The pion-photon transition form factor at two loops in QCD

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Zoominar, Loop and phase space integrals

$\gamma\gamma^* \to \pi^0$ form factor

- $\gamma\gamma^* \to \pi^0$ **production: theoretically clean and simple**

$$\langle \pi^0(p)|\, j_\mu^{\rm em}\, |\gamma(p')\rangle = g_{\rm em}^2 \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu(p')\, q^\rho p^\sigma\, F_{\gamma\pi}(Q^2) \qquad Q^2 \equiv -q^2 = -(p-p')^2$$

◇ **Theory motivations**

- **QCD precision study**
- **investigating collinear factorization properties (OPE)**

◇ **Phenomenological motivations**

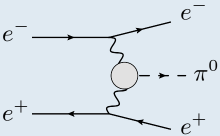
- **necessary for confronting future experimental precision**
- **axial-vector contribution in DVCS**
- **one-loop and next-leading-power known**

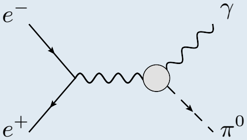
  [F. del Aguila, M. Chase (1981); E. Braaten (1983); E. Kadantseva, S. Mikhailov, A. Radyushkin (1986); Y-L. Shen, Y-M. Wang (2017)]

- **"Babar" puzzle**

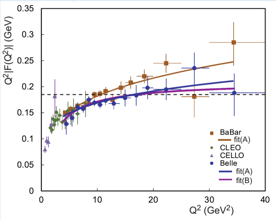
Experimental status

- **Measurement channel** $e^+e^- \to \pi^0$





- **Experiment vs. Theory**



asymptotic limit $\lim_{Q^2\to\infty} F_{\gamma\pi}(Q^2) \propto \sqrt{2} f_\pi$

[Brodsky, Lepage (1980)]

-  **Scale violation ?**

$\gamma\gamma^* \to \pi^0$ transition form factor

$$\langle \pi^0(p)|\, j_\mu^{\rm em}\, |\gamma(p')\rangle = g_{\rm em}^2 \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu(p')\, q^\rho p^\sigma F_{\gamma\pi}(Q^2) \qquad Q^2 \equiv -q^2 = -(p-p')^2$$

$$j_\mu^{\rm em} = \sum_q g_{\rm em} Q_q\, \bar{q}\gamma_\mu q$$

- **Naturally, two null-vectors n and $\bar{n}$ emerge ($(n\cdot\bar{n}) = 2$)**

$$p'_\mu = (n\cdot p')\frac{\bar{n}_\mu}{2}\,, \qquad p_\mu = (\bar{n}\cdot p)\frac{n_\mu}{2}\,, \qquad n\cdot p' \sim \bar{n}\cdot p \sim \mathcal{O}(\sqrt{Q^2})$$

- **Leading-power factorization**

$$F_{\gamma\pi}^{\rm LP}(Q^2) = \frac{(e_u^2 - e_d^2) f_\pi}{\sqrt{2}\, Q^2}\int_0^1 dx\, T_2(x, Q^2, \mu_F)\, \phi_\pi(x,\mu_F)\,, \quad T_2 = \sum_{\ell=0}^{\infty} a_s^\ell T_2^{(\ell)}\,, \quad a_s = \frac{\alpha_s}{4\pi}$$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \qquad m_u = m_d = 0 \quad \text{exact isospin}$$

Computation of T_2: tree-level

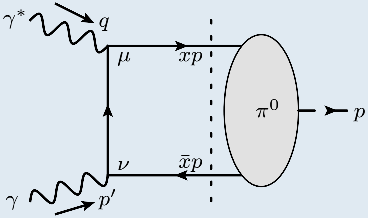
- **Leading-power factorization**

$$F_{\gamma\pi}^{\mathrm{LP}}(Q^2) = \frac{(e_u^2 - e_d^2) f_\pi}{\sqrt{2}\, Q^2} \int_0^1 dx\, T_2(x, Q^2, \mu_F)\, \phi_\pi(x, \mu_F)$$

- **Non-perturbative ingredient: $\phi_\pi(x,\mu)$ — leading-twist pion LCDA**

$$\langle \pi(p)|\, [\bar{q}(z\bar{n})[z\bar{n}, 0]\gamma_\mu \gamma_5 q(0)]_R\, |0\rangle = -i f_\pi p_\mu \int_0^1 dx\, e^{ixz\bar{n}\cdot p} \phi_\pi(x, \mu_F)\,,$$

- **T_2 from four-point correlator $\Pi_{\mu\nu}$ (tree-level):**



$$\Pi_{\mu\nu} = i \int d^d z\, e^{-iq\cdot z} \langle \bar{q}(\bar{x}p) q(xp)|\, \mathrm{T}\{j_\mu^{\mathrm{em}}, j_\nu^{\mathrm{em}}\}\, |0\rangle$$

Computation of T_2: beyond tree-level

- $\Pi_{\mu\nu}$ **perturbatively calculable:** [Y-M. Wang, Y-L. Shen (2017); J. Gao, T. Huber, YJ, Y-M. Wang (2021)]

$$\Pi^{\mu\nu} = \sum_{k=A,B} \sum_{\ell=0}^{\infty} \underbrace{(Z_a a_s)^{\ell}}_{\text{bare}} A_k^{(\ell)}(x) \langle \bar{q}(\bar{x}p)\Gamma_k^{\mu\nu} q(xp)\rangle_{\text{tree}} \,,$$

$$\Gamma_A^{\mu\nu} = \gamma_\perp^\mu \not{\bar{n}} \gamma_\perp^\nu \,, \qquad \Gamma_B^{\mu\nu} = \gamma_\perp^\nu \not{\bar{n}} \gamma_\perp^\mu \,, \quad \Leftarrow \quad \textbf{QED WI \& C-symmetry}$$

$$v_\perp^\mu \equiv g_\perp^{\mu\nu} v_\nu \,, \quad g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{1}{2} n^\mu \bar{n}^\nu - \frac{1}{2}\bar{n}^\mu n^\nu \,, \quad \epsilon_\perp^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{n}_\rho n_\sigma$$

$\Downarrow$ **matching**

$$\Pi^{\mu\nu} = \sum_{a=1,2,E} T_a \otimes \langle \mathcal{O}_a^{\mu\nu}\rangle \,, \qquad \mathcal{O}_a^{\mu\nu}(x) = \frac{\bar{n}\cdot p}{2\pi} \int d\tau\, e^{i\bar{x}\tau\bar{n}\cdot p} \bar{q}(\tau\bar{n})[\tau\bar{n},0]\Gamma_a^{\mu\nu} q(0) \,,$$

$$\Gamma_1^{\mu\nu} = g_\perp^{\mu\nu}\not{\bar{n}} \,, \quad \Gamma_2^{\mu\nu} = i\epsilon_\perp^{\mu\nu}\not{\bar{n}}\gamma^5 \,, \quad \Gamma_E^{\mu\nu} = \not{\bar{n}}\left(\frac{1}{2}[\gamma^\mu,\gamma^\nu] - i\epsilon_\perp^{\mu\nu}\gamma^5\right)$$

◇ T_1 **decouples from** $\gamma\gamma^* \to \pi^0$ **due to parity: vector contribution in DVCS**

[V. Braun, A. Manashov, S. Moch, J. Schoenleber (2020)]

◇ T_2 **odd parity responsible for** $\gamma\gamma^* \to \pi^0$**: compute in** $\overline{\text{MS}}$**/NDR scheme**

◇ T_E **(**$\mathcal{O}_E^{\mu\nu}$**) evanescent coefficient function (operator): indispensable in dim. reg.**

Extracting T_2 from $\Pi_{\mu\nu}$: matching

- **Matching two formulas of $\Pi^{\mu\nu}$**

$$\Pi^{\mu\nu} = \sum_{a=1,2,E} T_a \otimes \langle \mathcal{O}_a^{\mu\nu} \rangle ,$$

$$\Updownarrow$$

$$\Pi^{\mu\nu} = \sum_{k=A,B} \sum_{\ell=0}^{\infty} (Z_a a_s)^\ell A_k^{(\ell)}(x) \langle \bar{q}(\bar{x}p)\Gamma_k^{\mu\nu} q(xp)\rangle_{\text{tree}}$$

- **Identity:** $\Gamma^{\mu\nu}_{A,B} = -(\Gamma_1^{\mu\nu} \pm \Gamma_2^{\mu\nu} \pm \Gamma_E^{\mu\nu}) \implies \bar{q}\Gamma^{\mu\nu}_{A,B} q \mapsto \mathcal{O}^{\mu\nu}_{a,\text{tree}}, A^{(\ell)}_{A,B} \mapsto A^{(\ell)}_{1,2,E}$
- **Relating $\mathcal{O}^{\mu\nu}_{a,\text{tree}}$ and $\mathcal{O}^{\mu\nu}_a$:**

$$\langle \mathcal{O}_1^{\mu\nu}\rangle = Z_{11} \langle \mathcal{O}^{\mu\nu}_{1,\text{tree}}\rangle , \quad \langle \mathcal{O}_i^{\mu\nu}\rangle = \sum_{j=2,E} Z_{ij} \langle \mathcal{O}^{\mu\nu}_{k,\text{tree}}\rangle$$

◇ Z_{11}: **obtainable from ERBL kernel: known to three-loop**
[G. Lepage, S. Brodsky (1980); A. Efremov and A. Radyushkin (1980); S. Mikhailov and A. Radyushkin (1985); A. Belitsky, D. Mueller, A. Freund(1999); V. Braun, A. Manashov, S. Moch, M. Strohmaier (2017)]

◇ $Z_{22} = Z_{11}$ **in $\overline{\text{MS}}$/NDR**

◇ Z_{2E} **calculable via diagrammatic approach: we need $Z^{(2)}_{2E}$**

Extracting T_2 from $\Pi_{\mu\nu}$: matching

- Z_{ij} **expandable in** a_s

$$Z_{ij} = \delta_{ij} + a_s Z^{(1)}_{ij} + a_s^2 Z^{(2)}_{ij} + \mathcal{O}(a_s^3)$$

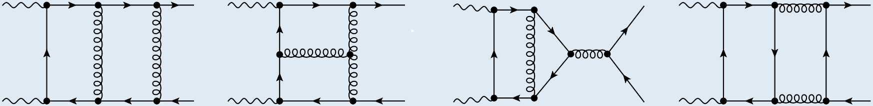
- **Matching yields "master formula"** $(A^{(\ell)}_2 = A^{(\ell)}_E, Z^{(1)}_{EE} = Z^{(1)}_{22}, Z^{(1)}_{2E} = 0)$ **:**

$$T^{(0)}_2 = T^{(0)}_E = A^{(0)}_2 \,,$$
$$T^{(1)}_2 = A^{(1)}_2 - \sum_{a=2,E} Z^{(1)}_{a2} \otimes T^{(0)}_a = T^{(1)}_E - Z^{(1)}_{E2} \otimes T^{(0)}_E \,,$$
$$T^{(2)}_2 = A^{(2)}_2 + Z^{(1)}_{\alpha} A^{(1)}_2 - \sum_{a=2,E} \sum_{k=0}^{1} Z^{(2-k)}_{a2} \otimes T^{(k)}_a$$

- **Task at hand:** $A^{(2)}_2, Z^{(2)}_{E2}$**; all done dim. reg.** $d = 4 - 2\epsilon$

Computation of $A_2^{(2)}$

- **Step 1: generating two-loop diagrams for $\Pi^{\mu\nu}$**
  - $\sim 100+$ **diagrams are generated via** `Feynarts` **and in-house routine**
  - **considering color/flavor/Furry theorem, total # of diagrams reduce to** 42×2



Sample diagrams

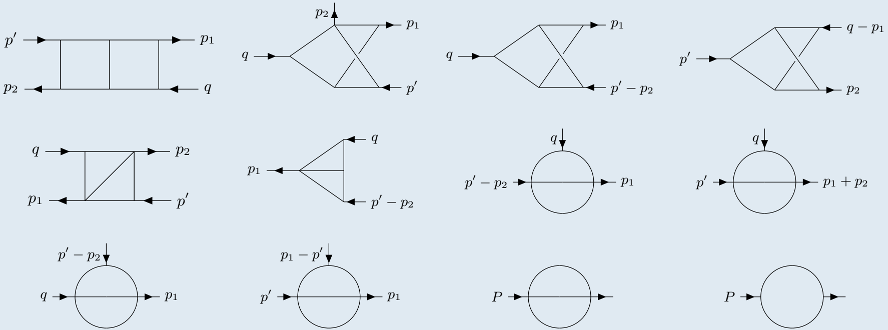
- **Step 2: translating diagrams to loop-integrals: in-house routine**
- **Step 3: reducing loop-integrals to scalar integrals: in-house routine (no trace used!)**
- **Step 4: IBP reduction (via** `FIRE`**) to master integrals**

[S. Laporta (2000); A. Smirnov (2008); A. Smirnov, F. Chukharev (2019)]

- **Intermediate check: only $\gamma_\perp^\mu \not{n} \gamma_\perp^\nu$ and $\gamma_\perp^\nu \not{n} \gamma_\perp^\mu$ present after summing up all diagrams: manifestation of QED WI and C-symmetry**

Computation of $A_2^{(2)}$: masters (minimum set)

- **A total of 12 masters needed**



◇ **the number of MIs reduced by exploiting collinear condition** $p_1 = xp \propto \bar{x}p = p_2$

- **MIs computed using DE and Mellin-Barnes representation**

[A. Kotikov (1991); E. Remiddi (1997); M. Argeri and P. Mastrolia (2007); M. Czakon (2006)]

- **all MIs are presentable through HPLs with indices** 0 **and** 1 **and argument** x**, or equivalently** $\mathrm{Li}_n(z)$ **with** $z \in \{x, \bar{x}, -x/\bar{x}\}$
- **combining IBP and MIs to get** $A_2^{(2)}$ **to** $\mathcal{O}(\epsilon^0)$**; also need** $A_1^{(1)}$ **to** $\mathcal{O}(\epsilon)$

Computation of Z_{E2}

- **Diagrams identical to two-loop ERBL kernel; in total** 29

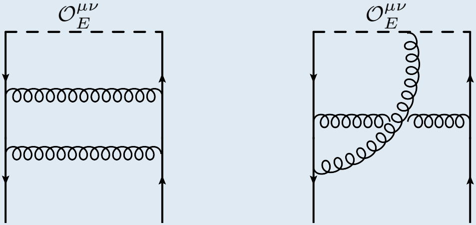


Diagram A **Diagram B** $\cdots$

$$I_A = -g^4 C_F^2 \int \frac{d^d l_1 d^d l_2}{(2\pi)^{2d}} \bar{u}(k_1)\gamma^\rho \frac{\not{k}_1 - \not{l}_2}{(k_1 - l_2)^2}\gamma^\alpha \frac{\not{k}_1 - \not{l}_1}{(k_1 - l_1)^2}$$

$$\times \mathcal{O}_E^{\mu\nu} \frac{\not{l}_1 + \not{k}_2}{(l_1 + k_2)^2}\gamma_\alpha \frac{\not{l}_2 + \not{k}_2}{(l_2 + k_2)^2}\gamma_\rho v(k_2) \frac{e^{iz(k_1 - l_1)\cdot\bar{n}}}{(l_1 - l_2)^2 l_2^2} \,,$$

$$I_B = -ig^4 z C_F \left(C_F - \frac{C_A}{2}\right) \int_0^1 du \int \frac{d^d l_1 d^d l_2}{(2\pi)^{2d}} \bar{u}(k_1)\not{\bar{n}} \frac{\not{k}_1 - \not{l}_2}{(k_1 - l_2)^2}\gamma^\rho \frac{\not{k}_1 + \not{l}_1}{(k_1 + l_1)^2}$$

$$\times \mathcal{O}_E \frac{\not{k}_2 - \not{l}_{12}}{(k_2 - l_{12})^2}\gamma_\rho v(k_2) \frac{e^{iz(k_1 + l_1 + u l_2)\cdot\bar{n}}}{l_2^2 (l_1 + l_2)^2}$$

Computation of Z_{E2}

- Introducing a single m for all particles to separate UV and IR
- Calculations done in position space
  - ◇ analytic results
  - ◇ simpler expression
  - ◇ constrained by conformal symmetry
- Fourier transform to momentum space to get Z_{E2} for extracting $T_2^{(2)}$

$$I_A = 8a_s^2 C_F^2 \left\{ \frac{\bar{t}}{\bar{x}} \left[3 + 2L_m\right] + \frac{t \ln t}{\bar{x}} \left[2(1 + L_m) + \ln t\right] + \frac{\bar{t}}{x} \ln \bar{x} \left[2(1 + L_m + \ln \bar{t}) - \ln \bar{x}\right] \right\} \theta(t - x) + (t \to \bar{t}, x \to \bar{x})$$

$$I_B = (\cdots)\theta(t - x) + (\cdots)\,\theta(x - t) + (\cdots)\,\theta(t - \bar{x}) + (\cdots)\,\theta(\bar{x} - t)\,, \qquad \frac{1}{\epsilon} \text{ divergent}$$

  - ◇ t and x: incoming and outgoing momentum fraction ($Z(x,t) \otimes f(t)$)
- Byproduct: leading-twist evolution kernel in Larin scheme at two loop

[V. Braun, A. Manashov, S. Moch, M. Strohmaier (2021)]

- Also need $Z_{22}^{(1)}$ to $\mathcal{O}(\epsilon^0)$ due to cross term $Z \otimes Z$
- Adding everything up to get $Z_{E2}^{(2)}$, all m dependence cancel

$T_2^{(2)}$ at α_s^2 order

- **Putting all pieces together to get** [J. Gao, T. Huber, YJ, Y-M. Wang (2021)]

$$T_2^{(2)}(x) = \left[\beta_0 C_F \left(\mathcal{K}_\beta^{(2)}(x)/x\right) + C_F^2 \left(\mathcal{K}_F^{(2)}(x)/x\right) + C_F/N_c \left(\mathcal{K}_N^{(2)}(x)/x\right)\right] + [x \mapsto \bar{x}]$$

$$\mathcal{K}_\beta^{(2)}(x) = f_1(L, \mathrm{H}_{\leqslant \vec{3}}), \qquad \mathcal{K}_F^{(2)}(x) = f_2(L^2, L, \mathrm{H}_{\leqslant \vec{4}}), \qquad \mathcal{K}_N^{(2)}(x) = f_3(L, \mathrm{H}_{\leqslant \vec{3}}),$$

  - ◇ **equivalently expressible by** $\mathrm{Li}_n(z)\,, z \in \{x, \bar{x}, -x/\bar{x}\}$

- **Full agreement with results obtained via conformal OPE** [V. Braun, A. Manashov, S. Moch, J. Schoenleber (2021)]
  - ◇ ↰ **nontrivial relations among HPLs needed**

Numerical analysis: models

- **Models for ϕ_π needed for $F_{\gamma\pi}$ prediction**

$$F_{\gamma\pi}^{\rm LP}(Q^2) = \frac{(e_u^2 - e_d^2) f_\pi}{\sqrt{2}\, Q^2} \int_0^1 dx\, T_2(x, Q^2, \mu_F)\, \phi_\pi(x, \mu_F)\,, \qquad \mu_F = \mu_{\rm UV}$$

- **Expanding $\phi_\pi(x)$ in Gegenbauer polynomials $a_{2n}(\mu)$**

$$\phi_\pi(x,\mu) = 6x\bar{x}\left(1 + \sum_{n=1}^{\infty} a_{2n}(\mu)\, C_{2n}^{3/2}(2x-1)\right)$$

Model I : $\phi_\pi(x,\mu_0) = \dfrac{\Gamma(2+2\,\alpha_\pi)}{\Gamma^2(1+\alpha_\pi)}\,(x\,\bar{x})^{\alpha_\pi}\,,$

with $\alpha_\pi(\mu_0) = 0.422^{+0.076}_{-0.067}$;

Model II : $\{a_2, a_4\}(\mu_0) = \{0.269(47), 0.185(62)\}\,,$

$\{a_6, a_8\}(\mu_0) = \{0.141(96), 0.049(116)\}\,;$

Model III : $\{a_2, a_4\}(\mu_0) = \{0.203^{+0.069}_{-0.057}\,, -0.143^{+0.094}_{-0.087}\}$

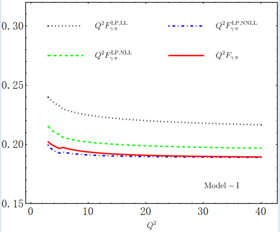
◇ $\mu_0 = 1.0$ GeV

[S. Brodsky, G. de Teramond (2008) & G. Bali, et. al (2019)]; [S. Cheng, A. Khodjamirian (2020)];

[A. Bakulev, S. Mikhailov, N. Stefanis (2008); S. Mikhailov, A. Pimikov, and N. Stefanis (2016); N. Stefanis (2020)]

Numerical analysis: two-loop effect

- $\alpha_s(Q^2)$ **corrections to** $F_{\gamma\pi}$ **at various orders for Model I**



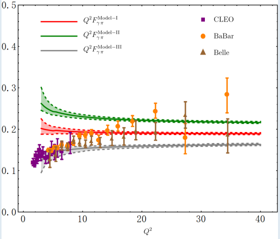
evolution of $\phi_\pi(x,\mu)$ **done one order higher in** α_s

— **includes higher power corrections** [Y-L. Shen, Y-M. Wang (2017)]

◇ **hierarchy holds for other models**

Numerical analysis: model dependence

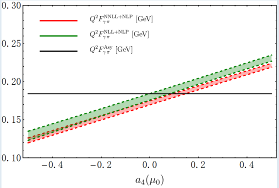
- **Model dependence**



color bands from scale variation $\mu_F^2 \in (1/4, 3/4)Q^2$

◇ well separated theory predictions $\Longrightarrow$ better model constraint with higher quality data

- **constraining less-known $a_4(\mu_0)$**



$a_2(\mu_0)$ determined from lattice; $a_{n\geq 6} = 0$

Summary

- Pion-photon transition FF to two loops in QCD/axial-vector for DVCS
  - Ideal hard exclusive process for QCD factorization
  - Two-loop bare amplitude from standard multi-loop technology
  - Nontrivial IR subtraction procedure due to evanescent operator
  - Non-negligible two-loop numerical impact to TFF, precision LCDA from data
- Future studies
  - Inclusion of massive quark loops [in preparation, T. Huber, YJ, Y-M. Wang (2022)]
  - $\gamma\gamma^* \to gg$ process for gluon GPD (identical MIs)

    [in preparation, V. Braun, YJ, A. Manashov, S. Moch, J. Schoenleber (2022)]
  - Double-virtual TFF

Back up slides

$F_{\gamma\pi}$ with asymptotic LCDA

$$F_{\gamma\pi}^{\mathrm{LP,asy}}(Q^2) = \frac{(e_u^2 - e_d^2) f_\pi}{\sqrt{2}\,Q^2} \left\{ 6 - 30\,a_s\,C_F - a_s^2 \left[C_F \beta_0 \left(\left(31 + 12 \log \frac{\mu^2}{Q^2} \right) \zeta_2 + 6\zeta_3 + 7 \right) \right.\right.$$
$$+ C_F^2 \left(24(\zeta_2 + \zeta_3) \log \frac{\mu^2}{Q^2} + 42\zeta_4 + 54\zeta_3 + 37\zeta_2 - \frac{85}{2} \right)$$
$$\left.\left. - \frac{C_F}{N_c} \left(6\zeta_4 - 12\zeta_3 - 2\zeta_2 + 13 \right) \right] + \mathcal{O}(\alpha_s^3) \right\} .$$