Gluonic Tetracharm Configuration of X(6900)

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Dec 13, 2020

Bing-Dong Wan and Cong-Feng Qiao,

arXiv:2012.00454

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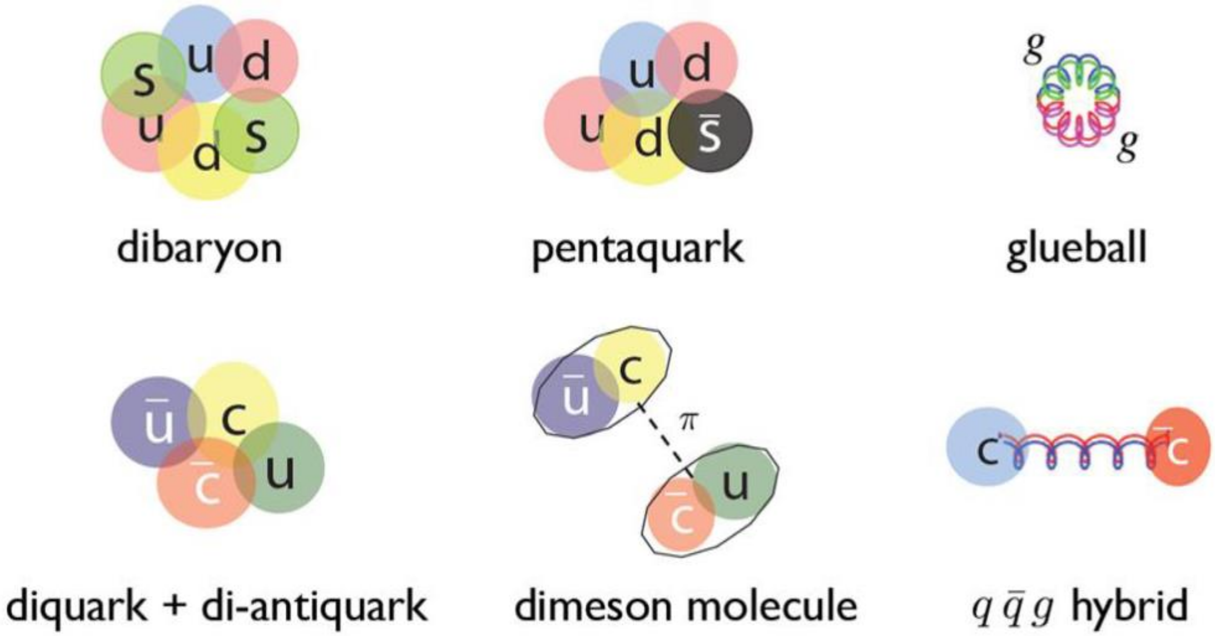
I. Exotic hadron studies

- ➢Conventional quark model
- ➢Exotic hadron states (multiquark, hybrid, glueball)



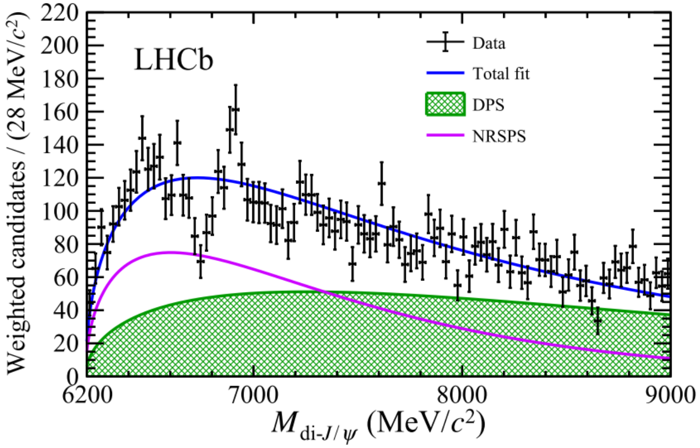
I. Exotic hadron studies

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	References
$X(3872)$	3871.69±0.17	<1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [10,32], BaBar [36],
					LHCb [34,73]
				$p\bar{p} \to (\pi^+\pi^- J/\psi) + \ldots$	CDF [31,74,75], D0 [76]
				$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	BES III [77]
				$B \to K(\omega J/\psi)$	Belle [78], BaBar [33]
				$B \to K(D^{*0}\bar{D}^0)$	Belle [38,79], BaBar [37]
				$B \to K(\gamma J/\psi)$ and	Belle [29], BaBar [30],
				$B \to K(\gamma\psi(2S))$	LHCb [40]
$Z_c(3900)^+$	3888.7 ± 3.4	35±7	1^+	$e^+e^- \to (J/\psi\,\pi^+)\pi^-$	Belle [43], BES III [55]
				$e^+e^- \to (D\bar{D}^*)^+\pi^-$	BES III [56]
$X(3915)$	3915.6 ± 3.1	28±10	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [80], BaBar [33]
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [81], BaBar [82]
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi(DD^*)$	Belle [83]
				$e^+e^- \to J/\psi\,(\ldots)$	Belle [84]
$Y(4008)$	3891 ± 42	255±42	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	Belle [42,43]
$Z_c(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B \to K(\pi^+\chi_{c1}(1P))$	Belle [53], BaBar [54]
$X(4050)^+$	4054 ± 3	45	?	$e^+e^- \to (\pi^+\psi(2S))\pi^-$	Belle [85]
$Y(4140)$	4143.4 ± 3.0	15^{+11}_{-7}	$?^{?+}$	$B \to K(\phi J/\psi)$	CDF [72],D0 [86]
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$	Belle [83]
$Z_c(4200)^+$	4196^{+35}_{-32}	370^{+99}_{-149}	?	$B \to K(\pi^+ J/\psi)$	Belle [87]
$Z_c(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	?	$B \to K(\pi^+\chi_{c1}(1P))$	Belle [53], BaBar [54]
$Y(4260)$	4263 ± 5	108±14	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	BaBar [41,88], CLEO [89],
					Belle [42,43]
				$e^+e^- \to (\pi^+\pi^- J/\psi)$	CLEO [47], BES III [56]
				$e^+e^- \to (\pi^0\pi^0 J/\psi)$	CLEO [47]
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$?^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [90]
$Y(4360)$	4361 ± 13	74±18	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^-\psi(2S))$	BaBar [44], Belle [45,85]
$Z_c(4430)^+$	4485^{+36}_{-25}	200^{+49}_{-58}	1^+	$B \to K(\pi^+\psi(2S))$	Belle [49,51,52],
					BaBar [50], LHCb [21]
				$B \to K(\pi^+ J/\psi)$	Belle [87], BaBar [50]
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to \gamma(\Lambda_c^+\Lambda_c^-)$	Belle [91]
$Y(4660)$	4664±12	48±15	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^-\psi(2S))$	Belle [45]
$Z_b(10610)^+$	10607.2±2.0	18.4±2.4	1^+	$e^+e^- \to (b\bar{b}\,\pi^+)\pi^-$	Belle [20]
$Z_b(10610)^0$	10609±4±4	NA	1^{+-}	$e^+e^- \to (\Upsilon(2,3S)\pi^0)\pi^0$	Belle [23]
$Z_b(10650)^+$	10652.2±1.5	11.5±2.2	1^+	$e^+e^- \to (b\bar{b}\,\pi^+)\pi^-$	Belle [20]
$Y_b(10888)$	10888.4±3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [60,62]

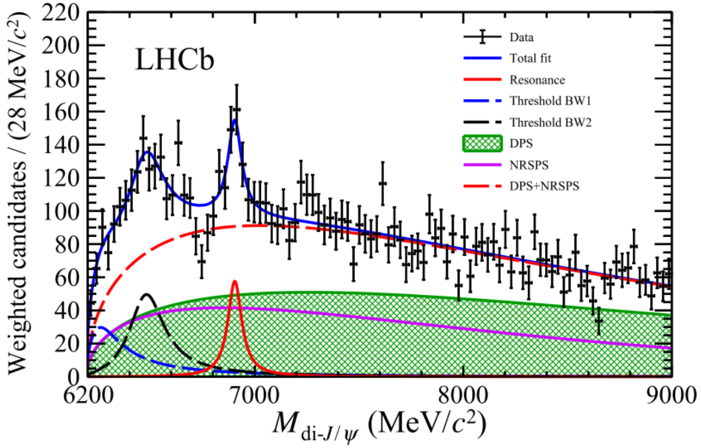
A. Hosaka, et al.,
PTEP (2016) 062C01

I. Exotic hadron studies

(a)



(b)



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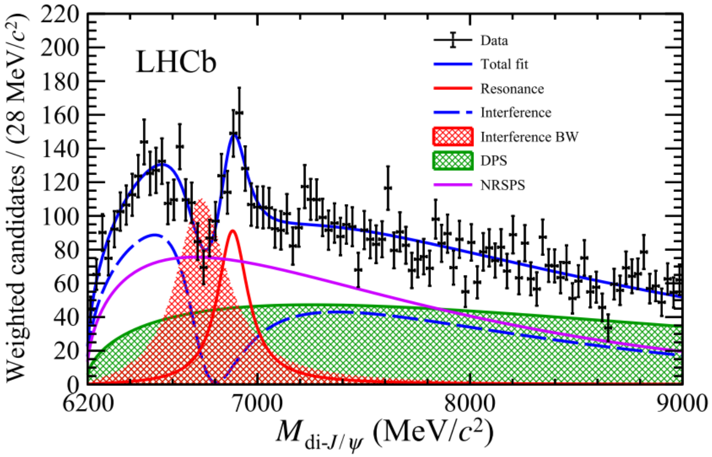


R. Aaij *et al.*, Sci. Bull. **65**, 1983 (2020)

I. Exotic hadron studies

Model I:

$$\begin{cases} m[X(6900)] = 6905 \pm 11 \pm 7 \mathrm{Me}V/c^2 \\ \Gamma[X(6900)] = 80 \pm 19 \pm 33 \mathrm{Me}V \end{cases}$$

Model II:

$$\begin{cases} m[X(6900)] = 6886 \pm 11 \pm 11 \mathrm{Me}V/c^2 \\ \Gamma[X(6900)] = 168 \pm 33 \pm 69 \mathrm{Me}V \end{cases}$$

- Consisting of four charm quarks
- Without soft photon
  - ✓ Exclude exotic quantum number
  - ✓ Do not contain light quarks

I. Exotic hadron studies

TABLE XI. The mass spectra (in units of GeV) of the tetraquark states $cc\bar{c}\,\bar{c}$, $bb\bar{b}\,\bar{b}$, and $bb\bar{c}\,\bar{c}$ in different frameworks. The $M^1_{\rm th}$ and $M^2_{\rm th}$ are the numerical results from the quark models I and II in this work, respectively.

	J^{PC}	$M^1_{\rm th}$	$M^2_{\rm th}$	[43]	[44]	[47]	[34]	[33]	[41]	[49]	[37,57]
$cc\bar{c}\,\bar{c}$	0^{++}	6.377	6.371	5.966	6.192 ± 0.025	6.001	⋯	⋯	6.038	6.470	6.44 ± 0.15
		6.425	6.483							6.558	
	1^{+-}	6.425	6.450	6.051	⋯	6.109	⋯	⋯	6.101	6.512	6.37 ± 0.18
	2^{++}	6.432	6.479	6.223	⋯	6.166	⋯	⋯	6.172	6.534	6.37 ± 0.19
$bb\bar{b}\,\bar{b}$	0^{++}	19.215	19.243	18.754	18.826 ± 0.025	18.815	18.72 ± 0.02	18.69 ± 0.03	⋯	19.268	18.45 ± 0.15
		19.247	19.305							19.305	
	1^{+-}	19.247	19.311	18.808	⋯	18.874	⋯	⋯	⋯	19.285	18.32 ± 0.17
	2^{++}	19.249	19.325	18.916	⋯	18.905	⋯	⋯	⋯	19.295	18.32 ± 0.17
$bb\bar{c}\,\bar{c}(cc\bar{b}\,\bar{b})$	0^{++}	12.847	12.886	⋯	⋯	12.571	⋯	⋯	⋯	12.935	⋯
		12.866	12.946							13.023	
	1^{+-}	12.864	12.924	⋯	⋯	12.638	⋯	⋯	⋯	12.945	⋯
	2^{++}	12.868	12.940	⋯	⋯	12.673	⋯	⋯	⋯	12.956	⋯

G.-J. Wang, L. Meng, and S.-L. Zhu, PRD **100**, 096013 (2019)

I. Exotic hadron studies

TABLE VI. Predicted mass spectra for the $cc\bar c\bar c$, $bb\bar b\bar b$ and $bb\bar c\bar c$ systems.

$J^{P(C)}$	Configuration	$\langle H\rangle$ (MeV)	Mass (MeV)	Eigenvector
0^{++}	$\vert\{cc\}^6_0\{\bar c\bar c\}^{\bar 6}_0\rangle^0_0$	6518 -0.2371	6518	$(1,0)$
	$\vert\{cc\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_0$	-0.2371 6487	6487	$(0,1)$
1^{+-}	$\vert\{cc\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_1$	(6500)	6500	1
2^{++}	$\vert\{cc\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_2$	(6524)	6524	1
0^{++}	$\vert\{bb\}^6_0\{\bar b\bar b\}^{\bar 6}_0\rangle^0_0$	19338 -0.1102	19338	$(1,0)$
	$\vert\{bb\}^{\bar 3}_1\{\bar b\bar b\}^{3}_1\rangle^0_0$	-0.1102 19322	19322	$(0,1)$
1^{+-}	$\vert\{bb\}^{\bar 3}_1\{\bar b\bar b\}^{3}_1\rangle^0_1$	(19329)	19329	1
2^{++}	$\vert\{bb\}^{\bar 3}_1\{\bar b\bar b\}^{3}_1\rangle^0_2$	(19341)	19341	1
0^{+}	$\vert\{bb\}^6_0\{\bar c\bar c\}^{\bar 6}_0\rangle^0_0$	13032 -0.1105	13032	$(1,0)$
	$\vert\{bb\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_0$	-0.1105 12953	12953	$(0,1)$
1^{+}	$\vert\{bb\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_1$	(12960)	12960	1
2^{+}	$\vert\{bb\}^{\bar 3}_1\{\bar c\bar c\}^{3}_1\rangle^0_2$	(12972)	12972	1

M.-S. Liu, Qi-Fang Lü, Xian-Hui Zhong, and Qiang Zhao, PRD **100**, 016006 (2019)

I. Exotic hadron studies

Table 1
Masses of the doubly hidden-charm $cc\bar{c}\bar{c}$ and doubly hidden-bottom $bb\bar{b}\bar{b}$ tetraquarks with various quantum numbers.

J^{PC}	Currents	m_{X_c} (GeV)	m_{X_b} (GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17

W. Chen *et al.*, PLB **773**, 247 (2017)

I. Exotic hadron studies

TABLE IV. Comparison of the values of the 0^{++} scalar tetraquark masses and couplings from different QSSR approaches. Our predictions are at LO (only the central value is quoted) and up to NLO of PT series where the errors come from Table II. The predictions of Ref. [77] are from Moments at LO and of Ref. [78] from LSR at LO. As already mentioned earlier, we notice that the choice of the numerical values of the $\overline{MS}$ running quark masses used at LO is not justified due to the ambiguous quark mass definition to that order. One may also have equally used a pole / on-shell mass which naturally appears in the expression of the spectral function evaluated using on-shell quark mass.

Scalar	$M_{\bar{c}\bar{c}cc}$ [GeV]					$M_{\bar{b}\bar{b}bb}$ [GeV]				
$\bar{q}\bar{q}qq$	LO	NLO	NLO $\oplus$ G3	LO [77]	LO [78]	LO	NLO	NLO $\oplus$ G3	LO [77]	LO [78]
Eq. (26)										
$\bar{S}_q S_q$	6.59	6.39 ± 0.08	6.41 ± 0.08	6.44 ± 0.15		19.51	19.13 ± 0.08	19.22 ± 0.12	18.45 ± 0.15	
$\bar{A}_q A_q$	6.52	6.49 ± 0.07	6.45 ± 0.08	6.46 ± 0.16	–	19.51	19.93 ± 0.15	19.87 ± 0.16	18.46 ± 0.14	–
$\bar{V}_q V_q$	6.55	6.61 ± 0.09	6.46 ± 0.18	6.59 ± 0.17		19.49	19.53 ± 0.07	19.49 ± 0.08	18.59 ± 0.17	
$\bar{P}_q P_q$	7.37	7.05 ± 0.07	6.80 ± 0.27	6.82 ± 0.18		19.96	19.78 ± 008	19.75 ± 0.08	19.64 ± 0.14	
Eq. (27)										
$\bar{A}_q A_q$	6.50	6.51 ± 0.06	6.47 ± 0.07		5.99 ± 0.08	19.49	19.75 ± 0.11	19.72 ± 0.12		18.84 ± 0.09

R. M. Albuquerque, et al., PRD 102 (2020) 094001

I. Exotic hadron studies

	$M_B^2(\mathrm{GeV}^2)$	$\sqrt{s_0}(\mathrm{GeV})$	PC	$R_i^{\langle GG\rangle}$	M_X (GeV)
0^{++} case A	$4.7-5.3$	7.0 ± 0.2	$(61-50)\%$	$(0.37-0.24)\%$	6.44 ± 0.11
0^{++} case B	$4.0-4.6$	7.3 ± 0.2	$(62-50)\%$	$(28.98-23.60)\%$	6.87 ± 0.10
0^{++} case C	$4.9-5.6$	7.1 ± 0.2	$(60-50)\%$	$(0.05-0.02)\%$	6.52 ± 0.11
0^{++} case D	$6.1-7.0$	7.7 ± 0.2	$(62-50)\%$	$(2.77-2.48)\%$	6.96 ± 0.11

TABLE I: The windows of the Borel parameter, continuum thresholds, pole contributions, two-gluon contributions, and predicted masses for $[c\bar{c}][c\bar{c}]$ tetraquark satates .

B.-C. Yang, L. Tang, and C.-F. Qiao, arXiv:2012.04463

I. Exotic hadron studies

- It was argued by Czarnecki that stable tetrons, such as a double charmonium $cc\bar{c}\bar{c}$ system, may not exist.

A. Czarnecki, B. Leng, and M.B. Voloshin, PLB **778**, 233 (2018)

- It is higher than double *J/ψ* threshold by about 700 MeV.

- X(6900) may not be stable as a tetracharm.

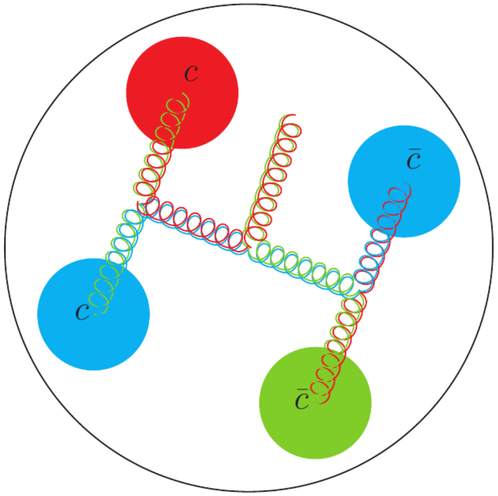
Contents:

- Exotic hadron studies
- **Hybrid explanation of X(6900)**
- Double hidden-charm hybrid via QCDSR
- Summary

II. Hybrid explanation of X(6900)

- Double hidden-charm hybrid

  - ✓ Constituent
    - ◆ Diquark
    - ◆ Antidiquark
    - ◆ Dynamic Gluon



  - ✓ The mass is heavier
  - ✓ Avoid what Czarnecki have argued
  - ✓ OZI suppress

II. Hybrid explanation of X(6900)

- Color structure

  - ✓ Quark= fundamental representation 3
  - ✓ Gluon= Adjoint representation 8
  - ✓ Observable particles=color singlet 1

    - ◆ Mesons $3 \otimes \bar{3} = 1 \oplus 8$
    - ◆ Diquark $3 \otimes 3 = \bar{3} \oplus 6$

II. Hybrid explanation of X(6900)

- Color structure

✓Molecular $1 \otimes 1 = 1$

✓Diquark-antidiquark
$$\begin{cases} 3 \otimes \bar{3} = 1 \oplus 8 \\ 6 \otimes \bar{6} = 1 \oplus 8 \end{cases}$$

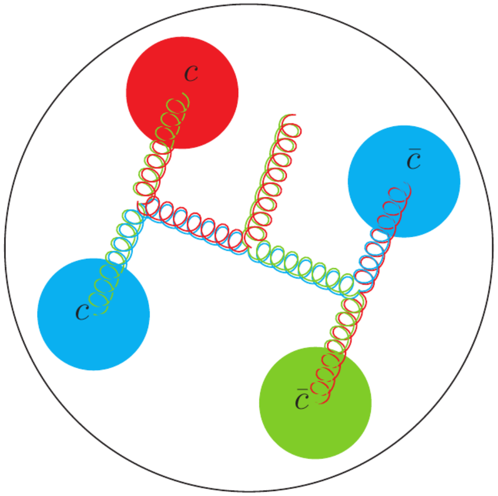
✓Hybrid $3 \otimes \bar{3} \otimes 8 = 1 \oplus \cdots$

II. Hybrid explanation of X(6900)

- Double hidden-charm hybrid

  ✓ Constituent

  - Diquark $\bar{3}_c$
  - Antidiquark 3_c
  - Dynamic Gluon 8_c



$$3 \otimes \bar{3} \otimes 8 = 1 \oplus \cdots$$

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- **Double hidden-charm hybrid via QCDSR**
- Summary

III. Double hidden-charm hybrid via QCDSR

- Interpolating currents for double hidden-charm hybrid

$$j^{A}_{0^{++}}(x) = g_s \epsilon_{ikl}\epsilon_{jmn}[c_k^T C\gamma_\mu c_l]\frac{\lambda^a_{ij}}{2}G^a_{\mu\nu}[\bar{c}_m\gamma_\nu C\bar{c}_n^T]$$

$$j^{B}_{0^{++}}(x) = g_s \epsilon_{ikl}\epsilon_{jmn}[c_k^T C\gamma_\mu\gamma_5 c_l]\frac{\lambda^a_{ij}}{2}G^a_{\mu\nu}[\bar{c}_m\gamma_\nu\gamma_5 C\bar{c}_n^T]$$

$$j^{A}_{0^{-+}}(x) = g_s \epsilon_{ikl}\epsilon_{jmn}[c_k^T C\gamma_\mu c_l]\frac{\lambda^a_{ij}}{2}\tilde{G}^a_{\mu\nu}[\bar{c}_m\gamma_\nu C\bar{c}_n^T]$$

$$j^{B}_{0^{-+}}(x) = g_s \epsilon_{ikl}\epsilon_{jmn}[c_k^T C\gamma_\mu\gamma_5 c_l]\frac{\lambda^a_{ij}}{2}\tilde{G}^a_{\mu\nu}[\bar{c}_m\gamma_\nu\gamma_5 C\bar{c}_n^T]$$

III. Double hidden-charm hybrid via QCDSR

- The two-point correlation function

$$\Pi^k_{JPC}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j^k_{JPC}(x), j^k_{JPC}(0)^\dagger\}|0\rangle$$

- The OPE side of the correlation function

$$\Pi^{k,\,OPE}_{JPC}(q^2) = \int_{s_{min}}^{\infty} ds \frac{\rho^{k,\,OPE}_{JPC}(s)}{s-q^2}$$

$$\rho^{OPE}(s) = \rho^{pert}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{\langle G^3\rangle}(s)$$

- The phenomenological side of the correlation function

$$\Pi^{k,\,phen}_{JPC}(q^2) = \frac{(\lambda^k_{JPC})^2}{(m^k_{JPC})^2 - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\rho^k_{JPC}(s)}{s-q^2}$$

III. Double hidden-charm hybrid via QCDSR

- The Borel transformation

$$\hat{B}_{M_B^2} \equiv \lim_{\substack{Q^2\to\infty, n\to\infty \\ Q^2/n=M_B^2}} \frac{(-Q^2)^n}{(n-1)!} \left(\frac{d}{dQ^2}\right)^n$$

- The quark-hadron duality approximation

$$\int_{s_0}^{\infty} ds \rho_{J^{PC}}^{k,\,OPE}(s) e^{-s/M_B^2} \simeq \int_{s_0}^{\infty} ds \rho_{J^{PC}}^{k,\,phen}(s) e^{-s/M_B^2}$$

- The mass function

$$m_{J^{PC}}^{k}(s_0, M_B^2) = \sqrt{-\frac{L_{J^{PC},\,1}^{k}(s_0, M_B^2)}{L_{J^{PC},\,0}^{k}(s_0, M_B^2)}}$$

III. Double hidden-charm hybrid via QCDSR

- The moments

$$L^{k}_{J^{PC},\,0}(s_0, M_B^2) = \int_{s_{min}}^{s_0} ds\ \rho^{k,\,OPE}_{J^{PC}}(s) e^{-s/M_B^2}$$

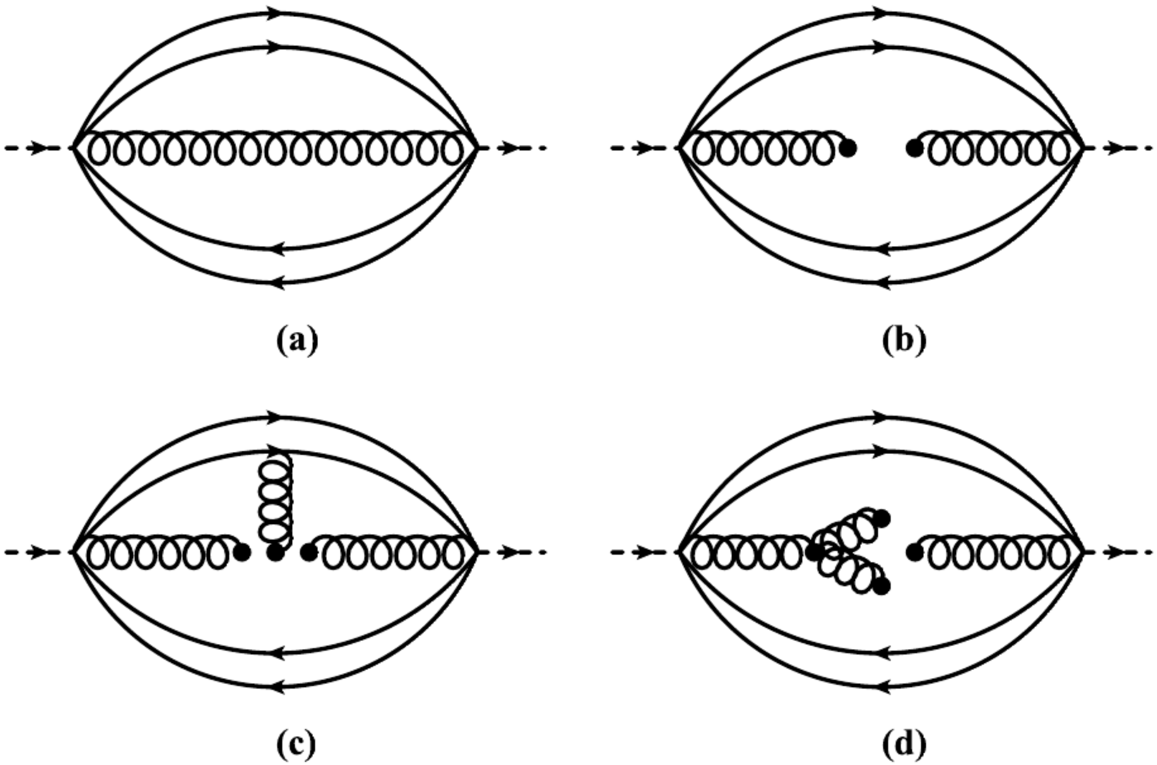
$$L^{k}_{J^{PC},\,1}(s_0, M_B^2) = \frac{\partial}{\partial \frac{1}{M_B^2}} L^{k,\,OPE}_{J^{PC}}(s_0, M_B^2)$$

- Ratios to constrain the windows of

$$R^{k,\,OPE}_{J^{PC}} = \left| \frac{L^{k,\,\langle g_s^3 G^3\rangle}_{J^{PC},\,0}(s_0, M_B^2)}{L^{k}_{J^{PC},\,0}(s_0, M_B^2)} \right| \quad , \quad R^{k,\,PC}_{J^{PC}} = \frac{L^{k}_{J^{PC},\,0}(s_0, M_B^2)}{L^{k}_{J^{PC},\,0}(\infty, M_B^2)}$$

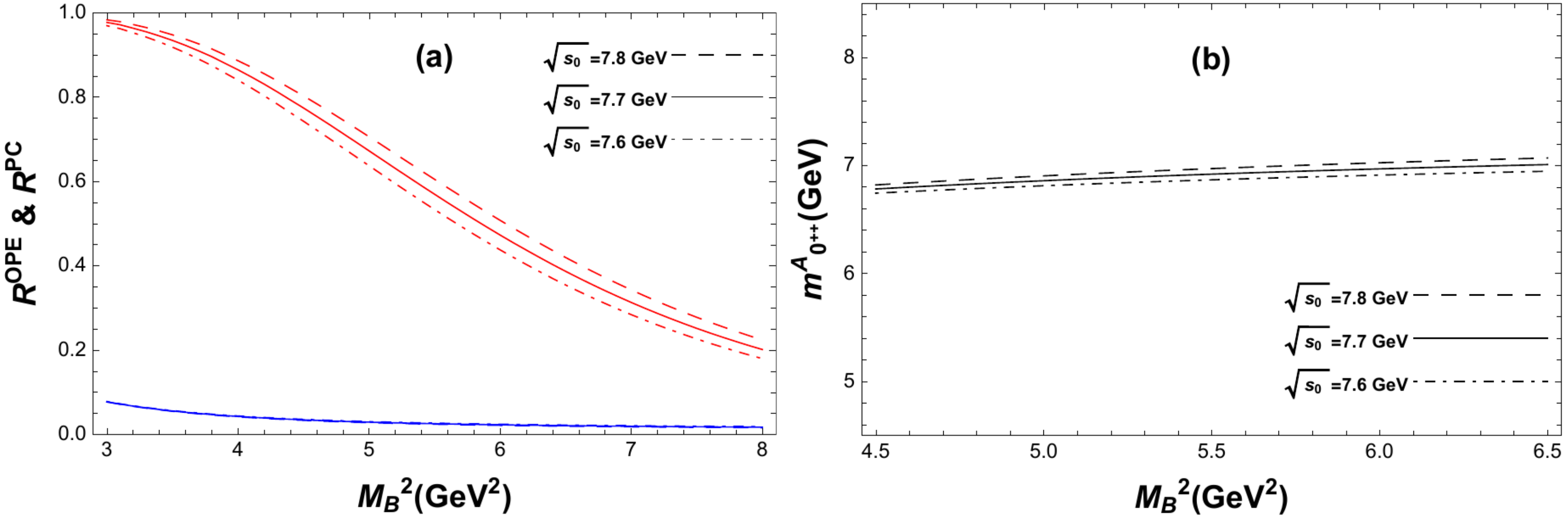
III. Double hidden-charm hybrid via QCDSR

- Typical Feynman diagrams of double hidden-charm hybrid



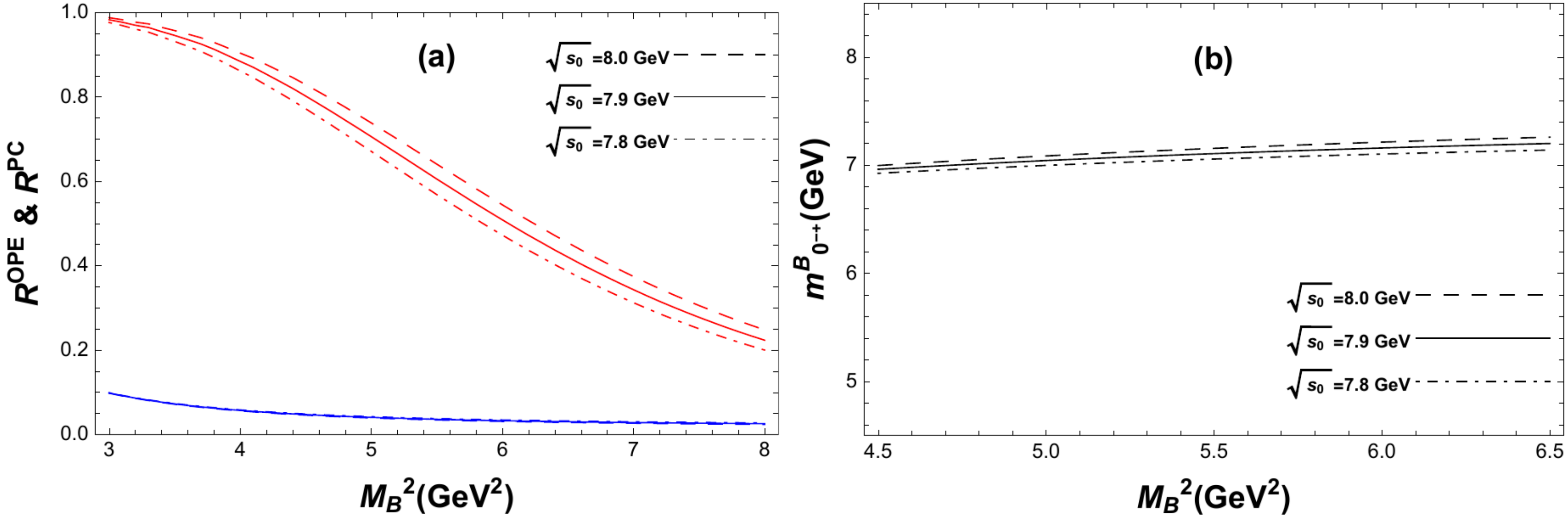
III. Double hidden-charm hybrid via QCDSR

0^{++} double hidden-charm hybrid with current A



III. Double hidden-charm hybrid via QCDSR

0^{-+} double hidden-charm hybrid with current B



III. Double hidden-charm hybrid via QCDSR

$$m^{A}_{0^{++}} = (6.92 \pm 0.14)\,\text{GeV} \quad , \quad m^{B}_{0^{-+}} = (7.10 \pm 0.12)\,\text{GeV} \; .$$

$$m^{A,\,b}_{0^{++}} = (19.30 \pm 0.23)\,\text{GeV} \quad , \quad m^{B,\,b}_{0^{-+}} = (19.46 \pm 0.20)\,\text{GeV} \; .$$

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IV. Summary

- A novel double hidden-charm hybrid configuration, i.e. $[3_c]_{cc} \otimes [8_c]_G \otimes [3_c]_{\bar{c}\bar{c}}$, is proposed to interpret the hadronic structure X(6900) recently observed in LHCb experiment.
- We obtain two stable double hidden-charm hybrid with masses about 6.92 GeV and 7.10 GeV with $J^{PC}=0^{++}$ and 0^{-+}, respectively.
- Their b-sector partners are also evaluated with masses (19.30 ± 0.23) and (19.46 ± 0.20) GeV for 0^{++} and 0^{-+}, respectively.

Thank you for your attention!

Back up

➢QCDSR

CFQ & Liang Tang, PRL113 (2014) 221601

CFQ & Liang Tang, NPB904 (2016) 282

CFQ & GH, PLB642 (2006) 53

- Field strength tensor

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(x) + g_s f^{abc} A^b_\mu(x) A^c_\nu(x)$$

- In coordinate gauge

$$A^a_\mu(x) \simeq \frac{1}{2} x^\nu G^a_{\nu\mu}(0); \qquad A^a_\mu(0) \simeq 0$$

$$G^a_{\mu\nu}(x) = G^a_{0\mu\nu}(0) + \frac{1}{4} g_s f^{abc} x^\rho x^\sigma G^b_{\rho\mu}(0) G^c_{\sigma\nu}(0)$$

$$G^a_{0\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x)$$

$$G^a_{\mu\nu}(0) = G^a_{0\mu\nu}(0) = G^a_{0\mu\nu}(x)$$

- Some contractions

$$\overbracket{G^a_{0\mu\nu}(x)G^i_{0\alpha\beta}}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\Gamma_{\mu\nu\alpha\beta}(p)e^{-ip\cdot(x-y)}$$

$$\Gamma_{\mu\nu\alpha\beta}(p) = p_\mu p_\alpha g_{\nu\beta} + p_\nu p_\beta g_{\mu\alpha} - p_\mu p_\beta g_{\nu\alpha} - p_\nu p_\alpha g_{\mu\beta}$$

$$\overbracket{A^m_\mu(x)G^j_{0\beta\gamma}}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{\delta^{mj}}{p_1^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$\overbracket{G^j_{0\beta\gamma}(x)A^m_\mu}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-\delta^{jm}}{p^2}(p_\beta g_{\mu\gamma} - p_\gamma g_{\mu\beta})e^{-ip\cdot(x-y)}$$

$$\overbracket{\widetilde{G}^a_{0\mu\nu}(x)\widetilde{G}^i_{0\rho\sigma}}(y) = \int \frac{d^4p}{(2\pi)^4}\frac{-i\delta^{ai}}{p^2}\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p)e^{-ip\cdot(x-y)}$$

$$\widetilde{\Gamma}_{\mu\nu\rho\sigma}(p) = p_\mu p_\rho g_{\nu\sigma} + p_\nu p_\sigma g_{\mu\rho} - p_\mu p_\sigma g_{\nu\rho} - p_\nu p_\rho g_{\mu\sigma} + p^2(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})$$

- Gluon condensates

$$\delta^{ab}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)|0\rangle = \frac{1}{D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$\delta^{ab}\langle 0|\widetilde{G}^a_{\mu\nu}(0)\widetilde{G}^b_{\rho\sigma}(0)|0\rangle = \frac{2-D}{2D(D-1)}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\langle GG\rangle$$

$$f^{abc}\langle 0|G^a_{\mu\nu}(0)G^b_{\rho\sigma}(0)G^c_{\alpha\beta}(0)|0\rangle = \frac{1}{D(D-1)(D-2)}T^3_{\mu\nu\rho\sigma\alpha\beta}\langle GGG\rangle$$

$$\begin{aligned} T^3_{\mu\nu\rho\sigma\alpha\beta} \quad = \quad & g_{\mu\rho}g_{\nu\alpha}g_{\sigma\beta} - g_{\mu\rho}g_{\nu\beta}g_{\sigma\alpha} - g_{\mu\sigma}g_{\nu\alpha}g_{\rho\beta} + g_{\mu\sigma}g_{\nu\beta}g_{\rho\alpha} \\ & - g_{\mu\alpha}g_{\nu\rho}g_{\sigma\beta} + g_{\mu\alpha}g_{\nu\sigma}g_{\rho\beta} + g_{\mu\beta}g_{\nu\rho}g_{\sigma\alpha} - g_{\mu\beta}g_{\nu\sigma}g_{\rho\alpha} \end{aligned}$$

- The spectral densities for 0^{++} double hidden-charm hybrid

$$
\begin{aligned}
\rho_i^{pert}(s) &= \frac{g_s^2}{2^{10}\times 5\times\pi^8}\int_{x_-}^{x_+}dx\int_{y_-}^{y_+}dy\int_{z_-}^{z_+}dz\int_{w_-}^{w_+}dw A_{xyzw}H_{xyzw}^3\\
&\times\Big\{4A_{xyzw}H_{xyzw}^3 xyzw-2H_{xyzw}^2[18xyzwA_{xyzw}s+\mathcal{N}_i(xy+zw)m_c^2]\\
&+20A_{xyzw}(m_c^4-xyzws^2)+5H_{xyzw}[12xyzwA_{xyzw}s^2\\
&+\mathcal{N}_i(xy+zw)m_c^2 s+2(A_{xyzw}-1)m_c^4]\Big\}\\
\rho_i^{\langle G^2\rangle}(s) &= \frac{\langle g_s^2G^2\rangle}{2^9\times 3\times\pi^6}\int_{x_-}^{x_+}dx\int_{y_-}^{y_+}dy\int_{z_-}^{z_+}dzF_{xyz}^2\\
&\times\Big\{6m_c^4+\mathcal{N}_i m_c^2(3s-2F_{xyz})(xy+zB_{xyz})\Big\}\\
\rho_i^{\langle G^3\rangle}(s) &= \frac{\langle g_s^3G^3\rangle}{2^8\times\pi^6}\int_{x_-}^{x_+}dx\int_{y_-}^{y_+}dy\int_{z_-}^{z_+}dz\Big\{\frac{1}{2}\big[4xyzB_{xyz}F_{xyz}^3\\
&- s(m_c^2+\mathcal{N}_i xys)(m_c^2+\mathcal{N}_i zsB_{xyz})-3F_{xyz}^2\big(6sxyzB_{xyz}+\mathcal{N}_i m_c^2(xy+zB_{xyz})\big)\\
&+2F_{xyz}\big(m_c^4+6s^2xyzB_{xyz}+3\mathcal{N}_i sm_c^2(xy+zB_{xyz})\big)\big]\\
&+\frac{m_c^2F_{xyz}}{x}\big[\mathcal{N}_i(F_{xyz}-s)(xy+zB_{xyz})-2m_c^2\big]\Big\}
\end{aligned}
$$

$$A_{xyzw} = (1-x-y-z-w)\,, B_{xyz} = (1-x-y-z)\,,$$

$$H_{xyzw} = \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)m_c^2 - s\,,$$

$$F_{xyz} = \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{1-x-y-z}\right)m_c^2 - s\,,$$

$$x_\pm = \left[\left(1-\frac{8m_c^2}{s}\right) \pm \sqrt{\left(1-\frac{8m_c^2}{s}\right)^2 - \frac{4m_c^2}{s}}\right]\Big/2\,,$$

$$y_\pm = \left[1+2x+\frac{3sx^2}{m_c^2-sx} \pm \sqrt{\frac{[m_c^2+sx(x-1)][(8x+1)m_c^2+sx(x-1)]}{(m_c^2-sx)^2}}\right]\Big/2\,,$$

$$z_\pm = \left[(1-x-y) \pm \sqrt{\frac{(x+y-1)[m_c^2(x+y-(x-y)^2)+sxy(x+y-1)]}{sxy-(x+y)m_c^2}}\right]\Big/2$$

$$w_- = \frac{xyzm_c^2}{sxyz-(xy+yz+xz)m_c^2}\,, w_+ = 1-x-y-z\,.$$

- The spectral densities for 0^{-+} double hidden-charm hybrid

$$
\begin{aligned}
\rho_i^{pert}(s) &= \frac{g_s^2}{2^{10}\times 5\times \pi^8}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy \int_{z_-}^{z_+} dz \int_{w_-}^{w_+} dw A_{xyzw} H_{xyzw}^3 \\
&\times \Big\{ -4A_{xyzw}H_{xyzw}^3 xyzw + 2H_{xyzw}^2[18xyzwA_{xyzw}s + \mathcal{N}_i(3A_{xyzw}-1) \\
&\times (xy+zw)m_c^2] + 20sA_{xyzw}(m_c^2+\mathcal{N}_i xys)(m_c^2+\mathcal{N}_i wzs) \\
&- 5H_{xyzw}[12xyzwA_{xyzw}s^2 + \mathcal{N}_i(6A_{xyzw}-1)(xy+zw)m_c^2 s \\
&+ 2(A_{xyzw}-1)m_c^4] \Big\} \\
\rho_i^{\langle G^2\rangle}(s) &= \frac{\langle g_s^2 G^2\rangle}{2^9\times 3\times \pi^6}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy \int_{z_-}^{z_+} dz F_{xyz}^2 \\
&\times \Big\{ 6m_c^4 + \mathcal{N}_i m_c^2(3s-2F_{xyz})(xy+zB_{xyz}) \Big\} \\
\rho_i^{\langle G^3\rangle}(s) &= \frac{\langle g_s^3 G^3\rangle}{2^8\times \pi^6}\int_{x_-}^{x_+} dx \int_{y_-}^{y_+} dy \int_{z_-}^{z_+} dz \Big\{ -\frac{1}{2}[4xyzB_{xyz}F_{xyz}^3 \\
&+ s(m_c^4 - s^2 xyzB_{xyz}) - 18F_{xyz}^2 sxyzB_{xyz} + 2F_{xyz}(6s^2xyzB_{xyz} - m_c^4)] \\
&+ \frac{m_c^2 F_{xyz}}{x}[\mathcal{N}_i(F_{xyz}-s)(xy+zB_{xyz}) - 2m_c^2] \Big\}
\end{aligned}
$$