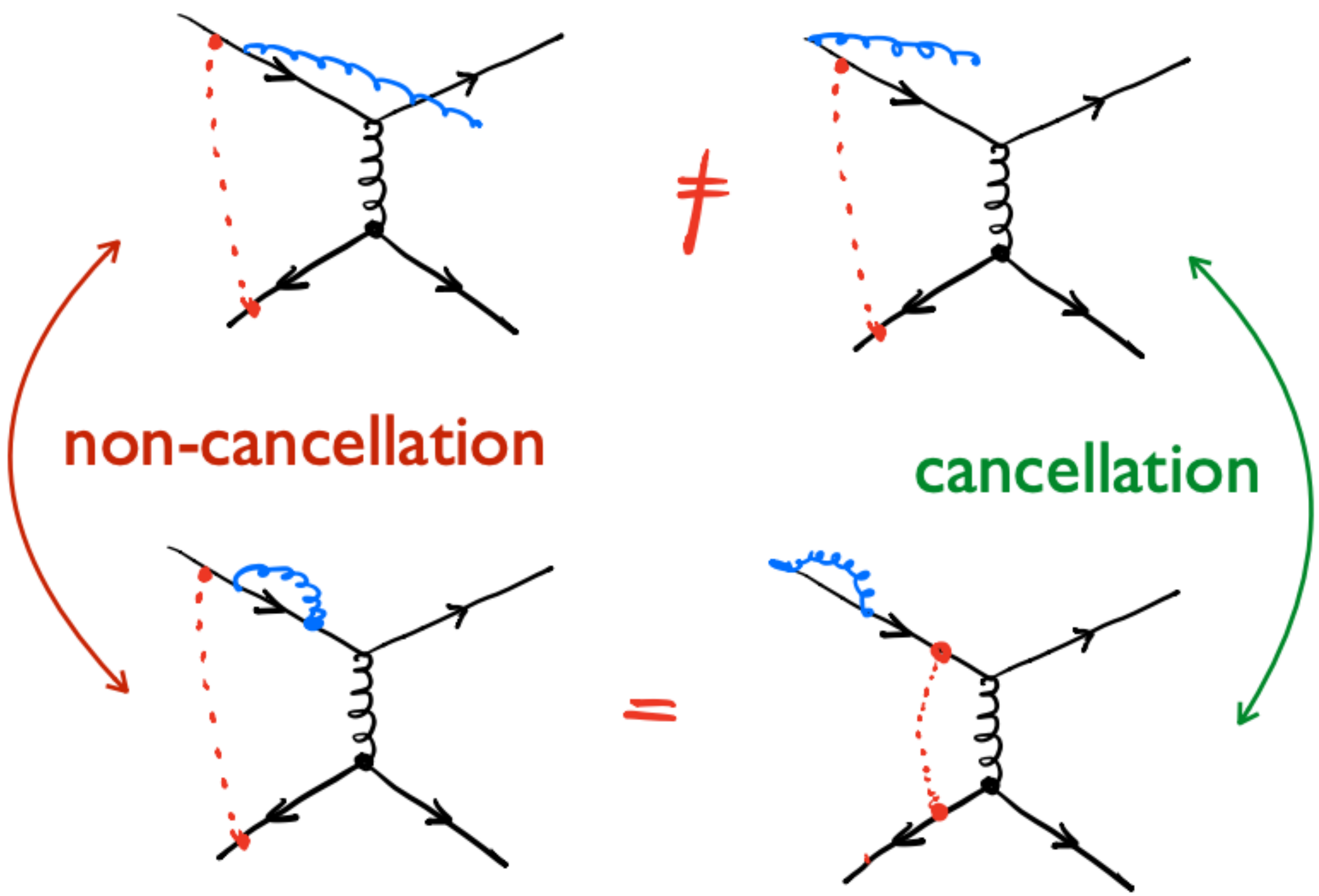
Action of the anomalous dimension



Compute $\mathcal{H}_4\, \boldsymbol{U}(\mu_s,\mu_h) = \mathcal{H}_4\, \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(Q,\mu)\right]$

$$= \mathcal{H}_4 + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \mathcal{H}_4\, \boldsymbol{\Gamma}^H(Q,\mu) + \int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \int_{\mu}^{\mu_h} \frac{d\mu'}{\mu'} \mathcal{H}_4\, \boldsymbol{\Gamma}^H(Q,\mu')\, \boldsymbol{\Gamma}^H(Q,\mu)$$

Non-cancellation of collinear logs



Blue: collinear emission $\mathbf{\Gamma}^{L}$

Red: Glauber phase $\mathbf{\Gamma}^{I}$

Leading super-leading logs

1. **Want the maximum numbers of logs, i.e. the maximum power of Γ^c**
2. **Need two imaginary parts V^G to spoil cancellation of collinear singularities**
3. **Need at least one real emission $\bar{\Gamma}$ to resolve the gap region**

Three properties of the anomalous dimension greatly simplify the calculations

- **Color coherence**

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m \, \overline{\boldsymbol{\Gamma}} \, \boldsymbol{\Gamma}^c$$

- **Cyclicity of the trace**

$$\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$$
$$\big\langle \boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}^G \otimes \mathbf{1} \big\rangle = 0$$

Leading super-leading logs

The super-leading logs at (3+n) order are associated with color traces of the form

$$C_{rn} = \big\langle \boldsymbol{\mathcal{H}}_4 \, (\boldsymbol{\Gamma}^c)^r \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{n-r} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \boldsymbol{1} \big\rangle \qquad\qquad 0 \le r \le n$$

The SLLs first appear at four loop (n=1)

The three loop terms (n=0) can be numerically significant

We consider the case where particles 1 and 2 transform in the fundamental representation of SU(N_c)

$$C_{rn} = 2^{8-r} \pi^2 \, (4N_c)^n \Big\{ \sum_{j>2} J_j \langle \mathcal{H}_4 [(\boldsymbol{T}_2 - \boldsymbol{T}_1) \cdot \boldsymbol{T}_j + 2^{r-1} N_c \, (\sigma_1 - \sigma_2) \, d_{abc} \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c] \rangle$$
$$+ 2 \, (1 - \delta_{r0}) \, J_2 \, \langle \mathcal{H}_4 \, [C_F + (2^r - 1) \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2] \rangle \Big\}$$

with the angular integrals: $J_j = \int \frac{d\Omega(n_k)}{4\pi} \, \big(W_{1j}^k - W_{2j}^k\big) \, \Theta_{\text{veto}}(n_k)$

All-order results of leading SLLs

(Becher, Neubert, **DYS** '21)

$$\omega \sim \alpha_s L^2$$

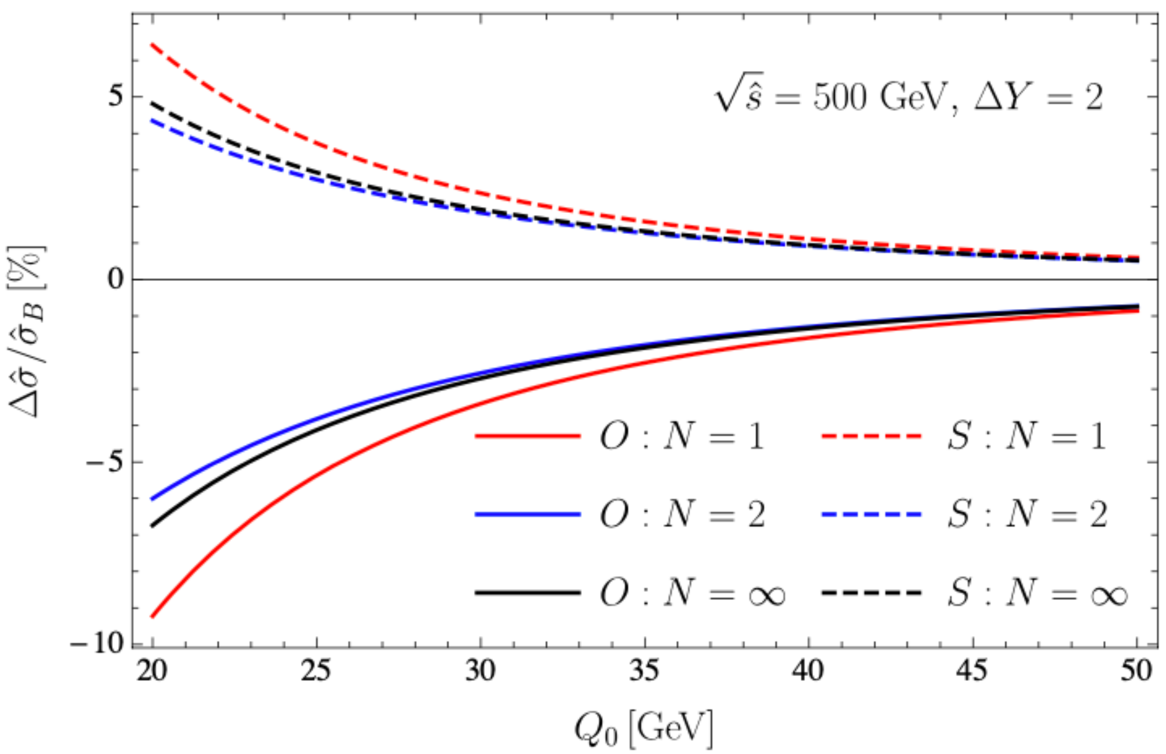
$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3 \frac{Q}{\mu_s} \frac{1}{N_c} \left[N_c^2 \left(4f_1(w) - 2f_\delta(w)\right] - 4f_2(\omega) + 2f_\delta(w)\right) \Delta Y \, \sigma_0$$

Numerical results

Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs → $e^{-\omega}$

Superleading logs $\xrightarrow{\omega \to \infty}$ $\frac{1}{\omega}$



Red: Four loop **Blue: Five loop** **Black: all order**

All-order results of leading SLLs

(Becher, Neubert, **DYS** '21)

hypergeometric function

$$f_\delta(w) = \frac{1}{3}\, {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right)$$

$$\omega \sim \alpha_s L^2$$

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3\frac{Q}{\mu_s}\,\frac{1}{N_c}\left[N_c^2\left(4f_1(w) - 2f_\delta(w)\right] - 4f_2(\omega) + 2f_\delta(w)\right)\Delta Y\,\sigma_0$$

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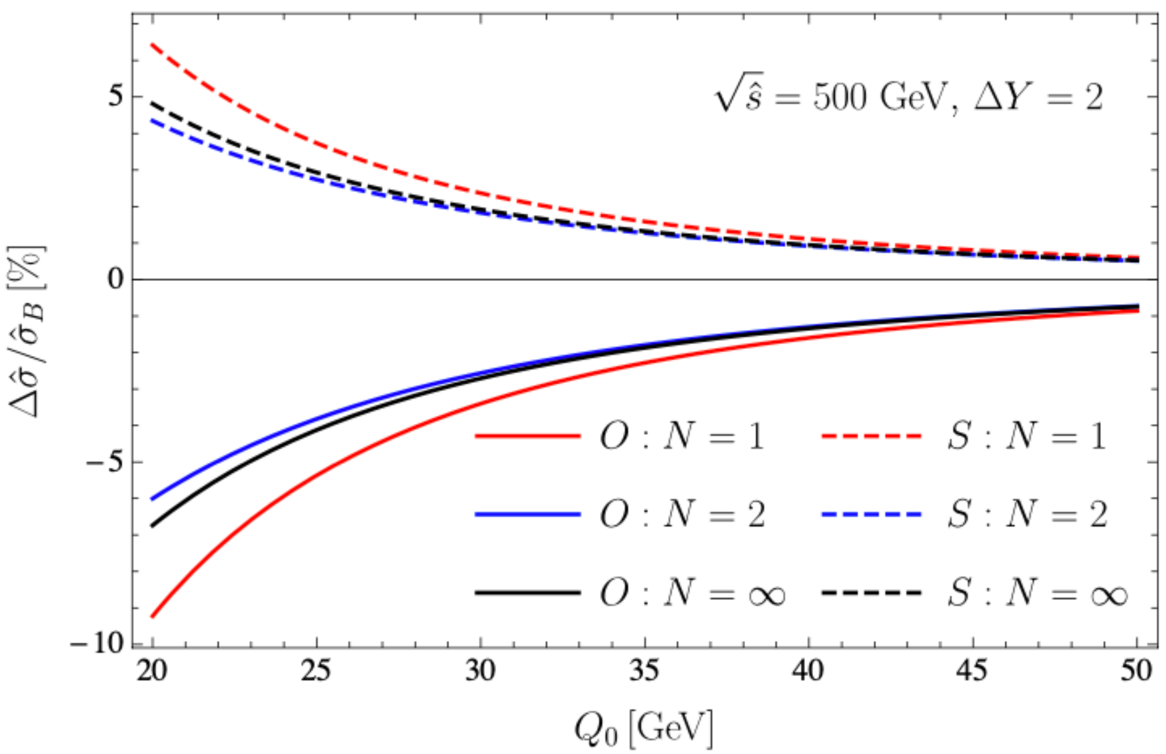

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$$f_\delta(w) = \frac{1}{3}\, {}_2F_2\left(1,1;2,\frac{5}{2};-w\right)$$

error function

$$f_2(w) = \frac{1}{w} - \frac{\sqrt{\pi}}{2w^{3/2}}\,\mathrm{erf}(\sqrt{w})$$

$$\omega \sim \alpha_s L^2$$

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3\frac{Q}{\mu_s}\frac{1}{N_c}\left[N_c^2\left(4f_1(w) - 2f_\delta(w)\right] - 4f_2(\omega) + 2f_\delta(w)\right)\Delta Y\,\sigma_0$$

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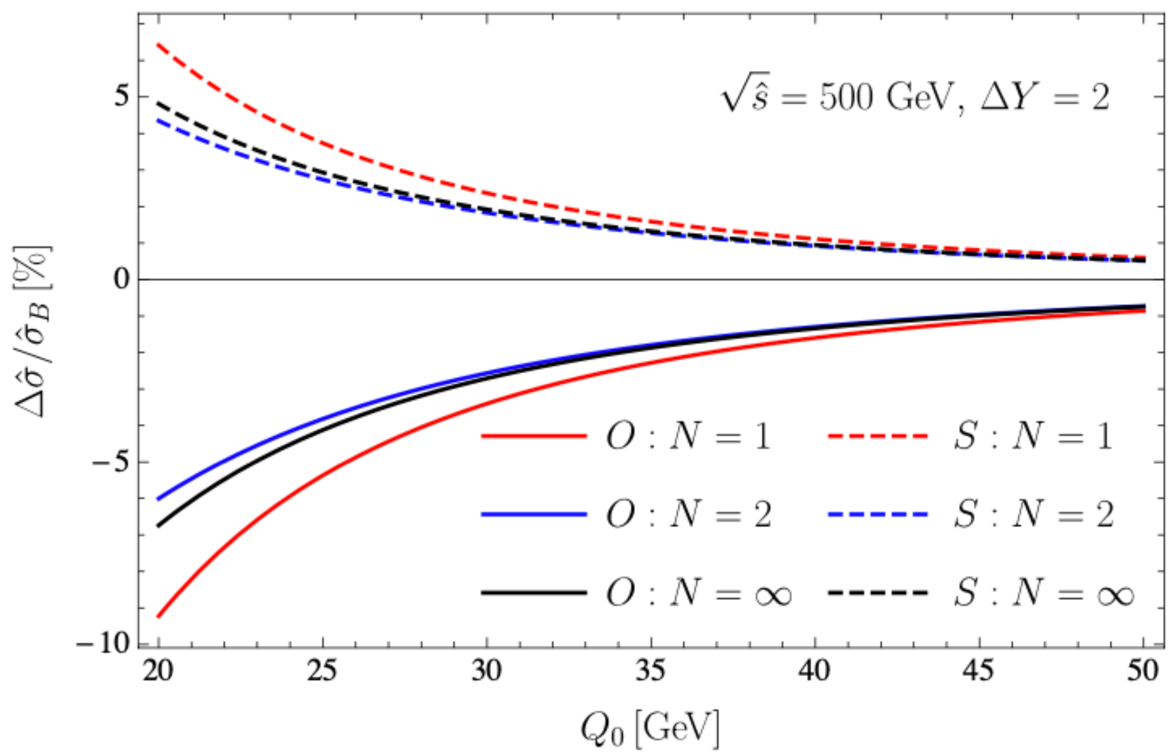

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All-order results of leading SLLs

(Becher, Neubert, **DYS** '21)

Owen's T function	hypergeometric function	error function
$f_1(w) = \frac{\sqrt{\pi}}{2w}\int_0^{\sqrt{\frac{w}{2}}}\frac{dz}{z^2}\left[\text{erf}(z) - \frac{e^{-2z^2}}{i}\text{erf}(iz)\right]$	$f_\delta(w) = \frac{1}{3}\,{}_2F_2\left(1,1;2,\frac{5}{2};-w\right)$	$f_2(w) = \frac{1}{w} - \frac{\sqrt{\pi}}{2w^{3/2}}\,\text{erf}(\sqrt{w})$

$$\omega \sim \alpha_s L^2$$

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3\frac{Q}{\mu_s}\frac{1}{N_c}\left[N_c^2\left(4f_1(w) - 2f_\delta(w)\right] - 4f_2(\omega) + 2f_\delta(w)\right)\Delta Y\,\sigma_0$$

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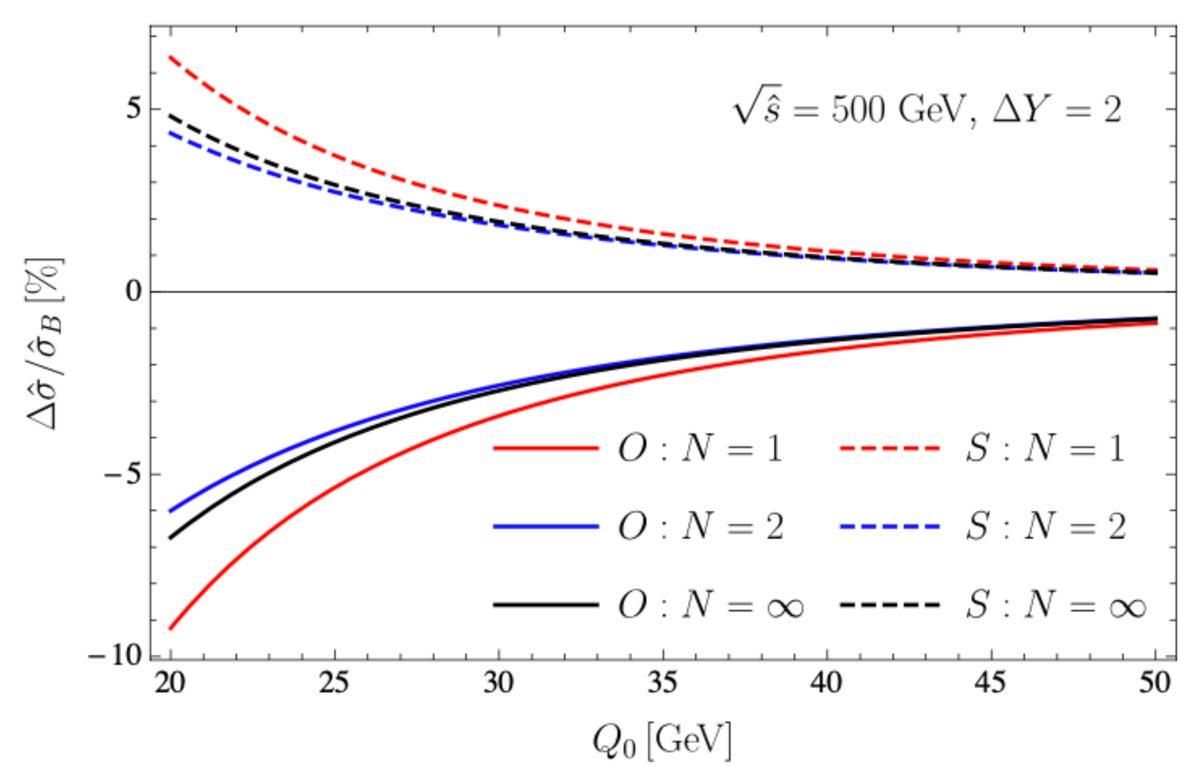
Numerical results



Red: Four loop   Blue: Five loop   Black: all order

SLLs in Drell-Yan and V+j

We find that SLLs also arise for processes with less than two final state jets

For Drell-Yan processes:

$$C_{rn} = -\hat{\sigma}_B \, 2^{9-r} \pi^2 C_F \, (4N_c)^n (2^r - 2)(1 - \delta_{r0}) \, J_2$$

SLLs start at 5-loop order

For V+j :

$$C_{rn} = \hat{\sigma}_B \, 2^{10-r} \, \pi^2 \, (4N_c)^{n-1} \left(N_c^2 + 2^r - 2\right) (1 - \delta_{r0}) \, J_2$$

SLLs start at 4-loop order

Summary

- If the radiation in a high-energy scattering process is restricted by experimental cuts, higher-order terms are enhanced by large logs
- The simple structure of these emissions often makes it possible to resum them to all orders.
- For exclusive jet cross sections at hadron colliders, even the LLs have a complicated structure
- We derive the all-order structure of SLLs at hadron colliders and resum them in closed form
- Our RG-based approach provides a transparent understanding of the underlying physics, and the analytical results should be useful in the ongoing effort to generalize parton showers
- Our findings indicate that SLLs could have an appreciable effect on precision observables, e.g. in Higgs production via VBF

Thank you