SCET factorization for the radiative leptonic B-meson decays

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Lectures on Loop Integrals and Phase-Space Integrals

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Outline

- ⋇ Introduction to the $B^- \to \gamma \ell \bar{\nu}_\ell$ decay
- ⋇ Method of regions
- ⋇ SCET calculation
- ⋇ Subleading power corrections

Introduction

Decay amplitude of the $B^- \to \gamma \ell \bar{\nu}_\ell$ process

$$A(B^- \to \gamma \ell \bar{\nu}_\ell) = \frac{G_F V_{ub}}{\sqrt{2}} \langle (\ell \bar{\nu}_\ell)(q) \gamma(p) | \bar{\ell} \gamma^\nu (1-\gamma_5) \nu_\ell \bar{u} \gamma_\nu (1-\gamma_5) b | B^-(p_B) \rangle$$

The amplitude is determined by two from factors F_V and F_A

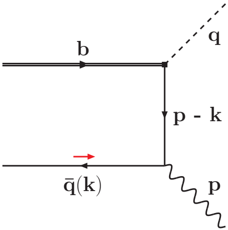
$$\begin{aligned} T_{\mu\nu}(p,q) &= -\, i \int d^4x \, e^{ipx} \langle 0 | T\{ j_\mu^{\mathrm{em}}(x) \, \bar{u} \gamma_\nu (1-\gamma_5) b \} | B^-(m_B v) \rangle \\ &= \epsilon_{\mu\nu p v} F_V + i \big[v_\mu p_\nu - v \cdot p g_{\mu\nu} \big] F_A + \cdots \end{aligned}$$

Theoretically clean: factorization properties of exclusive B decays, extract important non-perturbative parameters of B meson.

[Korchemsky, Pirjol and Yan hep-ph/9911427; Descotes-Genon and Sachrajda hep-ph/0209216; Wang and Shen 1803.06667; Beneke, Braun, Ji and Wei 1804.04962; Belle, 1810.12976; Galda and Neubert 200605428; Shen, Wei, Zhao and Zhou 2009.03480; $\cdots$]

Introduction

$E_\gamma \sim \mathcal{O}(m_b)$, light-cone vectors n and $\bar{n}$: $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$.
Power-expansion parameter $\lambda = \mathcal{O}(\sqrt{\Lambda_{\mathrm{QCD}}/m_b})$.



We have momenta:

- hard p_b: $(n \cdot p_b, \bar{n} \cdot p_b, p_{b\perp}) \sim m_b(1, 1, 1)$
- soft k: $m_b(\lambda^2, \lambda^2, \lambda^2)$
- collinear p: $m_b(1, \lambda^4, \lambda^2)$
- hard-collinear $p - k$: $m_b(1, \lambda^2, \lambda)$

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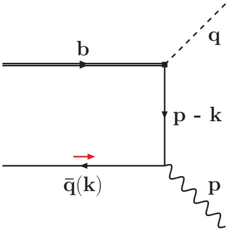

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Two perturbative scales: hard scale $\mathcal{O}(m_b)$, hard-collinear scale $\mathcal{O}(\lambda m_b)$

$B^- \to \gamma \ell \bar{\nu}_\ell$ decay form factors at LP

$$F_{V/A,\,\rm LP} = \frac{Q_u m_B}{n \cdot p} \tilde{f}_B(\mu) C(n \cdot p, m_b, \mu) \int_0^\infty \frac{d\omega}{\omega} J(n \cdot p\, \omega, \mu) \phi_B^+(\omega, \mu)$$

ω: projection of the soft-antiquark momentum on the light cone $\bar{n} \cdot k$.

Method of regions

With the method of regions, one could calculate contributions from different momentum regions [Beneke and Smirnov hep-ph/9711391]
A simple example $M \gg m$ [Becher, Broggio and Ferroglia 1410.1892]

$$I = \int_0^\infty dk \frac{k}{(k^2+m^2)(k^2+M^2)} = \frac{\ln\frac{M}{m}}{M^2-m^2} = \frac{\ln\frac{M}{m}}{M^2} + \mathcal{O}\left(\frac{m^2}{M^2}\right)$$

Reproduce the LP result with $M \gg \Lambda \gg m$

$$I = \underbrace{\int_0^\Lambda dk \frac{k}{(k^2+m^2)(k^2+M^2)}}_{I_{(\mathrm{s})}} + \underbrace{\int_\Lambda^\infty dk \frac{k}{(k^2+m^2)(k^2+M^2)}}_{I_{(\mathrm{h})}}$$

Factorization formula $I = I_{(\mathrm{s})} + I_{(\mathrm{h})}$, cancellation of Λ

$$I_{(\mathrm{s})} = \int_0^\Lambda dk \frac{k}{(k^2+m^2)}\left(\frac{1}{M^2} - \frac{k^2}{M^4} + \cdots\right) = \frac{1}{M^2}\ln\left(\frac{\Lambda}{m}\right) - \frac{\Lambda^2}{2M^4} + \cdots$$

$$I_{(\mathrm{h})} = \int_\Lambda^\infty dk \frac{k}{k^2(k^2+M^2)}\left(\frac{1}{k^2} - \frac{m^2}{k^4} + \cdots\right) = \frac{1}{M^2}\ln\left(\frac{M}{\Lambda}\right) + \frac{\Lambda^2}{2M^4} + \cdots$$

Method of regions

$$I = \int_0^\infty dk \frac{k}{(k^2+m^2)(k^2+M^2)} = \frac{\ln\frac{M}{m}}{M^2} + \cdots$$

Separate the low-energy and high-energy regions with dim-reg

$$I_{(\mathrm{s})}(0,\infty) = \int_0^\infty dk k^{-\epsilon} \frac{k}{(k^2+m^2)} \Big(\frac{1}{M^2} - \frac{k^2}{M^4} + \cdots \Big) = \frac{1}{M^2}\Big(\frac{1}{\epsilon} + \ln\frac{1}{m} \Big) + \cdots$$

$$I_{(\mathrm{h})}(0,\infty) = \int_0^\infty dk k^{-\epsilon} \frac{k}{(k^2+M^2)} \Big(\frac{1}{k^2} - \frac{m^2}{k^4} + \cdots \Big) = \frac{1}{M^2}\Big(-\frac{1}{\epsilon} + \ln M \Big) + \cdots$$

No double counting between low-energy and high-energy regions

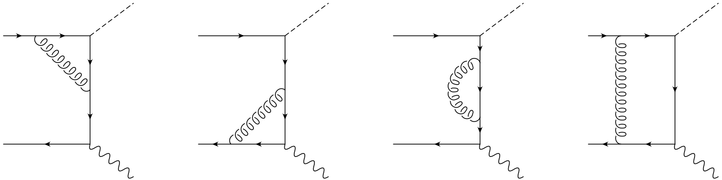
$$I_{(\mathrm{s})}(0,\infty) = I_{(\mathrm{s})}(0,\Lambda) + I_{(\mathrm{s})}(\Lambda,\infty)\,, \qquad I_{(\mathrm{h})}(0,\infty) = I_{(\mathrm{h})}(\Lambda,\infty) + I_{(\mathrm{h})}(0,\Lambda)$$

$$I_{(\mathrm{s})}(\Lambda,\infty) = \int_\Lambda^\infty dk k^{-\epsilon} k \Big(\frac{1}{M^2} - \frac{k^2}{M^4} + \cdots \Big)\Big(\frac{1}{k^2} - \frac{m^2}{k^4} + \cdots \Big)$$

$$I_{(\mathrm{h})}(0,\Lambda) = \int_0^\Lambda dk k^{-\epsilon} k \Big(\frac{1}{k^2} - \frac{m^2}{k^4} + \cdots \Big)\Big(\frac{1}{M^2} - \frac{k^2}{M^4} + \cdots \Big)$$

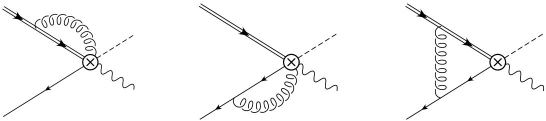
NLO factorization formula

QCD one-loop diagrams: expand the loop momentum in different regions



Three loop-momentum regions:

hard region $m_b(1, 1, 1)$, hard-collinear region $m_b(1, \lambda^2, \lambda)$ and
soft region $m_b(\lambda^2, \lambda^2, \lambda^2)$ ⇔ matrix element of operator



$$F_{V/A,\,\mathrm{LP}} = \frac{Q_u m_B}{n \cdot p} \tilde{f}_B(\mu) C(n \cdot p, m_b, \mu) \int_0^\infty \frac{d\omega}{\omega} J(n \cdot p\, \omega, \mu) \phi_B^+(\omega, \mu)$$

Soft collinear effective theory

effective fields in SCET $\Leftrightarrow$ regions in method of regions

- prove factorization formula to all order in α_s
- systematically extend to subleading power
- derive RG equations of operators and resum large logs

- Below hard scale $\mathcal{O}(m_b)$ we need SCET_I: multipole expansion

  $\xi_{hc} \sim \lambda$, $\quad A^\mu_{hc} \sim (1, \lambda^2, \lambda)$,

  $q_s \sim \lambda^3$, $\quad h_v \sim \lambda^3$, $\quad A^\mu_s \sim (\lambda^2, \lambda^2, \lambda^2)$

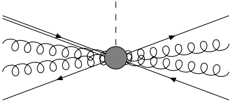
  $\mathcal{L}_{\text{SCET}_\text{I}} =$

  $(\mathcal{L}_\xi^{(0)} + \mathcal{L}_\xi^{(1)} + \cdots) + \mathcal{L}_q^{(0)} + (\mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi q}^{(2)} + \cdots) + (\mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)} + \cdots) + \mathcal{L}_{\text{HQET}}$

- Below hard-collinear scale $\mathcal{O}(\lambda m_b)$: SCET_II

  $\boxed{\xi_c \sim \lambda^2}$, $\quad \boxed{A^\mu_c \sim (1, \lambda^4, \lambda^2)}$, $\quad q_s,\ h_v,\ A_s$

  $\mathcal{L}_{\text{SCET}_\text{II}} = (\mathcal{L}_c^{(0)} + \cdots) + (\mathcal{L}_s^{(0)} + \cdots) +$ ~~$(\mathcal{L}_{c+s}^{(2)} + \cdots)$~~



$B^- \to \gamma \ell \bar{\nu}_\ell$ form factors at LP

We consider the matrix element of heavy-to-light current

$$\langle \gamma(p) | \bar{u}(1+\gamma_5)\gamma_\nu b | B^-(p_B) \rangle$$

A tree-level analysis: integrate out hard and hard-collinear fields by EOMs [Beneke and Feldmann hep-ph/0311335]

$$b = h_v + \mathcal{O}(\lambda h_v), \qquad \bar{u} = \bar{\xi}_c + \bar{q}_s \left(\frac{\not{\bar{n}}}{2} \not{A}^{\text{em}}_{\perp c} \frac{1}{i \bar{n} \cdot \overleftarrow{\partial}} + 1 \right) + \bar{u}^{(4)} + \cdots$$

LP SCET$_{\text{II}}$ operator scales as λ^6

$$O_{\text{LP}} = \bar{q}_s \frac{\not{\bar{n}}}{2} \not{A}^{\text{em}}_{\perp c} \frac{1}{i \bar{n} \cdot \overleftarrow{\partial}} \gamma_\nu (1 - \gamma_5) h_v$$

$B^- \to \gamma \ell \bar{\nu}_\ell$ form factors at LP

- Operators in SCET_I scale as λ^4

$$\text{T}\{\bar{\xi}_{hc}(1+\gamma_5)\gamma_\nu h_v, \mathcal{L}^{(1)}_{\xi q}\}$$

  Translating $A^{\text{em}}_{\perp hc}$ to $A^{\text{em}}_{\perp c}$ will generate an additional λ factor.
  $\mathcal{O}(\lambda^5)$ operators with field contents $\bar{\xi}_{hc} A_{\perp hc} h_v$ will not contribute at LP.
  $\mathcal{O}(\lambda^6)$ operators with field contents $\bar{q}_s h_v$ will also not contribute at LP.

- Operators in SCET_II contain power enhanced factor $1/(i\bar{n}\cdot\partial)$ which acts only on soft fields. [Beneke and Feldmann hep-ph/0311335]
  The # of $1/(i\bar{n}\cdot\partial)$ is constrained by RPI

$$n \to \alpha n, \quad \bar{n} \to \alpha^{-1}\bar{n}, \quad \text{with } \alpha \sim \mathcal{O}(1)$$

  Factorizable SCET_II operator: all orders in α_s

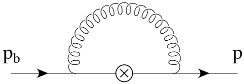
$$O_{\text{LP}} = \bar{q}_s \frac{\not{\bar{n}}}{2} \not{A}^{\text{em}}_{\perp c} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} \gamma_\nu (1-\gamma_5) h_v$$

QCD to $\mathrm{SCET_I}$: hard function

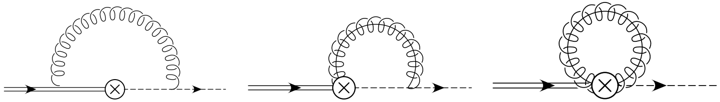
From QCD to $\mathrm{SCET_I}$

$$\bar{u}(1+\gamma_5)\gamma_\nu b \Rightarrow \int d\hat{s}\tilde{C}(\hat{s})[(\bar{\xi}_{hc}W_{hc})(sn)(1+\gamma_5)\gamma_\nu h_v]$$

QCD diagram at NLO



$\mathrm{SCET_I}$ diagrams at NLO



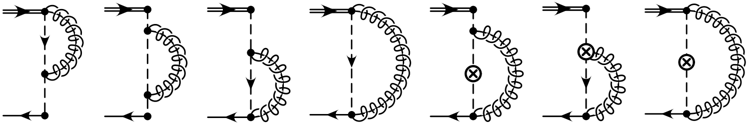
C at NNLO [Bonciani and Ferroglia 0809.4687; Asatrian, Greub and Pecjak 0810.0987; Beneke, Huber and Li 0810.1230; Bell 0810.5695]

Renormalization of the composite $\mathrm{SCET_I}$ operator.

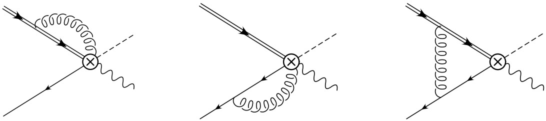
From SCET$_{\rm I}$ to SCET$_{\rm II}$

$$\mathrm{T}\{(\bar{\xi}_{hc}W_{hc})(1+\gamma_5)\gamma_\nu h_v, j^{\rm em}\} \Rightarrow \int \frac{d\omega}{\omega} J(\omega)[(\bar{q}_s Y_s)(t\bar{n})\frac{\not{\bar{n}}}{2} \not{A}^{\rm em}_{\perp c}\gamma_\nu(1-\gamma_5)Y_s^\dagger h_v]_{\rm FT}$$

SCET$_{\rm I}$ diagrams at NLO



SCET$_{\rm II}$ diagrams at NLO



Jet function at NNLO [Liu and Neubert 2003.03393]

$B^- \to \gamma \ell \bar{\nu}_\ell$ form factors at NNLO

$$F_{V/A,\,\mathrm{LP}} = \frac{Q_u m_B}{n\cdot p}\tilde{f}_B(\mu) C(n\cdot p, m_b, \mu) \int_0^\infty \frac{d\omega}{\omega} J(n\cdot p\,\omega, \mu)\phi_B^+(\omega,\mu)$$

Factorization scale $\mu \to$ hard-collinear scale

Large Logs

- $\ln(m_b/\mu)$, $\ln(n\cdot p/\mu)$ from C and $\tilde{f}_B$
- $\ln(\Lambda_{\mathrm{QCD}}/\mu)$ from ϕ_B^+ (two-loop γ [Braun, Ji and Manashov 1905.04498])

Numerical result [Galda and Neubert 2006.05428]

$$I_{\mathrm{NLO}} = \frac{1}{\lambda_B}\left[0.731 + 0.035\,\sigma_2 - 0.003\,\sigma_3 + \dots\right],$$

$$I_{\mathrm{NNLO}} = \frac{1}{\lambda_B}\left[0.664\,^{+0.026}_{-0.040} + 4.36\,^{+0.17}_{-0.46}\cdot 10^{-2}\,\sigma_2 + 0.35^{+2.97}_{-1.99}\cdot 10^{-3}\,\sigma_3 + \dots\right],$$

NNLO result is about 10% smaller than the NLO result.

Subleading power corrections

- Power corrections are numerically important in B decays

$$\Lambda_{\rm QCD}/m_b \quad \sim \quad \alpha_s(\mu)/\pi$$

$$\text{NLP@LO} \sim \text{LP@NLO}, \qquad \text{NNLP@LO} \sim \text{NLP@NLO} \sim \text{LP@NNLO}$$

- New phenomena will appear [Beneke and Rohrwild 1110.3228]

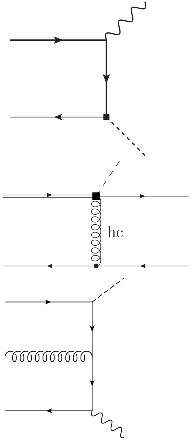
$$F_{V/A} = \underbrace{\frac{Q_u\, m_B}{2E_\gamma}\, R(E_\gamma)}_{\rm LP} + \left[\xi(E_\gamma) \pm \left(\frac{Q_b\, m_B f_B}{2E_\gamma m_b} + \frac{Q_u\, m_B f_B}{(2E_\gamma)^2}\right)\right]$$

  - Local symmetry breaking terms
  - End-point singularity: same as $B \to V$ form factors

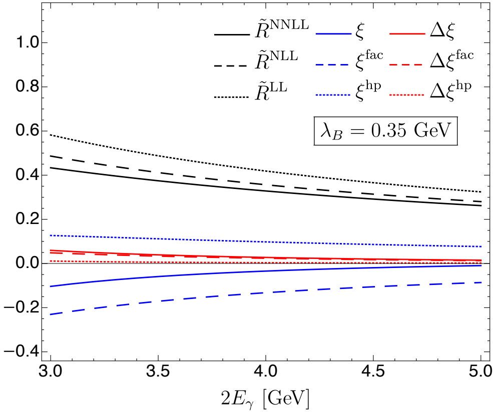
Subleading power corrections

NLP $\mathrm{SCET}_{\mathrm{II}}$ operators scale as λ^8

- Power suppressed operators
  - Local $\mathrm{SCET}_{\mathrm{II}}$ operator $\bar{q}_s A^{\mathrm{em}}_{\perp c} h_v$
  - $\mathrm{SCET}_{\mathrm{I}}$ operators $\bar{\xi}_{hc} A_{\perp hc} h_v$: hadronic structure of the photon
- LP operator with power suppressed Lagrangian



NLP numerical results



[Cui, Shen, Wang, Wang and Wei]

$$F_{V/A} = \tilde{R} + \xi \pm \Delta\xi$$

$\tilde{R}$: LP

ξ: symmetry conserving

$\Delta\xi$: symmetry breaking

ξ^{hp}, $\Delta\xi^{\text{hp}}$: hadronic photon

NLP contributions bring about $\mathcal{O}(20\%)$ correction

Summary

※ Introduction to method of regions

※ $B^- \to \gamma \ell \bar{\nu}_\ell$ form factors F_V and F_A at LP

- Calculation from method of regions
- SCET: $\boxed{\text{QCD} \underset{\mathcal{O}(m_b)}{\to} \text{SCET}_\text{I} \underset{\mathcal{O}(\lambda m_b)}{\to} \text{SCET}_\text{II}}$

※ Subleading power corrections: numerical results

Thank you!