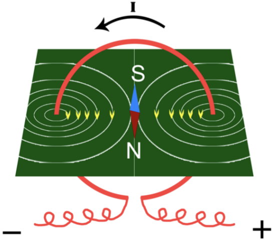
Theoretical calculations of muon g-2

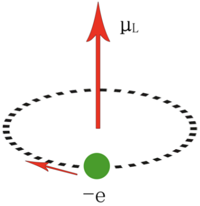
Xu Feng (冯旭)

PEKING UNIVERSITY

微扰量子场论研讨会，2021年05月15日

Circular current and magentic moment





- Magnetic moment

$$\mu = I \cdot S$$

- Circular current

$$I = \frac{e}{T}, \quad T = \frac{2\pi R}{v}, \quad S = \pi R^2 \quad \rightarrow \quad \mu = \frac{1}{2} Rev$$

- Angular momentum

$$L = Rmv$$

- A relation between μ and L

$$\mu = g \frac{e}{2m} L$$

with a dimensionless coefficient $g = 1$ called as Landé g factor

Electron anomalous magnetic moment

- For an electron with mass m, charge e, spin s, its magnetic dipole moment μ is given by

$$\mu = g_e \frac{e}{2m} s$$

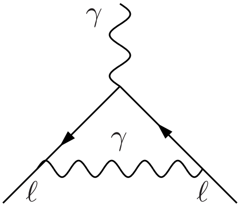
Dirac theory predicts the Landé factor $g = 2$

- The quantum fluctuation causes the anomalous magnetic moment

$$a_e = \frac{g_e - 2}{2} \neq 0$$

g-2 receives the largest contribution from QED

$$a_e \approx \frac{\alpha}{2\pi} \approx 0.001\,16$$



Electron g-2 lays a foundation for QED

Experimental measurements (CODATA 2018)

$$a_e^{\mathrm{exp}} = 1\,159\,652\,181.28(18) \times 10^{-12}$$

Theretical (QED) predication - $O(\alpha^5)$ (Aoyama, Kinoshita, Nio, 2018)

$$a_e^{\mathrm{th}} = 1\,159\,652\,181.61(23) \times 10^{-12}$$

Experiments and theory match to the 10th digits, successfully verifying QED



Tomonaga, Schwinger & Feynman won Nobel Prize (1965) for developing QED

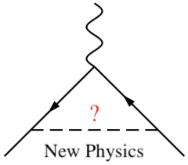
Three generation of leptons

e vs μ vs τ: same properties in Standard Model with only different masses

$$m_\tau : m_\mu : m_e \approx 3500 : 200 : 1$$

Muon is not stable, experimental precision is much better for a_e than that for a_μ

So why muon?



- In lowest order, heavy virtual particle with scale Λ_{NP} contributes to a_ℓ as

$$a_\ell^{NP} \propto \frac{m_\ell^2}{\Lambda_{NP}^2} \quad \rightarrow \quad \frac{a_\mu^{NP}}{a_e^{NP}} \propto \frac{m_\mu^2}{m_e^2} \approx 4 \times 10^4$$

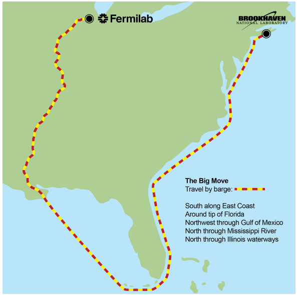
- Loose a factor of 800 in experimental precision $\rightarrow$ a_μ is still 50 times more sensitive to NP
- τ is more senstitive to NP than muon, but life time is 7×10^6 times shorter than μ

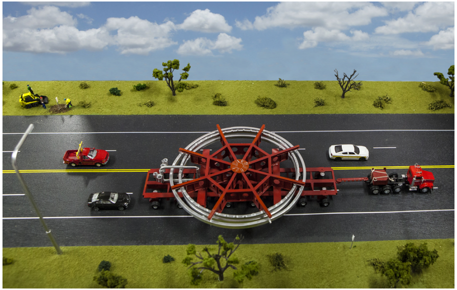
Muon g-2

3.7 times of standard deviation between BNL experiment and Standard Model

BNL Exp. [0.54 ppm]	$a_\mu^{\mathrm{exp}} = 116592080(63) \times 10^{-11}$	Muon G-2, PRD 2006
SM Total [0.32 ppm]	$a_\mu^{\mathrm{SM}} = 116591810(43) \times 10^{-11}$	White paper 2020
Deviation [3.7 σ]	$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 279(76) \times 10^{-11}$	

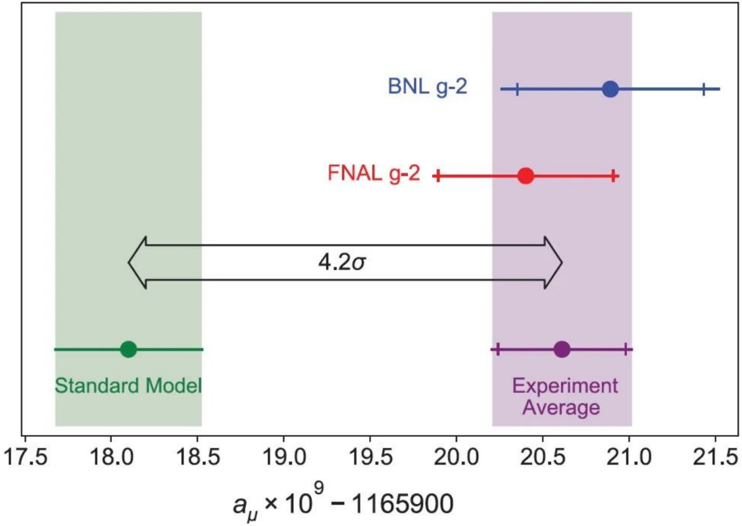
New experiment: main device is a 15 meter superconducting electromagnet





Move from BNL to FNAL ⇒ reduce the experimental error by a factor of 1/4

Experimental efforts: BNL 2006 $\rightarrow$ FNAL 2021

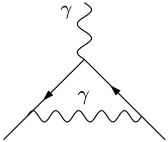


- Combine new experiment, the deviation changes from 3.7 to 4.2 σ
- Analyzed $< 6\%$ of the data that the experiment will eventually collect

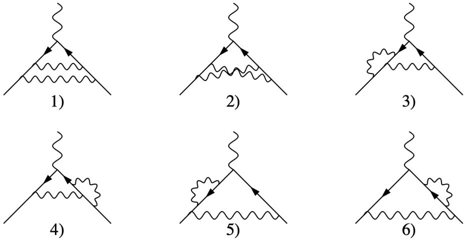
Perturbative calculations

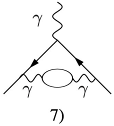
QED contribution summary

	value	#diagrams	publications
1-loop	$0.5\left(\frac{\alpha}{\pi}\right)$	1	Schwinger 1948
2-loop	$0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2$	7	Petermann 1957, Elend 1966
3-loop	$24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3$	72	Kinoshita 1995, Laporta & Remiddi 1996
4-loop	$130.8796(63)\left(\frac{\alpha}{\pi}\right)^4$	891	Aoyama et.al. 2015, Laporta 2017
5-loop	$753.29(1.04)\left(\frac{\alpha}{\pi}\right)^5$	12 672	Aoyama, Kinoshita & Nio 2018



One loop





Two loop

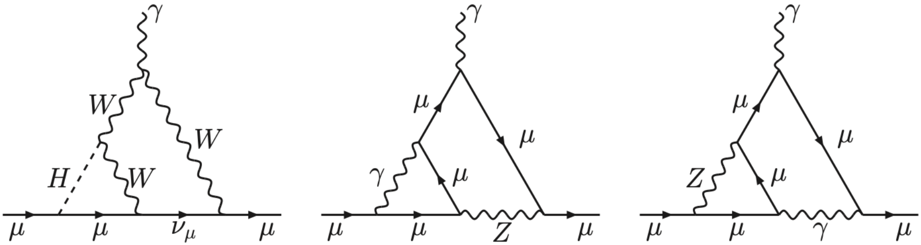
QED contribution summary

$$
\begin{aligned}
a_\mu^{\text{QED}} &= 0.5 \times \left(\frac{\alpha}{\pi}\right) + 0.765\,857\,425 \underbrace{(17)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^2 \\
&\quad + 24.050\,509\,96 \underbrace{(32)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^3 + 130.8796 \underbrace{(63)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^4 \\
&\quad + 753.29 \underbrace{(1.04)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^5 \\
&= 116\,584\,718.853 \underbrace{(9)}_{m_\mu/m_{e,\tau}} \underbrace{(19)}_{c_4} \underbrace{(7)}_{c_5} \underbrace{(29)}_{\alpha(a_e)} [36] \times 10^{-11}
\end{aligned}
$$

- All terms up to $O(\alpha^4)$ are cross checked by different groups
- Entire $O(\alpha^5)$ contribution has been calculated only by one group $\rightarrow$ need a cross check

Weak contribution summary

Calculation up to two loops with sample diagrams



	value	publications
QED incl. 5-loops	$116\,584\,718.853(36) \times 10^{-11}$	Aoyama et.al. 2018
Weak incl. 2-loops	$153.6(1.0) \times 10^{-11}$	Gnendiger et.al. 2013

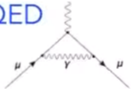
Compared to QED, the weak contribution is suppressed by a factor of $m_\mu^2/M_W^2 \sim 10^{-6}$

Non-perturbative calculations

Standard Model contributions to muon $g-2$

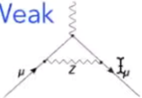
$$\boxed{a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})}$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

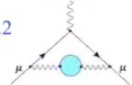


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

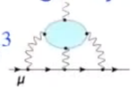
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

…Light-by-Light (HLbL)

α^3



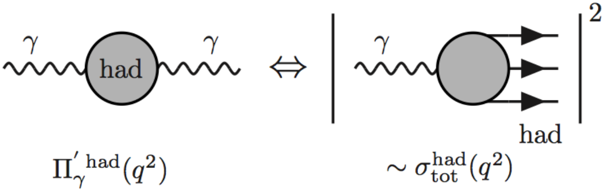
$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

FNAL exp targets on precision of 0.14 ppm $\rightarrow$ HVP with error 0.2-0.3%

Hadronic vacuum polarization

V_μ  V_ν $= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_V(q^2)$

- Optical theorem



$$\mathrm{Im}\,\Pi_V(s) = \frac{s}{4\pi\alpha}\sigma(e^+e^- \to \mathrm{hadrons})$$

- Dispersion relation

$$\Pi_V(q^2) - \Pi_V(0) = \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty} ds\, \frac{\mathrm{Im}\,\Pi_V(s)}{s(s - q^2 - i\varepsilon)}$$

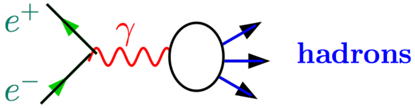
Although HVP function is non-perturbative at low energy, it can be computed using experimental cross section as inputs

R value from experiment

Experimental measurement of R value

$$R = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

- As $\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$ is known, R value is equivalent to the measurement of $\sigma(e^+e^- \to \gamma^* \to \text{hadrons})$



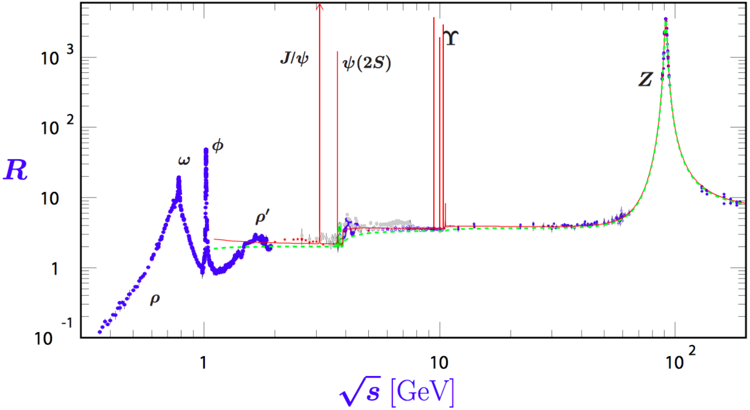
In the high energy region, R can be calculated perturbatively

$$R(s)^{\text{pert}} = N_c \sum_f Q_f^2 \frac{v_f}{2}(3 - v_f^2)\Theta(s - 4m_f^2) \times (1 + \alpha_s c_1 + \alpha_s^2 c_2 + \cdots)$$

- $N_c = 3$ provide evidence of three colors for QCD in the history
- Velocity $v_f = \sqrt{1 - \frac{4m_f^2}{s}}$

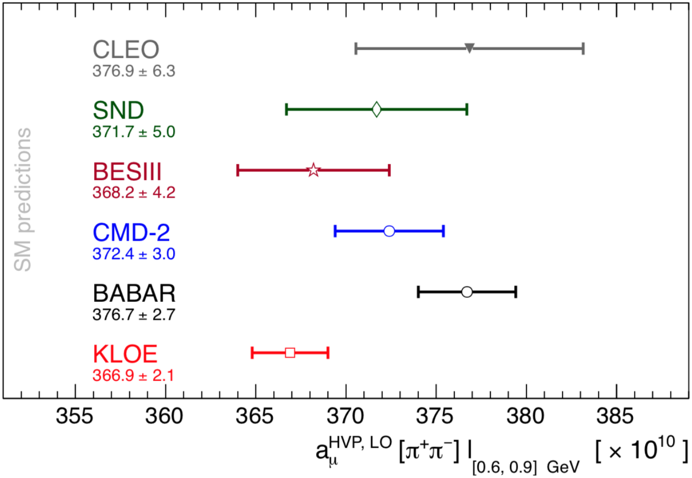
R value from experiment

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Data-driven calculations of HVP

Combined exp. data + dispersion theory → HVP with error 0.6%



- Integral range: 0.6 - 0.9 GeV → involving the ρ resonance peak
- 2.9 σ tension between KLOE and BABAR $\sim$ 2.45$\times$ HVP error

Outlook from BESIII

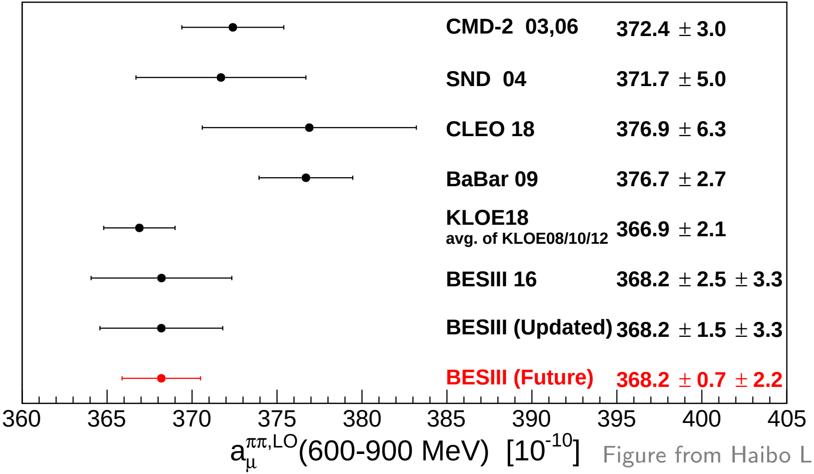
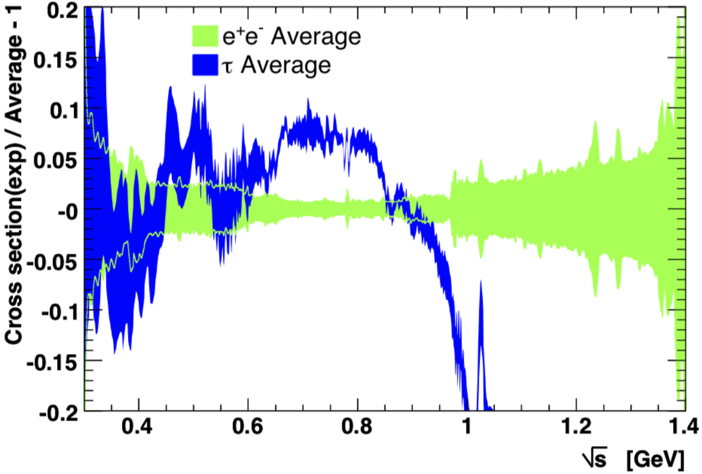


Figure from Haibo Li

- Current result published in 2016 and updated in 2020
- Untill 2024, accumalate 7$\times$ data $\rightarrow$ reduce error to $\pm 2.2 \times 10^{-10}$

e^+e^- collision vs τ decay

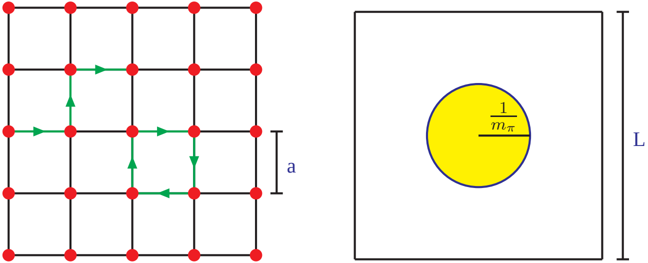


- ρ-γ mixing correction has been applied to τ decay
- Discrepancy at 0.6 - 0.9 GeV
- Need better understanding of the IB corrections to τ decay

Lattice QCD calculations

Lattice discretization

- quark fields live on the lattice sites, $\psi(x)$, $x_\mu = n_\mu a$
- gluons represented as links between lattice sites, $U_\mu(x) = e^{iagA_\mu(x)}$



With finite a and L, quarks and gluons can be simulated on supercomputer

Euclidean path integral:

- Minkowski time replaced by $x_0 \to -it \quad \Rightarrow \quad e^{-iHx_0} \to e^{-Ht} = e^{-S[\psi,\bar{\psi},A]}$

$$\langle O \rangle \sim \int [d\psi][d\bar{\psi}][dA]\, O\, e^{-S[\psi,\bar{\psi},A]}$$

Integrate out the quark fields using Grassmann Algebra

$$\langle O \rangle \sim \int [dU] O[U] \det(\not{D} + m) e^{-S_g[U]}$$

Importance sampling: generate gauge configurations with probability distribution

$$p[U] \propto \det(\not{D} + m) e^{-S_g[U]}$$

this can be achieved by hybrid Monte Carlo simulation: Monte Carlo + Molecular Dynamics

Integration is approximated by average over gauge configurations

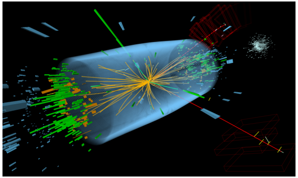
$$\int [dU] \det(\not{D} + m) e^{-S_g[U]} \quad \rightarrow \quad \frac{1}{N} \sum_{\{U\}}$$

statistical error is reduced by $1/\sqrt{N}$

Experiment vs Lattice QCD

HEP Experiment

BEPC collider(Energy、Luminosity)



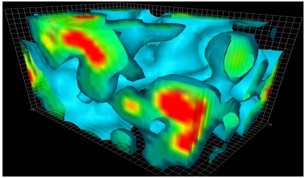
Collision， Events

↓

BES III Detector, measurement

LQCD simulation

Super Computer(Performance、Memory)



Simulation, QCD vacuum

↓

Lattice QCD calculation

Summary of HLbL

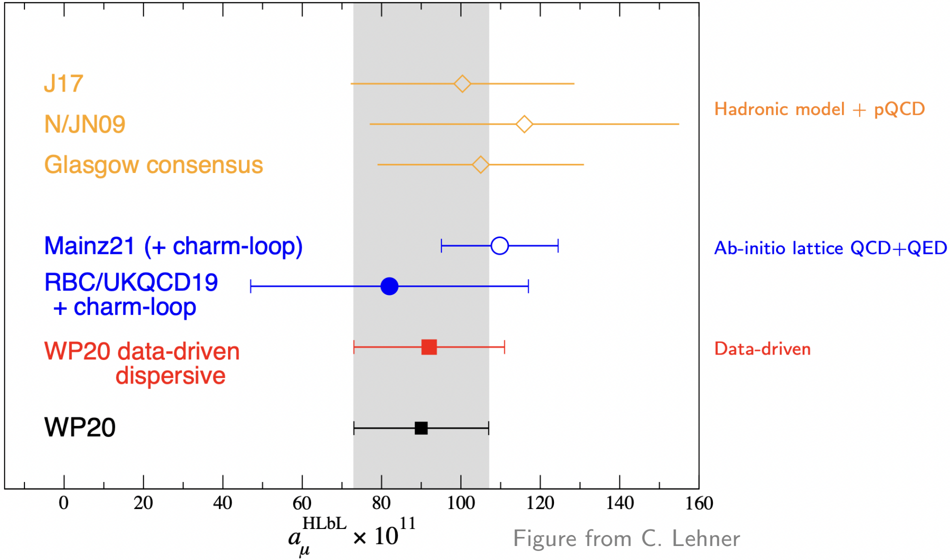
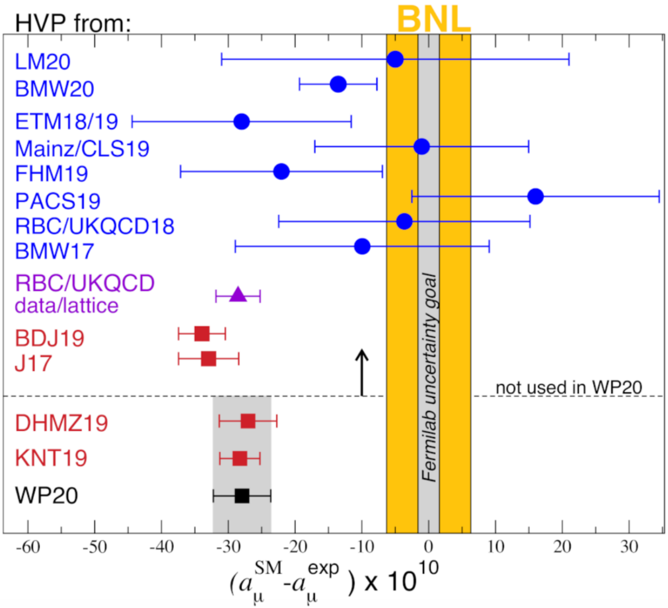
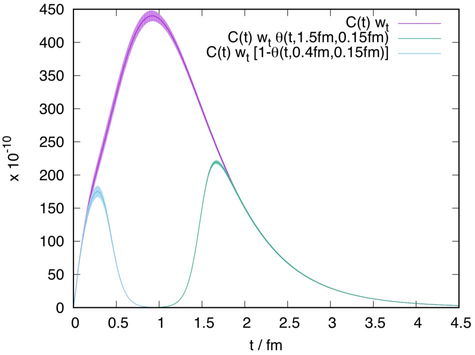


Figure from C. Lehner


HVP from:
BNL
LM20
BMW20
ETM18/19
Mainz/CLS19
FHM19
PACS19
RBC/UKQCD18
BMW17
RBC/UKQCD
data/lattice
BDJ19
J17
not used in WP20
DHMZ19
KNT19
WP20
Fermilab uncertainty goal
-60 -50 -40 -30 -20 -10 0 10 20 30
$(a_\mu^{SM} - a_\mu^{exp}) \times 10^{10}$

Joint analysis: data+lattice

R ratio data recompiled in Euclidean position space



$$a_\mu^{\mathrm{HVP,\,LO}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}\,,$$

$$a_\mu^{\mathrm{SD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[1-\Theta(x_0,t_0,\Delta)]\,,$$

$$a_\mu^{\mathrm{W}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)[\Theta(x_0,t_0,\Delta)-\Theta(x_0,t_1,\Delta)]\,,$$

$$a_\mu^{\mathrm{LD}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, C(x_0)\widetilde{f}(x_0)\Theta(x_0,t_1,\Delta)\,,$$

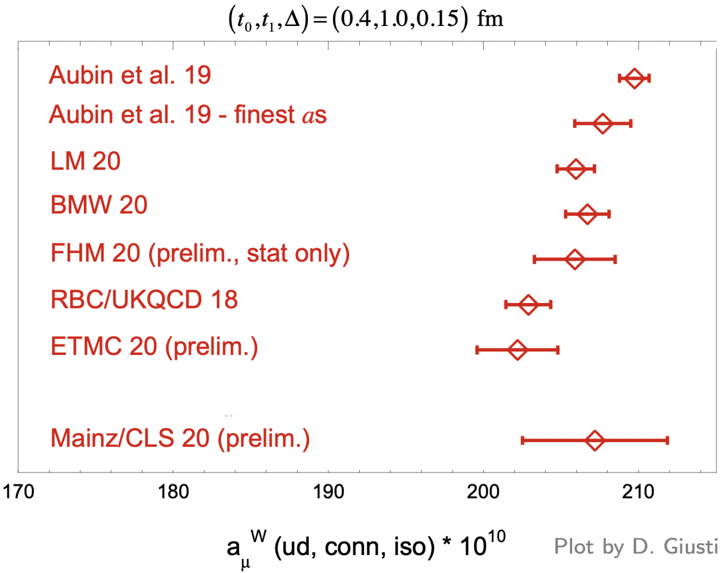
Here $\Theta(t,t',\Delta) = [1+\tanh[(t-t')/\Delta]]/2$ is a smooth step function

R ratio provides short and very long-distance physics + lattice data provide the mediate-scale physics

Question: short-distance part may be determined by pQCD?

Cross check between different lattice groups

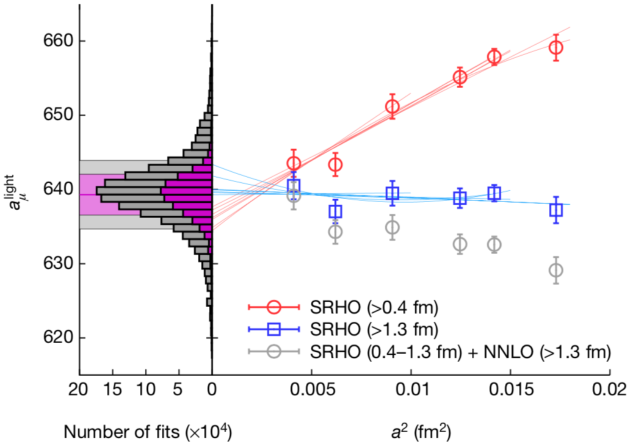
Important to have a cross check



Plot by D. Giusti

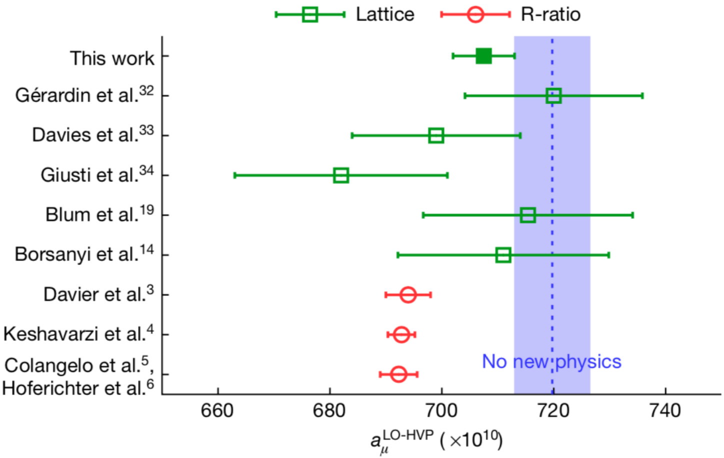
BMW result

First result reach subpercent precision $a_\mu^{\mathrm{HVP,LO}} = 707.5(5.5)_{\mathrm{tot}} \times 10^{-10}$



- Large lattice artifacts
- Final analysis involves 500,000 different continuum extrapolations
- Variance of cont. extrapolations are taken into account in the error

BMW result



"Our lattice result shows some tension with the R-ratio determinations of refs.3– 6. Obviously, our findings should be confirmed - or refuted - by other studies using different discretizations of QCD. Those investigations are underway." - quoted from BMW's paper - Nature (2021)

Outlook

- Expect more lattice HVP calculation at sub-percent level in the coming years
- Important to have comparison and global average with more accurate results from different group
- Data-driven dispersive results will improve with expected experimental results from BESIII and others