

# Dijet correlation and factorization violation at hadron colliders

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#### Standard Model Production Cross Section Measurements Sta

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# **Collinear factorization for inclusive observables**

For inclusive observables, sensitive only to a single high-energy scale Q, we have

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 \,\hat{\sigma}_{ab}(Q, x_1, x_2, \mu_f) \,f_a(x_1, \mu_f) \,f_b(x_2, \mu_f) + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

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partonic cross sections: perturbation theory parton distribution functions (PDFs): nonperturbative

power corrections nonperturbative

$$\sigma = \sum_{a,b} \int_0^1 dx_1 dx_2 C_{ab}(Q, x_1, x_2, \mu) \langle P(p_1) | O_a(x_1) | P(p_1) \rangle \langle P(p_2) | O_b(x_2) | P(p_2) \rangle + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

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Wilson coefficient: matching at  $\mu \approx Q$ perturbation theory

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low-energy matrix elements nonperturbative

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Wilson coefficient: matching at  $\mu \approx Q$ perturbation theory



low-energy matrix elements nonperturbative

power suppressed operators The matching coefficient  $C_{ab}$  is independent of external states and insensitive to physics below the matching scale  $\mu$ .

Can use quark and gluon states to perform the matching.

• Trivial matrix elements

 $\langle q_{a'}(x'p)|O_a(x)|q_{a'}(x'p)\rangle = \delta_{aa'}\,\delta(x'-x)$ 

• Wilson coefficients are partonic cross section

 $C_{ab}(Q, x_1, x_2) = \hat{\sigma}_{ab}(Q, x_1, x_2)$ 

Bare Wilson coefficients have divergencies.
 Renormalization induces dependence on μ.

Quite nontrivial that the low-energy matrix element factorizes into a product

$$\langle P(p_1)|O_a(x_1)|P(p_1)\rangle \langle P(p_2)|O_b(x_2)|P(p_2)\rangle$$

One should be worried about long-distance interactions mediated by soft gluons



All proton collisions include forward component (proton remnants)



Absence of factorization-violation due to Glauber gluons is important element of factorization proof for Drell-Yan process. Bodwin '85; Collins, Soper, Sterman '85 '88 ...

#### e.g. TMD factorization is violated in di-jet/di-hadron production

Collins, Qiu `07; Collins `07, Vogelsang, Yuan `07; Rogers, Mulders `10, Schwartz, Yan, Zhu ,'17,'18 ...



FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

We remark that, because the TMD factorization breaking effects are due to the Glauber region where all components of gluon momentum are small, the interactions responsible for breaking TMD factorization are associated with large distance scales.

Rogers, Mulders `10

### **Tools: Soft-Collinear Effective Theory**

- Technical challenges
  - Glauber gluons are offshell
  - Must be included as potential, not dynamical field in the effective Lagrangian
  - Glauber region is not well defined without additional rapidity regulator (on top of dim.reg.) (Rothsten & Stewart '20)





### **Tools: Soft-Collinear Effective Theory**

- Technical challenges
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- study QCD factorization without Glauber region
  - Assign scaling behavior to fields
  - Expand Lagrangian to leading power
  - Resummation with Renormalization Group



### Jet radius and $q_T$ joint resummation for boson-jet correlation

(Chien, DYS & Wu '19 JHEP)



#### **Construction of the theory formalism**

- Multiple scales in the problem
- Rely on effective field theory: SCET + Jet Effective Theory (Becher, Neubert, Rothen, DYS '16 PRL)

$$\frac{d\sigma}{d^2 q_T d^2 p_T d\eta_J dy_V} = \sum_{ijk} \int \frac{d^2 x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} \mathcal{S}_{ij \to Vk}(\vec{x}_T, \epsilon) \mathcal{B}_{i/N_1}(\xi_1, x_T, \epsilon) \mathcal{B}_{j/N_2}(\xi_2, x_T, \epsilon)$$
$$\times \mathcal{H}_{ij \to Vk}(\hat{s}, \hat{t}, m_V, \epsilon) \sum_{m=1}^{\infty} \langle \mathcal{J}_m^k(\{\underline{n}_J\}, R \, p_J, \epsilon) \otimes \mathcal{U}_m^k(\{\underline{n}_J\}, R \, \vec{x}_T, \epsilon) \rangle$$

(also see Sun, Yuan, Yuan '14; Buffing, Kang, Lee, Liu '18, ...)

# **Numerical results**



- NLL resummation is consistent with the LHC data ( $q_T \& \Delta \Phi$ )
- ΔΦ distribution for dijet production can be a clean probe of *factorization violation* (Collins & Qiu '07, Rogers & Mulders '10, .....)
- NLL result has 20-30% scale uncertainties. Higher-order resummation is necessary

### **Recoil-free azimuthal angle for boson-jet correlation**

(Chien, Rahn, Schrignder, DYS, Waalewijn & Wu '21 PLB)



Following the standard steps in SCET<sub>2</sub> we obtain the following factorization formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_{x,V}\,\mathrm{d}p_{T,J}\,\mathrm{d}y_V\,\mathrm{d}\eta_J} = \int \frac{\mathrm{d}b_x}{2\pi} \,e^{\mathrm{i}p_{x,V}b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) H_{ij\to Vk}(p_{T,V}, y_V - \eta_J) J_k(b_x)$$

**Fourier transformation in 1-D** 

# **Numerical results**



- first N<sup>2</sup>LL resummation including full jet algorithms
- good perturbative convergence
- Pythia agrees well
- Our work serves as a baseline for pinning down the factorization violation effects

![](_page_19_Picture_0.jpeg)

# Gap fraction at the LHC

![](_page_20_Figure_1.jpeg)

leading logs:

$$e^+e^-, ep: \quad \alpha_s^n \ln^n\left(\frac{Q}{Q_0}\right)$$

$$pp: \qquad \cdots \qquad + \alpha_s^3 (i\pi)^2 \ln^3 \left(\frac{Q}{Q_0}\right) \times \alpha_s^n \ln^{2n} \left(\frac{Q}{Q_0}\right)$$

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- Such events was originally suggested on the basis of color flow considerations in QCD Bjorken '93
- Global Logs resummation is first done by Oderda & Sterman '98
- Forshaw, Kyrieleis, Seymour '06 have analyzed the effect of Glauber phases in nonglobal observables directly in QCD
  - Non-zero contributions starting at 3 loops
  - Collinear logarithms starting at 4 loops: Super-leading logs

#### wide angle soft gluon emission developing a sensitivity to emission at small angles

# Gap fraction at the LHC

![](_page_21_Figure_1.jpeg)

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#### wide angle soft gluon emission developing a sensitivity to emission at small angles

# Fixed order calculation

- Gluons are added in all possible ways to trace diagrams and colour factors calculated using COLOUR
- Diagrams are then cut in all ways consistent with strong ordering
- At fourth order there are 10,529 diagrams and 1,746,272 after cutting.
- SLL terms are confirmed at fourth order and computed for the first time at 5<sup>th</sup> order

![](_page_22_Figure_5.jpeg)

Keates and Seymour arXiv:0902.0477 [hep-ph]

#### Simone Marzani's slide

### Factorization for gap between jets in e+e-

(Becher, Neubert, Rothen, DYS, '15 PRL, '16 JHEP; Caron-Huot '15 JHEP)

Hard function *m* hard partons along fixed directions {n<sub>1</sub>, ..., n<sub>m</sub>}  $\mathcal{H}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$ 

Soft function squared amplitude with *m* Wilson lines

$$\sigma(Q, Q_{\Omega}) \sim \sum_{m=2}^{\infty} \prod_{i=1}^{m} \int \frac{d\Omega(\vec{n}_i)}{4\pi} \operatorname{Tr}_{c} \left[ \mathcal{H}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q, \mu) \mathcal{S}_m(\{\vec{n}_1, \cdots, \vec{n}_m\}, Q_{\Omega}, \mu) \right]$$

## Factorization for gap between jets in e+e-

(Becher, Neubert, Rothen, DYS, '15 PRL, '16 JHEP; Caron-Huot '15 JHEP)

![](_page_24_Figure_2.jpeg)

**One-loop anomalous dimension:**  $\Gamma^{(1)} = \begin{pmatrix} V_2 \ R_2 \ 0 \ 0 \ \dots \\ 0 \ V_3 \ R_3 \ 0 \ \dots \\ 0 \ 0 \ V_4 \ R_4 \ \dots \\ 0 \ 0 \ 0 \ V_5 \ \dots \\ \vdots \ \vdots \ \vdots \ \vdots \ \vdots \ \ddots \end{pmatrix}$ 

$$\begin{split} \boldsymbol{V}_{m} &= -2\sum_{(ij)} \int \frac{d\Omega(n_{k})}{4\pi} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) W_{ij}^{k} \left[ \Theta_{\mathrm{in}}^{n\bar{n}}(k) + \Theta_{\mathrm{out}}^{n\bar{n}}(k) \right] \\ &+ 2i\pi \sum_{(ij)} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij}, \\ \boldsymbol{R}_{m} &= 4\sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} W_{ij}^{m+1} \Theta_{\mathrm{in}}(n_{m+1}) \,. \end{split} \qquad \begin{aligned} \Pi_{ij} &= 1 \quad \text{if both incomised} \\ \text{or outgoing} \end{split}$$

![](_page_25_Picture_2.jpeg)

**One-loop anomalous dimension:** 

$$\begin{split} \boldsymbol{V}_{m} &= -2\sum_{(ij)} \int \frac{d\Omega(n_{k})}{4\pi} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) W_{ij}^{k} \left[ \Theta_{\mathrm{in}}^{n\bar{n}}(k) + \Theta_{\mathrm{out}}^{n\bar{n}}(k) \right] \\ &+ 2i\pi \sum_{(ij)} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij}, \\ \boldsymbol{R}_{m} &= 4\sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} W_{ij}^{m+1} \Theta_{\mathrm{in}}(n_{m+1}) \,. \end{split} \qquad \begin{split} \Pi_{ij} &= 1 \quad \text{if both incoming} \\ &\text{or outgoing} \end{split}$$

![](_page_26_Picture_2.jpeg)

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Imaginary part of the anomalous dimension:

For e+e-:

For pp:

$$\sum_{i \neq j} \mathbf{T}_i \cdot \mathbf{T}_j = -\sum_i \mathbf{T}_i^2 = -\sum_i C_i \qquad \sum_{(ij)} T_i \cdot T_j \prod_{ij} = 2 T_1 \cdot T_2 + \sum_{i=3} T_i \cdot (-T_1 - T_2 - T_i)$$
$$= 2 T_1 \cdot T_2 + (T_1 + T_2) \cdot (T_1 + T_2) - \sum_{i=3}^m C_i^2$$
$$= 4 T_1 \cdot T_2 + C_1^2 + C_2^2 - \sum_{i=3}^m C_i^2$$
Non trivial

#### **Extracting the collinear singularities**

$$\begin{split} \boldsymbol{V}_{m} &= 2\sum_{(ij)} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \left\{ -\ln \frac{\mu^{2}}{Q^{2}} + \int \frac{d\Omega(n_{k})}{4\pi} \overline{W}_{ij}^{k} \right\} \\ &- 2i\pi \sum_{(ij)} \left( \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R} \right) \Pi_{ij}, \\ \boldsymbol{R}_{m} &= 4\sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,R} \left\{ \frac{1}{2} \left[ \delta(n_{k} - n_{i}) + \delta(n_{k} - n_{j}) \right] \ln \frac{\mu^{2}}{Q^{2}} - \overline{W}_{ij}^{m+1} \Theta_{\mathrm{in}}(n_{m+1}) \right\} \end{split}$$

The one-loop anomalous dimension is

$$\boldsymbol{V}_{m} = \overline{\boldsymbol{V}}_{m} + \boldsymbol{V}^{I} + \sum_{i=1}^{m} \boldsymbol{V}_{i}^{C} \ln \frac{\mu^{2}}{Q^{2}},$$
$$\boldsymbol{\mathcal{H}}_{m} \boldsymbol{\mathcal{R}}_{C} = \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \vdots \underbrace{1}_{2} + \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \underbrace{1}_{2} \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \underbrace{1}_{2} \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \underbrace{1}_{2} \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \underbrace{1}_{2} \underbrace{1}_{2} \underbrace{1}_{2} \underbrace{\mathcal{M}}_{i}^{C} \underbrace{1}_{2} \underbrace{1}$$

### Super-leading logs from RG evolution

(Becher, Neubert, DYS, 2105.XXXX)

Consider the processes  $q_1 q_2' 
ightarrow q_3 q_4'$ 

**LO hard function:**  $\mathcal{H}_4 = t^a_{\alpha_3\alpha_1} t^a_{\alpha_4\alpha_2} t^b_{\beta_1\beta_3} t^b_{\beta_2\beta_4} \sigma_0$ 

Expand the expansion kernel (Becher, Neubert, Rothen, DYS '16 PRL)

$$\mathcal{H}_{4} U(\mu_{s}, \mu_{h}) = \mathcal{H}_{4} \mathbf{P} \exp\left[\int_{\mu_{s}}^{\mu_{h}} \frac{d\mu}{\mu} \mathbf{\Gamma}^{H}(Q, \mu)\right] \qquad \mathbf{\mathcal{B}}$$
$$= \mathcal{H}_{4} + \int_{\mu_{s}}^{\mu_{h}} \frac{d\mu}{\mu} \mathcal{H}_{4} \mathbf{\Gamma}^{H}(Q, \mu) + \int_{\mu_{s}}^{\mu_{h}} \frac{d\mu}{\mu} \int_{\mu}^{\mu_{h}} \frac{d\mu'}{\mu'} \mathcal{H}_{4} \mathbf{\Gamma}^{H}(Q, \mu') \mathbf{\Gamma}^{H}(Q, \mu)$$

The first non-zero contribution of has factor arises at 3-loop order

$$S^{(3)} = \left\langle \mathcal{H}_4 \mathbf{V}^I \mathbf{V}^I (\overline{\mathbf{V}}_4 + \overline{\mathbf{R}}_4) \right\rangle \left(\frac{\alpha_s}{4\pi}\right)^3 \frac{1}{3!} \ln^3 \left(\frac{Q}{\mu}\right) = -\left(\frac{\alpha_s}{4\pi}\right)^3 \frac{16 C_F}{3} \pi^2 L_Q^3 J_1 \sigma_0$$
$$J_1 = 2\Delta Y \operatorname{sign}(\eta_J)$$

Super-leading logs at 4-loop order:

$$S_0^{(4)} = \left\langle \mathcal{H}_4 \mathbf{V}^I \left[ \sum_{i=1}^4 \mathbf{V}_i^L \mathbf{V}^I (\overline{\mathbf{V}}_4 + \overline{\mathbf{R}}_4) + \sum_{i=1}^4 \mathbf{R}_i^L \mathbf{V}^I (\overline{\mathbf{V}}_5 + \overline{\mathbf{R}}_5) \right] \right\rangle \left( \frac{\alpha_s}{4\pi} \right)^4 \frac{(-2)}{5!} \ln^5 \left( \frac{Q}{\mu} \right)$$

$$\omega \sim \alpha_s L^2$$

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3 \frac{Q}{\mu_s} \frac{1}{N_c} \left[ N_c^2 \left(4f_1(w) - 2f_\delta(w)\right) - 4f_2(\omega) + 2f_\delta(w) \right) \Delta Y \sigma_0$$

Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs 
$$\longrightarrow e^{-\omega}$$
  
Superleading logs  $\xrightarrow{\omega \to \infty} \frac{1}{\omega}$ 

![](_page_29_Figure_5.jpeg)

#### **Numerical results**

#### hypergeometric function

$$f_{\delta}(w) = \frac{1}{3} {}_{2}F_{2}\left(1, 1; 2, \frac{5}{2}; -w\right)$$

 $\omega \sim \alpha_s L^2$ 

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3 \frac{Q}{\mu_s} \frac{1}{N_c} \left[N_c^2 \left(4f_1(w) - 2f_\delta(w)\right) - 4f_2(\omega) + 2f_\delta(w)\right) \Delta Y \sigma_0$$

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Global logs 
$$\longrightarrow e^{-\omega}$$
  
Superleading logs  $\xrightarrow{\omega \to \infty} \frac{1}{\omega}$ 

![](_page_30_Figure_7.jpeg)

 $\begin{aligned} \text{hypergeometric function} & \text{error function} \\ f_{\delta}(w) = \frac{1}{3} {}_{2}F_{2}\left(1, 1; 2, \frac{5}{2}; -w\right) & f_{2}(w) = \frac{1}{w} - \frac{\sqrt{\pi}}{2w^{3/2}} \operatorname{erf}(\sqrt{w}) \\ \omega \sim \alpha_{s}L^{2} \\ \\ S_{O} = \left(\frac{\alpha_{s}}{\pi}\right)^{3} \pi^{2} \ln^{3} \frac{Q}{\mu_{s}} \frac{1}{N_{c}} \left[N_{c}^{2} \left(4f_{1}(w) - 2f_{\delta}(w)\right) - 4f_{2}(\omega) + 2f_{\delta}(w)\right) \Delta Y \sigma_{0} \end{aligned}$ 

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#### 1.0 0.8 0.6 $R(Q_0)$ $R_{\rm NGL}(Q_0)$ 0.4 $R_{\text{NGL}\times\text{SLL}}(Q_0)$ $R_{\text{NGL+SLL}}(Q_0)$ 0.2 0.0 20 40 60 80 100 $Q_0$

#### **Numerical results**

![](_page_32_Figure_1.jpeg)

Sudakov suppression of the superleading logarithms is weaker than the one present for global observables

Global logs 
$$\longrightarrow e^{-\omega}$$
  
Superleading logs  $\xrightarrow{\omega \to \infty} \frac{1}{\omega}$ 

#### Numerical results

![](_page_32_Figure_5.jpeg)

# Summary

- Factorization is at the heart of any quantitative prediction using pQFT at hadron colliders
- We investigate naive factorization violation effects using QCD jets production
  - Azimuthal decorrelation: first NNLL results with jet algorithms
  - Gap fraction: all-order results of superseding logs
- Understand low energy theory from Glauber gluons ?
- How to include Non-Perturbative corrections? Lattice QCD input?
- Forward jet, small-x, BFKL, BK ...

![](_page_33_Figure_8.jpeg)

![](_page_34_Picture_0.jpeg)

# **Recoil and the jet axis**

![](_page_35_Figure_1.jpeg)

Jet axis is along jet momentum: recoiled by soft radiation in jet

- TH challenge: Non-global log resummation; non-linear evolution
- EX challenge: Contamination

**Recoil absent for the p<sub>T</sub>-weighted recombination** (Ellis, Soper '93)

$$p_{t,r} = p_{t,i} + p_{t,j},$$
  
 $\phi_r = (w_i \phi_i + w_j \phi_j) / (w_i + w_j)$   
 $y_r = (w_i y_i + w_j y_j) / (w_i + w_j)$   
 $w_i = p_t^n$ 

 $n \rightarrow \infty$  (Winner-take-all scheme) (Bertolini, Chan, Thaler '13)