Hadron Exotics on Lattice QCD: A survey talk

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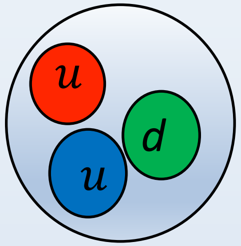
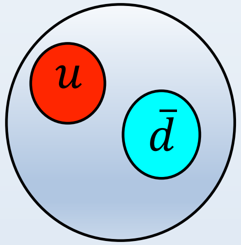
Hadrons: color singlets made of quarks and gluons

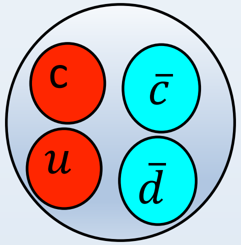
- Ordinary Hadrons
- Exotic Hadrons



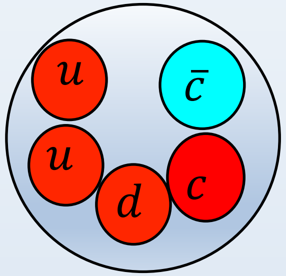
baryon



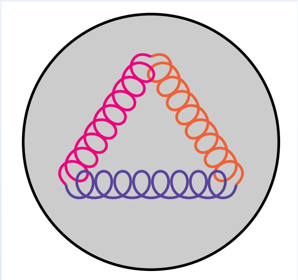
meson



tetra-quark



penta-quark

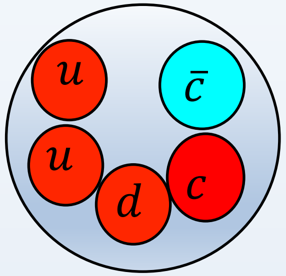


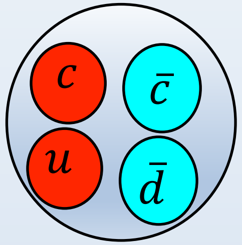
Glueball

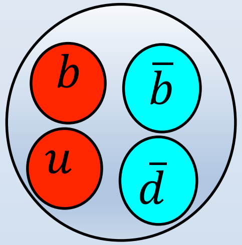
$$|p >= |uud > +|uud\bar{q}q > + \cdots \qquad < 0|O_{uud}\ |p > \neq 0$$

Many Experimental Progress on Hadron Exotics

Heavy Exotics







2020 Z_{cs} (3985)firstly by BESIII

2020 X(6900) firstly by LHCb

2015 P_c(4380/4450) firstly by LHCb

2013 Z_c(3900) firstly by BESIII and Belle

2012 Z_b(10610/10650) firstly by Belle

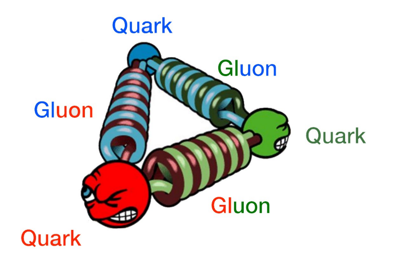
2005 Y(4260) firstly by BaBar

2003 X(3872) firstly by Belle

Theoretical Tools to study Hadrons

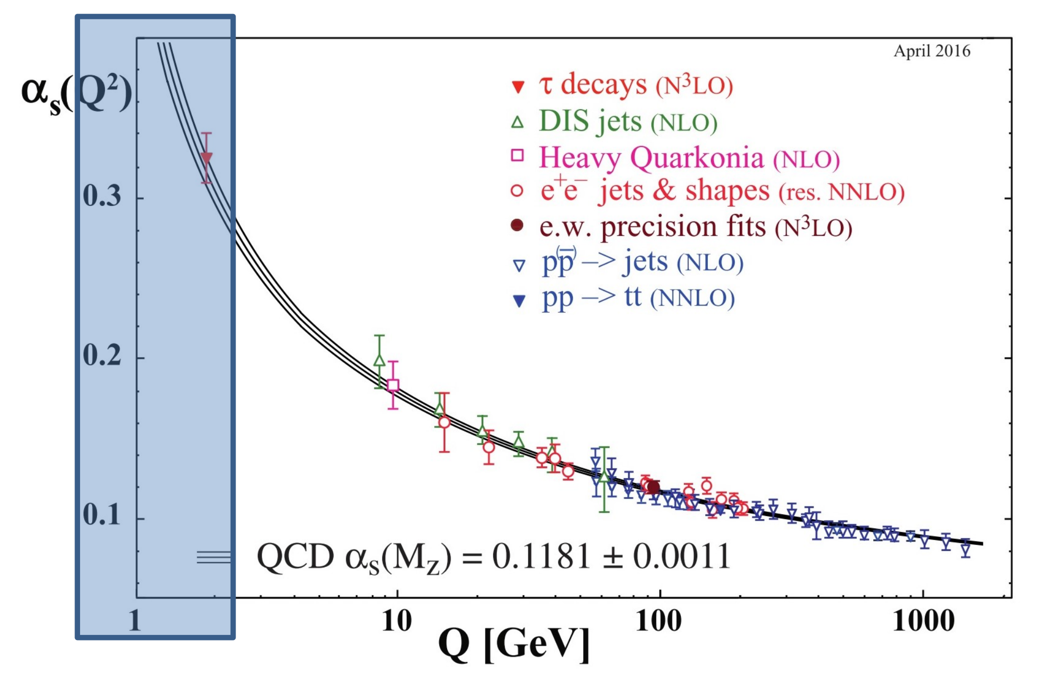
夸克禁闭：**夸克永远是以强子的方式束缚在一起，不存在自由夸克：**

- Quark Model
- QCD sum rules
- Hadron-level effective theory
- Lattice QCD



Quantum Chromodynamics

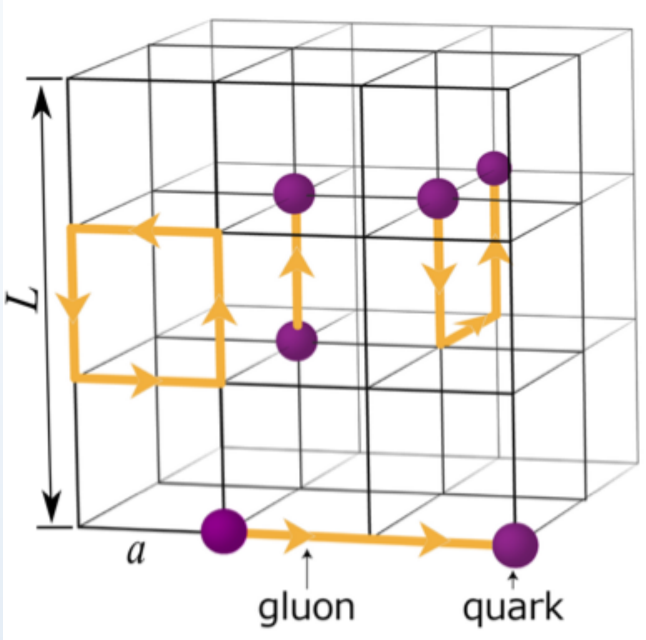
Many Excellent Reviews:
Chen, et.al, *Phys.Rept.* 639,1(2016)
Guo, et.al, *Rev.Mod.Phys.* 90,015004(2018)
Brambilla, et.al, *Phys.Rept.* 873,1(2020)



渐近自由：高能标（短距离），强相互作用耦合常数变小，微扰理论 --- 2004, Nobel Prize

Lattice QCD

- 将QCD 的基本自由度定义在离散的四维欧氏时空格子上。
- 夸克和反夸克场被定义在格点上，规范场则定义在相邻两格点间的链接上。
- 超立方格子的体积为$(N_s a)^3$ x $(N_T a)$, 其中格点间距a 和空间方向上长度$L=N_s a$ 提供了量子场论的紫外和红外截断。



ϕ是场变量， $S[\phi]$是有效作用量， $O[\phi]$是物理量算符，$O[\phi]$ 的期望值$\langle O\rangle$为，

$$\langle O\rangle = \int D\phi O[\phi]P[\phi] \qquad P[\phi] = \frac{1}{Z}e^{-S[\phi]} \qquad Z = \int D\phi e^{-S[\phi]}$$

利用路径积分量子化进行表述，格点QCD 形式上类似于一个统计物理模型。其中$P[\phi]$ 是场应当服从的玻尔兹曼分布。使得Monte Carlo方法的使用成为可能。

Lattice QCD

典型的格点QCD 数值计算可以分为几个步骤：

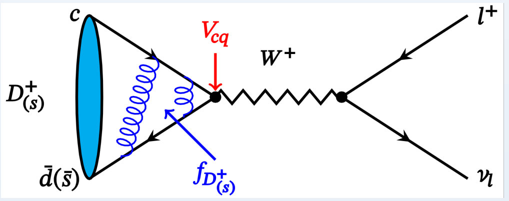
1. 利用Monte Carlo 算法产生正确分布的规范场组态；
2. 从正确分布的组态中取样对感兴趣的物理量$O[\phi]$ 进行测量:

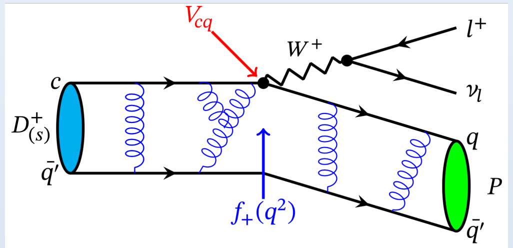
$$\sum_{(\vec{y}-\vec{x})\in Z^3} e^{i\vec{p}.(\vec{y}-\vec{x})} \left\langle T\left(\psi(t;\vec{y}),\psi^{\dagger}(t;\vec{y})\right)\right\rangle \sim \sum_{\Gamma,i} Z_i^{\Gamma} e^{-E_i^{\Gamma} t}$$

3. 把有限体积内的物理量和真实的物理量相联系。

What can study from Lattice QCD

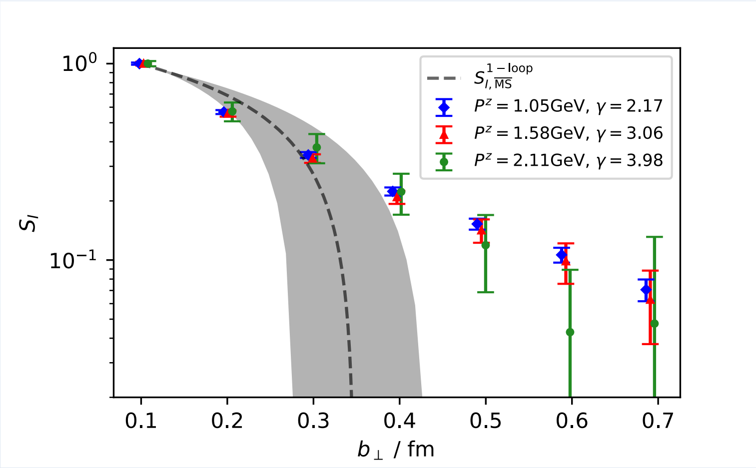
弱衰变矩阵元:
衰变常数和半轻衰变形状因子





电磁矩阵元:
muon g-2

部分子分布函数



Soft-function:
LPC:PRL 125,192001(2020)

强子质量谱：介子、重子、奇特强子态

Different steps for Lattice QCD analysis of Hadron Exotics

- Conventional approach: Hadron spectrum from 2pt
- Lüscher Formula
- Non-Lüscher Formula

Hadron spectrum from 2pt: conventional approach

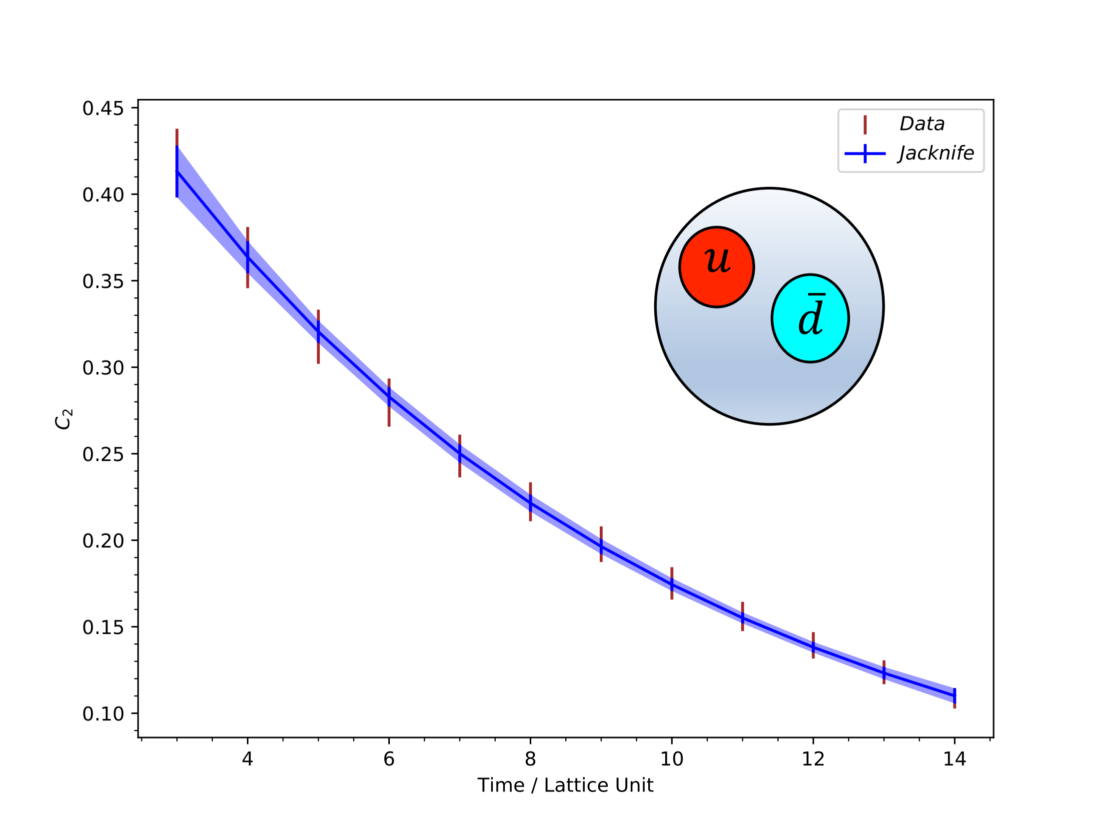
- Choose the operator O_{α} with correct quantum number
- Calculate correlation function matrix

$$\mathrm{C}_{\alpha\beta}(\mathrm{t}) = < \Omega | O_{\alpha}(t) O_{\beta}^{\dagger}(0) \, | \Omega >$$

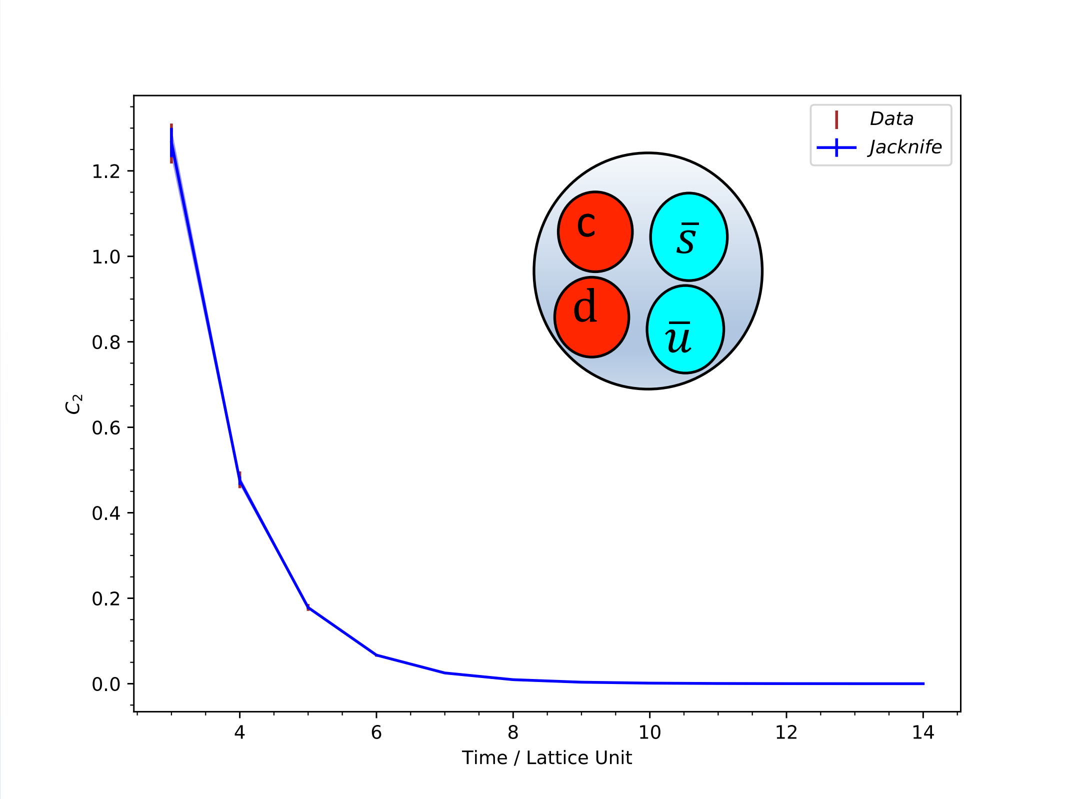
Many brilliant results in the past decades, but I will show some toy results.

Some TOY results: 2-point correlation

$O_\pi = \bar{u}\gamma_5 d$



$O_{scud} = [\bar{u}\gamma_5 c][\bar{s}\gamma_5 d]$

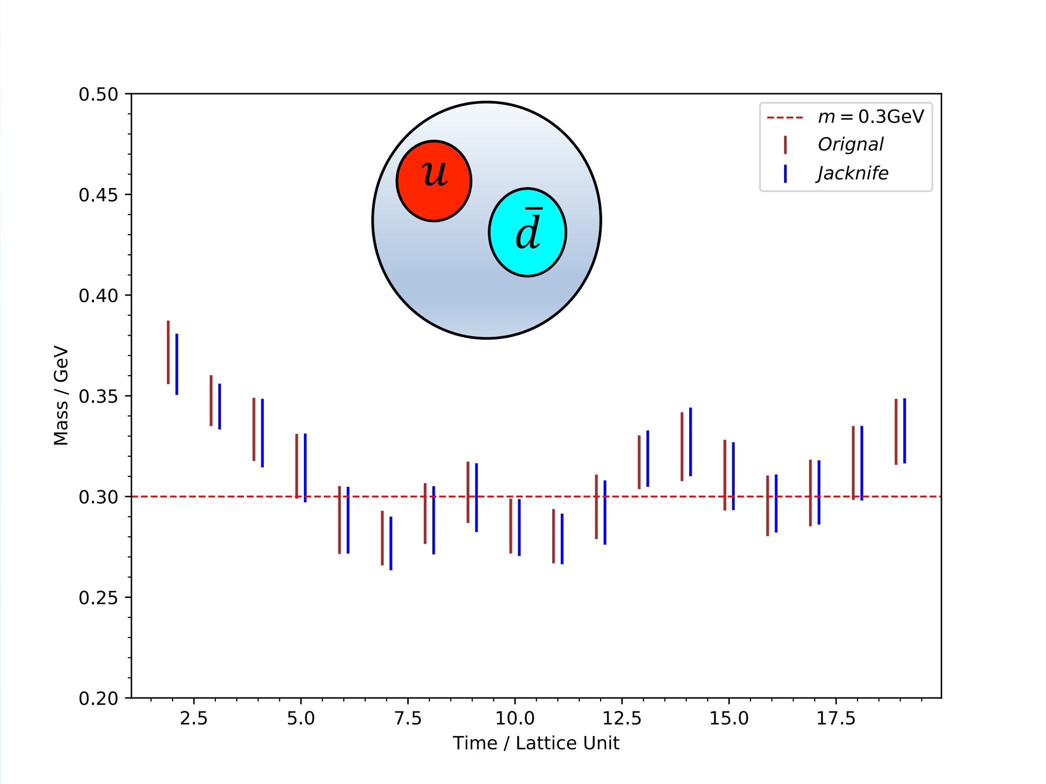


$C_2 = < 0|O_i O_i^+|0 >$

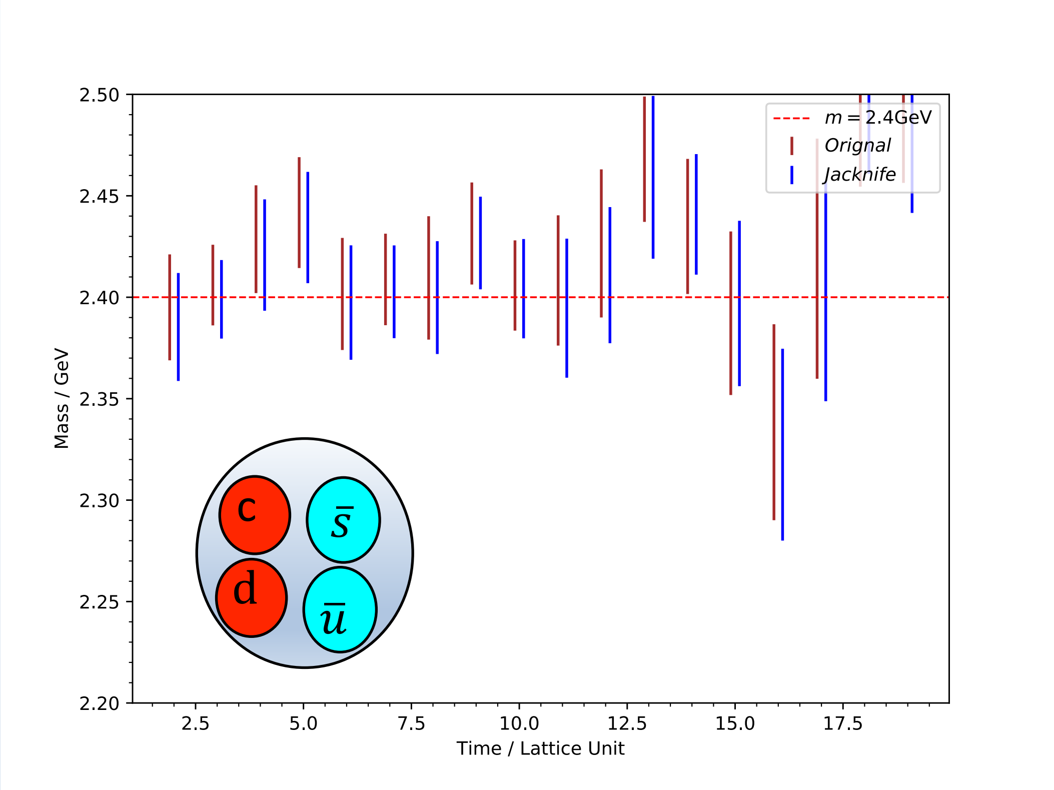
20 configurations

Some TOY results: 2-point correlation

$O_\pi = \bar{u}\gamma_5 d$



$O_{scud} = [\bar{u}\gamma_5 c][\bar{s}\gamma_5 d]$



$$m = \log \frac{C_2(t)}{C_2(t+1)}$$

20 configurations

Hadron spectrum from 2pt: conventional approach

- Choose the operator O_α with correct quantum number
- Calculate correlation function matrix

$$C_{\alpha\beta}(t) = < \Omega|O_\alpha(t)O_\beta^\dagger(0)\,|\Omega >$$

- Calculate the eigenvalue of $C_{\alpha\beta}(t)$ as $e^{-E_i t}$, E_i as discrete energy

Below the threshold, a bound state, direct corresponding to the hadron states,

such as proton, pion

Above the threshold, just the discrete energy of continuum scattering state,

not just the energy of hadron, such as ρ， N*

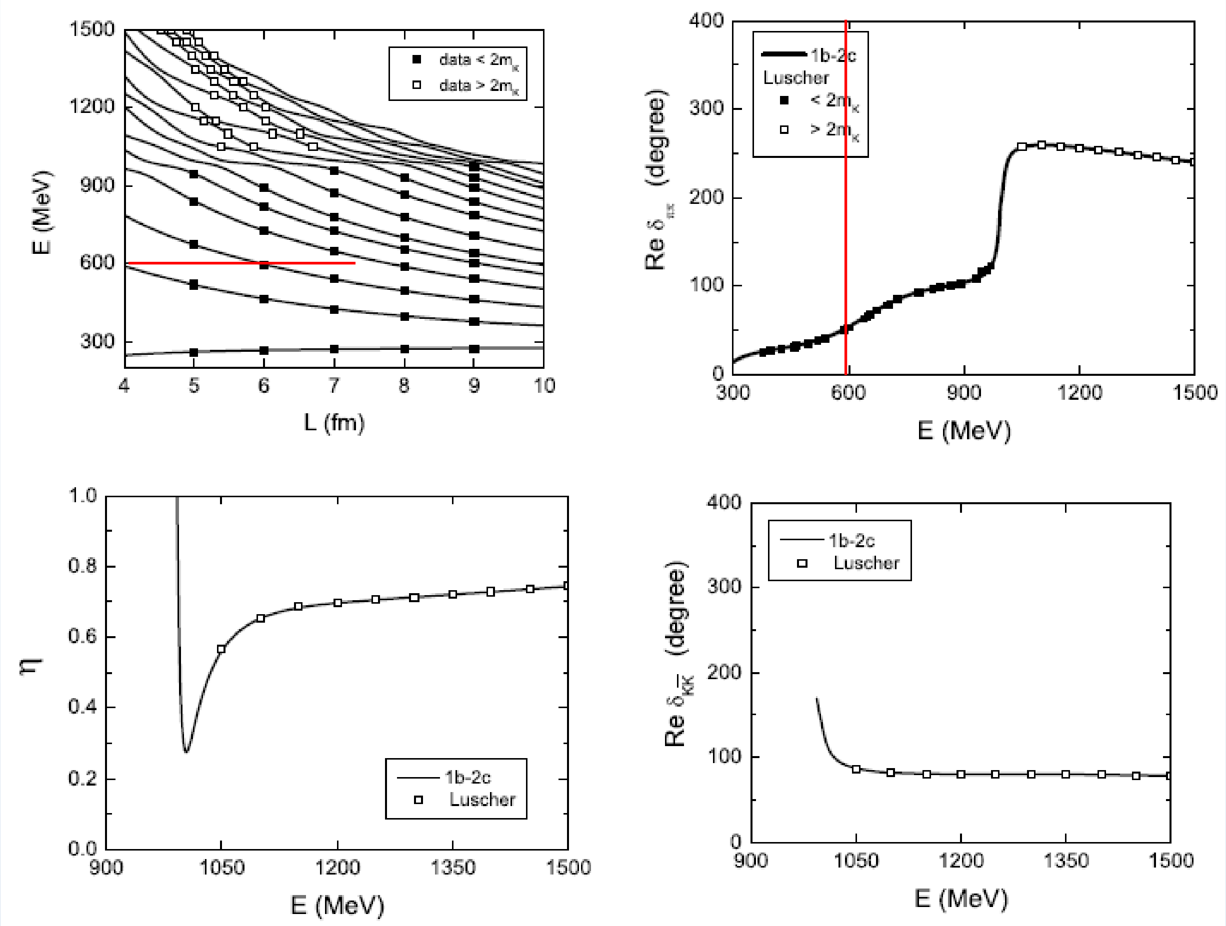
[Note: These states can roughly corresponding to the hadron state just for the small lattice size since it maybe only one state around the hadron energy and therefore takes the most information of hadron]

Question: How to get the physical information exactly from these energy levels ?

Lüscher Formula

M. Lüscher:
Commun. Math. Phys., 105:153-188, 1986.
Commun. Math. Phys., 104:177, 1986.
Nucl. Phys., B354:531-578, 1991.

ππ –> ππ & ππ –> $\bar{K}K$ & $\bar{K}K$–> $\bar{K}K$



$\bar{K}K$ 阈下

L —— E —— $\delta_{\pi\pi}(E)$ One to One

$$\delta_{\pi\pi}(k) = \Delta_{\pi\pi}(L) \bmod \pi$$

$$\Delta_\alpha(L) = \tan^{-1}\left(\frac{q_\alpha \pi^{3/2}}{Z_{00}(1, q_\alpha^2)}\right) \qquad q_\alpha = \sqrt{\frac{E(L)^2}{4} - m_\alpha^2}$$

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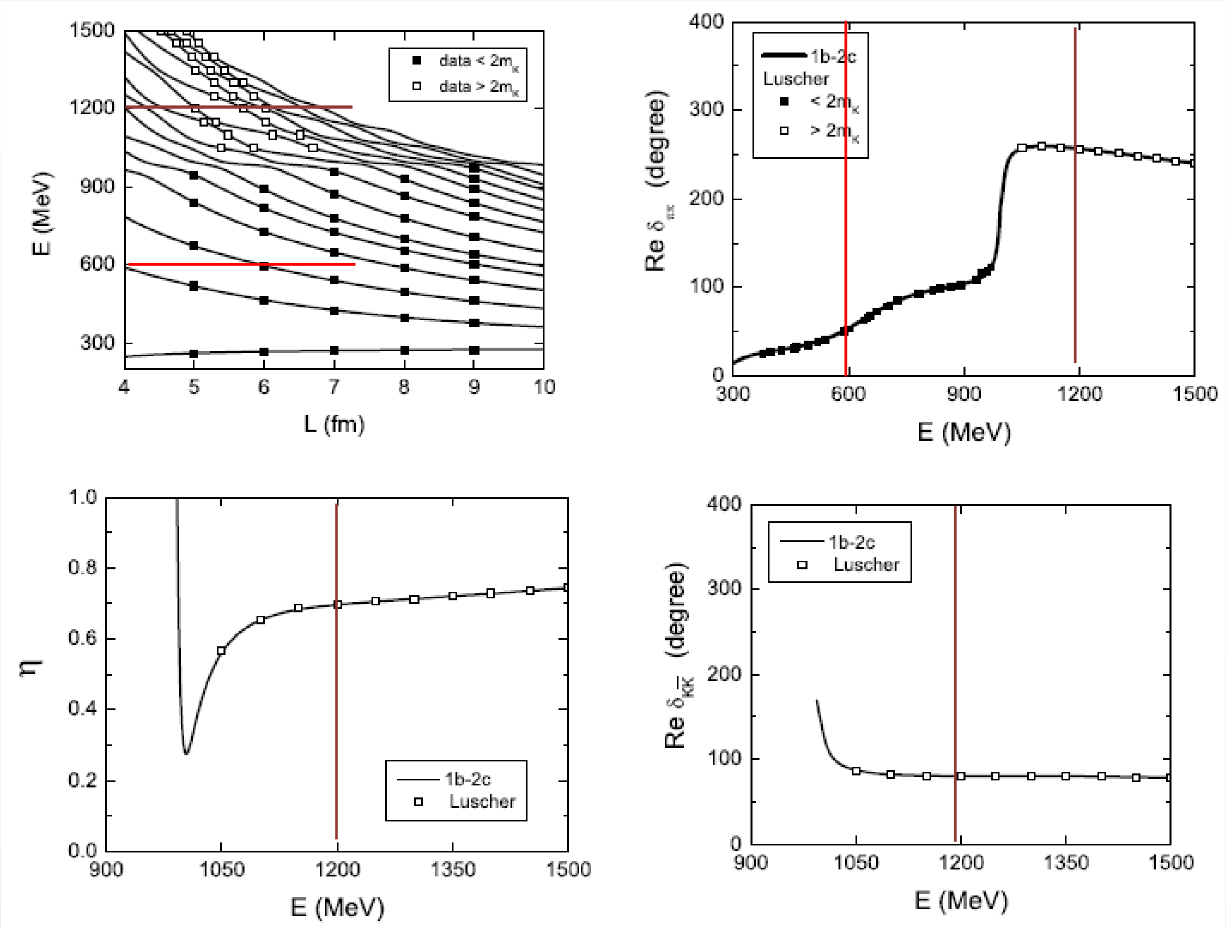

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$\bar{K}$K 阈上

L_1, L_2, L_3 —— E Many to One

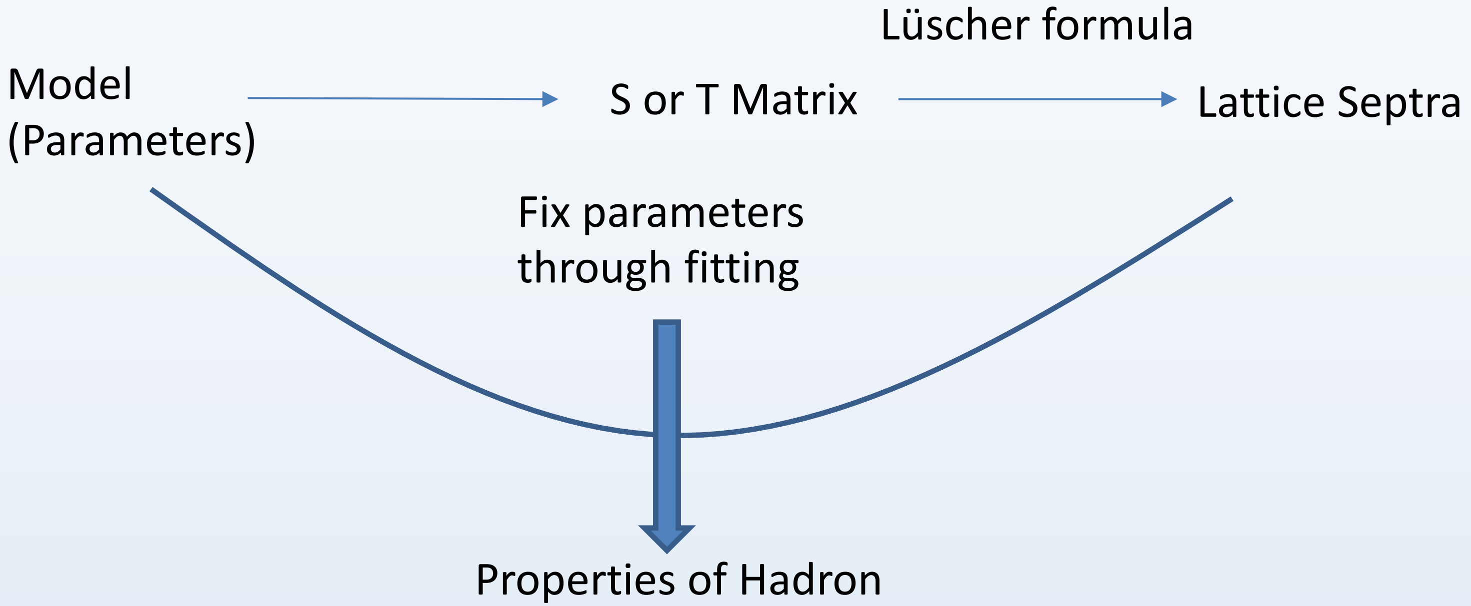
$\delta_{\pi\pi}(E)$, $\delta_{K\bar{K}}(E)$, $\eta(E)$

To find many Lattice size for the same E is almost impossible now.

$$0 = \cos\left(\Delta_{\pi\pi}(L) + \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) - \delta_{K\bar{K}}(E)\right) - \eta\cos\left(\Delta_{\pi\pi}(L) - \Delta_{K\bar{K}}(L) - \delta_{\pi\pi}(E) + \delta_{K\bar{K}}(E)\right)$$

S.He,X.Feng,andC.Liu, JHEP 07(2005)011

- Lüscher formula + Model（such as K matrix）



- 文献: (Jlab 强子组)

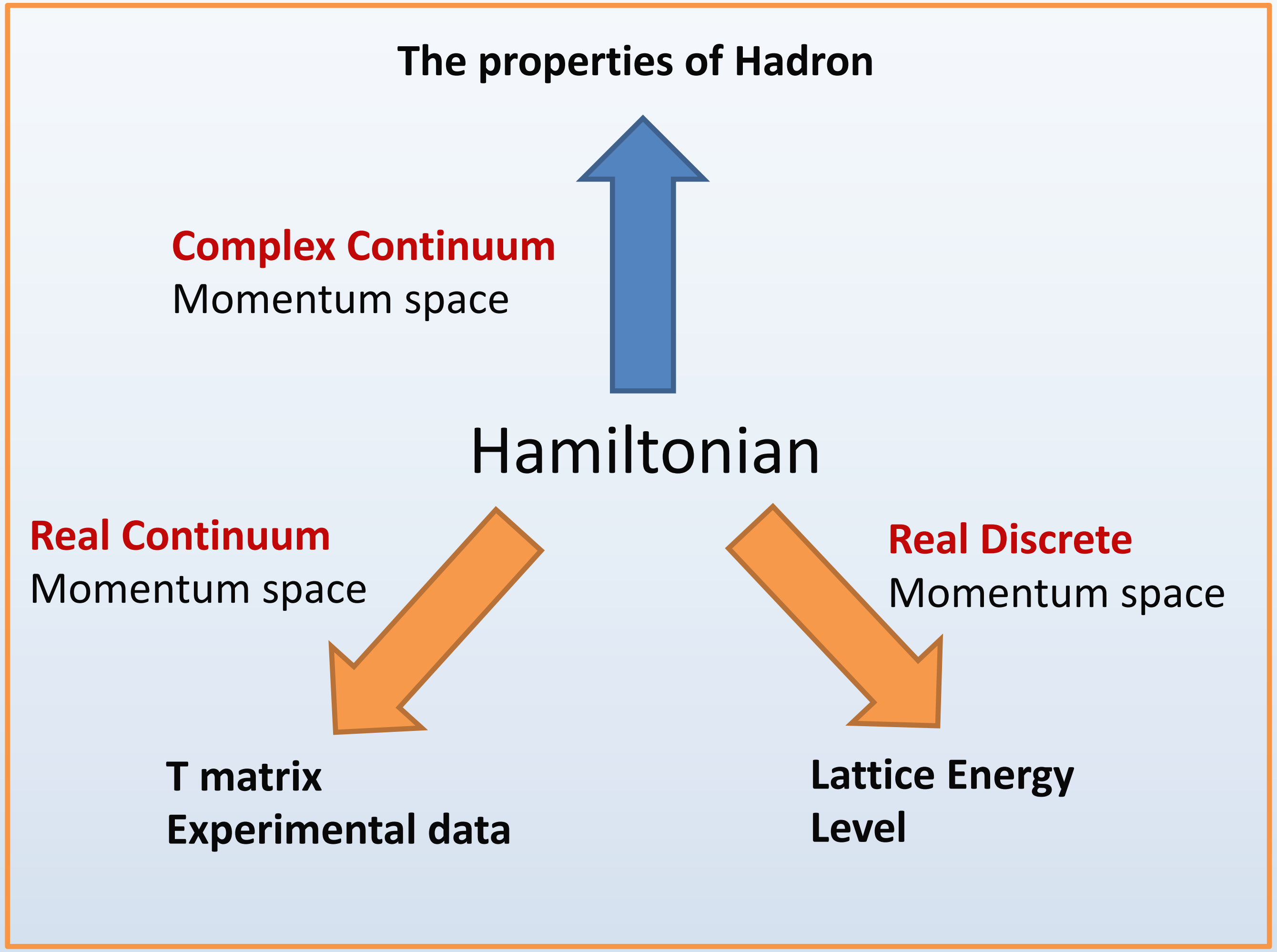
J.J. Dudek ect., PRL 113, 182001 (2014); PRD 93, 094506 (2016);PRD 92, 094502 (2015); PRD 93, 094506 (2016).

G. Moir, M. Peardon, S.M. Ryan, C.E. Thomas, and D.J. Wilson, J. High Energy Phys. 10 (2016) 011

Non-Luscher Formula

- Hamiltonian Approach (HEFT)
- T matrix method
- Hal QCD

哈密顿有效方法介绍 (HEFT)



- Hamiltonian Matrix for discrete momentum

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0 + H_I)\,|\Psi> = E\,|\Psi>$$

Eigen-Value ⟷ **Lattice Spectrum**

Eigen-Vector

文献: (Adelaide and UCAS 强子组)

哈密顿有效方法介绍 (HEFT)

1. **Finite-volume matrix Hamiltonian model for a Δ→Nπ system**
   J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W.Thomas, R.D. Young
   Phys.Rev. D87 (2013) no.9, 094510
2. **Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD**
   J-J Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young
   Phys.Rev. C90 (2014) no.5, 055206
3. **Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev.Lett. 116 (2016) no.8, 082004
4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
   J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young
   Phys.Rev.Lett. 114 (2015) no.13, 132002
5. **Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD**
   Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.3, 034034
6. **Structure of the Λ(1405) from Hamiltonian effective field theory**
   Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu
   Phys.Rev. D95 (2017) no.1, 014506
7. **Nucleon resonance structure in the finite volume of lattice QCD**
   Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas
   Phys.Rev. D95 (2017) no.11, 114507
8. **Structure of the Roper Resonance from Lattice QCD Constraints**
   Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W. Thomas
   Phys.Rev. D97 (2018) no.9, 094509
9. **Partial Wave Mixing in Hamiltonian Effective Field Theory**
   Yan Li, Jia-jun Wu, C. D. Abell, Derek B. Leinweber, Anthony W. Thomas
   Phys.Rev. D101 (2020) no.11, 114501

Main point is try to understand N*(1535), N*(1440), Λ(1405)

From the eigenvector of Hamiltonian to check the compositeness of these state, and make the conclusion,

N*(1535) N*(1440), Λ(1405)

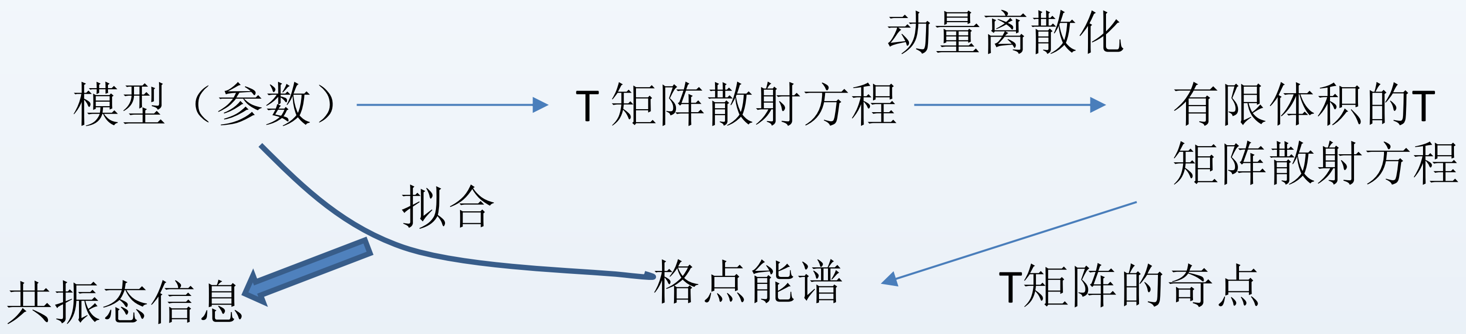
The main components (at least 50%) of N*(1535) is from the 3 quark core.

The Λ*(1405) is predominantly a molecular $\bar{K}$ N bound State.

The N*(1440) seems a rescattering state.

T matrix method

- 直接寻找有限体积下的T矩阵的奇点
- 基本思路是：



文献: (Valencia, GWU, Bonn, 北航)

M. Döring, U.-G. Meissner, E. Oset, and A. Rusetsky, EPJA 47, 139 (2011),
M. Döring, U.-G. Meissner, E. Oset, and A. Rusetsky, EPJA 48, 114 (2012),
A.M. Torres, L.R. Dai, C. Koren, D. Jido, and E. Oset, PRD 85, 014027 (2012),
M. Döring and U.-G. Meissner, JHEP. 01 (2012) 009,
M. Döring,M.Mai,andU.-G.Meissner, PLB722,185 (2013),
L.S. Geng, X.L. Ren, Y. Zhou, H.X. Chen, and E. Oset, PRD 92, 014029 (2015).
Dan Zhou, E.-L. Cui, H.-X. Chen, L.-S. Geng, L.-H. Zhu PRD 91 (2015) no.9,094505
R. Molina, M. Döring PRD94 (2016) no.5, 056010,
Dehua Guo, A. Alexandru, R. Molina, M. Döring PRD94 (2016) no.3, 034501

HAL QCD 方法

- HAL QCD 方法: *N. Ishii, S. Aoki, T. Hatsuda PRL 99 022001(2007)*
- 基本思路是:

$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$

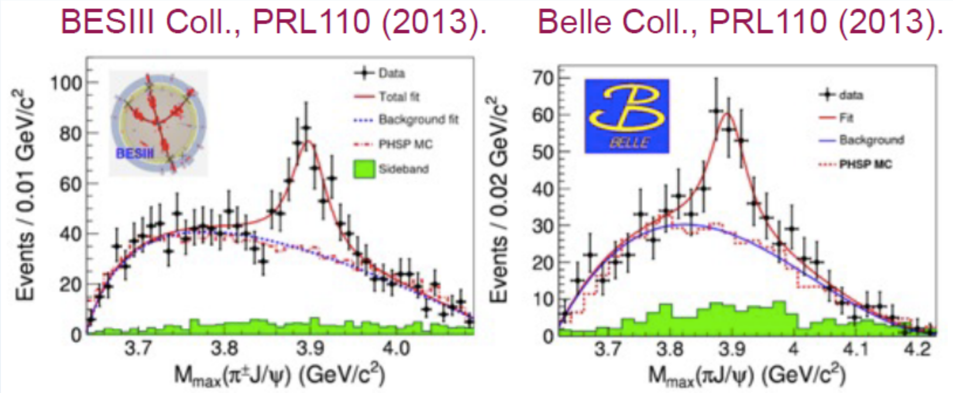
格点的本征波函数，
等价于NBS 波函数 → 时间无关的非定域势能 → S 矩阵

$$\langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\Phi^\dagger(0)|0\rangle = \sqrt{Z_1Z_2}\sum_n A_n\psi_n(\vec{r})e^{-W_n\tau}$$

时间无关的非定域势能 ↓

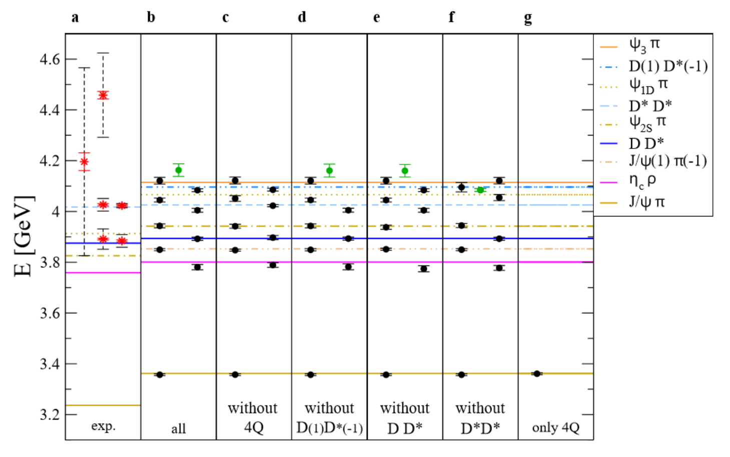
共振态信息

Some Examples： Zc states



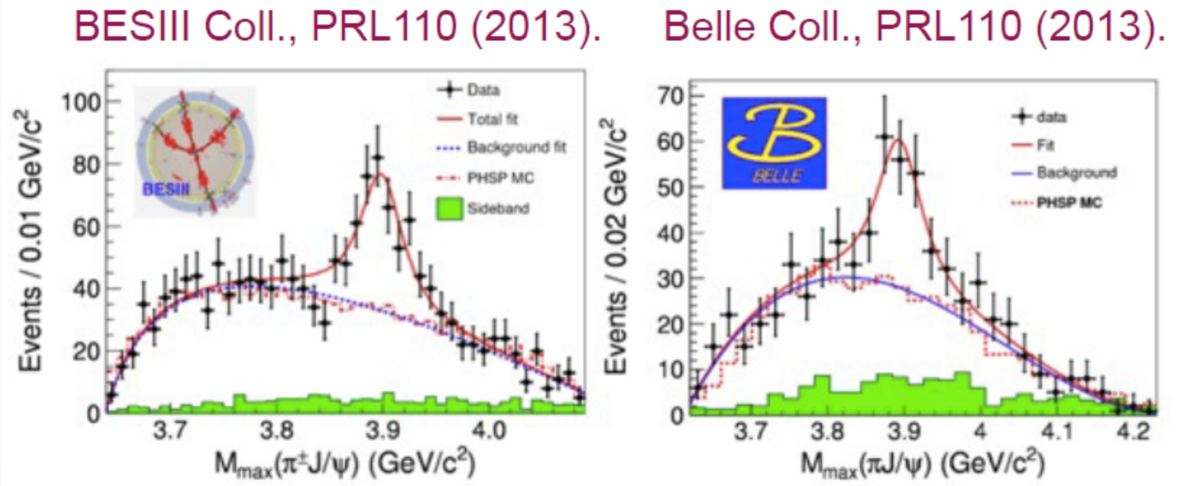
Lattice studies :

1. $\boldsymbol{DD^*}$bar scattering (single channel)
   - $\boldsymbol{Zc}$ **3900** : (CLQCD), PRD89(2014)094506
   - $\boldsymbol{Zc}$ **4020** : (CLQCD), PRD92(2015)054507
   - $\boldsymbol{Zc}$ **4430** : (CLQCD), PRD 70(2009) 034503, 93(2016)114501
2. $\boldsymbol{DD^*}$bar + $\boldsymbol{J/\psi\ \pi}$ (2 channel)
   - $\boldsymbol{Zc}$(**3900**) (CLQCD), CPC43 (2019) 103103
3. $\boldsymbol{DD^*}$bar + $\boldsymbol{J/\psi\ \pi}$ +$\boldsymbol{\eta_c \rho}$ (3 channel) … to continue …
   - $\boldsymbol{Zc}$(**3900**) C. Liu et al., PRD101(2020) , 054502
4. Spectroscopy study (S. Prelovsek et al. PRD91(2015)014504)
5. Potential matrix and scattering amplitudes
   (HAL Collab.), PRL117(2016) 242001



interpolating fields. For our pion mass of 266 MeV we find all the expected two-meson states but no additional candidate for Z_c^+ below 4.2 GeV. Possible reasons for not seeing an additional eigenstate

Zc(3900): Still not clear!!!

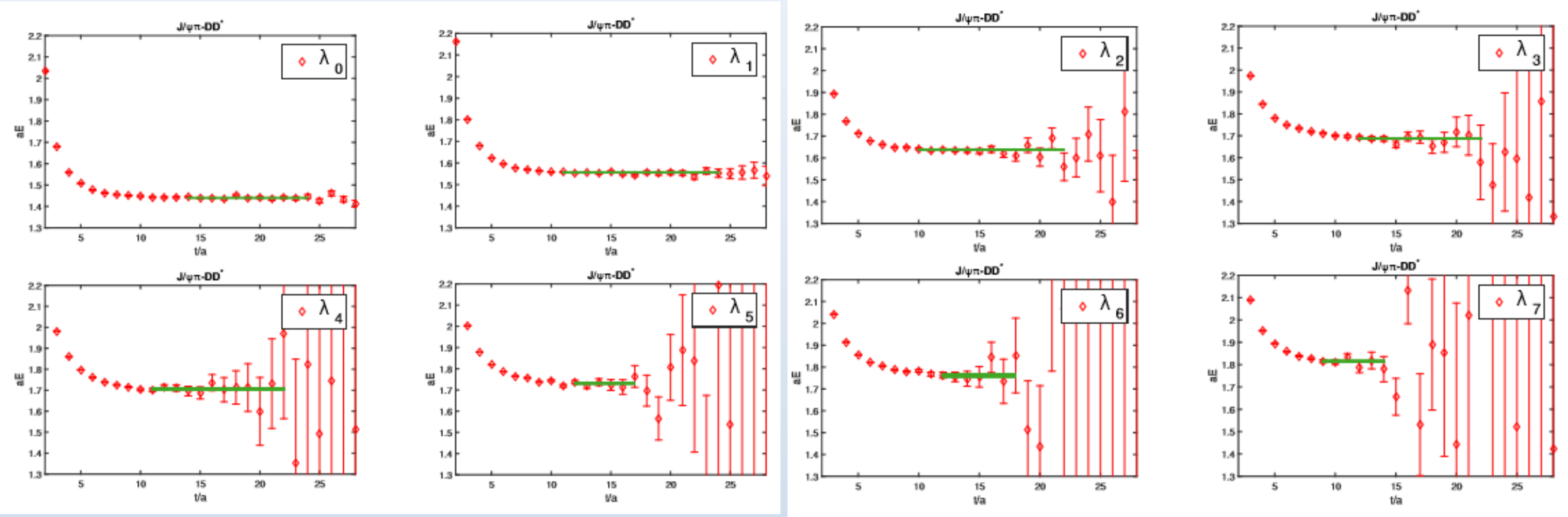


Lattice studies: $Zc(3900)$

1. DD^*bar: (CLQCD), PRD89(2014)094506
2. DD^*bar + $J/\psi\ \pi$: (CLQCD), CPC43 (2019) 103103
3. DD^*bar + $J/\psi\ \pi$ + $\eta_c\rho$ (3 channel) … to continue …

   C. Liu et al., PRD101(2020) , 054502
4. Potential matrix and scattering amplitudes

   (HAL Collab.), PRL117(2016) 242001



- Unlike what the HALQCD collaboration finds out, our results do not support a narrow resonance-like peak close to the threshold by taking into account the most relevant two coupled channels in the problem.
- However, one has to keep in mind that we have not estimated the systematic uncertainties. All error estimates are purely statistical and the systematics could be due to finite lattice spacing, non-physical pion and charm quark masses, finite volume effects, etc. which needs to be clarified in future studies.

CPC43 (2019) 103103

Conclusion

- Lattice QCD can provide a systematic approach to study hadron spectrum.
- Many progresses have been made in the past years.
- To draw a definite conclusion on the internal structure, many more detailed (mostly expensive) analyses are requested.

Thank you for your attention.