Double logarithmic corrections at subleading power

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微扰量子场论研讨会

Beyond Sudakov approximation

- Next-to-eikonal soft gluon radiation

  D.Bonocore, E.Laenen, L.Magnea, L.Vernazza, C.D.White JHEP 1612, 121(2016)

  ...

- Jets and jettiness

  R.Boughezal, X.Liu and F.Petriello JHEP 1703, 160(2017)

  ...

- Power corrections in SCET

  I.Moult, I.W.Stewart, G.Vita and H.X.Zhu, JHEP 1707(2017), 067

  M.Beneke, M.Garny, R.Szafron, J.Wang JHEP 1803(2018) 001

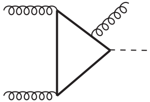
  Z.Liu, B.Mecaj, M.Neubert and X.Wang JHEP 2101(2012) 077

  ...

- many other studies ...

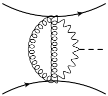
Our work

- T. Liu, A. Penin and N. Zerf, Phys. Lett. B **771** (2017), 492-496
- T. Liu and A. Penin, Phys. Rev. Lett. **119** (2017) no.26, 262001
- T. Liu and A. Penin, JHEP **11** (2018), 158



Still Working

- T. Liu, K. Melnikov and A. Penin, Phys. Rev. Lett. **123** (2019) no.12, 122002



Recently we start to look at the amplitudes at $\mathcal{O}(m_q^3)$. Results unpublished.

Outline

- Introduction

  Dirac and Pauli form factor

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- $\mathcal{O}(m_q^{1,2})$ double logarithms

  1. Physical origin
  2. Infrared factorization and Resummation
  3. All order results

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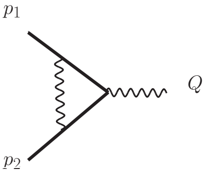
- Conclusion

Form factor



$$\mathcal{F} = \bar{q}(p_2)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_q}F_2\right)q(p_1)$$

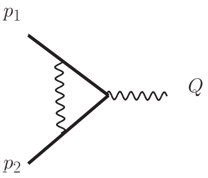
Three-loop massless [Baikov,Chetyrkin,Smirnov^2,Steinhauser 2009]
[Gehrmann,Glover,Huber,Ikizlerli, Studerus 2010]

Four-loop massless [Henn,Lee,Smirnov^2,Steinhauser 2016; Manteuffel,Schabinger 2016
Lee,Smirnov^2,Steinhauser 2019; Manteuffel,Schabinger 2019]

Two-loop massive [Mastrolia,Remiddi 2003; Bernreuther et al 2005]

Three-loop massive [Henn,Smirnov^2,Steinhauser 2016; Lee,Smirnov^2,Steinhauser 2018;
Blumlein,Marquard,Rana,Schneider 2019]

Dirac form factor



In high energy limit $p_1^2 = p_2^2 = m_q^2 \ll Q^2$

$$\rho = m_q^2/Q^2$$

$$F_1 = \exp\left[-\frac{\alpha_s}{2\pi}\frac{C_F \ln\rho(1+\mathcal{O}(\rho^2))}{\varepsilon}\right]\sum_{n=0}^{\infty}\rho^n F_1^{(n)}$$

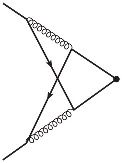
Leading power:

Sudakov logs [Sudakov 1956; Frenkel, Taylor 1976]
Subleading logs also exponentiate [Muller 1979; Collins 1980; Sen 1981;]

$\mathcal{O}(\rho)$ corrections:

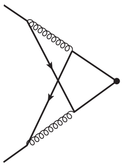
Expansion by regions [Beneke, Smirnov 1998]
Double logs come from soft quark exchange [Penin 2014; Liu, Penin 2017]

Two loop



- Sudakov parameterization: $l_i = u_i p_1 + v_i p_2 + l_{i\perp}$
- eikonal glue: $\frac{1}{(p_1+l_i)^2} \approx \frac{1}{Q^2 v_i}$
- soft quark: $\frac{m_q}{(l_i^2 - m_q^2)} \approx -i\pi m_q \delta(Q^2 u_i v_i + l_{i\perp}^2 - m_q^2)$
- $\eta_i = \ln v_i / \ln \rho$, $\xi_i = \ln u_i / \ln \rho$

Two loop



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- $\eta_i = \ln v_i / \ln \rho$, $\xi_i = \ln u_i / \ln \rho$

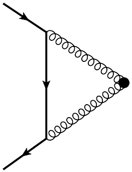
Result:

$$x = \frac{\alpha_s}{4\pi} \ln^2 \rho$$

$$K = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2)$$

$$F_1^{(1,2l)} = 2\,(C_A - 2C_F)\,x^2 \times \int K(\eta_1,\eta_2,\xi_1,\xi_2)\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2$$

Toy model



- quark scattering by $(G^a_{\mu\nu})^2$ operator
- mass suppression from helicity flip
- soft quark propagator: m_q^2/l^2
- eikonal gluon propagators: $g_{\mu\nu}/(2p_i l)$

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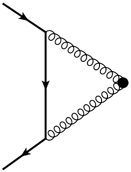

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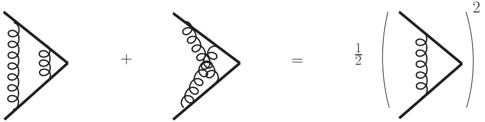
Eikonal(Sudakov) factorization:

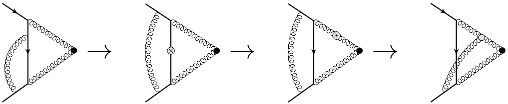
Eikonal identity: $\frac{1}{p_i l_1}\frac{1}{p_i(l_1+l_2)} + \frac{1}{p_i(l_1+l_2)}\frac{1}{p_i l_2} = \frac{1}{p_i l_1}\frac{1}{p_i l_2}$



$$\Rightarrow \quad \texttt{n-loop} = \frac{(\texttt{1-loop})^n}{n!}$$

QED Resummation

- Ward identity:



- crossed vertex: $S(l) \to S(l) - S(l + l_g^+)$
- 3-photon vertex: $2e_q p_1^\mu$

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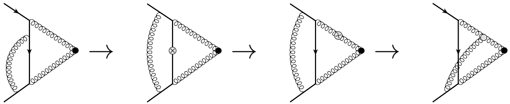

- crossed vertex: $S(l) \to S(l) - S(l + l_g^+)$
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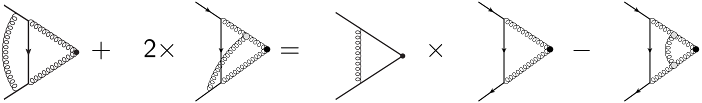
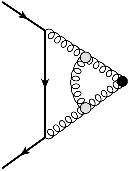
- Sudakov factorization



$\Uparrow$

- Non-Sudakov logs: multiple soft photons exponentiate

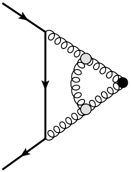
QCD Resummation



$e_q^2 \to C_F g_s^2$ and glue self-coupling bring C_A

color weight $C_A - C_F$

QCD Resummation



$e_q^2 \to C_F g_s^2$ and glue self-coupling bring C_A

color weight $C_A - C_F$

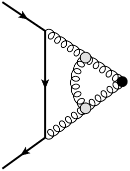
Factorized formula: $\mathcal{G} = Z_q^2 g(-z)\mathcal{G}^{(0)}$

quark Sudakov factor: $Z_q^2 = \exp\left[-C_F\left(\frac{\alpha_s}{2\pi}\frac{\ln\rho}{\varepsilon} + x\right)\right]$

non-Sudakov double-logarithmic function: $g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta e^{2z\eta\xi}$

Here $x = \frac{\alpha_s}{4\pi}\ln^2\rho$, $\quad z = (C_A - C_F)x$, $\quad g(0) = 1$

QCD Resummation



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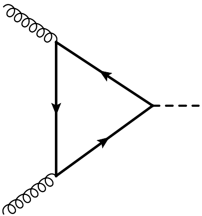
[Anastasiou et al, 2020]

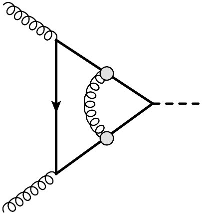


Two-loop analytical result.

[Anastasiou et al. 2007]

Three-loop semi-analytical result.

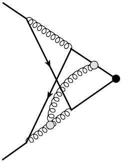
[Czakon et al. 2020]



$$Z_g^2 = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\right]$$

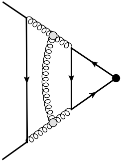
$$\mathcal{M}^b_{gg\to H} = -Z_g^2 g(z)\left(\frac{3}{2}\ln^2\rho\,\rho\right)\mathcal{M}^{(0)}_{gg\to H}$$

Vector and Scalar form factors

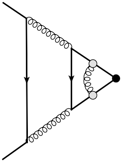


$$F_V = Z_q^2 \sum_n \rho^n F_V^{(n)} \qquad F_V^{(1)} = \frac{C_F(C_A - 2C_F)}{6} x^2 f(-z)\,,$$

$$f(z) = 12 \int_0^1 \mathrm{d}\eta_1 \int_{\eta_1}^1 \mathrm{d}\eta_2 \int_0^{1-\eta_2} \mathrm{d}\xi_2 \int_{\xi_2}^{1-\eta_1} \mathrm{d}\xi_1 \, e^{2z\eta_1(\xi_1-\xi_2)} \times e^{2z\xi_2(\eta_2-\eta_1)}$$



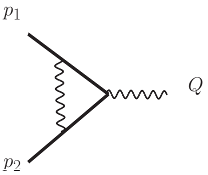
$$F_S = Z_q^2 \sum_n \rho^n F_S^{(n)} \qquad F_S^{(1)} = -\frac{C_F T_F}{3} x^2 f_S(-z)\,,$$



$$f_S(z) = 24 \int_0^1 \mathrm{d}\eta_1 \int_0^{1-\eta_1} \mathrm{d}\xi_1 \int_{\eta_1}^{1-\xi_1} \mathrm{d}\eta_2 \int_{\xi_1}^{1-\eta_2} \mathrm{d}\xi_2 \, e^{2z\eta_2\xi_2} e^{-2z\eta_1\xi_1}$$

Note that $f_S(z) \equiv f(z)$.

Pauli form factor



In high energy limit $p_1^2 = p_2^2 = m_q^2 \ll Q^2, \rho = m_q^2/Q^2$

$$\mathcal{F} = \bar{q}(p_2)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}Q^\nu}{2m_q}F_2\right)q(p_1)$$

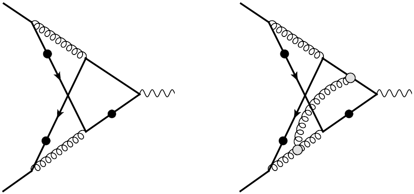
$$F_2 = \exp\left[-\frac{\alpha_s}{2\pi}\frac{C_F \ln\rho(1+\mathcal{O}(\rho^2))}{\varepsilon}\right]\sum_{n=0}^{\infty}\rho^n F_2^{(n)}$$

- Chirality flip: $F_2^{(0)} = 0$.
- $\left[F_2^{(1)}\right]_{1-loop} \propto \int \frac{\mathrm{d}^4 l}{\pi^2}\frac{(p_1 l)+(p_2 l)}{l^2\left((p_1-l)^2-m_q^2\right)\left((p_2-l)^2-m_q^2\right)}$
- $\left[F_2^{(1)}\right]_{1-loop} \propto -\frac{C_F\alpha_s}{\pi}\ln\rho + \mathcal{O}(\rho\ln\rho)$

The exchange of massless soft gauge boson doesn't produce double logarithms in the first order.

Pauli form factor

$\mathcal{O}(m_q^3)$ amplitude $F_2^{(2)}$ could get DLs at two loops, which works almost the same way as Dirac form factor.



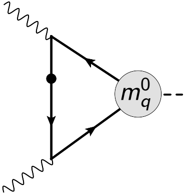
$$F_2^{(2)} = \frac{2C_F(C_A-2C_F)}{3}x^2 f(-z)$$

$H\gamma\gamma$ amplitude

$$\mathcal{M}^{q}_{\gamma\gamma H} = T_F \frac{e_q^2}{4\pi^2}\frac{y_q m_q}{m_H^2}\left[p_1^{\mu} p_2^{\nu} - g^{\mu\nu}(p_1 p_2)\right] A_{\nu}(p_1) A_{\mu}(p_2) H M^{q}_{\gamma\gamma H}$$

$$m_q \ll m_H,\ \rho = m_q^2/m_H^2.$$

$$M^{q}_{\gamma\gamma H} = \sum_{n=0}^{\infty} \rho^n M^{(n)}_{\gamma\gamma H}$$



$$\left[M^{(0)}_{\gamma\gamma H}\right]_{1-loop} = \ln^2\rho$$

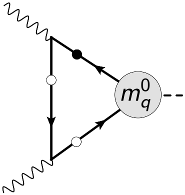
$H\gamma\gamma$ amplitude

massive soft quark exchange

$$\int \frac{\mathrm{d}^4 l}{\pi^2} \frac{l^2 \quad - \quad 4(lp_1)(lp_2)/(p_1 p_2)}{(l^2 - m_q^2)\big((p_1 - l)^2 - m_q^2\big)\big((p_2 - l)^2 - m_q^2\big)}$$

$$\frac{1}{(p_i - l)^2 - m_q^2} = -\frac{1}{2(p_i l)} \left(1 + \frac{l^2 - m_q^2}{2(p_i l)} + \dots\right)$$

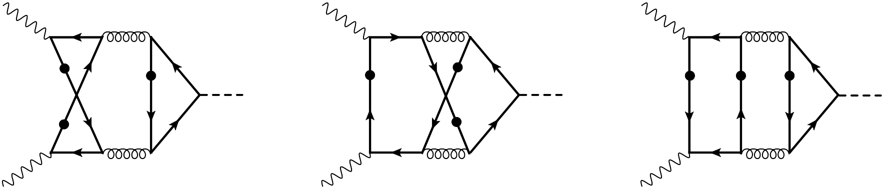
$\Longrightarrow \mathcal{O}(m_q^2)$ double logarithms.

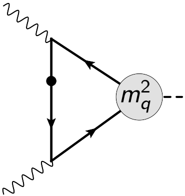


More tricks in higher loop, but DLs could be resummed.

$H\gamma\gamma$ amplitude

Additional m_q^2 terms provided by two soft quarks start to appear at three-loop level. After taking fermion spinor trace only the last one left.





Effectively off-shell scalar form factor obtained before.

$H\gamma\gamma$ amplitude

$$
\begin{aligned}
\mathcal{C}^{(2,0)} + \mathcal{C}^{(2,1)} = \Big(&54.56087661 - 105.9626626\, L_s + 8.887259013\, L_s^2 + 6.645715659\, L_s^3 \\
&+ 0.8289545430\, L_s^4 + 0.03333333333\, L_s^5 - \boxed{0.001851851852\, L_s^6}\Big)\, z^{-1} \\
&+ \Big(85.66611966 + 12.14331252\, L_s - 34.87431988\, L_s^2 - 11.29224411\, L_s^3 \\
&- 0.8617012615\, L_s^4 + 0.01718750000\, L_s^5 + \boxed{0.001099537037\, L_s^6}\Big)\, z^{-2}
\end{aligned}
$$

[Niggetiedt, 2020]

$H\gamma\gamma$ amplitude

$$\mathcal{C}^{(2,0)} + \mathcal{C}^{(2,1)} = \Big(54.56087661 - 105.9626626\, L_s + 8.887259013\, L_s^2 + 6.645715659\, L_s^3 + 0.8289545430\, L_s^4 + 0.03333333333\, L_s^5 - \boxed{0.001851851852\, L_s^6}\Big)\, z^{-1} + \Big(85.66611966 + 12.14331252\, L_s - 34.87431988\, L_s^2 - 11.29224411\, L_s^3 - 0.8617012615\, L_s^4 + 0.01718750000\, L_s^5 + \boxed{0.001099537037\, L_s^6}\Big)\, z^{-2}$$

[Niggetiedt, 2020]

- Another new source of DLs at three loops, which we haven't encountered before.

$H\gamma\gamma$ amplitude

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[Niggetiedt, 2020]

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- Three-loop coefficients agrees at $\mathcal{O}(m_q^1)$ and $\mathcal{O}(m_q^3)$.

$H\gamma\gamma$ amplitude

$$\begin{aligned}\mathcal{C}^{(2,0)} + \mathcal{C}^{(2,1)} = \Big(&54.56087661 - 105.9626626\, L_s + 8.887259013\, L_s^2 + 6.645715659\, L_s^3 \\ &+ 0.8289545430\, L_s^4 + 0.03333333333\, L_s^5 - \boxed{0.001851851852\, L_s^6}\Big)\, z^{-1} \\ &+ \Big(85.66611966 + 12.14331252\, L_s - 34.87431988\, L_s^2 - 11.29224411\, L_s^3 \\ &- 0.8617012615\, L_s^4 + 0.01718750000\, L_s^5 + \boxed{0.001099537037\, L_s^6}\Big)\, z^{-2}\end{aligned}$$

[Niggetiedt, 2020]

- Another new source of DLs at three loops, which we haven't encountered before.
- Three-loop coefficients agrees at $\mathcal{O}(m_q^1)$ and $\mathcal{O}(m_q^3)$.
- Currently only the Abelian part(relevant C_F term) are resummed.

Conclusion

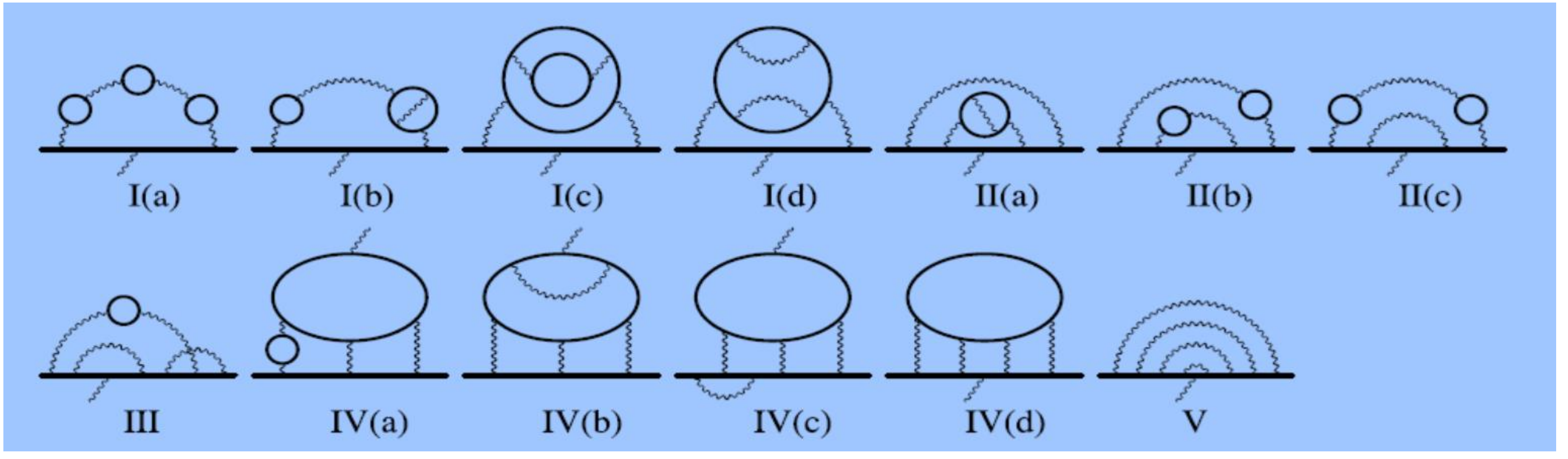
- $\mathcal{O}(m_q^1)$ DLs confirmed by explicit calculations.
- Some progress on understanding the DL structure of $\mathcal{O}(m_q^3)$ amplitudes.
  - Pauli form factor seems good.
  - $H\gamma\gamma$ at three-loop level agrees with literature.
  - Still no clear physical picture of all order results for some non-abelian parts.

Thanks for your attention!

QED修正

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

Order	with $\alpha(\text{Cs})$	with $\alpha(a_e)$
2	116 140 973.321(23)	116 140 973.233(28)
4	413 217.6258(70)	413 217.6252(70)
6	30 141.90233(33)	30 141.90226(33)
8	381.004(17)	381.004(17)
10	5.0783(59)	5.0783(59)
$a_\mu(\text{QED})$	116 584 718.931(30)	116 584 718.842(34)



QED部分的独立检验

Anomalous magnetic moment with heavy virtual leptons

Alexander Kurz (KIT, Karlsruhe, TTP and DESY, Zeuthen), Tao Liu (KIT, Karlsruhe, TP), Peter Marquard (DESY, Zeuthen), Matthias Steinhauser (KIT, Karlsruhe, TTP) (Nov 11, 2013)

Published in: *Nucl.Phys.B* 879 (2014) 1-18 • e-Print: 1311.2471 [hep-ph]

Light-by-light-type corrections to the muon anomalous magnetic moment at four-loop order

Alexander Kurz (DESY, Zeuthen and KIT, Karlsruhe, TTP), Tao Liu (KIT, Karlsruhe, TTP), Peter Marquard (DESY, Zeuthen), Alexander V. Smirnov (Moscow, ITEP), Vladimir A. Smirnov (SINP, Moscow) et al. (Aug 4, 2015)

Published in: *Phys.Rev.D* 92 (2015) 7, 073019 • e-Print: 1508.00901 [hep-ph]

Electron contribution to the muon anomalous magnetic moment at four loops

Alexander Kurz (DESY, Zeuthen and KIT, Karlsruhe, TTP), Tao Liu (Alberta U.), Peter Marquard (DESY, Zeuthen), Alexander Smirnov (Moscow State U.), Vladimir Smirnov (SINP, Moscow and Humboldt U., Berlin and Humboldt U., Berlin, Inst. Math.) et al. (Feb 8, 2016)

Published in: *Phys.Rev.D* 93 (2016) 5, 053017 • e-Print: 1602.02785 [hep-ph]

High-precision calculation of the 4-loop contribution to the electron g-2 in QED

Stefano Laporta (INFN, Bologna and Bologna U.) (Apr 23, 2017)

Published in: *Phys.Lett.B* 772 (2017) 232-238 • e-Print: 1704.06996 [hep-ph]

QED检验结果

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

	universal	e^-	τ	$e^- + \tau$
Steinhauser:	$-5.44(35)$	$+386.77(1.40)$	$+0.12371(15)$	$+0.182592(29)$
Kinoshita:	$-5.56894(245)$	$+386.264(17)$	$+0.12326(35)$	$+0.18259(12)$
Laporta:	$-5.56679893738506\ldots$			

1. 四圈图贡献每部分至少有**两个**组用**不同**的方法计算过
2. 最大的理论误差比费米实验室预期的实验误差小**一个量级**

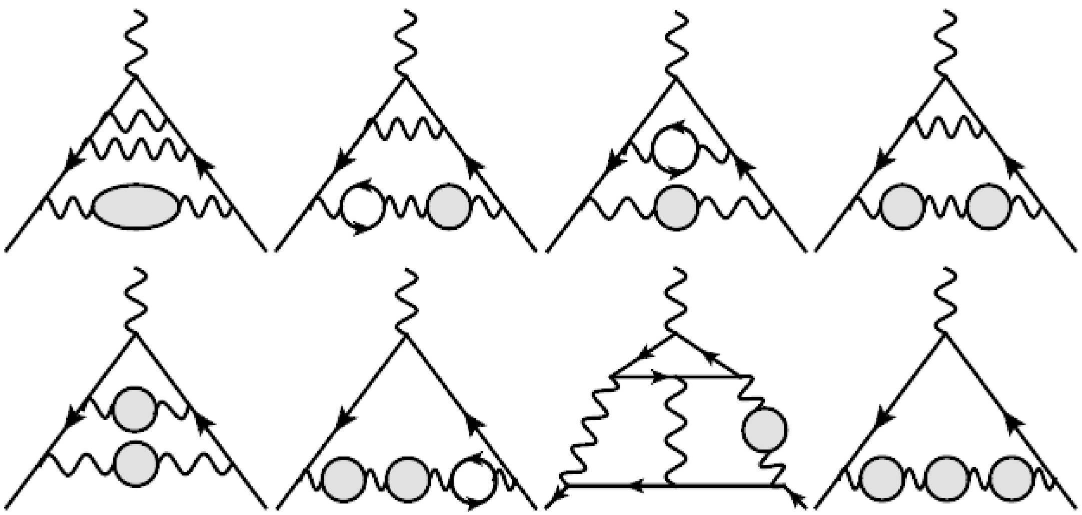
NNLO HVP

$$a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11} \quad (0.46 \text{ ppm})$$

Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order

Alexander Kurz (DESY, Zeuthen and KIT, Karlsruhe, TTP), Tao Liu (KIT, Karlsruhe, TTP), Peter Marquard (DESY, Zeuthen), Matthias Steinhauser (KIT, Karlsruhe, TTP) (Mar 25, 2014)

Published in: *Phys.Lett.B* 734 (2014) 144-147 • e-Print: 1403.6400 [hep-ph]



NNLO贡献：

$12.4 \pm 0.1 \times 10^{-11}$

与将来预期的实验误差在同一个量级上