

北京航空航天大学
BEIHANG UNIVERSITY



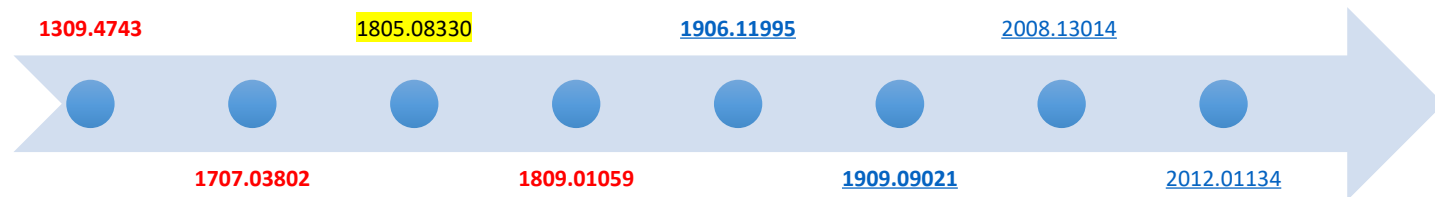
How to test the picture of

Hadronic Molecules

from 2 to 3

Lisheng Geng (耿立升) @ Beihang U.

lisheng.geng@buaa.edu.cn



Contents

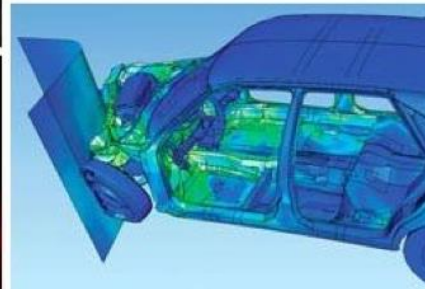
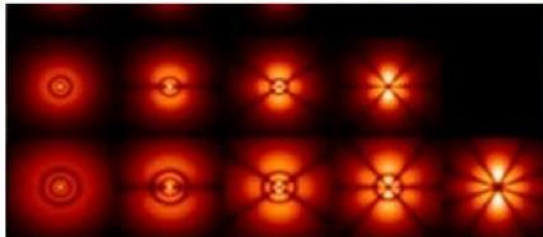
- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$ and $Ds1(2460)$ as DK/D^*K molecules: theory & lattice
- DDK molecule: $R^{++}(4140)$
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- Where to search for these 3-body molecules
- Summary and outlook

Contents

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$ and $Ds1(2460)$ as DK/D^*K molecules: theory & lattice
- DDK molecule: $R^{++}(4140)$
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- Where to search for these 3-body molecules
- Summary and outlook

Reductionism vs. Emergence

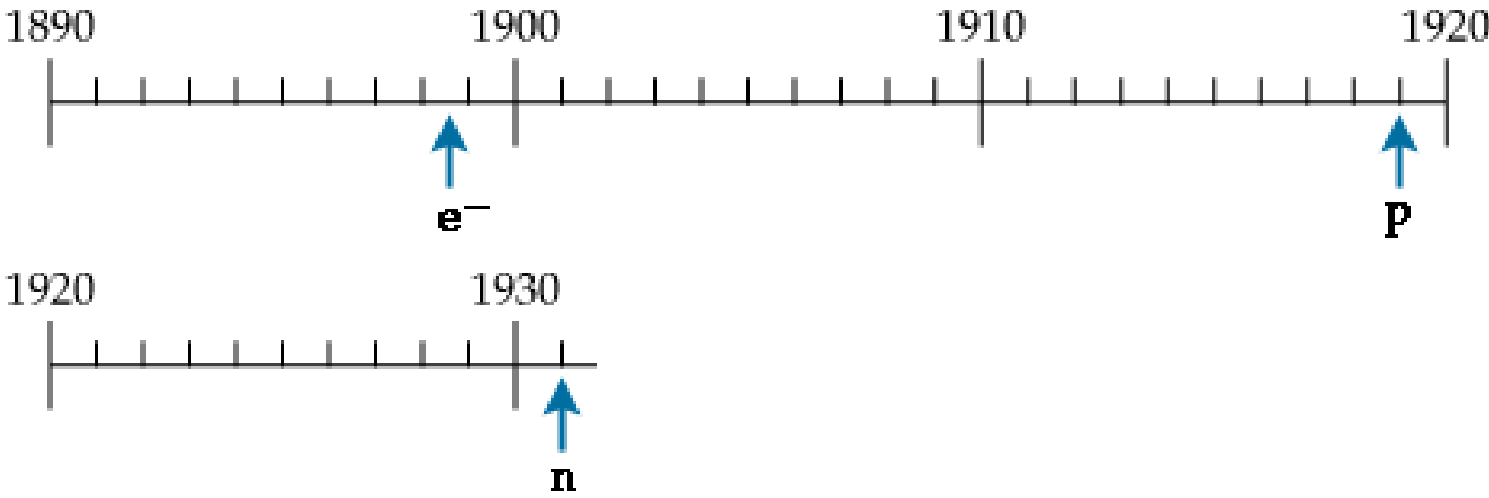
Top
Down



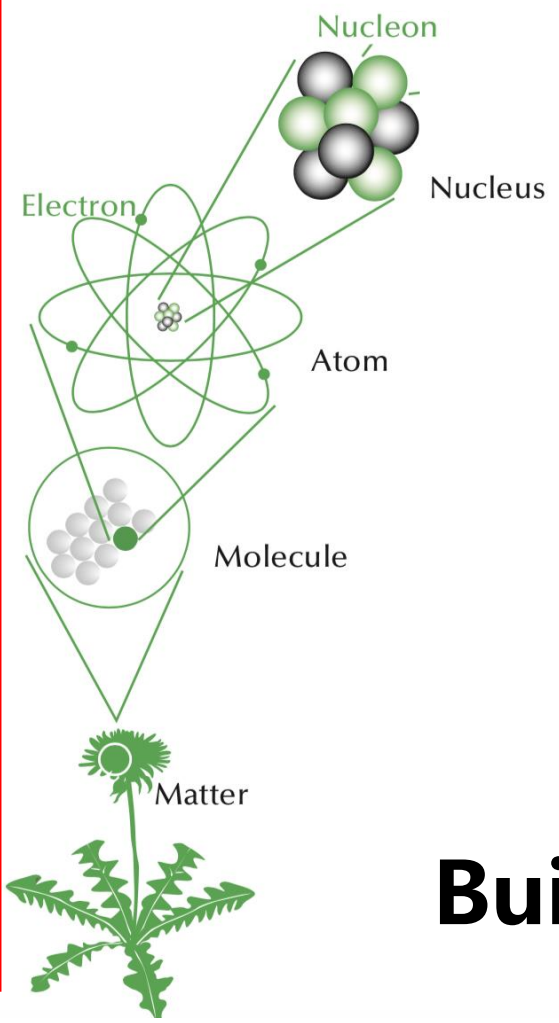
Bottom
Up

Search for the fundamental building blocks

Particles discovered before 1932



10^{-15}
 10^{-14}
 10^{-10}
 10^{-6}
 10^{-1}



IUPAC Periodic Table of the Elements

1																		2																		13																		14																		15																		16																		17																		18																																																																																																																																																																																																					
1 H Hydrogen 1.00784(7)																		2 He Helium 4.002602																		5 B Boron 10.811																		6 C Carbon 12.011																		7 N Nitrogen 14.00643																		8 O Oxygen 15.999																		9 F Fluorine 18.9984032																		10 Ne Neon 20.1797																																																																																																																																																																																																					
3 Li Lithium 6.941																		4 Be Beryllium 9.0122																		11 Na Sodium 22.98976928																		12 Mg Magnesium 24.304																		13 Al Aluminum 26.9815386																		14 Si Silicon 28.0855																		15 P Phosphorus 30.973762																		16 S Sulfur 32.06																		17 Cl Chlorine 35.45																		18 Ar Argon 39.948																																																																																																																																																																	
19 K Potassium 39.0983																		20 Ca Calcium 40.078																		21 Sc Scandium 44.955912																		22 Ti Titanium 47.867																		23 V Vanadium 50.9415																		24 Cr Chromium 51.9961																		25 Mn Manganese 54.938045																		26 Fe Iron 55.845																		27 Co Cobalt 58.933195																		28 Ni Nickel 58.6934																		29 Cu Copper 63.546																		30 Zn Zinc 65.38																		31 Ga Gallium 69.723																		32 Ge Germanium 72.6305																		33 As Arsenic 74.9216																		34 Se Selenium 78.9718																		35 Br Bromine 79.904																		36 Kr Krypton 83.796																	
37 Rb Rubidium 85.468																		38 Sr Strontium 87.62																		39 Y Yttrium 88.90584																		40 Zr Zirconium 91.224																		41 Nb Niobium 92.90638																		42 Mo Molybdenum 95.94																		43 Tc Technetium 98																		44 Ru Ruthenium 101.072																		45 Rh Rhodium 102.9055																		46 Pd Palladium 106.42																		47 Ag Silver 107.8682																		48 Cd Cadmium 112.411																		49 In Indium 114.818																		50 Sn Tin 118.710																		51 Sb Antimony 121.757																		52 Te Tellurium 127.6																		53 I Iodine 126.905																		54 Xe Xenon 131.29																	
55 Cs Cesium 132.90545196																		56 Ba Barium 137.327																		57-71 Lanthanoids																		72 Hf Hafnium 178.49																		73 Ta Tantalum 180.94788																		74 W Tungsten 183.84																		75 Re Rhenium 186.207																		76 Os Osmium 190.23																		77 Ir Iridium 192.222																		78 Pt Platinum 195.084																		79 Au Gold 196.966569																		80 Hg Mercury 200.59																		81 Tl Thallium 204.3833																		82 Pb Lead 207.2																		83 Bi Bismuth 208.9804																		84 Po Polonium 209																		85 At Astatine 210																		86 Rn Radon 222																	
87 Fr Francium 223																		88 Ra Radium 226																		89-103 Actinoids																		104 Rf Rutherfordium 261																		105 Db Dubnium 262																		106 Sg Seaborgium 263																		107 Bh Bohrium 264																		108 Hs Hassium 265																		109 Mt Meitnerium 266																		110 Ds Darmstadtium 267																		111 Rg Roentgenium 268																		112 Cn Copernicium 269																		113 Nh Nihonium 270																		114 Fl Flerovium 271																		115 Mc Moscovium 272																		116 Lv Livermorium 273																		117 Ts Tennessine 274																		118 Og Oganesson 276																	
57 La Lanthanum 138.90547																		58 Ce Cerium 140.12																		59 Pr Praseodymium 140.90766																		60 Nd Neodymium 144.242																		61 Pm Promethium 144.9127																		62 Sm Samarium 150.36																		63 Eu Europium 151.964																		64 Gd Gadolinium 157.25																		65 Tb Terbium 158.92534																		66 Dy Dysprosium 162.50015																		67 Ho Holmium 164.93032																		68 Er Erbium 167.259																		69 Tm Thulium 168.93048																		70 Yb Ytterbium 173.05468																		71 Lu Lutetium 174.96706																																																																							
89 Ac Actinium 227																		90 Th Thorium 232.0377																		91 Pa Protactinium 231.03688																		92 U Uranium 238.02891																		93 Np Neptunium 237																		94 Pu Plutonium 244																		95 Am Americium 243																		96 Cm Curium 247																		97 Bk Berkelium 247																		98 Cf Californium 251																		99 Es Einsteinium 252																		100 Fm Fermium 257																		101 Md Mendelevium 258																		102 No Nobelium 259																		103 Lr Lawrencium 260																																																																							

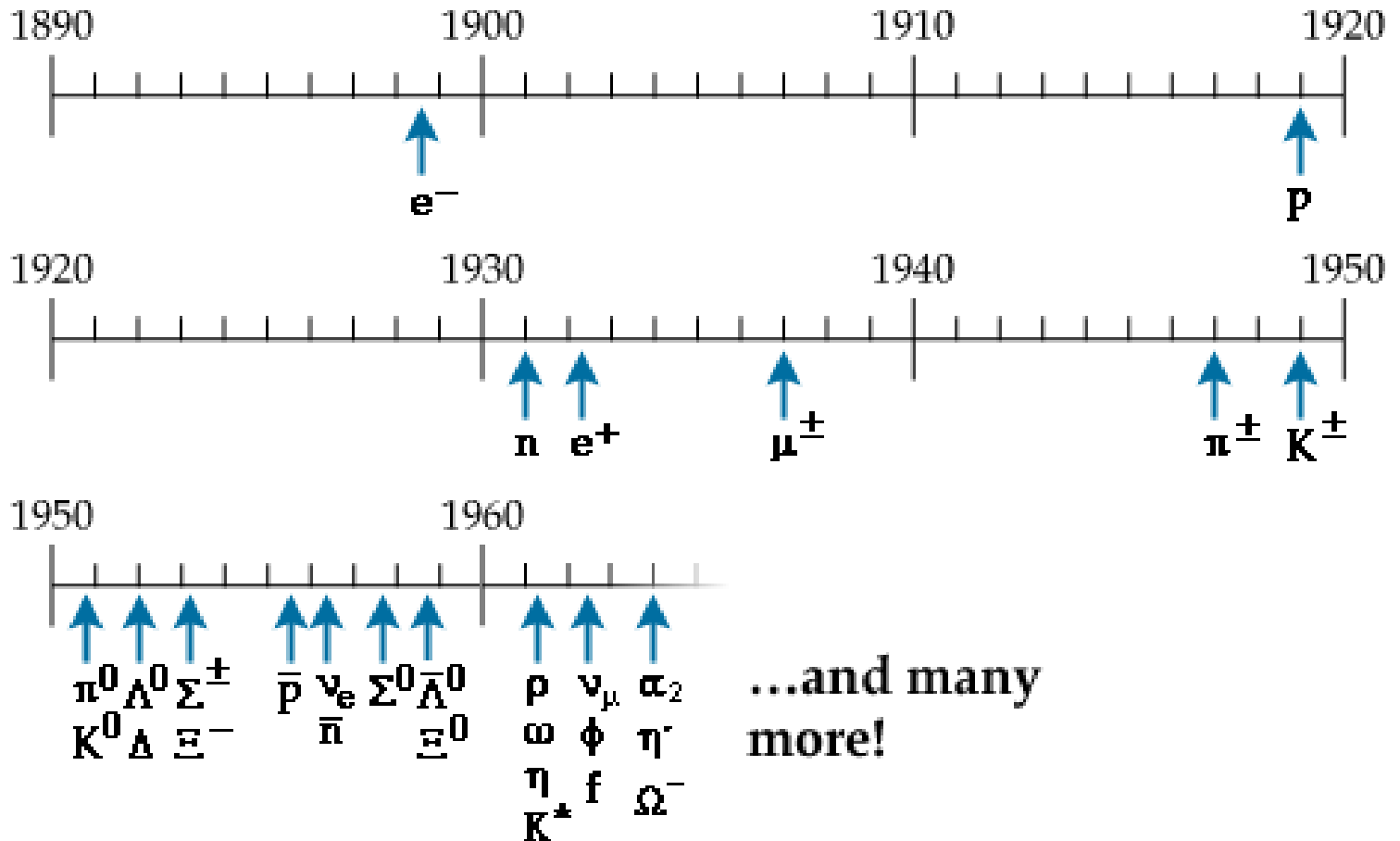


For notes and updates to this table, see www.iupac.org. This version is dated 1 December 2018. Copyright © 2018 IUPAC, the International Union of Pure and Applied Chemistry.



Building up the atomic world

Nothing good lasts for ever



Hadron spectroscopy—QM—QCD

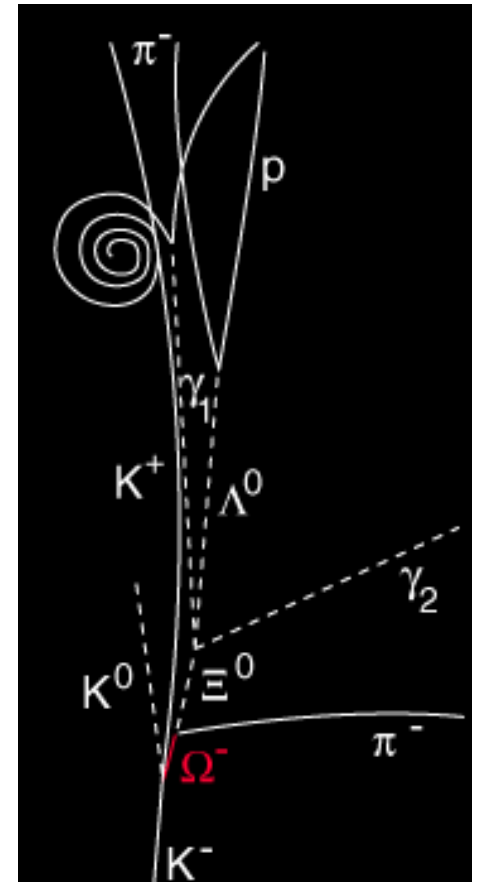
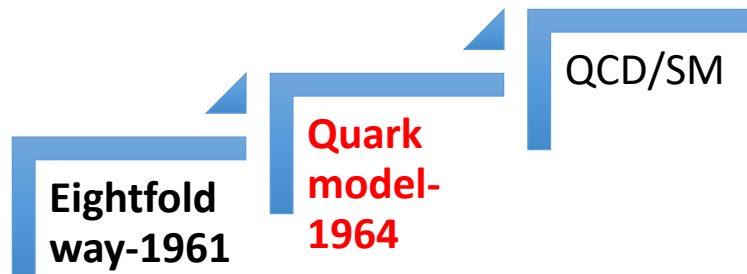
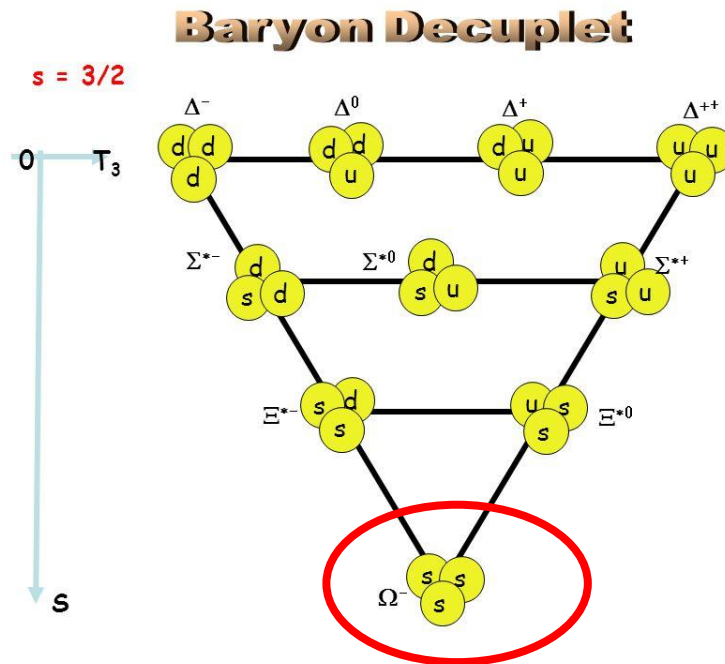
Put an end to the then chaotic situation



Murray Gell-Mann



Yuval Ne'eman.



V. E. Barnes et al., Phys. Rev. Lett. 12, 204 (1964)

More quarks, more leptons, SM completed at 2012

Quarks

u up	c charm	t top
d down	s strange	b bottom

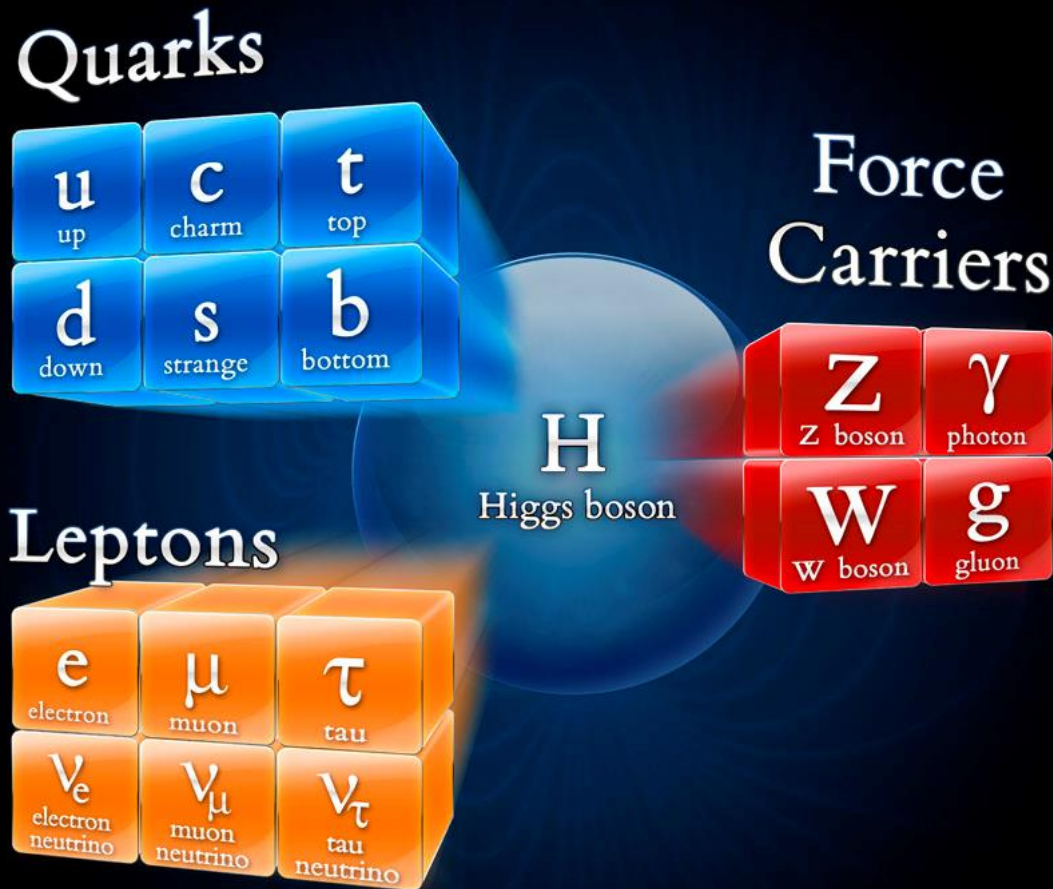
Leptons

e electron	μ muon	τ tau
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino

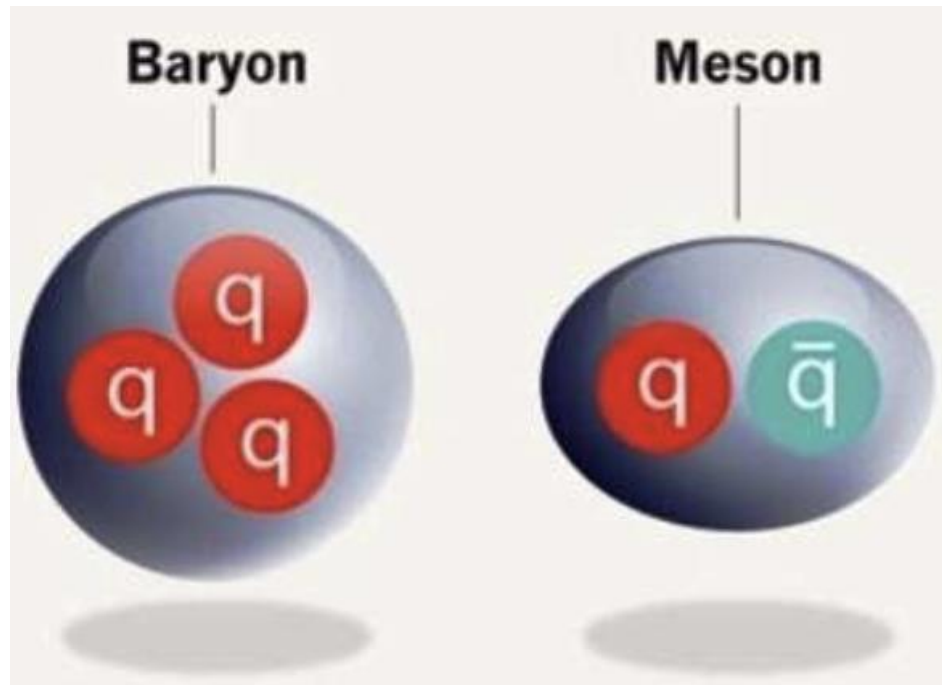
Force Carriers

Z Z boson	γ photon
W W boson	g gluon

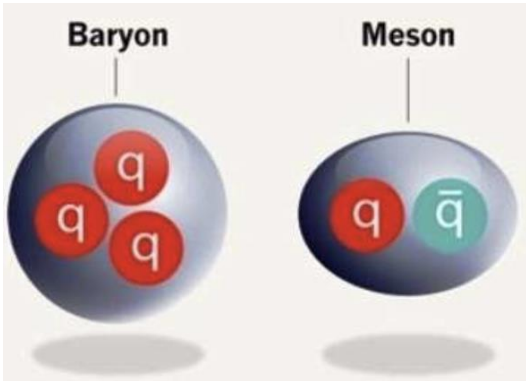
H
Higgs boson



Naive QM: hadron structure



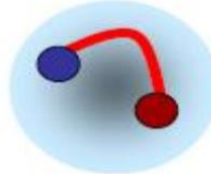
Beyond Naïve QM, more complicated structures allowed



In the naïve quark model

In principle, QCD **allows**

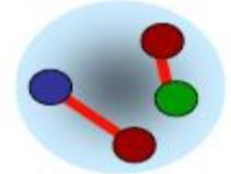
Hybrid



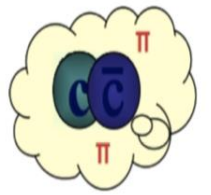
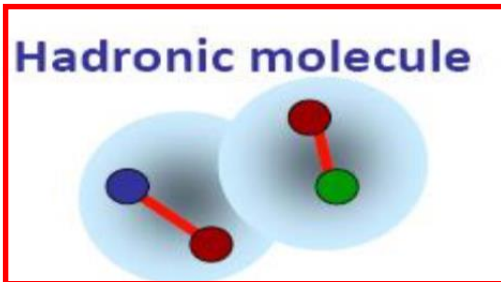
Glueball



Tetraquark

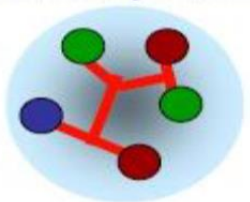


Hadronic molecule



Hadro-quarkonium

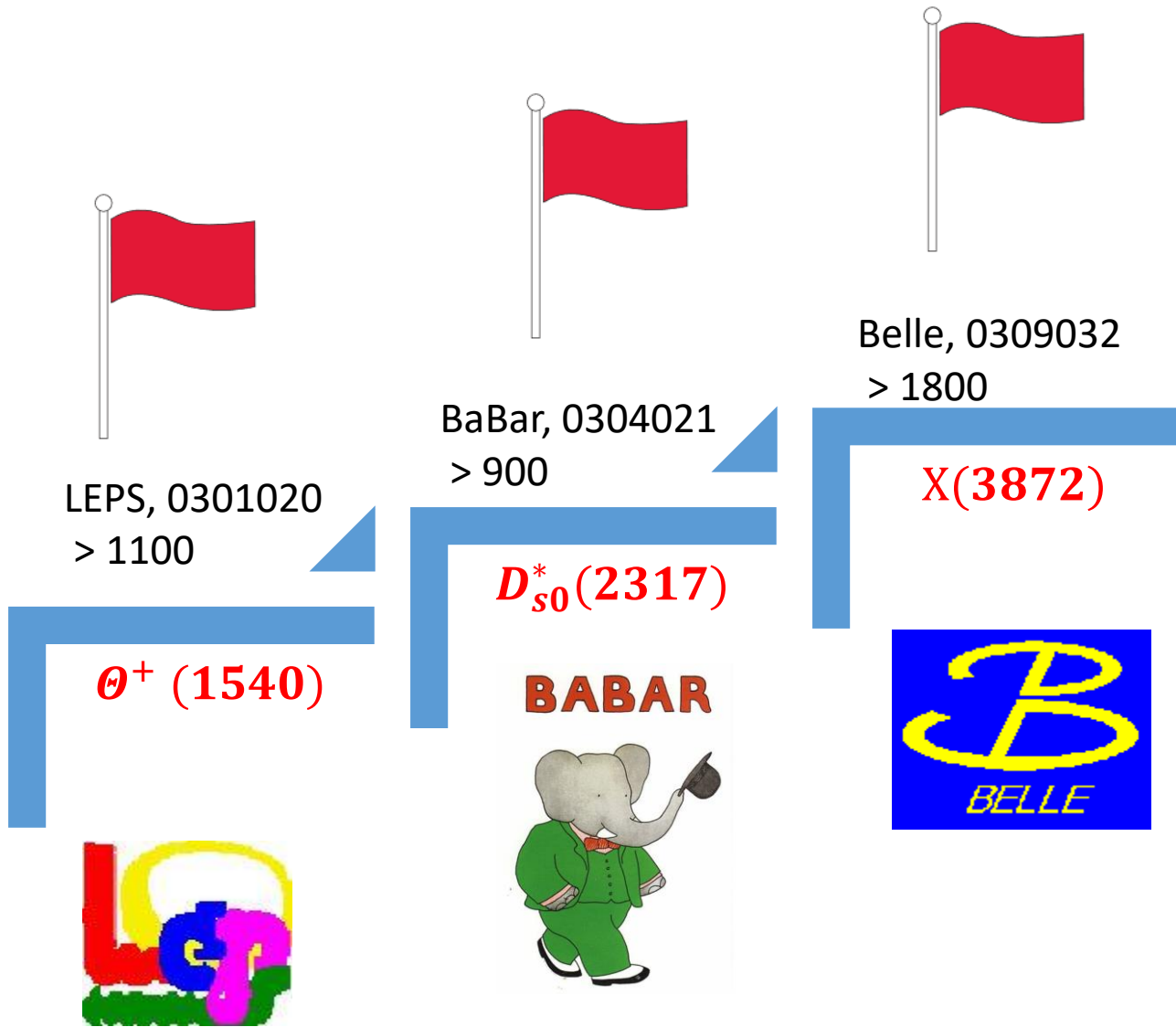
Pentaquark



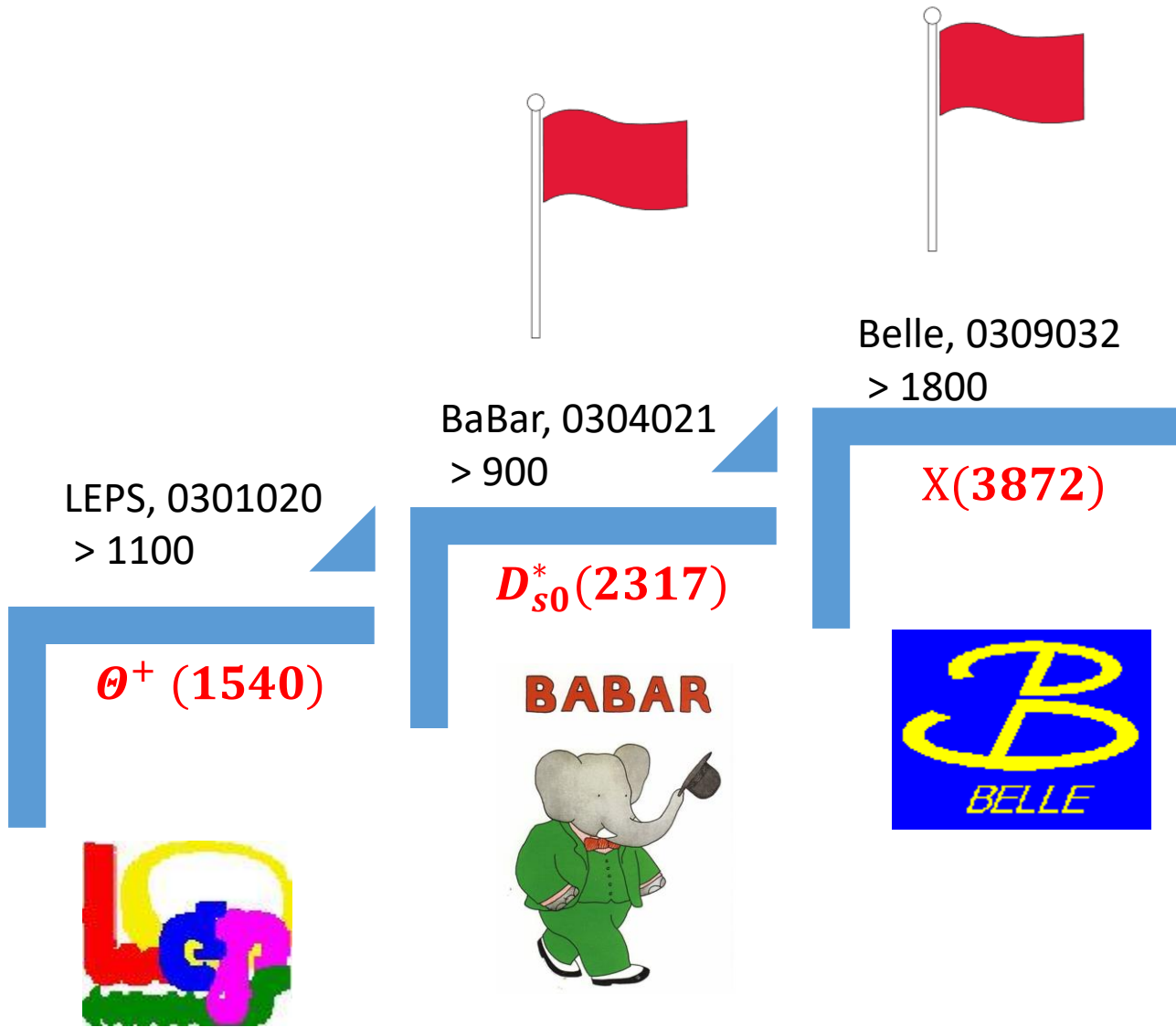
Not much happened until 2003

$\Lambda(1405)$, $N^*(1535)$, ...
 $f_0(500)$, $f_0(980)$, $a_0(980)$, ...

Beginning of a new era: 2003



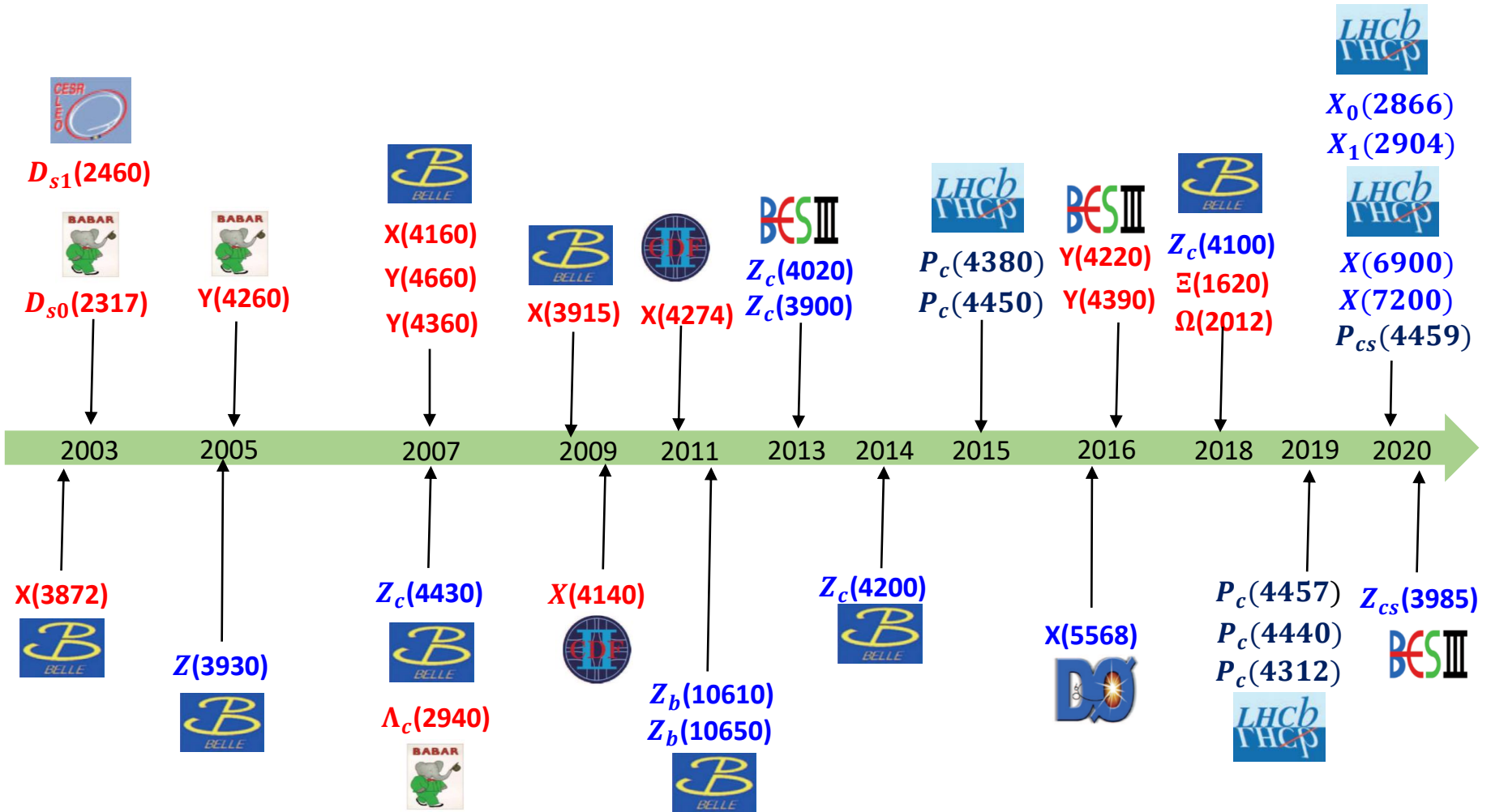
Beginning of a new era: 2003



Exotic mesons or baryons

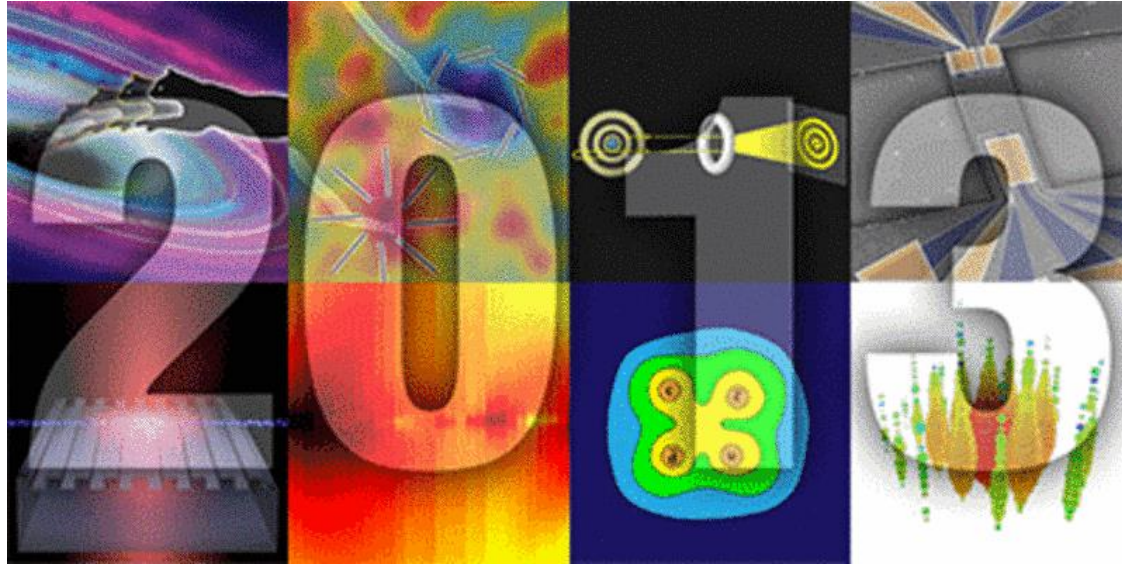
Tetraquark states

Pentaquark states



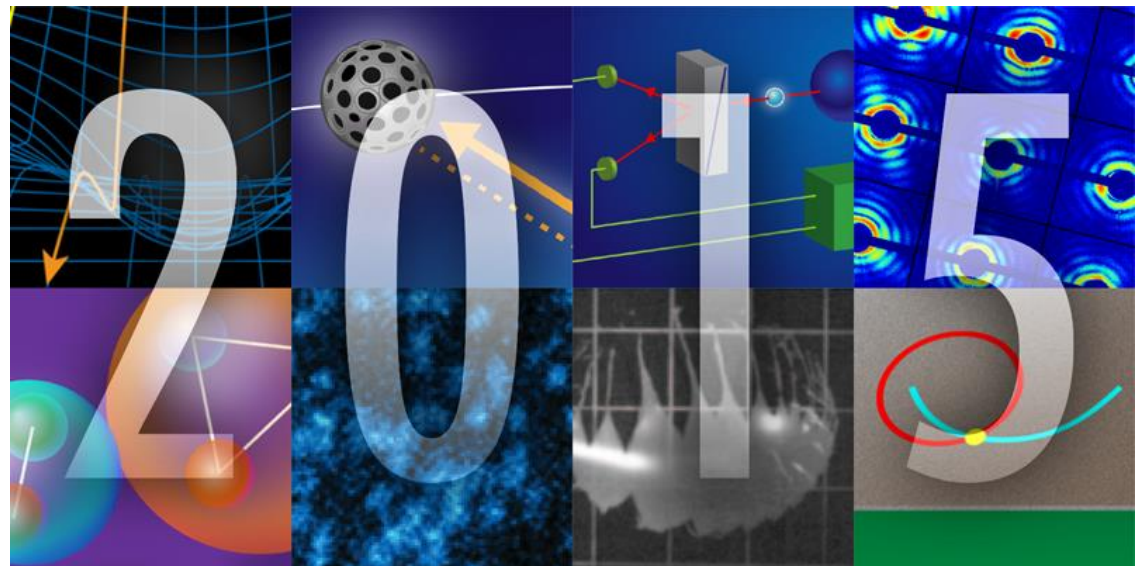
Highlights of the year

the research covered in Physics that **really made waves in and beyond the physics community.**

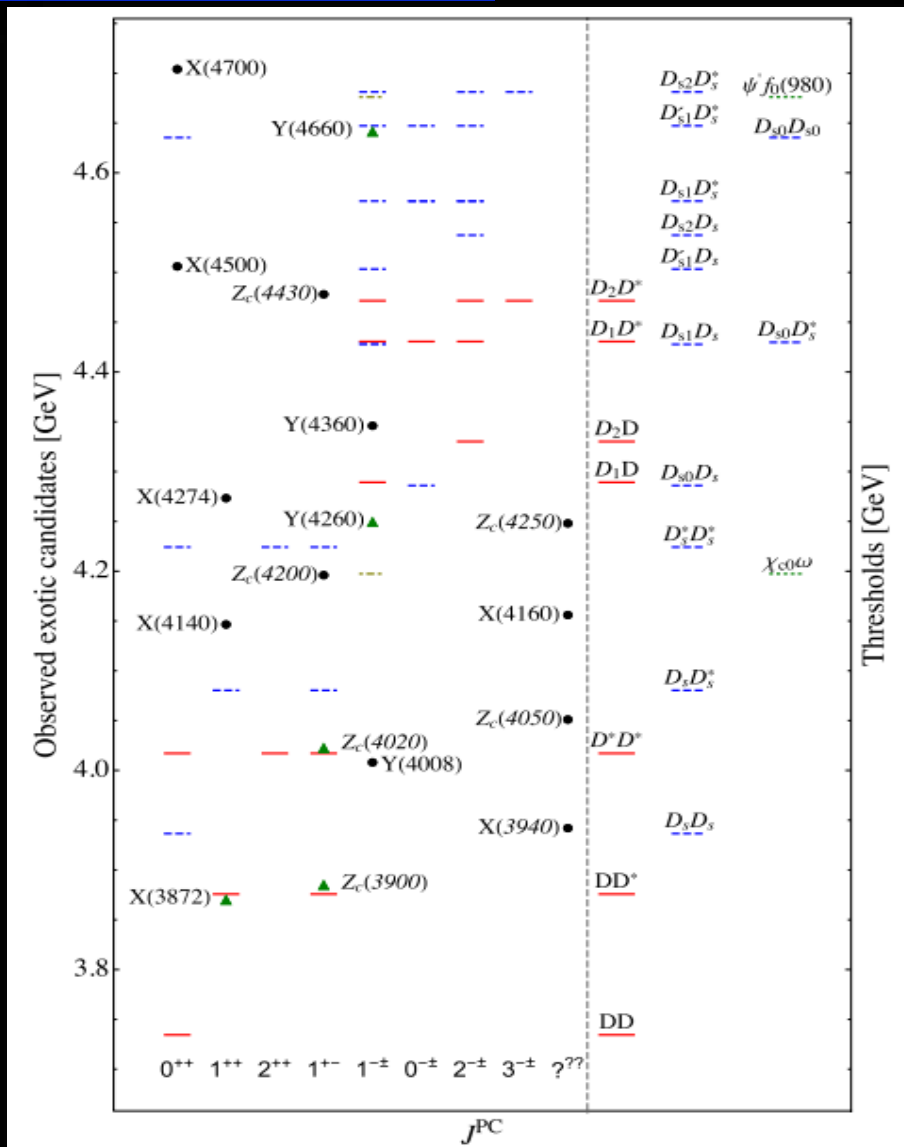


Four-Quark Matter/BESIII

Particle High Five/LHCb

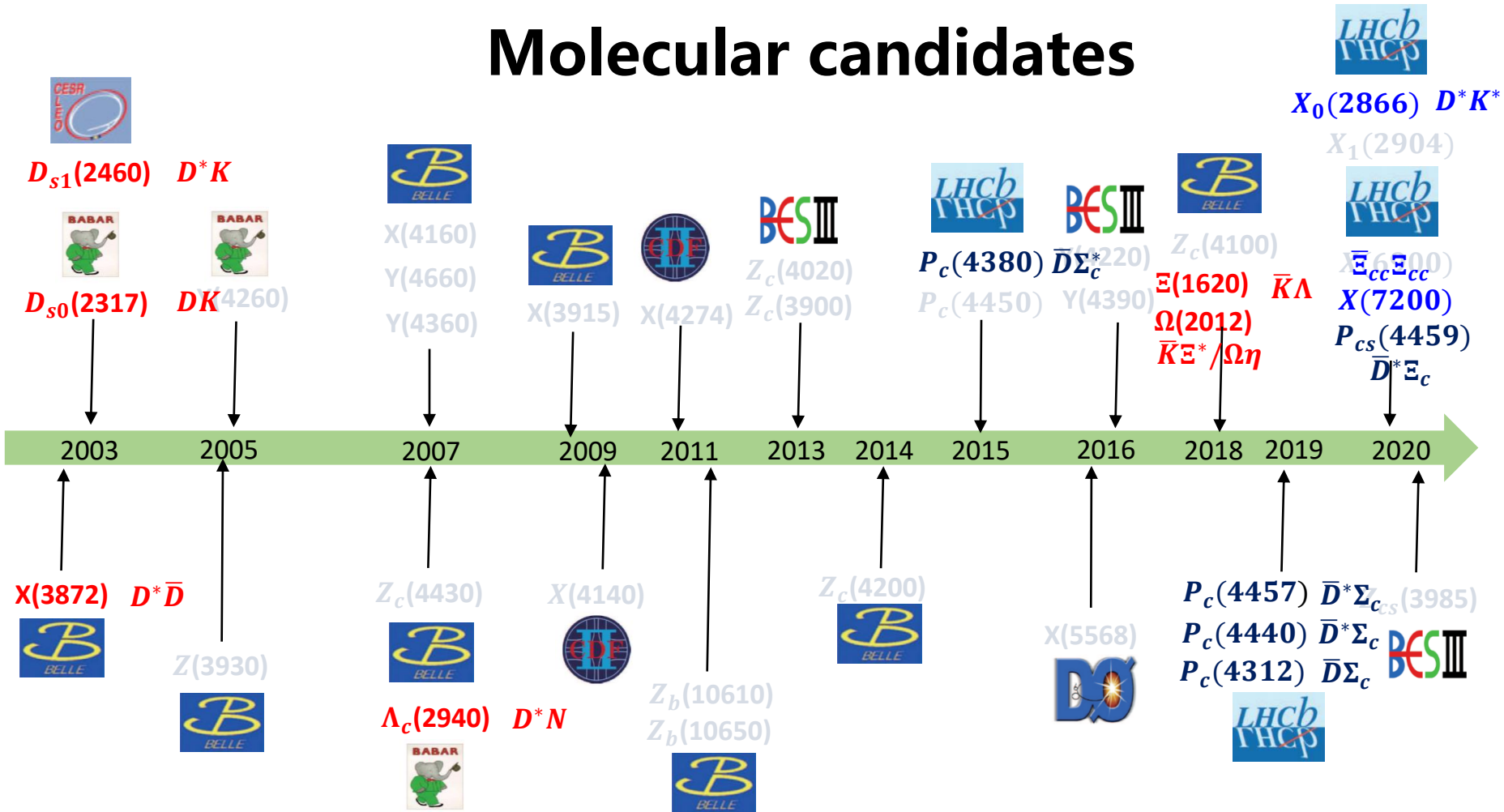


Many (if not all) of them close to thresholds



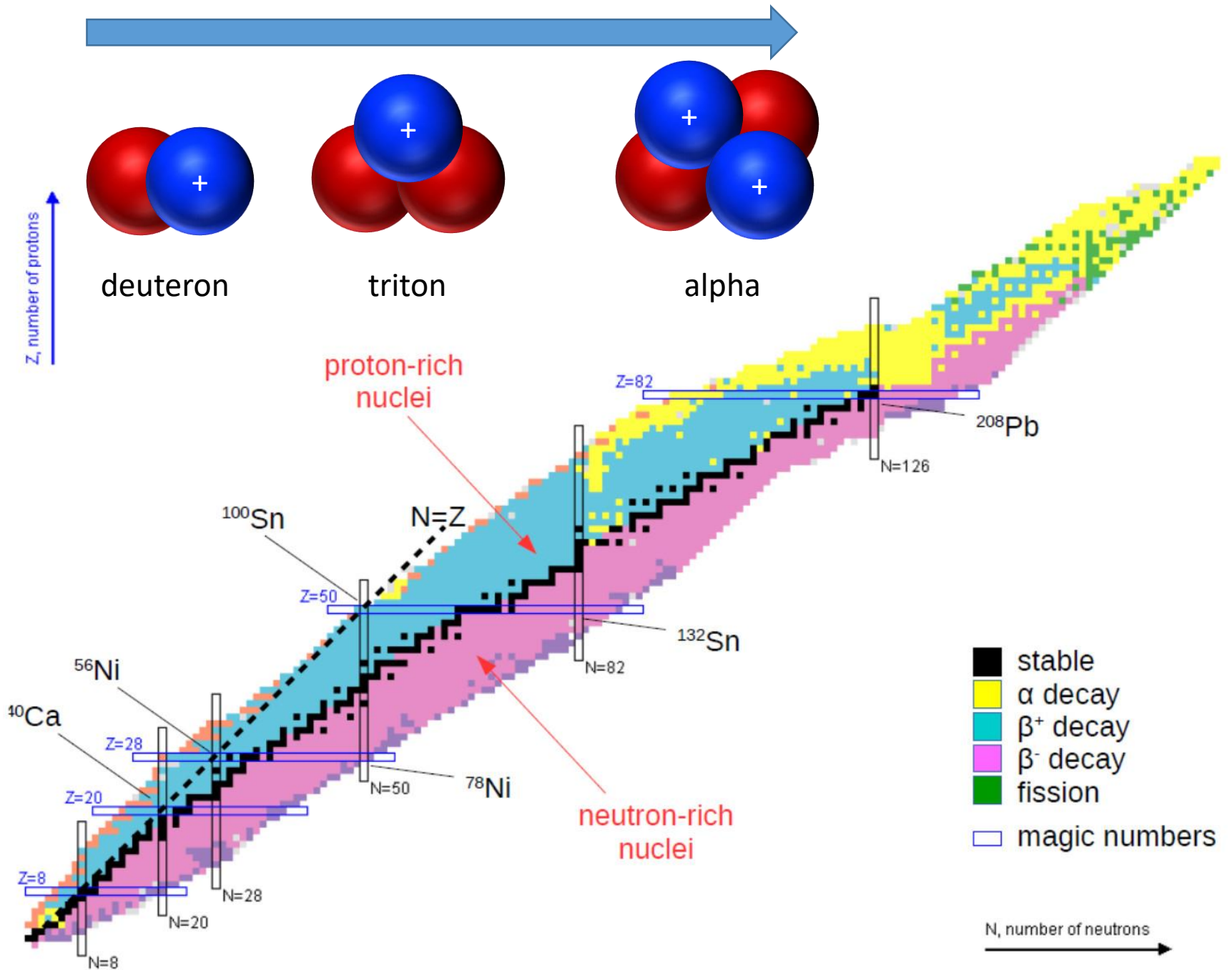
Feng-Kun Guo, Christoph Hanhart,
 Ulf-G. Meißner, Qian Wang,
 Qiang Zhao, Bing-Song Zou.
 Rev.Mod.Phys. 90 (2018) 015004.

Molecular candidates

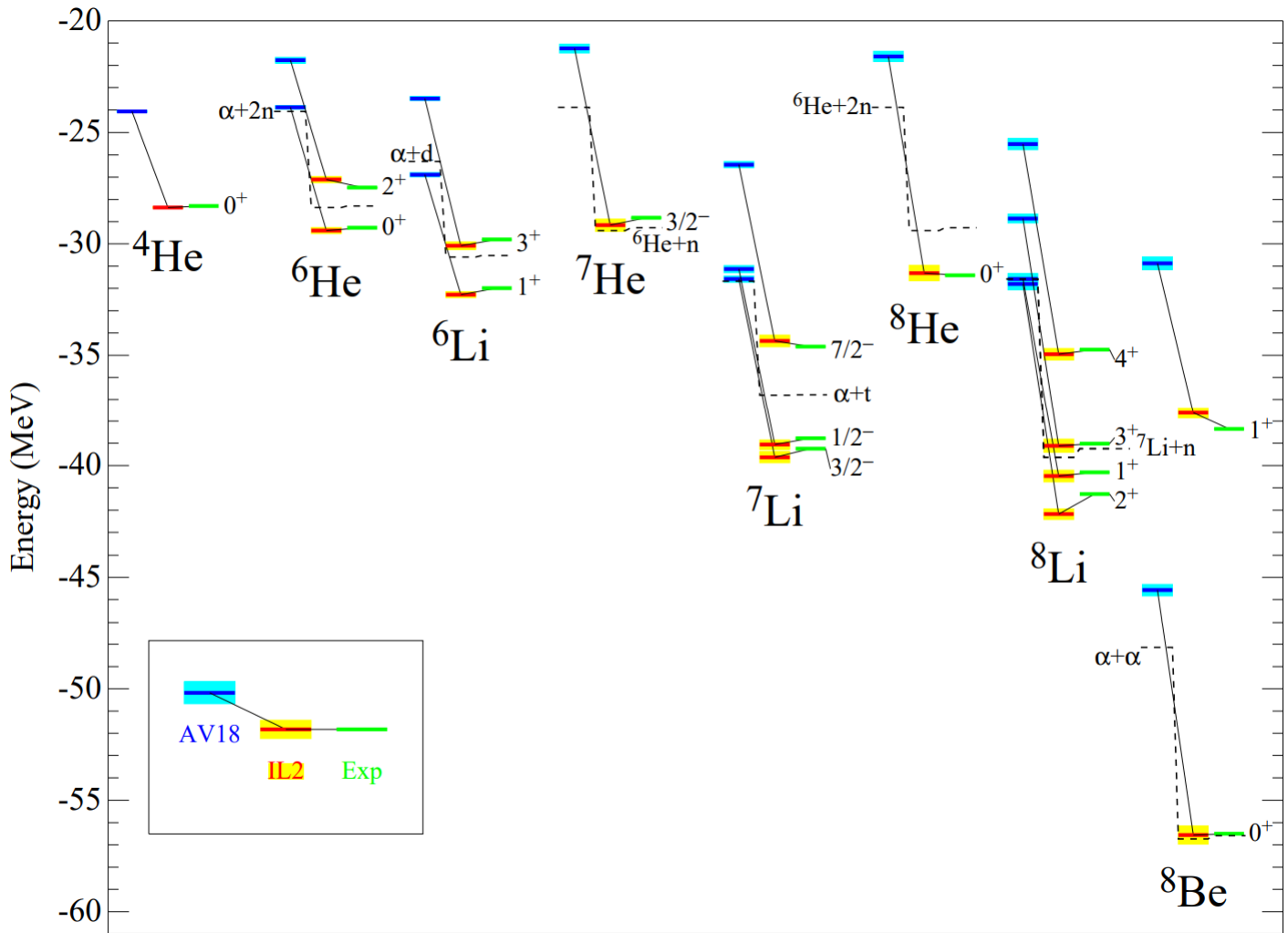


How to check the molecular picture?
our naïve answer—go to many body





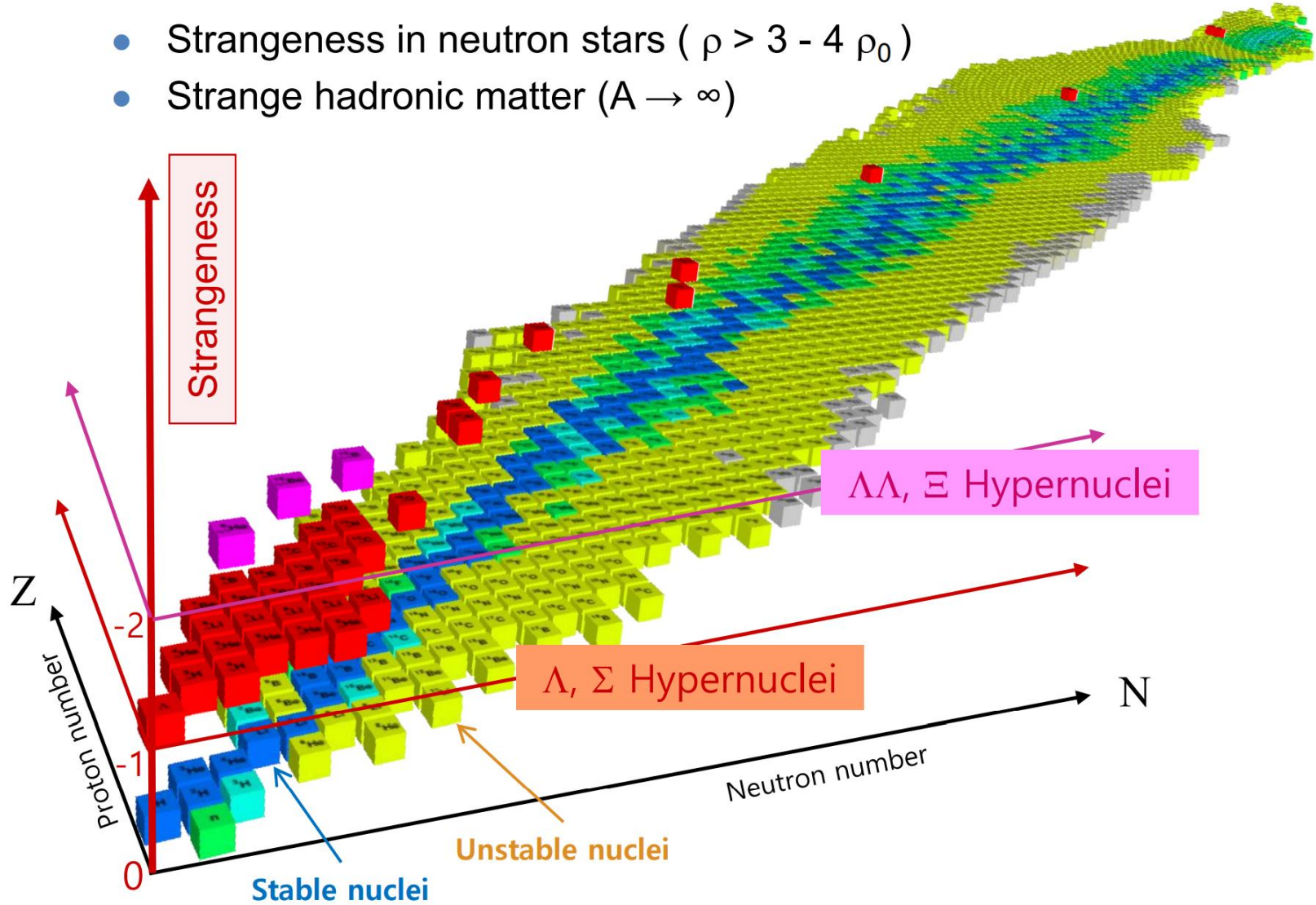
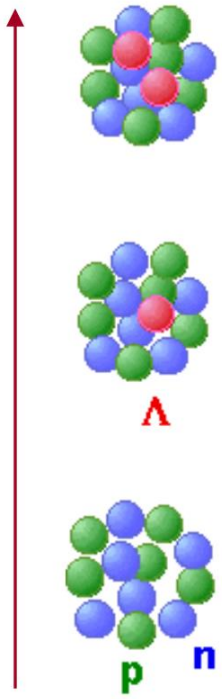
Quantum Monte Carlo Calculations of light Nuclei



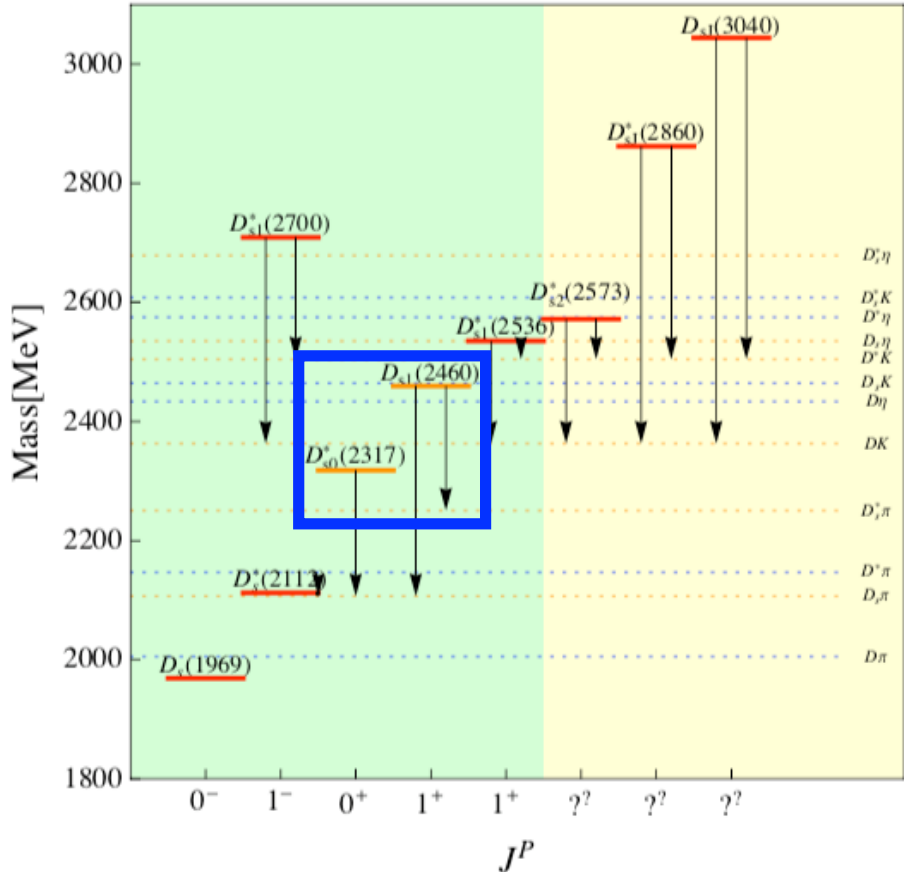
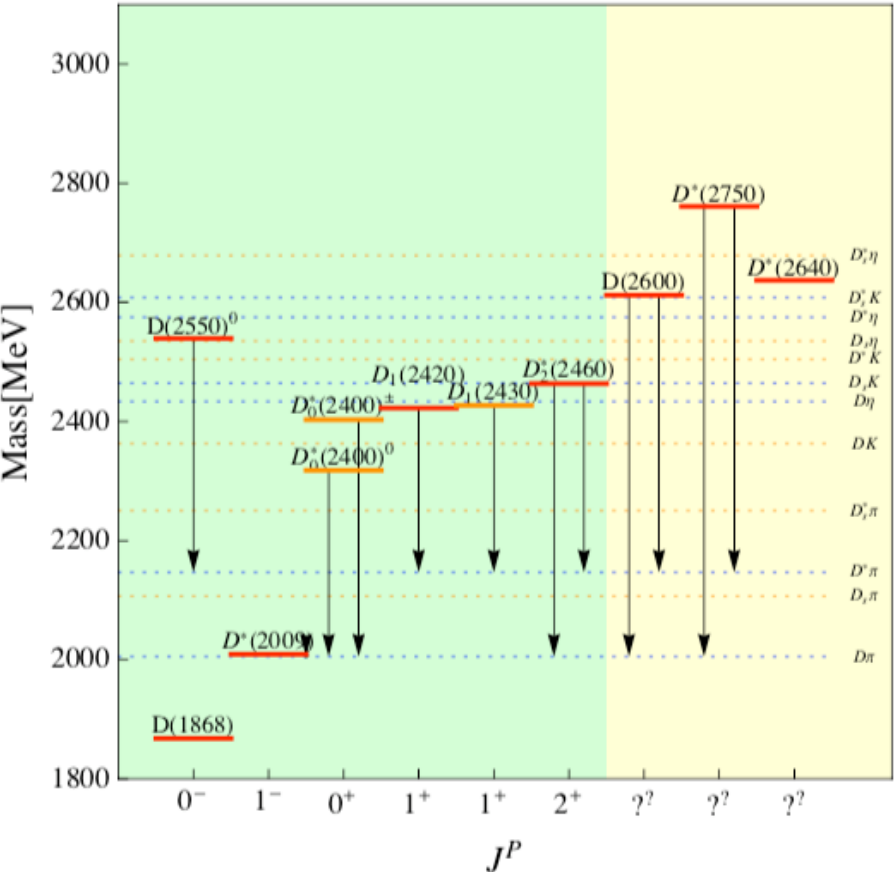
Adding hyperons → three D nuclear chart

- Strangeness in neutron stars ($\rho > 3 - 4 \rho_0$)
- Strange hadronic matter ($A \rightarrow \infty$)

Higher density



Next best two-body molecule candidates

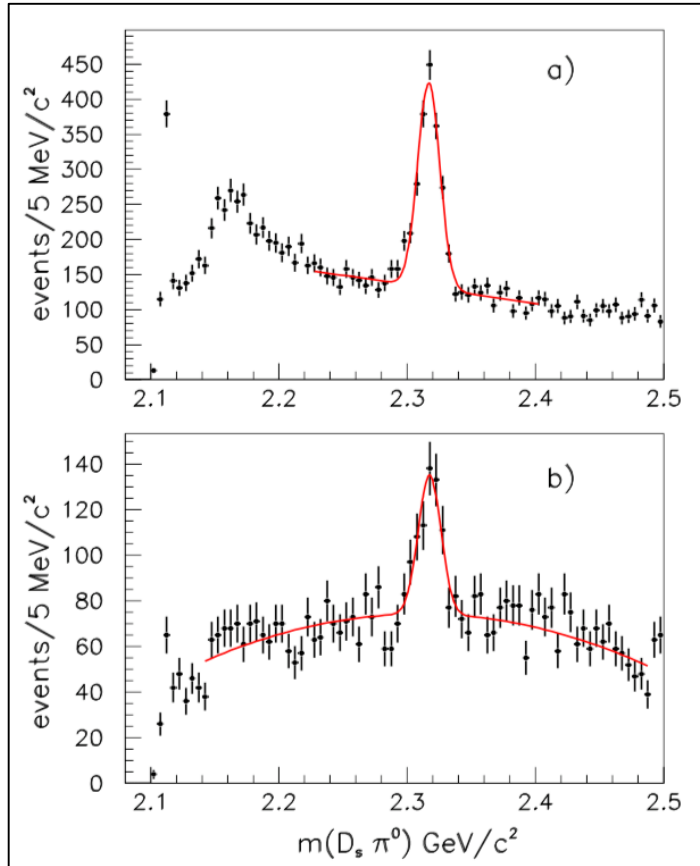


Ds0*(2317)

Ds1(2460)

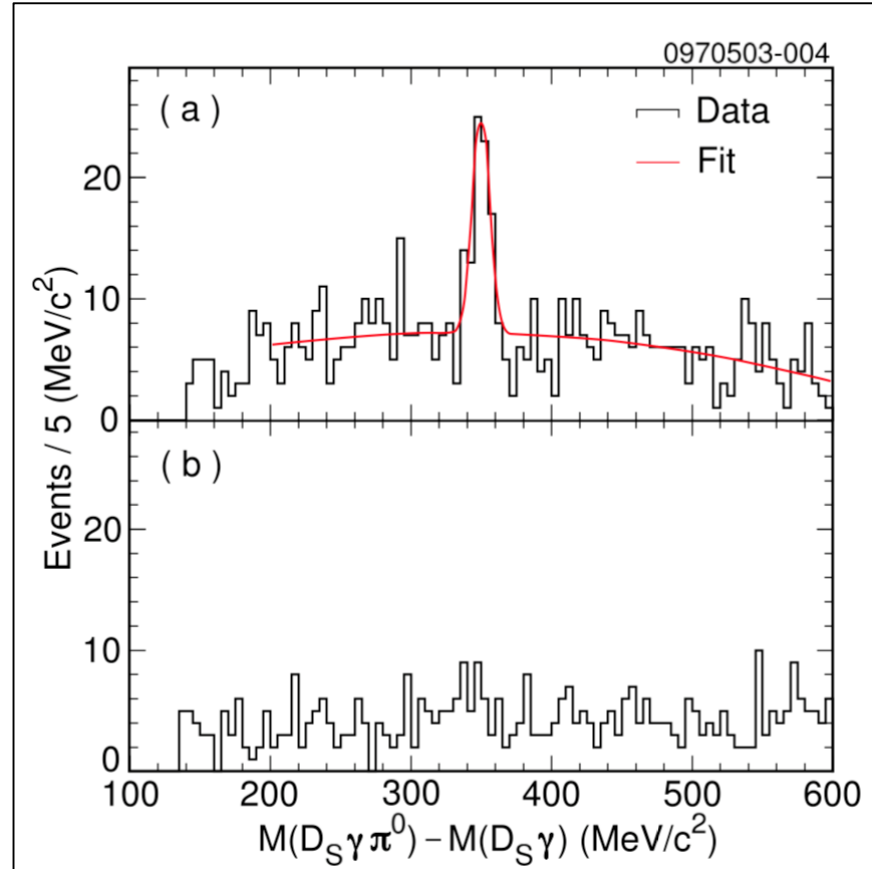
Discovery channels

$D_s0^*(2317)$



BaBar PRL90,242001(2003)

$D_s1(2460)$



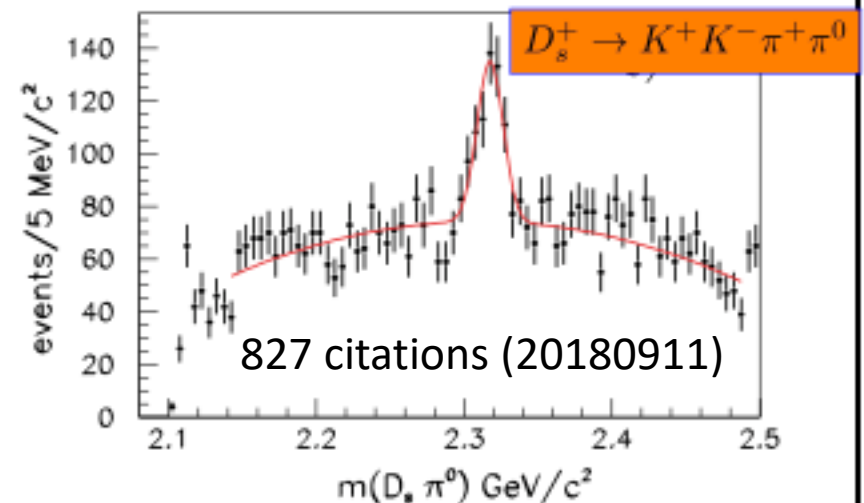
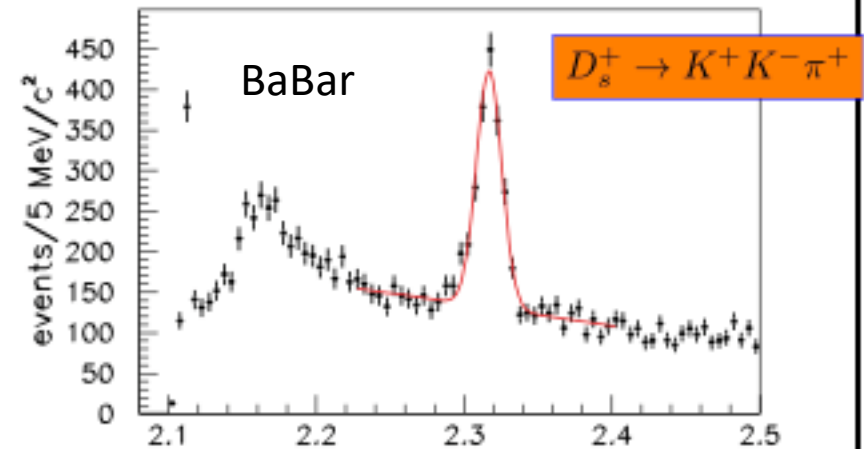
CLEO PRD68,032002(2003)

What are special about these two states

- D_{s0}^* (2317), D_{s1} (2460)
- 160/70 MeV lower than quark model predictions--difficult to be understood as conventional $c\bar{s}$ states.
- “Dynamically generated” from strong DK interaction
 - ✓ E. E. Kolomeitsev 2004, [SEP]
 - ✓ F. K. Guo 2006,
 - ✓ D. Gamermann 2007

$$m_{D_{s1}(2460)} - m_{D_{s0}^*(2317)} \approx m_{D^*} - m_D$$

$M = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV

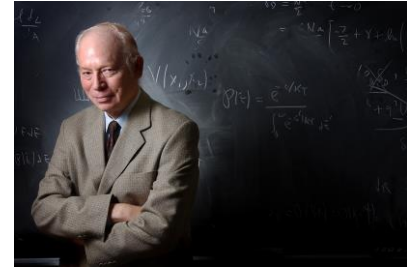


PRL90,242001(2003)

Contents

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- **$Ds_0^*(2317)$ and $Ds_1(2460)$ as DK/D^*K molecules: theory & lattice**
- DDK molecule: $R^{++}(4140)$
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- (Future) experimental searches
- Summary and outlook

UChPT in Bethe-Salpeter equation

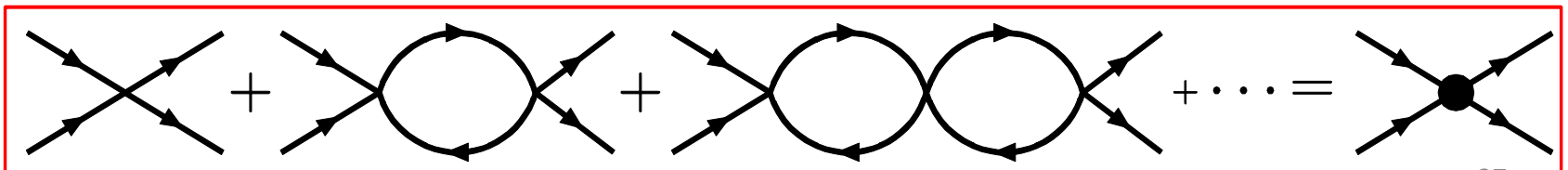


□ Model independent DK interaction from ChPT

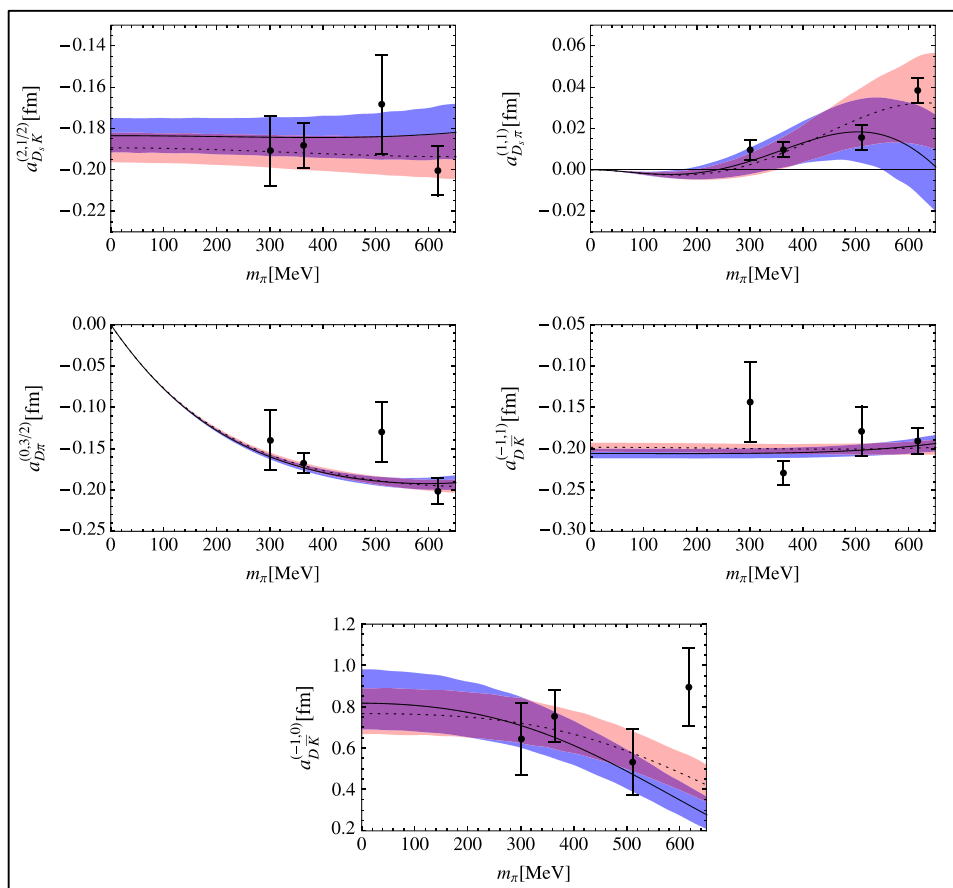
$$\mathcal{V}_{\text{WT}}(P(p_1)\phi(p_2) \rightarrow P(p_3)\phi(p_4)) = \frac{1}{4f_0^2} \mathcal{C}_{\text{LO}} (s - u) \quad \text{Weinberg-Tomazawa}$$

$$\begin{aligned} \mathcal{V}_{\text{NLO}}(P(p_1)\phi(p_2) \rightarrow P(p_3)\phi(p_4)) = & -\frac{8}{f_0^2} \mathcal{C}_{24} \left(c_2 p_2 \cdot p_4 - \frac{c_4}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & -\frac{4}{f_0^2} \mathcal{C}_{35} \left(c_3 p_2 \cdot p_4 - \frac{c_5}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 + p_1 \cdot p_2 p_3 \cdot p_4) \right) \\ & -\frac{4}{f_0^2} \mathcal{C}_6 \frac{c_6}{m_P^2} (p_1 \cdot p_4 p_2 \cdot p_3 - p_1 \cdot p_2 p_3 \cdot p_4) \\ & -\frac{8}{f_0^2} \mathcal{C}_0 c_0 + \frac{4}{f_0^2} \mathcal{C}_1 c_1, \end{aligned} \quad (11)$$

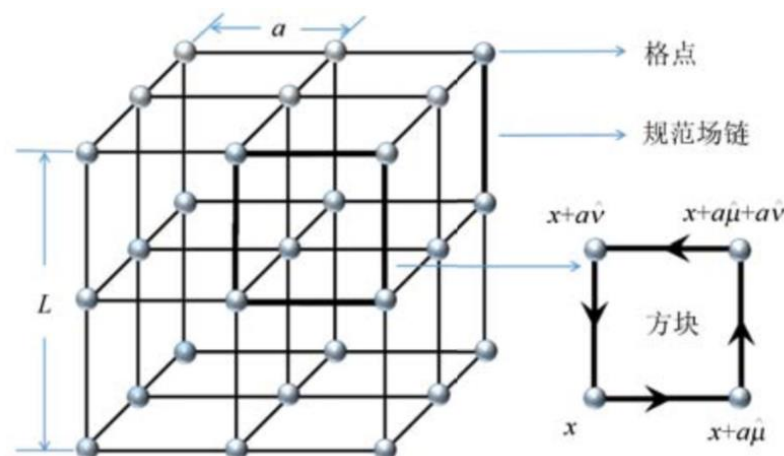
□ Resummed in the Bethe-Salpeter equation (two-body elastic unitarity)



Fixing the LECs using latest LQCD* data



- NLO ChPT kernel: 5 LECs
- A quite good description of the 20 Lattice scattering lengths of pseudoscalar mesons and D mesons (I=0 DK excluded) can be achieved.



Ds0 and Ds1 dynamically generated

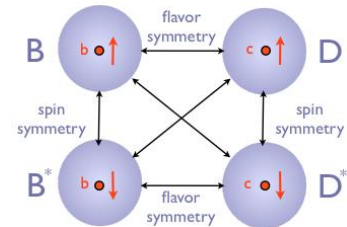
“Post-diction”

● Charm sector

D_{s0}^* (2317), D_{s1} (2460)

TABLE V. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
$(1, 0)$	2317 ± 10	2457 ± 17
$(0, 1/2)$	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$



● Bottom Sector

TABLE VI. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of bottom mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
$(1, 0)$	5726 ± 28	5778 ± 26
$(0, 1/2)$	$(5537 \pm 14) - i(118 \pm 22)$	$(5586 \pm 16) - i(124 \pm 25)$

Predicted Bs0 and Bs1 states

Physics Letters B 750 (2015) 17–21



Contents lists available at [ScienceDirect](#)

Physics Letters B

www.elsevier.com/locate/physletb



Predicting positive parity B_s mesons from lattice QCD



C.B. Lang^a, Daniel Mohler^{b,*}, Sasa Prelovsek^{c,d}, R.M. Woloshyn^e

^a Institute of Physics, University of Graz, A-8010 Graz, Austria

^b Fermi National Accelerator Laboratory, Batavia, IL 60510-5011, USA

^c Department of Physics, University of Ljubljana, 1000 Ljubljana, Slovenia

^d Jozef Stefan Institute, 1000 Ljubljana, Slovenia

^e TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

Table 5

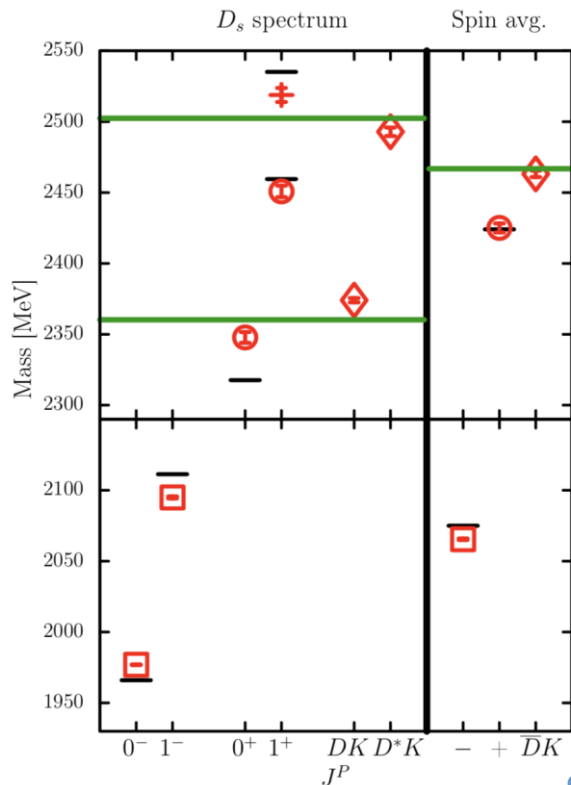
Comparison of masses from this work to results from various model based calculations; all masses in MeV.

J^P	0^+	1^+
Covariant (U)ChPT [24]	5726(28)	5778(26)
NLO UHMChPT [19]	5696(20)(30)	5742(20)(30)
LO UChPT [17,18]	5725(39)	5778(7)
LO χ -SU(3) [16]	5643	5690
HQET + ChPT [20]	5706.6(1.2)	5765.6(1.2)
Bardeen, Eichten, Hill [15]	5718(35)	5765(35)
rel. quark model [5]	5804	5842
rel. quark model [22]	5833	5865
rel. quark model [23]	5830	5858
HPQCD [30]	5752(16)(5)(25)	5806(15)(5)(25)
this work	5713(11)(19)	5750(17)(19)

In agreement with IQCD

More support from recent IQCD studies

- [G.K.C. Cheung et al., arXiv:2008.06432\[hep-lat\].](#)
- [G. S. Bali et al., arXiv:1706.01247 \[hep-lat\].](#)
- [C. B. Lang et al., arXiv:1403.8103 \[hep-lat\].](#)
- [D. Mohler et al., arXiv:1308.3175 \[hep-lat\].](#)



“DK components substantial”

FIG. 12. On the left, our final results for the lower lying D_s spectrum as detailed in Table VII. The short horizontal black lines indicate the corrected experimental values (see Section II) while the green horizontal lines give the positions of the DK and D^*K non-interacting thresholds. Our lattice results for the finite volume thresholds are labelled DK and D^*K , respectively. The errors indicated are statistical only. On the right, the negative parity spin-averaged $1S$ mass $m_- = \frac{1}{4}(m_{0^-} + 3m_{1^-})$ is shown and denoted $-$, while the same spin-average of the positive parity 0^+ and 1^+ states is labelled with $+$ and the weighted average of the threshold is labelled as \overline{DK} .

Further **tests** of the DK interaction

- Experiments, theory, and lattice QCD all show that DK or D^*K interaction is strong enough to form $Ds0^*(2317)$ or $Ds1(2460)$
- A natural question is: if we add one more $D(\bar{D})$ or $D^*(\bar{D}^*)$, can they form molecules of three hadrons?
- This seems to **be a rather straightforward and naive question**, but **remains unexplored** until quite recently

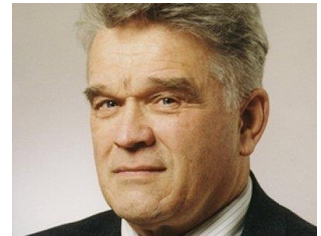
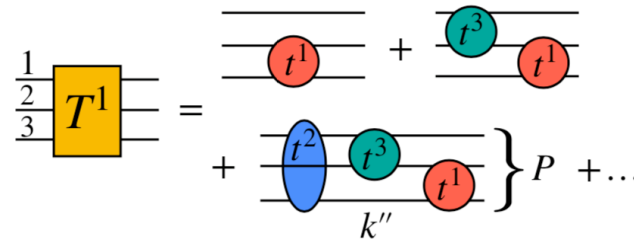
Contents

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds_0^*(2317)$ and $Ds_1(2460)$ as DK/D^*K molecules: theory & lattice
- **DDK molecule: $R^{++}(4140)$**
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- (Future) experimental searches
- Summary and outlook

An explicit three-body study of DDK

- Coupled-three-channel problem: $D(\text{DK} - D_S\pi - D_S\eta)$
- Three-body scattering matrix (Faddeev)

$$T = \sum_{i=1}^3 T^i$$



$$T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + \sum_{j \neq i=1}^3 T_R^{ij}, \quad i = 1, 2, 3,$$

$$T_R^{ij} = t^i g^{ij} t^j + t^i \left[G^{iji} T_R^{ji} + G^{ijk} T_R^{jk} \right],$$

A. Martínez Torres, K. P. Khemchandani, and E. Oset PRC **77**, 042203(R)

A. Martínez Torres, K.P. Khemchandani, **LSG**, M. Napsuciale, E. Oset, PRD78 (2008) 074031

Two-body inputs

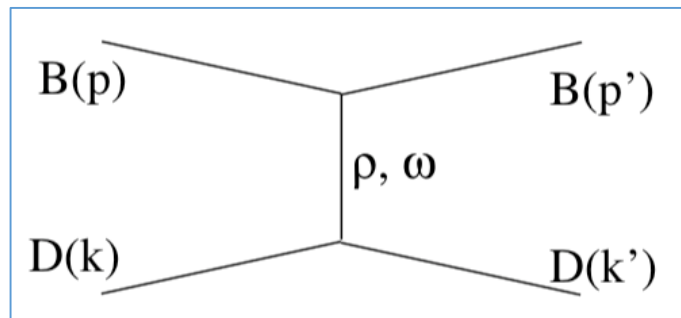
- **DK: leading order UChPT** DK , $D_s\eta$ and $D_s\pi$

$$V_{ij} = -\frac{C_{ij}}{4f^2}(s - u)$$

$$a(\mu) = -1.846, \mu = 1000 \text{ MeV} \Rightarrow \text{Pole} = 2318 \text{ MeV}$$

F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, PL B641, 278 (2006).

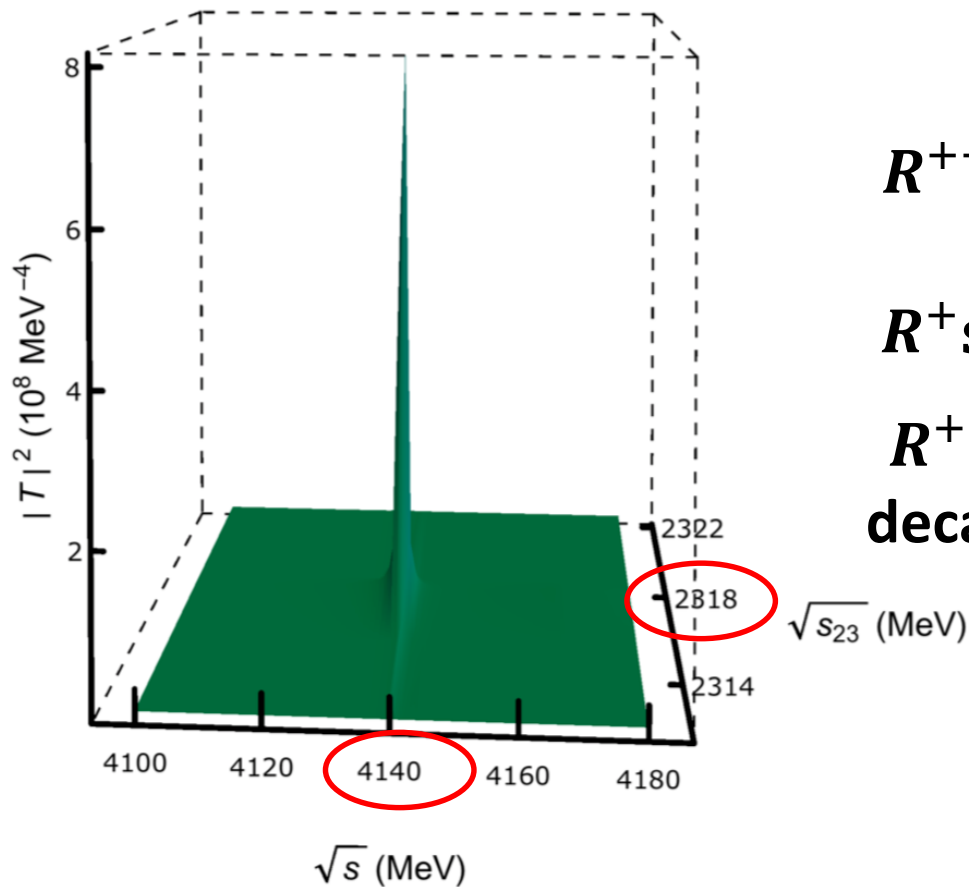
- **DD(Ds): local hidden gauge theory**



$$a(\mu) = -1.3 \sim -1.5, \mu = 1500 \text{ MeV} \Leftarrow \text{fixed from } DD\bar{D}/D\bar{D}^* \rightarrow X(3700) / X(3872)$$

S. Sakai, L. Roca, and E. Oset, PRD96, 054023 (2017).

Three-body amplitudes

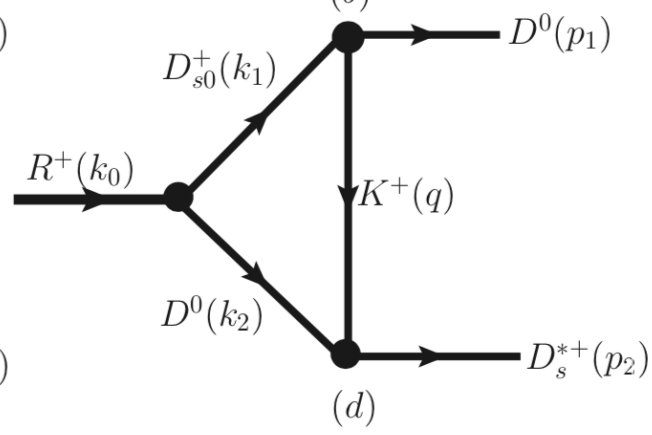
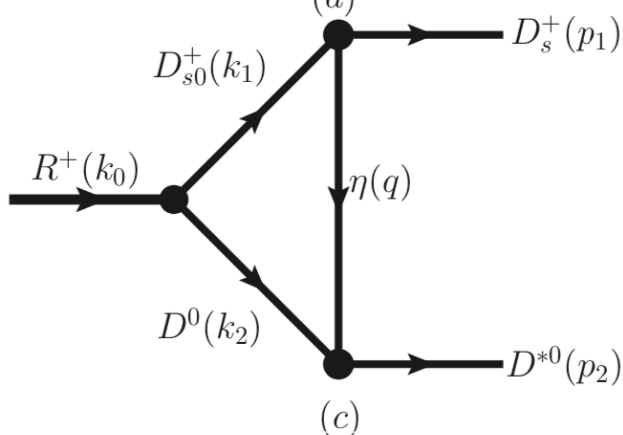
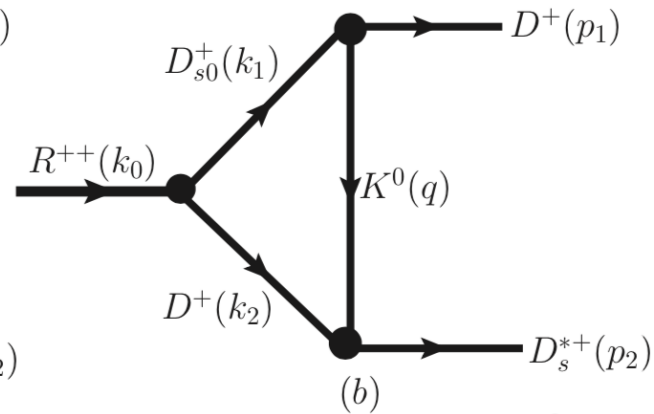
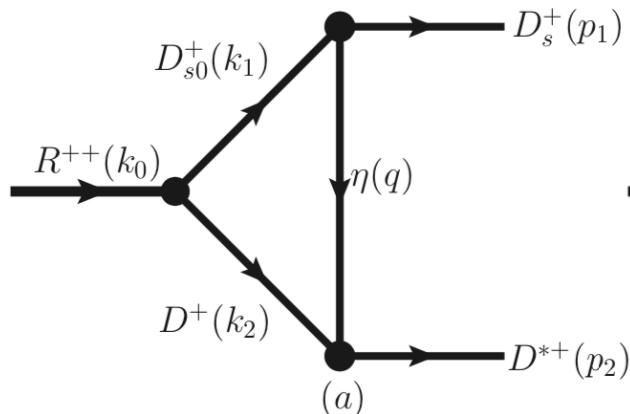
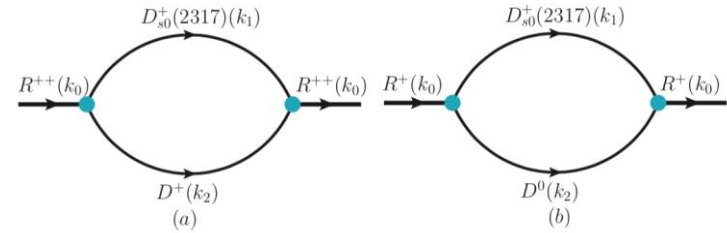


$$R^{++} = (I, I_{23}) = \left(\frac{1}{2}, 0\right)$$

R^+ should also exist

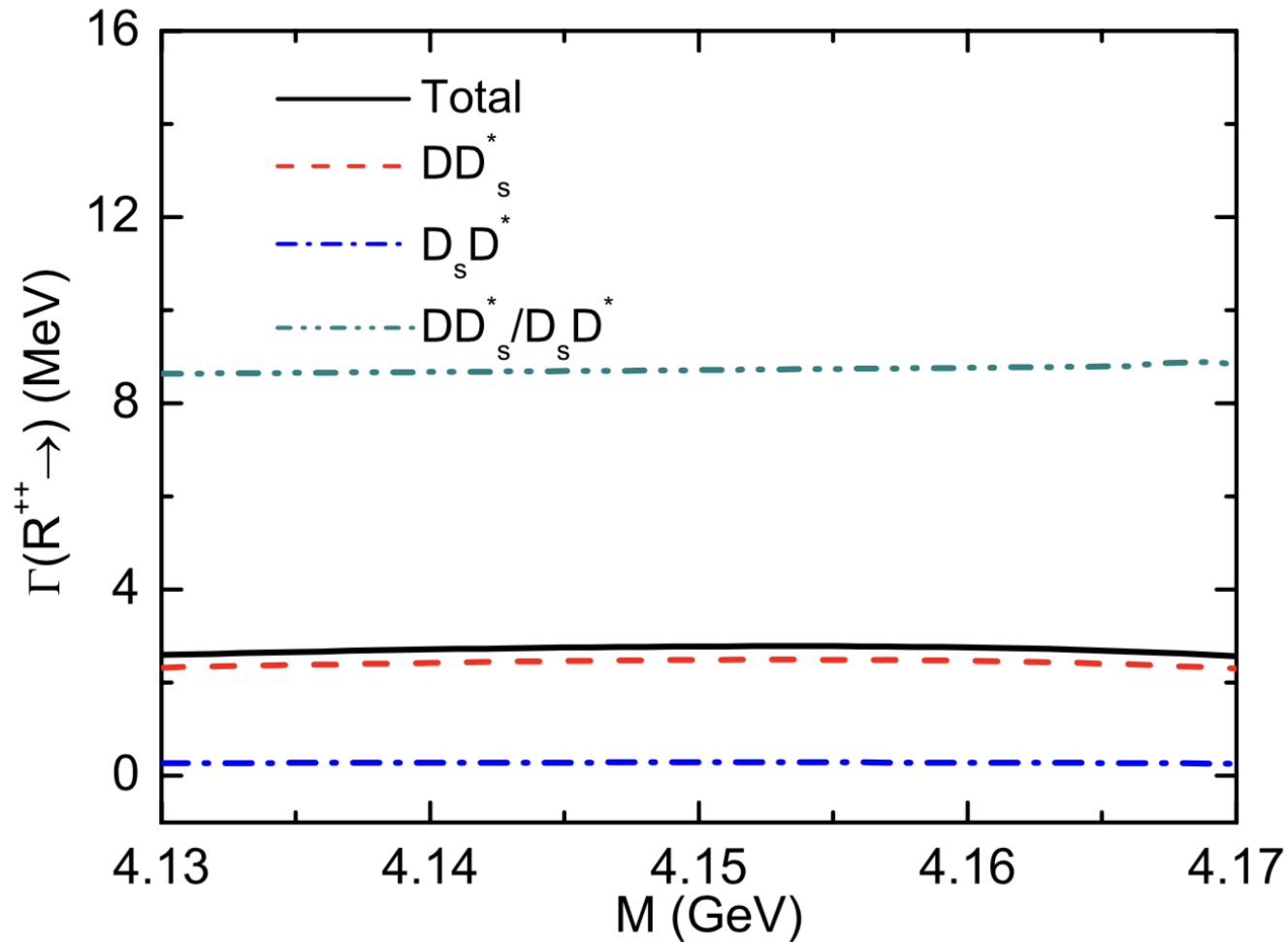
R^{++} is a bound state, but can decay strongly

Two-body decay width



Two-body decay width

Kaon-Exchange Dominant

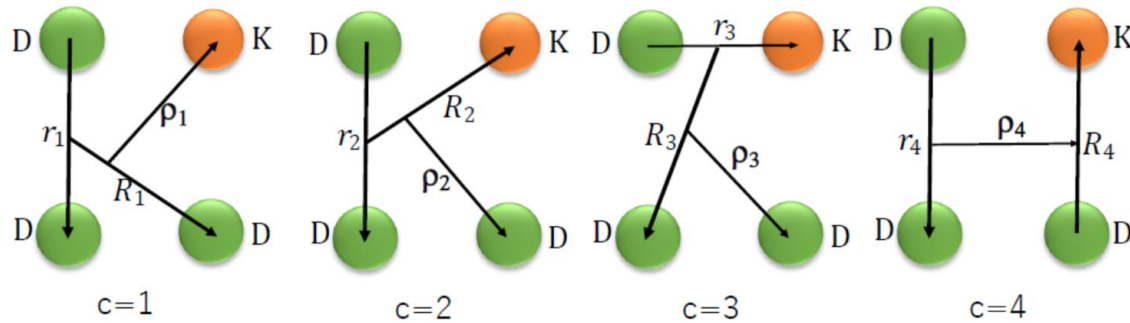
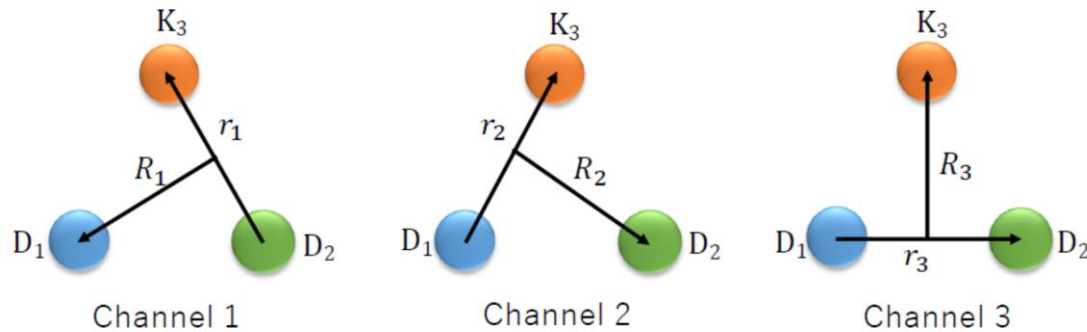


A DDDK state

 $1(0^+)$

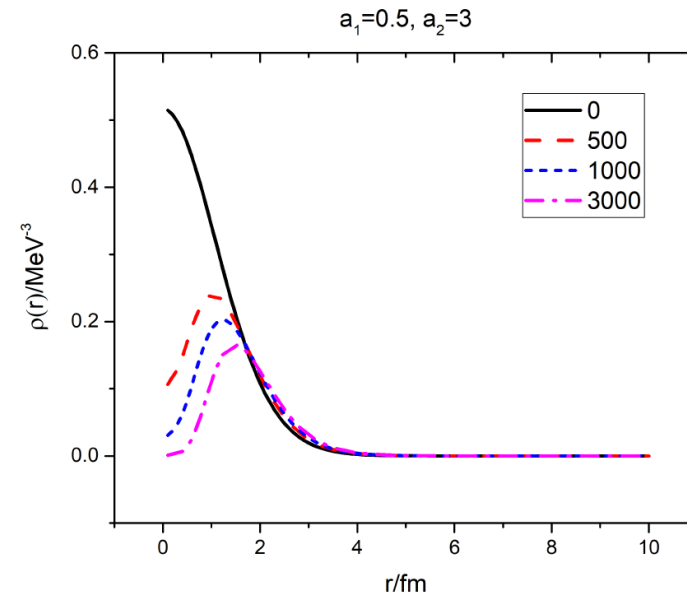
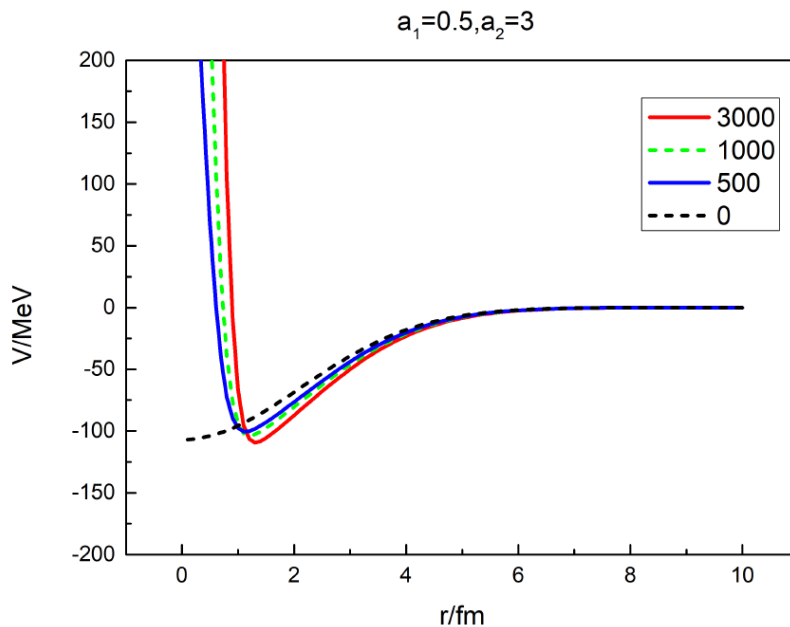
Gaussian Expansion Method

What if we add one more D?



A DDDK state $1(0^+)$

What if we add one more D? **Our study** shows that such a state exists as well



Uncertainties are at **the order of 10-20 MeV**

$$V_{DK}(r) = C_1 e^{-r^2/a_1^2} + C_2 e^{-r^2/a_2^2}$$

	DK*	DDK	DDDK
Binding	45 MeV	(67-71) MeV	91-107 MeV

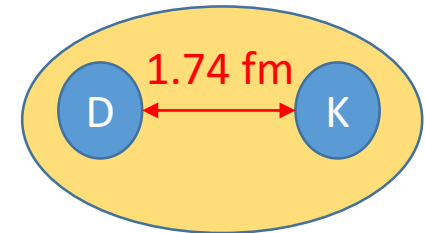
DD interactions play a minor role

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	E_2	$E_3(\text{only } V_{DK})$	$E_3(V_{DK} + V_{DD})$	$E_4(\text{only } V_{DK})$	$E_4(V_{DK} + V_{DD})$
		$R_S = 0.5\text{fm}$		$R_c = 1\text{fm}$		
0	-320.1	-45.0	-65.8	-71.2	-89.4	-106.8
500	-455.4	-45.0	-65.8	-70.4	-89.2	-103.5
1000	-562.6	-45.0	-65.7	-69.7	-88.8	-101.4
3000	-838.7	-45.0	-65.0	-68.4	-87.0	-97.3
		$R_S = 0.5\text{fm}$		$R_c = 2\text{fm}$		
0	-149.1	-45.0	-66.0	-68.8, -45.1	-88.7, -66.3	-97.6, -70.7
500	-178.4	-45.0	-65.9	-68.2, -45.5	-88.5, -66.7	-95.5, -70.9
1000	-195.0	-45.0	-65.8, -45.2	-67.9, -45.8	-88.2, -66.9	-94.5, -71.2
3000	-225.9	-45.0	-65.3, -45.6	-67.2, -46.6	-87.0, -67.0	-92.6, -71.7
		$R_S = 0.5\text{fm}$		$R_c = 3\text{fm}$		
0	-107.0	-45.0	-66.2, -47.3	-68.0, -48.3	-88.8, -70.2	-94.4, -74.3
500	-119.4	-45.0	-66.2, -48.2	-67.7, -49.3	-88.7, -71.0	-93.2, -74.8
1000	-125.6	-45.0	-66.1, -48.7	-67.5, -49.8	-88.4, -71.3	-92.5, -75.2
3000	-136.2	-45.0	-65.8, -49.4	-67.1, -50.7	-87.6, -71.7	-91.4, -75.7

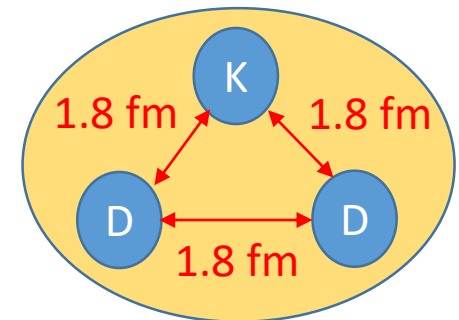
Spatial distributions

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$\langle T \rangle$	$\langle V_{DK} \rangle$	$\langle V_{DD} \rangle$
$R_S = 0.5\text{fm } R_c = 1\text{fm}$							
0	-320.1	1.28	1.32	1.36	124.37	-189.61	-5.98
500	-455.4	1.39	1.44	1.47	99.51	-164.83	-5.03
1000	-562.6	1.46	1.53	1.54	91.43	-156.67	-4.51
3000	-838.7	1.61	1.69	1.68	93.24	-157.80	-3.82
$R_S = 0.5\text{fm } R_c = 2\text{fm}$							
0	-149.1	1.74	1.80	1.80	60.20	-125.74	-3.23
500	-178.4	1.91	1.98	1.96	51.00	-116.59	-2.64
1000	-195.0	1.99	2.07	2.04	50.63	-116.12	-2.43
3000	-225.9	2.13	2.22	2.15	53.61	-118.59	-2.24
$R_S = 0.5\text{fm } R_c = 3\text{fm}$							
0	-107.0	2.13	2.19	2.17	39.49	-105.35	-2.13
500	-119.4	2.31	2.38	2.34	34.80	-100.73	-1.77
1000	-125.6	2.37	2.47	2.42	34.90	-100.77	-1.65
3000	-136.2	2.53	2.61	2.53	36.66	-102.24	-1.54

Ds0*(2317)



R(4140)



Contents

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds_0^*(2317)$ and $Ds_1(2460)$ as DK/D^*K molecules: theory & lattice
- DDK molecule: $R^{++}(4140)$
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- (Future) experimental searches
- Summary and outlook

Instead of a D , adding a \overline{D}^* to the DK pair

- Fixed center approximation (FCA):

$$K(D\overline{D}^* + \overline{D}D^*) \sim KX(3872)/Zc(3900)$$

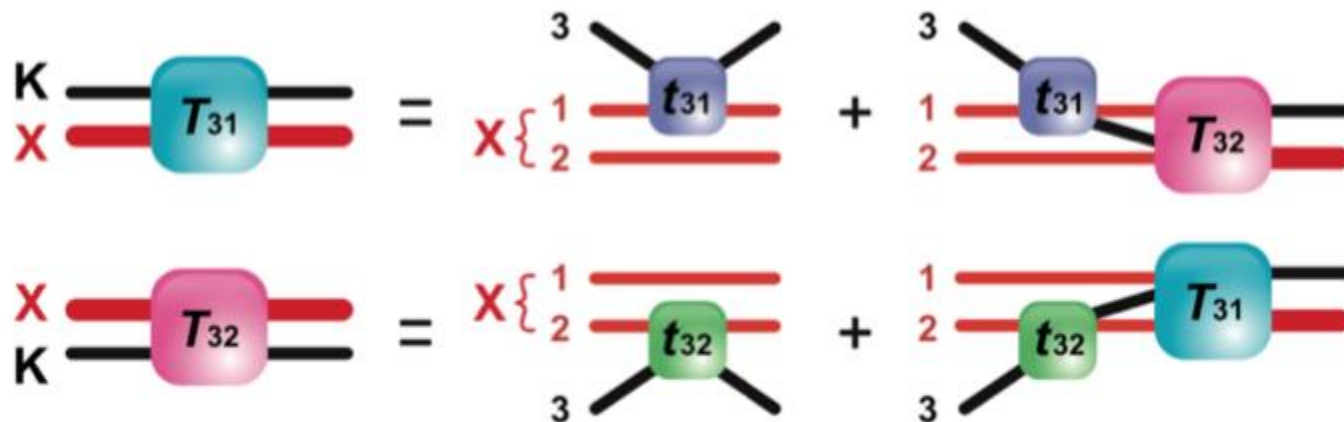
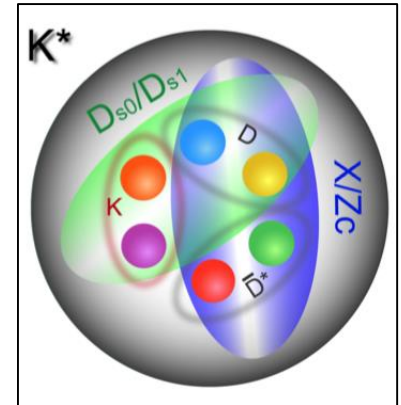
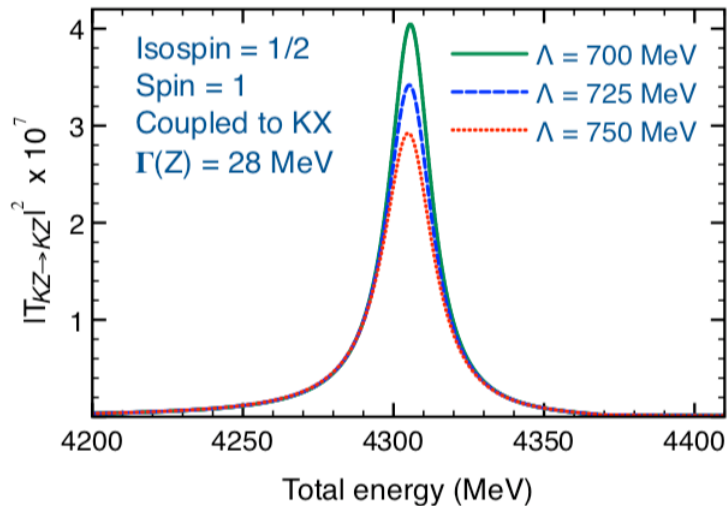
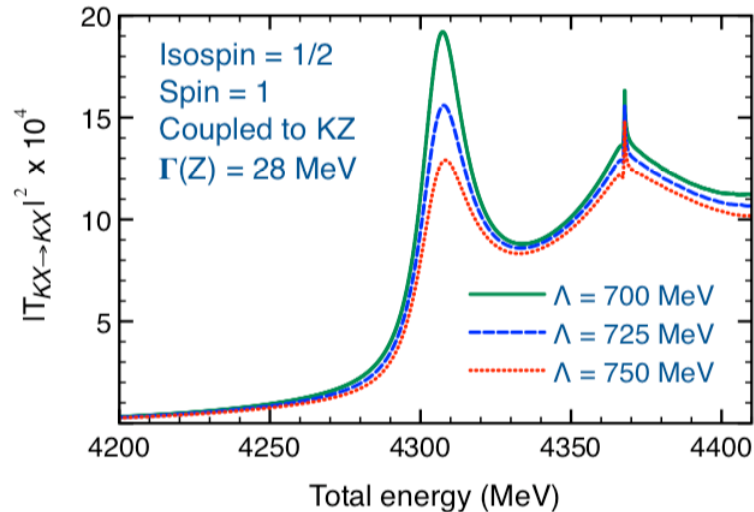


Figure 2: Diagrams showing the scattering of the particle labeled “3” (K) on a cluster (X) made of particles 1 (D) and 2 (\overline{D}^*).

$K^*(4307)$



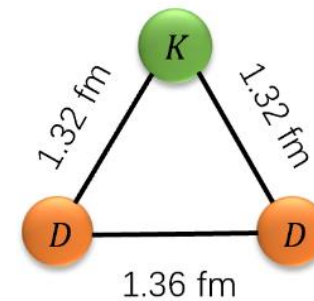
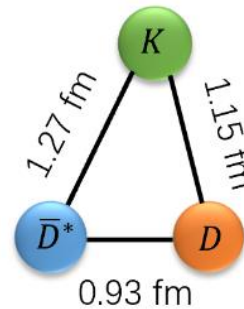
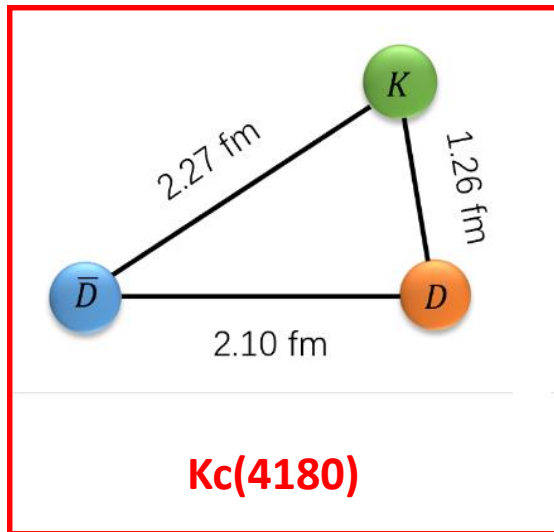
- Treating KX and KZ as coupled channel systems
- A resonance with $M=(4307 \pm 2) - i(9 \pm 2) \text{ MeV}$ with $I(J^P) = 1/2(1^-)$

In agreement with Li Ma, Qian Wang, Ulf-G. Meißner, 1711.06143, but with completely different dynamics

Instead of a D , adding a \bar{D} to the DK pair

2012.01134

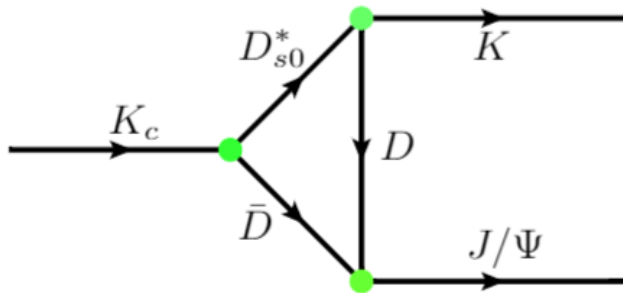
The Three Musketeers



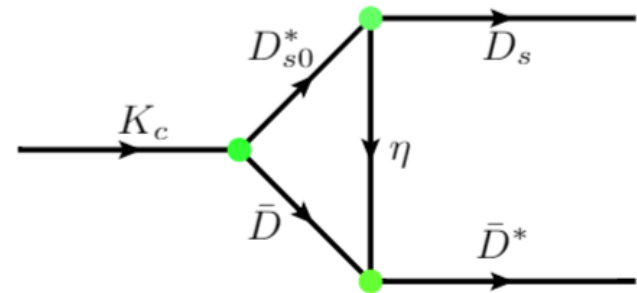
	This work	Ref [28]	Ref [29]
Method	GEM(SE)	BOA(SE)	FCA(FE)
Interaction Models	$\chi^{\text{EFT+OBE}}$	delocalized π bond	$\chi^{\text{EFT+OBE}}$
$\frac{1}{2}(0^-) D\bar{D}K$	$4181.2^{+2.4}_{-1.4} (B_3 \simeq 48.9^{+1.4}_{-2.4})$	-	-
$\frac{1}{2}(1^-) D\bar{D}^*K$	$4294.1^{+6.6}_{-3.1} (B_3 \simeq 77.3^{+3.1}_{-6.6})$	$4317.92^{+6.13}_{-6.55} (B_3 \simeq 53.52^{+6.55}_{-6.13})$	$4307 \pm 2 (B_3 \simeq 64 \pm 2)$

$K_c(4180)$ decay

[2012.01134](#)

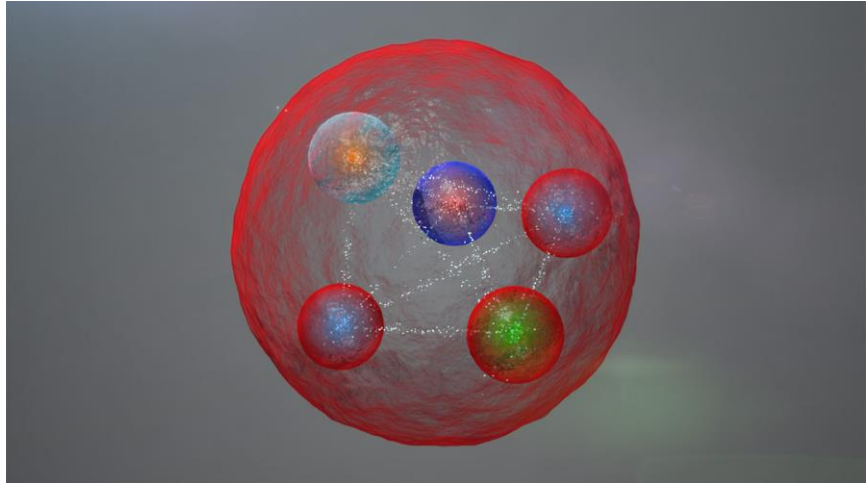
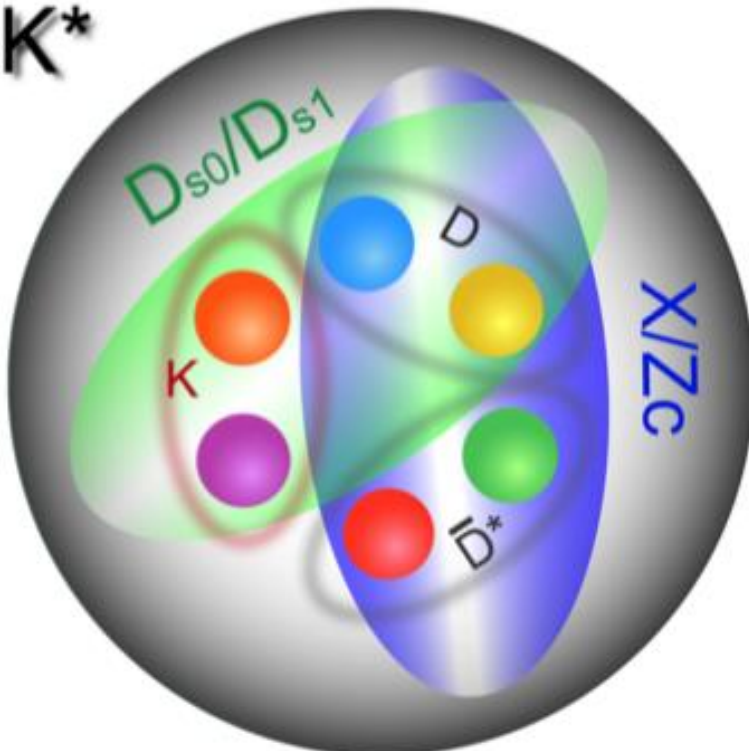


~ 1 MeV



~ 1 MeV

Kc(4180)—**bosonic counterpart** of Pc



Pentaquark (N*) by LHCb

Phys.Rev.Lett. 115 (2015) 072001

Prediction of narrow N* and Λ* resonances with hidden charm above 4 GeV,
Jia-Jun Wu, R. Molina, E. Oset, B.S. Zou, 1007.0573

Contents

- Motivation: new types of clusters of color singlets in addition to atomic nuclei—as a nontrivial test of the molecule picture
- $Ds0^*(2317)$ and $Ds1(2460)$ as DK/D^*K molecules: theory & lattice
- DDK molecule: $R^{++}(4140)$
- $D\bar{D}^*K$ and $D\bar{D}K$ molecules: $K^*(4307)$ and $K_c(4180)$
- Where to search for these 3-body molecules
- Summary and outlook

Analogy between KD and $\bar{K}N$

$D_{s0}^*(2317)$

- DK bound state
- **Dynamically generated**--
Unitary heavy hadron
chiral perturbation theory

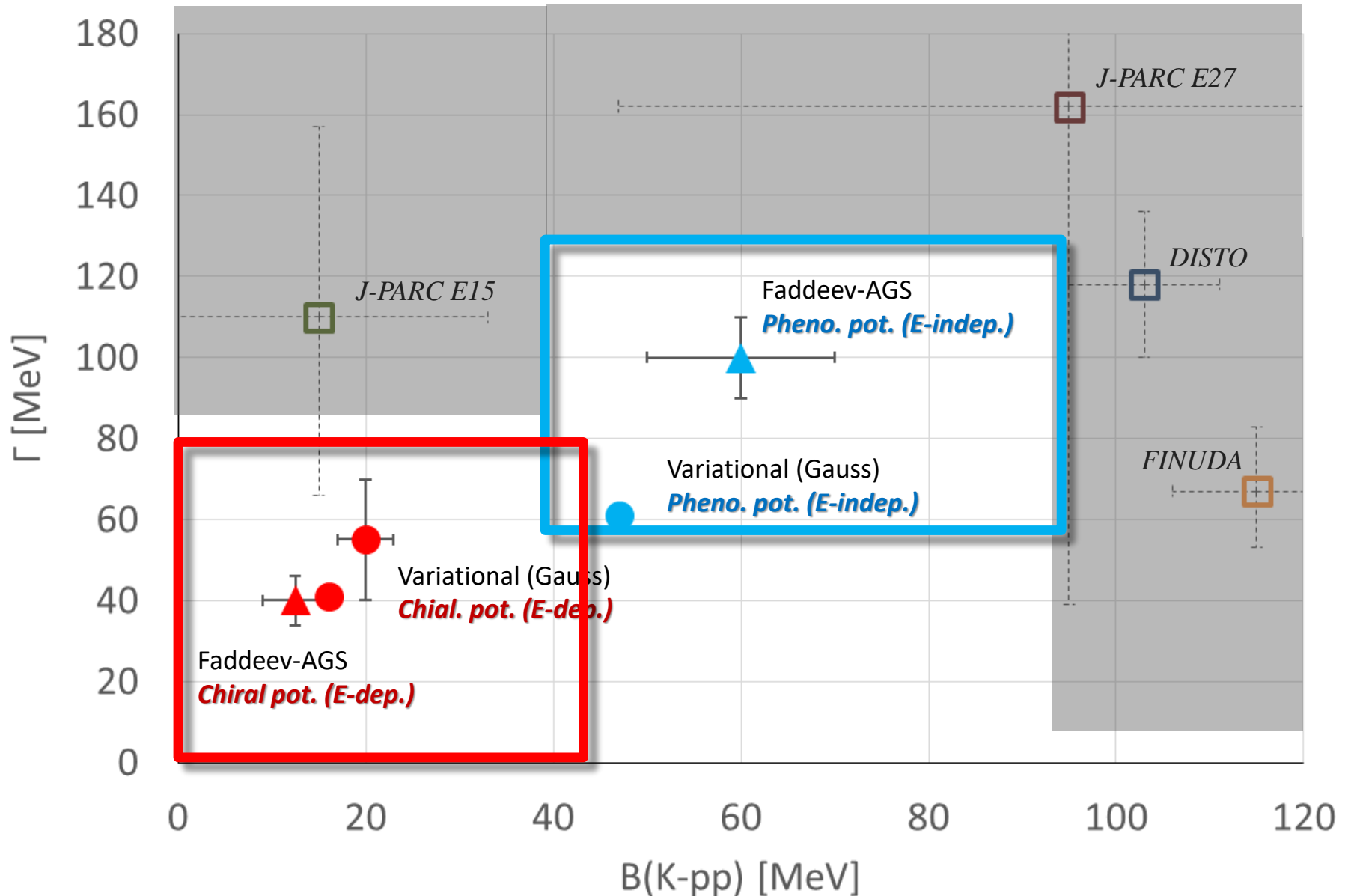
$\Lambda(1405)$

- N-Kbar bound state
- **Dynamically generated**--
Unitary baryon chiral
perturbation theory

The interaction between a **kaon** and a **heavy particle** seems to play an important role

Current status on " K^-pp "

A. Dote, Menu2019



Search for the DDK bound state by Belle

2008.13341

Search for a doubly-charged DDK bound state in $\Upsilon(1S, 2S)$ inclusive decays and via direct production in e^+e^- collisions at $\sqrt{s} = 10.520, 10.580, \text{ and } 10.867$ GeV

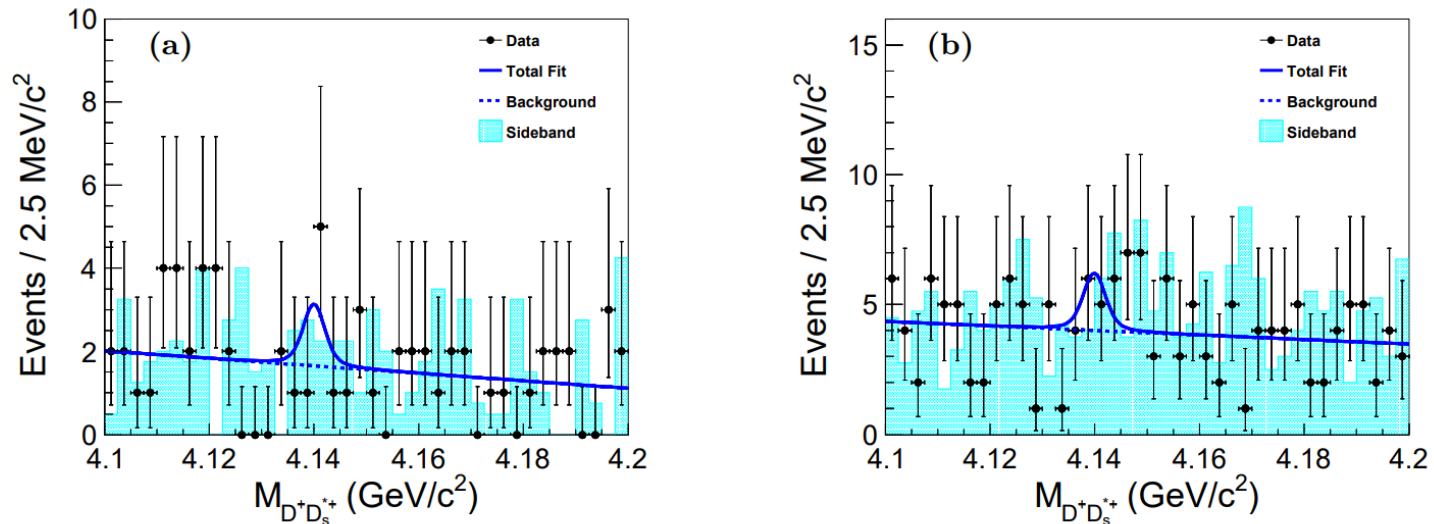


FIG. 4: The invariant-mass spectra of $D^+D_s^{*+}$ in the (a) $\Upsilon(1S)$ and (b) $\Upsilon(2S)$ data samples. The cyan shaded histograms are from the normalized M_{D^+} and $M_{D_s^{*+}}$ sideband events. The blue solid curves show the fitted results with the R^{++} mass fixed at $4.14 \text{ GeV}/c^2$ and width fixed at 2 MeV , and the blue dashed curves are the fitted backgrounds.

Search for the DDK bound state by Belle

2008.13341

Search for a doubly-charged DDK bound state in $\Upsilon(1S, 2S)$ inclusive decays and via direct production in e^+e^- collisions at $\sqrt{s} = 10.520, 10.580, \text{ and } 10.867$ GeV

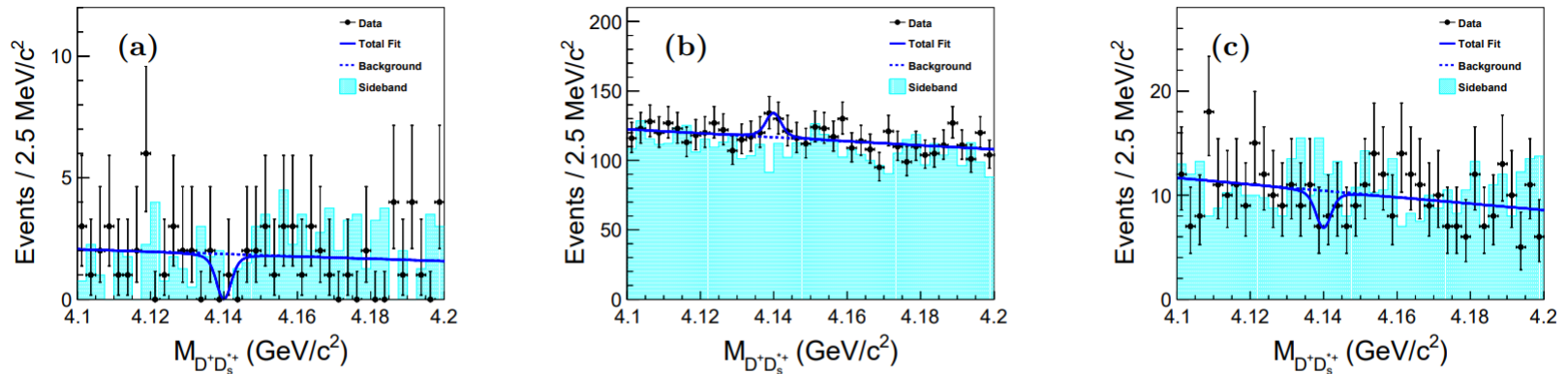


FIG. 10: The invariant-mass spectra of the $D^+D_s^{*+}$ from e^+e^- annihilations at (a) $\sqrt{s} = 10.520$ GeV, (b) $\sqrt{s} = 10.580$ GeV, and (c) $\sqrt{s} = 10.867$ GeV data samples. The cyan shaded histograms are from the normalized M_{D^+} and $M_{D_s^{*+}}$ sideband events. The blue solid curves show the fitted results with the R^{++} mass fixed at 4.14 GeV/c² and width fixed at 2 MeV, and the blue dashed curves are the fitted backgrounds.

Where to search for $K_c(4180)$

$$\Lambda_b \rightarrow J/\psi K P$$

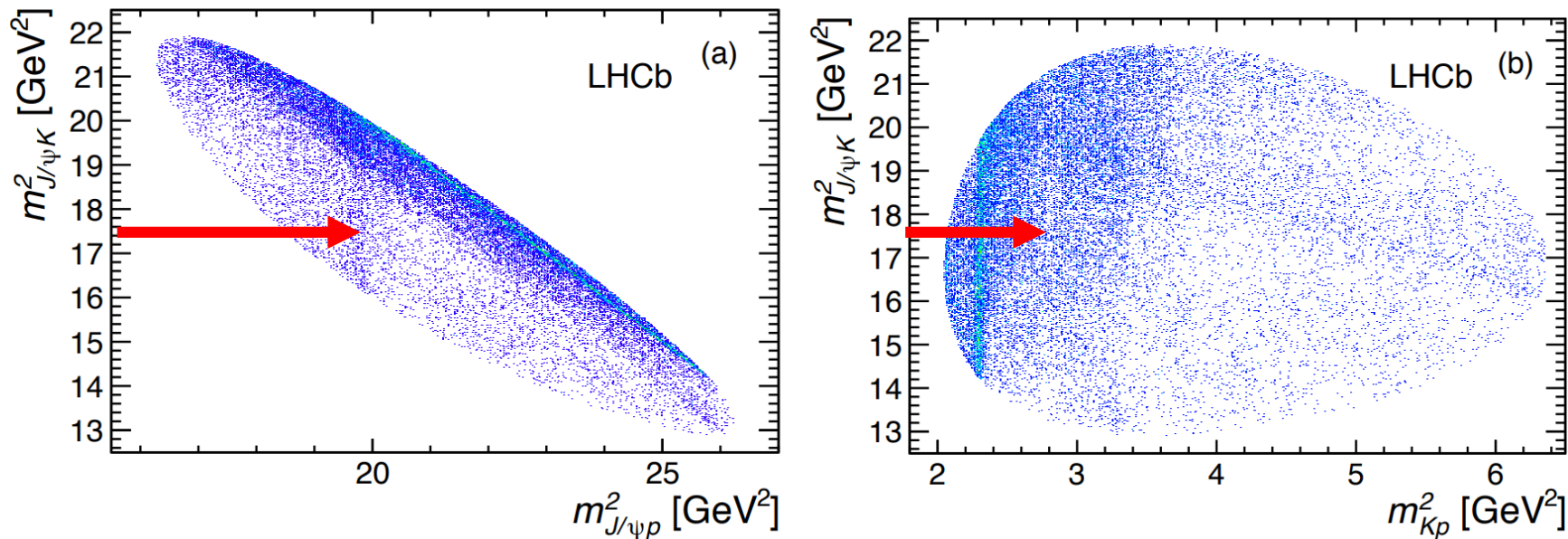


Figure 10: (a) Invariant mass squared of $J/\psi K^-$ versus $J/\psi p$ and (b) of $J/\psi K^-$ versus $K^- p$ for candidates within $\pm 15 \text{ MeV}$ of the Λ_b^0 mass.

LHCb:1507.03414: an integrated luminosity of 3 fb^{-1}

Where to search for **Kc(4180)**

$$B_s \rightarrow J/\psi K K$$

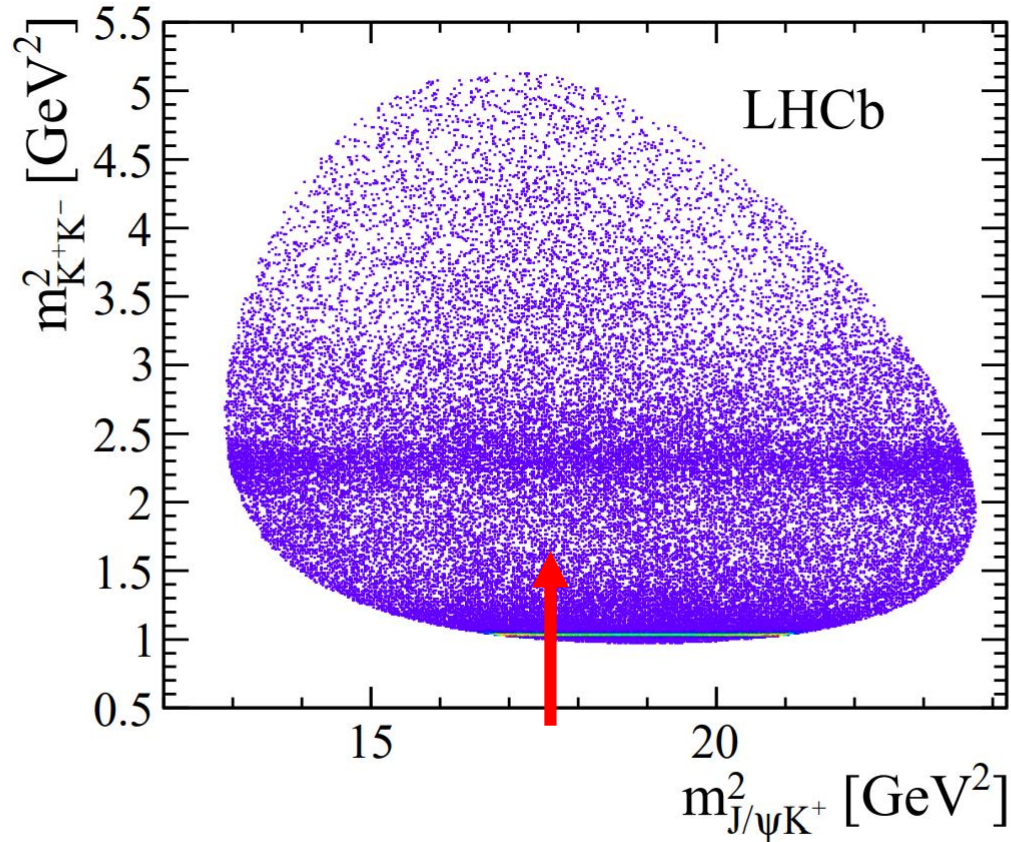
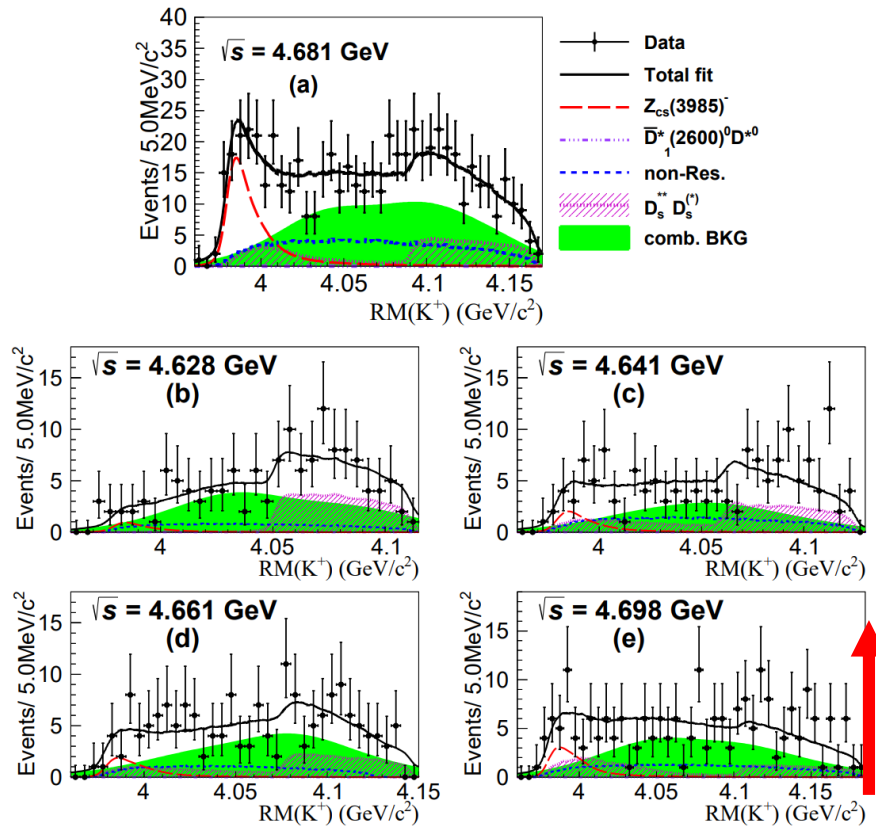


Figure 3: Invariant mass squared of K^+K^- versus $J/\psi K^+$ for $B_s^0 \rightarrow J/\psi K^+ K^-$ candidates within ± 15 MeV of the B_s^0 mass peak. The high intensity $\phi(1020)$ resonance band is shown with a line (light green).

LHCb: 1704.08217: an integrated luminosity of 3 fb^{-1}

Where to search for $K_c(4180)$



$$e^+e^- \rightarrow KD^*\bar{D}_s$$

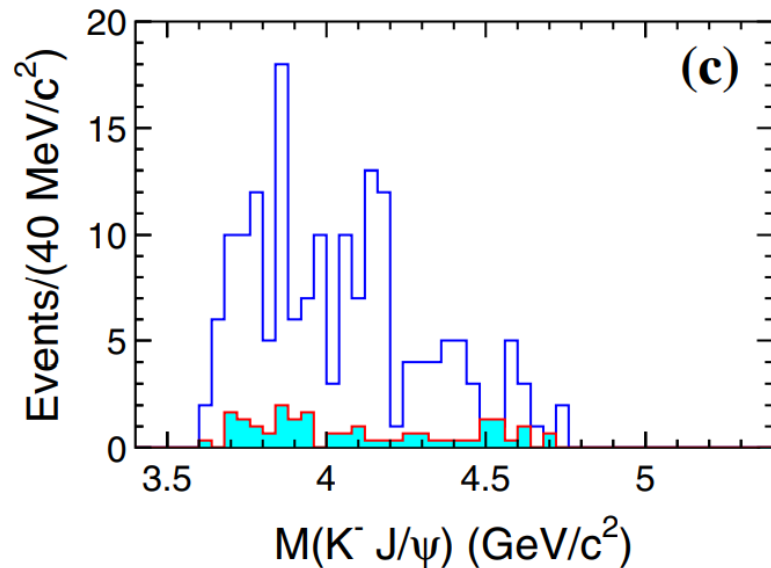
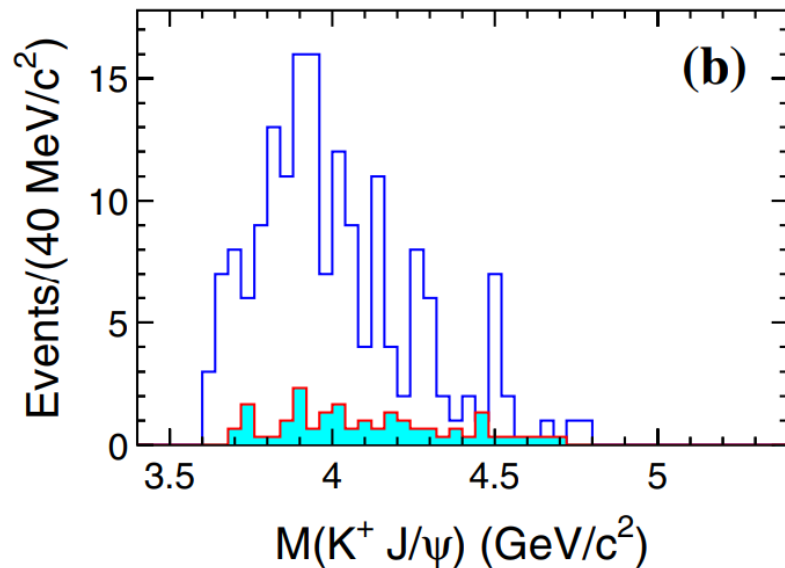
FIG. 3. Simultaneous unbinned maximum likelihood fit to the K^+ recoil-mass spectra in data at $\sqrt{s}=4.628, 4.641, 4.661, 4.681$ and 4.698 GeV. Note that the size of the $D^{*0}\bar{D}_1^{*0}(\rightarrow D_s^- K^+)$ component is consistent with zero.

Where to search for $K_c(4180)$

a few pb

$(ISR) e^+e^- \rightarrow K^+K^-J/\psi$

$4.4 < M(K^+K^-J/\psi) < 5.5 \text{ GeV}/c^2$



Belle: 1402.6578 : 980 fb⁻¹

Summary and outlook

- Many interpretations exist for newly discovered exotic hadrons and are difficult to be distinguished from each other
- We proposed a **novel way to validate the molecule picture**. Taking the $Ds_0^*(2317)$ as an example, if it **is indeed a molecule of DK**, then new forms of matter may be built upon them, similar to the build up of the periodic table.
- **We have performed explicit few-body studies**—demonstrating that indeed both $DDK, D\bar{D}^*K, D\bar{D}K$ and $DDDK$ states **bind**
- Now we need experimental or lattice QCD confirmations and further theoretical studies on their production and decay mechanisms

道生一，一生二，二生三

三生万物

Thanks for your attention!

DDK system in finite volume

Jin-Yi Pang, Jia-Jun Wu, and Li-Sheng Geng, 2008.13014

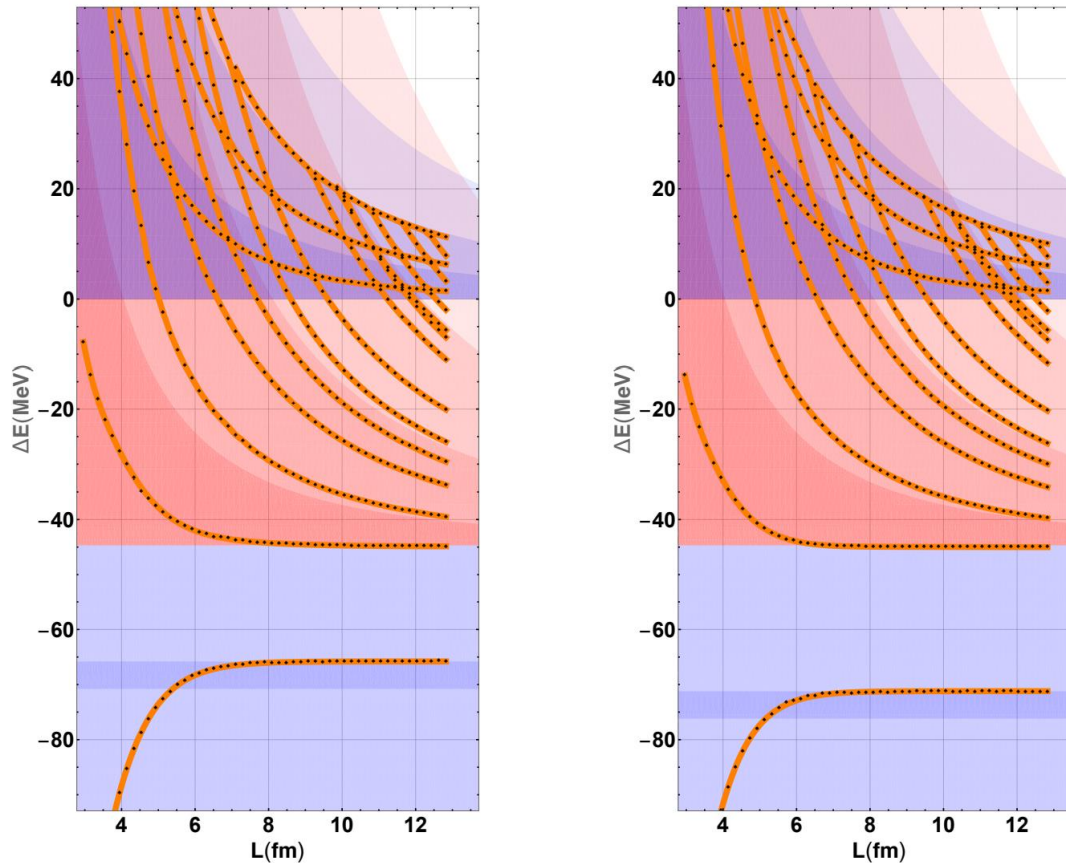
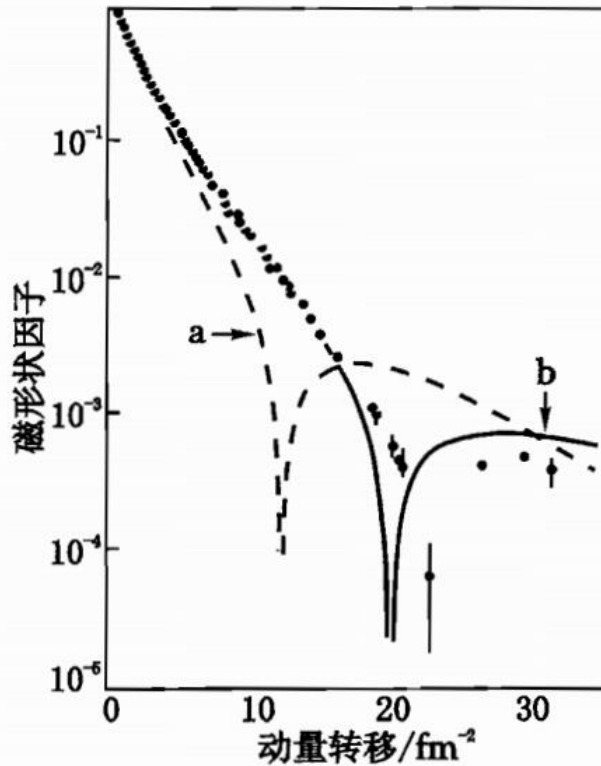


Figure 5: DDK states in finite volume. Left: only the DK interaction is considered. Right: both DK and DD interactions are taken into account. The upper blue regions indicate the case of 3 free particles in finite volume. The red regions indicate the case of free $D_{s0}^*(2317)$ and D . The lower blue regions indicate the DDK bound state below the $DD_{s0}^*(2317)$ threshold.

Magnetic formfactor of Triton—textbook example



a: only nucleon degrees of freedom
b: pion cloud

图 8-30 ${}^3\text{H}$ 的磁矩及其理论解释^①