

Introduction to FORM

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IHEP-CAS

websites for FORM

- <https://github.com/vermaseren/form>
- <https://www.nikhef.nl/~form/maindir/documentation/documentation.html>

Installation

Examples

```
*** test1.frm  
S a,b;  
L F = (a+b)^2;  
P;  
.end
```

```
Symbols,a,b;  
Symbols,a b;  
Symbols ,a b;  
S a,b;  
Sym a b;  
sym a b;  
sYmB a b;  
SYMBOL a,b;
```

```
*** modify test1.frm
```

```
S a,b;
```

```
L F=(a+b)^2;
```

```
P;
```

```
.end
```

```
*** test2.frm
```

```
Symbols a,b,c,d,e,f,g;
```

```
Local F=(a+b+c+d+e+f+g)^32;
```

```
.end
```

```
*** Try this in MMA?
```

```
*** test3.frm
Symbols a,b,c,d;
Local F = (a+b+c+1)^6;
id a = -c+d+1;
*** .sort
id b=-d+1;
Print;
.end

*** .sort splits module
```

```

*** test4.frm
Symbols a,b,c,d;
On HighFirst;
Local F = a+b+c+d;
.sort
id a = (a+b)^2;
id c = b+d;
id b = b+1;
Print;
.end

```

```

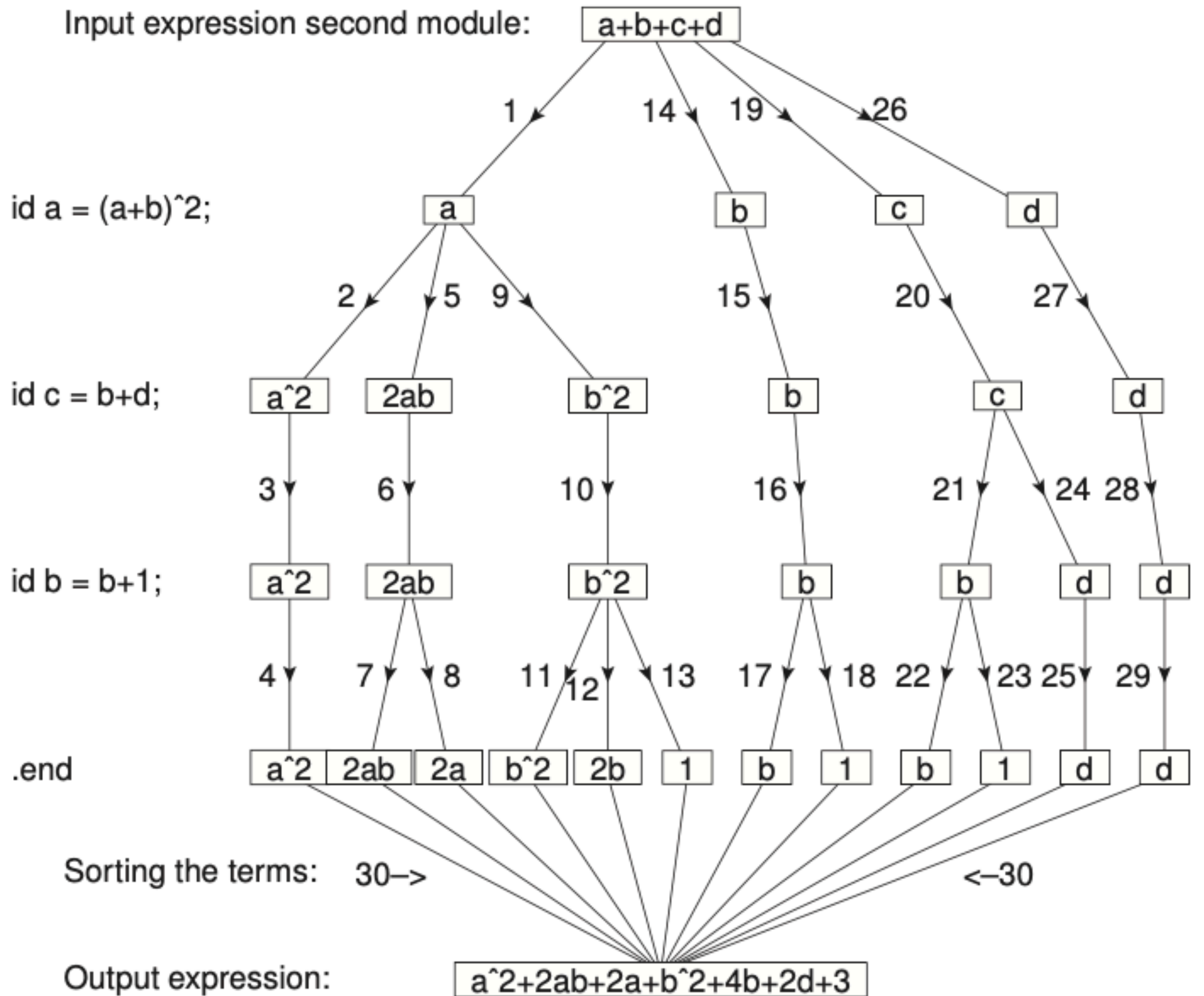
*** test5.frm
Symbols a,b,c;
Local F = (a+b)^6;
id a^2*b = c;
Print +s;
.end

```

```

S a,b,c,d;
On HighFirst;
L F = a+b+c+d;
.sort

```



Functions

```
*** test6.frm  
Functions A1,B1;  
CFunctions A2,B2;  
Local F1 = (A1+B1)^3;  
Local F2 = (A2+B2)^3;  
Print;  
.end
```

```
*** test7.frm  
Functions A1,B1;  
CFunctions A2,B2;  
Local F1 = (A1+B1+A2+B2)^3;  
Local F2 = ((A1+B1)+A2+B2)^3;  
Print;  
.end
```

```
*** test8.frm  
Functions A1,B1,C1,D1;  
CFunctions A2,B2,C2,D2;  
Local F1 = (A1+B1+C1+D1+A2+B2+C2+D2)^7;  
Local F2 = ((A1+B1+C1+D1)+A2+B2+C2+D2)^7;  
.end
```



```
*** test9.frm  
CFunction f,S,R;  
Symbol x,N;  
Local F = f(x)+f(x^2)+f(x,x+1)+f;  
Local G = S(R(3,1,-2),N+1);  
Print;  
.end
```

Index & Vector

```
*** test10.frm  
Index i1,i2,i3;  
Vector p1,p2,p3;  
Local F = p1(i1)*(p2(i1)+p3(i3))*(p1(i2)+p2(i3));  
Print;  
.end
```

```
*** test11.frm  
Index i1;  
Vector p1,p2,p3,p4;  
Local F = p1(i1)*p3(i1)*p2(i1)*p4(i1);  
Print;  
.end
```

```
*** test12.frm
Symbol x,D;
Index i1=3,i2 = 4,i3=D,i4=0,i5,i6,i7;
Local F = x*d_(i1,i1)
      +x^2*d_(i2,i2)+x^3*d_(i3,i3)
      +x^4*d_(i1,i2)*d_(i2,i1)+x^5*d_(i2,i1)*d_(i1,i2)
      +x^6*d_(i5,i6)*d_(i6,i7)
      +x^7*d_(i4,i4)
      +x^8*d_(i5,i4)*d_(i4,i7);
Print +s;
.end
```

```
*** test13.frm
Symbol D;
Index m1=2,m2=2,m3=2,m4=2;
Index n1=3,n2=3,n3=3,n4=3,n5=3,n6=3;
Index r1=D,r2=D,r3=D,r4=D,r5=D,r6=D;
Local F1 = e_(m1,m2)*e_(m3,m4);
Local F2 = e_(m1,m2)*e_(m2,m3);
Local F3 = e_(m1,m2)*e_(m1,m2);
Local G1 = e_(n1,n2,n3)*e_(n4,n5,n6);
Local G2 = e_(n1,n2,n3)*e_(n3,n4,n5);
Local G3 = e_(n1,n2,n3)*e_(n2,n3,n4);
Local G4 = e_(n1,n2,n3)*e_(n1,n2,n3);
Local H1 = e_(r1,r2,r3)*e_(r4,r5,r6);
Local H2 = e_(r1,r2,r3)*e_(r3,r4,r5);
Local H3 = e_(r1,r2,r3)*e_(r2,r3,r4);
Local H4 = e_(r1,r2,r3)*e_(r1,r2,r3);
Contract;
Print;
.end
```

```
*** test14.frm  
Index i1,i2,i3,i4;  
Vector p1,p2;  
Local F = e_(i1,i2,i3,i4)*p1(i1)*p2(i2);  
Print;  
.end
```

```
*** test15.frm  
Index i1,i2;  
Vector p1,p2,p3,p4;  
Local F = e_(i1,i2,p1,p2)*e_(i1,i2,p3,p4);  
Contract;  
Print;  
.end
```

Gamma Matrix

```
*** test16.frm
Index mu1,mu2,mu3,mu4;
Vector p1,p2,p3,p4;
Local F1 = g_(1,mu1)*g_(1,mu2)*g_(1,mu3)*g_(1,mu4);
Local F2 = g_(2,p1)*g_(2,p2)*g_(2,p3)*g_(2,p4);
Print;
.sort
Tracen,1;
Trace4 2;
Print;
.end
```

$$\gamma_5 \rightarrow g_{5_}(1) \text{ or } g_(1,5_)$$

$$\gamma_6 = (1 + \gamma_5) \rightarrow g_{6_}(1) \text{ or } g_(1,6_)$$

$$\gamma_7 = (1 - \gamma_5) \rightarrow g_{7_}(1) \text{ or } g_(1,7_)$$

In addition there is the unit matrix $g_{i_}(1)$.

```
*** test17.frm
Vector p1,p2,p3,p4,q1,q2,q3,q4;
Index i1,i2,i3,i4;
Local F = g_(1,p1,i1,7_,p2,i2,7_,p3,i3,7_,p4,i4,7_)
          *g_(2,q1,i1,7_,q2,i2,7_,q3,i3,7_,q4,i4,7_);
Trace4,1;
Trace4,2;
Print;
.end
```

Pattern Matching

```
*** test18.frm  
Symbol x,y,z,a,b;  
CFunction f;  
Local F = f(x)+f(y)+f(4)+f(a+b);  
id f(z?) = z^2;  
Print;  
.end
```

```
*** test19.frm  
Symbol x,y,z,a,b;  
CFunction f;  
Local F = f(x)*f(y)*f(y)*f(4)*f(a+b)*f(a+b);  
id f(z?)*f(z?) = z^2*f(3,z);  
Print +s;  
.end
```



```
*** test20.frm
Symbol a1,a2,a3,a4,a5,a6;
CFunction f,g;
Local F = f(a1,a2,a3,a4)*f(a1,a3,a2,a1,a4,a5,a6);
id f(?a,a3,?b) = g(?b,a3,?a);
Print +s;
.end
```

How to do this?

$$f(x1)*f(x2)*f(x3)*f(x4)*f(x5) \Rightarrow f(x1,x2,x3,x4,x5)$$

```
*** test21.frm
Symbol x1,...,x10;
auto symbol y;
CFunction f;

Local expr = 1
#do i = 1, 10
    *f(x`i')
#enddo
;

repeat id f(?y1)*f(?y2) = f(?y1,?y2);
.sort

print;
.end
```

```
*** test22.frm
Symbol x1,...,x10;
auto symbol y;
CFunction f;

Local expr = 1
#do i = 1, 10
    *f(x`i')
#enddo
;

chainin f;
.sort
print;
.end
```

```
*** test23.frm
Symbol a1,a2,a3,a4,a5,a6;
CFunction f,g;
Local F = f(a1,a2,a3,a4)*f(a1,a3,a2,a1,a4,a5,a6);

repeat id f(a1?,a2?,?a) = f(a1)*f(a2,?a);
***chainout f;

Print +s;
.end
```

fibonacci数列

```
*** test24.frm
Symbol n,x1,x2;
CFunction f,fib;
Local F = f(10,1,1);
repeat;
  id f(0,?a) = fib(?a);
  id f(n?,?a,x1?,x2?) = f(n-1,?a,x1,x2,x1+x2);
endrepeat;
Print;
.end
```

Table

```
*** test25.frm
symbol a1,...,a9;
Table,sparse,coefftable(1);

Local expr = (a1+...+a9)^10;

bracket a1;
.sort

fillexpression coefftable = expr(a1);
.sort

printable coefftable;
.end
```

C++内部直接调用FORM

- Please watch the example.