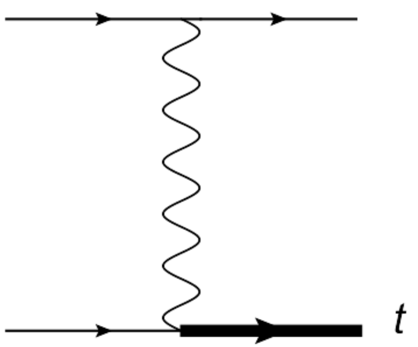
Calculation of master integrals for single top quark hadron production

Long-Bin Chen

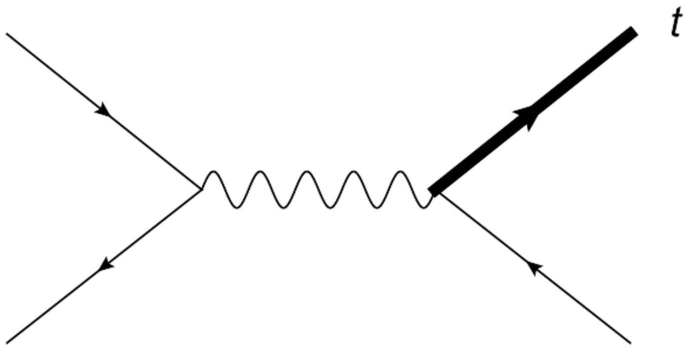
In collaboration with Jian Wang

2020-05-16

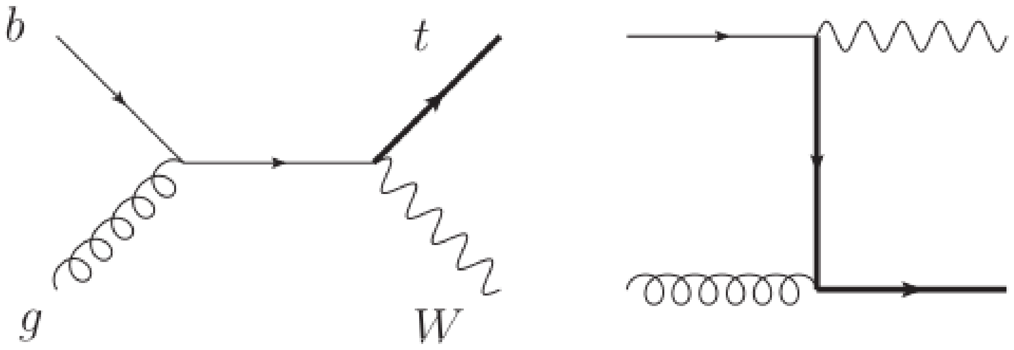
Single top quark hadron production



t-channel



s-channel



t+W

Measurement of V_{tb}

arXiv:2105.04464 [pdf, other]

Single Top Quark Production with and without a Higgs Boson

Qing-Hong Cao, Hao-ran Jiang, Guojin Zeng

QCD corrections:

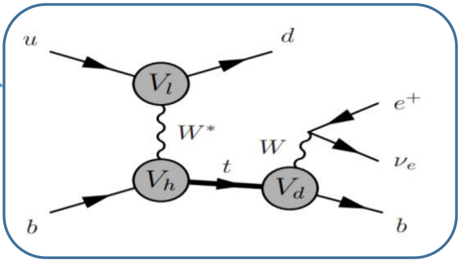
Shouhua Zhu, *Phys. Lett. B, 2002, 524:283-288.* t W 1-loop QCD with K-factor 1.4~1.7

R. Schwienhorst, C.-P. Yuan, C. Muller, Q.-H. Cao, *Phys. Rev. D, 2011, 83:034019.*

Edmond L. Berger, Jun Gao, C.-P. Yuan, Hua Xing Zhu, *Phys.Rev. D, 2016, 94:071501.*

Edmond L. Berger, Jun Gao, and Hua Xing Zhu, *JHEP, 2017, 11:158.*

Najam ul Basat, Zhao Li, and Yefan Wang, *arXiv:2102.08225.*



Ze Long Liu, Jun Gao, Phys. Rev. D, 2018, 98:071501

....

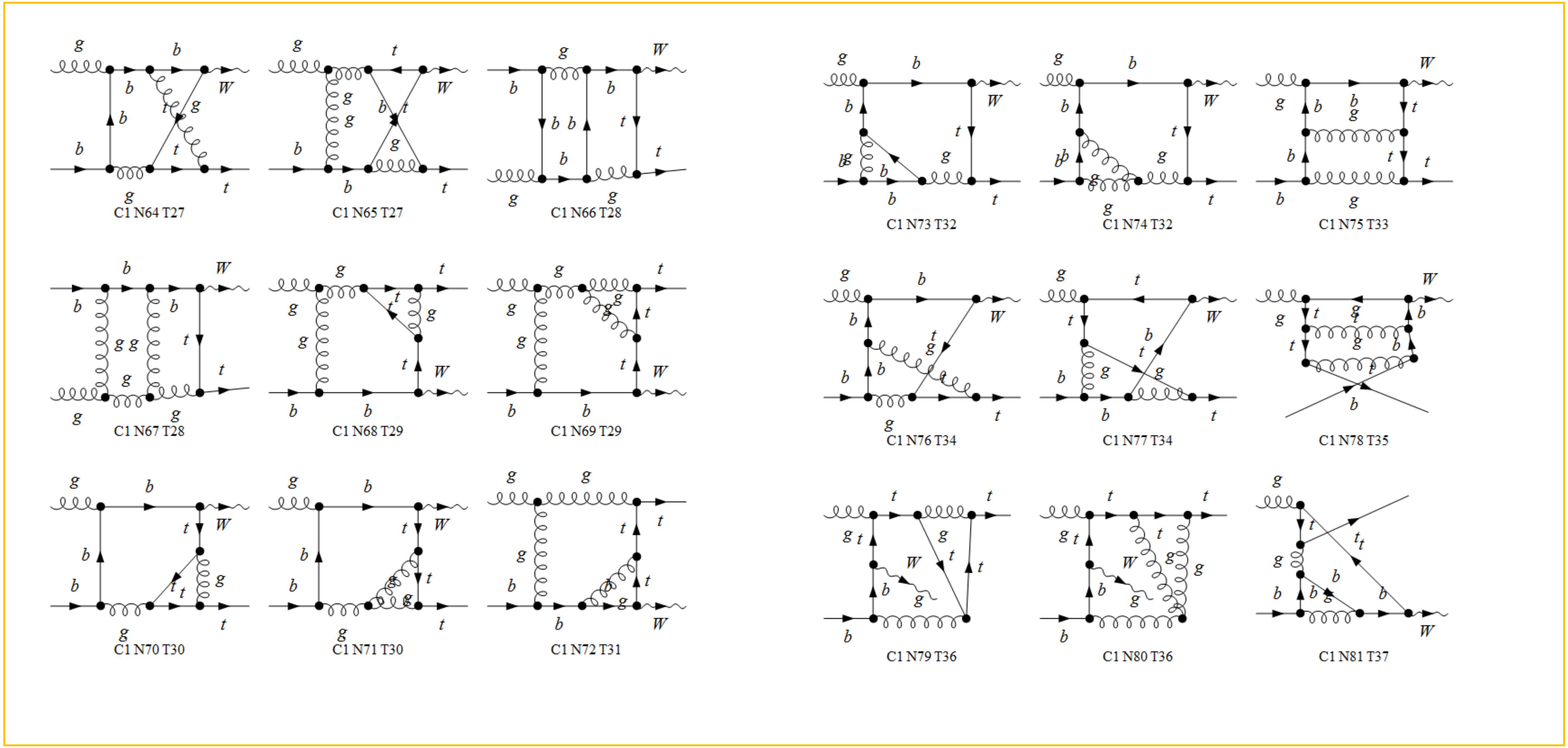
The need for higher order QCD corrections

QED $\alpha \sim O(0.01)$

QCD $\alpha_s \sim O(0.1)$ $(\mu \gg \Lambda_{QCD})$

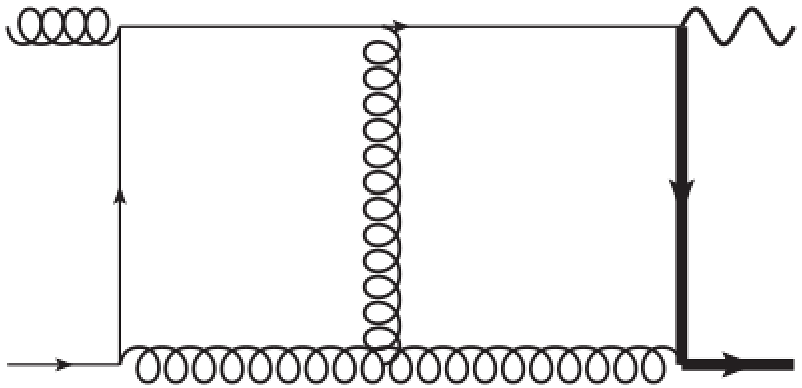
QED are more convergent
in perturbative expansion than **QCD**

For higher order corrections, one of the difficult parts is the calculation of multi-loop integrals.

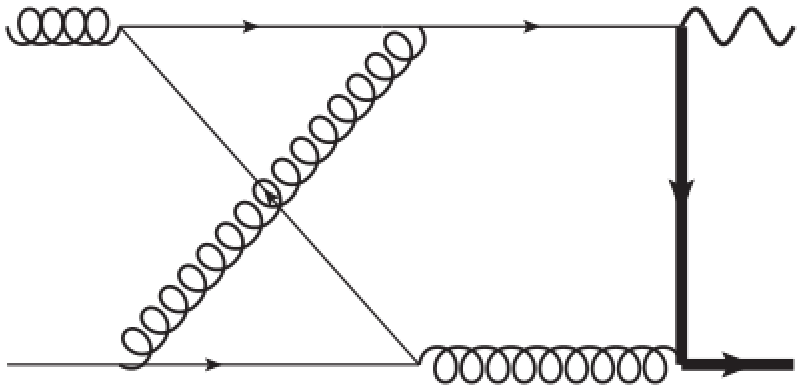


Sample of two-loop Feynman diagrams for t+W hadron production

Sample of two-loop Feynman diagrams for single top quark hadron production
(one massive propagator)



(1)

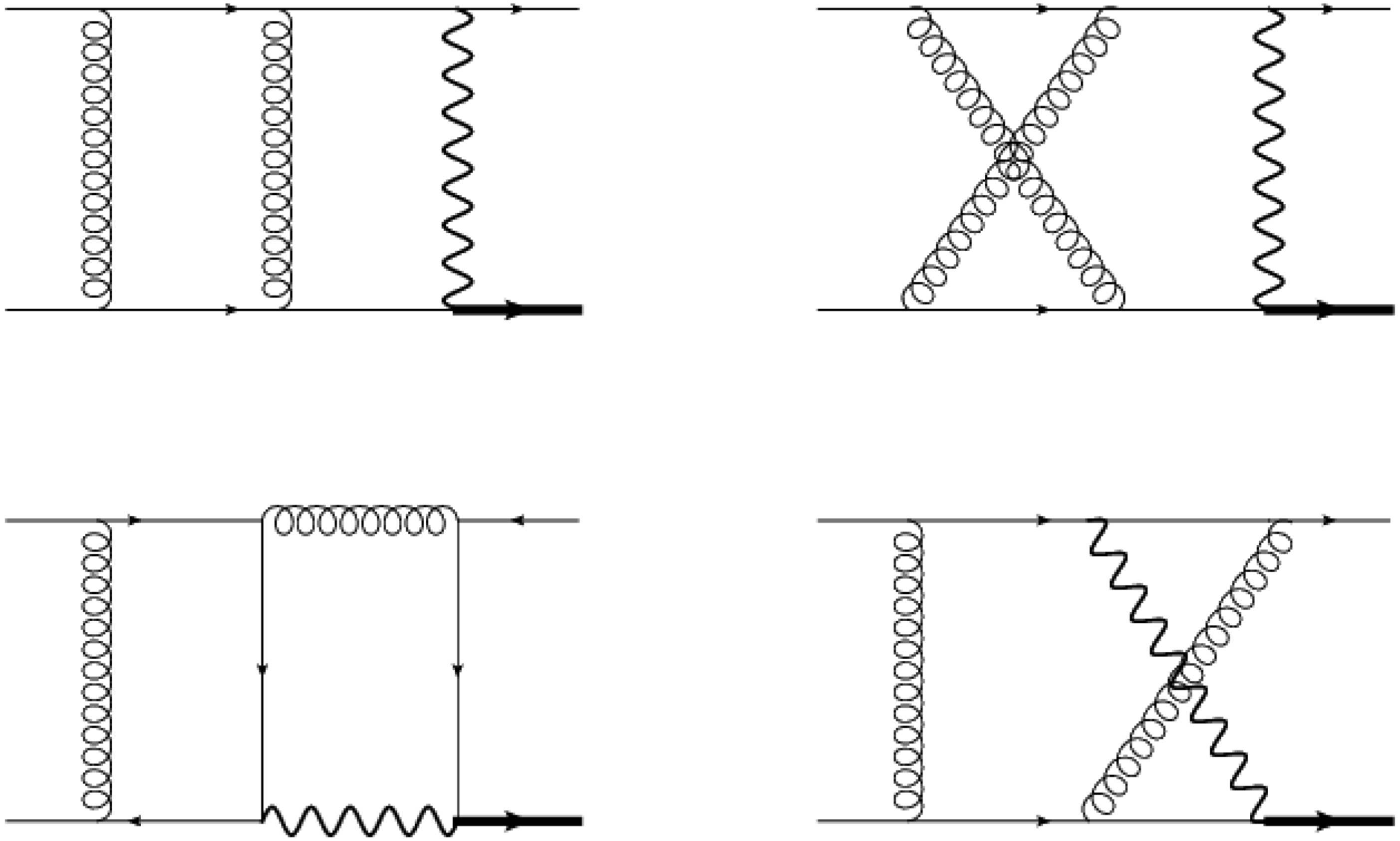


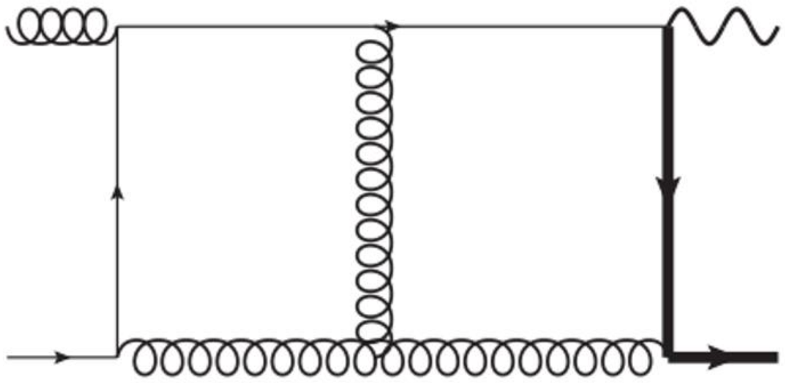
(2)

$Scales: s, t, u, m_W, m_t$

Sort the feynman diagrams by number of massive propagators

Sample of s-channel and t-channel two-loop diagrams



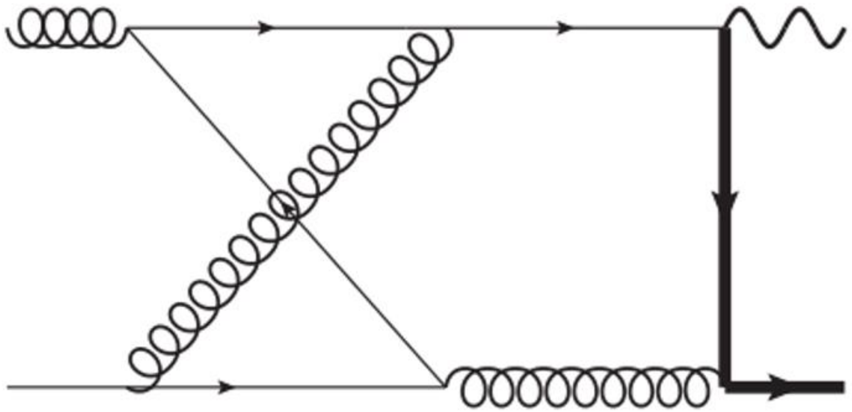


(1)

$$I_{n_1,n_2,\ldots,n_9} = \int \mathcal{D}^D q_1 \; \mathcal{D}^D q_2 \frac{1}{D_1^{n_1} \; D_2^{n_2} \; D_3^{n_3} \; D_4^{n_4} \; D_5^{n_5} \; D_6^{n_6} \; D_7^{n_7} D_8^{n_8} \; D_9^{n_9}}$$

$$\begin{aligned} D_1 &= q_1^2, D_2 = q_2^2, D_3 = (q_1 - k_1)^2, D_4 = (q_1 + k_2)^2, D_5 = (q_1 + q_2 - k_1)^2, \\ D_6 &= (q_2 - k_1 - k_2)^2, D_7 = (q_2 - k_3)^2 - m_t^2, \\ D_8 &= (q_1 + k_1 + k_2 - k_3)^2 - m_t^2, D_9 = (q_2 - k_1)^2. \end{aligned}$$

31MIs



(2)

$$J_{n_1,n_2,\ldots,n_9} = \int \mathcal{D}^D q_1 \; \mathcal{D}^D q_2 \frac{1}{P_1^{n_1} \; P_2^{n_2} \; P_3^{n_3} \; P_4^{n_4} \; P_5^{n_5} \; P_6^{n_6} \; P_7^{n_7} P_8^{n_8} \; P_9^{n_9}}.$$

$$\begin{aligned} P_1 &= q_1^2, P_2 = (q_1 - q_2)^2, P_3 = q_2^2, P_4 = (q_1 + k_1)^2, P_5 = (q_1 - q_2 - k_2)^2, \\ P_6 &= (q_2 + k_1 + k_2)^2, P_7 = (q_2 + k_1 + k_2 - k_3)^2 - m_t^2, \\ P_8 &= (q_1 - k_3)^2, P_9 = (q_2 + k_1)^2. \end{aligned}$$

34MIs

Integrals Reduction: IBP (integration-by-parts)

$$F(a) = \int \frac{\mathrm{d}^d k}{(k^2 - m^2)^a}.$$

↓

$$\int \mathrm{d}^d k \frac{\partial}{\partial k} \cdot k \frac{1}{(k^2 - m^2)^a} = 0,$$

↓

$$(d - 2a) F(a) - 2am^2 F(a+1) = 0.$$

↓

$$F(a) = \frac{d - 2a + 2}{2(a-1)m^2} F(a-1).$$

→

$$F(q_1, \ldots, q_n; a_1, \ldots, a_N; d) = \int \cdots \int \prod_{i=1}^{h} \mathrm{d}^d k_i \frac{1}{\prod_{j=1}^{N} E_j^{a_j}},$$

$$\int \cdots \int \prod_{i'=1}^{h} \mathrm{d}^d k_{i'} \frac{\partial}{\partial k_i} \left(p_j \prod_{j'=1}^{N} E_j^{-a_{j'}} \right) = 0$$

$$\sum \alpha_i F(a_1 + b_{i,1}, \ldots, a_N + b_{i,N}) = 0.$$

K.G. Chetyrkin, F.V. Tkachov, Nucl. Phys. B **192**, 159 (1981)

P.A. Baikov, Phys. Lett. B **385**, 404 (1996)
P.A. Baikov, Nucl. Instrum. Methods A **389**, 347 (1997)
S. Laporta, Int. J. Mod. Phys. A **15**, 5087 (2000)
S. Laporta, E. Remiddi, Phys. Lett. B **379**, 283 (1996)

- Reduction Packages:
  FIRE, **Reduze**, **Kira**⋯

FIRE5: a C++ implementation of Feynman Integral REduction
Alexander V. Smirnov (Moscow State U.) (Aug 11, 2014)
Published in: *Comput.Phys.Commun.* 189 (2015) 182-191 • e-Print: 1408.2372 [hep-ph]
pdf DOI cite

FIRE6: Feynman Integral REduction with Modular Arithmetic
A.V. Smirnov (Moscow State U. and KIT, Karlsruhe, TTP), F.S. Chuharev (Moscow State U.) (Jan 23, 2019)
e-Print: 1901.07808 [hep-ph]
pdf DOI cite

Differential Equations (DE):

$$\frac{d}{dx}\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} = \begin{pmatrix} A_{11} & \cdots & A_{1,n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{n,n} \end{pmatrix}\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}$$

x are Lorentz invariant kinematics

Differential equations method: New technique for massive Feynman diagrams calculation

A.V. Kotikov (BITP, Kiev) (Jun, 1990)

Published in: *Phys.Lett.B* 254 (1991) 158-164

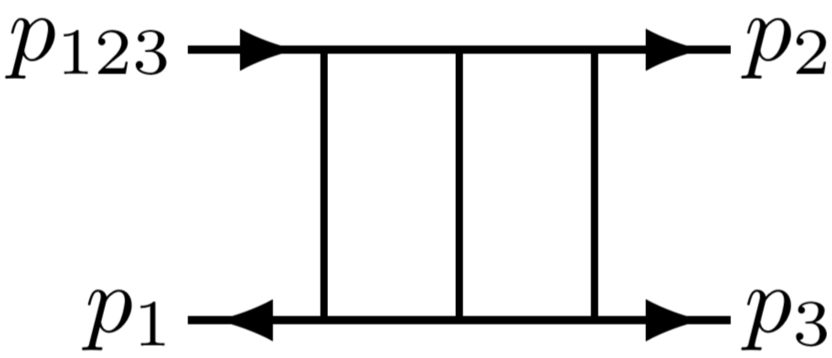
Differential equation method: The Calculation of N point Feynman diagrams

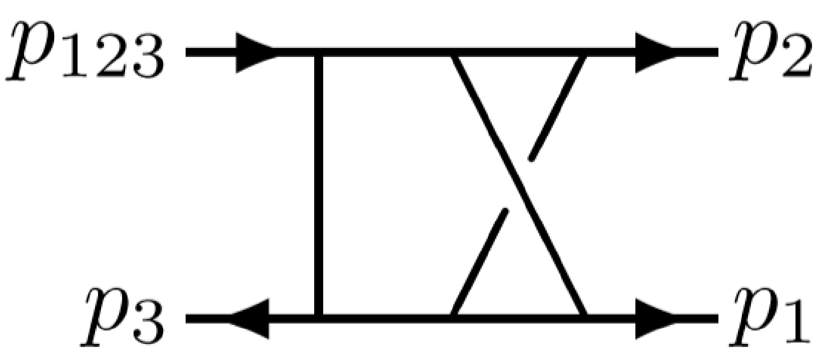
A.V. Kotikov (BITP, Kiev) (1991)

Published in: *Phys.Lett.B* 267 (1991) 123-127, *Phys.Lett.B* 295 (1992) 409-409 (erratum)

Progress in DE method:

T. Gehrmann, E. Remiddi, Nucl. Phys. B **580**, 485 (2000)
T. Gehrmann, E. Remiddi, Nucl. Phys. B **601**, 248 (2001)
T. Gehrmann, E. Remiddi, Nucl. Phys. B **601**, 287 (2001)





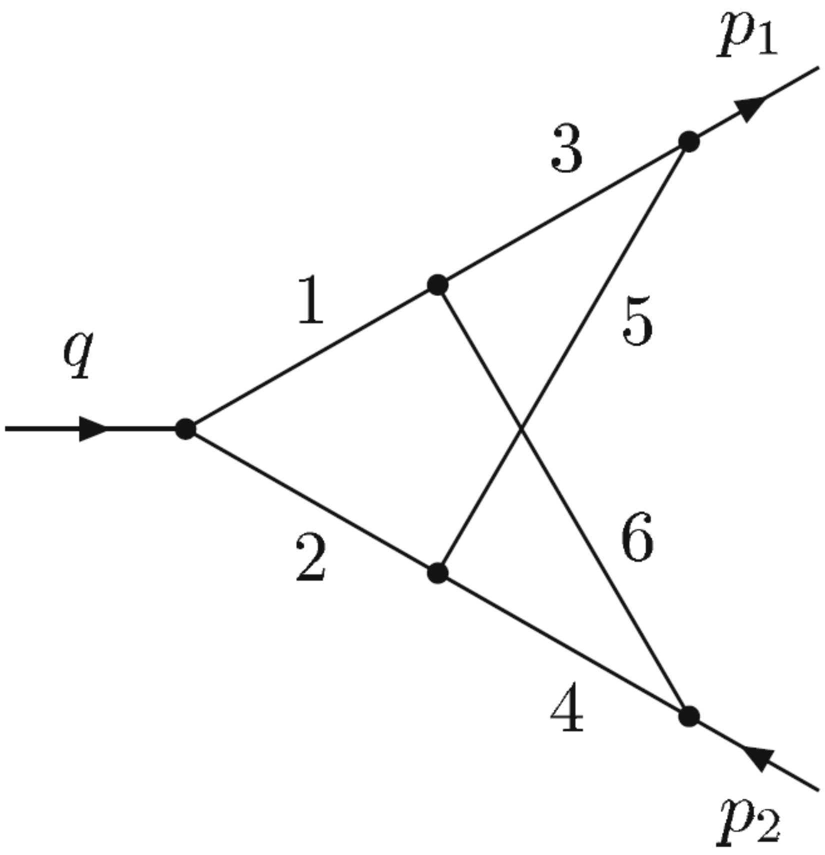
Weight / Transcendentality

Log[x]^n, π^n, ζ(n),Li$_n$[x]

Uninform weight-n.

Example

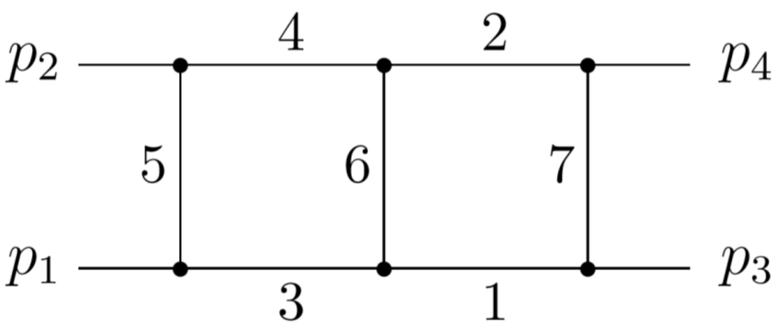
R. J. Gonsalves, Phys. Rev. D **28**, 1542 (1983)



Can be evaluated by Feynman parameters

$$F(Q^2;1,\ldots,1;d)=\frac{\left(\mathrm{i}\pi^{d/2}\mathrm{e}^{-\gamma_{\mathrm{E}}\varepsilon}\right)^2}{(Q^2)^{2+2\varepsilon}}\times\left(\frac{1}{\varepsilon^4}-\frac{\pi^2}{\varepsilon^2}-\frac{83\zeta(3)}{3\varepsilon}-\frac{59\pi^4}{120}\right)+O(\varepsilon).$$

Based on MB representation



Planar

$$
\begin{aligned}
f(x,\varepsilon) = & -\frac{4}{\varepsilon^4} + \frac{5\ln x}{\varepsilon^3} - \left(2\ln^2 x - \frac{5}{2}\pi^2\right)\frac{1}{\varepsilon^2} \\
& - \left(\frac{2}{3}\ln^3 x + \frac{11}{2}\pi^2 \ln x - \frac{65}{3}\zeta(3)\right)\frac{1}{\varepsilon} \\
& + \frac{4}{3}\ln^4 x + 6\pi^2 \ln^2 x - \frac{88}{3}\zeta(3)\ln x + \frac{29}{30}\pi^4 \\
& - [2\,\mathrm{Li}_3(-x) - 2\ln x\,\mathrm{Li}_2(-x) - (\ln^2 x + \pi^2)\ln(1+x)]\frac{2}{\varepsilon} \\
& - 4[S_{2,2}(-x) - \ln x\, S_{1,2}(-x)] + 44\,\mathrm{Li}_4(-x) \\
& - 4\,[\ln(1+x) + 6\ln x]\,\mathrm{Li}_3(-x) \\
& + 2\left(\ln^2 x + 2\ln x\ln(1+x) + \frac{10}{3}\pi^2\right)\mathrm{Li}_2(-x) \\
& + \left(\ln^2 x + \pi^2\right)\ln^2(1+x) \\
& - \frac{2}{3}[4\ln^3 x + 5\pi^2 \ln x - 6\zeta(3)]\ln(1+x) + O(\varepsilon),
\end{aligned}
$$

Analytical result for dimensionally regularized massless on shell double box #1

Vladimir A. Smirnov (Moscow State U.) (May 13, 1999)

Published in: *Phys.Lett.B* 460 (1999) 397-404 • e-Print: hep-ph/9905323 [hep-ph]

Multiloop integrals in dimensional regularization made simple

Johannes M. Henn (Princeton, Inst. Advanced Study) (Apr 5, 2013)

Published in: *Phys.Rev.Lett.* 110 (2013) 251601 • e-Print: 1304.1806 [hep-th]

Choosing canonical basis (basis with uniform transcendentality)

For random basis g, we may have:

$$\partial_x \vec{g}(x;\epsilon) = B(x,\epsilon)\,\vec{g}(x;\epsilon)$$

We can choose new basis f:

d=4-2∈

$$\vec{f} = T\vec{g},$$

$$d\,\vec{f}(x,\epsilon) = \epsilon\,\left(d\,\tilde{A}\right)\,\vec{f}(x;\epsilon)$$

$$\tilde{A} = \left[\sum_k A_k \log \alpha_k(x)\right].$$

arXiv:1412.2296 [pdf, other] hep-ph hep-th math-ph doi 10.1088/1751-8113/48/15/153001

Lectures on differential equations for Feynman integrals

Authors: Johannes M. Henn

How to Find Canonical Basis

Fuchsia: a tool for reducing differential equations for Feynman master integrals to epsilon form

Oleksandr Gituliar (Hamburg U., Inst. Theor. Phys. II), Vitaly Magerya (Novomoskovsk U.) (Jan 16, 2017)

Published in: *Comput.Phys.Commun.* 219 (2017) 329-338 • e-Print: 1701.04269 [hep-ph]

Libra: a package for transformation of differential systems for multiloop integrals

Roman N. Lee (Novosibirsk, IYF) (Dec 1, 2020)

e-Print: 2012.00279 [hep-ph]

Constructing canonical Feynman integrals with intersection theory

Jiaqi Chen (Peking U. and Peking U., SKLNPT), Xuhang Jiang (Peking U., SKLNPT), Xiaofeng Xu (U. Bern (main)), Li Lin Yang (Zhejiang U., Inst. Mod. Phys.) (Aug 7, 2020)

Published in: *Phys.Lett.B* 814 (2021) 136085 • e-Print: 2008.03045 [hep-th]

Leading singularities in Baikov representation and Feynman integrals with uniform transcendental weight

Christoph Dlapa (Munich, Max Planck Inst.), Xiaodi Li (Zheijiang U.), Yang Zhang (PCFT, Hefei and Hefei, CUST) (Mar 8, 2021)

e-Print: 2103.04638 [hep-th]

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H. Frellesvig, F. Gasparotto, S. Laporta, M. K. Mandal, P. Mastrolia, L. Mattiazzi, and S. Mizera, *Decomposition of Feynman Integrals by Multivariate Intersection Numbers*, arXiv:2008.04823.

C. Meyer, *Transforming differential equations of multi-loop Feynman integrals into canonical form*, *JHEP* **04** (2017) 006, [arXiv:1611.01087].

C. Meyer, *Algorithmic transformation of multi-loop master integrals to a canonical basis with CANONICA*, *Comput. Phys. Commun.* **222** (2018) 295–312, [arXiv:1705.06252].

O. Gituliar and V. Magerya, *Fuchsia: a tool for reducing differential equations for Feynman master integrals to epsilon form*, *Comput. Phys. Commun.* **219** (2017) 329–338, [arXiv:1701.04269].

M. Prausa, *epsilon: A tool to find a canonical basis of master integrals*, *Comput. Phys. Commun.* **219** (2017) 361–376, [arXiv:1701.00725].

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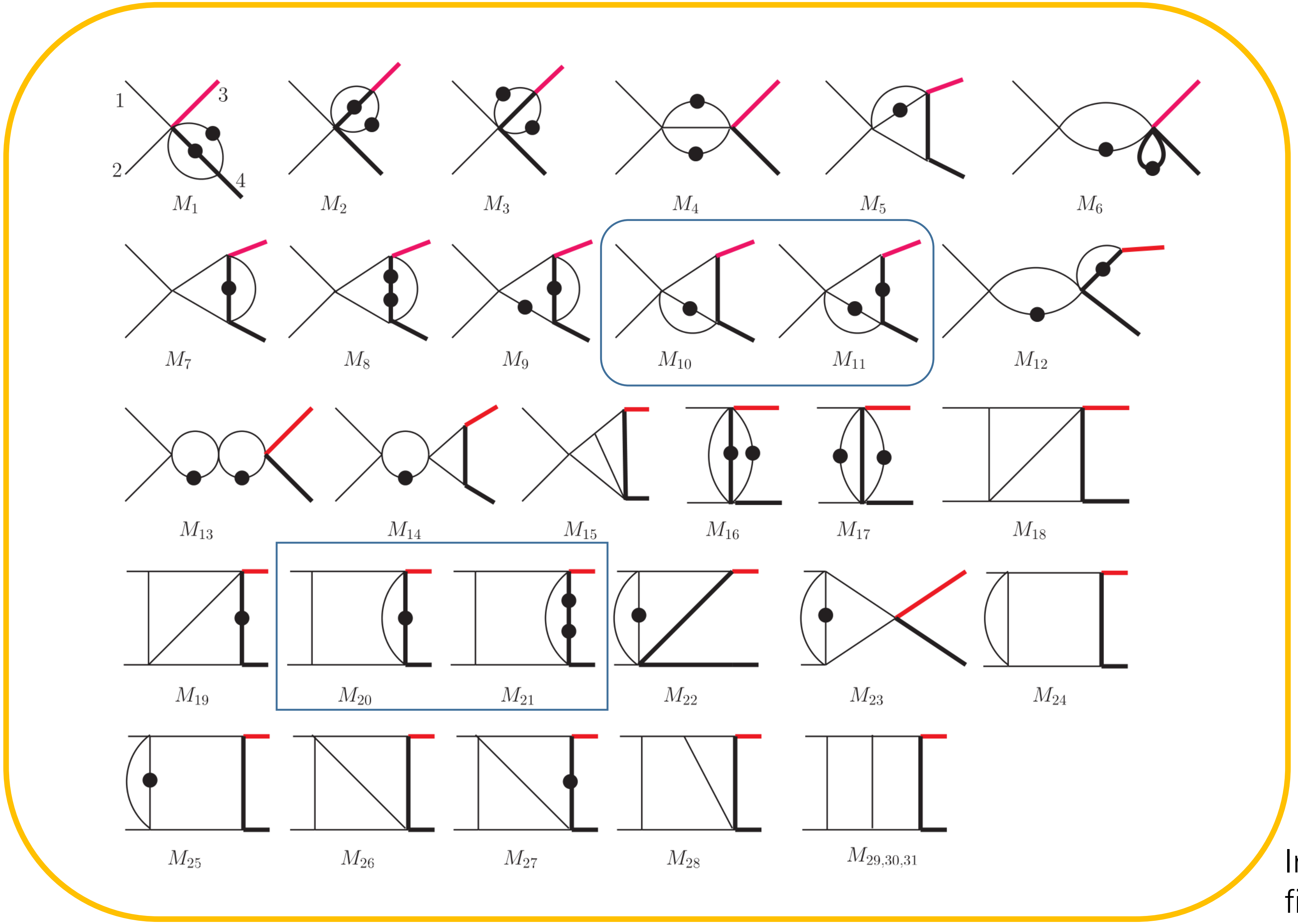
Choosing good integrals



Integrals for first family

$$
\begin{aligned}
&\mathrm{M}_1 = \epsilon^2\, I_{0,0,0,1,2,0,2,0,0}\,, && \mathrm{M}_2 = \epsilon^2\, I_{0,0,1,0,2,0,2,0,0}\,, && \mathrm{M}_3 = \epsilon^2\, I_{0,0,2,0,2,0,1,0,0}\,,\\
&\mathrm{M}_4 = \epsilon^2\, I_{0,0,1,0,2,2,0,0,0}\,, && \mathrm{M}_5 = \epsilon^3\, I_{0,0,1,0,2,1,1,0,0}\,, && \mathrm{M}_6 = \epsilon^2\, I_{0,0,1,2,0,0,2,0,0}\,,\\
&\mathrm{M}_7 = \epsilon^3\, I_{0,0,1,1,1,0,2,0,0}\,, && \mathrm{M}_8 = \epsilon^2\, I_{0,0,1,1,1,0,3,0,0}\,, && \mathrm{M}_9 = \epsilon^2\, I_{0,0,2,1,1,0,2,0,0}\,,\\
&\mathrm{M}_{10} = \epsilon^3\, I_{0,1,0,1,2,0,1,0,0}\,, && \mathrm{M}_{11} = \epsilon^2\, I_{0,1,0,1,2,0,2,0,0}\,, && \mathrm{M}_{12} = \epsilon^2\, I_{0,1,1,2,0,0,2,0,0}\,,\\
&\mathrm{M}_{13} = \epsilon^2\, I_{0,1,1,2,0,2,0,0,0}\,, && \mathrm{M}_{14} = \epsilon^3\, I_{0,1,1,2,0,1,1,0,0}\,, && \mathrm{M}_{15} = \epsilon^4\, I_{0,1,1,1,1,0,1,0,0}\,,\\
&\mathrm{M}_{16} = \epsilon^2\, I_{1,0,0,0,2,0,2,0,0}\,, && \mathrm{M}_{17} = \epsilon^2\, I_{2,0,0,0,2,0,1,0,0}\,, && \mathrm{M}_{18} = \epsilon^4\, I_{1,0,1,0,1,1,1,0,0}\,,\\
&\mathrm{M}_{19} = \epsilon^3\, I_{1,0,1,0,1,1,2,0,0}\,, && \mathrm{M}_{20} = \epsilon^3\, I_{1,0,1,1,1,0,2,0,0}\,, && \mathrm{M}_{21} = \epsilon^2\, I_{1,0,1,1,1,0,3,0,0}\,,\\
&\mathrm{M}_{22} = \epsilon^3\, I_{1,1,0,0,2,0,1,0,0}\,, && \mathrm{M}_{23} = \epsilon^3\, I_{1,1,0,0,2,1,0,0,0}\,, && \mathrm{M}_{24} = \epsilon^3(1-2\epsilon)\, I_{1,1,0,0,1,1,1,0,0}\,,\\
&\mathrm{M}_{25} = \epsilon^3\, I_{1,1,0,0,2,1,1,0,0}\,, && \mathrm{M}_{26} = \epsilon^4\, I_{1,1,0,1,1,0,1,0,0}\,, && \mathrm{M}_{27} = \epsilon^3\, I_{1,1,0,1,1,0,2,0,0}\,,\\
&\mathrm{M}_{28} = \epsilon^4\, I_{1,1,1,1,1,0,1,0,0}\,, && \mathrm{M}_{29} = \epsilon^4\, I_{1,1,1,1,1,1,1,0,0}\,, && \mathrm{M}_{30} = \epsilon^4\, I_{1,1,1,1,1,1,1,0,-1}\,,\\
&\mathrm{M}_{31} = \epsilon^4\, I_{1,1,1,1,1,1,1,-1,0}\,,\,.
\end{aligned}
$$

Basis for first family:

$$
\begin{aligned}
F_1 &= m_t^2 M_1 , \\
F_2 &= m_W^2 M_2 , \\
F_3 &= (m_W^2 - m_t^2) M_3 - 2 m_t^2 M_2 , \\
F_4 &= (-s) M_4 , \\
F_5 &= \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_5 , \\
F_6 &= (-s) M_6 , \\
F_7 &= \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_7 , \\
F_8 &= m_t^2 \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_8 , \\
F_9 &= m_W^2 s\, M_9 + m_t^2 (m_t^2 - m_W^2 - s) M_8 + \frac{3}{2}(m_t^2 - m_W^2 - s) M_7 , \\
F_{10} &= \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_{10} , \\
F_{11} &= m_t^2 (-s) M_{11} - \frac{3}{2}(m_t^2 - m_W^2 - s) M_{10} , \\
F_{12} &= m_W^2\, s\, M_{12} , \\
F_{13} &= s^2 M_{13} , \\
F_{14} &= s \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_{14} , \\
F_{15} &= \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_{15} , \\
F_{16} &= t\, M_{16} , \\
F_{17} &= (t - m_t^2) M_{17} - 2 m_t^2 M_{16} , \\
F_{18} &= (m_t^2 - m_W^2 - s) M_{18} ,
\end{aligned}
$$

$$
\begin{aligned}
F_{19} &= m_t^2 (-s) M_{19} , \\
F_{20} &= t\,(-s) M_{20} , \\
F_{21} &= m_t^2 (-s) \left((t - m_t^2) M_{21} - M_{20} \right) , \\
F_{22} &= (t - m_W^2) M_{22} , \\
F_{23} &= (-s) M_{23} , \\
F_{24} &= \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_{24} , \\
F_{25} &= (t - m_t^2)(-s) M_{25} , \\
F_{26} &= (m_t^2 - m_W^2 - s) M_{26} , \\
F_{27} &= -(m_W^2\, t - m_t^2 (s + t + m_W^2) + m_t^4) M_{27} , \\
F_{28} &= (t - m_W^2)(-s) M_{28} , \\
F_{29} &= -(t - m_t^2) s^2 M_{29} , \\
F_{30} &= (-s) \sqrt{(s-(m_W-m_t)^2)(s-(m_W+m_t)^2)}\, M_{30} , \\
F_{31} &= s^2 (M_{31} + M_{14}) + s\left(-M_{15} - M_{10} + 2M_7 - \frac{3}{2} M_5 + 3 m_t^2 M_8\right) \\
&+ (s + t - m_W^2)\left(s\, M_{25} - \frac{1}{4} M_{17}\right) - \frac{s + t - m_W^2}{4(t - m_t^2)} \left(2(m_t^2 + 2 m_W^2) M_2 - 3 s\, M_4\right. \\
&+ \left.(m_t^2 - m_W^2) M_3 - 2(2t + m_t^2) M_{16} + 12 (s + t - m_W^2) M_{18} + 8 m_t^2\, s\, M_{19}\right).
\end{aligned}
$$

Introduce new variables: $s = m_t^2 \frac{(x+z)(1+xz)}{x}, \; t = y\, m_t^2, \; m_W = z\, m_t.$

$$d\,\mathbf{F}(x,y,z;\epsilon) = \epsilon\,(d\,\tilde{A})\,\mathbf{F}(x,y,z;\epsilon),$$

$$d\,\tilde{A} = \sum_{i=1}^{15} R_i\, d\log(l_i),$$

(1)

For non-planar family (2)

$$d\,\mathbf{B}(x,y,z;\epsilon) = \epsilon\,(d\,\tilde{C})\,\mathbf{B}(x,y,z;\epsilon),$$

$$d\,\tilde{C} = \sum_{i=1}^{17} Q_i\,d\log(l_i),$$

(2)

Letters:

$$l_1 = x\,,$$
$$l_2 = x + 1\,,$$
$$l_3 = x - 1\,,$$
$$l_4 = x + z\,,$$
$$l_5 = x\,z + 1\,,$$
$$l_6 = x + y\,z,\,,$$
$$l_7 = x\,z + y\,,$$
$$l_8 = y\,,$$
$$l_9 = y - 1\,,$$
$$l_{10} = y - z^2\,,$$
$$l_{11} = z\,,$$
$$l_{12} = z^2 - 1\,,$$
$$l_{13} = x^2 z + xy + x + z\,,$$
$$l_{14} = x^2 z + x\left(y + z^2\right) + z\,,$$
$$l_{15} = x^2 z + x\left(-yz^2 + y + 2z^2\right) + z\,,$$
$$l_{16} = x^2 z + xy + z\,,$$
$$l_{17} = x^2 z^3 + xy\left(z^2 - 1\right) + 2xz^2 + z^3.$$

Goncharov Polylogarithms

A. B. Goncharov, *Multiple polylogarithms, cyclotomy and modular complexes*, Math. Res. Lett. **5**, (1998) 497–516, [arXiv:1105.2076].

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,,$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!}\ln^n x\,.$$

$$G_{a_1,\ldots,a_m}(x) G_{b_1,\ldots,b_n}(x) = \sum_{c\in a \amalg b} G_{c_1,c_2,\ldots,c_{m+n}}(x)\,.$$

Numerical evaluation of multiple polylogarithms

Jens Vollinga (Mainz U., Inst. Phys.), Stefan Weinzierl (Mainz U., Inst. Phys.) (Oct, 2004)

Published in: *Comput.Phys.Commun.* 167 (2005) 177 • e-Print: hep-ph/0410259 [hep-ph]

GINAC

handyG —Rapid numerical evaluation of generalised polylogarithms in Fortran

L. Naterop (Zurich U.), A. Signer (Zurich U. and PSI, Villigen), Y. Ulrich (Zurich U. and PSI, Villigen) (Sep 4, 2019)

Published in: *Comput.Phys.Commun.* 253 (2020) 107165 • e-Print: 1909.01656 [hep-ph]

Fortran

Boundary conditions:

1.Integrate directly
2.Regular conditions

$$F_1 = -\frac{1}{4} - \frac{\pi^2 \epsilon^2}{6} - 2\zeta(3)\epsilon^3 - \frac{8\pi^4\epsilon^4}{45} + O(\epsilon^5)$$

F_{24}=0 at x=1

$$\frac{dF_{25}}{dx} = -\frac{(2F_{16}+F_{17}-6(F_{22}-F_{23}-F_{24}+F_{25}))\epsilon}{4\left(x+\frac{z}{y}\right)} - \frac{(2F_{16}+F_{17}-6(F_{22}-F_{23}+F_{24}+F_{25}))\epsilon}{4\left(x+\frac{y}{z}\right)} - \frac{(2F_{16}+F_{17}+6(F_{22}-F_{23}-F_{24}+F_{25}))\epsilon}{4(x+z)} - \frac{(2F_{16}+F_{17}+6(F_{22}-F_{23}+F_{24}+F_{25}))\epsilon}{4\left(x+\frac{1}{z}\right)} + \frac{(2F_{16}+F_{17})\epsilon}{2x}$$

Numerical check:

family1-res	2021/3/16 13:59	Wolfram Mathema...	9,923 KB
family2-res	2021/3/29 11:18	Wolfram Mathema...	103,514 KB

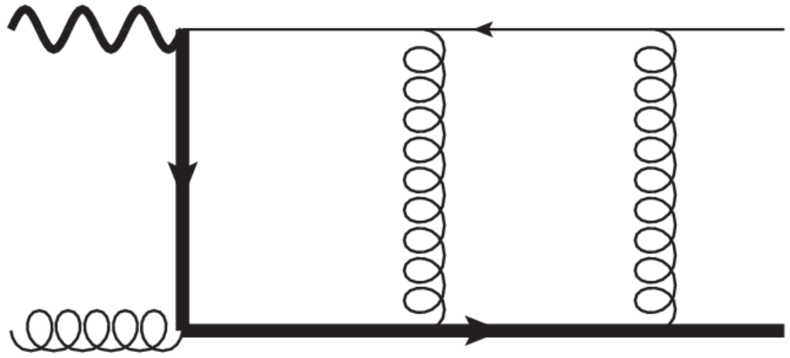
For $(s=-2, t=-1, m_W^2=-9, m_t=1)$, the analytic results gives

$$Analytic : I_{30} = \frac{0.45675312}{\epsilon^2} + \frac{1.0750424}{\epsilon} + 3.1672532,$$

and FIESTA packages gives

$$FIESTA : I_{30} = \frac{0.456753 \pm 0.000004}{\epsilon^2} + \frac{1.07504 \pm 0.000026}{\epsilon} + 3.16726 \pm 0.000148.$$

Integrals beyond mutilple polylogarithms



Applying Maximal Cuts to obtain homogeneous solutions:

$$\frac{2\,K\!\left(-\frac{16\,\mathrm{mt}\,\sqrt{-\mathrm{mt}^2}\,(\mathrm{mw}^2-t)\,\sqrt{-\mathrm{mt}^2\,(\mathrm{mt}^2+\mathrm{mw}^2-t-u)\,(\mathrm{mt}^2\,\mathrm{mw}^2-t\,u)}}{\mathrm{mt}^8-2\,(2\,\mathrm{mw}^2-t+u)\,\mathrm{mt}^6+(-8\,\mathrm{mw}^4+4\,(3\,t+u)\,\mathrm{mw}^2-3\,t^2+u^2)\,\mathrm{mt}^4-2\,t\,u\,(-2\,\mathrm{mw}^2+3\,t+u)\,\mathrm{mt}^2-8\,\sqrt{-\mathrm{mt}^2}\,(\mathrm{mw}^2-t)\,\sqrt{-\mathrm{mt}^2\,(\mathrm{mt}^2+\mathrm{mw}^2-t-u)\,(\mathrm{mt}^2\,\mathrm{mw}^2-t\,u)}\,\mathrm{mt}+t^2\,u^2}\right)}{\sqrt{\frac{\mathrm{mt}^8-2\,\mathrm{mt}^6\,(2\,\mathrm{mw}^2-t+u)+\mathrm{mt}^4\,(-8\,\mathrm{mw}^4+4\,\mathrm{mw}^2\,(3\,t+u)-3\,t^2+u^2)-2\,\mathrm{mt}^2\,t\,u\,(-2\,\mathrm{mw}^2+3\,t+u)-8\,\sqrt{-\mathrm{mt}^2}\,\mathrm{mt}\,(\mathrm{mw}^2-t)\,\sqrt{-\mathrm{mt}^2\,(\mathrm{mt}^2+\mathrm{mw}^2-t-u)\,(\mathrm{mt}^2\,\mathrm{mw}^2-t\,u)}+t^2\,u^2}{\mathrm{mt}^2\,(\mathrm{mw}^2-t)^2}}}$$

$$K(k)=\int_0^{\frac{\pi}{2}}\frac{\mathrm{d}\theta}{\sqrt{1-k^2\sin^2\theta}}$$

Outlook

- **Analytic calculation for several families could be achieved**
- **Full analytic calculation for all families is still very challenging**
- **Numerical way is promising** see Ma's talk

Thanks!