Light-ray OPE in QCD

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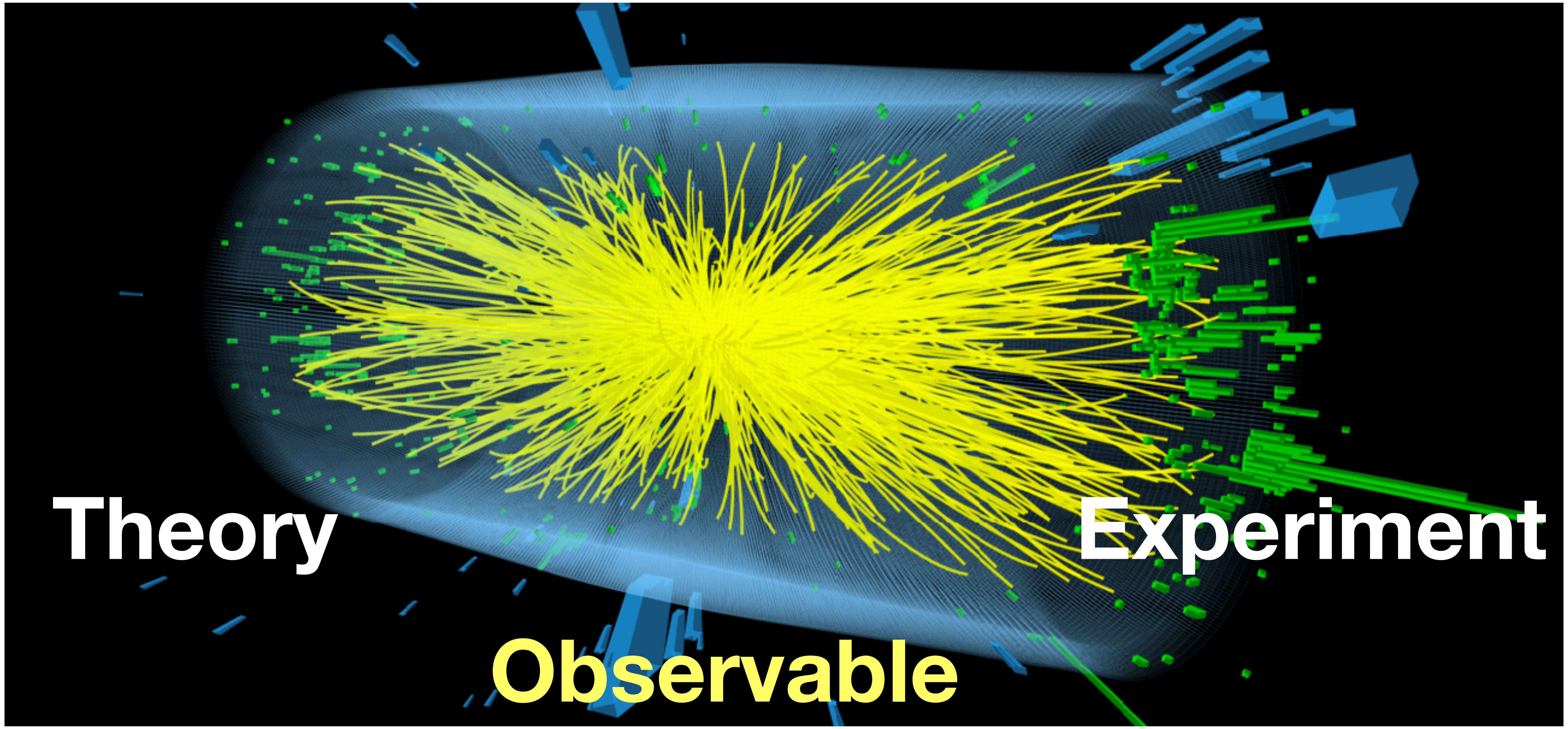
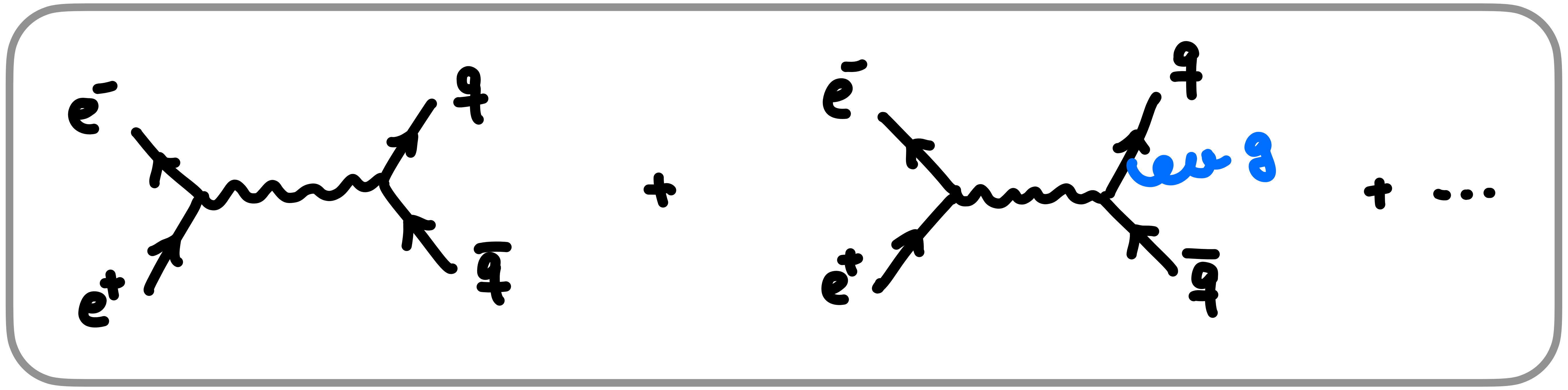
Collider Physics



- Experimentally viable
- Easy to calculate
- Have clean theoretical understanding

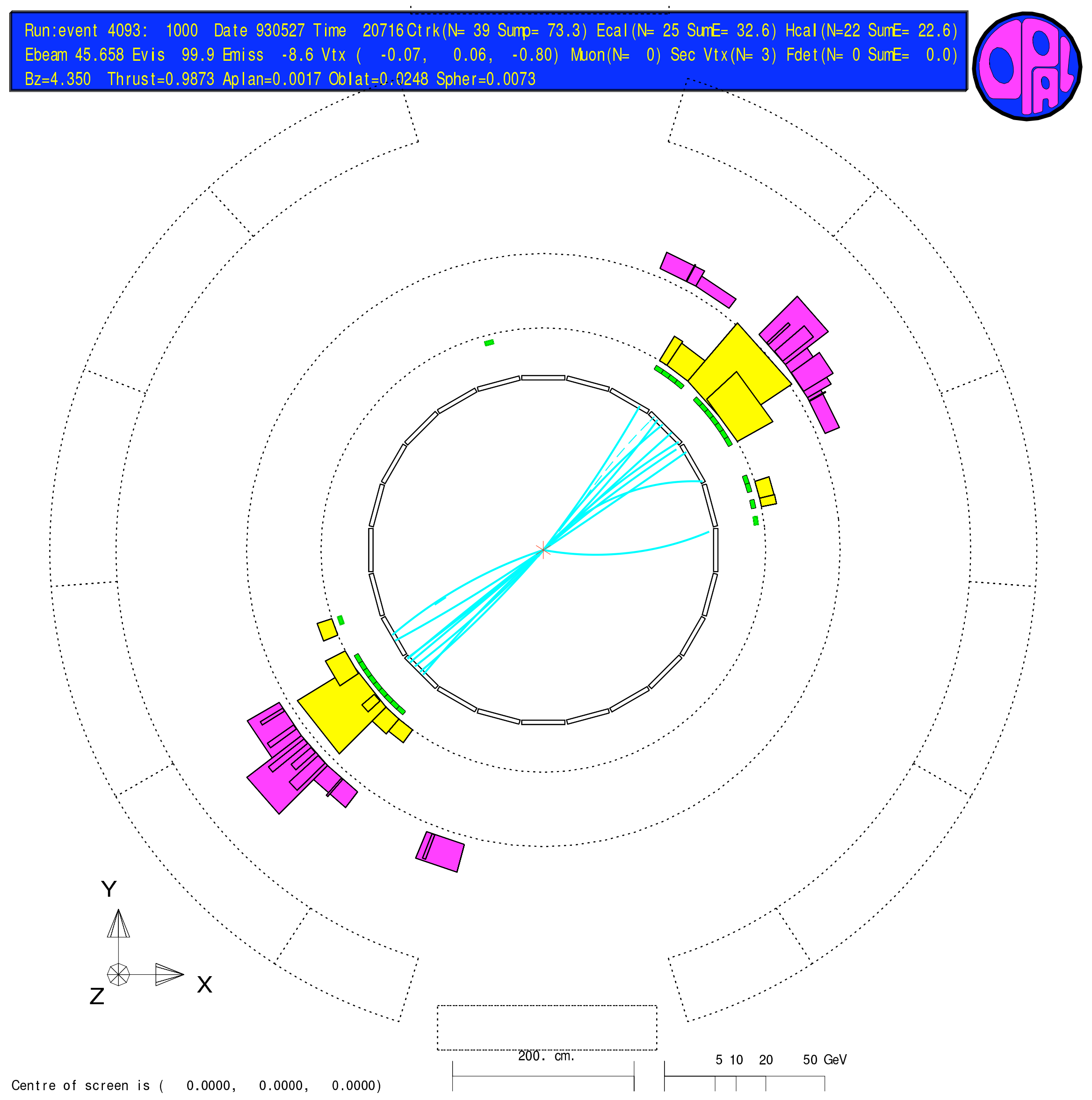
Event Shapes

e^+e^- annihilation



+ hadronization

pencil-like distribution vs (more) spherical distribution



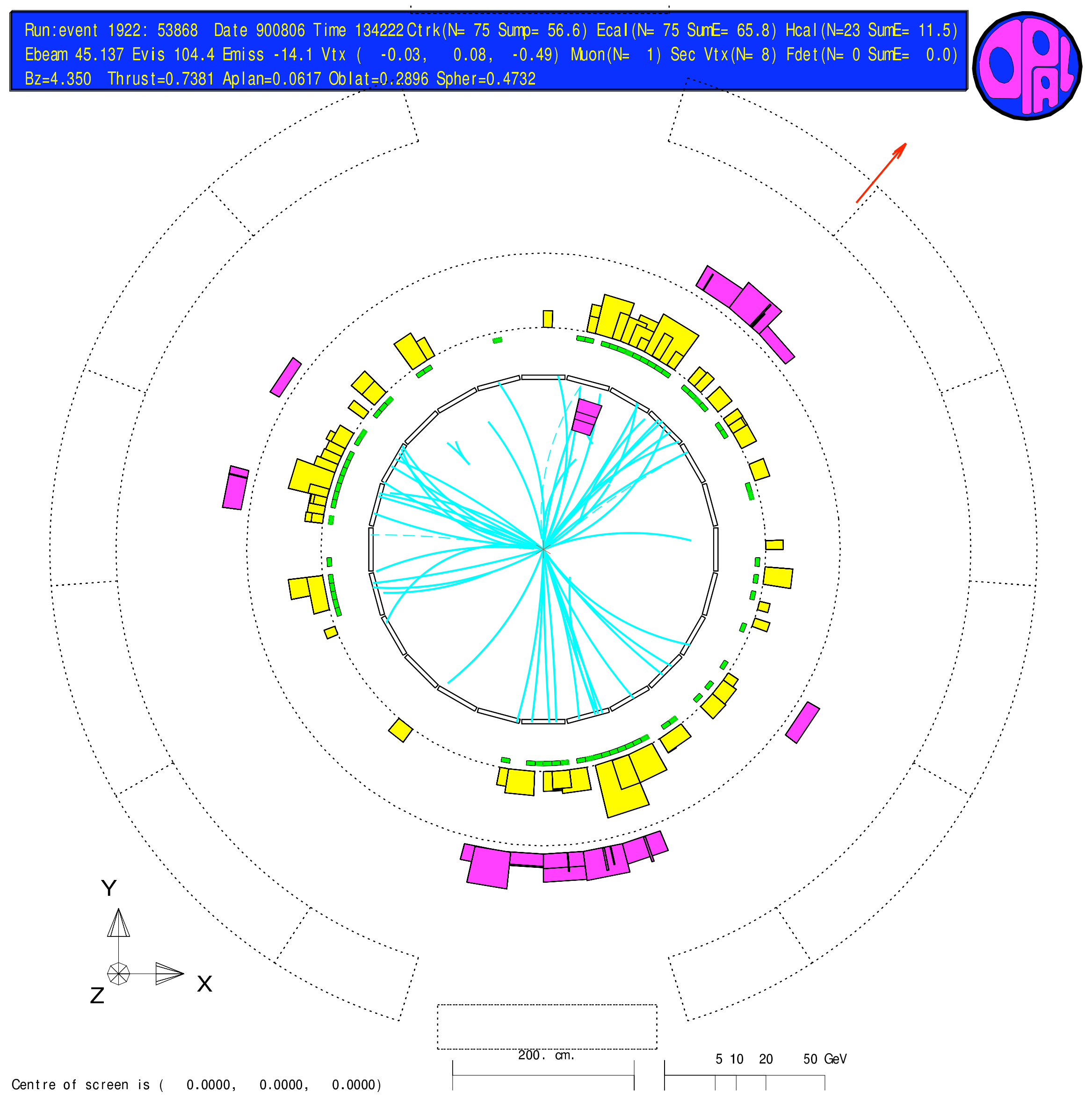
Example: Thrust

[Farhi, 1977]

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q}$$

pencil-like $T \sim 1$

spherical $T \sim 1/2$



Energy-energy correlator

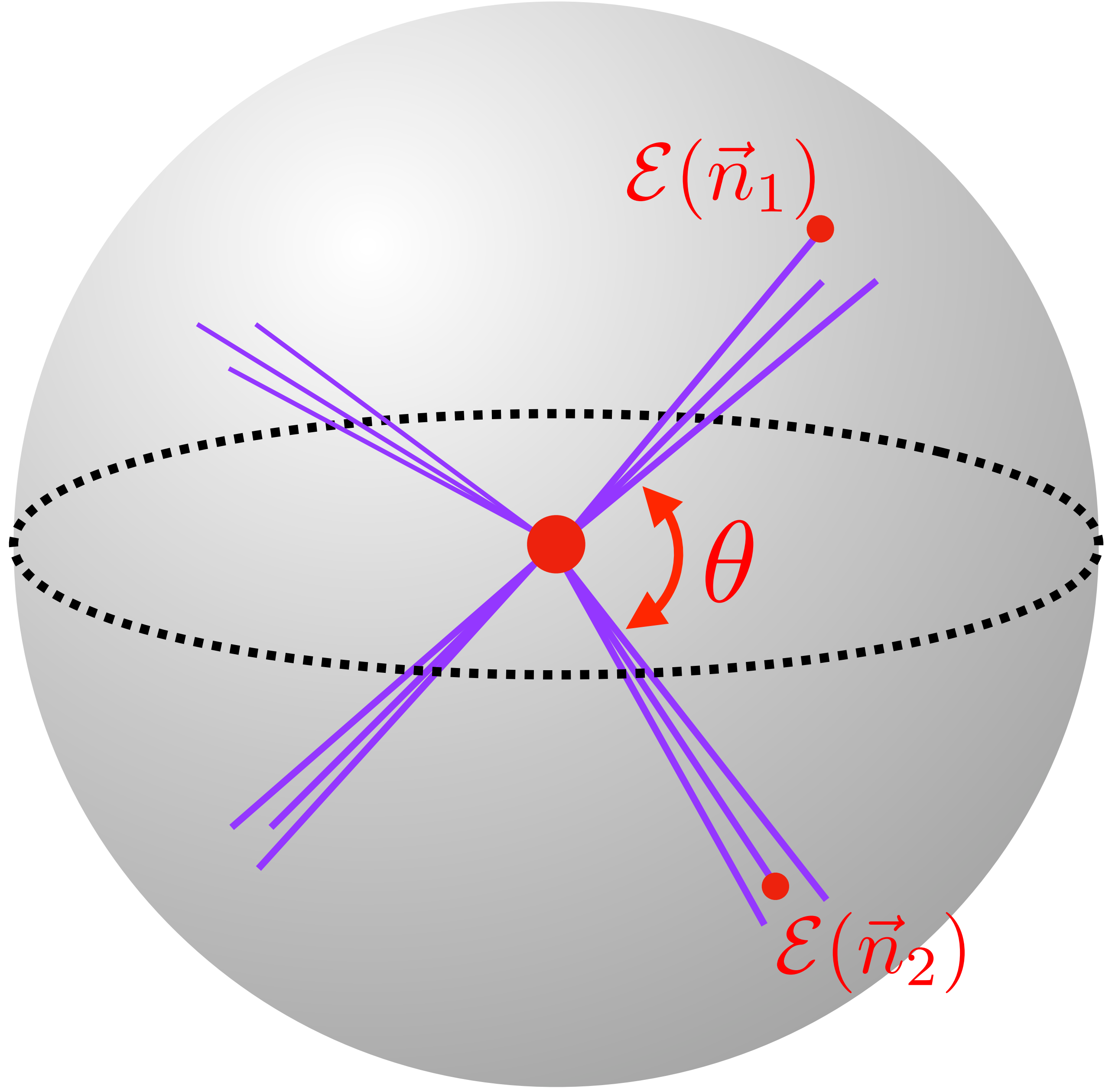
[Basham, Brown, Ellis and Love, 1978]

introduced energy-energy correlation

$$\frac{d\Sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$

which characterizes the correlation of two energy detectors at spatial infinity (celestial sphere).

Energy Correlation on the celestial sphere



Familiar concept in statistical mechanics!

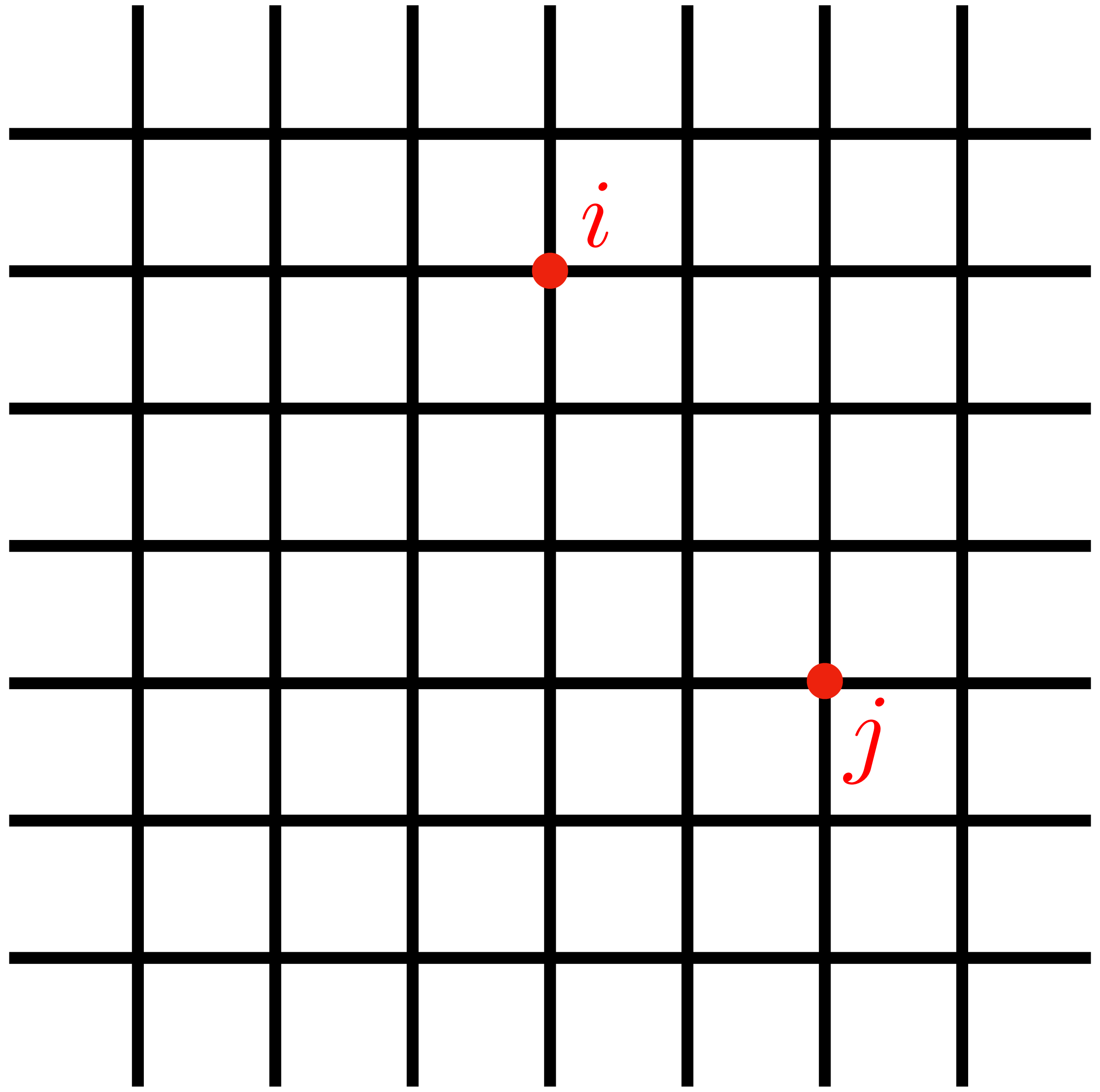
Probability Distribution

differential cross section $d\sigma$	Boltzmann factor $e^{-\beta H}$

Weighting Factor

eigenvalues of energy	eigenvalues of spin

Spin Correlation on the plane (2D Ising)

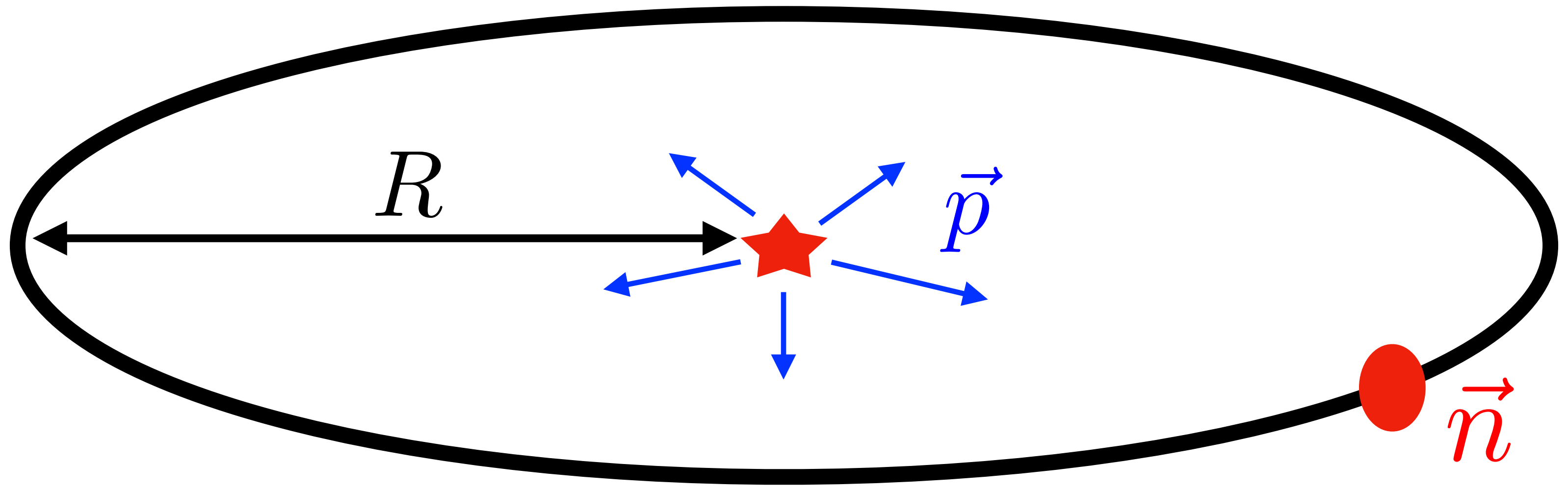


Energy Flow Operator $\mathcal{E}(\vec{n})$

"Perturbative" vs "Non-perturbative"

A calorimeter only detects particles flowing along direction $\vec{n}$, and weight with its energy E, e.g.

$$\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} E_{\vec{p}}\, a^\dagger_{\vec{p}}\, a_{\vec{p}}\, \delta^{(2)}(\vec{n}-\hat{p})$$



The radiation power passing the detector (located at $R\vec{n}$) at time t is

$$n^i\, T^0_i(t, R\vec{n}) R^2 d\Omega$$

[Korchemsky, Sterman, 1999; Hofman, Maldacena, 2008; Bauer, Fleming, Lee, Sterman, 2008; …]

- Integrate t to get the total received energy
- Detector is effectively located at infinity

$$\boxed{\mathcal{E}(\vec{n}) = \lim_{R\to\infty} R^2 \int_0^\infty dt\, n_i T^{0i}(t, R\vec{n})}$$

non-perturbative definition

Energy Flow Operator

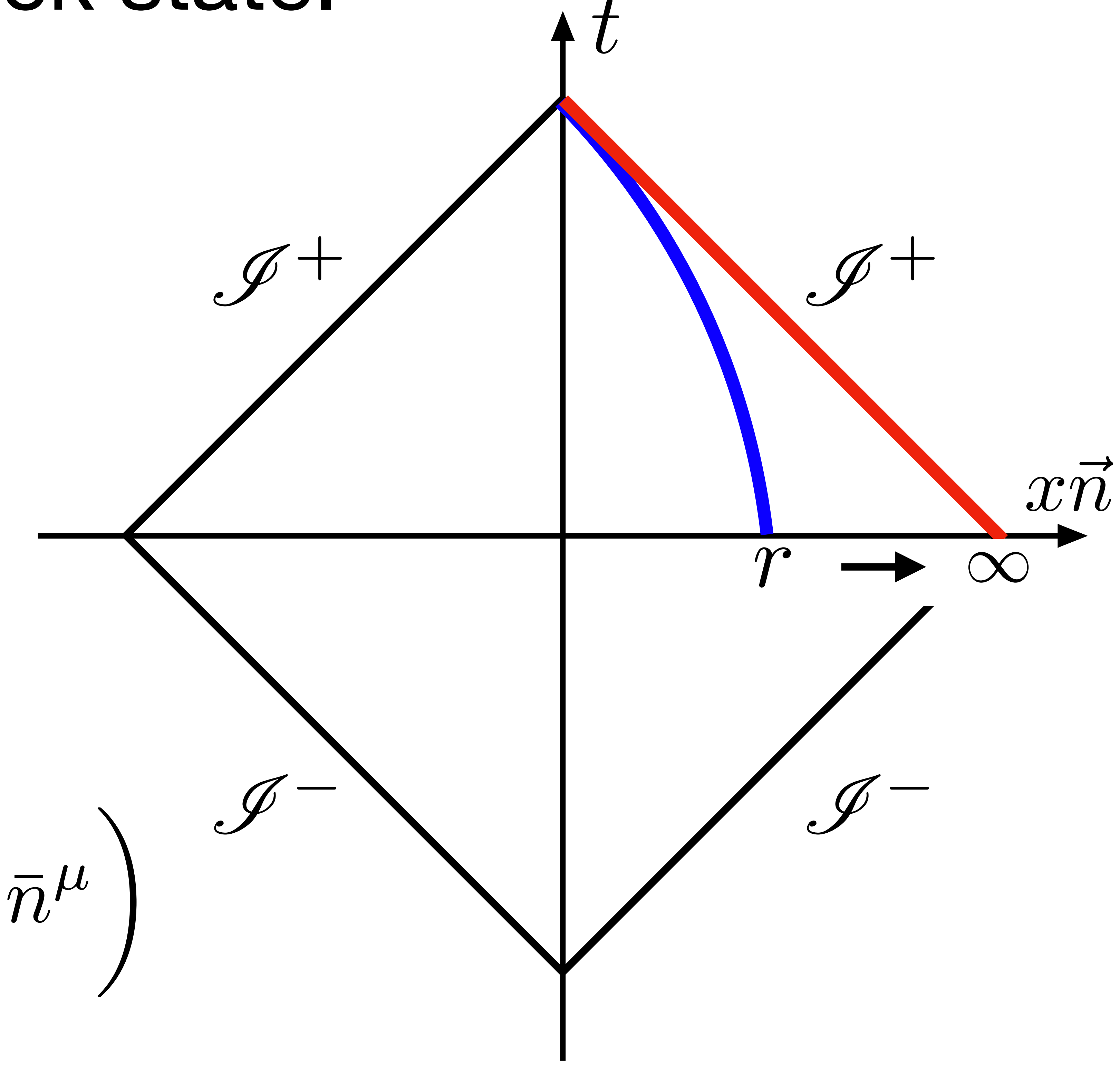
- For free theory, we can use mode expansion [see, e.g. Bauer, Fleming, Lee and Sterman, 2008]

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n}) \xrightarrow[T_{0i} = \partial_t\phi\partial_i\phi]{\textbf{Free scalar}} \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} E_{\vec{p}}\, a_{\vec{p}}^\dagger a_{\vec{p}}\, \delta^{(2)}(\vec{n} - \hat{p})$$

so they are equivalent when acting on asymptotic Fock state.

- The energy flow operator is a non-local operator defined on a light-ray located at future null infinity

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$



- Equivalent form in lightcone coordinate

$$\mathcal{E}(\vec{n}) = \frac{1}{4} \lim_{x^+\to\infty} \left(\frac{x^+}{2}\right)^2 \int_{-\infty}^{\infty} dx^-\ \bar{n}_{\mu_1}\bar{n}_{\mu_2} T^{\mu_1\mu_2}\left(\frac{x^+}{2} n^\mu + \frac{x^-}{2}\bar{n}^\mu\right)$$

- This is an example of light-ray operators.

Energy Correlators

Energy correlators are correlation function of multiple energy flow operators inside some non-vacuum states

$$\langle \Psi' | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \ldots | \Psi \rangle$$

Looks like a correlation function in a fictitious 2D field theory on S^2

e.g. created by local operators

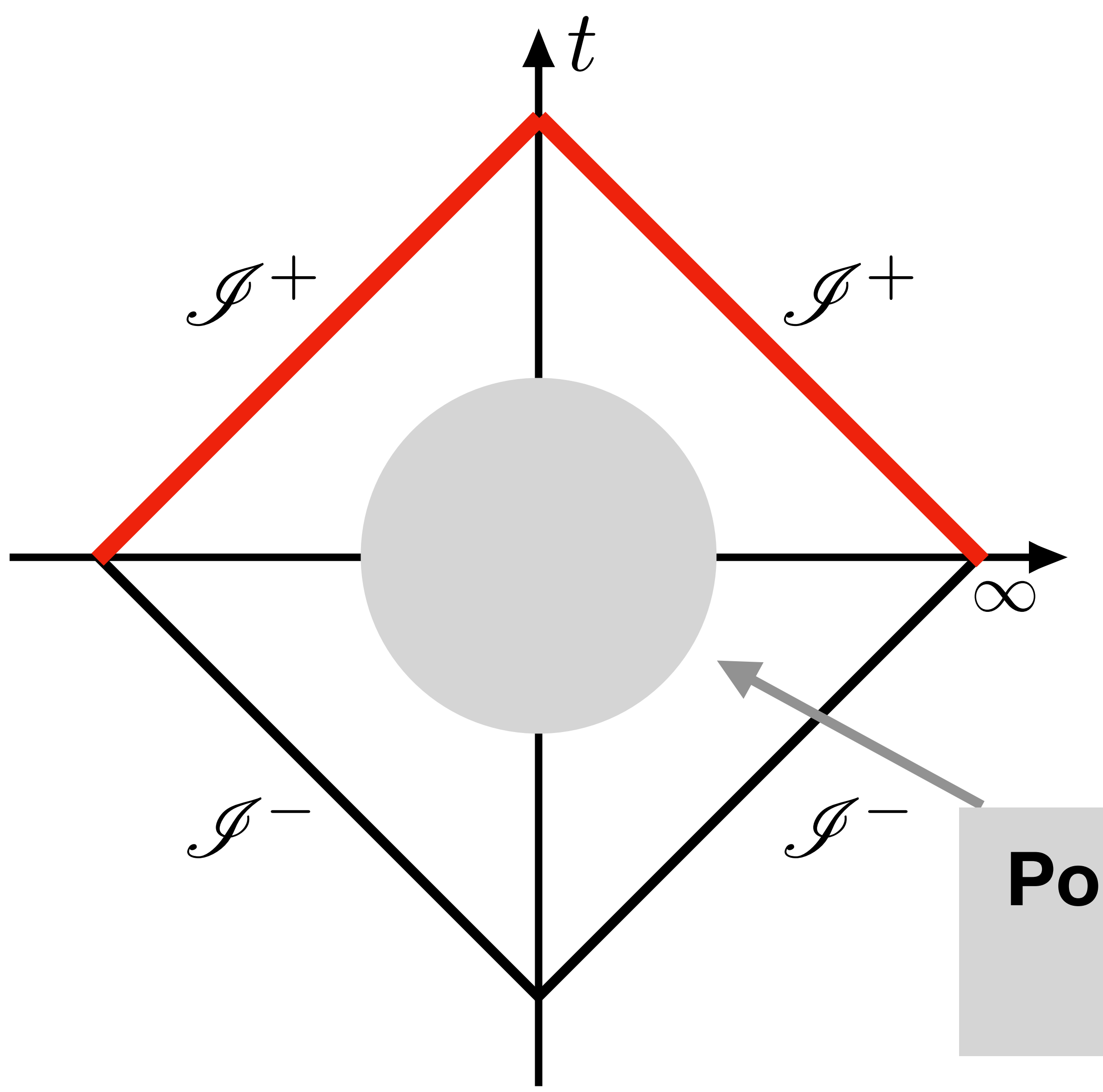
[momentum space] $\langle O'(-q) | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \ldots | O(q) \rangle$

Source with total momentum $q = (Q,0,0,0)$

[coordinate space] $\langle \Omega | O'(x) \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \ldots O(0) | \Omega \rangle$

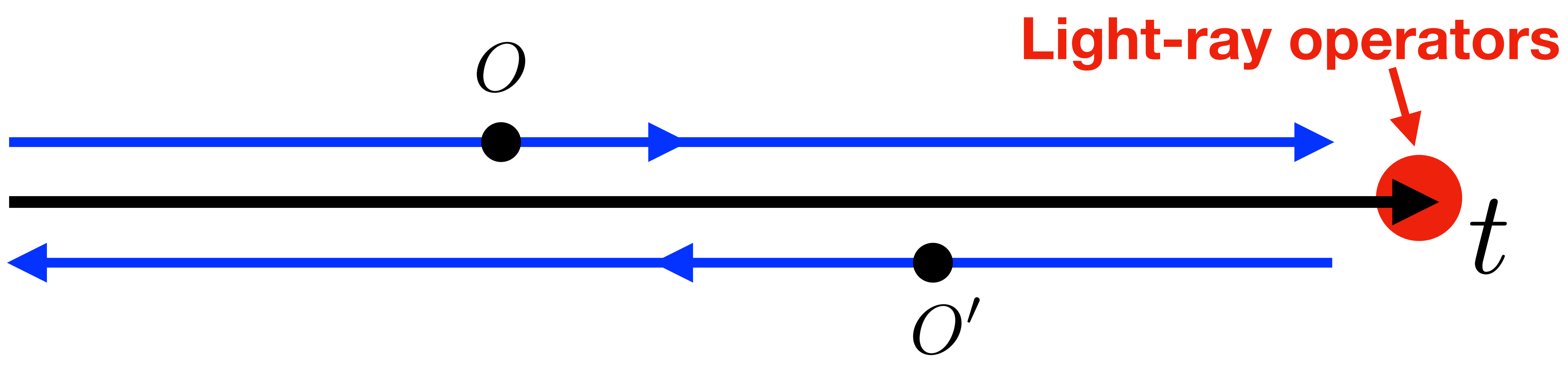
Wightman function, not time-ordered function

Light-ray operators lie in the future of all local sources



Positions of local operators
O, O'

Schwinger-Keldysh contour



Light-ray Operators

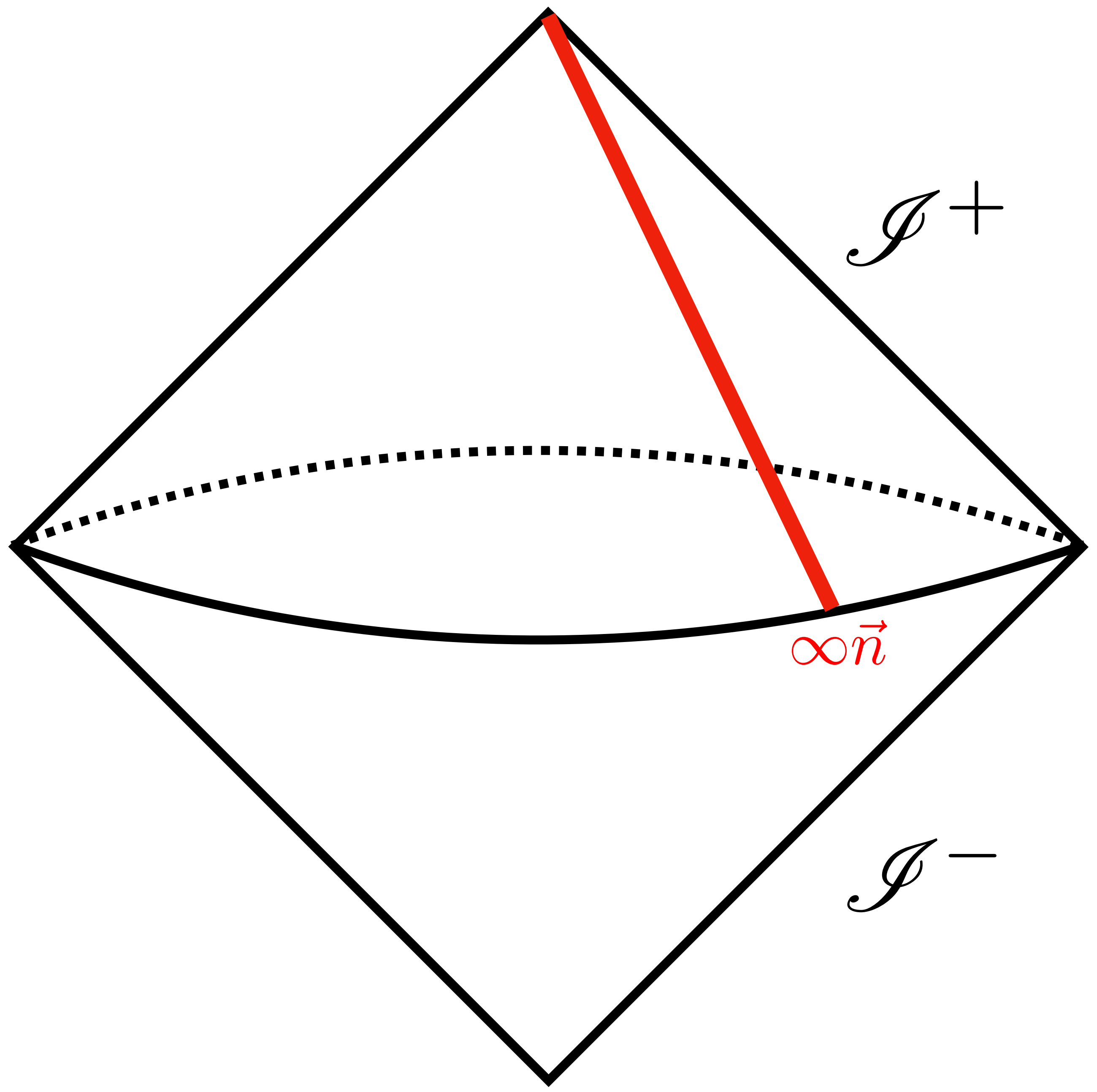
light transform of **local operators**

Energy flow operator $$\mathcal{E}(\vec{n}) = \boxed{\lim_{r\to\infty} r^2 \int_0^\infty dt}\, \vec{n}_i T^{0i}(t, r\vec{n})$$

Energy radiation fall with inverse square law, r^2 compensates this effect to be non-vanishing.

More general light-ray operators

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\overset{\text{twist}}{\Delta - J}} \int_0^\infty dt\, O^{\mu_1\cdots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$



Lesson from CFT : $O(x,z) = O^{\mu_1\mu_2\cdots\mu_n}(x)z_{\mu_1}z_{\mu_2}\ldots z_{\mu_n}$

$$\langle O(0,z_1)O(x,z_2)\rangle = \frac{\left(z_1\cdot z_2 - 2\frac{z_1\cdot x z_2\cdot x}{x^2}\right)^J}{(x^2)^\Delta}$$

$$\xrightarrow[\text{send } x \text{ to null infinity}]{(x^+\to+\infty)} \sim (x^+)^{J-\Delta}$$

Light-ray Operators

light transform of local operators

Energy flow operator

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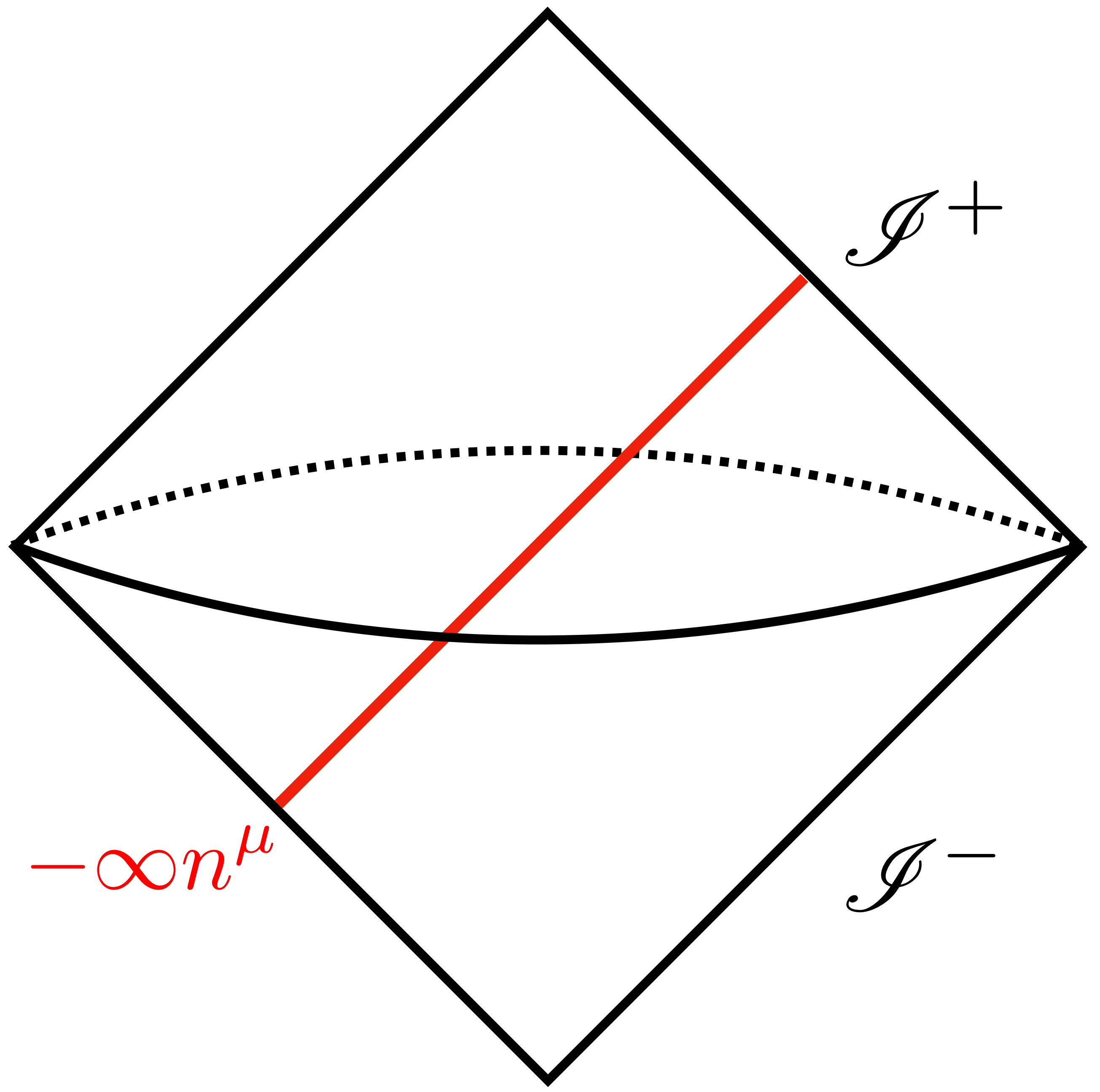
More general light-ray operators

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\overset{\text{twist}}{\Delta - J}} \int_0^\infty dt\, O^{\mu_1\ldots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

In other contexts of physics, light-ray operators are not necessarily at null infinity—they can live on any light-ray.



$$\mathbf{L}[\mathcal{O}](\underset{\text{starting point}}{x}, \overset{\text{direction}}{n}) = \int_{-\infty}^{\infty} d\alpha(-\alpha)^{-\Delta-J}\mathcal{O}\left(x - \frac{n}{\alpha}, n\right)$$

[Kravchuk, Simmons-Duffin, 2018]

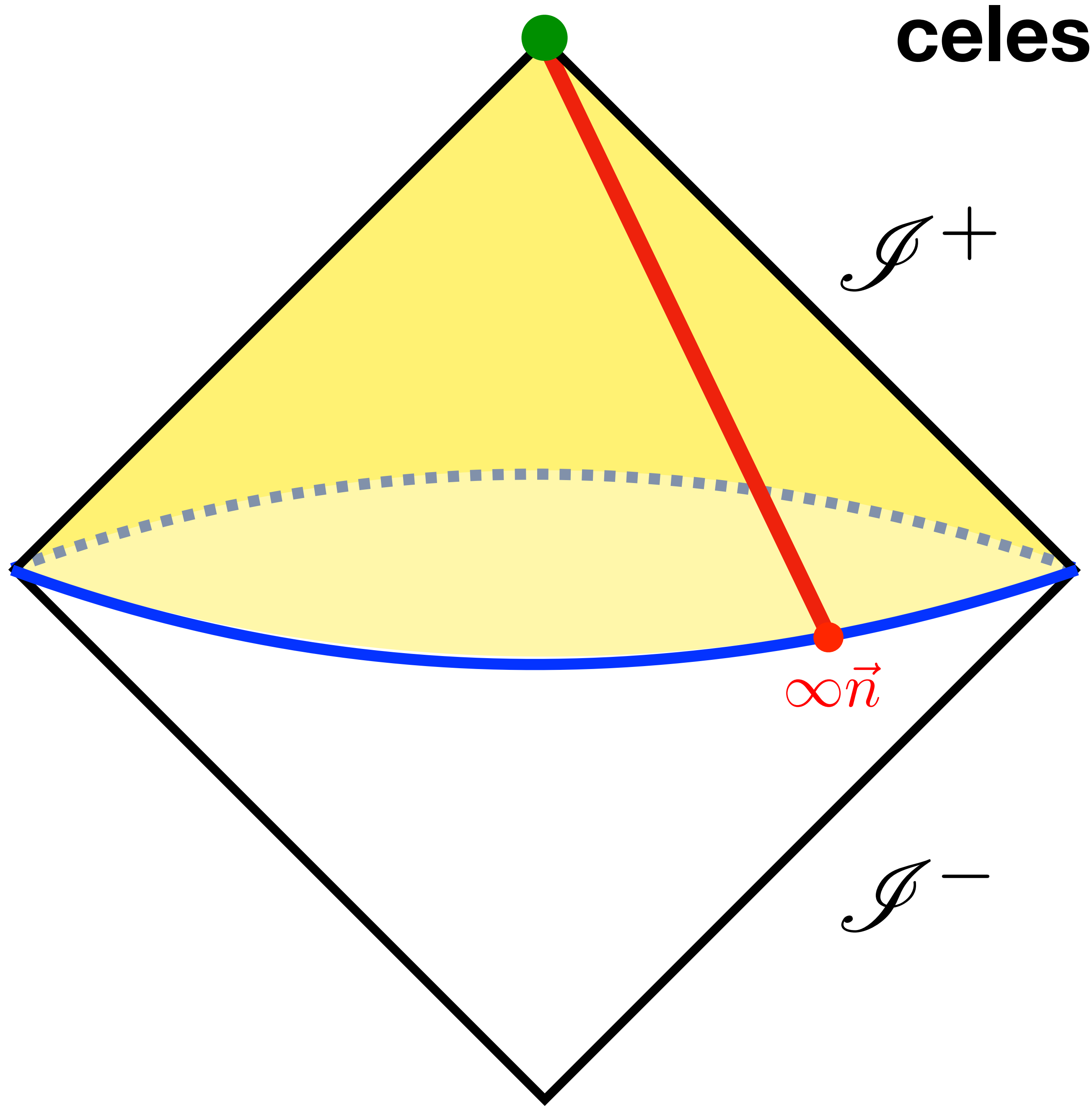
Light-ray Operators

In CFT, different configurations are related by conformal transformation.

[Hofman and Maldecena, 2008; Kravchuk, Simmons-Duffin, 2018]

null infinity ⟷ **null plane** $y^+ = 0$

celestial sphere S^2 ⟷ **transverse plane** $\mathbb{R}^2$

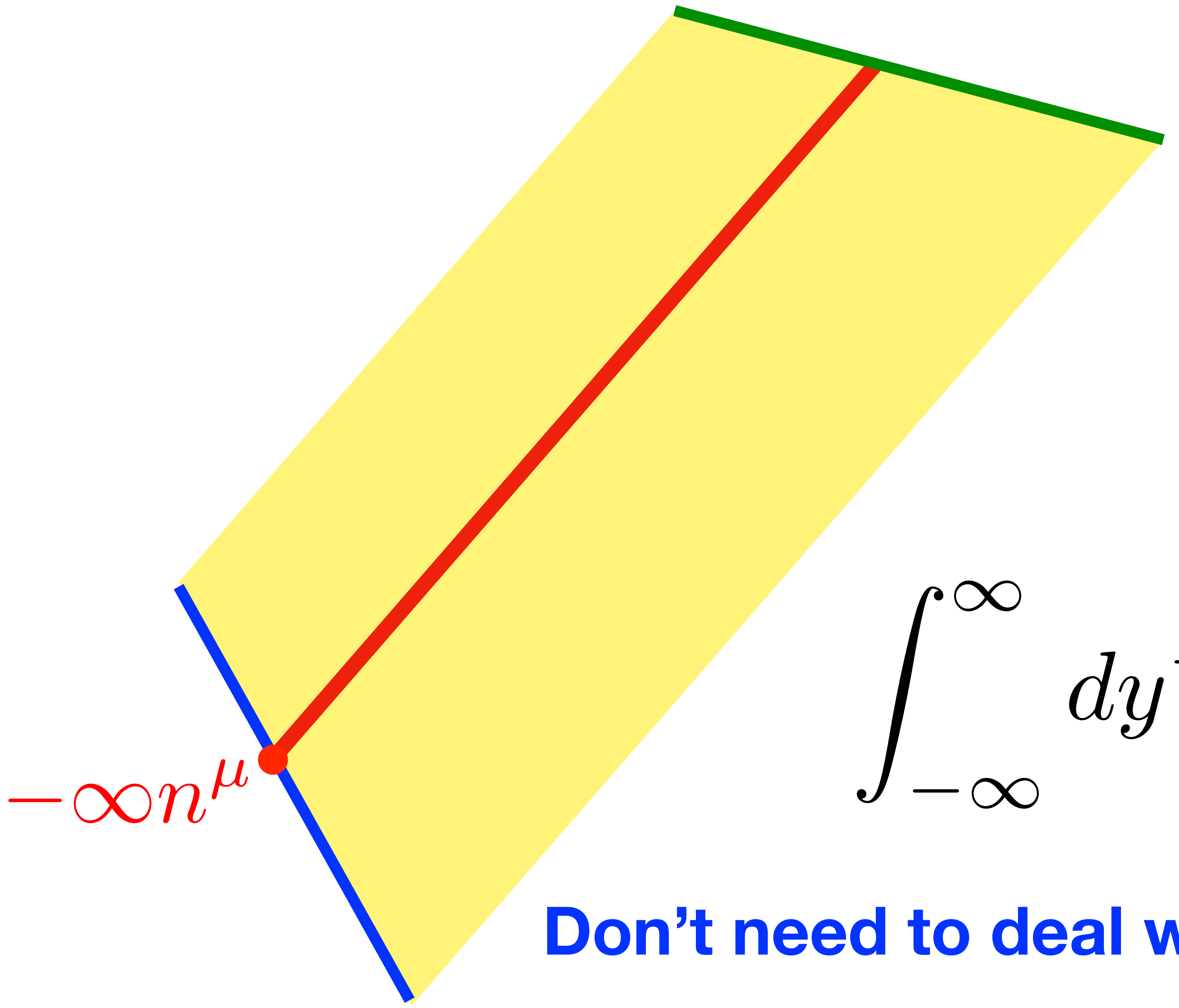


Conformal $\longrightarrow$ **Transformation**

$$y^+ = -\frac{1}{x^+}$$

$$y^- = x^- - \frac{\mathbf{x}^2}{x^+}$$

$$\mathbf{y} = \frac{\mathbf{x}}{x^+}$$



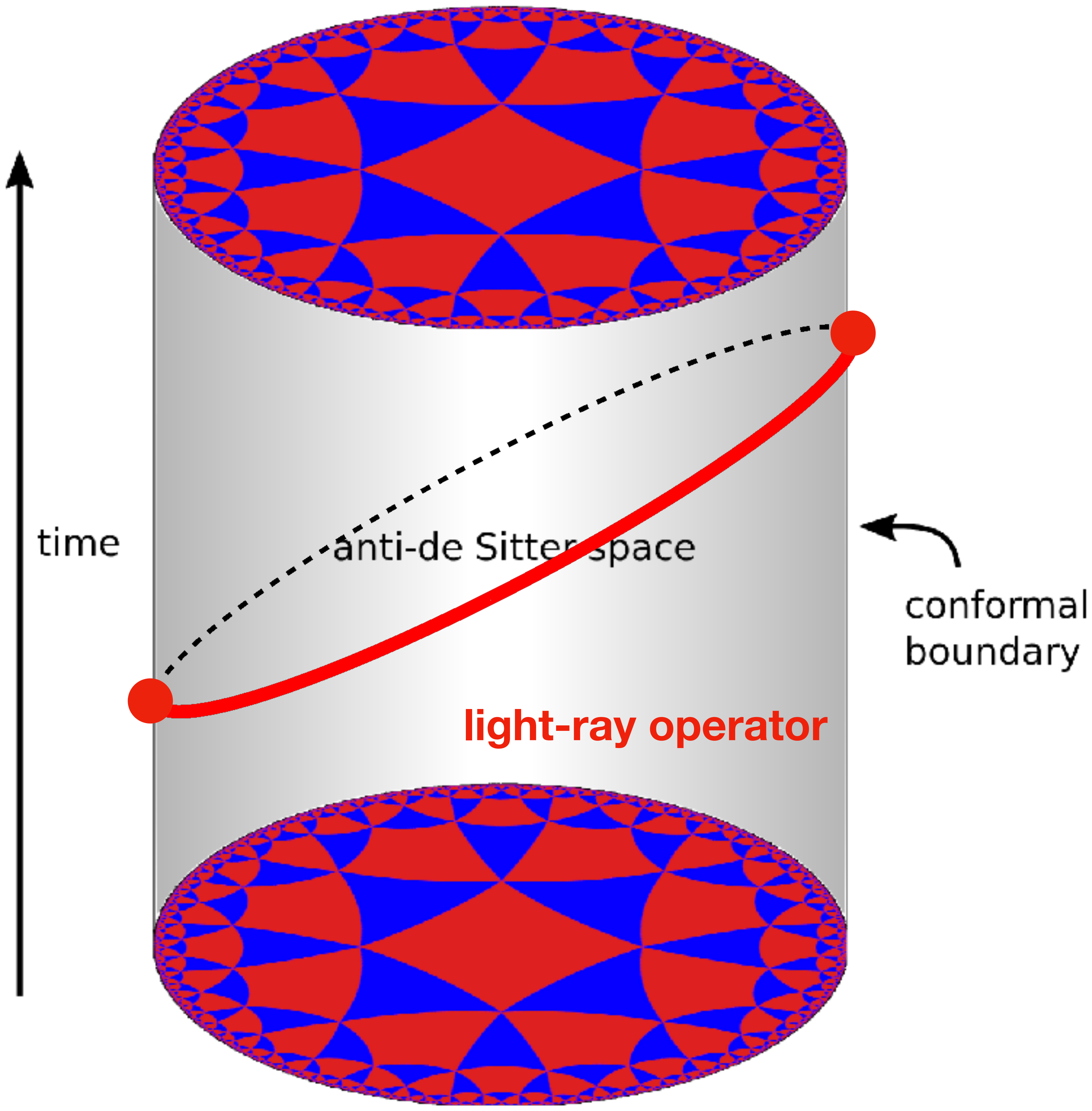
$$\int_{-\infty}^{\infty} dy^- O(y^+ = 0, y^-, \mathbf{y})$$

Don't need to deal with taking limit in this case

In embedding space formalism, they correspond to different gauge fixing.

[see Kologlu, Kravchuk, Simmons-Duffin and Zhiboedov, 2019]

AdS/CFT Correspondence

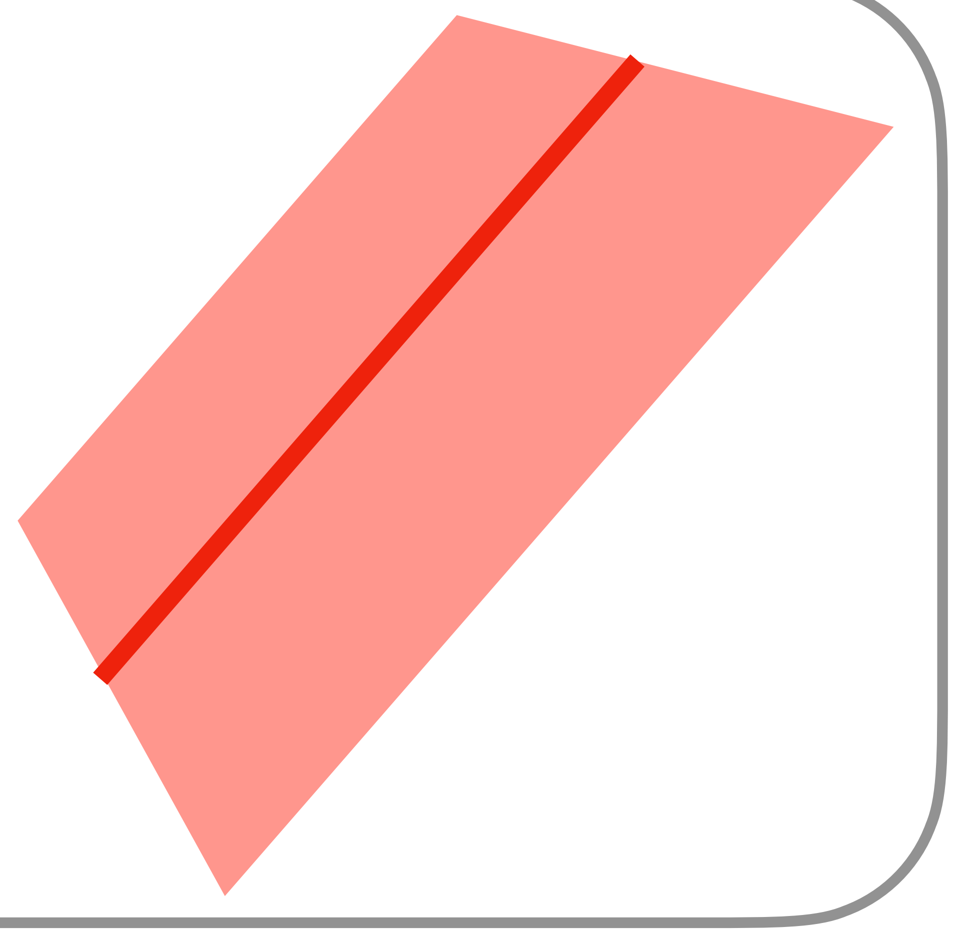


CFT	AdS
strong coupling	**weak coupling**
stress tensor insertion $T^{\mu\nu}$	**metric perturbation** $h_{\mu\nu}$
energy flow operator $\mathcal{E}$	**gravitational shockwave**
energy correlators	**propagation through shockwaves**

shockwave (flat space version)

$$h_{--} \propto \delta(y^-) \frac{1}{|\vec{y}_\perp|^{d-4}}$$

localized on the null plane



In strong coupling limit, energy correlations are uniform distributions. **[Hofman, Maldacena, 2008]**

Spacetime Symmetry

Little group that fixes a light-ray in future null infinity consists of

translations, collinear boost, transverse rotations, dilatation

Poincare group part

- Dimension = $J - 1$

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} \underbrace{\boxed{r^{\Delta-J}}}_{-(\Delta-J)} \underbrace{\boxed{\int_0^\infty dt}}_{-1} \underbrace{\boxed{O^{\mu_1\cdots\mu_J}(t, r\vec{n})}}_{+\Delta} \bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

- Collinear Spin = $1 - \Delta$

Boost quantum number

$$\lim_{\bar{n}\cdot x\to\infty} (\bar{n}\cdot x)^{\Delta-J} \int_{-\infty}^{\infty} d(n\cdot x)\; O^{\mu_1\cdots\mu_J}(x)\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

Boost along $\vec{n}$ $\quad n^\mu \to \lambda n^\mu, \quad \bar{n}^\mu \to \lambda^{-1}\bar{n}^\mu$

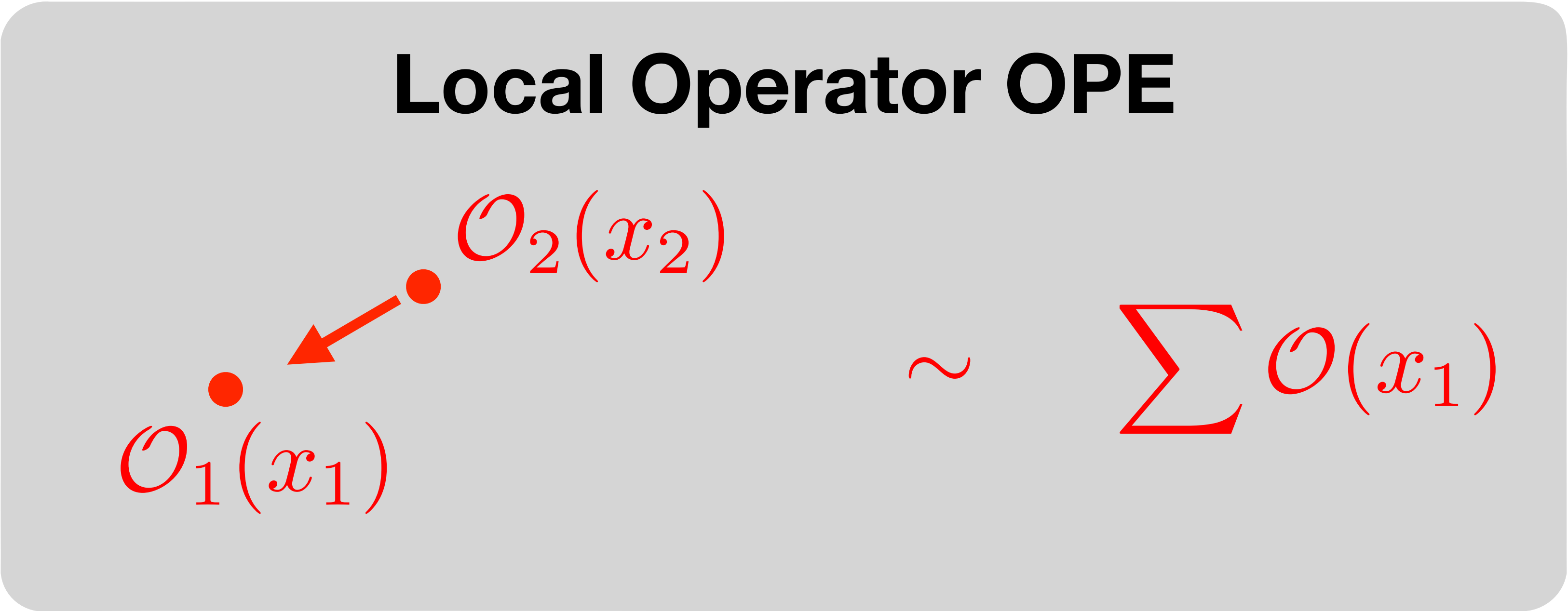
$$\mathbb{O}(\vec{n}) \to \lambda^{1-\Delta}\mathbb{O}(\vec{n}) \xrightarrow[\vec{n}\to n^\mu]{\text{promote}} \boxed{\mathbb{O}(\lambda n^\mu) = \lambda^{1-\Delta}\mathbb{O}(n^\mu)}$$

homogeneous

- Transverse Spin = transverse spin of the local operator
- Momentum = 0 (invariant under translations)

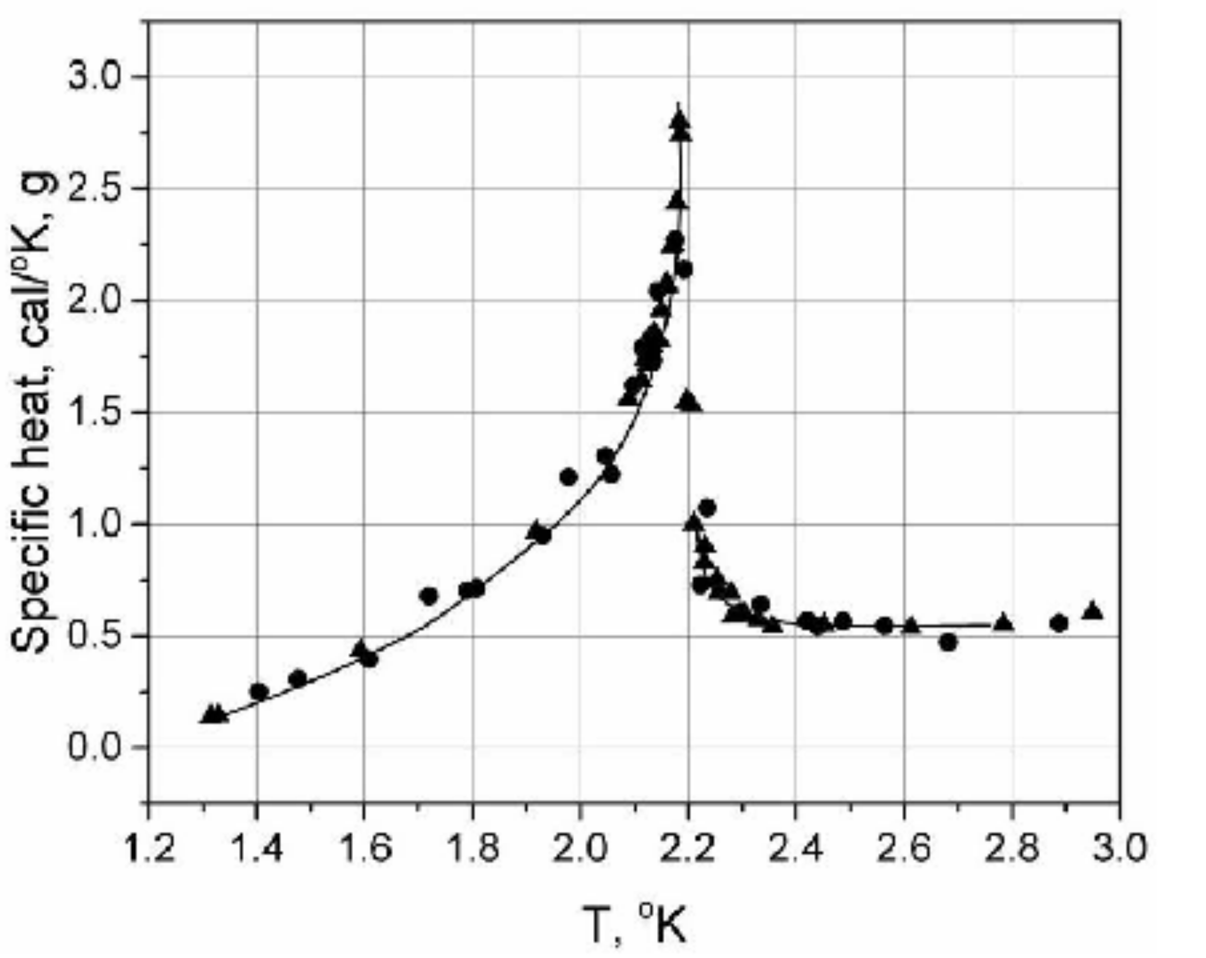
Light-ray OPE

Short distance scaling behavior is determined by local **Operator Product Expansion** (OPE).

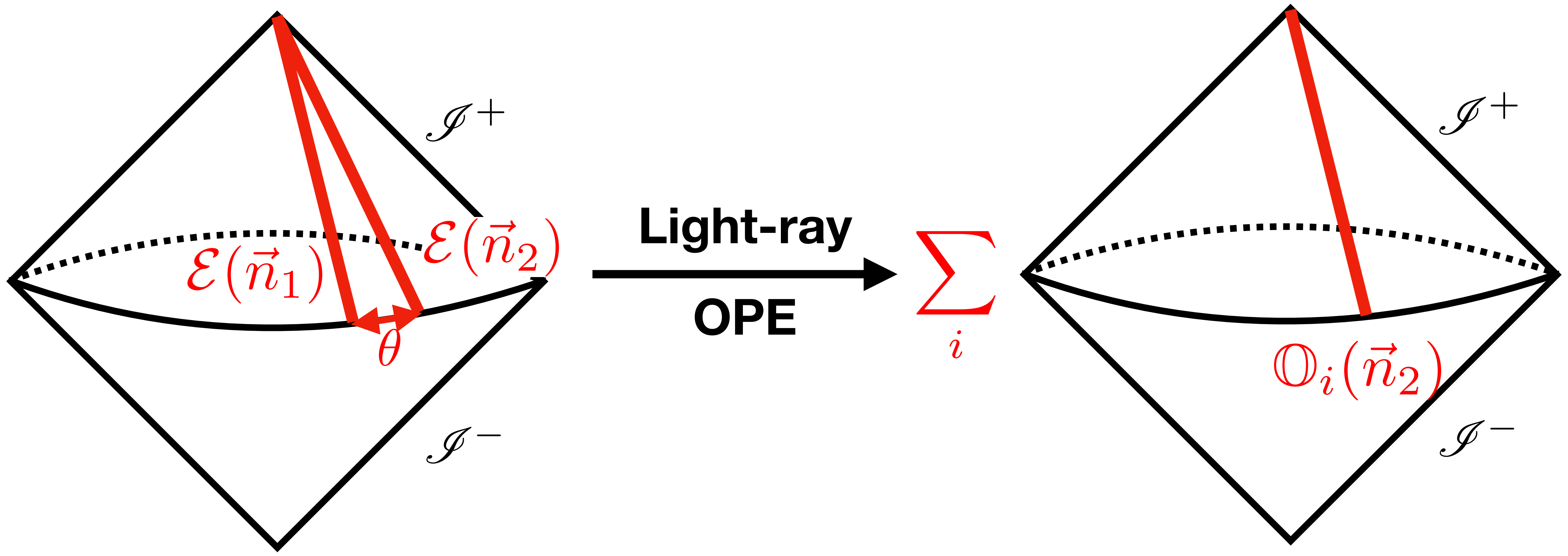


Local Operator OPE

liquid helium critical behavior

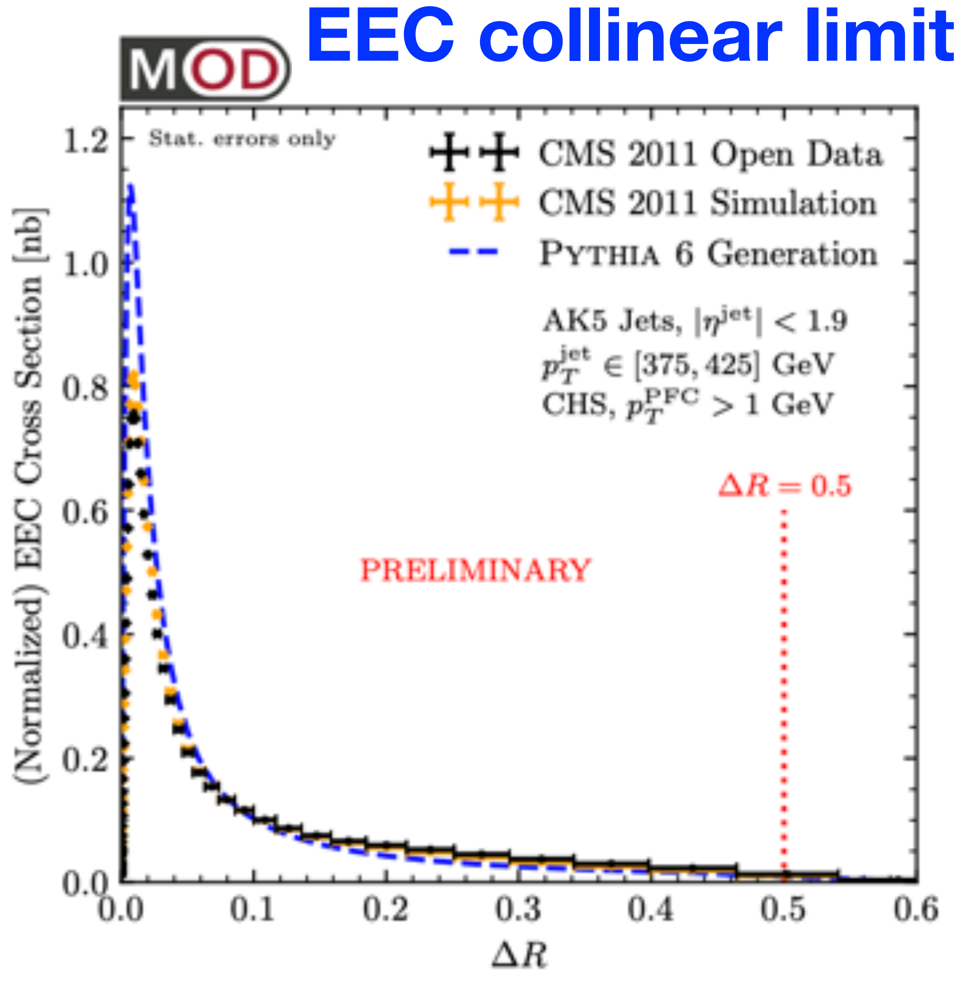


Small angle behavior is controlled by the OPE of these light-ray operators.



"local OPE on the celestial sphere"

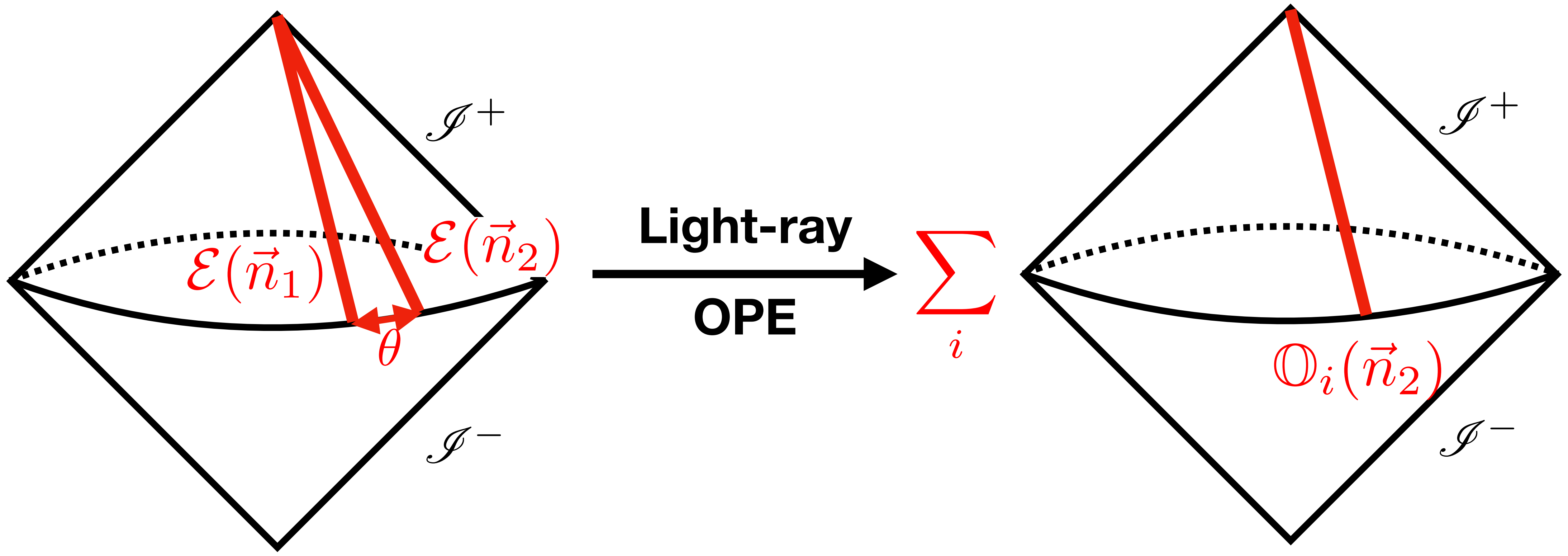
EEC collinear limit



[Komiske, Moult, Thaler, Zhu, in preparation]

Light-ray OPE

Symmetry and Power Counting



Expansion parameter θ is dimensionless, the dimensions simply add up.

(of light-ray operators)

Recall that the dimension of a light-ray operator is $J-1$

J is the collinear spin of the local operator

$$\boxed{\mathbb{O}_1\mathbb{O}_2 \supset \mathbb{O} \qquad (J_1-1)+(J_2-1)=J-1}$$

(Exact in CFT)

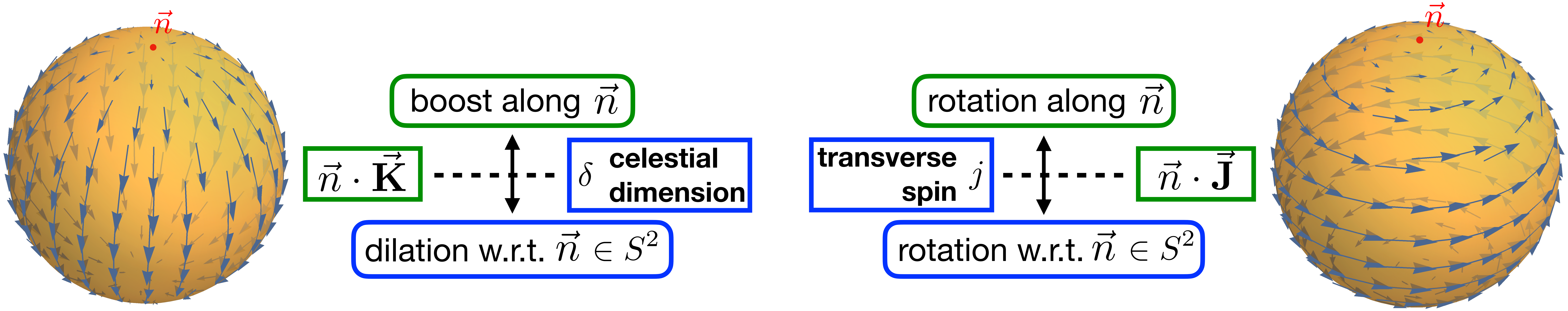
Example: in $\mathcal{E}\mathcal{E}$ OPE, $J_1=J_2=1 \Rightarrow J=3$

different from local OPE: $J=3$ operator is absent in the TT OPE

Light-ray OPE

Symmetry and Power Counting

Lorentz group is equivalent to the **conformal group** on the **celestial sphere**.



Angle θ plays the role of length on the celestial sphere

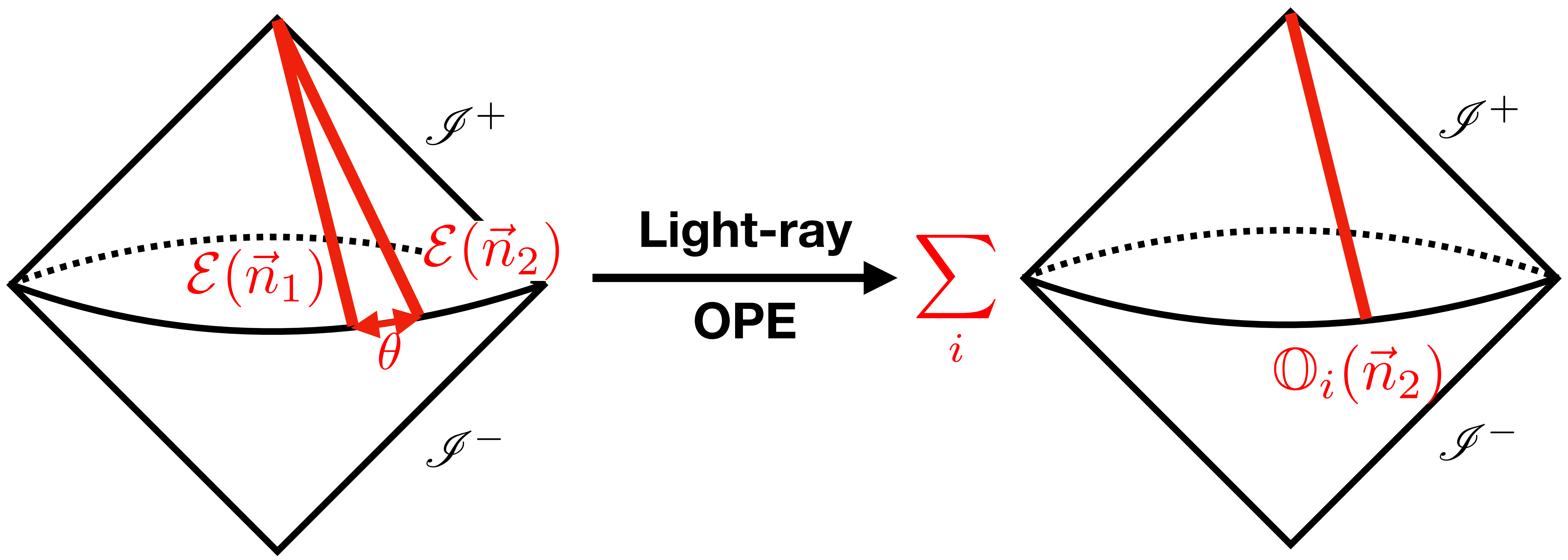
⟶ power counting on θ is related to celestial dimension (boost quantum number)

twist $\tau_i \equiv$ **dim** $\Delta_i -$ **spin** $(J_i = 3)$

Light-ray OPE $\quad \mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i - 4} \mathbb{O}_i(\vec{n}_2)$ **[Hofman, Maldacena, 2008]**

collinear spin $\quad (1-4) \;+\; (1-4) \quad = \quad (\tau_i - 4) \;+\; (1-\Delta_i)$

Light-ray OPE

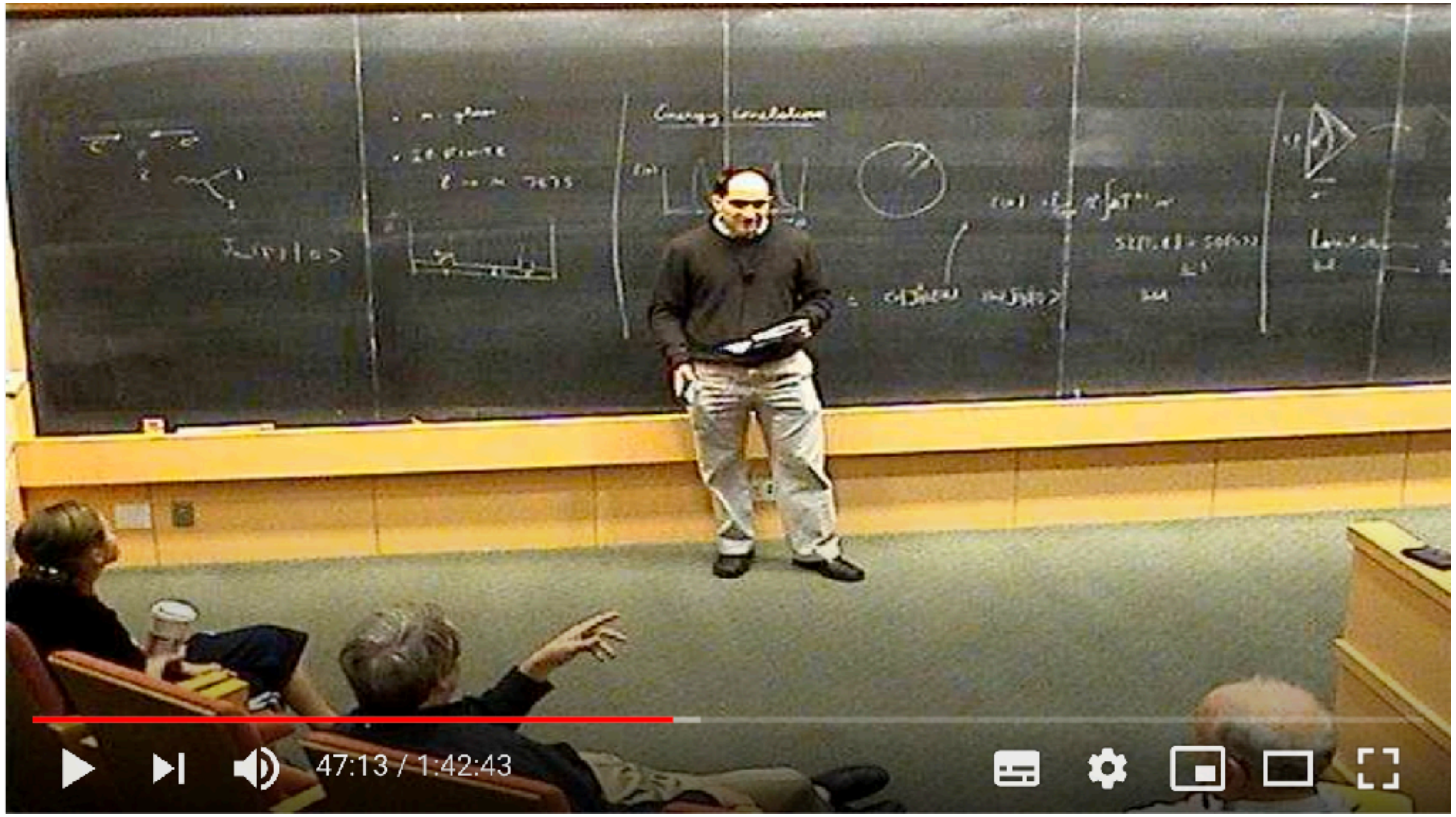


Light-ray OPE $$\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\,\theta^{\tau_i-4}\mathbb{O}_i(\vec{n}_2)$$

Small angle scaling is dominated by the leading twist operators.

Light-ray OPE in CFT is rigorous and convergent. **[Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]**
[Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020]

In QCD, things are less understood, but the leading power contribution is. **[HC, Moult, Zhu, 2020]**



Polchinski: There is a lot of QCD data, can you see this (scaling behavior) there?

Maldacena: People do not do this. I haven’t figured out why they don’t. I think they just haven’t thought about this. I was talking to people who did this calculation of two-point function at LEP, computing alpha_s and so on, and they focused mostly on the large angles. But they didn’t study the small angles. And I asked him whether they had a good reason for not studying the small angles and they said well we didn’t know the resummation formula, didn’t study it.

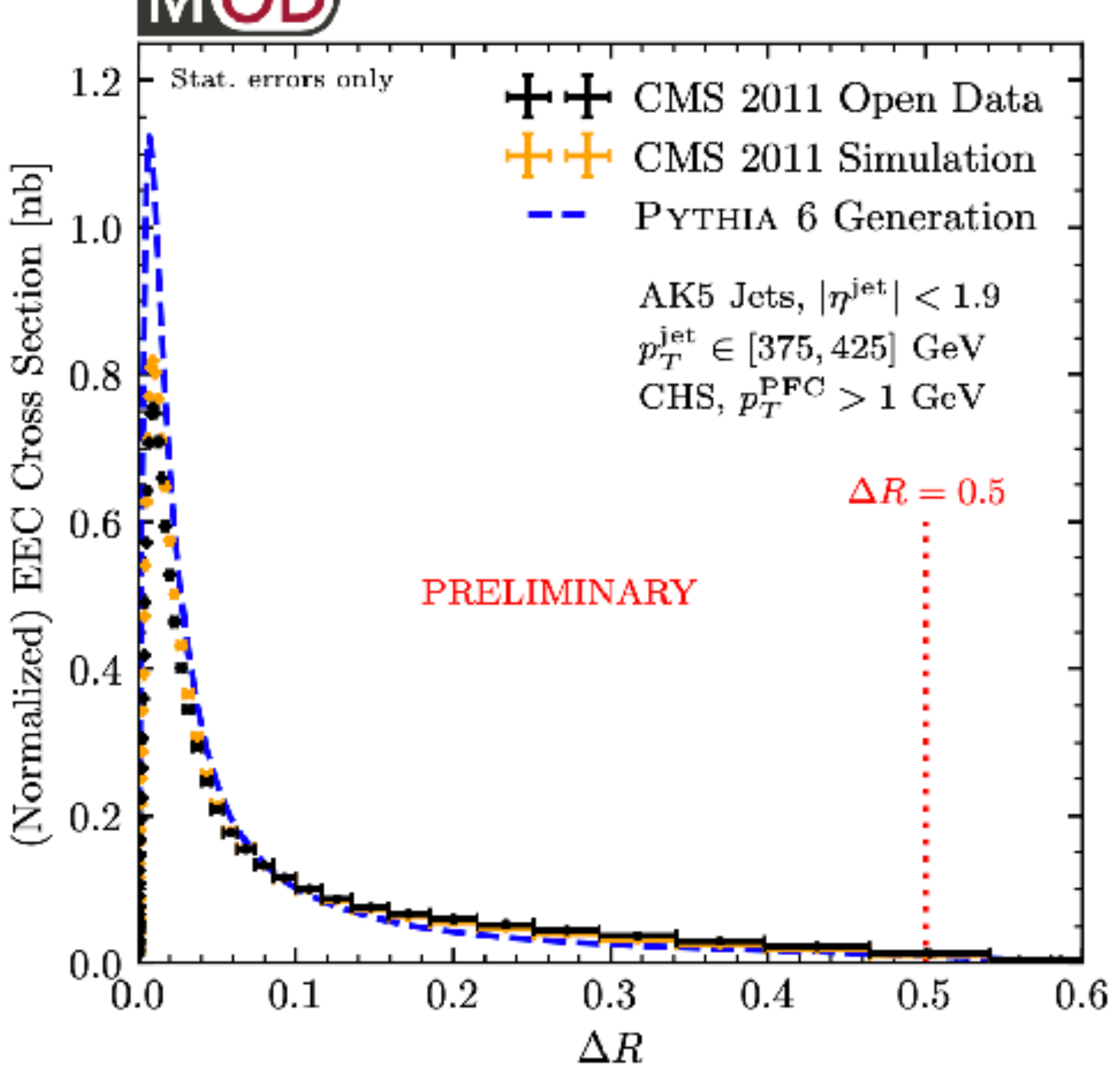
EEC with CMS Open Data

[Komiske, Moult, Thaler, Zhu, in preparation]

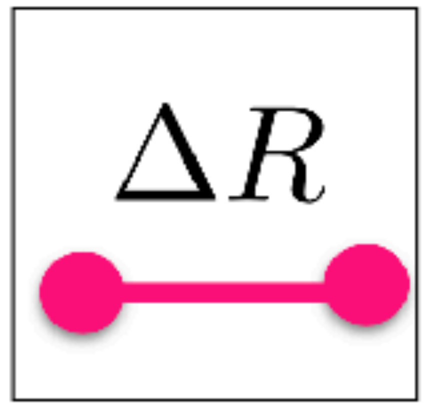
Packaged in “MIT Open Data”, provided by Jesse Thaler and Patrick Komiske

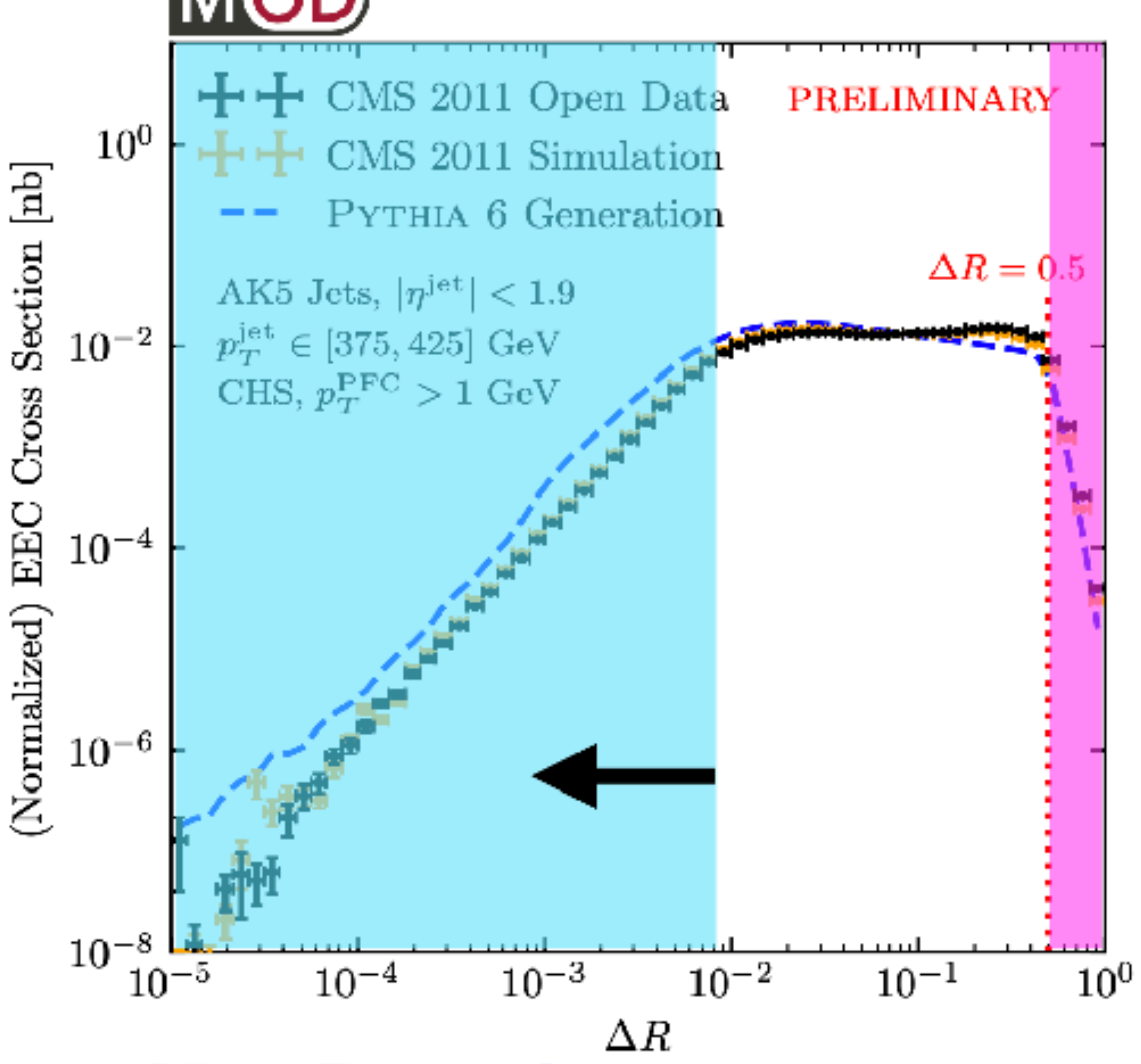
Nice scaling behavior in perturbative regime

$$\frac{d\sigma}{d\Delta R}$$



$$\Delta R \frac{d\sigma}{d\Delta R}$$





Non-Perturbative

Application in QCD

Leading Twist Operators in QCD

Local Operators

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity ±

$$\xrightarrow{\lim\limits_{r\to\infty} r^2 \int_0^\infty dt} \qquad \vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \boxed{\begin{matrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \end{matrix}} \\ \boxed{\begin{matrix} \mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n}) \end{matrix}} \end{bmatrix}$$

unpolarized

polarized

Mode Expansion

$$\mathbb{O}_q^{[J]}(\vec{n}) = \sum_s \int \frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\ E^{J-1}(b^\dagger_{\vec{p},s}b_{\vec{p},s} + d^\dagger_{\vec{p},s}d_{\vec{p},s})$$

$$\mathbb{O}_g^{[J]}(\vec{n}) = \sum_{\lambda,c} \int \frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\ E^{J-1}\ a^\dagger_{\vec{p},\lambda,c}a_{\vec{p},\lambda,c}$$

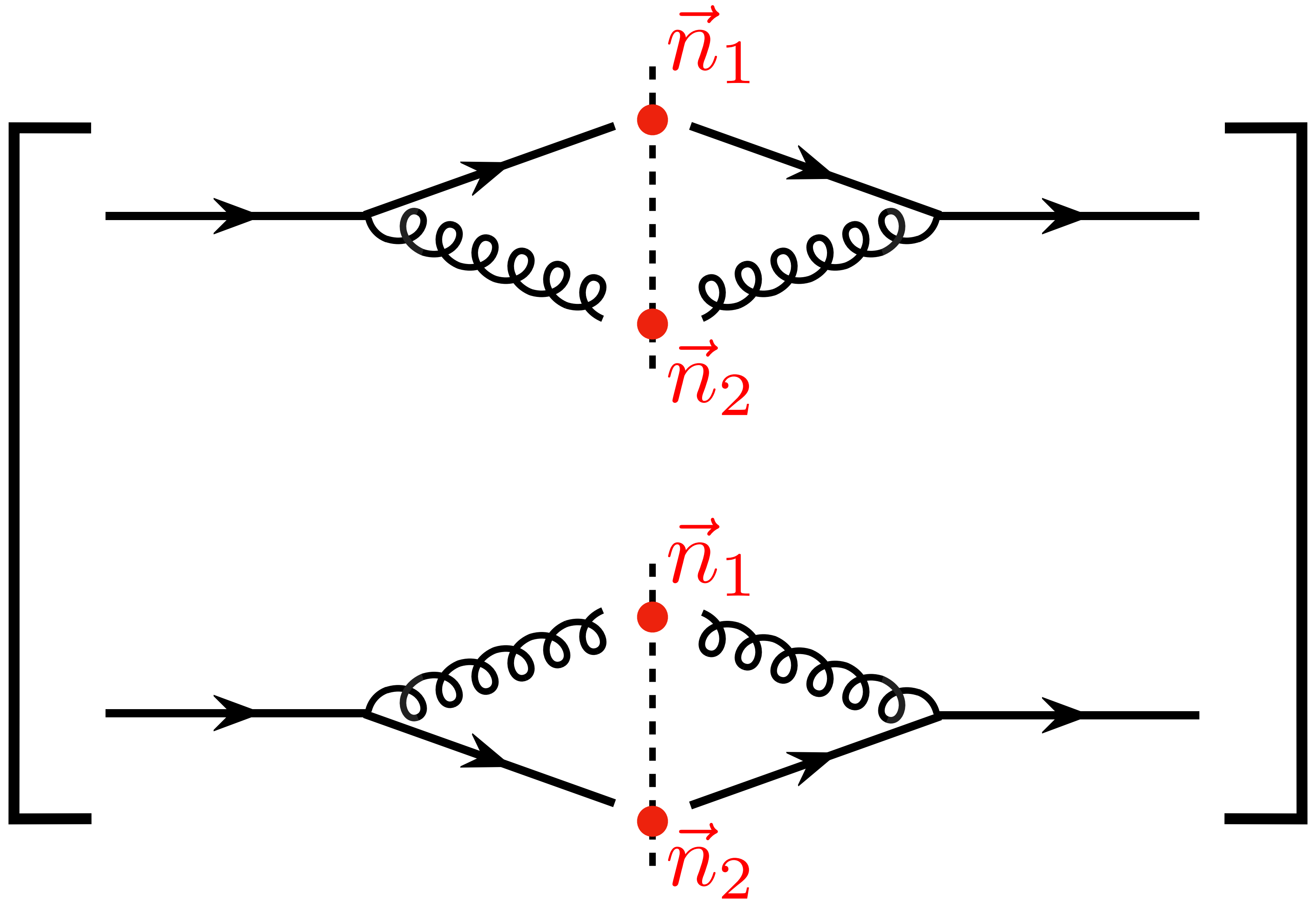
$$\mathbb{O}_{\tilde{g},\lambda}^{[J]}(\vec{n}) = -\sum_c \int \frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\ E^{J-1}\ a^\dagger_{\vec{p},\lambda,c}a_{\vec{p},-\lambda,c}$$

$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$ OPE

in quark state

$$\langle\Omega|\psi(x)\ \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ \bar{\psi}(0)|\Omega\rangle$$

$$= \int \frac{E_1^2 dE_1}{(2\pi)^3 2E_1}\frac{E_2^2 dE_2}{(2\pi)^3 2E_2}\ E_1 E_2\ e^{-i(p_1+p_2)\cdot x}$$



Wilson coefficient

$$\xrightarrow{\theta\to 0}\quad -\frac{1}{2\pi}\frac{2}{\theta^2}\boxed{[(\gamma_{qq}(2)-\gamma_{qq}(3))+(\gamma_{gq}(2)-\gamma_{gq}(3))]}\ \boxed{\langle\Omega|\psi(x)\ \mathbb{O}_q^{[3]}(\vec{n}_2)\bar{\psi}(0)|\Omega\rangle}$$

related to twist-2 Anom. Dim.

$$\int\frac{d^3p}{(2\pi)^3 2E}\delta^{(2)}(\hat{p}-\vec{n})\ \not{n}\ E^3\ e^{-ip\cdot x}$$

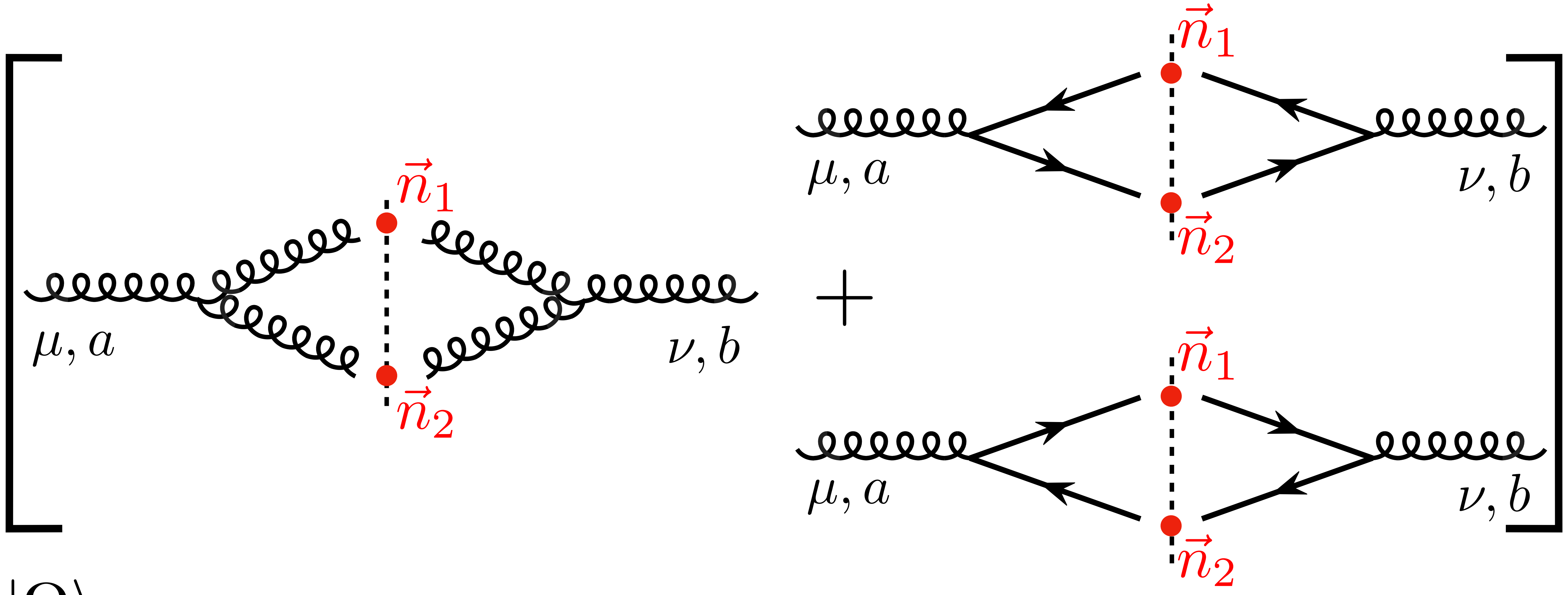
Confirms the general analysis: (1) spin-3 operator, (2) correct scaling behavior

$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$ **OPE**

in gluon state

$$\langle\Omega|A_b^\nu(x)\;\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\;A_a^\mu(0)|\Omega\rangle$$

$$=\int\frac{E_1^2dE_1}{(2\pi)^3 2E_1}\frac{E_2^2dE_2}{(2\pi)^3 2E_2}\;E_1E_2\;e^{-i(p_1+p_2)\cdot x}$$



$$\xrightarrow{\theta\to 0}\quad -\frac{1}{2\pi}\frac{2}{\theta^2}\Big[c_g\,\langle\Omega|A_b^\nu(x)\;\mathbb{O}_g^{[3]}\;A_a^\mu(0)|\Omega\rangle$$

$$+c_{\tilde{g}}\Big(e^{2i\phi}\,\langle\Omega|A_b^\nu(x)\;\mathbb{O}_{\tilde{g},-}^{[3]}\;A_a^\mu(0)|\Omega\rangle+e^{-2i\phi}\,\langle\Omega|A_b^\nu(x)\;\mathbb{O}_{\tilde{g},+}^{[3]}\;A_a^\mu(0)|\Omega\rangle\Big)\Big]$$

Wilson coefficients

unpolarized $c_g=(\gamma_{gg}(2)-\gamma_{gg}(3))+2n_f\,(\gamma_{qg}(2)-\gamma_{qg}(3))$

polarized $c_{\tilde{g}}=(\gamma_{g\tilde{g}}(2)-\gamma_{g\tilde{g}}(3))+2n_f\,(\gamma_{q\tilde{g}}(2)-\gamma_{q\tilde{g}}(3))$

$\vec{\mathbb{O}}^{[J]}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$ OPE

Introduce the Wilson coefficient matrix

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

OPE leading contribution

$$\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}$$

Energy flow operator corresponds to $J = 2$, and doesn't distinguish quark and gluon.

$$\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$$

$\mathcal{E}\mathcal{E}$ OPE

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\vec{\mathcal{J}}\left[\widehat{C}_\phi(2) - \widehat{C}_\phi(3)\right]\vec{\mathbb{O}}^{[3]}(\hat{n}_1) + \text{higher twist}$$

$$\vec{\mathcal{J}} = (1, 1, 0, 0)$$

3-point Energy Correlator

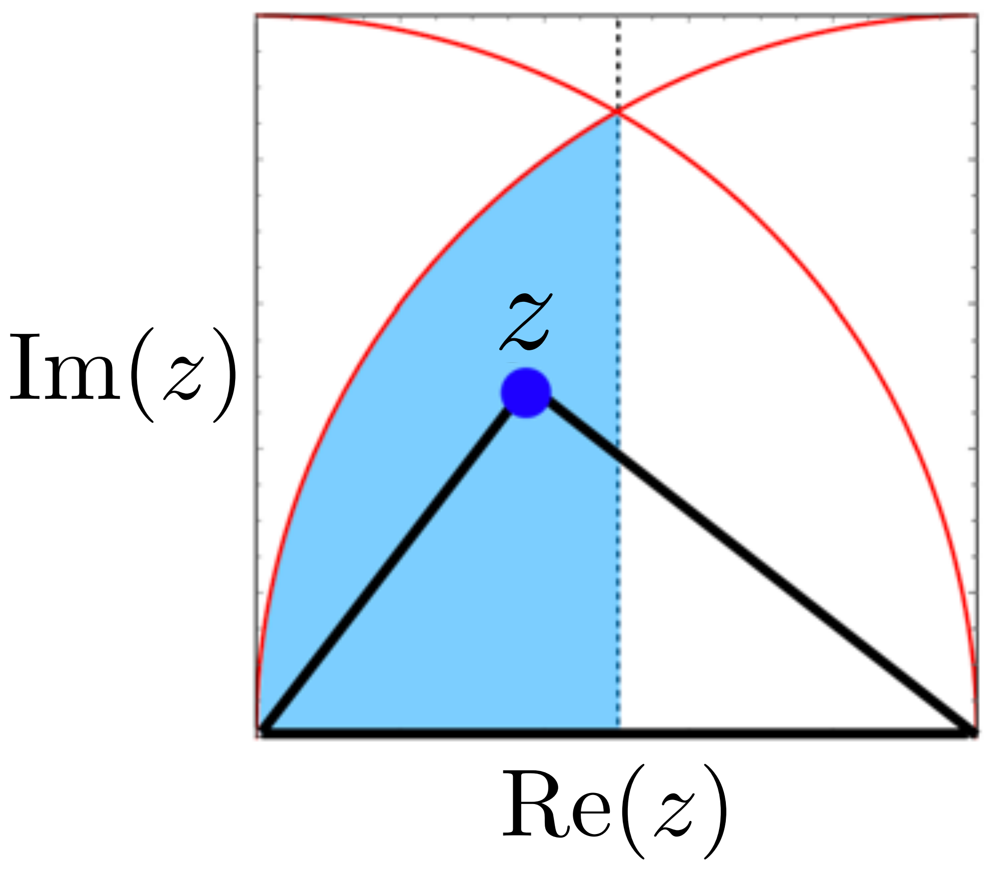
in the collinear limit

Kinematics

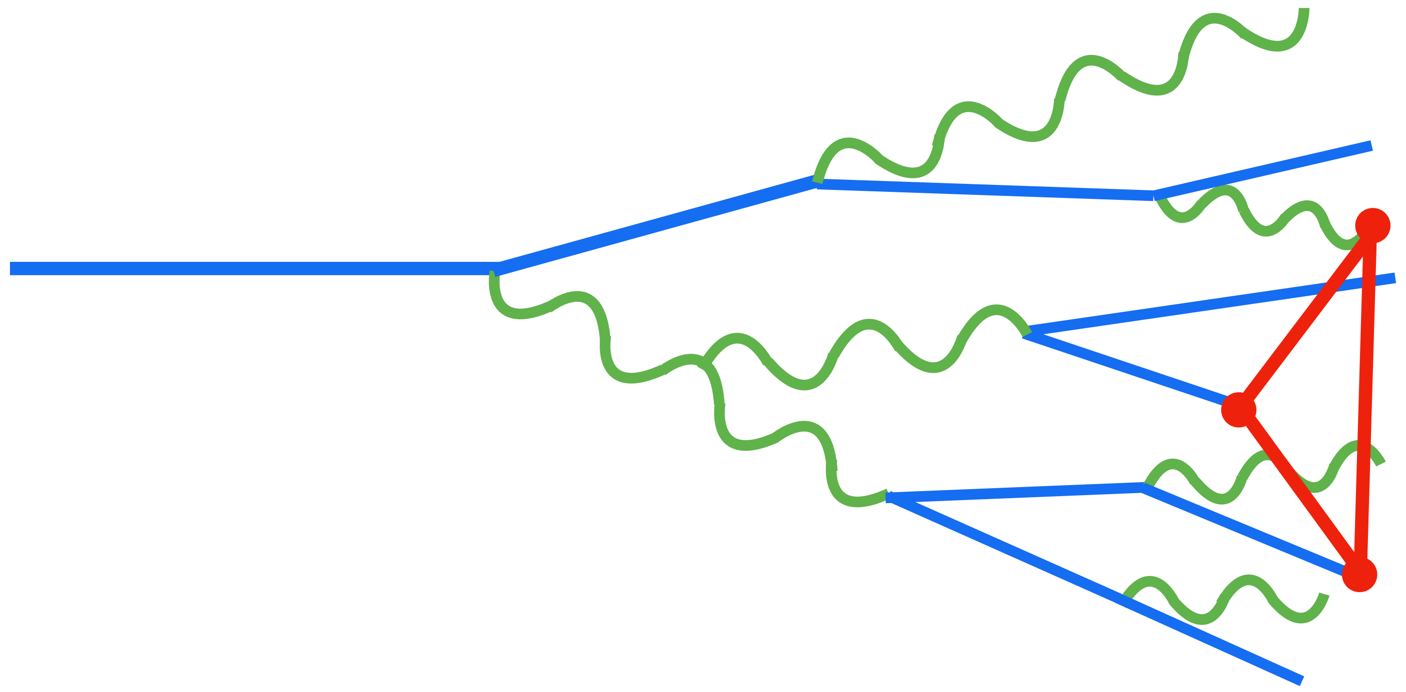
In the collinear limit, EEEC configuration can be approximated by a triangle.

[HC, Luo, Moult, Yang, Zhang, Zhu, 2019]

Moduli space of triangle shape



Parameterized in terms of
(1) the longest side^2 x_L **[Size]**
(2) a complex number z **[Shape]**



$$\boxed{\frac{E_aE_bE_c}{(Q/2)^3}\,\delta\left(x_1-\frac{s_{ab}}{4E_aE_b}\right)\delta\left(x_2-\frac{s_{bc}}{4E_bE_c}\right)\delta\left(x_3-\frac{s_{ac}}{4E_aE_c}\right)}$$

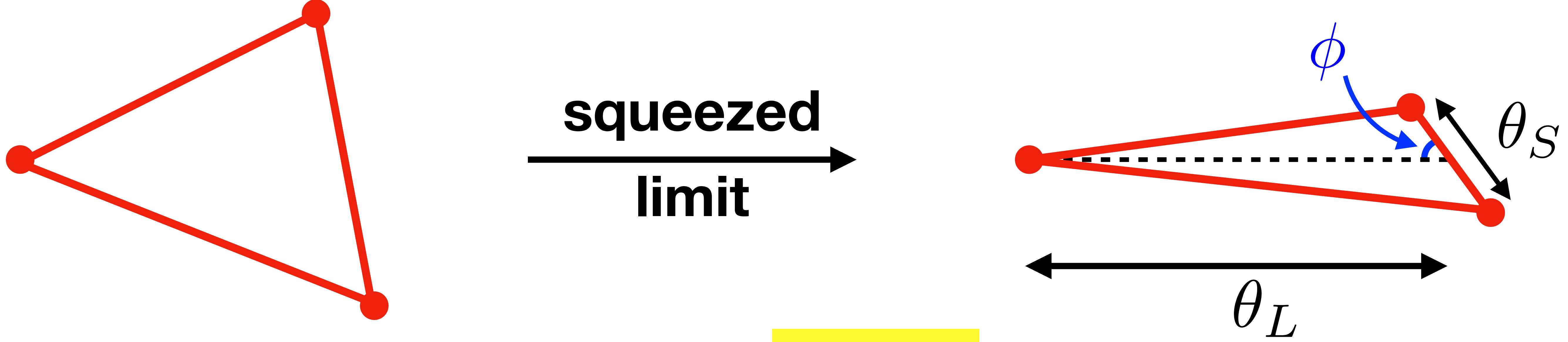
Definition

$$\frac{1}{\sigma_{\text{tot}}}\frac{d^3\Sigma_i}{dx_1dx_2dx_3}=\sum_{a,\,b,\,c}\int \overset{\text{collinear phase space}}{\boxed{d\Phi_c^{(3)}}}\,\underset{1\,\to\,3\text{ splitting probability}}{\boxed{P_{abc}^{(i)}}}\,\overset{\text{measurement}}{\boxed{\mathcal{M}_{\text{EEEC}}}}$$

x_1, x_2, x_3 **length^2 of each side ~ angle^2**

Squeezed Limit

Squeezed limit physically corresponds to bringing two detectors very close.



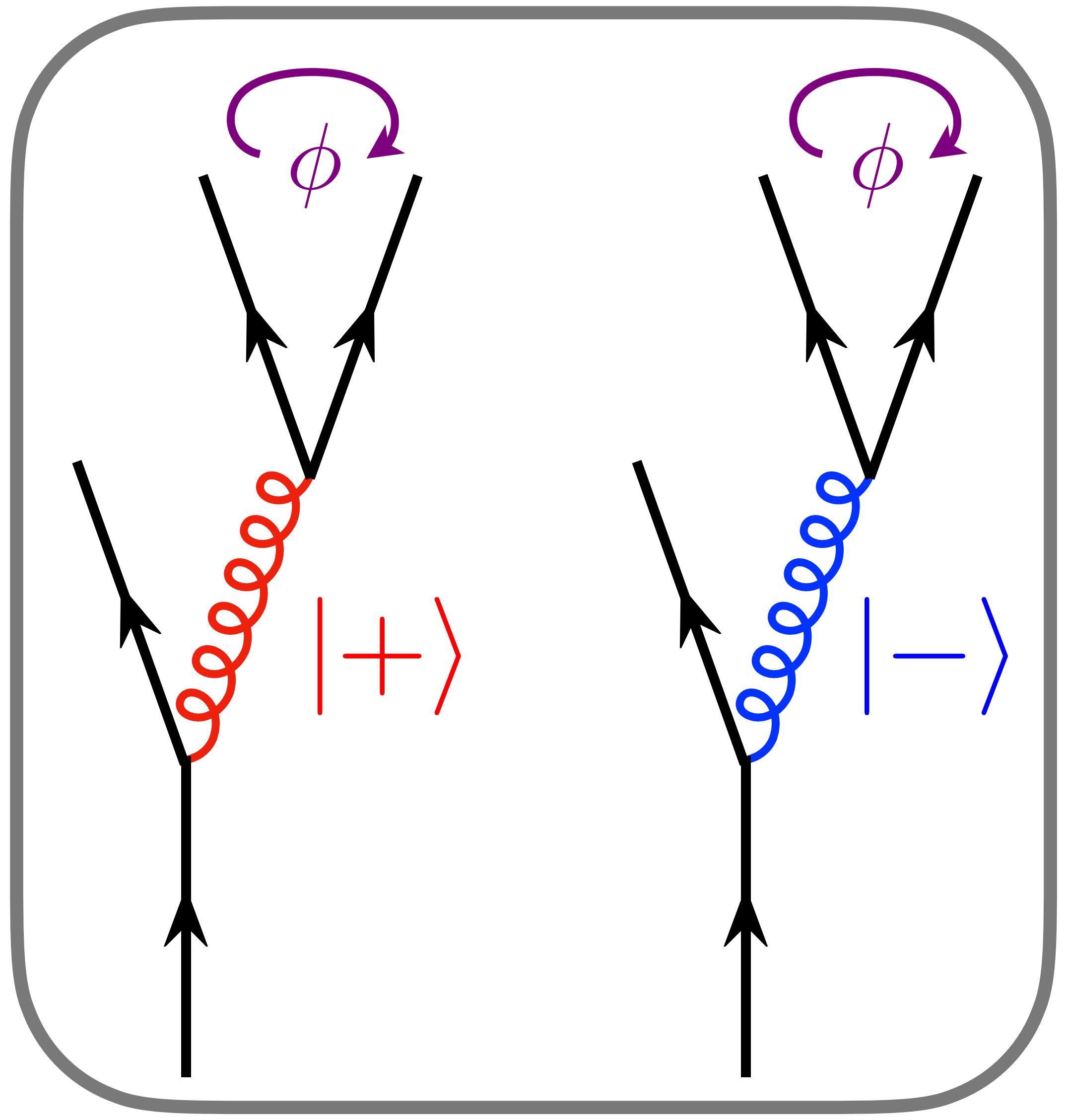
$$\frac{d^3\Sigma_i}{d\theta_L^2\, d\theta_S^2\, d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\text{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$

$$\text{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2 \frac{16}{5}$$

$$\text{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F \frac{3}{5}$$

Squeezed limit encodes spin correlation information and the Leading Power resummation is done. **[HC, Moult, Zhu, 2020]**

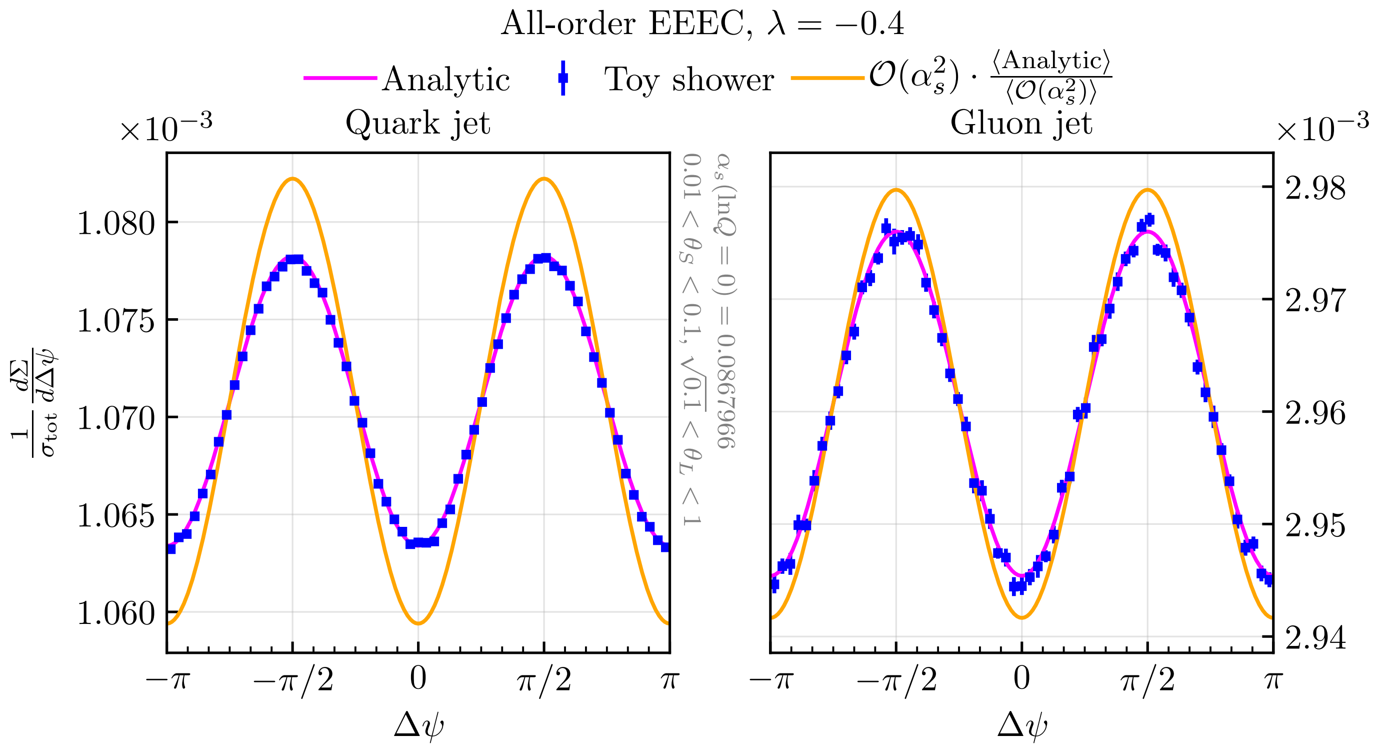
Interference Effect



Contributing Operator

$$F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

twist-2, transverse spin-2 gluonic operator

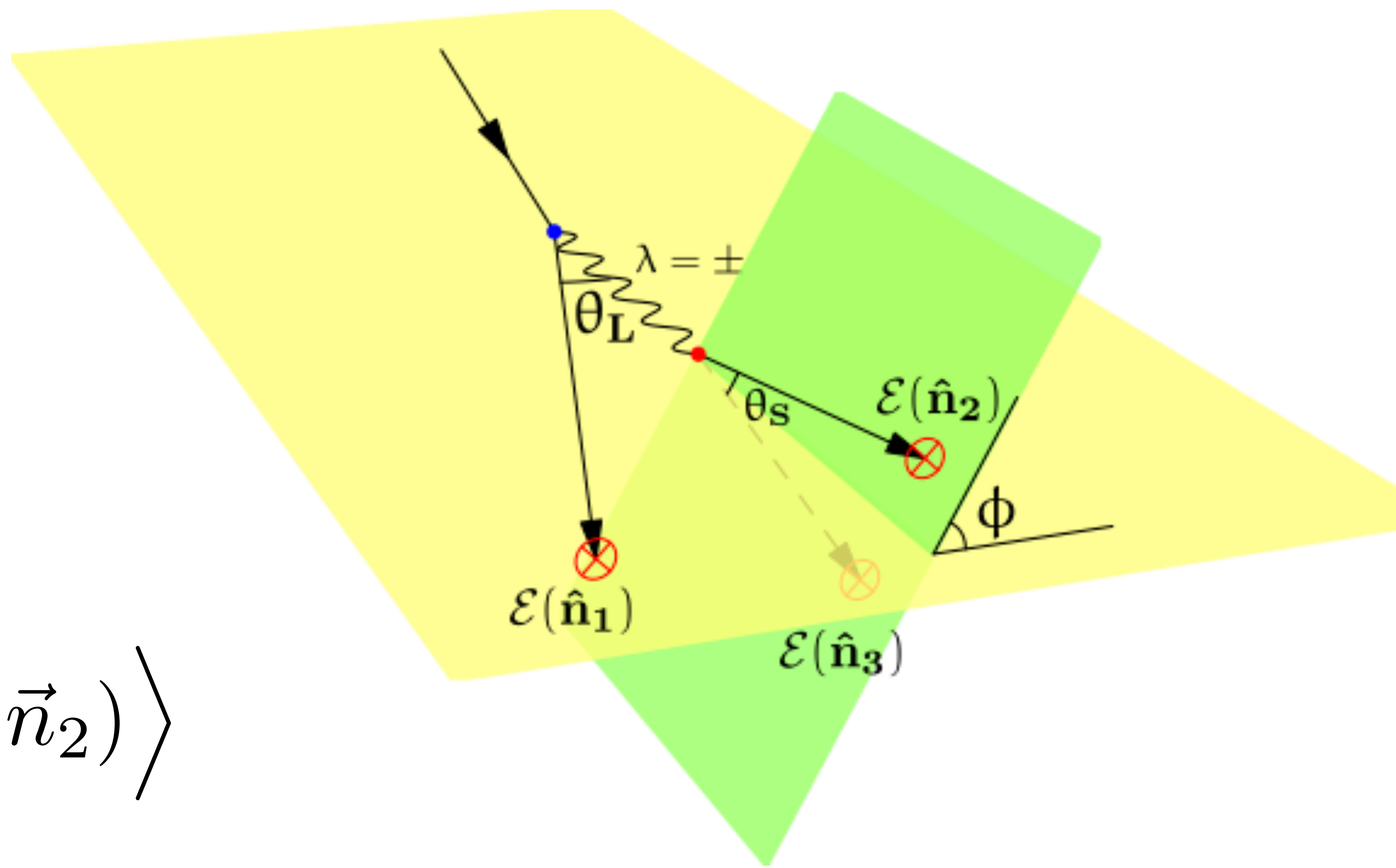


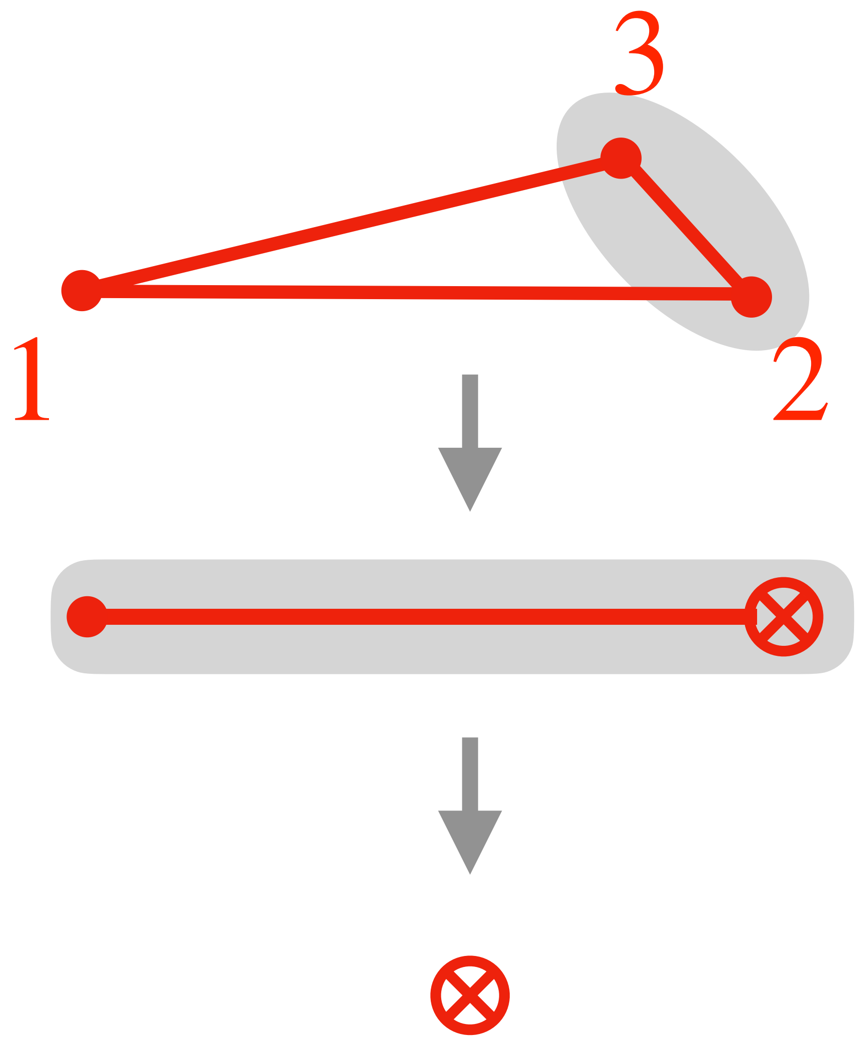
When collinear spin correlation is included in the **PanScales** family of parton showers, our resummed result provides validation of shower results.

[Karlberg, Salam, Scyboz, Verheyen, 2021]

Squeezed Limit

from light-ray OPE





$$\langle\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$

$$\longrightarrow -\frac{1}{2\pi}\frac{2}{\theta_S^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left\langle\mathcal{E}(\vec{n}_1)\vec{\mathbb{O}}^{[3]}(\vec{n}_2)\right\rangle$$

$$\longrightarrow \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left[\widehat{C}_{\phi_L}(3)-\widehat{C}_{\phi_L}(4)\right]\boxed{\left\langle\vec{\mathbb{O}}^{[4]}(\vec{n}_1)\right\rangle}$$

Hard function

This correctly reproduces the previous fixed order squeezed limit results.

Hierarchy $\theta_L \gg \theta_S$

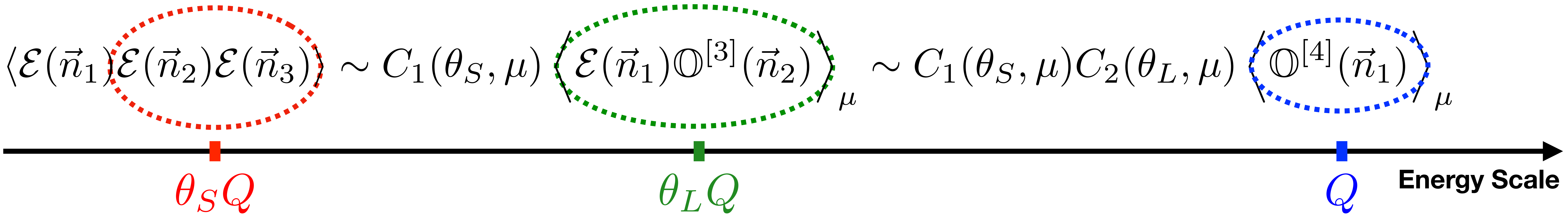
fixed order result needs to be resummed !

unpolarized jet $\left\langle\mathbb{O}^{[J]}_{\tilde{g},\pm}\right\rangle = 0$

quark jet $\left\langle\vec{\mathbb{O}}^{[4]}(\vec{n}_1)\right\rangle = (1,0,0,0)$

gluon jet $\left\langle\vec{\mathbb{O}}^{[4]}(\vec{n}_1)\right\rangle = (0,1,0,0)$

RG evolution



$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle \sim C_1(\theta_S,\mu)\left\langle \mathcal{E}(\vec{n}_1)\mathbb{O}^{[3]}(\vec{n}_2)\right\rangle_\mu \sim C_1(\theta_S,\mu)C_2(\theta_L,\mu)\left\langle \mathbb{O}^{[4]}(\vec{n}_1)\right\rangle_\mu$$

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g},ij}^{[J]} = -\frac{1}{2^J}F_a^{(i+}(iD^+)^{J-2}F_a^{j)+}$$

$$\left[\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}\right]$$

CANNOT MIX

RG equation:

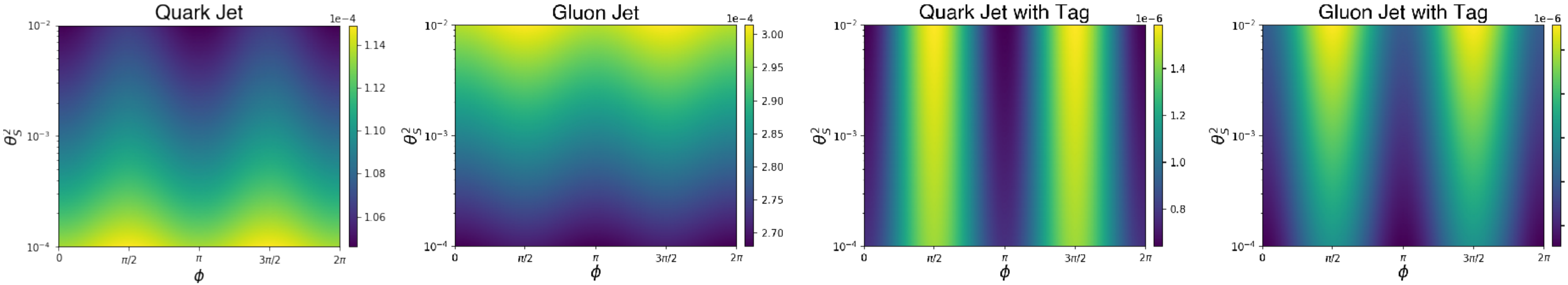
$$\frac{d}{d\ln\mu^2}\vec{\mathcal{O}}^{[J]} = -\widehat{\gamma}(J)\cdot\vec{\mathcal{O}}^{[J]}$$

$$\widehat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0 \\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1} \end{pmatrix}$$

LL Results for Squeezed Limit

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3)$$

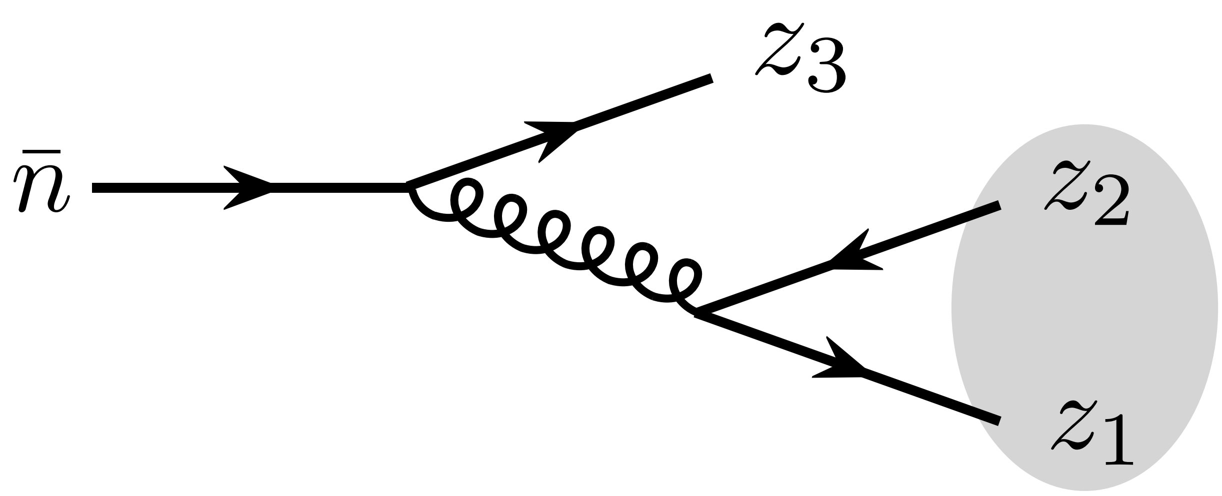
$$=\frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left[\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right]^{\frac{\widehat{\gamma}(3)}{\beta_0}}\left[\widehat{C}_{\phi_L}(3)-\widehat{C}_{\phi_L}(4)\right]\left[\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right]^{\frac{\widehat{\gamma}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1)$$



More Structures…

higher power expansion

For simplicity, tagging final state quarks



Squeezed limit: $z_1 \cdot z_2 \to 0$

Expanding the full result:

highest transverse spin series

[HC, Moult, Sandor, Zhu, forthcoming]

$$g(u,v) \equiv g(z,\bar{z}) \propto \underset{\cos 2\phi}{-z^3\bar{z}} + \frac{39}{10}z^2\bar{z}^2 - z\bar{z}^3 \qquad \textbf{LP}$$

$$\underset{\cos 3\phi}{-z^4\bar{z}} + \frac{39}{20}z^3\bar{z}^2 + \frac{39}{20}z^2\bar{z}^3 - z\bar{z}^4 \qquad \textbf{NLP}$$

$$\underset{\cos 4\phi}{-\frac{6}{7}z^5\bar{z}} + \frac{229}{140}z^4\bar{z}^2 - \frac{211}{140}z^3\bar{z}^3 + \frac{229}{140}z^2\bar{z}^4 - \frac{6}{7}z\bar{z}^5 \qquad \textbf{NNLP}$$

$$\underset{\cos 5\phi}{-\frac{5}{7}z^6\bar{z}} + \frac{207}{140}z^5\bar{z}^2 - \frac{233}{140}z^4\bar{z}^3 - \frac{233}{140}z^3\bar{z}^4 + \frac{207}{140}z^2\bar{z}^5 - \frac{5}{7}z\bar{z}^6 \qquad \textbf{NNNLP}$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$-z^3\bar{z}\ {}_2F_1\left(3,2,6,z\right)$$

$$\downarrow$$

Block Structure

Conformal Symmetry on the Celestial Sphere

Summary

- Light-ray operators play an important role in collider physics.
- Light-ray OPE, organized as twist expansion, governs the small angle scaling behavior.
- As an application in QCD, light-ray OPE correctly predicts the perturbative 3-point energy correlator in the squeezed limit and nicely organizes the RG.

Thanks!