Fully-charm tetraquark states and their strong decays into di-charmonia

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Based on: **PLB773 (2017), 247-251; Sci.Bull. 65 (2020), 1994-2000**

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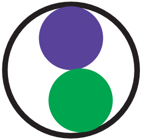
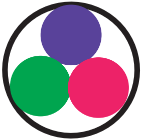
Outline

Quark model

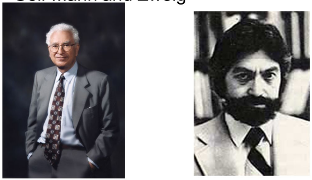


meson($q\bar{q}$)



baryon(qqq)

Gell-Mann and Zweig



- **Quark model** is established to classify hadrons: mesons ($q\bar{q}$) and baryons (qqq).
- Hadrons with exotic quantum numbers are exotic hadron states.
- **QCD** allows for hadrons outside the naive quark model. Hadron structures are more complicated in **QCD**: $\mathrm{N}_{\mathrm{quarks}} \neq 2, 3$.
- $SU(3)_c$ **gauge symmetry**: $(N_q - N_{\bar{q}})$ is divisible by 3, plus any number N_g of valence gluons can form a color singlet.

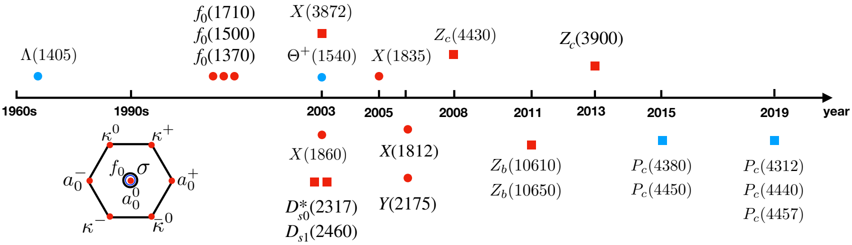
Searching for exotica

Light hadron sector:

- **Dibaryon**: Deuteron, H states, $d^*(2380)$.
- **Hybrid** candidates: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ (dispute).
- **Glueball** candidates: $a_0(980)$ and $f_0(980)$.
- **Tetraquark** candidates: light scalar mesons.
- **Pentaquark**: $\Theta^+(1540)$ ($S = 1$, long story of appeared and disappeared)

Heavy hadron sector: **breakthough in multiquarks!**

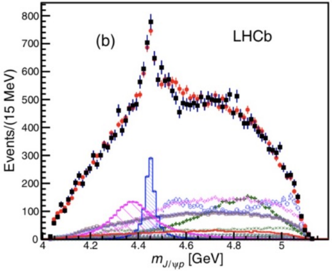
- $P_c(4380)$, $P_c(4312)$, $P_c(4440)$, $P_c(4457)$, $\mathbf{P_{cs}(4459)}$: hidden-charm pentaquark states.
- Plenty of **XYZ states**: candidates of molecules, tetraquarks, hybrids...



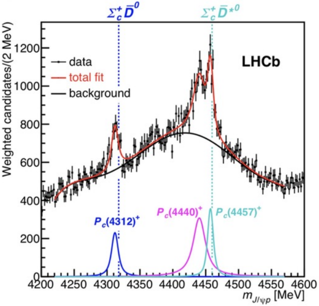
Pentaquarks: $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$

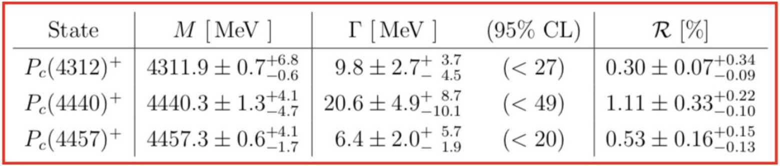
LHCb observed Pc states in 2015 and 2019:

PRL 115 (2015) 072001



PRL 122 (2019) 222001

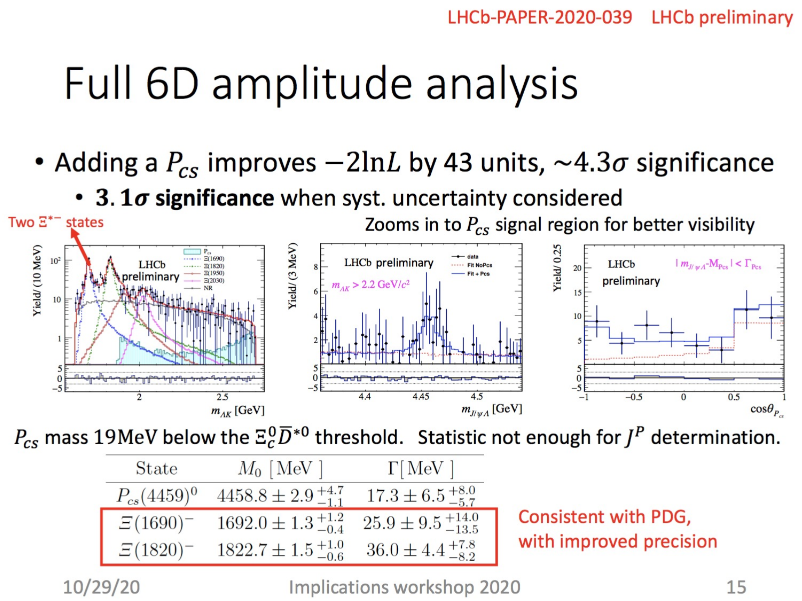




State	M [MeV]	Γ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

Pentaquark: $P_{cs}(4459)$

Slide from Mengzhen Wang



LHCb-PAPER-2020-039 LHCb preliminary

Full 6D amplitude analysis

- Adding a P_{cs} improves $-2\ln L$ by 43 units, ~4.3σ significance
  - **3.1σ significance** when syst. uncertainty considered

Two Ξ^{*-} states

Zooms in to P_{cs} signal region for better visibility

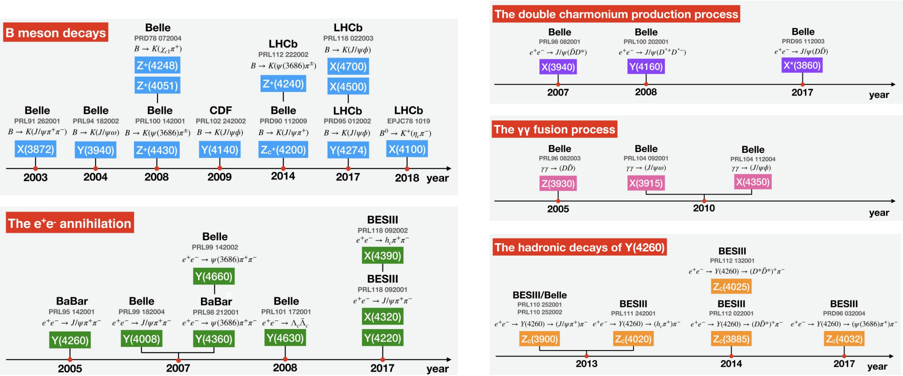
P_{cs} mass 19MeV below the $\Xi_c^0\overline{D}^{*0}$ threshold. Statistic not enough for J^P determination.

State	M_0 [MeV]	Γ[MeV]
$P_{cs}(4459)^0$	$4458.8 \pm 2.9\,^{+4.7}_{-1.1}$	$17.3 \pm 6.5\,^{+8.0}_{-5.7}$
$\Xi(1690)^-$	$1692.0 \pm 1.3\,^{+1.2}_{-0.4}$	$25.9 \pm 9.5\,^{+14.0}_{-13.5}$
$\Xi(1820)^-$	$1822.7 \pm 1.5\,^{+1.0}_{-0.6}$	$36.0 \pm 4.4\,^{+7.8}_{-8.2}$

Consistent with PDG, with improved precision

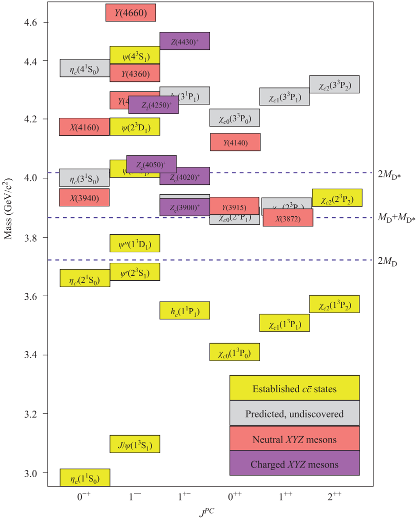
Overview of XYZ States

Experiments: Belle, BaBar, BESIII, CDF, CLEO, D0, LHCb...



Prog.Part.Nucl.Phys.107 (2019) 237-320.

Overview of XYZ States

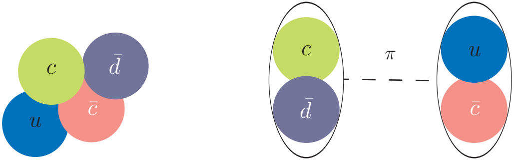


Front. Phys. 10 (2015) 101401

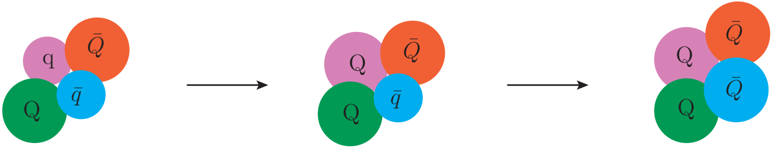
- Many charmonium-like states were discovered above the open-charm thresholds.
- Their masses and decay modes are different from the pure $c\bar{c}$ charmonium states.
- Some charged Z_c states were observed, which are evidences for four-quark states $(c\bar{c}u\bar{d})$.
- They are good candidates for exotic hadron states: **molecule, tetraquark, hybrid ...**

Theoretical Models

- Theoretical configurations: tetraquark, molecule, hybrid,...
- $Z_c^{\pm}$ states: tetraquark, molecule



- What happens as the mass of the light quarks is raised? Binding becomes stronger?

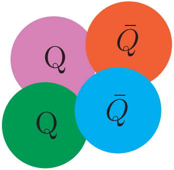


- QED analog: molecular positronium Ps_2 (bound state of $e^+e^-e^+e^-$) discovered in 2007 Nature 449 (09, 2007) 195–197.

Doubly hidden-flavor tetraquarks: $QQ\bar{Q}\bar{Q}$

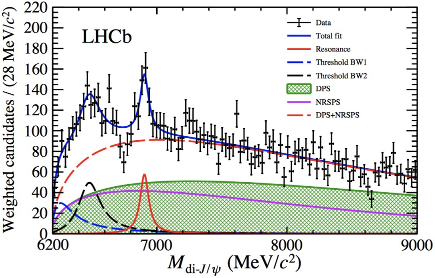
$QQ\bar{Q}\bar{Q}$ Tetraquarks:

- They are far away from the mass range of the observed conventional $q\bar{q}$ hadrons.
- Can be clearly distinguished experimentally from the normal states.
- The light mesons $(\pi, \rho, \omega, \sigma ...)$ can not be exchanged between two charmonia/bottomonia.
- The binding force comes from the short-range gluon exchange.
- A molecule configuration is not favored and thus the $QQ\bar{Q}\bar{Q}$ is a good candidate for compact tetraquark.



$X(6900)$: resonance structure in J/ψ-pair mass spectrum

LHCb observed several structures in the J/ψ-pair mass spectrum **(Sci. Bull., 2020, 2020, 65)**:



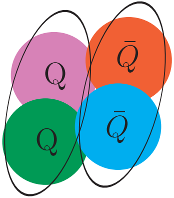
- The mass and width of $X(6900)$ are:(1) $M = 6905 \pm 11 \pm 7$ MeV, $\Gamma = 80 \pm 19 \pm 33$ MeV based on no-interference fit; (2) $M = 6886 \pm 11 \pm 11$ MeV, $\Gamma = 168 \pm 33 \pm 69$ MeV based on the simple model with interference.
- A broad structure next to threshold ranging from 6.2 to 6.8 GeV;
- Hint for another structure around 7.2 GeV with low significance.
- **These structures are consistent with predicted $T_{cc\bar{c}\bar{c}}$ (PLB773(2017), 247-251).**

Tetraquark Sum Rules

- Study two-point correlation function of current $J(x)$ with the same quantum numbers with hadron state:

$$\Pi(q^2) = i \int d^4x e^{iq\cdot x} \langle \Omega | T[J(x) J^\dagger(0)] | \Omega \rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle \Omega | J(x) | X \rangle \neq 0$
- Currents are probes of spectrum and might not overlap with state



Interpolating currents with $J^{PC} = 0^{++}$:

$$\begin{aligned}
J_1 &= Q_a^T C \gamma_5 Q_b \bar{Q}_a \gamma_5 C \bar{Q}_b^T \,, \\
J_2 &= Q_a^T C \gamma_\mu \gamma_5 Q_b \bar{Q}_a \gamma^\mu \gamma_5 C \bar{Q}_b^T \,, \\
J_3 &= Q_a^T C \sigma_{\mu\nu} Q_b \bar{Q}_a \sigma^{\mu\nu} C \bar{Q}_b^T \,, \\
J_4 &= Q_a^T C \gamma_\mu Q_b \bar{Q}_a \gamma^\mu C \bar{Q}_b^T \,, \\
J_5 &= Q_a^T C Q_b \bar{Q}_a C \bar{Q}_b^T \,,
\end{aligned}$$

- Hadron level: described by the dispersion relation

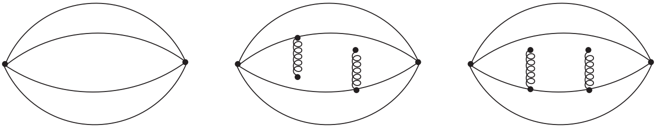
$$\Pi(q^2) = \frac{(q^2)^N}{\pi}\int\frac{\mathrm{Im}\Pi(s)}{s^N(s-q^2-i\epsilon)}ds+\sum_{n=0}^{N-1}b_n(q^2)^n,$$

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s)=\sum_n\delta(s-m_n^2)\langle 0|J|n\rangle\langle n|J^\dagger|0\rangle$$

$$= f_X^2\delta(s-m_X^2)+\mathrm{continuum},$$

- Quark-gluon level: evaluated via operator product expansion(OPE)

$$\Pi(s)=\Pi^{pert}(s)+\Pi^{\langle GG\rangle}(s)+...,$$



- Define moments in Euclidean region $Q^2 = -q^2 > 0$:

$$M_n(Q_0^2) = \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi(Q^2)|_{Q^2=Q_0^2}$$
$$= \int_{m_H^2}^{\infty} \frac{\rho(s)}{(s+Q_0^2)^{n+1}} ds = \frac{f_X^2}{(m_X^2+Q_0^2)^{n+1}}\left[1+\delta_n(Q_0^2)\right],$$

where $\delta_n(Q_0^2)$ contains the higher states and continuum.

- Ratio of the moments

$$r(n, Q_0^2) \equiv \frac{M_n(Q_0^2)}{M_{n+1}(Q_0^2)} = \left(m_X^2 + Q_0^2\right)\frac{1+\delta_n(Q_0^2)}{1+\delta_{n+1}(Q_0^2)}.$$

- Predict hadron mass

$$m_X = \sqrt{r(n, Q_0^2) - Q_0^2}$$

for sufficiently large n when $\delta_n(Q_0^2) \cong \delta_{n+1}(Q_0^2)$ for convergence.

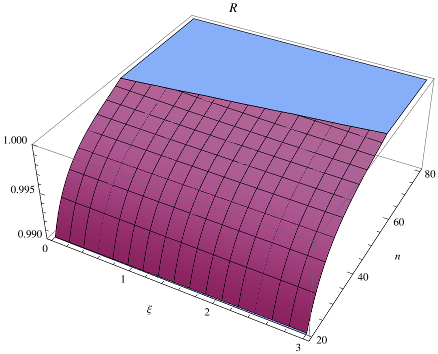
Limitations for $(n,\ \xi)$ parameter space:

$$\xi = Q_0^2/16m_c^2, \text{ for } cc\bar{c}\bar{c} \text{ system;}$$
$$\xi = Q_0^2/m_b^2, \text{ for } bb\bar{b}\bar{b} \text{ system.}$$

- Small ξ: higher dimensional condensates give large contributions to $M_n(Q_0^2)$, leading to bad OPE convergence.
- Large ξ: slower convergence of $\delta_n(Q_0^2)$. This can be compensated by taking higher derivative n for the lowest lying resonance to dominate.
- Large n: moving further away from the asymptotically free region. The OPE convergence would also become bad.
- Requiring $\Pi^{\langle GG\rangle}(s) \leq \Pi^{pert}(s)$ to obtain an upper limit n_{max}, which will increase with respect to ξ.
- Good $(n,\ \xi)$ region: the lowest lying resonance dominates the moments while the OPE series has good convergence.

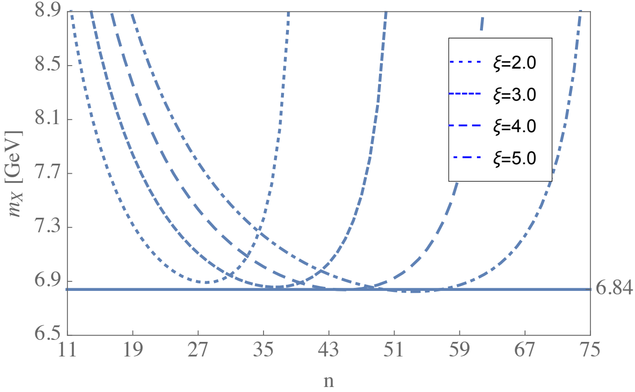
Hölder's inequality:

$$R = \frac{M_n(Q_0^2)^2}{M_r(Q_0^2)M_{2n-r}(Q_0^2)} \leq 1\,,$$



The boundary gives $(n,\xi) = (48, 0.2), (49, 0.4), (49, 0.6), (50, 0.8)$.

Mass for $cc\bar{c}\bar{c}$ tetraquark with $J^{PC} = 0^{-+}$: mass curves have plateaus at $(n, \xi) = (28, 2), (36, 3), (45, 4), (53, 5)$



$$m_{T_{cc\bar{c}\bar{c}}} = (6.84 \pm 0.18)\,\text{GeV}.$$

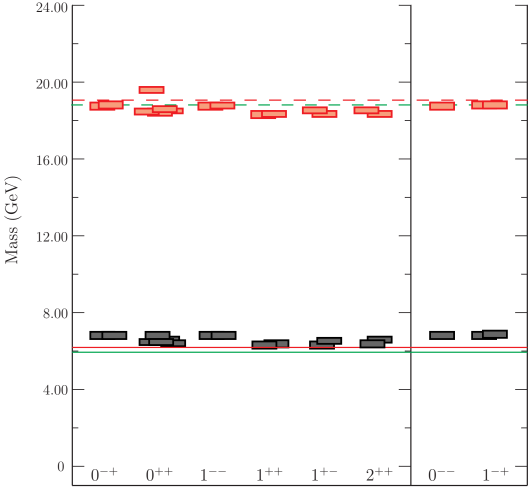
Good predictions for LHCb's observations:

PLB773(2017), 247-251

J^{PC}	Currents	m_{X_c}(GeV)	m_{X_b}(GeV)
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16

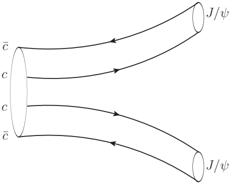
Our previous calculations in 2017 are consistent very good with the LHCb's observation:

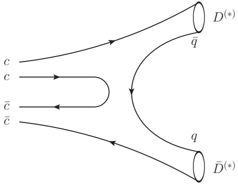
- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{++}$, 2^{++} are agree with the broad structure around 6.2-6.8 GeV;**
- **The masses of $cc\bar{c}\bar{c}$ tetraquarks with $J^{PC} = 0^{-+}$, 1^{-+} are consistent with the mass of X(6900).**

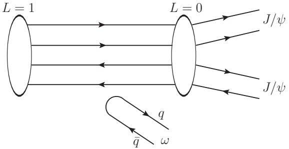

Mass (GeV)
24.00
20.00
16.00
12.00
8.00
4.00
0
0^{-+}
0^{++}
1^{--}
1^{++}
1^{+-}
2^{++}
0^{--}
1^{-+}

Decay behavior: $cc\bar{c}\bar{c}$ tetraquarks

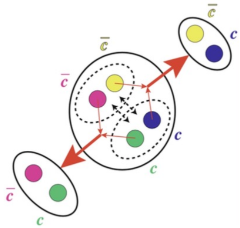
- $cc\bar{c}\bar{c} \to (ccq) + (\bar{c}\bar{c}\bar{q})$: kinematically forbidden.
- $cc\bar{c}\bar{c} \to (cqq) + (\bar{c}\bar{q}\bar{q})$: suppressed by two light quark pair creation.
- $cc\bar{c}\bar{c} \to (c\bar{c}) + (c\bar{c})$: charm quark pair rearrangement or annihilation (suppressed). Phase space is small.
- $cc\bar{c}\bar{c} \to (q\bar{c}) + (c\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation, with large phase space.
- $cc\bar{c}\bar{c}(L=1) \to cc\bar{c}\bar{c}(L=0) + (q\bar{q})_{I=0}$: OZI forbidden.







Strong decays into di-charmonia

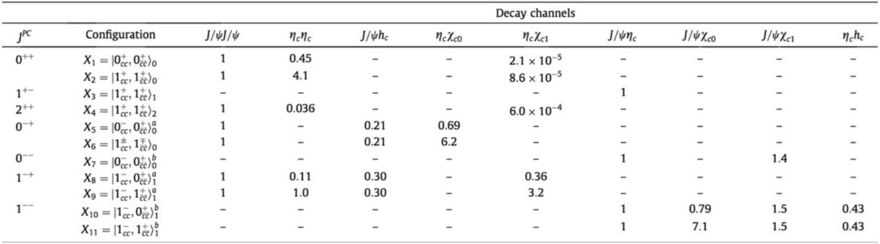


PLB773(2017), 247-251;
Sci.Bull.65,2020, 1994-2000

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S)$, $J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P)$, $J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P)$, $J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi\chi_{c1}(1P)$	$J/\psi\eta_c(1S)$
1^{++}	–	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$, $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi\eta_c(1S)$	$J/\psi\chi_{c0}(1P)$, $J/\psi\chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P)$, $\eta_c(1S)\chi_{c1}(1P)$	$J/\psi J/\psi$
1^{--}	$J/\psi\chi_{c0}(1P)$, $J/\psi\chi_{c1}(1P)$, $\eta_c(1S)h_c(1P)$	$J/\psi\eta_c(1S)$

Strong decays into di-charmonia

We calculate their relative branching ratios through the Fierz rearrangement:



		Decay channels								
J^{PC}	Configuration	$J/\psi J/\psi$	$\eta_c\eta_c$	$J/\psi h_c$	$\eta_c\chi_{c0}$	$\eta_c\chi_{c1}$	$J/\psi\eta_c$	$J/\psi\chi_{c0}$	$J/\psi\chi_{c1}$	$\eta_c h_c$
0^{++}	$X_1 = \|0^+_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0$	1	0.45	–	–	2.1×10^{-5}	–	–	–	–
	$X_2 = \|1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_0$	1	4.1	–	–	8.6×10^{-5}	–	–	–	–
1^{+-}	$X_3 = \|1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1$	–	–	–	–	–	1	–	–	–
2^{++}	$X_4 = \|1^+_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_2$	1	0.036	–	–	6.0×10^{-4}	–	–	–	–
0^{-+}	$X_5 = \|0^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0^a$	1	–	0.21	0.69	–	–	–	–	–
	$X_6 = \|1^\pm_{cc}, 1^\mp_{\bar{c}\bar{c}}\rangle_0$	1	–	0.21	6.2	–	–	–	–	–
0^{--}	$X_7 = \|0^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_0^b$	–	–	–	–	–	1	–	1.4	–
1^{-+}	$X_8 = \|1^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_1^a$	1	0.11	0.30	–	0.36	–	–	–	–
	$X_9 = \|1^-_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1^a$	1	1.0	0.30	–	3.2	–	–	–	–
1^{--}	$X_{10} = \|1^-_{cc}, 0^+_{\bar{c}\bar{c}}\rangle_1^b$	–	–	–	–	–	1	0.79	1.5	0.43
	$X_{11} = \|1^-_{cc}, 1^+_{\bar{c}\bar{c}}\rangle_1^b$	–	–	–	–	–	1	7.1	1.5	0.43

- We suggest **the broad structure around 6.2-6.8 GeV to be a S-wave $cc\bar{c}\bar{c}$ tetraquark with $J^{PC} = 0^{++}$ or 2^{++}**, while **the narrow structure around 6.9 Gev to be a P-wave $cc\bar{c}\bar{c}$ tetraquark with $J^{PC} = 0^{-+}$ or 1^{-+}**.
- We propose to confirm them in the **di-η_c, $J/\psi h_c$, $\eta_c\chi_{c0}$, $\eta_c\chi_{c1}$** channels. These channels are helpful to determine their quantum numbers.

Summary

- LHCb has observed two resonance structures in the J/ψ-pair mass spectrum: a narrow structure X(6900) and a broad structure around 6.2-6.8 GeV;
- We have calculated the mass spectra for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquark states. We also study strong decays of the possible fully-charm tetraquarks, and calculate their relative branching ratios through the Fierz rearrangement
- Our results suggest that **the broad structure around 6.2–6.8 GeV can be interpreted as an S-wave $cc\bar{c}\bar{c}$ tetraquark state with $J^{PC} = 0^{++}$ or 2^{++}**, while **the narrow structure $X(6900)$ to be a P-wave one with $J^{PC} = 0^{-+}$ or 1^{-+}**.
- We propose to confirm them in the di-η_c, $J/\psi h_c$, $\eta_c\chi_{c0}$ and $\eta_c\chi_{c1}$ channels. These channels are helpful to determine their quantum numbers.

Thank you for your attention!