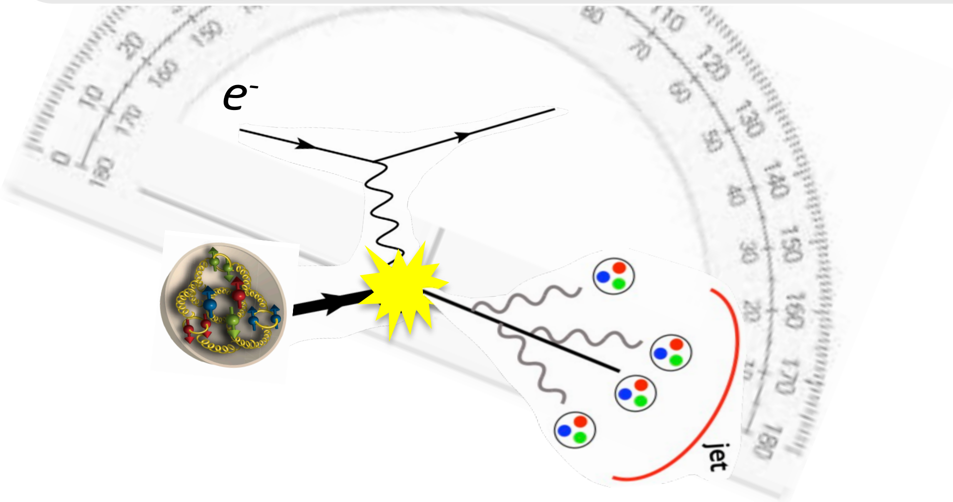
Angularity in Deep-Inelastic Scattering

[2106.xxxx]



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QCD and hadronic jet

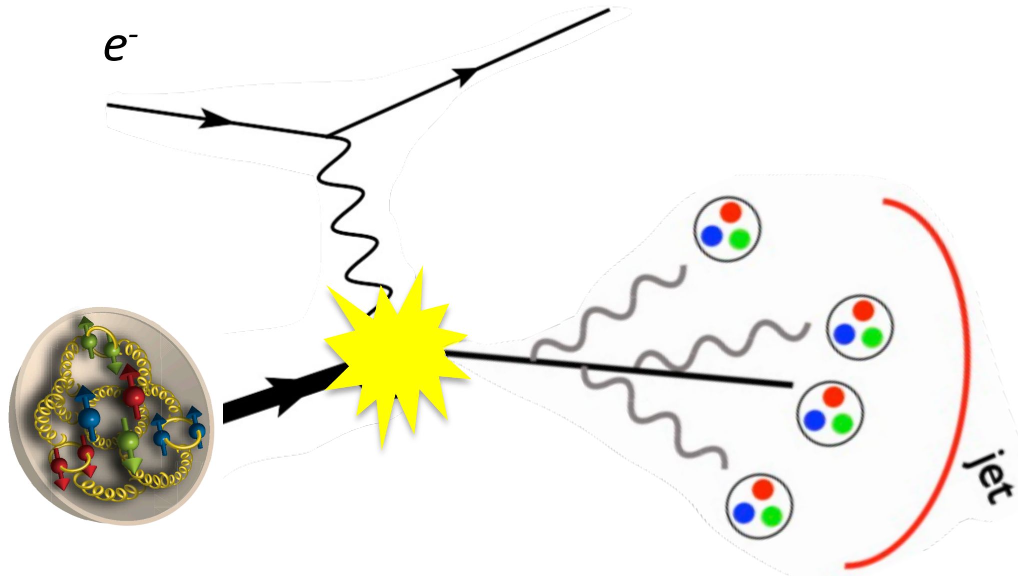
- QCD is dominated in collider experiments: Belle II, BESIII, ATLAS, CMS, sPHENIX, future EIC
- q, g turns into a group of hadrons travelling along the same direction
- jets ~ q, g at short distances
- collinear and soft modes dominate jets

$$p_c = (\lambda^2, 1\ , \lambda)Q$$

large energy, small angle

$$p_s = (\lambda^2, \lambda^2, \lambda^2)Q$$

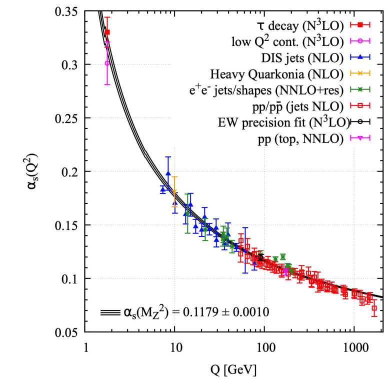
small energy, wide angle



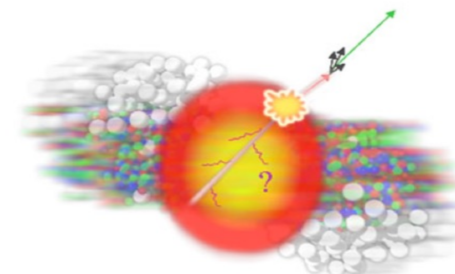
- hard, collinear, and soft parts by QCD factorization, or EFT factorization using soft-collinear effective theory (SCET)

Jet physics applications

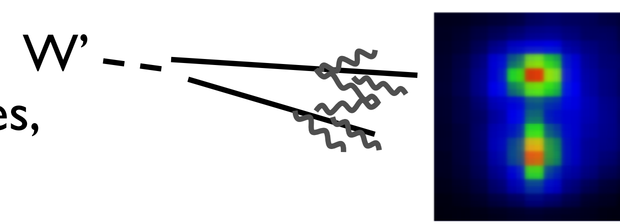
- precision test such as α_s , heavy-quark mass determinations using *event shape observables* defined as a function of final-state momenta
  ex) *thrust*, broadening, angularity, EEC and more



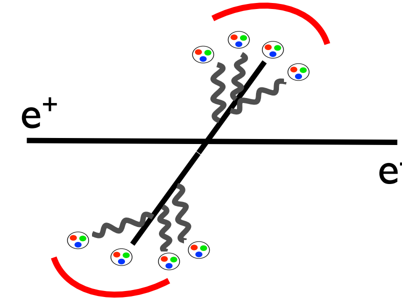
- probe of nucleon structure, hot/cold nuclear medium:
  di-jet asymmetry, energy loss, jet-quenching,
  TMD-related jet observables and more



- new physics search :
  jet substructure studies using shape observables,
  multi-differential distributions

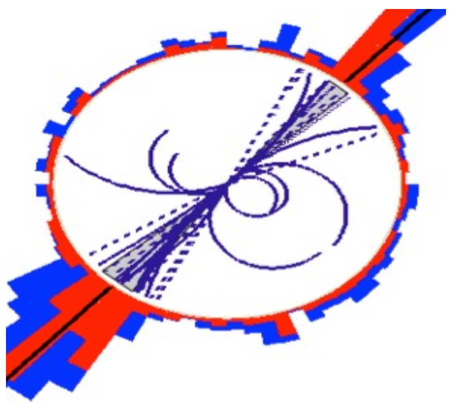


Event shape: thrust in e^+e^-

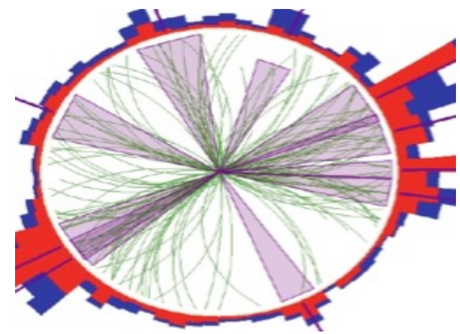


- Event shapes characterize a geometric feature of the event.
- Thrust is small for di-jet events while is large for multi-jet events
- One of most precisely studied event shapes: N^{3}LL'+N^{3}LO

$$\tau_{ee} = \frac{1}{Q_N} \min_{\vec{n}} \sum_i |p_\perp^i| e^{-|\eta_i|}$$



di-jet event
$\tau_{ee} \to 0$



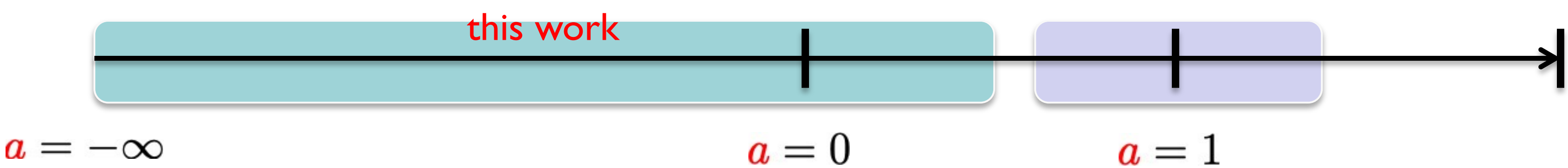
multi-jet event
$\tau_{ee} \to 1$

Angularity: generalized version of thrust

$$\tau_a = \frac{1}{Q_N} \sum_i |p_\perp^i| e^{-|\eta_i|(1-a)}$$

[C. Berger, T. Kucs, G. Sterman 0303051]

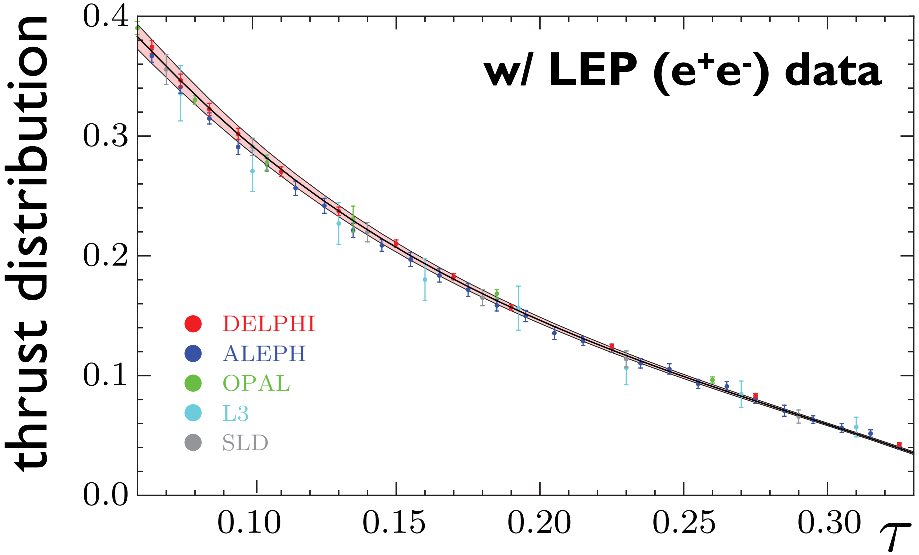
- a class of observables continuous parameter 'a' ranging between $-\infty$ and 2
- the collinear with large rapidity is sensitive to a while the soft is less sensitive
- reducing to well-known observables: thrust and broadening



- In e+e-, **SCET factorization ($a < 1$)** [Bauer, Fleming, Lee, Sterman '08] and resummation [Hornig, Lee, Ovaneyan '09, Bell, Hornig, Lee, Talbert '18], **factorization in $a \to 1$ region** [Budhraja, Jain, Procura '19]
- jet angularity [Hornig, Makris, Mehen '16], various axes [Larkoski, Neill, Thaler '14], double-differential ang [Procura, Waalewjin, Zeune '18], jet angularity in DIS [Z. Kang, K. Lee, Ringer '18]

$a = 0$: thrust

[Abbate, Fickinger, Hoang, Mateu, Stewart, 1006.3080]

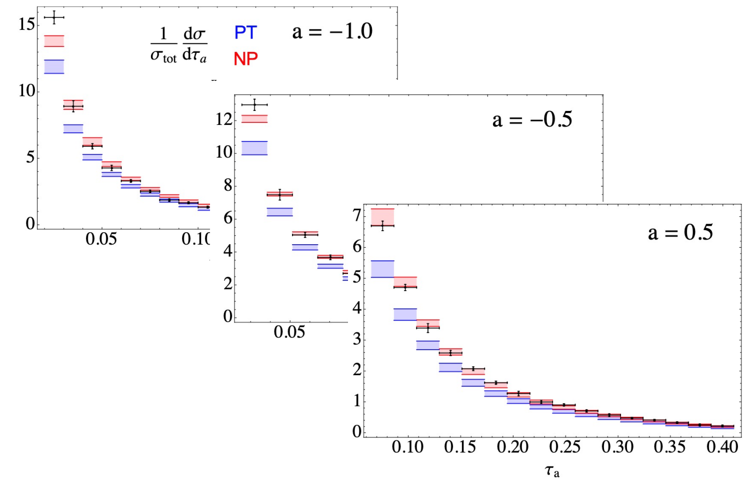


- SCET prediction at N^{3}LL'+N^{3}LO, good agreement with data
- α_s values at 1% precision

$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$

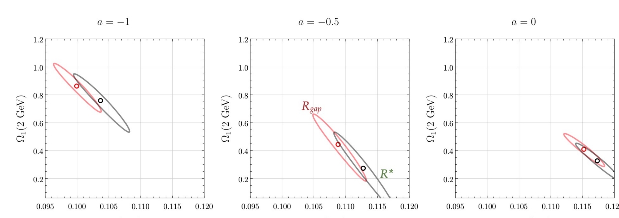
$a \neq 0$ case

[Bell, Hornig, Lee, Talbert, 1808.0767]

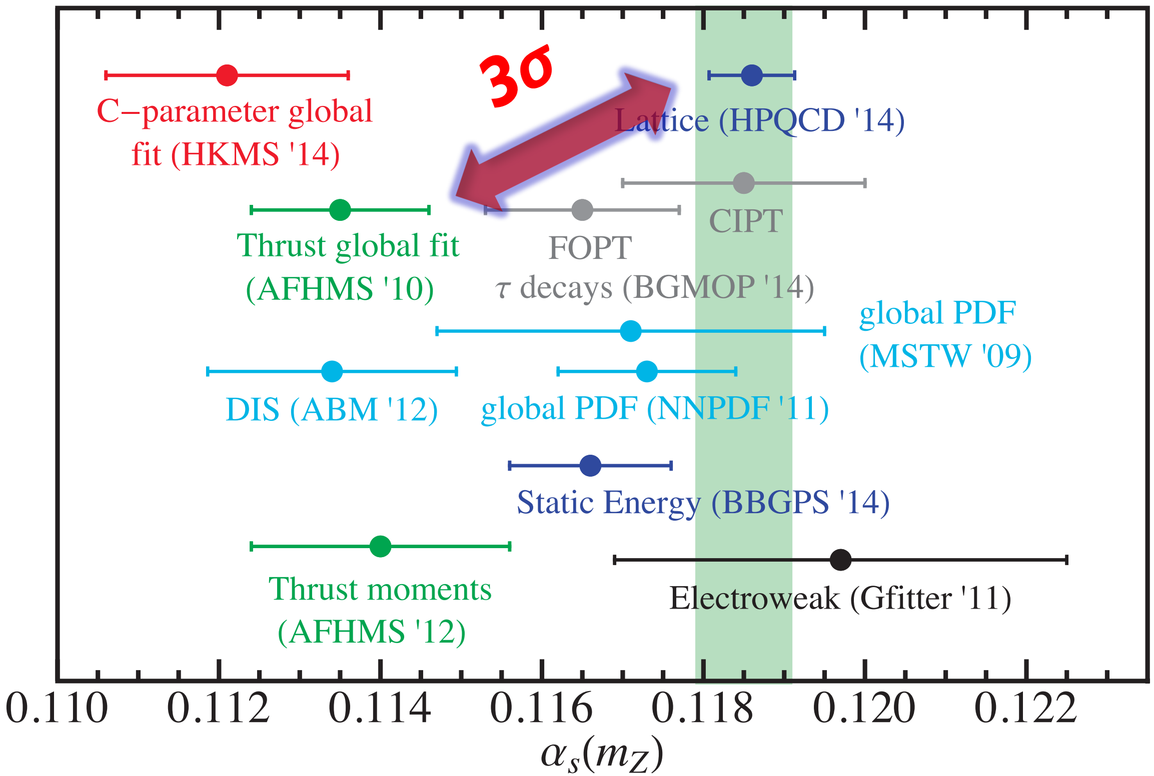


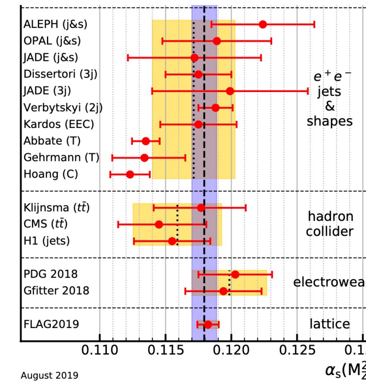
- NNLL'+ NLO level
- a careful study ongoing for α_s determination

from Chris Lee's talk in SCET 2021



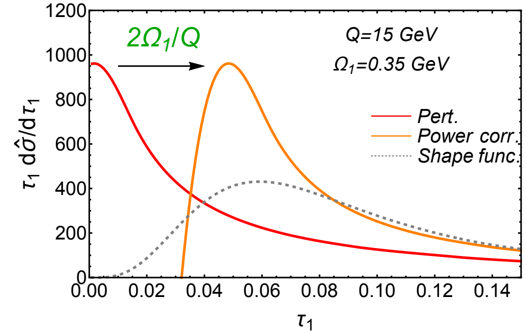
A tension: α_s from thrust VS lattice





- this tension lasts for a decay and no clear resolution.
- new data in future EIC and new determination in DIS may shed light on this issue.

Why angularity?



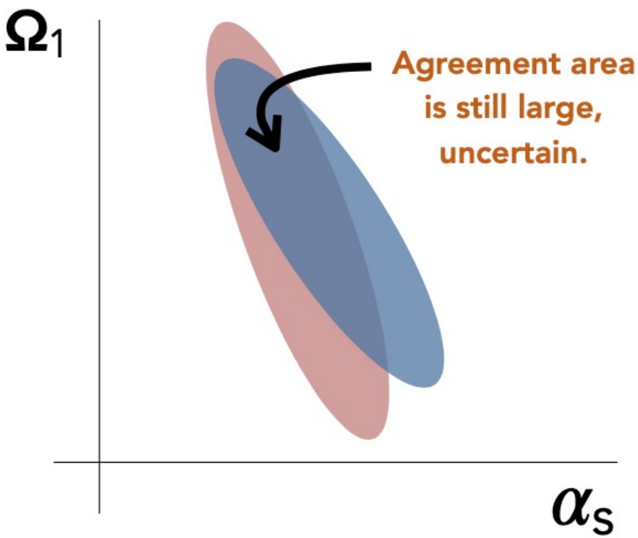
- correlations between α_s and nonperturbative (NP) effect
- leading NP effect shifts the distribution by NP parameter Ω_1.
  Ω_1 is universal for angularities and C parameter.

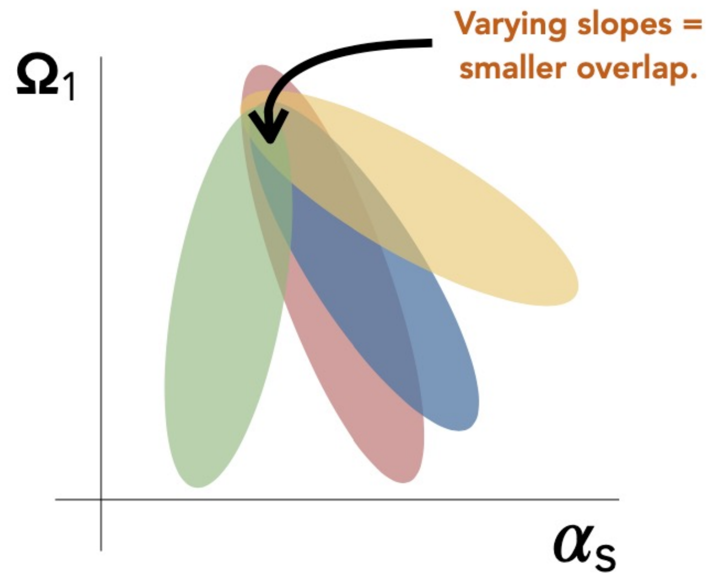
$$d\sigma(\tau_a) \to d\sigma\left(\tau_a - \frac{2}{1-a}\frac{\Omega_1}{Q}\right)$$

[Lee, Sterman, 0611061]

- Angularities with various 'a' and Q provides **better control on the degeneracies**

from Chris Lee's talk in SCET 2021



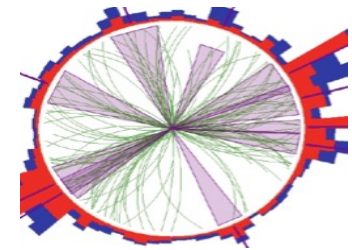


N-jettiness: thrust for N-jet event

[Stewart, Tackmann, Waalewijn, 09]

$$\tau_N = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_1\cdot p_i, \ldots, q_N\cdot p_i\}$$

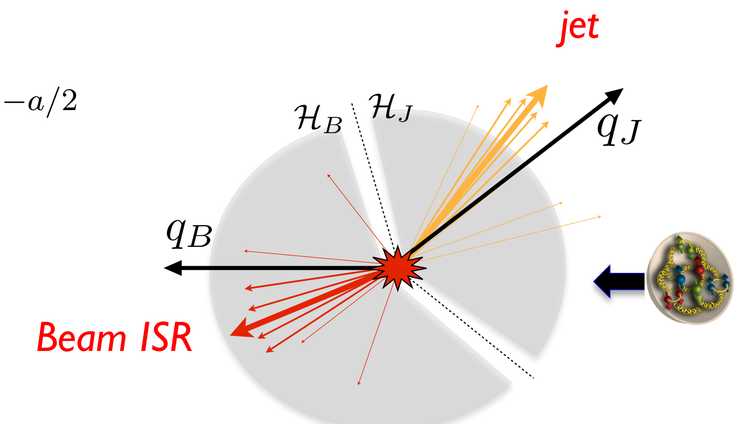
- *applicable for AB colliders A,B={e,p} and for N jets*
- min. groups particles into N-jet and beam regions
- *reduce to thrust when N=1 and AB=ee*



N-jet limit: $\tau_N \to 0$

Angularity for DIS: one final jet + ISR from beam

$$\tau_a = \frac{2}{Q^2}\sum_{i\epsilon\mathcal{H}_B}(q_B.p_i)\left(\frac{q_B.p_i}{q_J.p_i}\right)^{-a/2} + \frac{2}{Q^2}\sum_{i\epsilon\mathcal{H}_J}(q_J.p_i)\left(\frac{q_J.p_i}{q_B.p_i}\right)^{-a/2}$$



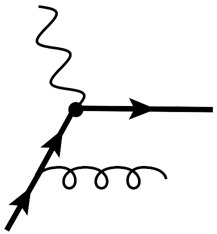
- *weighted by angularity parameter a*
- axes q_B, q_J group particles into beam and jet regions

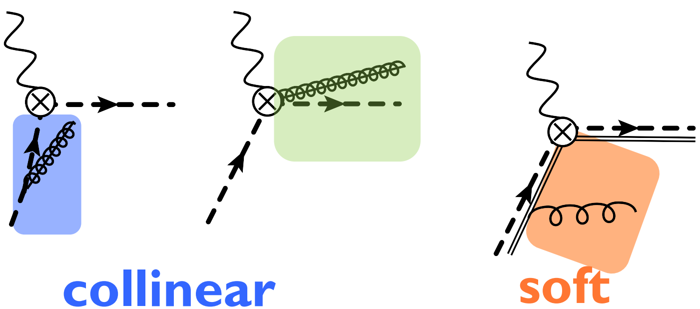
$$q_B = xP\,, \qquad q_J = (|\vec{P}_J|\,, \vec{P}_J)\,.$$

- reduces to 1-jettiness in DIS when $a = 0$

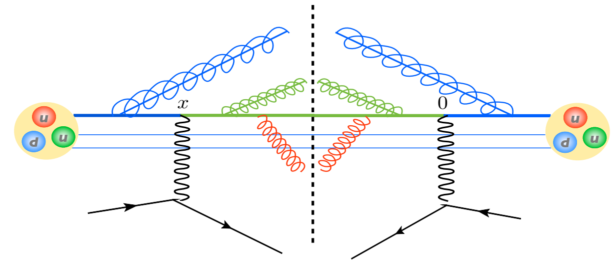
SCET factorization for DIS angularity

- small τ_a region is dominated by soft, jet-collinear, and beam-collinear modes

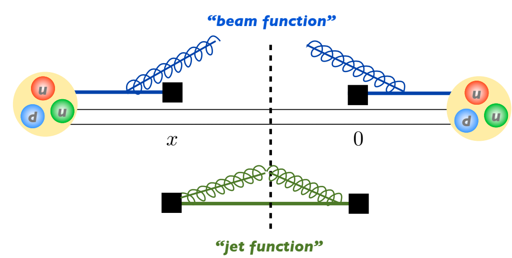


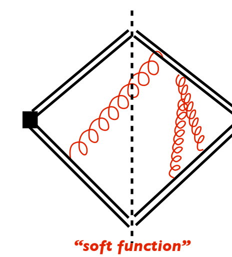


$+\ O(\lambda^4)$



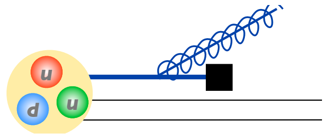
$$\sigma = H \times B \otimes J \otimes S$$



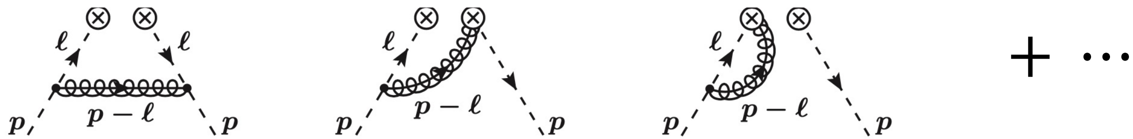


$$B = \mathcal{I} \otimes f$$ ← Parton Distribution Function

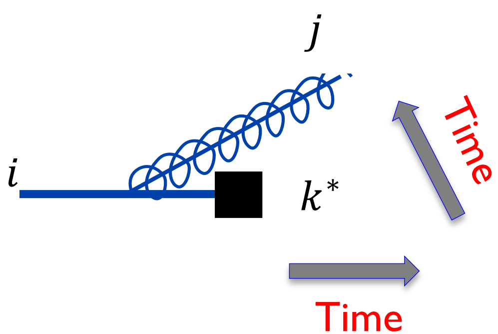
Angularity beam function at $\mathcal{O}(\alpha_s)$



- Hard, Jet, Soft functions known up to $\mathcal{O}(\alpha_s^2)$ from e^+e^- angularity while angularity beam function was not studied before
- One can literally compute one-loop diagrams



- We use crossing symmetry of splitting function from $k^* \to ij$ [Ritzmann, Waalewijin 1407.3272]

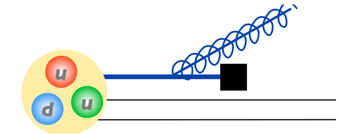


$$P_{i\to k^*j}(2p_i\cdot p_j, x) = (-1)^{\Delta_f} P_{k^*\to ij}(-2p_i\cdot p_j, 1/x)$$

- phase-space integral is one-body and simple
- the crossing breaks down at 3-loops. a recovery recipe recently is found.

[H. Chen, T-Z. Yang, H.X. Zhu, Y. J. Zhu 2006.10534]

Angularity beam function at $O(\alpha_s)$



- In Laplace space with $\nu_a \leftrightarrow \tau_a$

$$\widetilde{\mathcal{B}}_q^{\rm bare} = \frac{\alpha_s C_F}{2\pi}\frac{2}{2-a}\left(1-\epsilon^2\frac{\pi^2}{12}\right)\Gamma\left[-\frac{2\epsilon}{2-a}\right]\left(\frac{Q\nu_a^{-\frac{1}{2-a}}}{\mu}\right)^{-2\epsilon} h_q(z,\epsilon)\,.$$

$$\widetilde{B}_{q/P}(\nu,x,\mu) = \sum_{j=q,g}\widetilde{I}_{qj}(\nu,x)\otimes f_j(x)\,.$$

$$\widetilde{I}_{qj}^{(1)} = \frac{\alpha_s}{4\pi}\left[\left(-j_B\kappa_B\frac{\Gamma_0}{2}L_B^2 - \gamma_0^B L_B\right)\mathbb{1} + 4C_{qj}P_{qj}(z)L_B + \widetilde{c}_1^{qj}(z)\right] \qquad L_B = \ln\left[\frac{(\nu_a e^{\gamma_E})^{-\frac{1}{2-a}}Q}{\mu}\right]$$

- anomalous dim and RGE are same as those of (fragmenting) jet function. All log terms are known.
- constant terms $\tilde{c}_1(z,a)$ are newly obtained

$$\widetilde{c}_1^{qq}(z) = 2C_F\left[\frac{2(1-a)}{2-a}(1+z^2)\mathcal{L}_1(1-z) + \frac{\pi^2}{12}\frac{a(4-a)}{(2-a)(1-a)}\mathcal{L}_{-1}(1-z)\right.$$
$$\left. +1-z-\frac{2(1-a)}{2-a}\frac{1+z^2}{1-z}\log z\right]$$

$$\widetilde{c}_1^{qg}(z) = 2T_F\left[1-P_{qg}(z)+\frac{2(1-a)}{2-a}P_{qg}(z)\log\left(\frac{1-z}{z}\right)\right]$$

Comparison to Fragmenting Jet Function

- hadron-fragmenting jet function are closely related with the beam function in their crossing relations and in their factorization structures

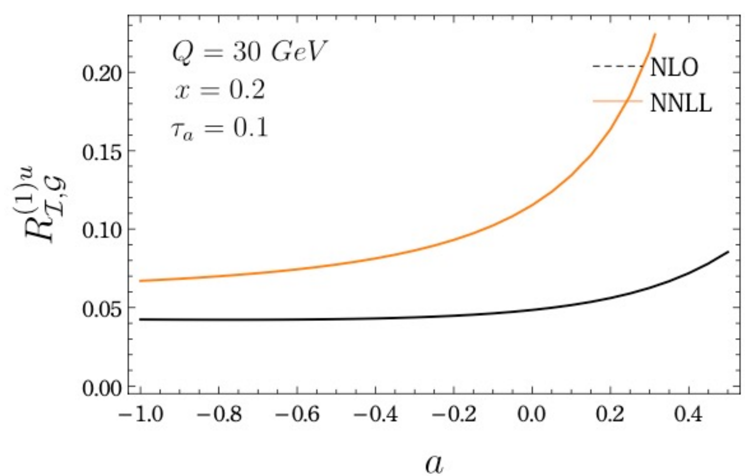
$$\widetilde{B}_{q/P}(\nu,x,\mu)=\sum_{j=q,g}\widetilde{I}_{qj}(\nu,x)\otimes f_j(x)\,. \qquad \tilde{\mathcal{G}}_q^h(\nu_a,x,\mu)=\sum_{j=q,g}\tilde{\mathcal{J}}_{qj}(\nu_a,x)\otimes D_j^h(x)$$

[Bain, Dai, Hornig, Hornig, Leibovich, Makris '16]

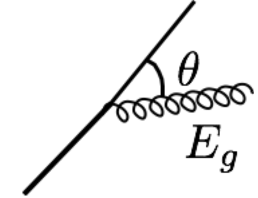
- all terms in the kernels are the same except for the constant term

$$\Delta_{qq}^{(1)}(a,z)=\mathcal{I}_{qq}^{(1)}-\mathcal{J}_{qq}^{(1)}$$
$$=\left[-\frac{2(1-a)}{2-a}\frac{1+z^2}{1-z}\log z\right]-\left[-\frac{2}{2-a}(1+z^2)\mathcal{L}_0(1-z)\log\left(1+(\frac{1-z}{z})^{(1-a)}\right)\right]$$

$$\Delta_{qg}^{(1)}(a,z)=\mathcal{I}_{qg}^{(1)}-\mathcal{J}_{qg}^{(1)}$$
$$=\left[\frac{2(1-a)}{2-a}P_{qg}(z)\log\left(\frac{1-z}{z}\right)\right]-\left[\frac{2}{2-a}P_{qg}(z)\log\left(\frac{z^{(1-a)}(1-z)^{(1-a)}}{z^{(1-a)}+(1-z)^{(1-a)}}\right)\right]$$



Log Log Log ...



probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$ $\longrightarrow$ $\mathrm{Log}(E_g)\,\mathrm{Log}(\theta)$

soft and collinear enhancements

Perturbation at risk by large log : $\alpha_s\, L \sim 1$

$$\sigma = 1 + \alpha_s + \alpha_s^2 + \cdots$$

$\mathrm{L} = \mathrm{Log}$

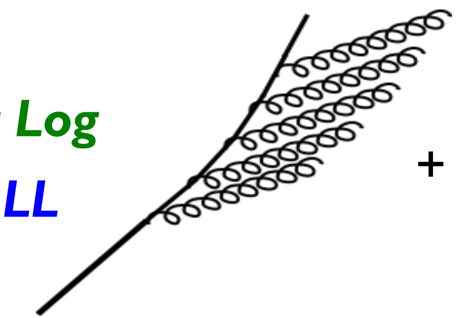
Resum large logs!

$$\log\sigma = \alpha_s L^2 + \alpha_s^2 L^3 + \cdots$$ ***Leading Log***

$$+\alpha_s L + \alpha_s^2 L^2 + \cdots$$ ***Next to LL***

$$+\alpha_s + \alpha_s^2 L + \cdots$$ ***NNLL***

$$+\alpha_s^2 + \cdots$$



+ other diagrams

Resummation by RG evolution

- RG evolution in Laplace space

similar to H,J,S

$$\mu \frac{d}{d\mu}\widetilde{B}(\mu) = \gamma_B(\mu)\,\widetilde{B}(\mu) \quad \Rightarrow \quad \widetilde{B}(\mu) = \widetilde{B}(\mu_b)\, \boxed{e^{K(\mu_b,\mu)-\eta(\mu_b,\mu)L_B}}$$

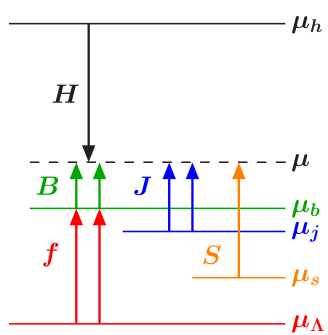
$$L = \ln(\mu/\mu_J)$$

$$\gamma_B(\mu) = j_B\,\kappa_B \Gamma_{\text{cusp}}(\alpha_s) L_B + \gamma_B(\alpha_s)$$

$$K = L\sum_{k=1}^{\infty}(\alpha_s L)^k + \sum_{k=1}^{\infty}(\alpha_s L)^k + \alpha_s \sum_{k=1}^{\infty}(\alpha_s L)^k + \cdots$$

LL ***NLL*** ***NNLL***

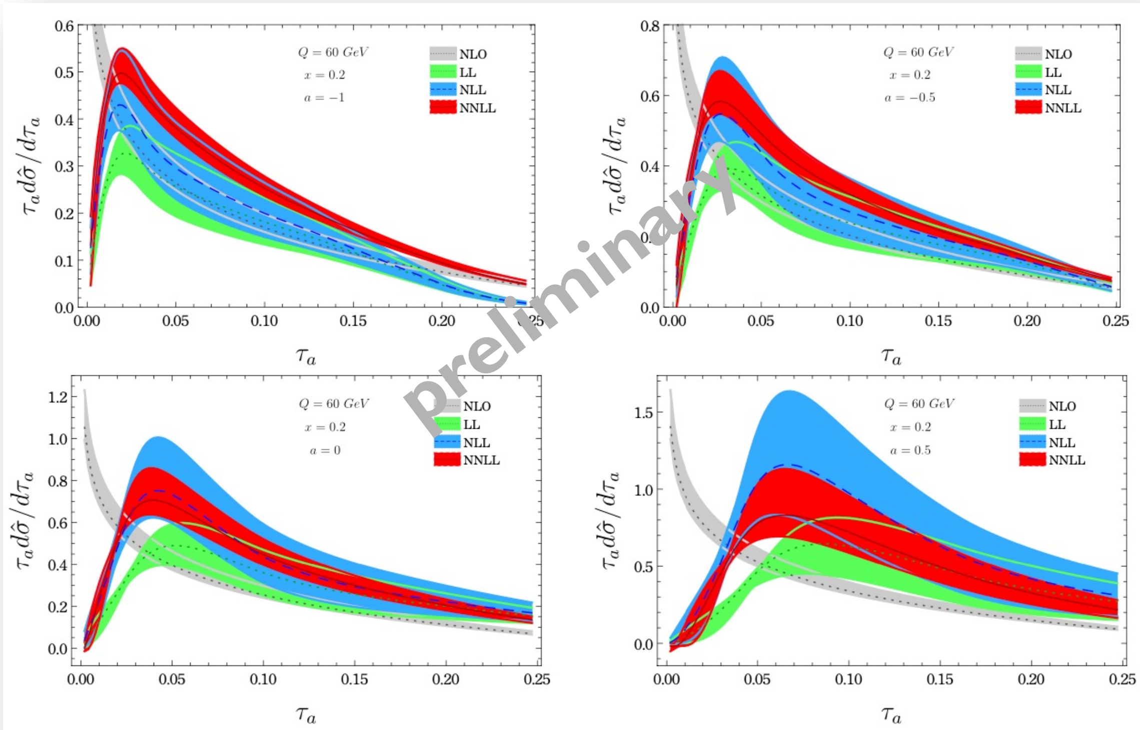
$$\tilde{\sigma}_q(\nu) = \sigma_0\, H_q(Q^2,\mu)\, \tilde{B}_q(\nu,\mu)\, \tilde{J}_q(\nu,\mu) \tilde{S}(\nu,\mu)\,,$$



	$\Gamma[\alpha_s]$	$\gamma[\alpha_s]$	$\beta[\alpha_s]$	$\{H,J,B,S\}[\alpha_s]$	
LL	α_s ✓	1	α_s ✓	1	
NLL	α_s^2 ✓	α_s ✓	α_s^2 ✓	1	
NNLL	α_s^3 ✓	α_s^2 ✓	α_s^3 ✓	✓ α_s	this work
N^3LL	α_s^4 ✓	α_s^3	α_s^4 ✓	α_s^2	$c_B^{(2)}$ missing

$\gamma_{J,S}^{(3)}$ missing

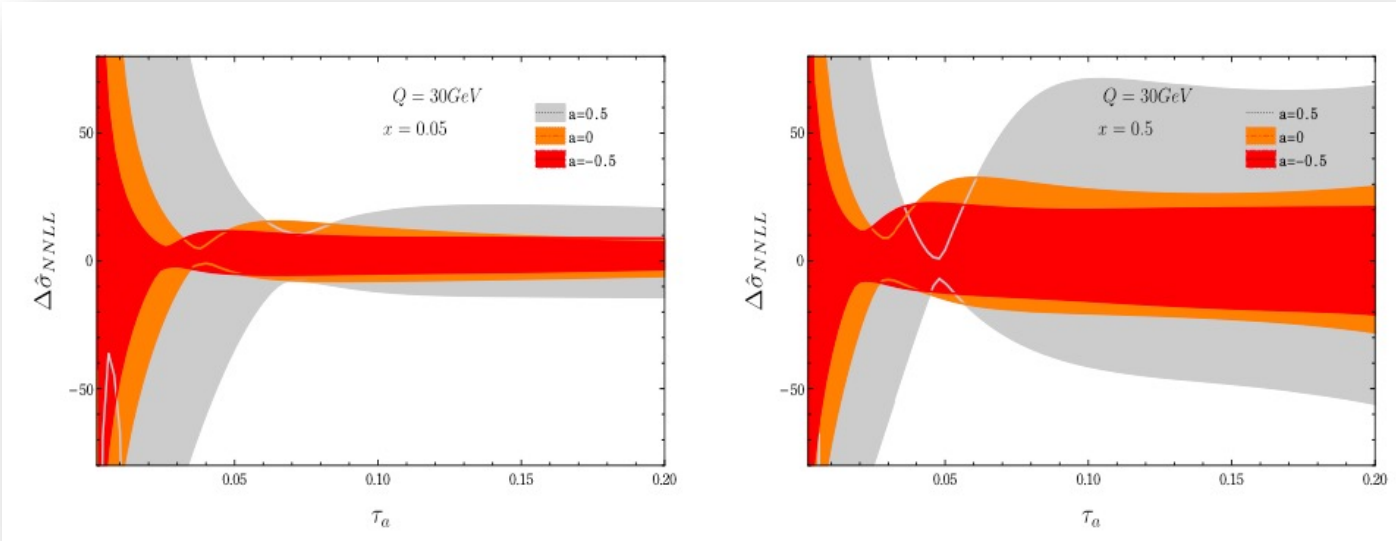
Preliminary results for DIS angularity



- fixed-order NLO are shown to show the effect of resummation
- with 'a' decreasing, distribution falls faster and its uncertainty tends to reduces.

Preliminary results for DIS angularity

relative uncertaintes at NNLL



- uncertainty seems to saturate as a decreases because anomalous dimension and constant terms approach to fixed value
- uncertainty reduces with x decreasing due to PDF

Summary and outlook

- A decay+ lasting tension (>3 sigma) in values of α_s between thrust w/ SCET and lattice results
- New data from EIC offers an independent test that may shed a light on this tension.
- DIS angularity with a continuous parameter 'a' provides various distributions useful to disentangle α_s from non-perturbative effect.
- New results:
  - factorization in $a < 1$ and angularity beam function at $\mathbf{O}(\boldsymbol{\alpha_s})$
  - resummed predictions at NNLL
  - uncertainties sensitive to the value of a

- our result valid $a < 1$ (SCET_{I}), need to study $a\sim 1$ (SCET_{II}) region
- Fixed-order results for large τ_a region and for scale profile setting
- For higher resummation, 2-loop constant of beam function and 3-loop $\gamma_{S,J}$

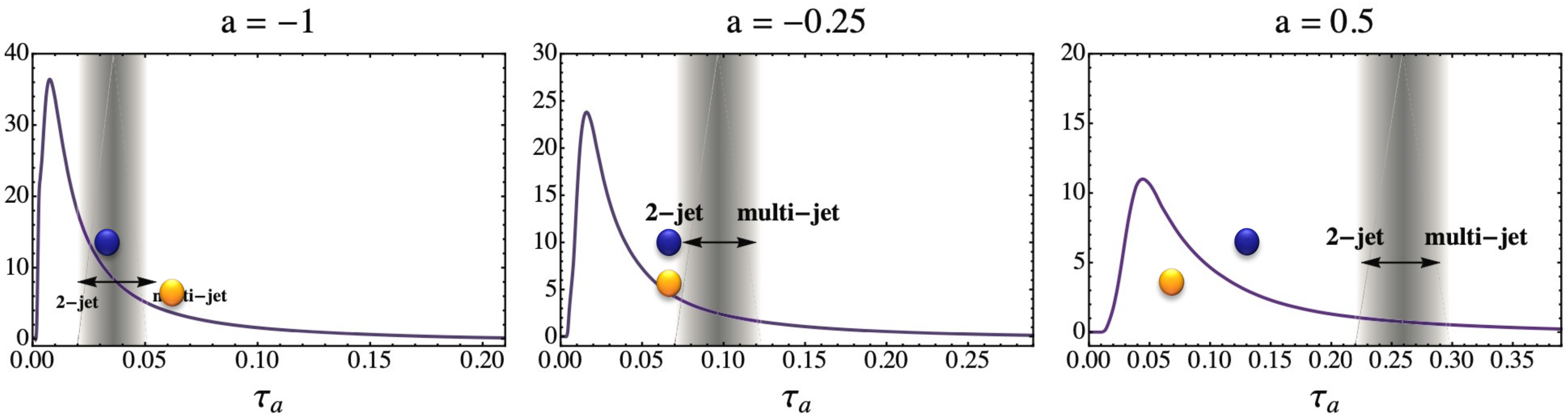
Thanks!

Angularity: soft and collinear modes

—C. F. Berger, T. Kucs and G. F. Sterman' 2003

$$\tau_a = \frac{2}{Q} \sum_{i \in \mathcal{X}} |\mathbf{p}_\perp^i| \, e^{-|\eta_i|(1-a)}$$

- angularity change much for collinear particles with large rapidity, while less from soft particles with small rapidity

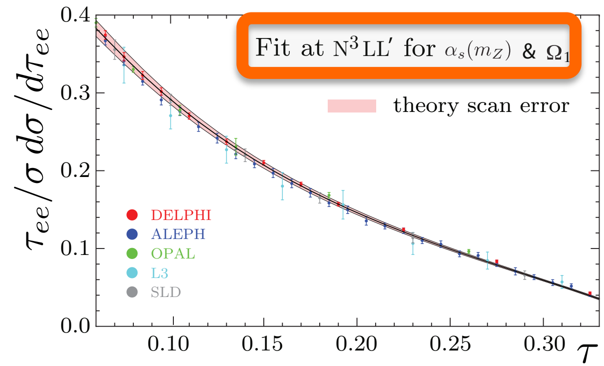


Thrust in ee and in DIS

$$\tau_{ee} = 1 - \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{p}_i \cdot \vec{n}|$$ [Farhi]

- ❑ Up to $O(\alpha_s^3)$+N³LL — Becher and Schwartz; Abbate, Fickinger, Hoang, Mateu, Stwart

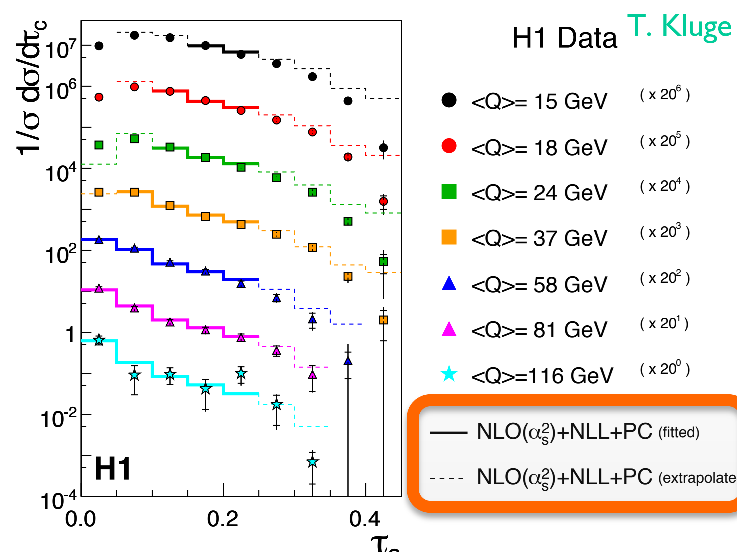
$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$



$$\tau_{\rm DIS} = 1 - \frac{1}{E_J} \sum_{i \in \mathcal{H}_J} |\vec{p}_i \cdot \vec{n}|$$

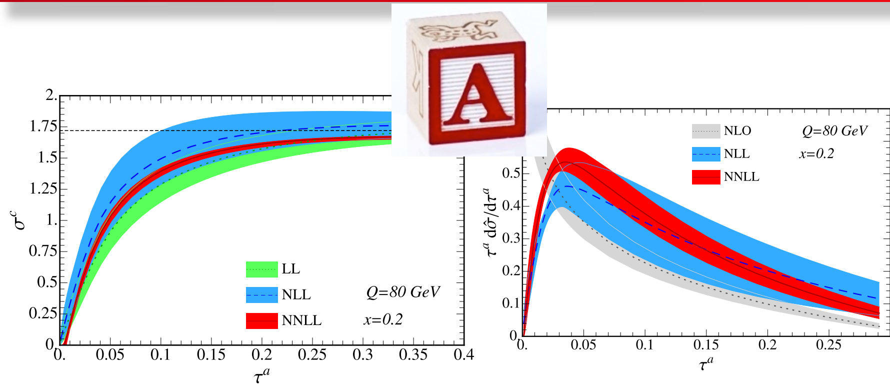
- ❑ one hemisphere measurement
- ❑ Up to $O(\alpha_s^2)$+NLL — Antonelli, Dasgupta, Salam

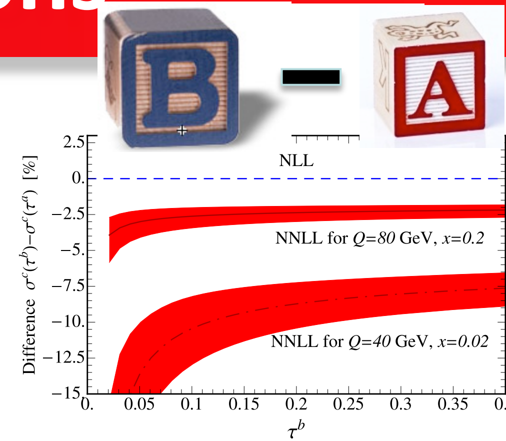
$$\alpha_s(m_Z) = 0.1198 \pm 0.0013(\text{exp.}) {}^{+0.0056}_{-0.0043}(\text{th.})$$



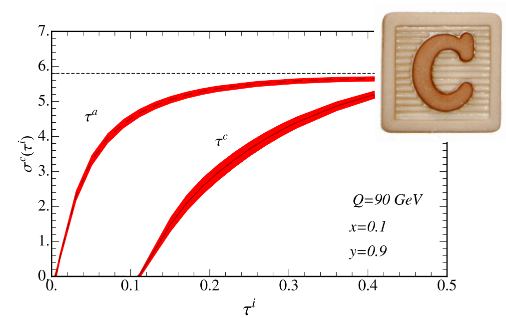
- ❑ *moving to higher precision is challenged due to Non-Global Logarithms*

NNLL predictions





DK, Lee, Stewart 2013



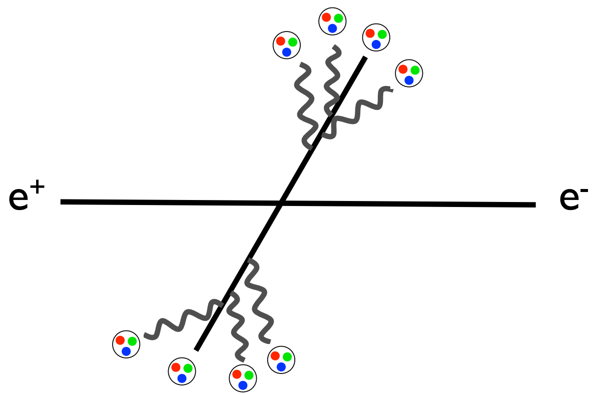
- One order higher than DIS thrust resummation (NLL)
- Higher precision?

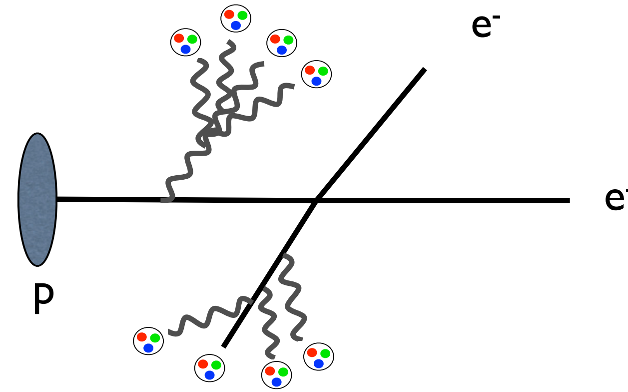
$$d\tilde{\sigma} = \exp\left[L\sum_{k=1}^{\infty}(\alpha_s L)^k + \sum_{k=1}^{\infty}(\alpha_s L)^k + \alpha_s \sum_{k=0}^{\infty}(\alpha_s L)^k + \cdots\right] + \mathrm{NS}(\alpha_s)$$

singular part: LL, NLL, NNLL, N^{3}LL,…

nonsingular part: $O(\alpha_s)$, $O(\alpha_s^2)$,…

SCET factorization for jets in *ep*





$$\sigma_{ee} = H_{ee} \times J \otimes J \otimes S_{ee}$$

$$\sigma_{ep} = H_{ep} \times B \otimes J \otimes S_{ep}$$

H quarks created at SD, $H_{ee}(Q^2) \leftrightarrow H_{ep}(-Q^2)$

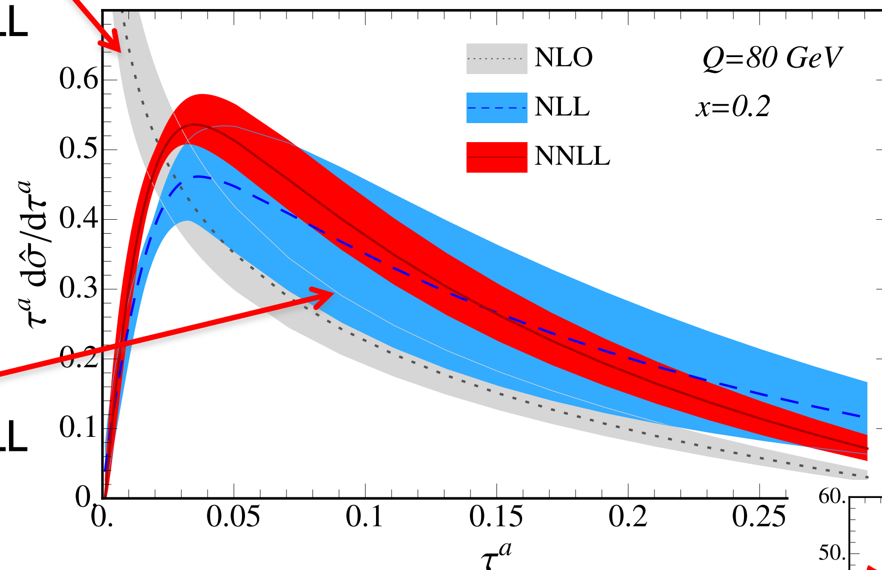
B, **J** universal for ee, ep and **S** universal up to α_s^2

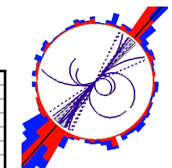
Universality in EFT, Not in QCD!

Resummed results

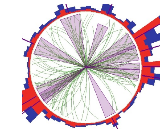
DK, Lee, Stewart, PRD2013
Kang, Liu, Mantry, Qiu PRD2013

Log singularity in NLO
cured in NLL, NNLL





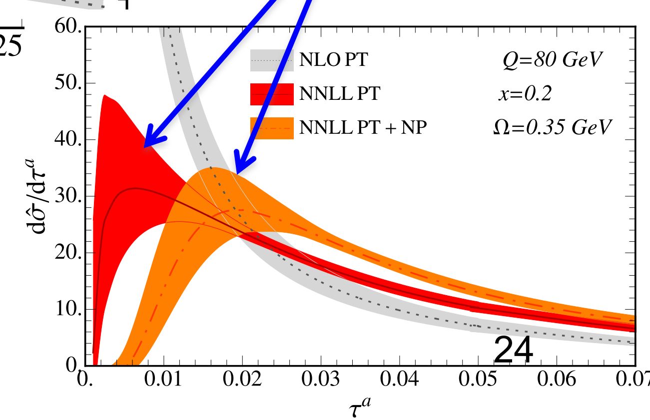
τ->0



τ->1

Good convergence from NLL to NNLL

Non-perturbative effect for small τ



First NNLL resummation accuracy in ep -> 2 jet!