EM Shower Profile

The Parameterized Simulation of Electromagnetic Showers in Homogeneous and Sampling Calorimeters

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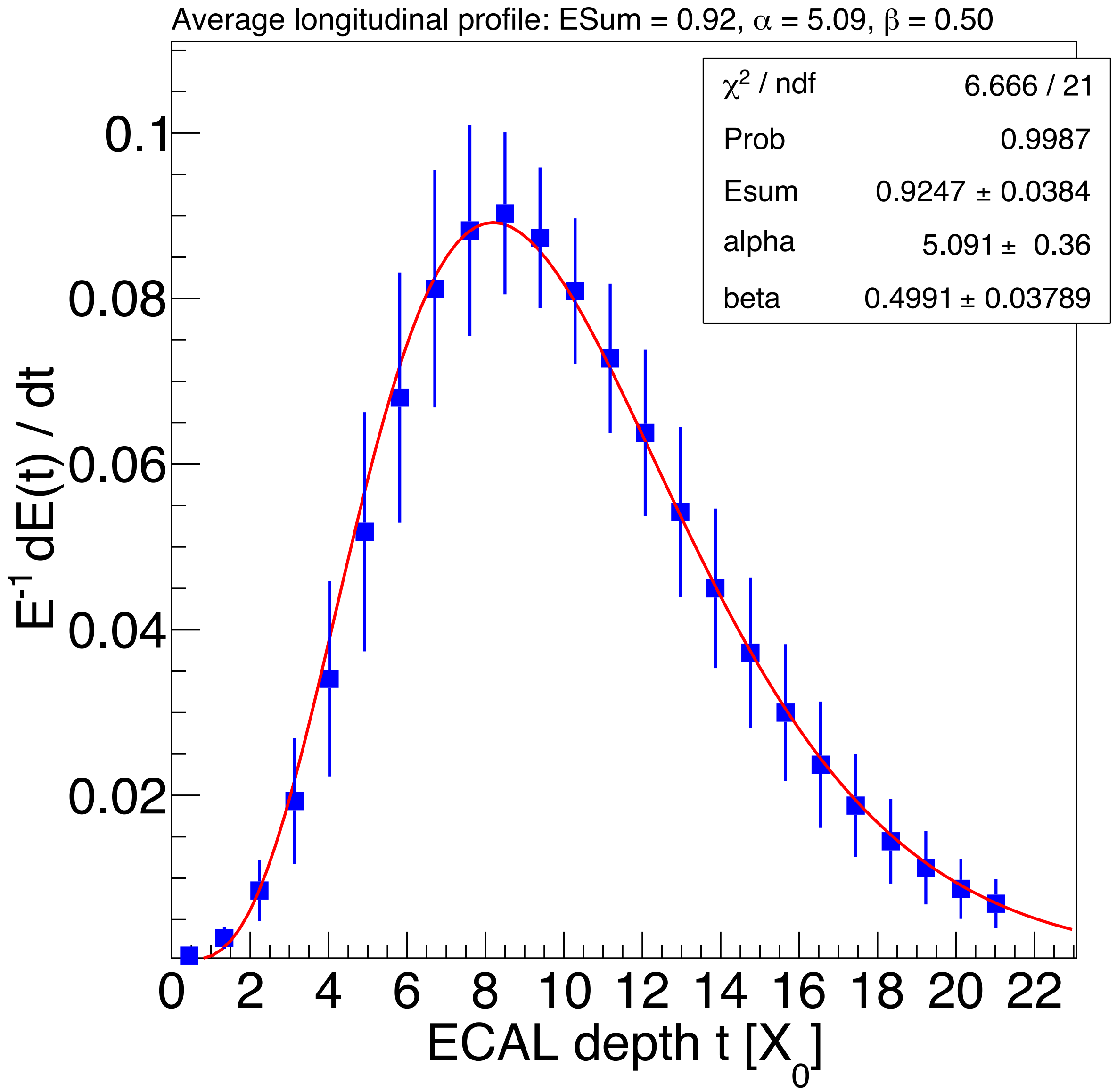
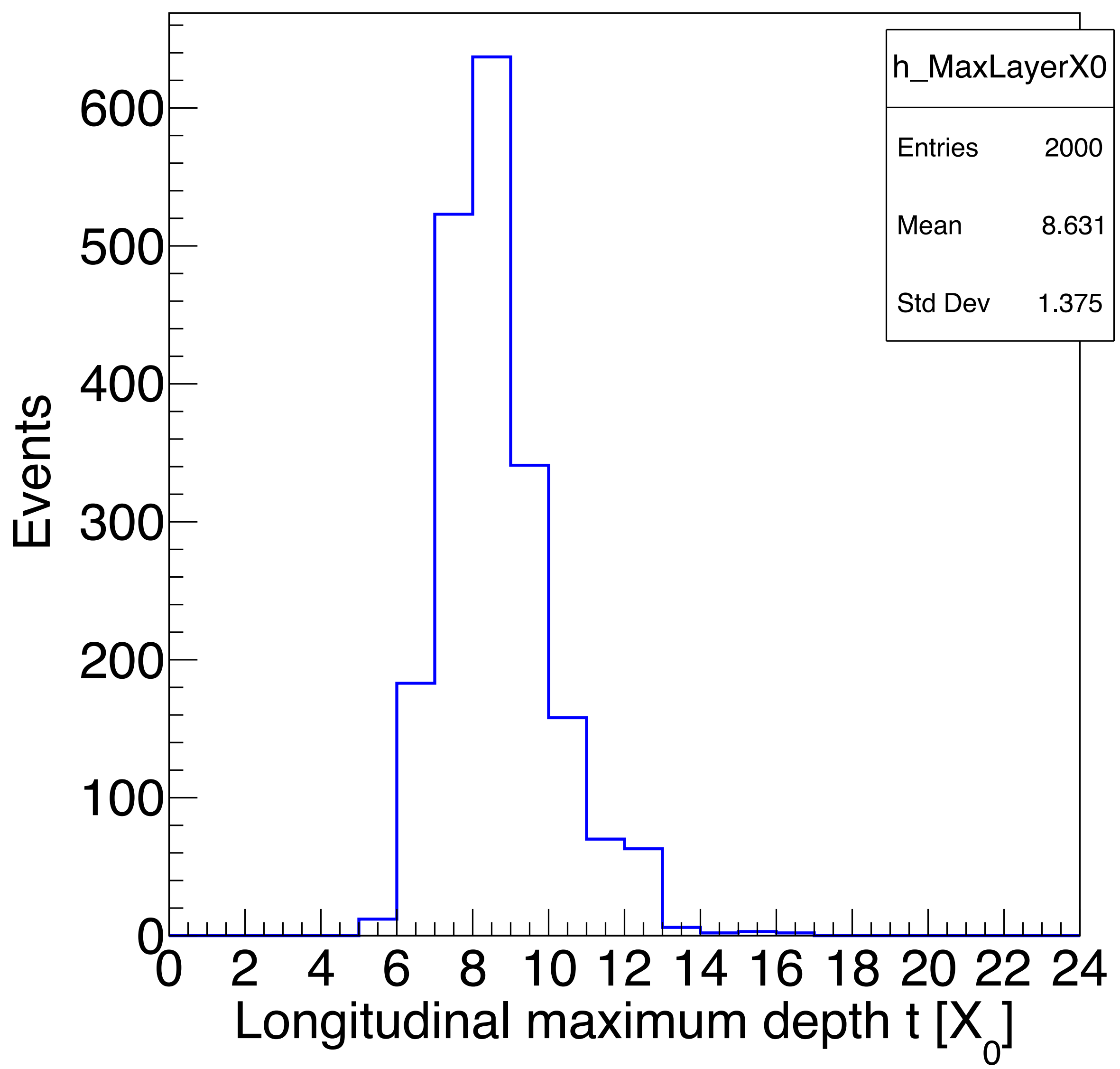
Abstract: A general approach to a fast simulation of electromagnetic showers using parameterizations of the longitudinal and radial profiles in homogeneous and sampling calorimeters is described. The dependence of the shower development on the materials used and the sampling geometry is taken into account explicitly. Comparisons with detailed simulations of various calorimeters and with data from the liquid argon calorimeter of the H1 experiment are made.

https://arxiv.org/abs/hep-ex/0001020v1

100GeV e-

Longitudinal Profile

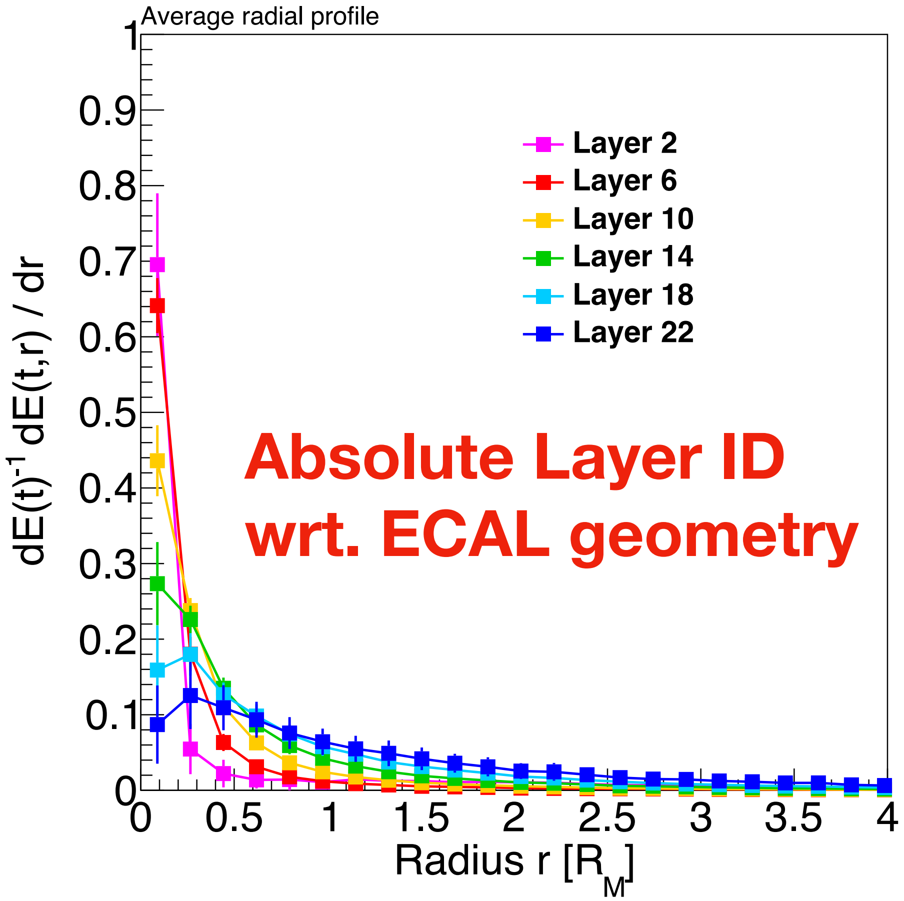


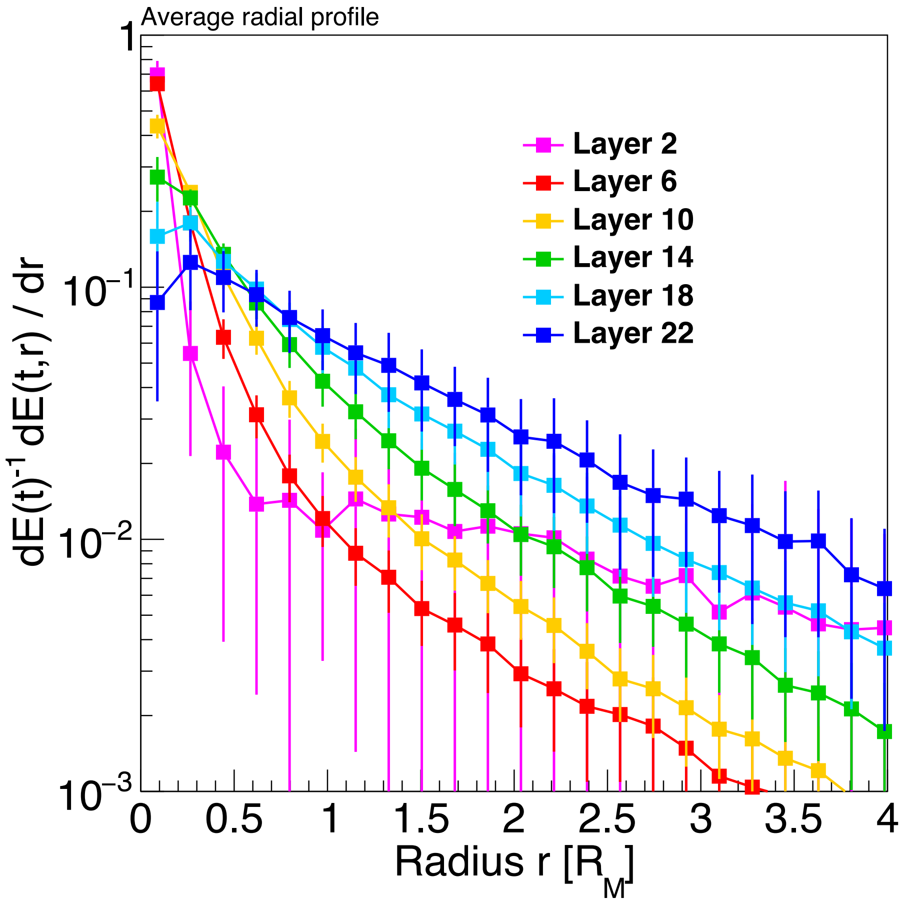


100GeV e-

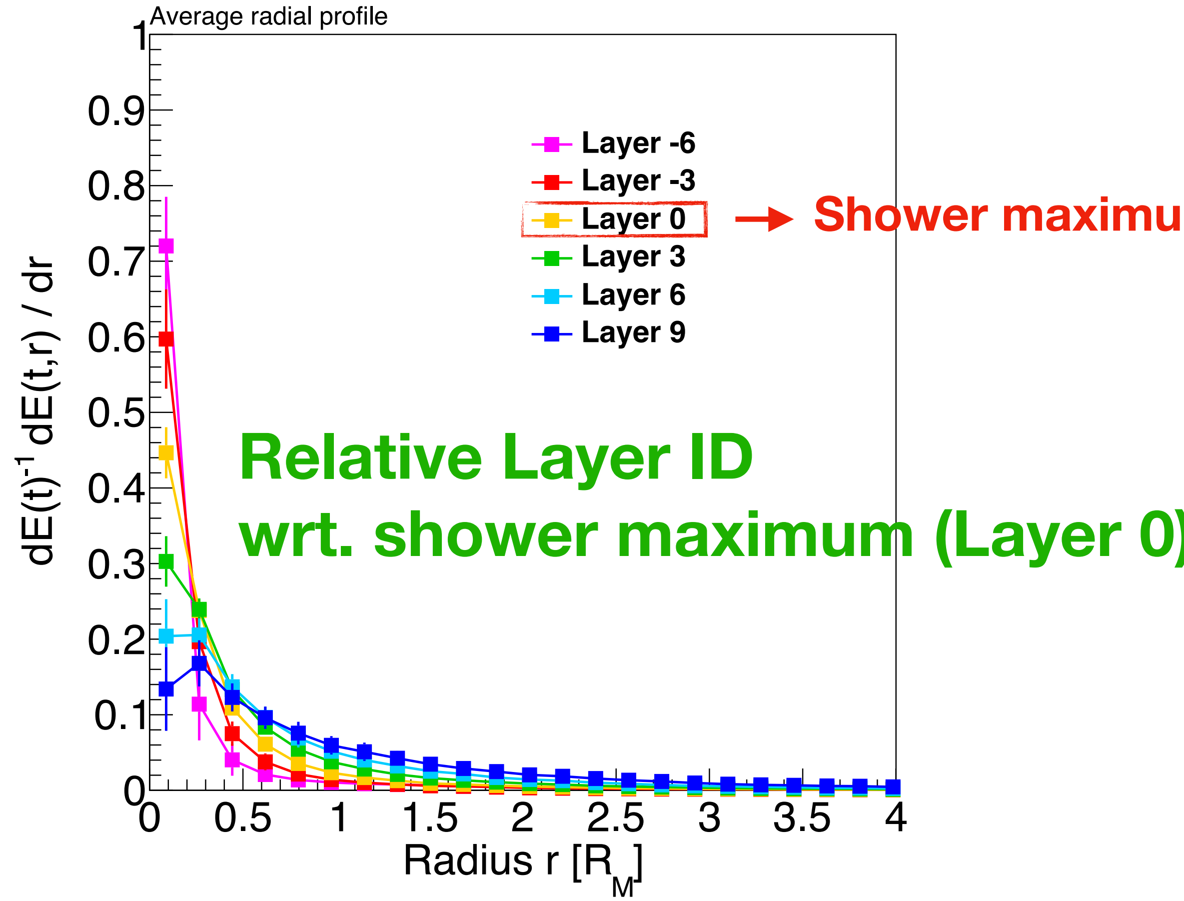
Radial Profile

Before

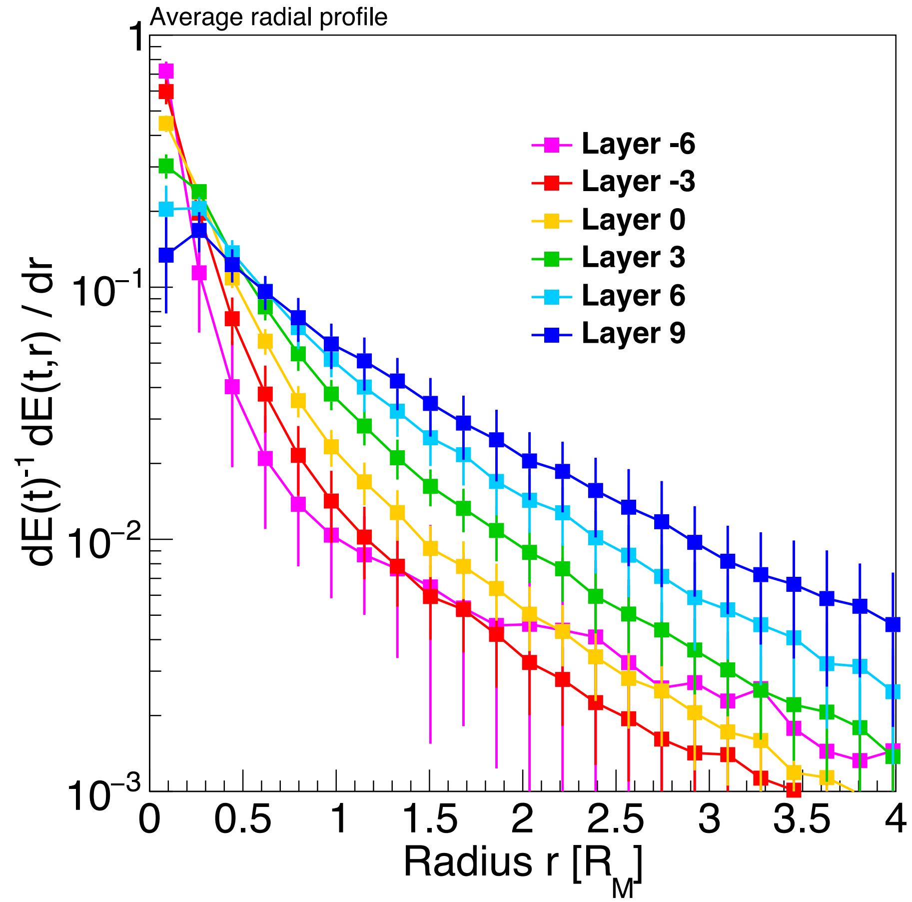




Update



→ Shower maximum



Radial Profile

Average radial energy profiles,

$$f(r) = \frac{1}{dE(t)} \frac{dE(t,r)}{dr}, \tag{22}$$

at different shower depths in pure uranium are presented in Fig.9. These profiles show a distinct maximum in the core of the shower which vanishes with increasing shower depth. In the tail ($r \gtrsim 1 R_M$) the distribution looks nearly flat at the beginning ($1-2X_0$), becomes steeper at moderate depths ($5-6X_0$, $13-14X_0$), and becomes flat again ($22-23X_0$). A variety of different functions can be found in the literature to describe radial profiles [14, 15, 16, 17, 18, 5]. We use the following two component Ansatz, an extension of [5]:

$$\begin{aligned} f(r) &= p f_C(r) + (1-p) f_T(r) \\ &= p \frac{2 r R_C^2}{(r^2 + R_C^2)^2} + (1-p) \frac{2 r R_T^2}{(r^2 + R_T^2)^2} \end{aligned} \tag{23}$$

with

$$0 \le p \le 1.$$

Here R_C (R_T) is the median of the core (tail) component and p is a probability giving the relative weight of the core component. For the shower depth $1-2X_0$ the distributions $f(r)$, $p f_C(r)$, and $(1-p) f_T(r)$ are also indicated in Fig.9.

The following formulae are used to parameterize the radial energy density distribution for a given energy and material:

$$\tau = t/T$$

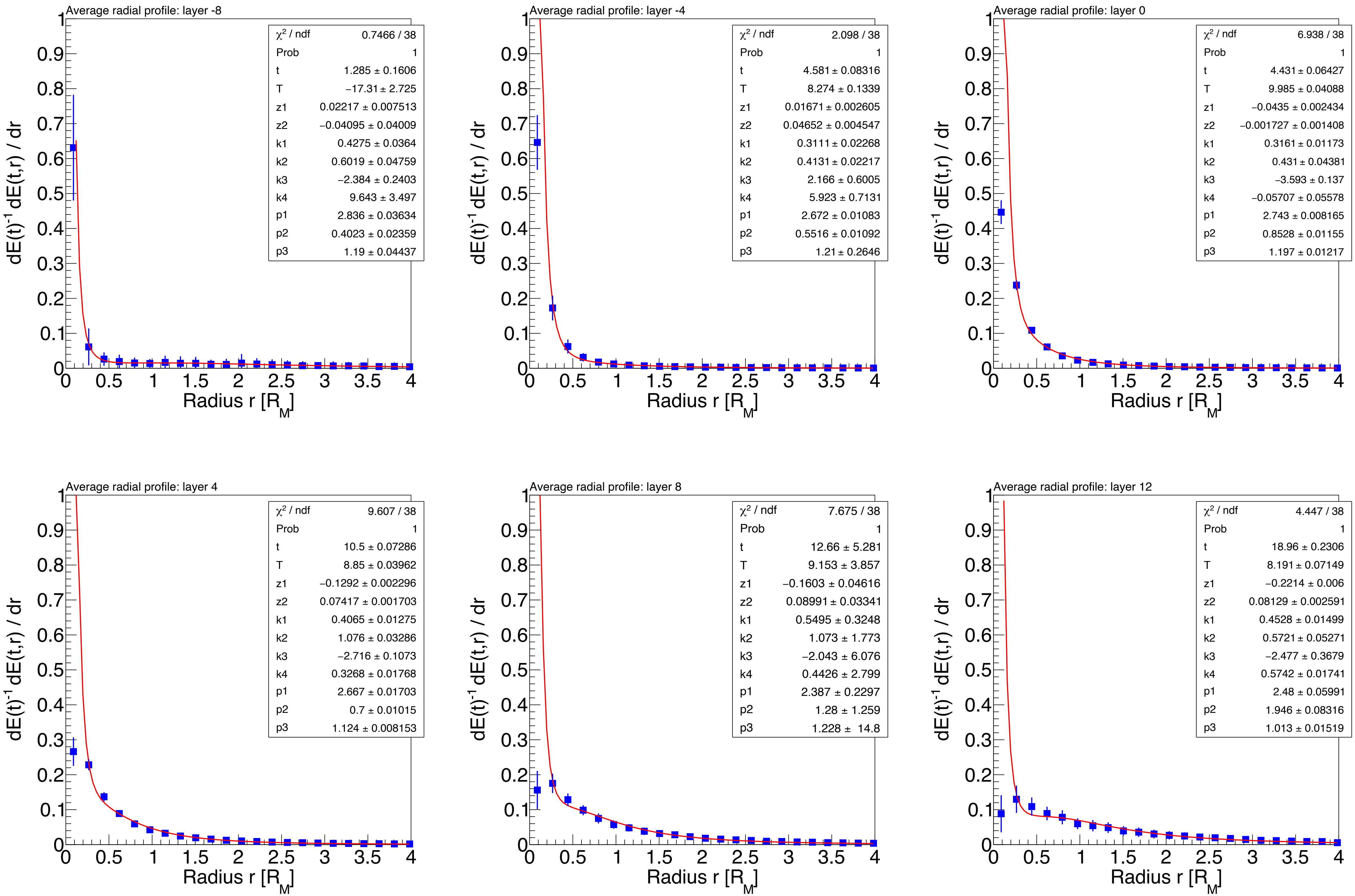
$$\begin{aligned} R_{C,hom}(\tau) &= z_1 + z_2 \tau \qquad (24) \\ R_{T,hom}(\tau) &= k_1 \{\exp(k_3(\tau - k_2)) + \exp(k_4(\tau - k_2))\} \qquad (25) \\ p_{hom}(\tau) &= p_1 \exp\left\{\frac{p_2 - \tau}{p_3} - \exp\left(\frac{p_2 - \tau}{p_3}\right)\right\} \qquad (26) \end{aligned}$$

with

$$\begin{aligned} z_1 &= 0.0251 + 0.00319 \ln E \\ z_2 &= 0.1162 + -0.000381 Z \\ k_1 &= 0.659 + -0.00309 Z \\ k_2 &= 0.645 \\ k_3 &= -2.59 \\ k_4 &= 0.3585 + 0.0421 \ln E \\ p_1 &= 2.632 + -0.00094 Z \\ p_2 &= 0.401 + 0.00187 Z \\ p_3 &= 1.313 + -0.0686 \ln E \end{aligned}$$

100GeV e-

Radial Profile



BACKUP

Longitudinal Profile

3.1 Longitudinal shower profiles – homogeneous media

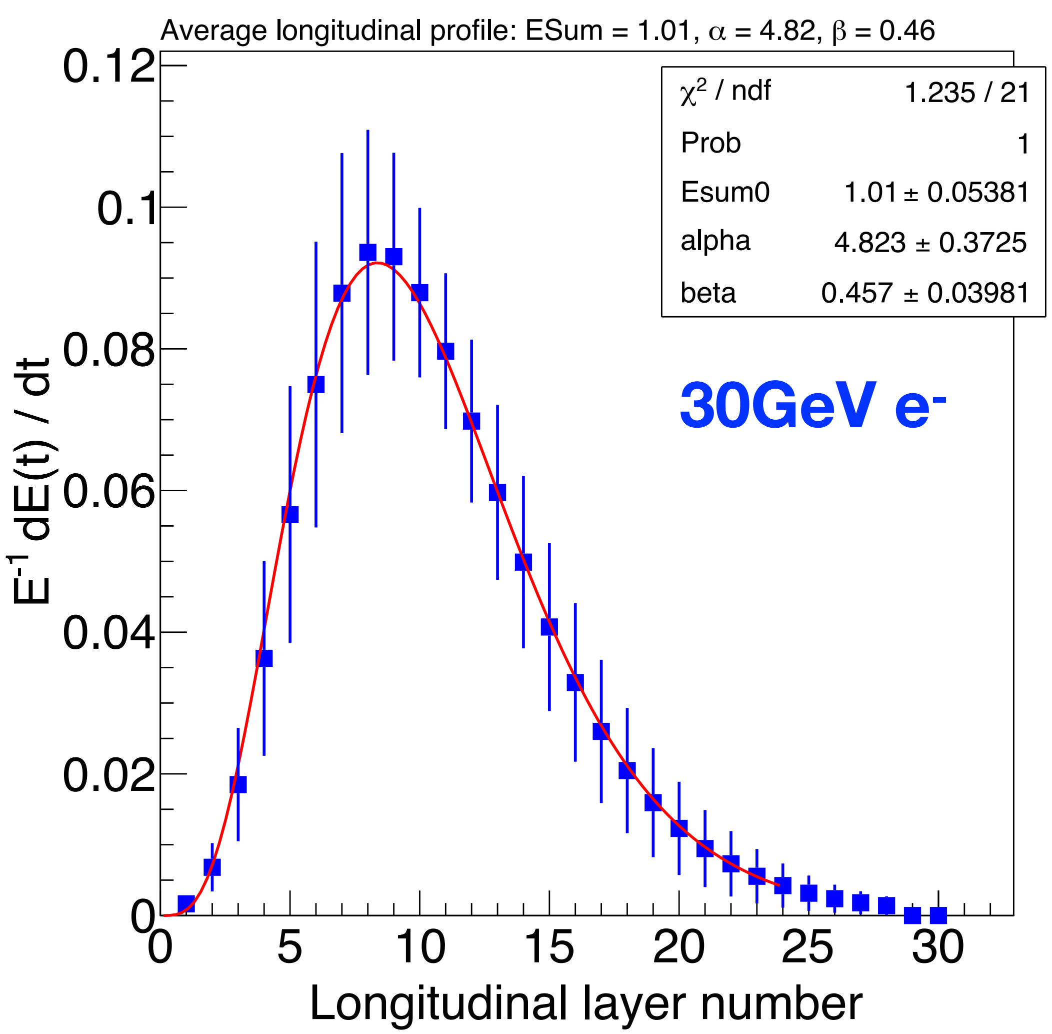
It is well known that average longitudinal shower profiles can be described by a gamma distribution [1]:

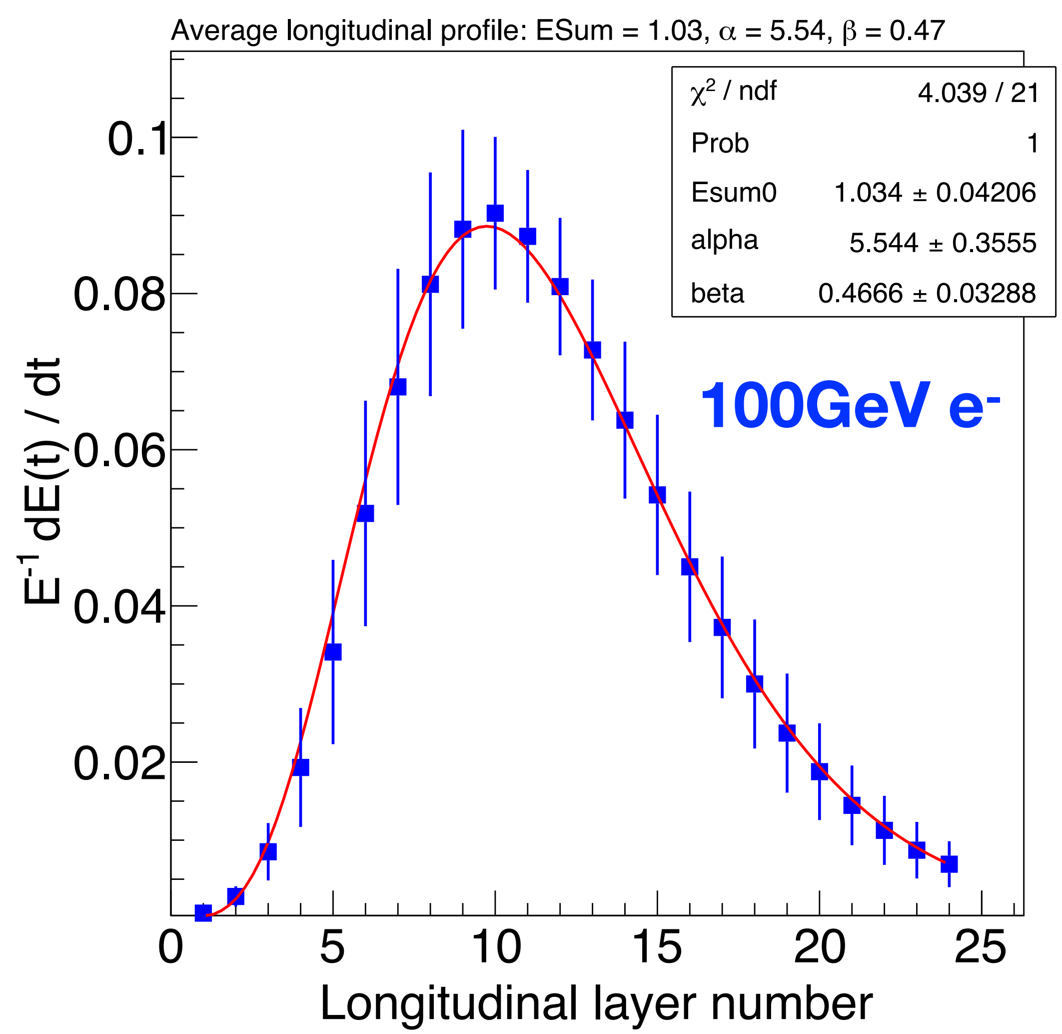
$$\left\langle \frac{1}{E}\frac{dE(t)}{dt} \right\rangle = f(t) = \frac{(\beta t)^{\alpha-1}\beta \exp(-\beta t)}{\Gamma(\alpha)}. \tag{2}$$

The center of gravity, $\langle t \rangle$, and the depth of the maximum, T, can be calculated from the shape parameter α and the scaling parameter β according to

$$\langle t \rangle = \frac{\alpha}{\beta} \tag{3}$$

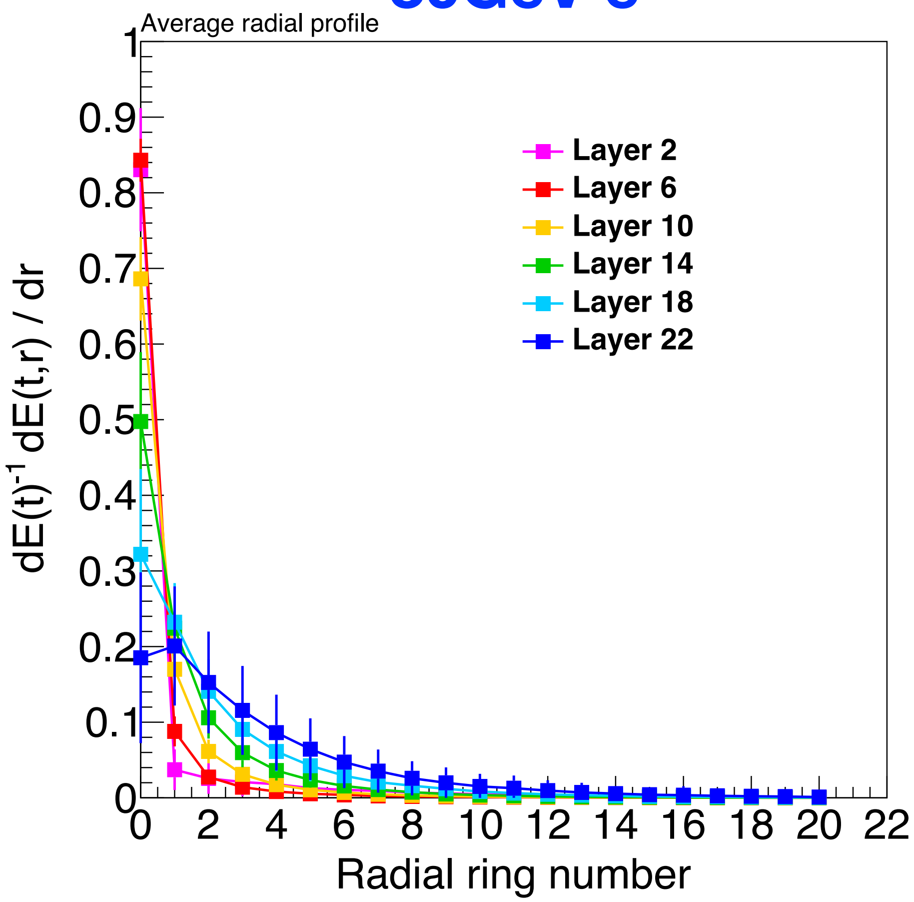
$$T = \frac{\alpha - 1}{\beta}. \tag{4}$$

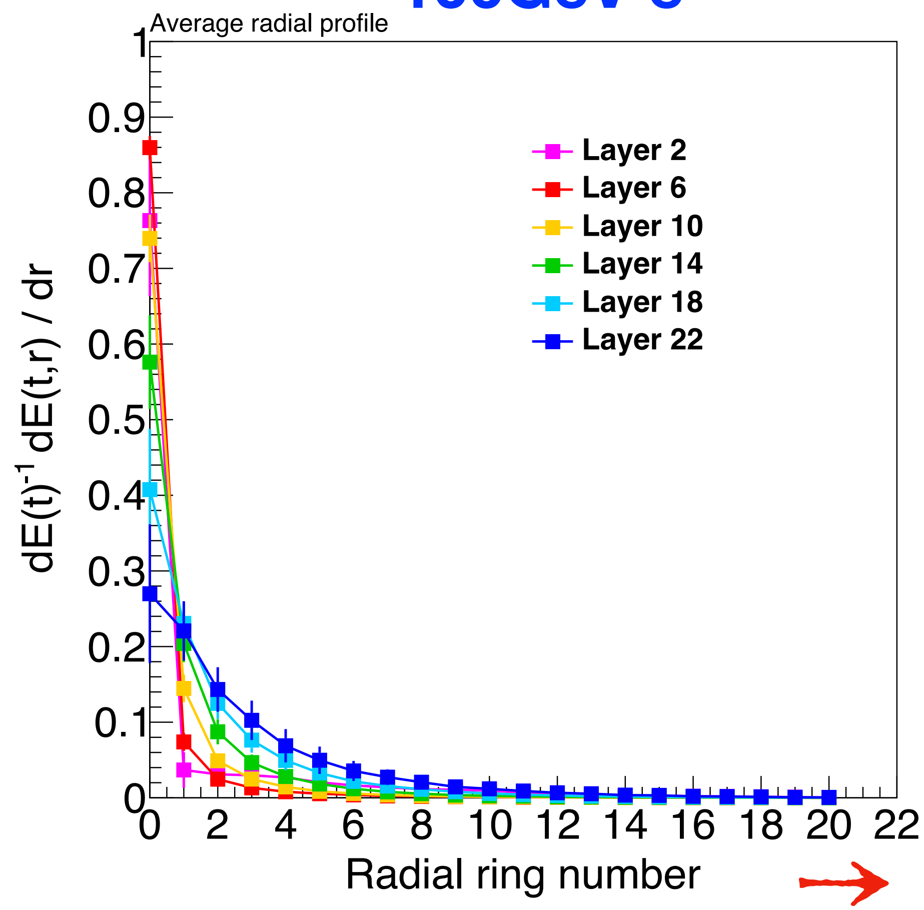




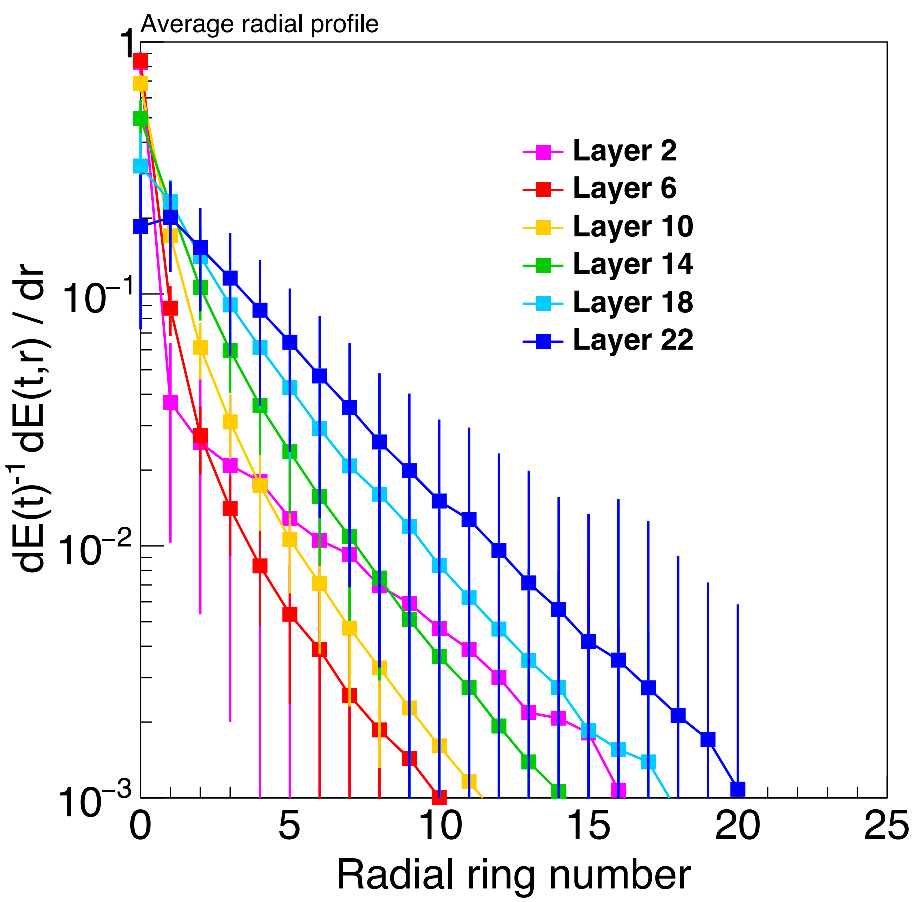
Radial Profile

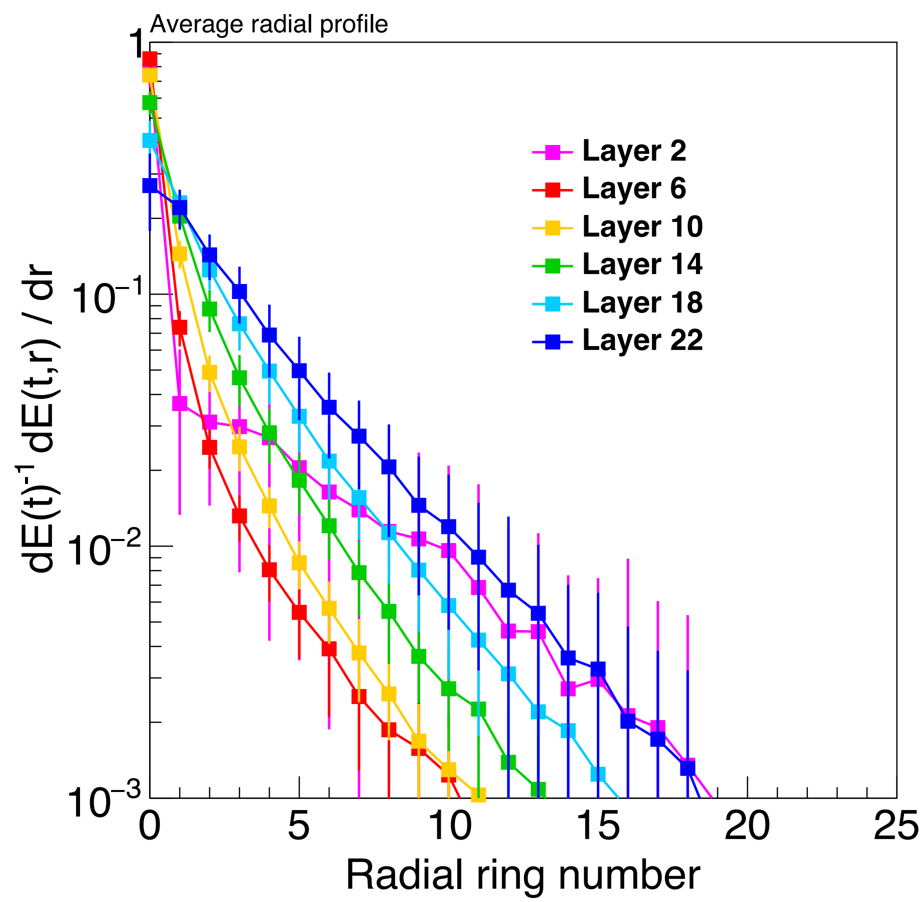
30GeV e⁻ | **100GeV e⁻**





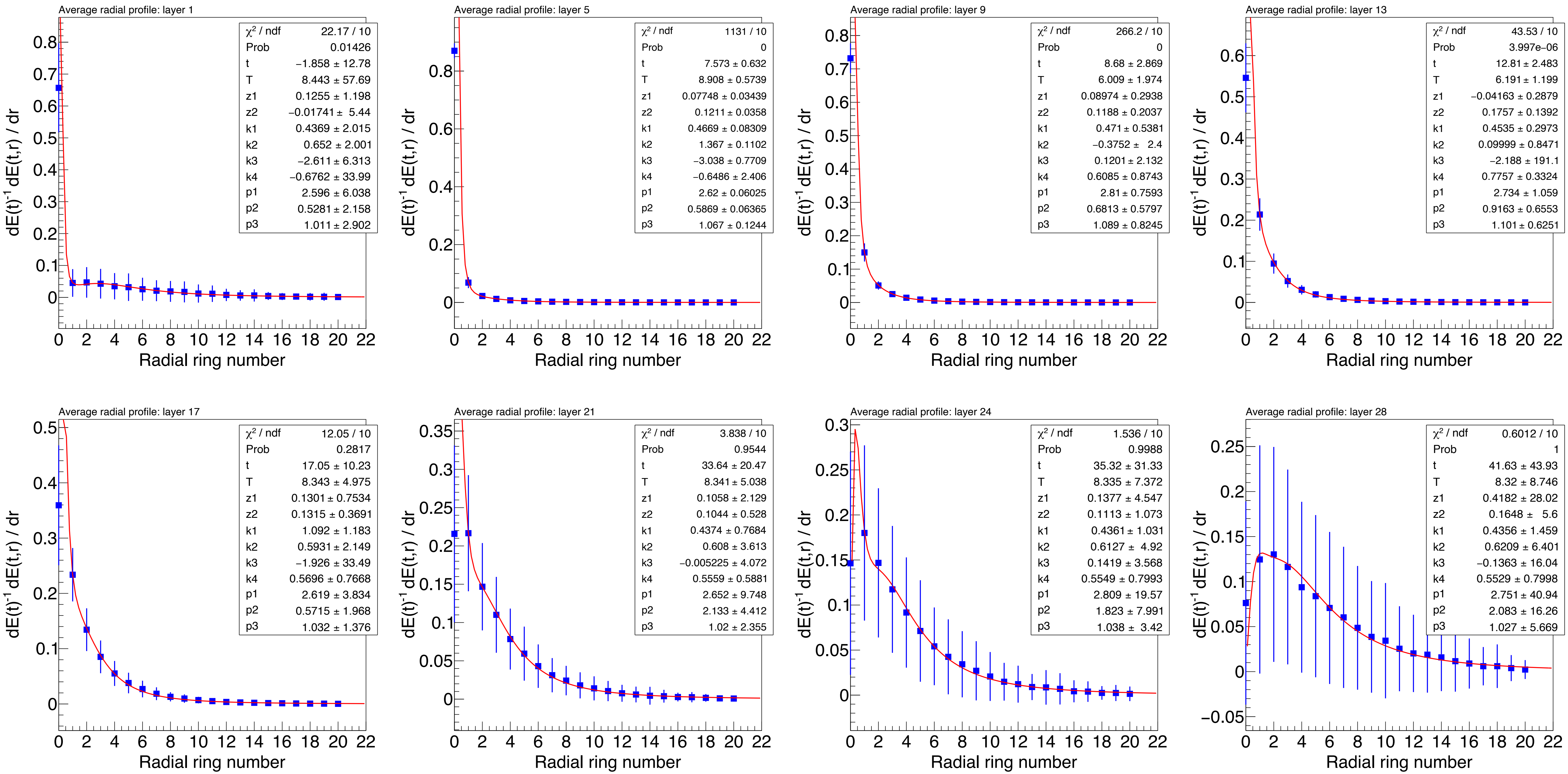
→ 10mm / ring





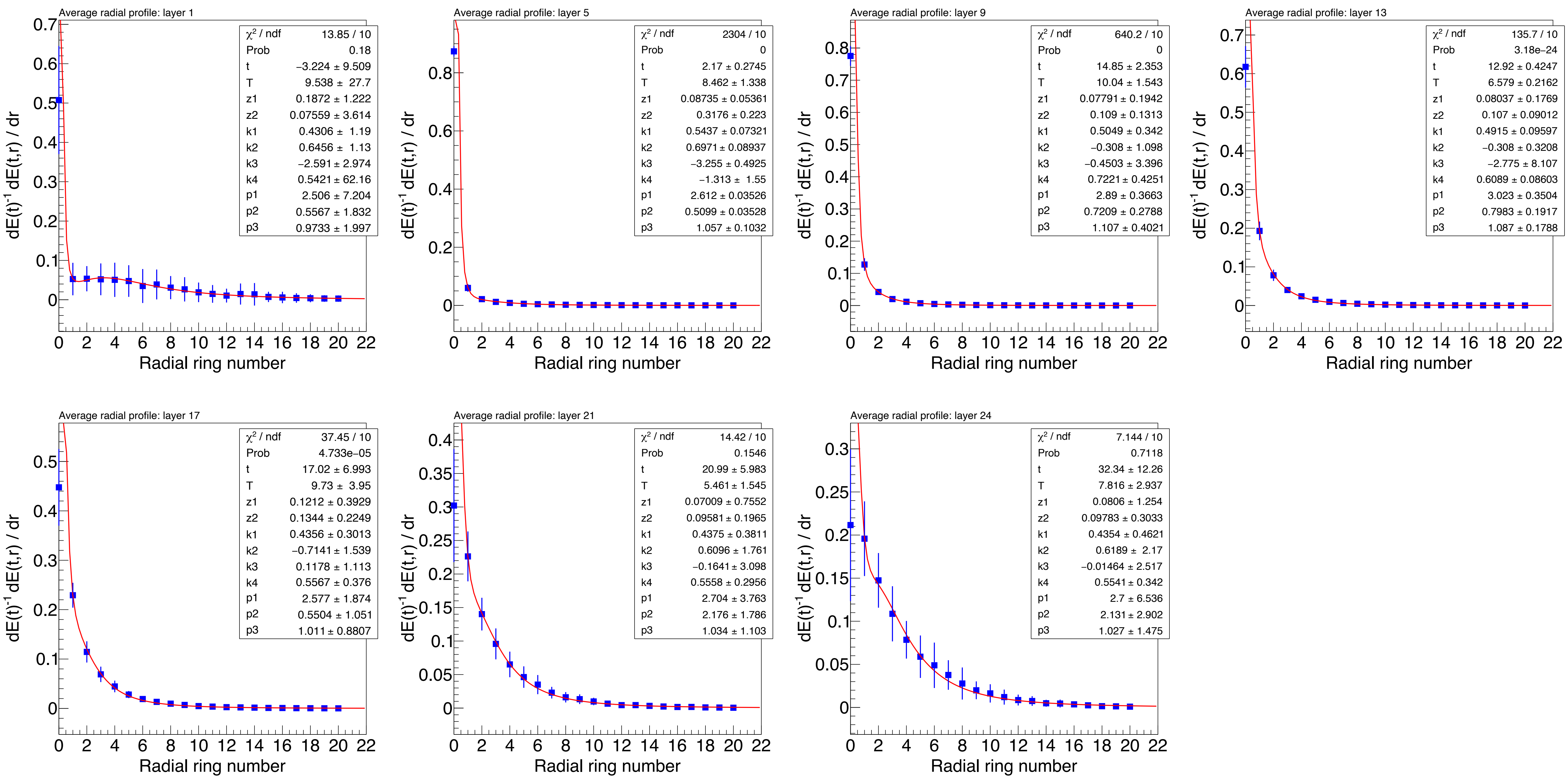
Radial Profile

30GeV e⁻



Radial Profile

100GeV e$^-$



REVIEW OF PARTICLE PHYSICS*

Particle Data Group 2018

The mean longitudinal profile of the energy deposition in an electromagnetic cascade is reasonably well described by a gamma distribution [59]:

$$\frac{dE}{dt} = E_0\, b\, \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)} \qquad (33.36)$$

Measurements of the lateral distribution in electromagnetic cascades are shown in Refs. 61 and 62. On the average, only 10% of the energy lies outside the cylinder with radius R_M. About 99% is contained inside of $3.5R_M$, but at this radius and beyond composition effects become important and the scaling with R_M fails. The distributions are characterized by a narrow core, and broaden as the shower develops. They are often represented as the sum of two Gaussians, and Grindhammer [60] describes them with the function

$$f(r) = \frac{2r\, R^2}{(r^2 + R^2)^2}\ , \qquad (33.40)$$

where R is a phenomenological function of x/X_0 and $\ln E$.

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Nuclear Instruments and Methods in Physics Research A290 (1990) 469–488
North-Holland

THE FAST SIMULATION OF ELECTROMAGNETIC AND HADRONIC SHOWERS *

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Received 3 November 1989

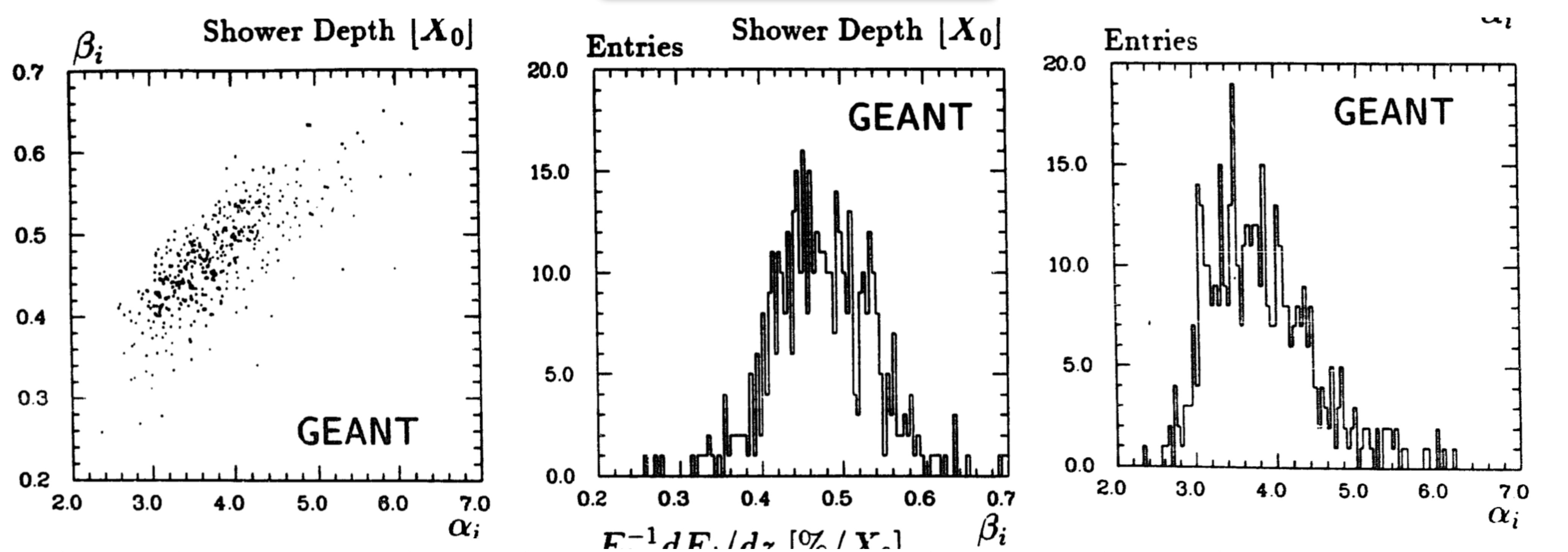
A program for the fast simulation of electromagnetic and hadronic showers using parameterizations for the longitudinal and lateral profile is described. The fluctuations and correlations of the parameters are taken into account in a consistent way. Comparisons with data over a wide energy range are made.

3. Parameterization of eiectromagnetic showers

3.1. Longitudinal shower profile

$$f_i(z) = \frac{x^{\alpha_i - 1} e^{-x}}{\Gamma(\alpha_i)}, \quad \text{with } x = \beta_i z,$$

parameters α_i and β_i.



3.2. Lateral shower profile

$$f(r) = \frac{2rR_{50}^2}{(r^2 + R_{50}^2)^2},$$

parameter R_{50} as a function of shower energy E [GeV] and shower depth z [in units of X_0 or λ_0]:

$$\langle R_{50}(E, z)\rangle = [R_1 + (R_2 - R_3 \ln E)z]^n,$$

$$V_{R_{50}}(E, z) = [(S_1 - S_2 \ln E)(S_3 + S_4 z)\langle R_{50}(E, z)\rangle]^2. \quad (7)$$

How to determine the three free parameters R1, R2, and R3?