Molecular nature of the P_c states

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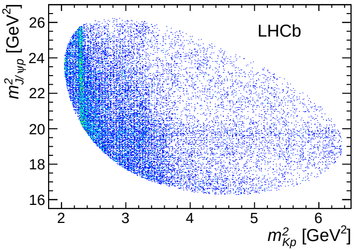
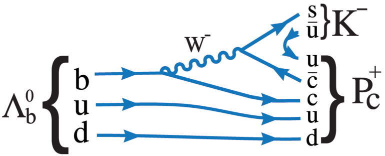
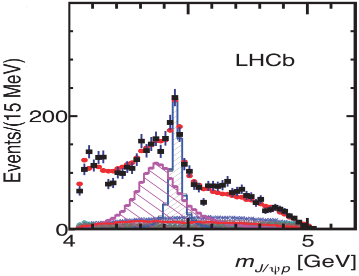
Charmonium-pentaquark states (I)

Observation of exotic structures (P_c) in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL 115, 072001 (2015)







$P_c(4380)^+$: $M = 4380 \pm 8 \pm 29$ MeV

$\Gamma = 205 \pm 18 \pm 86$ MeV

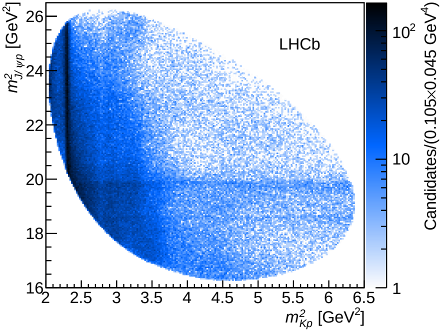
$P_c(4450)^+$: $M = 4449.8 \pm 1.7 \pm 2.5$ MeV

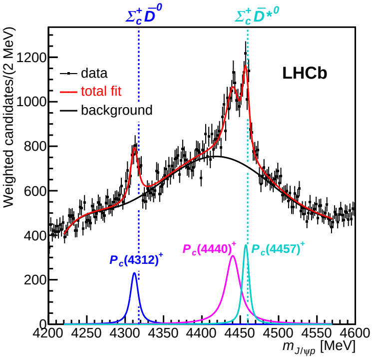
$\Gamma = 39 \pm 5 \pm 19$ MeV

Preferred Parity: Opposite

Charmonium-pentaquark states (II)

LHCb, PRL 122, 222001 (2019)





State	M [MeV]	Γ [MeV]	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	$0.53 \pm 0.16^{+0.15}_{-0.13}$

Charmonium-pentaquark (theoretical)

- Compact pentaquark — Cheng et al., PRD100(2019)054002

  $P_c(4312)$, $P_c(4440)$, $P_c(4457)$: $J^P = 3/2^-, 1/2^-, 3/2^-$

- Compact diquark model — Ali et al., JHEP1910(2019)256

$3/2^-$	4240 ± 29
$3/2^+$	4440 ± 35
$5/2^+$	4457 ± 35

- $P_c(4312)$: virtual state — Fernández-Ramírez et al., PRL123(2019)092001
- K-matrix: $J/\psi p - \Sigma_c \bar{D} - \Sigma_c \bar{D}^*$ — Kuang et al., EPJC80(2020)433

  $\hookrightarrow P_c(4312)$: $\Sigma_c \bar{D}$, $P_c(4457)$: ? cusp effect

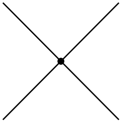
- Molecule (HQSS) — Liu et al., PRL122,242001 (2019)

	Molecule	J^P	M (MeV)		Molecule	J^P	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4311.8 − 4313.0	B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4306.3 − 4307.7
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4376.1 − 4377.0	B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4370.5 − 4371.7
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4440.3*	B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4457.3*
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4457.3*	B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4440.3*
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4500.2 − 4501.0	B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4523.2 − 4523.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4510.6 − 4510.8	B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4516.5 − 4516.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4523.3 − 4523.6	B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4500.2 − 4501.0

- quantum numbers? line shape? the existence of $P_c(4380)$?

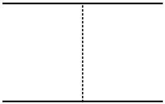
Effective Lagraingian $\Sigma_c^{(*)}\bar{D}^{(*)}$, $J/\psi p$, $\eta_c p$, $\Lambda_c\bar{D}^{(*)}$

- Contact Lagrangian



$$
\begin{aligned}
\mathcal{L} &= -C_a \vec{S}_c^\dagger \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] \\
&\quad -C_b i\epsilon_{ijk}(S_c^\dagger)_j (S_c)_k \mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c]. \\
&\quad +C_c\Big(S_{ab}^{i\dagger} T_{ca}\langle \bar{H}_c^\dagger \sigma^i \bar{H}_b\rangle - T_{ca}^\dagger S_{ab}^i \langle \bar{H}_b^\dagger \sigma^i \bar{H}_c\rangle\Big) \\
&\quad +C_d T_{ab}^\dagger T_{ba}\langle \bar{H}_c^\dagger \bar{H}_c\rangle
\end{aligned}
$$

- One-pion exchange (OPE)

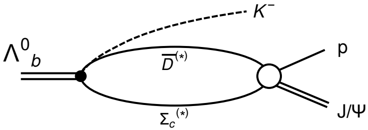


$$
\begin{aligned}
\mathcal{L}_{DD\pi} &= \frac{g}{4}\langle \sigma \cdot u_{ab}\bar{H}_b \bar{H}_a^\dagger\rangle, \\
\mathcal{L}_{\Sigma_c\Sigma_c\pi} &= i\frac{3}{2} g_1 \epsilon_{ijk}\langle \bar{S}_i u_j S_k\rangle. \\
\mathcal{L}_{\Sigma_c\Lambda_c\pi} &= -\frac{1}{\sqrt{2}} g_3 (S_{ab}^{i\dagger} u_{bc}^i T_{ca} + T_{ab}^\dagger u_{bc}^i S_{ca}^i),
\end{aligned}
$$

☞ Nonrelativistic superfield for the heavy-quark spin doublets

$$
\begin{aligned}
\vec{S}_c &= \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*, \\
\bar{H}_c &= \frac{1}{2}\Big(-\bar{D} + \vec{\sigma}\cdot\vec{\bar{D}}^*\Big).
\end{aligned}
$$

$\Lambda_b^0 \to K^- J/\psi p$



- ☞ $m_{J/\psi p} \sim 4440$ MeV
  $\hookrightarrow |\mathbf{p}| \sim 810$ MeV
  $\hookrightarrow J/\psi p(S),\ J/\psi p(D)$
- ☞ Effective Lagrangian for $\bar{D}^{(*)}\Sigma_c^{(*)} \to J/\psi p\ (\eta_c p)$: $J = -\eta_c + \sigma \cdot \psi$

$$\mathcal{L} = \frac{g_S}{\sqrt{3}} N^\dagger \sigma^i \bar{H} J^\dagger S^i - \sqrt{3} g_D N^\dagger \sigma^i \bar{H} (\partial^i \partial^j - \frac{1}{3}\delta^{ij}\partial^2) J^\dagger S^j,$$

- ☞

$$\text{channels} \quad \begin{cases} \underline{\Sigma_c^{(*)}\bar{D}^{(*)}(S/D)}, \Lambda_c \bar{D}^{(*)}(S/D) & \to \alpha, \beta, \gamma \\ J/\psi p(S/D), \eta_c p(S/D) & \to i, j, k \end{cases}$$

- ☞ Weak production: S-wave $\Sigma_c^{(*)}\bar{D}^{(*)}$.

Lippman-Schwinger equaitons

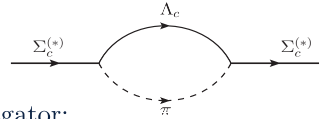
$$U_\alpha^J(E,p) \;=\; P_\alpha^J(E,p) - \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} V_{\alpha\beta}^J(E,p,q) G_\beta(E,q) U_\beta^J(q),$$

$$U_i^J(E,p) \;=\; \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} \mathcal{V}_{\beta i} G_\beta(E,q) U_\beta^J(q).$$

☞ $\Gamma(\Sigma_c^* \to \Lambda_c\pi) = 15.0$ MeV, $\Gamma(\Sigma_c \to \Lambda_c\pi) = 1.86$ MeV,

$\hookrightarrow \sim \Gamma(P_c)$

☞ The self-energy function $\tilde{\Sigma}_R^{(*)}(s) \sim ig^2 \frac{p^3}{\sqrt{s}}$



☞ Two-body propagator:

$$G_\beta(E,\mathbf{q}) = \frac{m_{\Sigma_c^{(*)}} m_{D^{(*)}}}{E_{\Sigma_c^{(*)}}(\mathbf{q}) E_{D^{(*)}}(\mathbf{q})} \frac{1}{E_{\Sigma_c^{(*)}}(\mathbf{q}) + E_{D^{(*)}}(\mathbf{q}) - E - \frac{\tilde{\Sigma}_R^{(*)}(s)}{2E_{\Sigma_c^{(*)}}(\mathbf{q})}},$$

$s = \left(E - E_{D^{(*)}}(\mathbf{q})\right)^2 - \mathbf{q}^2$ is the off-shellness of $\Sigma_c^{(*)}$.

Fit Schemes

☞ The effective potentials

$$V^J_{\alpha\beta} = V^J_{\mathrm{CT},\alpha\beta} + V^J_{\mathrm{OPE},\alpha\beta} + \mathcal{G}^J_{\alpha\beta},$$

The effective contributions from the $J/\psi p$ and $\eta_c p$ bubble loop ($k \sim 0.9$ GeV)

$$\mathcal{G}^J_{\alpha\beta} = \sum_i$$

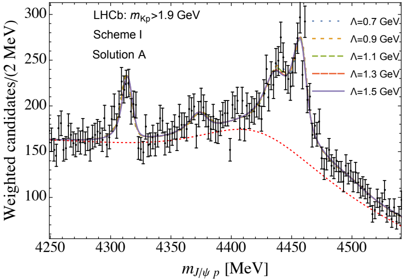
$$= R_{\mathcal{G}} - \sum_i i \frac{1}{2\pi E} m_{\psi(\eta_c)} m_p g^{(\prime)J}_{\alpha i} g^{(\prime)J}_{\beta i} k^{2l_i+1}.$$

☞ Fit schemes:

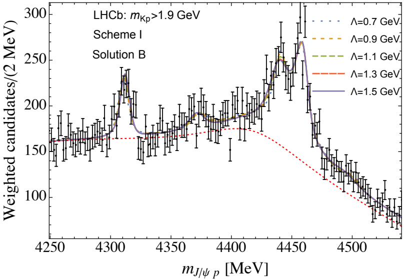
- Scheme I: pure contact potential w/o $\Lambda_c \bar{D}^{(*)}$
- Scheme II: Scheme I + elastic OPE w/o $\Lambda_c \bar{D}^{(*)}$ (+ CT for $\Lambda_c \bar{D}^{(*)}$)
- Scheme III: Scheme II + S-D counter term w/o $\Lambda_c \bar{D}^{(*)}$
  ↪ coupled channel
- Scheme IV: contact + OPE + S-D counter terms w $\Lambda_c \bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



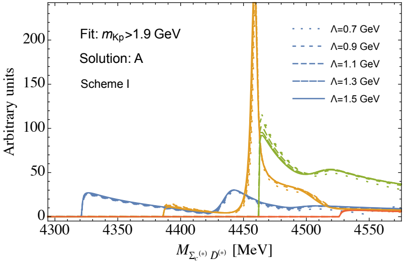
Solution B



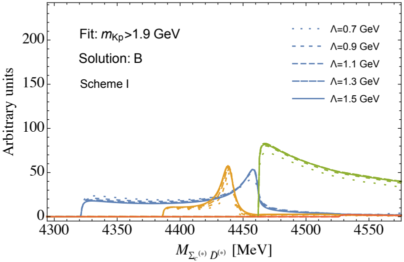
- ☞ $\Lambda_{\rm soft} \sim 0.7$ GeV
- ☞ Cutoff-independent for both solution A and B
- ☞ No need for $\Lambda_c\bar{D}^{(*)}$

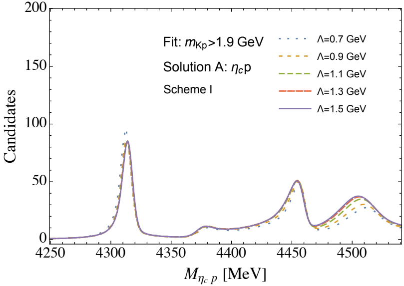
Scheme I: pure contact potential w/o $\Lambda_c \bar{D}^{(*)}$

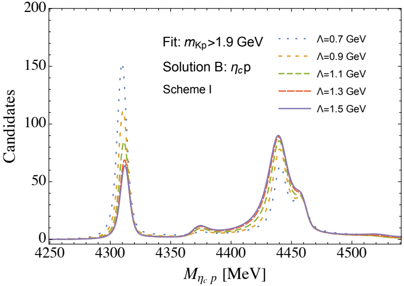
Solution A



Solution B







Scheme I: pole positions

		Solution A		Solution B	
	DC ([MeV])	J^P	Pole [MeV]	J^P	Pole [MeV]
$P_c(4312)$	$\Sigma_c \bar{D}$ (4321.6)	$\frac{1}{2}^-$	$4314(1) - 4(1)i$	$\frac{1}{2}^-$	$4312(2) - 4(2)i$
$\mathbf{P_c(4380)}^*$	$\Sigma_c^* \bar{D}$ (4386.2)	$\frac{3}{2}^-$	$4377(1) - 7(1)i$	$\frac{3}{2}^-$	$4375(2) - 6(1)i$
$P_c(4440)$	$\Sigma_c \bar{D}^*$ (4462.1)	$\frac{1}{2}^-$	$4440(1) - 9(2)i$	$\frac{3}{2}^-$	$4441(3) - 5(2)i$
$P_c(4457)$	$\Sigma_c \bar{D}^*$ (4462.1)	$\frac{3}{2}^-$	$4458(2) - 3(1)i$	$\frac{1}{2}^-$	$4462(4) - 5(3)i$
P_c	$\Sigma_c^* \bar{D}^*$ (4526.7)	$\frac{1}{2}^-$	$4498(2) - 9(3)i$	$\frac{1}{2}^-$	$4526(3) - 9(2)i$
P_c	$\Sigma_c^* \bar{D}^*$ (4526.7)	$\frac{3}{2}^-$	$4510(2) - 14(3)i$	$\frac{3}{2}^-$	$4521(2) - 12(3)i$
P_c	$\Sigma_c^* \bar{D}^*$ (4526.7)	$\frac{5}{2}^-$	$4525(2) - 9(3)i$	$\frac{5}{2}^-$	$4501(3) - 6(4)i$

☞ * NOT the broad $P_c(4380)$ reported by LHCb in 2015

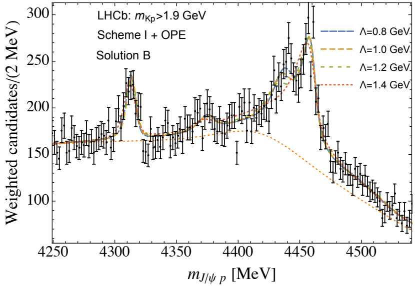
☞ Bound states with respect to the dominant channel (DC)

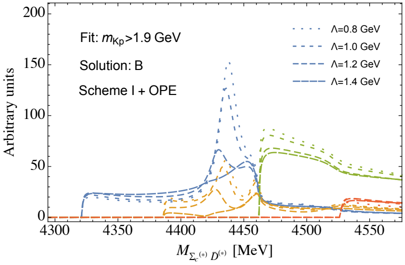
Scheme II: scheme I + OPE w/o $\Lambda_c\bar{D}^{(*)}$

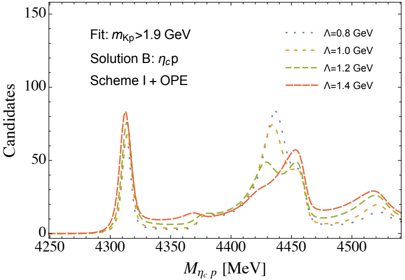
☞ No solution A

☞ Solution B:
Cut-off dependent

☞ $\Lambda_{\rm soft} \sim 700$ MeV

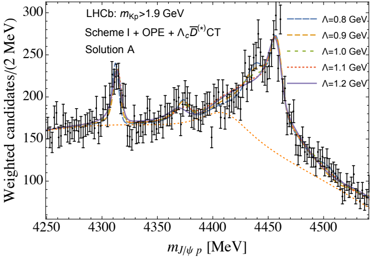


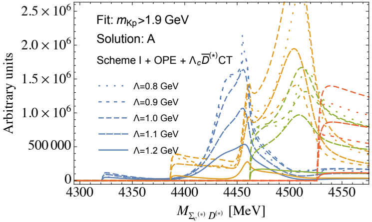




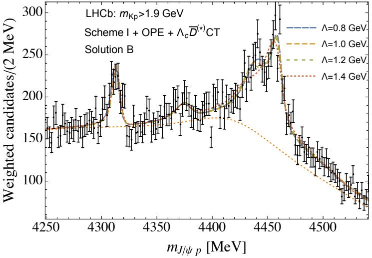
Scheme I + OPE + CT for $\Lambda_c \bar{D}^{(*)}$

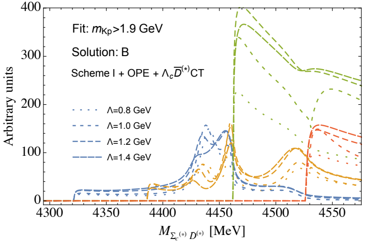
Solution A





Solution B

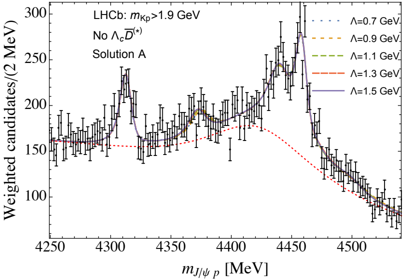




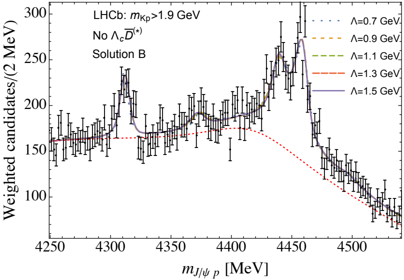
- Cut-off dependent
- $\Lambda_{\text{soft}} \sim 900$ MeV $\Lambda_c \bar{D}^{(*)}$

Scheme III: contact + OPE + S-D w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



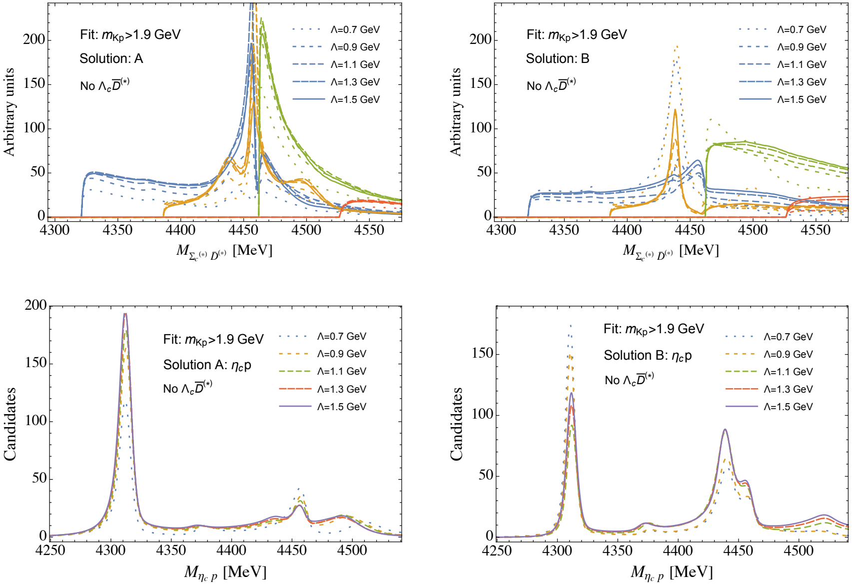
Solution B



☞ Cutoff-independent for both solution A and B

Scheme III: contact + OPE + S-D w/o $\Lambda_c \bar{D}^{(*)}$

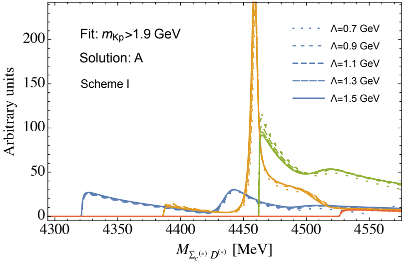
Solution A $\Lambda_{\rm soft} \sim 0.7$ GeV Solution B



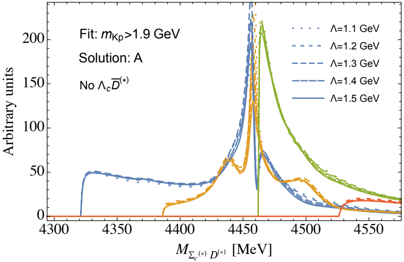
Scheme I vs Scheme III w/o $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.7$ GeV

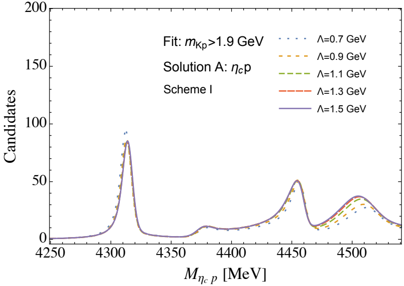
Solution A

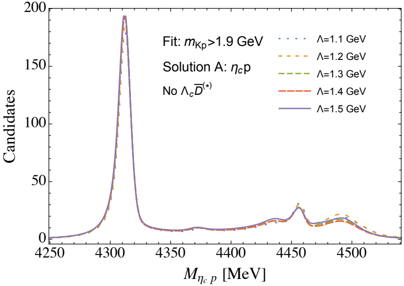
Scheme I: pure contact



Scheme III: contact + OPE + SD

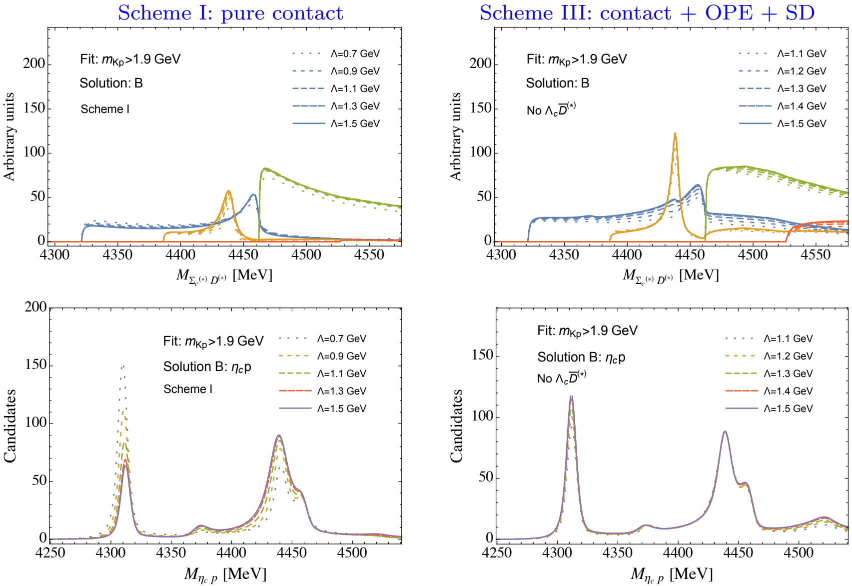






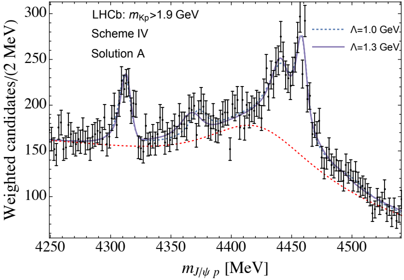
Scheme I vs Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ $\Lambda_{\rm soft}\sim 0.7$ GeV

Solution B

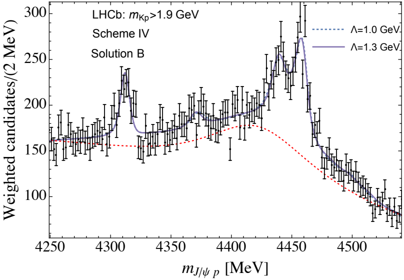


Scheme IV: CT + OPE + SD with $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.9$ GeV

Solution A



Solution B



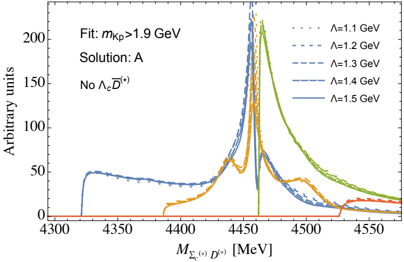
☞ Cutoff-independent for both solution A and B

☞ ? uncertainty

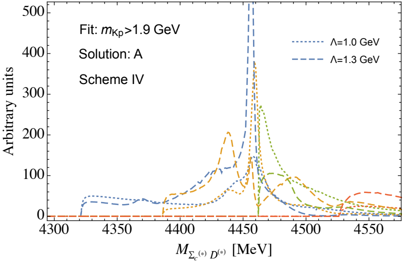
Scheme III w/o $\Lambda_c \bar{D}^{(*)}$ vs Scheme IV w $\Lambda_c \bar{D}^{(*)}$

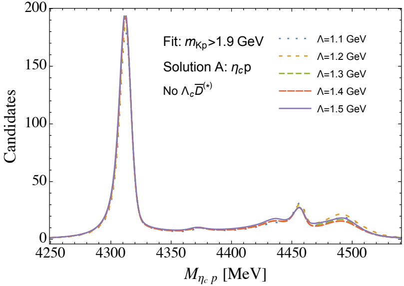
Solution A

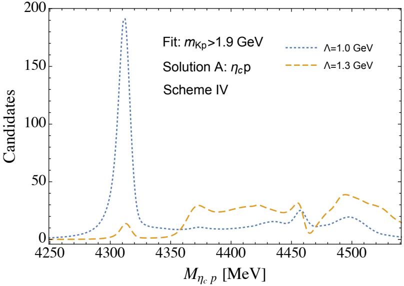
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



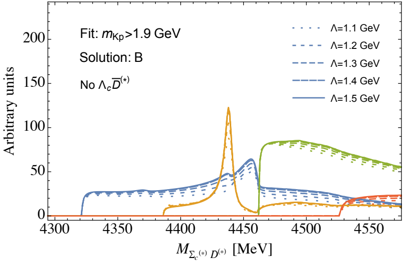




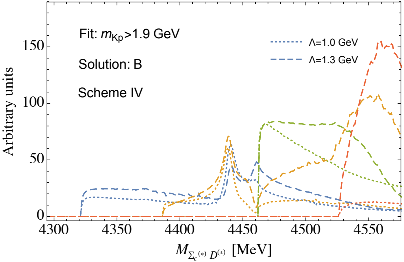
Scheme III w/o $\Lambda_c \bar{D}^{(*)}$ vs Scheme IV w $\Lambda_c \bar{D}^{(*)}$

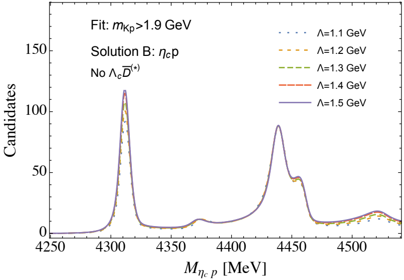
Solution B

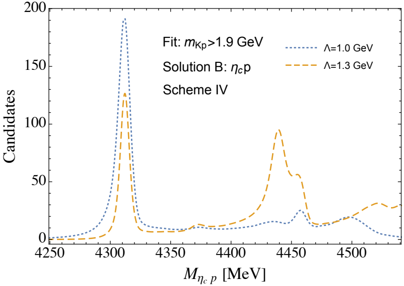
Scheme III: $\Lambda_{\mathrm{soft}} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\mathrm{soft}} \sim 0.9$ GeV



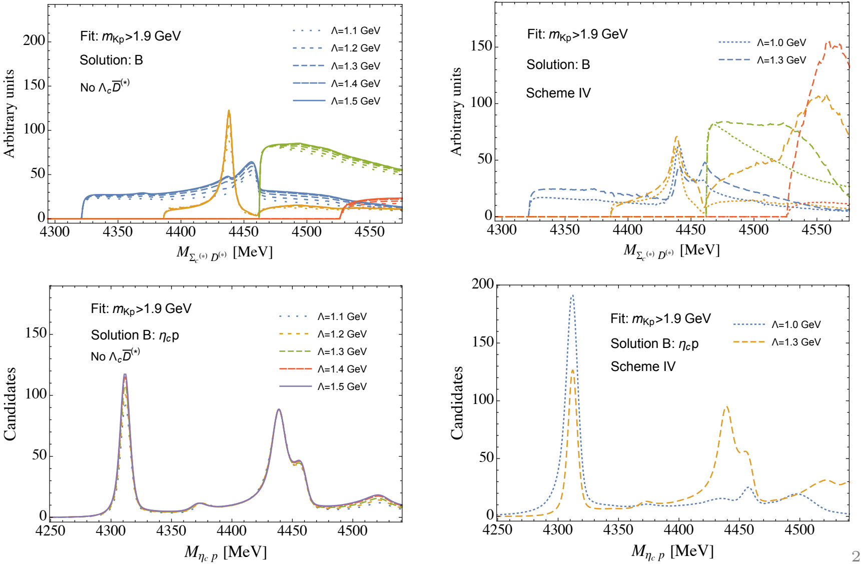




Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w $\Lambda_c\bar{D}^{(*)}$

Solution B :)

Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV — Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



Summary & Outlook

☞ Solving Lippmann-Schwinger equation with respect to

- Unitarity, three-body cut
  ↪ width of $\Sigma_c^{(*)}$
- Coupled-channels
  ↪ cut-off independence: OPE → SD counter term
- Heavy quark spin symmetry
  ↪ 7 $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecular states

☞ $\Lambda_{\text{cutoff}} = 1.3$ GeV
↪ $\Lambda_{\text{cutoff}} \gg \Lambda_{\text{soft}}$

- Solution A is scheme dependent
- Solution B is consistent for all cut-off independent schemes
  $P_c(4440)$: $J^P = 3/2^-$, $P_c(4457)$: $J^P = \frac{1}{2}^-$ preferred ?

☞ Formalism consistent
↪ we can not say much about $\Lambda_c\bar{D}^{(*)}$ interaction without data in this channel.

☞ A narrow $P_c(4380)$, different from the broad one reported by LHCb in 2015.

Thank you very much for your attention!

Isospin structures

The isospin wave function of $P_c^{(*)}$

$$P_c^{+(*)} = -\sqrt{\frac{1}{3}}\bar{D}^{(*)0}\Sigma_c^{(*)+} + \sqrt{\frac{2}{3}}\bar{D}^{(*)-}\Sigma_c^{(*)++}.$$

The corresponding potential

$$\begin{aligned}
V_{\Sigma_c^{(*)}\bar{D}^{(*)}} &= \frac{1}{3}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} + \frac{2}{3}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&\quad -\frac{\sqrt{2}}{3}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} - \frac{\sqrt{2}}{3}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} \\
&= 2V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} = -\sqrt{2}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&= -\sqrt{2}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}},
\end{aligned}$$

$$\begin{aligned}
V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}} &= 0 \\
V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)++}D^{(*)-}} &= -\frac{1}{\sqrt{2}}V_{\Sigma_c^{(*)+}\bar{D}^{(*)0}\to\Sigma_c^{(*)++}D^{(*)-}} \\
&= -\frac{1}{\sqrt{2}}V_{\Sigma_c^{(*)++}D^{(*)-}\to\Sigma_c^{(*)+}\bar{D}^{(*)0}}
\end{aligned}$$

Heavy-quark spin symmetry (HQSS)

- In the limit $\Lambda_{\rm QCD}/m_Q \to 0$
  $\hookrightarrow$ strong interactions are independent on heavy-quark spin
- S-wave $\bar{D}^{(*)}\Sigma_c^{(*)}$ $\Lambda_c\bar{D}^{(*)}$ spin decomposition $|s_Q \otimes j_\ell\rangle$

$$\begin{pmatrix} |\Sigma_c\bar{D}\rangle \\ |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{1}{2}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 & 0 \\ \frac{1}{2\sqrt{3}} & \frac{5}{6} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ \sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \\ |0 \otimes \frac{1}{2}\rangle' \\ |1 \otimes \frac{1}{2}\rangle' \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} |\Sigma_c\bar{D}^*\rangle \\ |\Sigma_c^*\bar{D}\rangle \\ |\Sigma_c^*\bar{D}^*\rangle \\ |\Lambda_c\bar{D}^*\rangle \end{pmatrix}_{\frac{3}{2}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{3} & \frac{\sqrt{5}}{3} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{3}} & \frac{1}{2}\sqrt{\frac{5}{3}} & 0 \\ \frac{1}{2}\sqrt{\frac{5}{3}} & \frac{\sqrt{5}}{3} & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |0 \otimes \frac{3}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle \\ |1 \otimes \frac{3}{2}\rangle \\ |1 \otimes \frac{1}{2}\rangle' \end{pmatrix}, \tag{2}$$

$$|\Sigma_c^*\bar{D}^*\rangle_{\frac{5}{2}} = |1 \otimes \frac{3}{2}\rangle. \tag{3}$$

Contact interactions

- Contact interaction: short-range interaction
- strong interaction: spin of light degrees of freedom

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Sigma_c^{(*)}\bar{D}^{(*)}$:

$$C_{\frac{1}{2}} \equiv \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle, \quad C_{\frac{3}{2}} \equiv \langle s_Q \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{3}{2}\rangle,$$

$\hookrightarrow$ $\Sigma_c^{(*)}\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C'_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle = \langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle'.$$

$\hookrightarrow$ $\Lambda_c\bar{D}^{(*)} \to \Lambda_c\bar{D}^{(*)}$:

$$C''_{\frac{1}{2}} \equiv {}'\langle s_Q \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|s_Q \otimes \frac{1}{2}\rangle',$$

- $J/\psi p$ $(\eta_c p)$ Heavy-Light spin decomposition:

$$|J/\psi p\rangle \begin{cases} S\text{-wave} : |1 \otimes \frac{1}{2}\rangle \\ D\text{-wave} : |1 \otimes \frac{3}{2}\rangle \end{cases}, \qquad |\eta_c p\rangle \begin{cases} S\text{-wave} : |0 \otimes \frac{1}{2}\rangle \\ D\text{-wave} : |0 \otimes \frac{3}{2}\rangle \end{cases}.$$

- $\Sigma_c^{(*)}\bar{D}^{(*)} \to J/\psi p$ $(\eta_c p)$:

$$g_S \equiv \langle 1 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_S = \langle 0 \otimes \frac{1}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_S,$$

$$g_D k^2 \equiv \langle 1 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|J/\psi p\rangle_D = \langle 0 \otimes \frac{3}{2}|\hat{\mathcal{H}}_I|\eta_c p\rangle_D$$

Contact potentials

$$V_C^{\frac{1}{2}^-} = \begin{pmatrix} \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ \frac{2}{3\sqrt{3}}C_{\frac{1}{2}} - \frac{2}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{7}{9}C_{\frac{1}{2}} + \frac{2}{9}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{2}{3}}C_{\frac{3}{2}} & -\frac{\sqrt{2}}{9}C_{\frac{1}{2}} + \frac{\sqrt{2}}{9}C_{\frac{3}{2}} & \frac{8}{9}C_{\frac{1}{2}} + \frac{1}{9}C_{\frac{3}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} \\ 0 & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & -\sqrt{\frac{2}{3}}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} & 0 \\ \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{2}{3}C'_{\frac{1}{2}} & \frac{\sqrt{2}}{3}C'_{\frac{1}{2}} & 0 & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{3}{2}^-} = \begin{pmatrix} \frac{1}{9}C_{\frac{1}{2}} + \frac{8}{9}C_{\frac{3}{2}} & -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & -\frac{1}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3\sqrt{3}}C_{\frac{1}{2}} + \frac{1}{3\sqrt{3}}C_{\frac{3}{2}} & \frac{1}{3}C_{\frac{1}{2}} + \frac{2}{3}C_{\frac{3}{2}} & +\frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} \\ -\frac{\sqrt{5}}{9}C_{\frac{1}{2}} + \frac{\sqrt{5}}{9}C_{\frac{3}{2}} & \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{1}{2}} - \frac{1}{3}\sqrt{\frac{5}{3}}C_{\frac{3}{2}} & \frac{5}{9}C_{\frac{1}{2}} + \frac{4}{9}C_{\frac{3}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} \\ -\frac{1}{3}C'_{\frac{1}{2}} & \frac{1}{\sqrt{3}}C'_{\frac{1}{2}} & \frac{\sqrt{5}}{3}C'_{\frac{1}{2}} & C''_{\frac{1}{2}} \end{pmatrix},$$

$$V_C^{\frac{5}{2}^-} = C_{\frac{3}{2}}.$$

channels

J^P	S-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$,$\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}^*$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c^*\bar{D}^*$

J^P	D-wave
$\left(\frac{1}{2}\right)^-$	$\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}^*$
$\left(\frac{3}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$
$\left(\frac{5}{2}\right)^-$	$\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*_{\frac{1}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{3}{2}}$, $\Sigma_c^*\bar{D}^*_{\frac{5}{2}}$, $\Lambda_c\bar{D}$, $\Lambda_c\bar{D}^*_{\frac{1}{2}}$, $\Lambda_c\bar{D}^*_{\frac{3}{2}}$

Next-leading order (NLO) effective Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{\rm NLO} = \\
&-D_a^{SS}\left(\partial^i \vec{S}^\dagger_{ab}\cdot\vec{S}_{ba}\langle\partial^i\bar{H}_c^\dagger\bar{H}_c\rangle + \vec{S}^\dagger_{ab}\cdot\partial^i\vec{S}_{ba}\langle\bar{H}_c^\dagger\partial^i\bar{H}_c\rangle\right)\\
&-D_b^{SS} i\epsilon_{jik}\left(\partial^\ell S^{j\dagger}_{ab}S^k_{ba}\langle\partial^\ell\bar{H}_c^\dagger\sigma^i\bar{H}_c\rangle + S^{j\dagger}_{ab}\partial^\ell S^k_{ba}\langle\bar{H}_c^\dagger\sigma^i\partial^\ell\bar{H}_c\rangle\right)\\
&-D_b^{SD} i\epsilon_{jik}\Big[\partial_i S_j^\dagger S_k\langle\partial_\ell\bar{H}^\dagger\sigma_\ell\bar{H}\rangle + \partial_\ell S_j^\dagger S_k\langle\partial_i\bar{H}^\dagger\sigma_\ell\bar{H}\rangle - \frac{2}{3}\partial_\ell S_j^\dagger S_k\langle\partial_\ell\bar{H}^\dagger\sigma_i\bar{H}\rangle\\
&+S_j^\dagger\partial_i S_k\langle\bar{H}^\dagger\sigma_\ell\partial_\ell\bar{H}\rangle + S_j^\dagger\partial_\ell S_k\langle\bar{H}^\dagger\sigma_\ell\partial_i\bar{H}\rangle - \frac{2}{3}S_j^\dagger\partial_\ell S_k\langle\bar{H}^\dagger\partial_\ell\sigma_i\bar{H}\rangle\Big]\\
&+\frac{4}{3}\sqrt{2}D_c^{SD}\Big[\partial^i S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}_c^\dagger\sigma^j\bar{H}_b\rangle + \partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^i\bar{H}_c^\dagger\sigma^j\bar{H}_b\rangle - \frac{2}{3}\partial^j S^{i\dagger}_{ab}T_{ca}\langle\partial^j\bar{H}_c^\dagger\sigma^i\bar{H}_b\rangle\\
&-\partial^i T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}_b^\dagger\sigma^j\bar{H}_c\rangle - \partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^i\bar{H}_b^\dagger\sigma^j\bar{H}_c\rangle + \frac{2}{3}\partial^j T^\dagger_{ca}S^i_{ab}\langle\partial^j\bar{H}_b^\dagger\sigma^i\bar{H}_c\rangle\\
&+S^{i\dagger}_{ab}\partial^i T_{ca}\langle\bar{H}_c^\dagger\sigma^j\partial^j\bar{H}_b\rangle + S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}_c^\dagger\sigma^j\partial^i\bar{H}_b\rangle - \frac{2}{3}S^{i\dagger}_{ab}\partial^j T_{ca}\langle\bar{H}_c^\dagger\sigma^i\partial^j\bar{H}_b\rangle\\
&-T^\dagger_{ca}\partial^i S^i_{ab}\langle\bar{H}_b^\dagger\sigma^j\partial^j\bar{H}_c\rangle - T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}_b^\dagger\sigma^j\partial^i\bar{H}_c\rangle + \frac{2}{3}T^\dagger_{ca}\partial^j S^i_{ab}\langle\bar{H}_b^\dagger\sigma^i\partial^j\bar{H}_c\rangle\Big]\\
&+D_d^{SS}\left(\partial^i T^\dagger_{ab}T_{ba}\langle\partial^i\bar{H}_c^\dagger\bar{H}_c\rangle + T^\dagger_{ab}\partial^i T_{ba}\langle\bar{H}_c^\dagger\partial^i\bar{H}_c\rangle\right)
\end{aligned}
\tag{4}
$$

NLO contact potential

$$V^J_{\text{NLO}}(p,p') = \begin{pmatrix} (p^2+p'^2)V^J_{SS} & p'^2 V^J_{SD} \\ p^2\left(V^J_{SD}\right)^T & 0 \end{pmatrix}.$$

$$V^{\frac{1}{2}}_{SD} = -\frac{128}{3}\begin{pmatrix} \frac{D_b^{SD}}{4\sqrt{6}} & 0 & -\frac{D_b^{SD}}{8\sqrt{30}} & -\frac{1}{8}\sqrt{\frac{3}{10}}D_b^{SD} & \frac{D_c^{SD}}{8\sqrt{3}} \\ -\frac{D_b^{SD}}{12\sqrt{2}} & -\frac{D_b^{SD}}{8\sqrt{6}} & \frac{D_b^{SD}}{6\sqrt{10}} & -\frac{D_b^{SD}}{8\sqrt{10}} & -\frac{D_c^{SD}}{24} \\ \frac{D_b^{SD}}{48} & -\frac{D_b^{SD}}{16\sqrt{3}} & \frac{7D_b^{SD}}{48\sqrt{5}} & \frac{D_b^{SD}}{8\sqrt{5}} & -\frac{D_c^{SD}}{24\sqrt{2}} \\ \frac{D_c^{SD}}{8\sqrt{3}} & 0 & \frac{D_c^{SD}}{8\sqrt{15}} & \frac{1}{8}\sqrt{\frac{3}{5}}D_c^{SD} & 0 \\ -\frac{D_c^{SD}}{24} & \frac{D_c^{SD}}{8\sqrt{3}} & -\frac{D_c^{SD}}{6\sqrt{5}} & \frac{D_c^{SD}}{8\sqrt{5}} & 0 \end{pmatrix}$$

Scheme I: pure contact potentials w/o $\Lambda_c\bar{D}^{(*)}$