NLO EW corrections to $e^+e^- \to Zh/H$ in 2HDM

(based on 1812.02597)

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Outline

1 Introduction to 2HDM

2 Higgs strahlung in e^+e^- colliders

3 Renormalization of 2HDM

4 Summary

A brief Introduction to 2HDM I

The potential [$SU(2)$ invariant; CP-conserving; Z_2 soft-breaking]: [Z_2: $(\Phi_1, \Phi_2) \to (\Phi_1, -\Phi_2)$]

$$
\begin{aligned}
V(\Phi_1, \Phi_2) =& m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
&+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right] \qquad (1)
\end{aligned}
$$

$$
\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}
$$

A brief Introduction to 2HDM I

The potential [$SU(2)$ invariant; CP-conserving; Z_2 soft-breaking]: [Z_2: $(\Phi_1, \Phi_2) \to (\Phi_1, -\Phi_2)$]

$$V(\Phi_1,\Phi_2) = m_1^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] \quad (1)$$

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

$$V|_{\text{bilinear}} = \frac{1}{2}(\rho_1\ \rho_2)\,M_\rho^2\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix} + \frac{1}{2}(\eta_1\ \eta_2)\,M_\eta^2\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} + \frac{1}{2}\left(\omega_1^+\ \omega_2^+\right)M_\omega^2\begin{pmatrix}\omega_1^-\\ \omega_2^-\end{pmatrix}$$

$$M_\rho^2 = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \lambda_1 v_1^2 & -m_{12}^2 + \lambda_{345}v_1v_2 \\ -m_{12}^2 + \lambda_{345}v_1v_2 & m_{12}^2\frac{v_1}{v_2} + \lambda_2 v_2^2 \end{pmatrix} + \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix}$$

$$M_\eta^2 = \left(\frac{m_{12}^2}{v_1v_2} - \lambda_5\right)\begin{pmatrix} v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2 \end{pmatrix} + \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix} \quad (2)$$

$$M_\omega^2 = \left(\frac{m_{12}^2}{v_1v_2} - \frac{\lambda_4+\lambda_5}{2}\right)\begin{pmatrix} v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2 \end{pmatrix} + \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{pmatrix}$$

A brief Introduction to 2HDM II

$$\Rightarrow V|_{\text{bilinear}} = \frac{1}{2}\begin{pmatrix} H^0 & h^0 \end{pmatrix} D_\rho^2 \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} G^0 & A^0 \end{pmatrix} D_\eta^2 \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} G^+ & H^+ \end{pmatrix} D_\omega^2 \begin{pmatrix} G^- \\ H^- \end{pmatrix}$$

$$D_\rho^2 = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}, \quad D_\eta^2 = \begin{pmatrix} m_{G^0}^2 & 0 \\ 0 & m_A^2 \end{pmatrix}, \quad D_\omega^2 = \begin{pmatrix} m_{G^\pm}^2 & 0 \\ 0 & m_{H^\pm}^2 \end{pmatrix} \tag{3}$$

$m_{G^0} = m_{G^\pm} = 0$

A brief Introduction to 2HDM II

$$\Rightarrow V|_{\text{bilinear}} = \frac{1}{2}\begin{pmatrix} H^0 & h^0 \end{pmatrix} D_\rho^2 \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} G^0 & A^0 \end{pmatrix} D_\eta^2 \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} G^+ & H^+ \end{pmatrix} D_\omega^2 \begin{pmatrix} G^- \\ H^- \end{pmatrix}$$

$$D_\rho^2 = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}, \quad D_\eta^2 = \begin{pmatrix} m_{G^0}^2 & 0 \\ 0 & m_A^2 \end{pmatrix}, \quad D_\omega^2 = \begin{pmatrix} m_{G^\pm}^2 & 0 \\ 0 & m_{H^\pm}^2 \end{pmatrix} \tag{3}$$

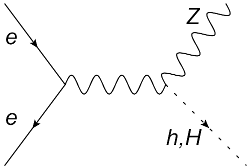
$m_{G^0} = m_{G^\pm} = 0$

$$\Rightarrow \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = R_\alpha^T \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = R_\beta^T \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = R_\beta^T \begin{pmatrix} \omega_1^\pm \\ \omega_2^\pm \end{pmatrix} \tag{4}$$

$$R_\theta \equiv \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}, \quad \tan 2\alpha = \frac{2(-m_{12}^2 + \lambda_{345} v_1 v_2)}{m_{12}^2 \frac{v_2}{v_1} + \lambda_1 v_1^2 - (m_{12}^2 \frac{v_1}{v_2} + \lambda_2 v_2^2)}$$

- H^0, h^0: CP-even Higgs
- $H^\pm$: charged Higgs
- A^0: CP-odd Higgs
- $G^0, G^\pm$: Goldstone boson
- Either h^0 or H^0 can be the SM-like Higgs
- $\{\lambda_{1-5}, m_{12}^2, T_1, T_2, v_1, v_2, g, g', Y_\psi\}$
- $\{m_h, m_H, m_A, m_{H^\pm}, \alpha, \beta, \lambda_5, T_h, T_H, e, m_W, m_Z, m_\psi\}$

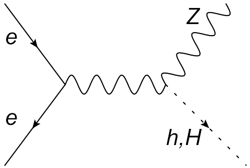
LO results



$$\sigma^0_{SM}\left(e^+e^- \to Zh\right) = \frac{\alpha_{em}^2 \pi}{192 s \sin^4\theta_W \cos^4\theta_W}\left[1 + \left(1 - 4\sin^2\theta_W\right)^2\right]\lambda^{\frac{1}{2}}\frac{\lambda + 12 m_Z^2/s}{\left(1 - m_Z^2/s\right)^2}$$

$$\lambda = \left(1 - \frac{(m_Z + m_h)^2}{s}\right)\left(1 - \frac{(m_Z - m_h)^2}{s}\right) \quad (5)$$

LO results

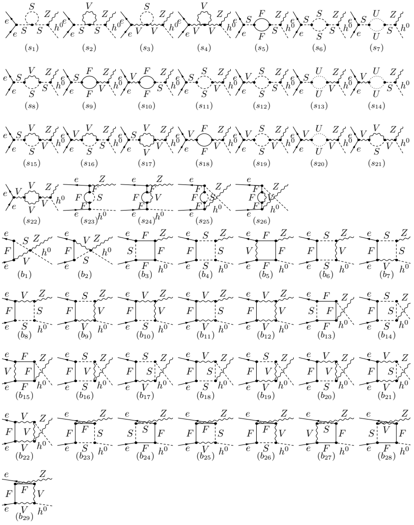


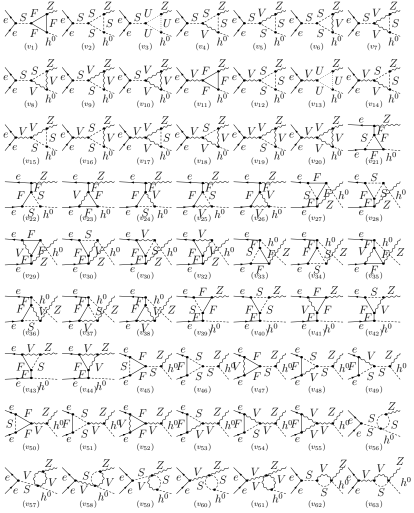
$$\sigma^0\left(e^+e^- \to Zh\right) = \sin^2(\beta-\alpha)\sigma^0_{SM}$$
$$\sigma^0\left(e^+e^- \to ZH\right) = \cos^2(\beta-\alpha)\sigma^0_{SM}$$

$$\sigma^0_{SM}\left(e^+e^- \to Zh\right) = \frac{\alpha_{em}^2\pi}{192s\sin^4\theta_W\cos^4\theta_W}\left[1+(1-4\sin^2\theta_W)^2\right]\lambda^{\frac{1}{2}}\frac{\lambda+12m_Z^2/s}{\left(1-m_Z^2/s\right)^2}$$

$$\lambda = \left(1-\frac{(m_Z+m_h)^2}{s}\right)\left(1-\frac{(m_Z-m_h)^2}{s}\right) \quad (5)$$

One-loop diagrams (Formcalc and Looptools)





Real corrections (Two cutoff method [Harris and Owens (2002)])

λ: photon mass Δ_E $(=\delta_s\sqrt{s}/2)$: soft cutoff $\Delta\theta$: collinear cutoff

$$d\sigma_S = -\frac{\alpha_{em}}{\pi} d\sigma^0 \times \left[\log\frac{4\Delta E^2}{\lambda^2}\left(1+\log\frac{m_e^2}{s}\right) + \frac{1}{2}\log^2\frac{m_e^2}{s} + \log\frac{m_e^2}{s} + \frac{1}{3}\pi^2\right]$$

$$d\sigma_{HC} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2 + 4m_e^2/s}{4m_e^2/s} - \frac{2z}{1-z}\frac{\Delta\theta^2}{\Delta\theta^2+4m_e^2/s}\right] d\sigma^0(zk_1)dz + (k_1 \Leftrightarrow k_2)$$

$$\xrightarrow{\Delta\theta^2 \gg m_e^2/s} \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2 s}{4m_e^2} - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right] dz$$

$$d\sigma_R(\lambda) = d\sigma_S(\lambda, \Delta E) + d\sigma_{HC}(\Delta E, \Delta\theta) + d\sigma_{H\overline{C}}(\Delta E, \Delta\theta)$$

Real corrections (Two cutoff method [Harris and Owens (2002)])

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$$d\sigma_{HC} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2+4m_e^2/s}{4m_e^2/s} - \frac{2z}{1-z}\frac{\Delta\theta^2}{\Delta\theta^2+4m_e^2/s}\right] d\sigma^0(zk_1)dz + (k_1 \Leftrightarrow k_2)$$

$$\xrightarrow{\Delta\theta^2 \gg m_e^2/s} \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2 s}{4m_e^2} - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right] dz$$

$$d\sigma_{CT} = -\frac{\alpha_{em}}{2\pi}\log\frac{s}{4m_e^2} P_{ee}^+(z,0) \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right] dz$$

$$d\sigma_R(\lambda) = d\sigma_S(\lambda,\Delta E) + d\sigma_{HC}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta) + d\sigma_{CT}$$

Real corrections (Two cutoff method [Harris and Owens (2002)])

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$$d\sigma_S = -\frac{\alpha_{em}}{\pi} d\sigma^0 \times \left[\log\frac{4\Delta E^2}{\lambda^2}\left(1+\log\frac{m_e^2}{s}\right) + \frac{1}{2}\log^2\frac{m_e^2}{s} + \log\frac{m_e^2}{s} + \frac{1}{3}\pi^2\right]$$

$$d\sigma_{HC} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2 + 4m_e^2/s}{4m_e^2/s} - \frac{2z}{1-z}\frac{\Delta\theta^2}{\Delta\theta^2+4m_e^2/s}\right] d\sigma^0(zk_1)dz + (k_1 \Leftrightarrow k_2)$$

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$$d\sigma_{HC+CT} \equiv d\sigma^*_{HC+CT}\,(\text{hard part}) + d\sigma_{SC}\,(\text{soft part})$$

$$d\sigma^*_{HC+CT} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\Delta\theta^2 - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right] dz$$

$$d\sigma_{SC} = -\frac{\alpha_{em}}{\pi}\log\frac{s}{4m_e^2}\left[\frac{3}{2} + 2\log\delta_s\right] d\sigma^0$$

$$d\sigma_R(\lambda) = d\sigma_S(\lambda, \Delta E) + d\sigma_{HC}(\Delta E, \Delta\theta) + d\sigma_{H\overline{C}}(\Delta E, \Delta\theta) + d\sigma_{CT}$$

Real corrections (Two cutoff method [Harris and Owens (2002)])

λ: photon mass Δ_E $(=\delta_s\sqrt{s}/2)$: soft cutoff $\Delta\theta$: collinear cutoff

$$d\sigma_S = -\frac{\alpha_{em}}{\pi} d\sigma^0 \times \left[\log\frac{4\Delta E^2}{\lambda^2}\left(1+\log\frac{m_e^2}{s}\right) + \frac{1}{2}\log^2\frac{m_e^2}{s} + \log\frac{m_e^2}{s} + \frac{1}{3}\pi^2\right]$$

$$d\sigma_{HC} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2+4m_e^2/s}{4m_e^2/s} - \frac{2z}{1-z}\frac{\Delta\theta^2}{\Delta\theta^2+4m_e^2/s}\right] d\sigma^0(zk_1)dz + (k_1 \Leftrightarrow k_2)$$

$$\xrightarrow{\Delta\theta^2 \gg m_e^2/s} \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2 s}{4m_e^2} - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right]dz$$

$$d\sigma_{CT} = -\frac{\alpha_{em}}{2\pi}\log\frac{s}{4m_e^2} P_{ee}^+(z,0) \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right]dz$$

$$d\sigma_{HC+CT} \equiv d\sigma^*_{HC+CT}(\text{hard part}) + d\sigma_{SC}(\text{soft part})$$

$$d\sigma^*_{HC+CT} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\Delta\theta^2 - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right]dz$$

$$d\sigma_{SC} = -\frac{\alpha_{em}}{\pi}\log\frac{s}{4m_e^2}\left[\frac{3}{2} + 2\log\delta_s\right] d\sigma^0$$

$$d\sigma_R(\lambda) = d\sigma_S(\lambda,\Delta E) + d\sigma_{HC}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta) + d\sigma_{CT}$$

$$\Rightarrow d\sigma_S(\lambda,\Delta E) + d\sigma_{SC}(\Delta E) + d\sigma^*_{HC+CT}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta)$$

Real corrections (Two cutoff method [Harris and Owens (2002)])

λ: photon mass Δ_E $(=\delta_s\sqrt{s}/2)$: soft cutoff $\Delta\theta$: collinear cutoff

$$d\sigma_S = -\frac{\alpha_{em}}{\pi} d\sigma^0 \times \left[\log\frac{4\Delta E^2}{\lambda^2}\left(1+\log\frac{m_e^2}{s}\right) + \frac{1}{2}\log^2\frac{m_e^2}{s} + \log\frac{m_e^2}{s} + \frac{1}{3}\pi^2\right]$$

$$d\sigma_{HC} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\frac{\Delta\theta^2+4m_e^2/s}{4m_e^2/s} - \frac{2z}{1-z}\frac{\Delta\theta^2}{\Delta\theta^2+4m_e^2/s}\right] d\sigma^0(zk_1)dz + (k_1 \Leftrightarrow k_2)$$

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$$d\sigma_{CT} = -\frac{\alpha_{em}}{2\pi}\log\frac{s}{4m_e^2} P_{ee}^+(z,0) \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right]dz$$

$$d\sigma_{HC+CT} \equiv d\sigma^*_{HC+CT}(\text{hard part}) + d\sigma_{SC}(\text{soft part})$$

$$d\sigma^*_{HC+CT} = \frac{\alpha_{em}}{2\pi}\left[\frac{1+z^2}{1-z}\log\Delta\theta^2 - \frac{2z}{1-z}\right] \times \left[d\sigma^0(zk_1) + d\sigma^0(zk_2)\right]dz$$

$$d\sigma_{SC} = -\frac{\alpha_{em}}{\pi}\log\frac{s}{4m_e^2}\left[\frac{3}{2} + 2\log\delta_s\right] d\sigma^0$$

$$d\sigma_R(\lambda) = d\sigma_S(\lambda,\Delta E) + d\sigma_{HC}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta) + d\sigma_{CT}$$

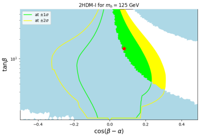
$$\Rightarrow d\sigma_S(\lambda,\Delta E) + d\sigma_{SC}(\Delta E) + d\sigma^*_{HC+CT}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta)$$

$$d\sigma^1 = d\sigma_{V+S}(\Delta E) + d\sigma_{SC}(\Delta E) + d\sigma^*_{HC+CT}(\Delta E,\Delta\theta) + d\sigma_{H\overline{C}}(\Delta E,\Delta\theta)$$

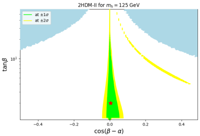
Benchmark points

h/H: different SM-like Higgs 1/2: different Yukawa coupling

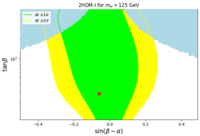
BP1h



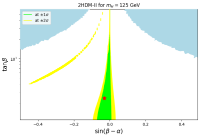
BP2h



BP1H



BP2H



Theoretical constraints

- Vacuum stability
- Perturbativity
- Tree-level unitarity

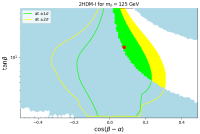
Experimental constraints

- EWPT (S, T, U parameters)
- B physics (SuperIso)
- LHC and other collider data (HiggsBounds and HiggsSignals)

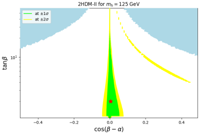
Benchmark points

h/H: different SM-like Higgs   1/2: different Yukawa coupling

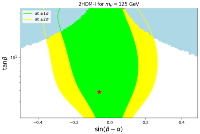
BP1h



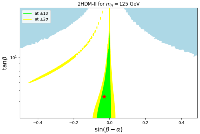
BP2h



BP1H



BP2H



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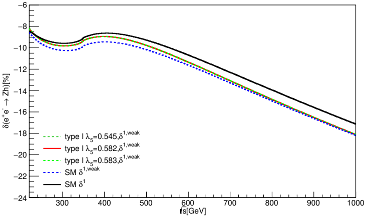
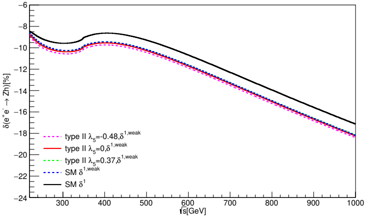
BPs	$\sin(\beta-\alpha)$	$\tan\beta$	m_h (GeV)	m_H (GeV)	m_A (GeV)	$m_{H^\pm}$ (GeV)	λ_5
BP1-h	0.99679	14.300	125.00	212.00	98.20	178.27	0.5819
BP2-h	0.99999	2.012	125.00	594.00	512.00	592.00	0.0000
BP1-H	-0.06000	2.830	95.00	125.00	169.00	170.00	-0.3220
BP2-H	-0.03000	2.160	95.00	125.00	600.00	600.00	-5.7800

NLO corrections (QED corrections are same in SM and 2HDM)

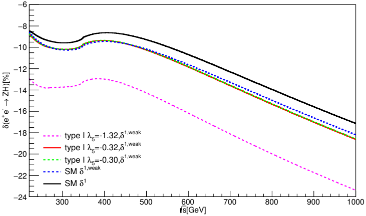
BP1h



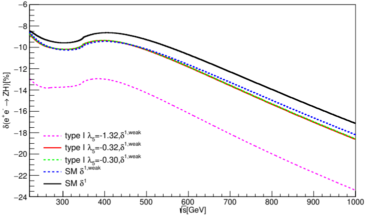
BP2h



BP1H

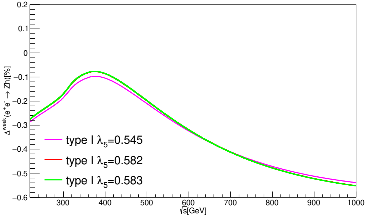


BP2H

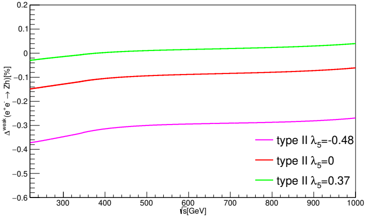


Effect of new physics

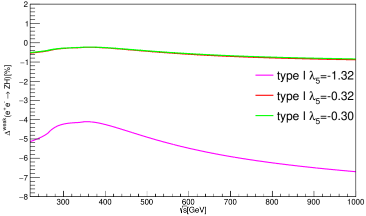
BP1h



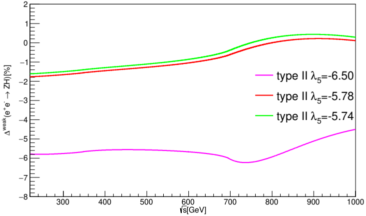
BP2h



BP1H



BP2H



Renormalization of charge (same as in the SM/QED [Denner (1993)])

renormalized in the Thomson limit → OS scheme

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{\sin\theta_W}{\cos\theta_W}\frac{1}{2}\delta Z_{ZA} = \frac{1}{2}\Pi(0) - \frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\Pi(0) = \lim_{s\to 0}\frac{\Sigma_T^{AA}(s)}{s} = \frac{\partial\,\Sigma_T^{AA}(s)}{\partial s}\bigg|_{s=0} \quad \left(\Pi(s) \equiv \frac{\Sigma_T^{AA}(s)}{s}\right)$$

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$$\Rightarrow \delta Z_e(0) = \frac{1}{2}\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \frac{1}{2}\Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \frac{1}{2}\Pi_{\mathrm{lepton}}(0) + \frac{1}{2}\Pi_{\mathrm{remaining}}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\Pi(0) = \lim_{s\to 0}\frac{\Sigma_T^{AA}(s)}{s} = \frac{\partial\Sigma_T^{AA}(s)}{\partial s}\bigg|_{s=0} \quad \left(\Pi(s) \equiv \frac{\Sigma_T^{AA}(s)}{s}\right)$$

$$= \Pi^{(5)}_{\mathrm{hadron}}(0) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Rightarrow \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

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$$\Pi(0) = \lim_{s\to 0}\frac{\Sigma_T^{AA}(s)}{s} = \frac{\partial\Sigma_T^{AA}(s)}{\partial s}\Big|_{s=0} \quad \left(\Pi(s) \equiv \frac{\Sigma_T^{AA}(s)}{s}\right)$$

$$= \Pi^{(5)}_{\mathrm{hadron}}(0) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Rightarrow \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Delta\alpha(\mu) \equiv \Pi_{f\neq \mathrm{top}}(0) - \mathrm{Re}\Pi_{f\neq\mathrm{top}}(\mu^2)$$

$$\delta Z_e(\mu) \equiv \delta Z_e(0) - \frac{1}{2}\Delta\alpha(\mu)$$

$$\alpha_{em}(\mu) \equiv \frac{\alpha_{em}(0)}{1-\Delta\alpha(\mu)}$$

Renormalization of charge (same as in the SM/QED [Denner (1993)])

renormalized in the Thomson limit → OS scheme

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{\sin\theta_W}{\cos\theta_W}\frac{1}{2}\delta Z_{ZA} = \frac{1}{2}\Pi(0) - \frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

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$$\Pi(0) = \lim_{s\to 0}\frac{\Sigma_T^{AA}(s)}{s} = \frac{\partial\Sigma_T^{AA}(s)}{\partial s}\Big|_{s=0} \quad \left(\Pi(s) \equiv \frac{\Sigma_T^{AA}(s)}{s}\right)$$

$$= \Pi^{(5)}_{\mathrm{hadron}}(0) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Rightarrow \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Delta\alpha(\mu) \equiv \Pi_{f\neq\mathrm{top}}(0) - \mathrm{Re}\Pi_{f\neq\mathrm{top}}(\mu^2)$$

$$= [\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) - \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(\mu^2)] + [\Pi_{\mathrm{lepton}}(0) - \mathrm{Re}\Pi_{\mathrm{lepton}}(\mu^2)]$$

$$\delta Z_e(\mu) \equiv \delta Z_e(0) - \frac{1}{2}\Delta\alpha(\mu)$$

$$= \frac{1}{2}\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(\mu^2) + \frac{1}{2}\mathrm{Re}\Pi_{\mathrm{lepton}}(\mu^2) + \frac{1}{2}\Pi_{\mathrm{remaining}}(0) - \frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\alpha_{em}(\mu) \equiv \frac{\alpha_{em}(0)}{1-\Delta\alpha(\mu)}$$

Renormalization of charge (same as in the SM/QED [Denner (1993)])

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$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{\sin\theta_W}{\cos\theta_W}\frac{1}{2}\delta Z_{ZA} = \frac{1}{2}\Pi(0) - \frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\Rightarrow \delta Z_e(0) = \frac{1}{2}\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \frac{1}{2}\Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \frac{1}{2}\Pi_{\mathrm{lepton}}(0) + \frac{1}{2}\Pi_{\mathrm{remaining}}(0) - \frac{s_W}{c_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\Pi(0) = \lim_{s\to 0}\frac{\Sigma_T^{AA}(s)}{s} = \frac{\partial\Sigma_T^{AA}(s)}{\partial s}\Big|_{s=0} \quad \left(\Pi(s) \equiv \frac{\Sigma_T^{AA}(s)}{s}\right)$$

$$= \Pi^{(5)}_{\mathrm{hadron}}(0) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Rightarrow \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) + \Pi_{\mathrm{lepton}}(0) + \Pi_{\mathrm{remaining}}(0)$$

$$\Delta\alpha(\mu) \equiv \Pi_{f\neq\mathrm{top}}(0) - \mathrm{Re}\Pi_{f\neq\mathrm{top}}(\mu^2)$$

$$= [\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(m_Z^2) + \Delta\alpha^{(5)}_{\mathrm{hadron}}(m_Z) - \mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(\mu^2)] + [\Pi_{\mathrm{lepton}}(0) - \mathrm{Re}\Pi_{\mathrm{lepton}}(\mu^2)]$$

$$\delta Z_e(\mu) \equiv \delta Z_e(0) - \frac{1}{2}\Delta\alpha(\mu)$$

$$= \frac{1}{2}\mathrm{Re}\Pi^{(5)}_{\mathrm{hadron}}(\mu^2) + \frac{1}{2}\mathrm{Re}\Pi_{\mathrm{lepton}}(\mu^2) + \frac{1}{2}\Pi_{\mathrm{remaining}}(0) - \frac{\sin\theta_W}{\cos\theta_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2}$$

$$\alpha_{em}(\mu) \equiv \frac{\alpha_{em}(0)}{1-\Delta\alpha(\mu)}$$

after resummation → $\overline{\mathrm{MS}}$-like scheme

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{H^0 H^0} & \frac{1}{2}\delta Z_{H^0 h^0} \\ \frac{1}{2}\delta Z_{h^0 H^0} & 1 + \frac{1}{2}\delta Z_{h^0 h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{G^0 G^0} & \frac{1}{2}\delta Z_{G^0 A^0} \\ \frac{1}{2}\delta Z_{A^0 G^0} & 1 + \frac{1}{2}\delta Z_{A^0 A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1 + \frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{H^0H^0} & \frac{1}{2}\delta Z_{H^0h^0} \\ \frac{1}{2}\delta Z_{h^0H^0} & 1+\frac{1}{2}\delta Z_{h^0h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \frac{1}{2}\delta Z_{G^0A^0} \\ \frac{1}{2}\delta Z_{A^0G^0} & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1+\frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

$$\Rightarrow R_{\alpha_0} = R_\alpha + \delta R_\alpha \delta\alpha$$

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{H^0H^0} & \frac{1}{2}\delta Z_{H^0h^0} \\ \frac{1}{2}\delta Z_{h^0H^0} & 1+\frac{1}{2}\delta Z_{h^0h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \frac{1}{2}\delta Z_{G^0A^0} \\ \frac{1}{2}\delta Z_{A^0G^0} & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1+\frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

$$\Rightarrow R_{\alpha_0} = R_\alpha + \delta R_\alpha \delta\alpha$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = R^T_{\alpha_0} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}_0 = (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{H^0H^0} & \frac{1}{2}\delta Z_{H^0h^0} \\ \frac{1}{2}\delta Z_{h^0H^0} & 1+\frac{1}{2}\delta Z_{h^0h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \frac{1}{2}\delta Z_{G^0A^0} \\ \frac{1}{2}\delta Z_{A^0G^0} & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1+\frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

$$\Rightarrow R_{\alpha_0} = R_\alpha + \delta R_\alpha \delta\alpha$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = R^T_{\alpha_0} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}_0 = (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha R^T_\alpha \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

$$= (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{H^0H^0} & \frac{1}{2}\delta Z_{H^0h^0} \\ \frac{1}{2}\delta Z_{h^0H^0} & 1+\frac{1}{2}\delta Z_{h^0h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \frac{1}{2}\delta Z_{G^0A^0} \\ \frac{1}{2}\delta Z_{A^0G^0} & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1+\frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

$$\Rightarrow R_{\alpha_0} = R_\alpha + \delta R_\alpha \delta\alpha$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = R^T_{\alpha_0} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}_0 = (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha R^T_\alpha \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

$$= (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\Rightarrow \delta\alpha = \frac{1}{4}(\delta Z_{H^0h^0} - \delta Z_{h^0H^0})$$

Renormalization of Higgs sector

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{H^0H^0} & \frac{1}{2}\delta Z_{H^0h^0} \\ \frac{1}{2}\delta Z_{h^0H^0} & 1+\frac{1}{2}\delta Z_{h^0h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \frac{1}{2}\delta Z_{G^0A^0} \\ \frac{1}{2}\delta Z_{A^0G^0} & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^\pm G^\pm} & \frac{1}{2}\delta Z_{G^\pm H^\pm} \\ \frac{1}{2}\delta Z_{H^\pm G^\pm} & 1+\frac{1}{2}\delta Z_{H^\pm H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

$$\alpha_0 = \alpha + \delta\alpha, \quad \beta_0 = \beta + \delta\beta$$

$$\Rightarrow R_{\alpha_0} = R_\alpha + \delta R_\alpha \delta\alpha$$

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = R^T_{\alpha_0} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}_0 = (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha R^T_\alpha \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

$$= (R^T_\alpha + \delta R^T_\alpha \delta\alpha)\sqrt{Z_\rho} R_\alpha \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}$$

$$\Rightarrow \delta\alpha = \frac{1}{4}(\delta Z_{H^0h^0} - \delta Z_{h^0H^0})$$

$$\delta\beta^{(1)} = \frac{1}{4}(\delta Z_{G^\pm H^\pm} - \delta Z_{H^\pm G^\pm})$$

$$\delta\beta^{(2)} = \frac{1}{4}(\delta Z_{G^0A^0} - \delta Z_{A^0G^0})$$

$$m_{\phi,0}^2 = m_\phi^2 + \delta m_\phi^2, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$
$$T_{H,0} = T_H + \delta T_H, \quad T_{h,0} = T_h + \delta T_h$$

$$m^2_{\phi,0} = m^2_\phi + \delta m^2_\phi, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$

$$T_{H,0} = T_H + \delta T_H, \quad T_{h,0} = T_h + \delta T_h$$

$$\widehat{T}_{h,H} = 0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi\phi}(m^2_\phi) = 0, \quad \phi = H^\pm, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}'_{\phi\phi}(k^2)|_{k^2=m^2_\phi} = 0, \quad \phi = G^\pm, H^\pm, G^0, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_1}) = \widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_2}) = 0, \quad (\phi_1, \phi_2) = (G^\pm, H^\pm), (G^0, A^0), (H^0, h^0)$$

$$m^2_{\phi,0} = m^2_\phi + \delta m^2_\phi, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$

$$T_{H,0} = T_H + \delta T_H, \quad T_{h,0} = T_h + \delta T_h$$

$$\widehat{T}_{h,H} = 0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi\phi}(m^2_\phi) = 0, \quad \phi = H^\pm, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}'_{\phi\phi}(k^2)|_{k^2=m^2_\phi} = 0, \quad \phi = G^\pm, H^\pm, G^0, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_1}) = \widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_2}) = 0, \quad (\phi_1, \phi_2) = (G^\pm, H^\pm), (G^0, A^0), (H^0, h^0)$$

$$\widehat{\Sigma}_{\phi\phi}(k^2) = \Sigma_{\phi\phi}(k^2) - \delta m^2_\phi + (k^2 - m^2_\phi)\,\delta Z_{\phi\phi} - \delta T_{\phi\phi}$$

$$\widehat{\Sigma}_{\phi_1\phi_2}(k^2) = \Sigma_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m^2_{\phi_1}) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m^2_{\phi_2}) - \delta T_{\phi_1\phi_2}$$

$$\left(\delta T_{\phi_1\phi_2}\right) = R^T \begin{pmatrix} \frac{\delta T_1}{v_1} & 0 \\ 0 & \frac{\delta T_2}{v_2} \end{pmatrix} R$$

$$m^2_{\phi,0} = m^2_\phi + \delta m^2_\phi, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$

$$T_{H,0} = T_H + \delta T_H, \quad T_{h,0} = T_h + \delta T_h$$

$$\widehat{T}_{h,H} = 0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi\phi}(m^2_\phi) = 0, \quad \phi = H^\pm, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}'_{\phi\phi}(k^2)|_{k^2=m^2_\phi} = 0, \quad \phi = G^\pm, H^\pm, G^0, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_1}) = \widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_2}) = 0, \quad (\phi_1, \phi_2) = (G^\pm, H^\pm), (G^0, A^0), (H^0, h^0)$$

$$\widehat{\Sigma}_{\phi\phi}(k^2) = \Sigma_{\phi\phi}(k^2) - \delta m^2_\phi + (k^2 - m^2_\phi)\,\delta Z_{\phi\phi} - \delta T_{\phi\phi}$$

$$\widehat{\Sigma}_{\phi_1\phi_2}(k^2) = \Sigma_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m^2_{\phi_1}) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m^2_{\phi_2}) - \delta T_{\phi_1\phi_2}$$

$$\left(\delta T_{\phi_1\phi_2}\right) = R^T \begin{pmatrix} \frac{\delta T_1}{v_1} & 0 \\ 0 & \frac{\delta T_2}{v_2} \end{pmatrix} R$$

$$\delta m^2_\phi = \mathrm{Re}\left[\Sigma_{\phi\phi}(m^2_\phi) - \delta T_{\phi\phi}\right]$$

$$\delta Z_{\phi\phi} = -\mathrm{Re}\Sigma'_{\phi\phi}(m^2_{\phi\phi})$$

$$\delta Z_{\phi_1\phi_2} = \frac{2\mathrm{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta T_{\phi_1\phi_2}\right]}{m^2_{\phi_1} - m^2_{\phi_2}}$$

$$m^2_{\phi,0} = m^2_\phi + \delta m^2_\phi, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$

$$T_{H,0} = T_H + \delta T_H, \quad T_{h,0} = T_h + \delta T_h$$

$$\widehat{T}_{h,H} = 0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi\phi}(m^2_\phi) = 0, \quad \phi = H^\pm, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}'_{\phi\phi}(k^2)|_{k^2=m^2_\phi} = 0, \quad \phi = G^\pm, H^\pm, G^0, A^0, H^0, h^0$$

$$\mathrm{Re}\widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_1}) = \widehat{\Sigma}_{\phi_1\phi_2}(m^2_{\phi_2}) = 0, \quad (\phi_1, \phi_2) = (G^\pm, H^\pm), (G^0, A^0), (H^0, h^0)$$

$$\widehat{\Sigma}_{\phi\phi}(k^2) = \Sigma_{\phi\phi}(k^2) - \delta m^2_\phi + (k^2 - m^2_\phi)\,\delta Z_{\phi\phi} - \delta T_{\phi\phi}$$

$$\widehat{\Sigma}_{\phi_1\phi_2}(k^2) = \Sigma_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m^2_{\phi_1}) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m^2_{\phi_2}) - \delta T_{\phi_1\phi_2}$$

$$\left(\delta T_{\phi_1\phi_2}\right) = R^T \begin{pmatrix} \frac{\delta T_1}{v_1} & 0 \\ 0 & \frac{\delta T_2}{v_2} \end{pmatrix} R$$

$$\delta m^2_\phi = \mathrm{Re}\left[\Sigma_{\phi\phi}(m^2_\phi) - \delta T_{\phi\phi}\right]$$

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Remarks

- $\delta\alpha$, $\delta\beta$, $\delta\lambda_5$
- complicated $T_{\phi_1\phi_2}$ (unlike in the SM)
- mission complete?

$$m^2_{\phi,0} = m^2_\phi + \delta m^2_\phi, \quad \phi = G^\pm, H^\pm, G^0, H^0, h^0$$

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Remarks

- $\delta\alpha$, $\delta\beta$, $\delta\lambda_5$
- complicated $T_{\phi_1\phi_2}$ (unlike in the SM)
- mission complete? NO!
- $\delta T_{h,H}$ is gauge dependent

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Remarks

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- complicated $T_{\phi_1\phi_2}$ (unlike in the SM)
- mission complete? NO!
- $\delta T_{h,H}$ is gauge dependent
- $\to$ gauge dependent counter terms

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Remarks

- $\delta\alpha$, $\delta\beta$, $\delta\lambda_5$
- complicated $T_{\phi_1\phi_2}$ (unlike in the SM)
- mission complete? NO!
- $\delta T_{h,H}$ is gauge dependent
- $\to$ gauge dependent counter terms
- how about final results?

FJ tadpole scheme [Fleischer and Jegerlehner (1981)], see also [Denner et al. (2016)]

$$\mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\ldots;v_{1,\mathrm{B}},\ldots;\ldots\right) \to \mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\ldots;v_{1,\mathrm{B}}+\Delta v_1,\ldots;\ldots\right)$$

$$\langle\phi_{i,\mathrm{B}}\rangle = t_i(\Delta v_1,\ldots)+T_i, \quad t_i(\Delta v_1,\ldots)\equiv \left.\frac{\partial\Delta\mathcal{L}}{\partial\phi_i}\right|_{\phi=0}, \quad \Delta\mathcal{L}\equiv\mathcal{L}-\left.\mathcal{L}\right|_{\Delta v=0}$$

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$$\mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\dots;v_{1,\mathrm{B}},\dots;\dots\right) \to \mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\dots;v_{1,\mathrm{B}}+\Delta v_1,\dots;\dots\right)$$

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$$\langle\phi_{i,\mathrm{B}}\rangle \overset{!}{=} 0 \Rightarrow t_i(\Delta v_1,\dots)+T_i=0$$

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$$t_{ij}(\Delta v_1,\ldots)\equiv\left.\frac{\partial^2\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j}\right|_{\phi=0},\quad t_{ijk}(\Delta v_1,\ldots)\equiv\left.\frac{\partial^3\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j\partial\phi_k}\right|_{\phi=0}$$

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$$\mathcal{L}_{\mathrm{B}} = (D_\mu\Phi_{\mathrm{B}})^\dagger(D^\mu\Phi_{\mathrm{B}}) - V_{\mathrm{B}}(\Phi_{\mathrm{B}}), \quad V_{\mathrm{B}}(\Phi_{\mathrm{B}}) = \lambda_{\mathrm{B}}(\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}})^2 - \mu_{\mathrm{B}}^2\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}} \supset V_{\mathrm{B}}^1 h_{\mathrm{B}} + V_{\mathrm{B}}^2 h_{\mathrm{B}}^2$$

$$V_{\mathrm{B}}^1 \equiv (v_{\mathrm{B}}+\Delta v)\left(\lambda_B\,(v_{\mathrm{B}}+\Delta v)^2-\mu_{\mathrm{B}}^2\right)$$

$$V_B^2 \equiv \frac{3\lambda_{\mathrm{B}}}{2}(v_{\mathrm{B}}+\Delta v)^2-\frac{1}{2}\mu_{\mathrm{B}}^2$$

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$$\mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\ldots;v_{1,\mathrm{B}},\ldots;\ldots\right) \to \mathcal{L}_{\mathrm{H,B}}\left(\phi_{1,\mathrm{B}},\ldots;v_{1,\mathrm{B}}+\Delta v_1,\ldots;\ldots\right)$$

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$$\Delta v_i = \Delta v_i^{(0)}+\Delta v_i^{(1)}+\ldots, \quad t_i = t_i^{(0)}+t_i^{(1)}+\ldots, \quad T_i = T_i^{(0)}+T_i^{(1)}+\ldots$$

$$t_{ij}(\Delta v_1,\ldots) \equiv \left.\frac{\partial^2\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j}\right|_{\phi=0}, \quad t_{ijk}(\Delta v_1,\ldots) \equiv \left.\frac{\partial^3\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j\partial\phi_k}\right|_{\phi=0}$$

$$\mathcal{L}_{\mathrm{B}} = (D_\mu\Phi_{\mathrm{B}})^\dagger(D^\mu\Phi_{\mathrm{B}})-V_{\mathrm{B}}(\Phi_{\mathrm{B}}), \quad V_{\mathrm{B}}(\Phi_{\mathrm{B}}) = \lambda_{\mathrm{B}}(\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}})^2-\mu_{\mathrm{B}}^2\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}} \supset V_{\mathrm{B}}^1 h_{\mathrm{B}}+V_{\mathrm{B}}^2 h_{\mathrm{B}}^2$$

$$V_{\mathrm{B}}^1 \equiv (v_{\mathrm{B}}+\Delta v)\left(\lambda_B\,(v_{\mathrm{B}}+\Delta v)^2-\mu_{\mathrm{B}}^2\right) = \frac{\mu_{\mathrm{B}}^2\Delta v}{v_{\mathrm{B}}^2}(2v_{\mathrm{B}}^2+3v_{\mathrm{B}}\Delta v+\Delta v^2)$$

$$V_B^2 \equiv \frac{3\lambda_{\mathrm{B}}}{2}(v_{\mathrm{B}}+\Delta v)^2-\frac{1}{2}\mu_{\mathrm{B}}^2 = \frac{\mu_{\mathrm{B}}^2}{2v_{\mathrm{B}}^2}(2v_{\mathrm{B}}^2+6v_{\mathrm{B}}\Delta v+3\Delta v^2)$$

$$T_h^{(0)} = -v_{\mathrm{B}}\left(\lambda_{\mathrm{B}}v_{\mathrm{B}}^2-\mu_{\mathrm{B}}^2\right) \to \lambda_{\mathrm{B}} = \mu_{\mathrm{B}}^2/v_{\mathrm{B}}^2$$

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$$\mathcal{L}_{\rm H,B}\left(\phi_{1,\rm B},\ldots;v_{1,\rm B},\ldots;\ldots\right) \to \mathcal{L}_{\rm H,B}\left(\phi_{1,\rm B},\ldots;v_{1,\rm B}+\Delta v_1,\ldots;\ldots\right)$$

$$\langle\phi_{i,\rm B}\rangle = t_i(\Delta v_1,\ldots)+T_i,\quad t_i(\Delta v_1,\ldots)\equiv \left.\frac{\partial\Delta\mathcal{L}}{\partial\phi_i}\right|_{\phi=0},\quad \Delta\mathcal{L}\equiv\mathcal{L}-\left.\mathcal{L}\right|_{\Delta v=0}$$

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$$\mathcal{L}_{\rm B}=(D_\mu\Phi_{\rm B})^\dagger(D^\mu\Phi_{\rm B})-V_{\rm B}(\Phi_{\rm B}),\quad V_{\rm B}(\Phi_{\rm B})=\lambda_{\rm B}(\Phi_{\rm B}^\dagger\Phi_{\rm B})^2-\mu_{\rm B}^2\Phi_{\rm B}^\dagger\Phi_{\rm B}\supset V_{\rm B}^1 h_{\rm B}+V_{\rm B}^2 h_{\rm B}^2$$

$$V_{\rm B}^1\equiv(v_{\rm B}+\Delta v)\left(\lambda_B\left(v_{\rm B}+\Delta v\right)^2-\mu_{\rm B}^2\right)=\frac{m_{h,\rm B}^2\Delta v}{2v_{\rm B}^2}(2v_{\rm B}^2+3v_{\rm B}\Delta v+\Delta v^2)$$

$$V_B^2\equiv\frac{3\lambda_{\rm B}}{2}(v_{\rm B}+\Delta v)^2-\frac{1}{2}\mu_{\rm B}^2=\frac{m_{h,\rm B}^2}{4v_{\rm B}^2}(2v_{\rm B}^2+6v_{\rm B}\Delta v+3\Delta v^2)$$

$$T_h^{(0)}=-v_{\rm B}\left(\lambda_{\rm B}v_{\rm B}^2-\mu_{\rm B}^2\right)\to\lambda_{\rm B}=\mu_{\rm B}^2/v_{\rm B}^2,\quad m_{h,\rm B}^2\equiv 2\mu_{\rm B}^2$$

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$$V_B^2\equiv\frac{3\lambda_{\mathrm{B}}}{2}(v_{\mathrm{B}}+\Delta v)^2-\frac{1}{2}\mu_{\mathrm{B}}^2 = \frac{m_{h,\mathrm{B}}^2}{4v_{\mathrm{B}}^2}(2v_{\mathrm{B}}^2+6v_{\mathrm{B}}\Delta v+3\Delta v^2)$$

$$T_h^{(0)}=-v_{\mathrm{B}}\left(\lambda_{\mathrm{B}}v_{\mathrm{B}}^2-\mu_{\mathrm{B}}^2\right)\to\lambda_{\mathrm{B}}=\mu_{\mathrm{B}}^2/v_{\mathrm{B}}^2, \quad m_{h,\mathrm{B}}^2\equiv 2\mu_{\mathrm{B}}^2$$

$$\Rightarrow t_h^{(1)} = -m_{h,\mathrm{B}}^2\Delta v^{(1)} \qquad t_{hh}^{(1)} = -\frac{3m_{h,\mathrm{B}}^2\Delta v^{(1)}}{v_{\mathrm{B}}}$$

FJ tadpole scheme [Fleischer and Jegerlehner (1981)], see also [Denner et al. (2016)]

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$$\langle\phi_{i,\mathrm{B}}\rangle \overset{!}{=} 0 \Rightarrow t_i(\Delta v_1,\ldots)+T_i=0, \quad T_i^{(0)}=0\Rightarrow t_i^{(0)}=\Delta v_i^{(0)}=0$$

$$\Delta v_i=\Delta v_i^{(0)}+\Delta v_i^{(1)}+\ldots, \quad t_i=t_i^{(0)}+t_i^{(1)}+\ldots, \quad T_i=T_i^{(0)}+T_i^{(1)}+\ldots$$

$$t_{ij}(\Delta v_1,\ldots)\equiv\left.\frac{\partial^2\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j}\right|_{\phi=0}, \quad t_{ijk}(\Delta v_1,\ldots)\equiv\left.\frac{\partial^3\Delta\mathcal{L}}{\partial\phi_i\partial\phi_j\partial\phi_k}\right|_{\phi=0}$$

$$\mathcal{L}_{\mathrm{B}}=(D_\mu\Phi_{\mathrm{B}})^\dagger(D^\mu\Phi_{\mathrm{B}})-V_{\mathrm{B}}(\Phi_{\mathrm{B}}), \quad V_{\mathrm{B}}(\Phi_{\mathrm{B}})=\lambda_{\mathrm{B}}(\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}})^2-\mu_{\mathrm{B}}^2\Phi_{\mathrm{B}}^\dagger\Phi_{\mathrm{B}}\supset V_{\mathrm{B}}^1 h_{\mathrm{B}}+V_{\mathrm{B}}^2 h_{\mathrm{B}}^2$$

$$V_{\mathrm{B}}^1\equiv(v_{\mathrm{B}}+\Delta v)\left(\lambda_B\,(v_{\mathrm{B}}+\Delta v)^2-\mu_{\mathrm{B}}^2\right)=\frac{m_{h,\mathrm{B}}^2\Delta v}{2v_{\mathrm{B}}^2}(2v_{\mathrm{B}}^2+3v_{\mathrm{B}}\Delta v+\Delta v^2)$$

$$V_B^2\equiv\frac{3\lambda_{\mathrm{B}}}{2}(v_{\mathrm{B}}+\Delta v)^2-\frac{1}{2}\mu_{\mathrm{B}}^2=\frac{m_{h,\mathrm{B}}^2}{4v_{\mathrm{B}}^2}(2v_{\mathrm{B}}^2+6v_{\mathrm{B}}\Delta v+3\Delta v^2)$$

$$T_h^{(0)}=-v_{\mathrm{B}}\left(\lambda_{\mathrm{B}}v_{\mathrm{B}}^2-\mu_{\mathrm{B}}^2\right)\to\lambda_{\mathrm{B}}=\mu_{\mathrm{B}}^2/v_{\mathrm{B}}^2, \quad m_{h,\mathrm{B}}^2\equiv 2\mu_{\mathrm{B}}^2$$

$$\Rightarrow t_h^{(1)}=-m_{h,\mathrm{B}}^2\Delta v^{(1)}\to\Delta v^{(1)}=-\frac{T_h^{(1)}}{m_{h,\mathrm{B}}^2}, \quad t_{hh}^{(1)}=-\frac{3m_{h,\mathrm{B}}^2\Delta v^{(1)}}{v_{\mathrm{B}}}=\frac{3T_h^{(1)}}{v_{\mathrm{B}}}$$

Comparison between FJ and conventional tadpole schemes

$$m^2_{h,\mathrm{B}} = m^2_{h,\mathrm{R}} + \delta m^2_h$$

FJ scheme

$$m^2_{h,\mathrm{B}} \equiv 2\mu^2_{\mathrm{B}}$$

$$\widehat{\Sigma}_{hh}(m^2_{h,\mathrm{R}}) = \Sigma^{\mathrm{1PI}}_{hh}(m^2_{h,\mathrm{R}}) - t^{(1)}_{hh} - \delta m^2_h$$

Conventional scheme

$$m^2_{h,\mathrm{B}} \equiv 3\lambda_{\mathrm{B}} v^2_{\mathrm{B}} - \mu^2_{\mathrm{B}} = 2\mu^2_{\mathrm{B}} - \frac{3T_h}{v_{\mathrm{B}}}$$

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Conventional scheme

$$m^2_{h,\mathrm{B}} \equiv 3\lambda_{\mathrm{B}} v^2_{\mathrm{B}} - \mu^2_{\mathrm{B}} = 2\mu^2_{\mathrm{B}} - \frac{3T_h}{v_{\mathrm{B}}}$$

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$$\delta m^2_h = 2\delta\mu^2 - \frac{3\delta T_h}{v}, \quad \delta T_h + T^{(1)}_h = 0 \to \delta T_h = -T^{(1)}_h$$

δT_h removed from δm^2_h, $m_{h,\mathrm{B}}$ defined in a gauge independent way

Renormalization of 2HDM in FJ tadpole scheme [Krause et al. (2016)]

all CTs (except $\delta\alpha$ and $\delta\beta$) do not spoil gauge invariance

$$
\begin{aligned}
v_1 &\to v_1 + \Delta v_1, \quad v_2 \to v_2 + \Delta v_2 \\
\hat{\Sigma}_{\phi\phi}(k^2) &= \Sigma_{\phi\phi}(k^2) - \delta m_\phi^2 + (k^2 - m_\phi^2)\,\delta Z_{\phi\phi} - \cancel{\delta T_{\phi\phi}} - t_{\phi\phi} \\
&= \Sigma^{\text{tad}}_{\phi\phi}(k^2) - \delta m_\phi^2 + (k^2 - m_\phi^2)\,\delta Z_{\phi\phi} \\
\hat{\Sigma}_{\phi_1\phi_2}(k^2) &= \Sigma_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m_{\phi_1}^2) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m_{\phi_2}^2) - \cancel{\delta T_{\phi_1\phi_2}} - t_{\phi_1\phi_2} \\
&= \Sigma^{\text{tad}}_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m_{\phi_1}^2) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m_{\phi_2}^2)
\end{aligned}
$$

Renormalization of 2HDM in FJ tadpole scheme [Krause et al. (2016)]

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$$v_1 \to v_1 + \Delta v_1, \quad v_2 \to v_2 + \Delta v_2$$

$$\widehat{\Sigma}_{\phi\phi}(k^2) = \Sigma_{\phi\phi}(k^2) - \delta m_\phi^2 + (k^2 - m_\phi^2)\,\delta Z_{\phi\phi} - \cancel{\delta T_{\phi\phi}} - t_{\phi\phi}$$
$$= \Sigma^{\text{tad}}_{\phi\phi}(k^2) - \delta m_\phi^2 + (k^2 - m_\phi^2)\,\delta Z_{\phi\phi}$$

$$\widehat{\Sigma}_{\phi_1\phi_2}(k^2) = \Sigma_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m_{\phi_1}^2) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m_{\phi_2}^2) - \cancel{\delta T_{\phi_1\phi_2}} - t_{\phi_1\phi_2}$$
$$= \Sigma^{\text{tad}}_{\phi_1\phi_2}(k^2) + \frac{1}{2}\delta Z_{\phi_1\phi_2}(k^2 - m_{\phi_1}^2) + \frac{1}{2}\delta Z_{\phi_2\phi_1}(k^2 - m_{\phi_2}^2)$$

$$\overline{\Sigma}(k^2) = \Sigma^{\text{tad}}|_{\xi=1}(k^2) + \Sigma^{\text{add}}(k^2)$$

$$\delta\alpha = \frac{\text{Re}\overline{\Sigma}_{H^0h^0}(m_{H^0}^2) + \text{Re}\overline{\Sigma}_{H^0h^0}(m_{h^0}^2)}{2(m_{H^0}^2 - m_{h^0}^2)}$$

$$\delta\beta = -\frac{\text{Re}\overline{\Sigma}_{G^0A^0}(0) + \text{Re}\overline{\Sigma}_{G^0A^0}(m_{A^0}^2)}{2m_{A^0}^2}$$

pinch technique [Binosi and Papavassiliou (2009)] is used to obtain pinched self-energies

Summary

- 2HDM is a simple extension of the SM, obtained by adding an extra Higgs doublet.
- Full NLO EW corrections to $e^+e^- \to Zh(H)$ in the 2HDM is studied and compared with the results in the SM. It is found that the effect of new physics can be sizable in some cases and hence could be measured in future e^+e^- colliders.
- The renormalization of 2HDM is a bit tricky. Tadpole contributions should be appropriately allocated to retain gauge invariance.

Thanks!

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