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Three-loop color-kinematics duality and Higgs amplitudes (Part 1)

based on work with Gang Yang and Guanda Lin, 2105.xxxxx

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Content

- Physical quantity: form factor (**Higgs amplitude**)
- Tools: **color-kinematics duality** & **unitarity**
- **New** 3-loop result

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- Physical quantity: form factor (**Higgs amplitude**)

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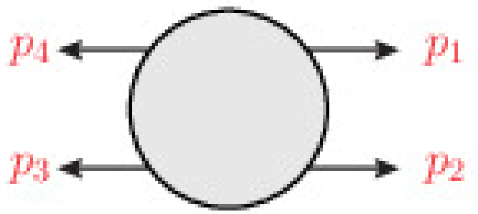
What is **form factor?**

Definition

Amplitude

Asymptotic States

E.g. $\langle g^{-}_{p_1}, g^{-}_{p_2}, g^{+}_{p_3}, g^{+}_{p_4} | \Omega \rangle$



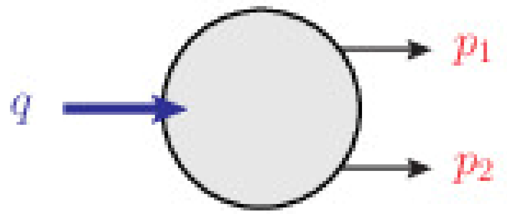
Form Factor

Operator

Asymptotic States

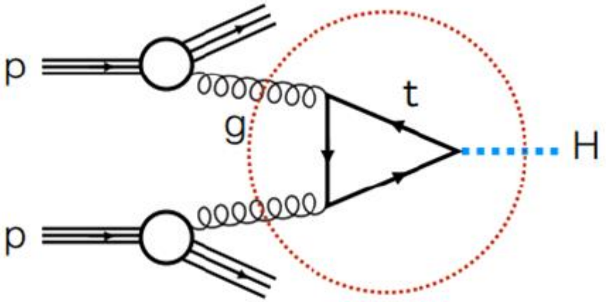
E.g. $\langle g^{-}_{p_1}, g^{-}_{p_2} | \mathcal{O}(q) | \Omega \rangle$

$$= \int \mathrm{d}^4 x e^{-iq\cdot x} \langle g^{-}_{p_1}, g^{-}_{p_2} | \mathcal{O}(x) | \Omega \rangle$$

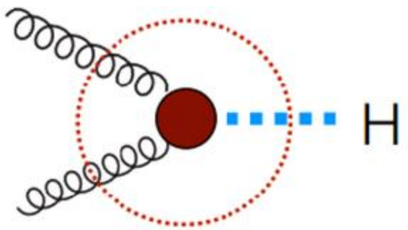


Why is form factor **important**?

Higgs



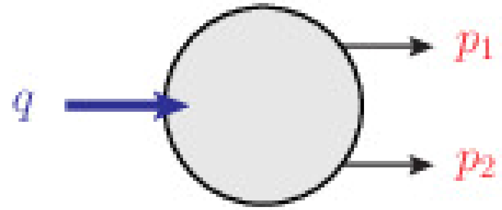
$m_t \to \infty$



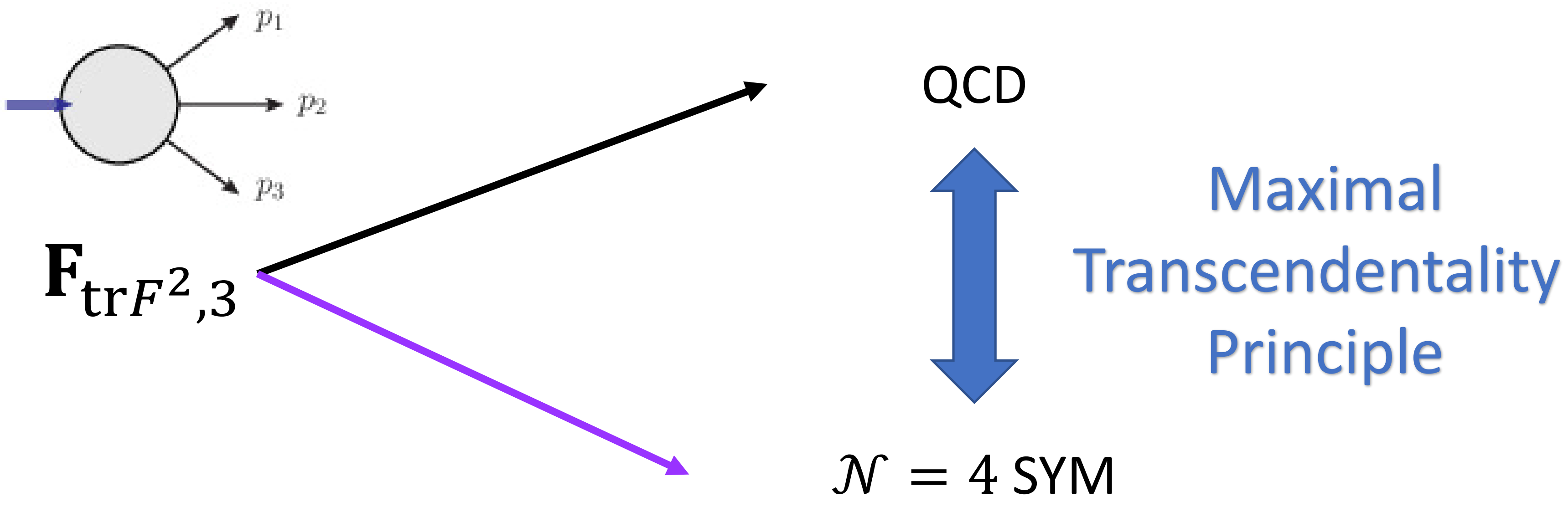
Effective gluon-Higgs vertex:

$$\mathcal{L}_{\rm eff} = C_0 H \,\mathrm{Tr}(F_{\mu\nu}^2) + \mathcal{O}(\tfrac{1}{m_t^2})$$

$\mathbf{F}_{\mathrm{tr}F^2}$



Calculate in $\mathcal{N}$=4 SYM



Our work:

- $\mathcal{N} = 4$ Super Yang-Mills Theory
- 3-loop 3-point full-color form factors with operators: $\mathrm{tr}\phi^2(\mathrm{tr}F^2), \mathrm{tr}\phi^3$
- With color-kinematics-dual structure

Content

- Physical quantity: form factor (**Higgs amplitude**)

- Tools: **color-kinematics duality** & **unitarity**

- **New** 3-loop result

What is **color-kinematics duality?**

$$\mathbf{A}^{(L)} = \sum_{trivalent\ \Gamma_i} \int \prod_j^L d^D l_j \frac{1}{S_i} \frac{C_i N_i}{\prod_a d_{i,a}}$$

color factor

kinematics factor

$$\tilde{f}^{abc} = Tr([T^a, T^b]T^c)$$

$$s_{ij} = (p_i + p_j)^2$$

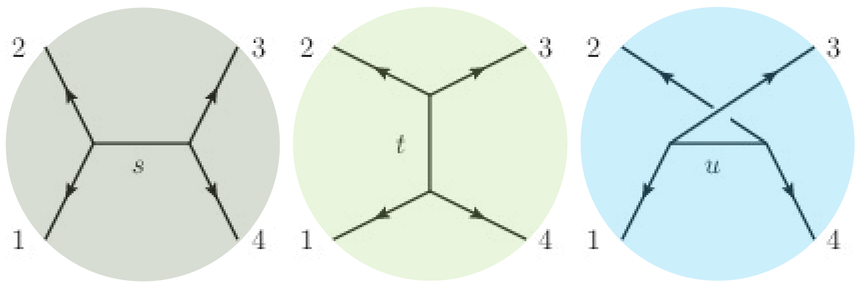
[Bern, Carrasco, Johansson 2008]

Color factor ⟷ Duality ⟷ Kinematics factor

What is **color-kinematics duality?**

$$\mathbf{A}^{(L)} = \sum_{trivalent\ \Gamma_i} \int \prod_j^L d^D l_j \frac{1}{S_i} \frac{C_i N_i}{\prod_a d_{i,a}}$$

Color factor ⟷ Duality ⟷ Kinematics factor



$$\mathbf{A}_4^{(0)}(p_1, p_2, p_3, p_4) = \frac{C_s N_s}{s} + \frac{C_t N_t}{t} + \frac{C_u N_u}{u}$$

$C_s = \tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4}$ $C_t = \tilde{f}^{a_1 a_4 b} \tilde{f}^{b a_2 a_3}$ $C_u = \tilde{f}^{a_1 a_3 b} \tilde{f}^{b a_2 a_4}$

Jacobi Relation $C_s + C_t = C_u$ ⟷ Duality ⟷ $N_s + N_t = N_u$ **Dual Jacobi Relation**

What is **color-kinematics duality?**

- Proved at tree-level [Bjerrum-Bohr et.al 2009; Stieberger 2009] [Feng, Huang, Jia 2010]

- Still a conjecture at loop-level, relying on explicit constructions:
  - 4-loop 4-point amplitudes in N=4 [Z. Bern, et al. 2012]
  - 5-loop Sudakov form factor in N=4 [G. Yang 2016]
  - 2-loop 5-point amplitudes in pure YM [O'Connell and Mogull 2015]

Usually non-trivial to find CK-dual solution at high loops

Tools: **color-kinematics duality**

Strategy of high-loop calculation:

Get integrand ansatz

Color-kinematics duality

Tools: **color-kinematics duality** & **unitarity**

Strategy of high-loop calculation:

Get integrand ansatz

Color-kinematics duality

Solve integrand ansatz

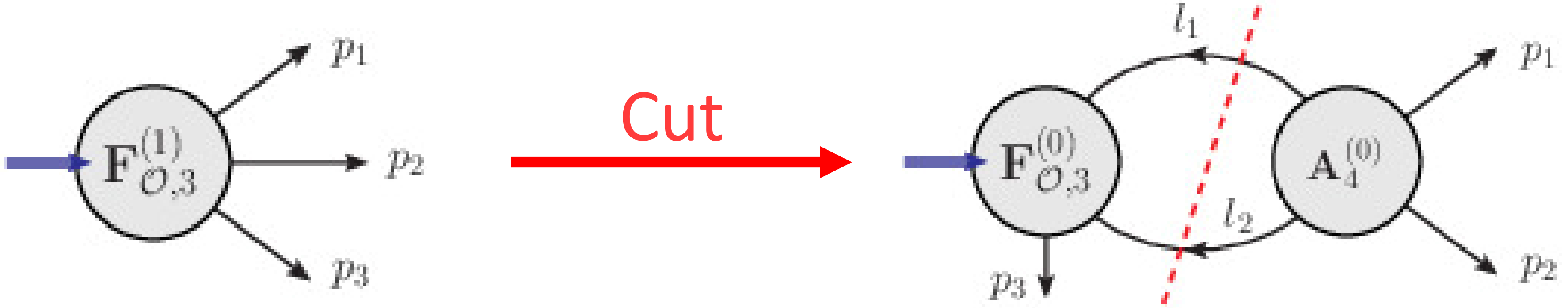
Symmetry

Unitarity

What is **unitarity?**

Unitarity cut: Set certain internal momenta on shell $\frac{i}{l^2} \to 2\pi\delta_+(l^2)$

$\mathbf{F}^{(1)}_{\mathcal{O},3}$ $\mathbf{F}^{(0)}_{\mathcal{O},3} \times \mathbf{A}^{(0)}_4$



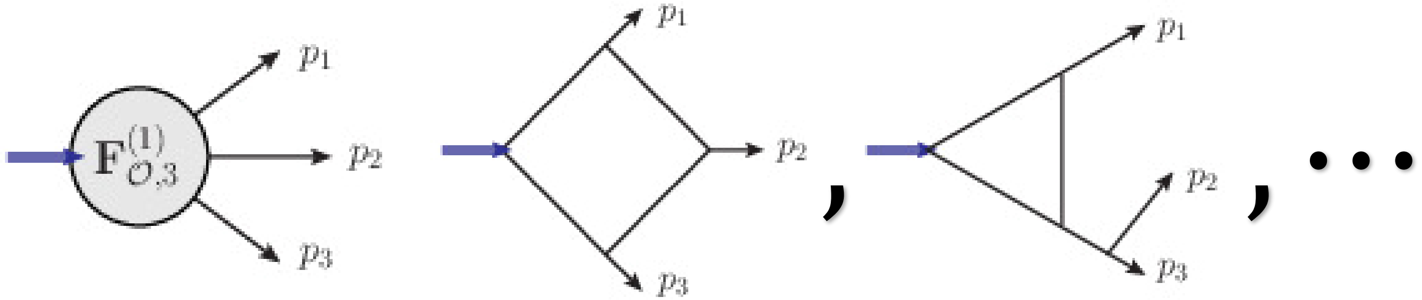
How to use **unitarity?**

$$\mathbf{F}^{(L)} = \sum_{trivalent\ \Gamma_i} \int \prod_j^L d^D l_j \frac{1}{S_i} \frac{C_i N_i}{\prod_a d_{i,a}} = \sum_i \frac{C_i}{S_i} I_i[N_i]$$

e.g. 1 loop, 3 point, $\mathcal{O} = \mathrm{tr}\phi^2$

color factor   kinematics factor

$$\mathbf{F}^{(1)}_{\mathcal{O},3} = C_1 I_1[N_1] + C_2 I_2[N_2] + \cdots$$



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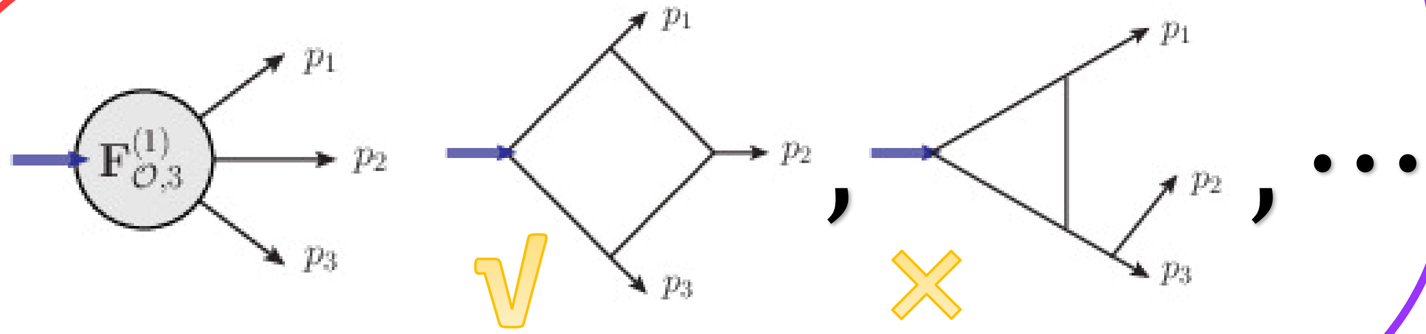
e.g. 1 loop, 3 point, $\mathcal{O} = \mathrm{tr}\phi^2$

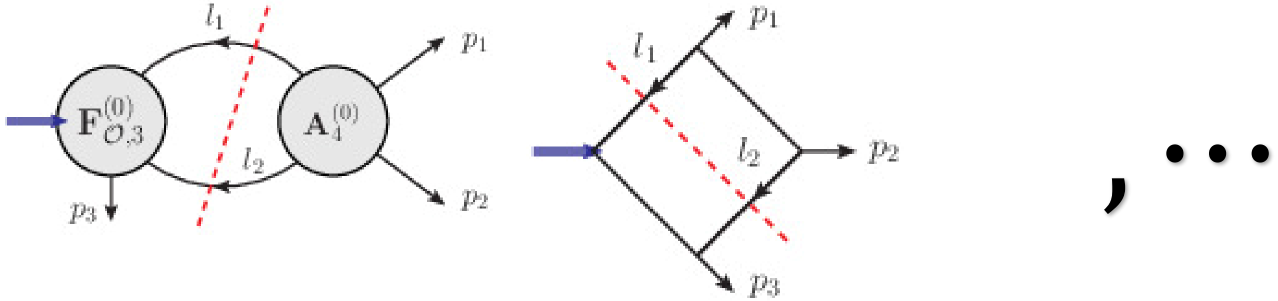
Cut

$$\mathbf{F}^{(1)}_{\mathcal{O},3} = C_1 I_1[N_1] + C_2 I_2[N_2] + \cdots$$



$$\mathbf{F}^{(0)}_{\mathcal{O},3} \times \mathbf{A}^{(0)}_4 = C_1 \mathrm{Cut}\big[I_1[N_1]\big] + \cdots$$

Get constraints on Ns



Content

- Physical quantity: form factor (**Higgs amplitude**)
- Tools: **color-kinematics duality** & **unitarity**
- **New** 3-loop result

New 3-loop result (3-loop 3-point case)

Procedure

Get integrand ansatz Color-kinematics duality

⇩

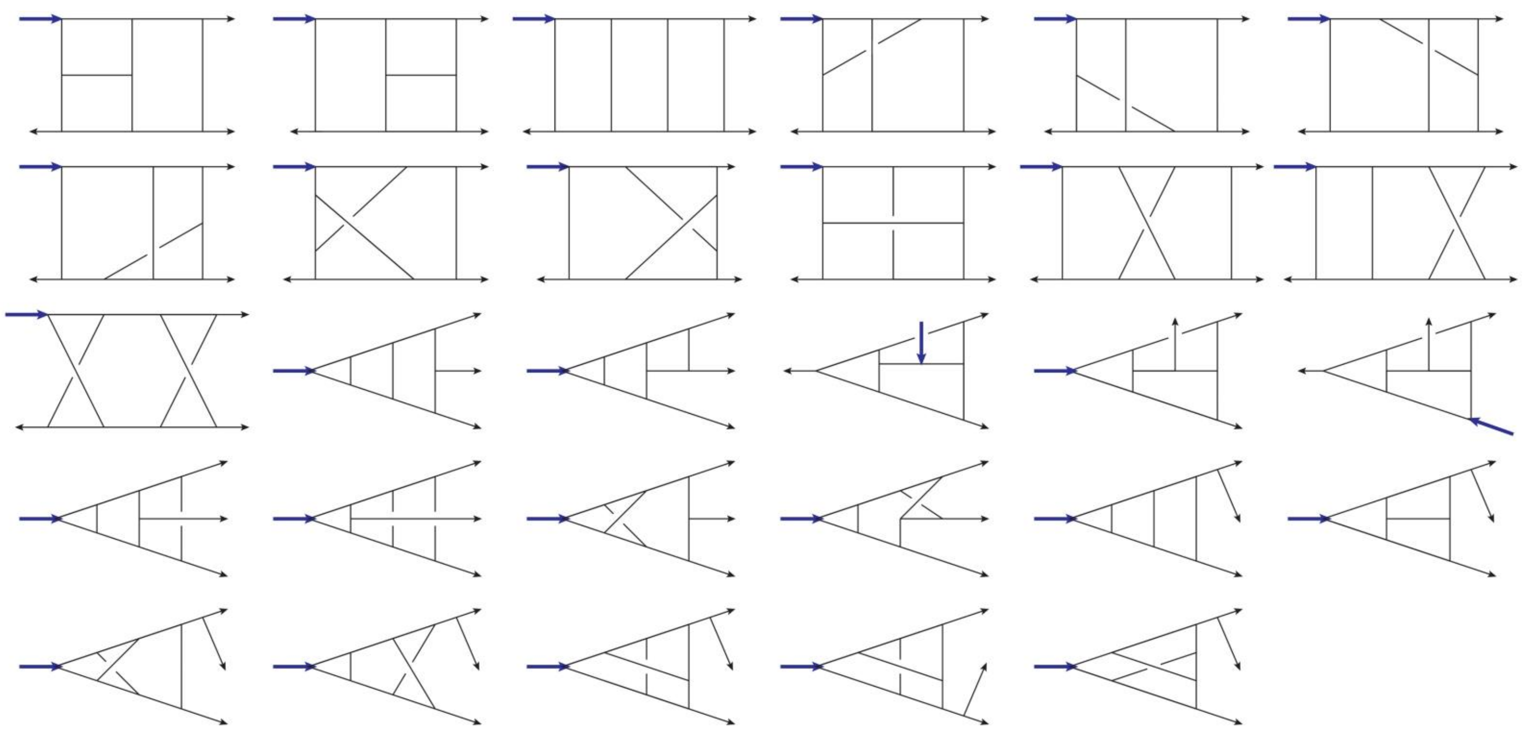
Solve integrand ansatz Symmetry

Unitarity

New 3-loop result (3-loop 3-point case)

Procedure

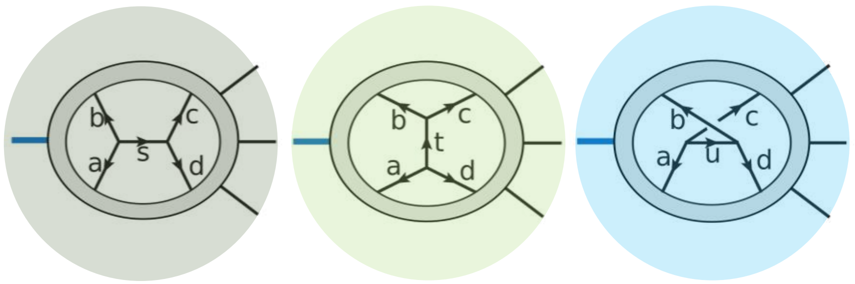
Get integrand ansatz Color-kinematics duality $\mathcal{O} = \mathrm{tr}\phi^2$



New 3-loop result (3-loop 3-point case)

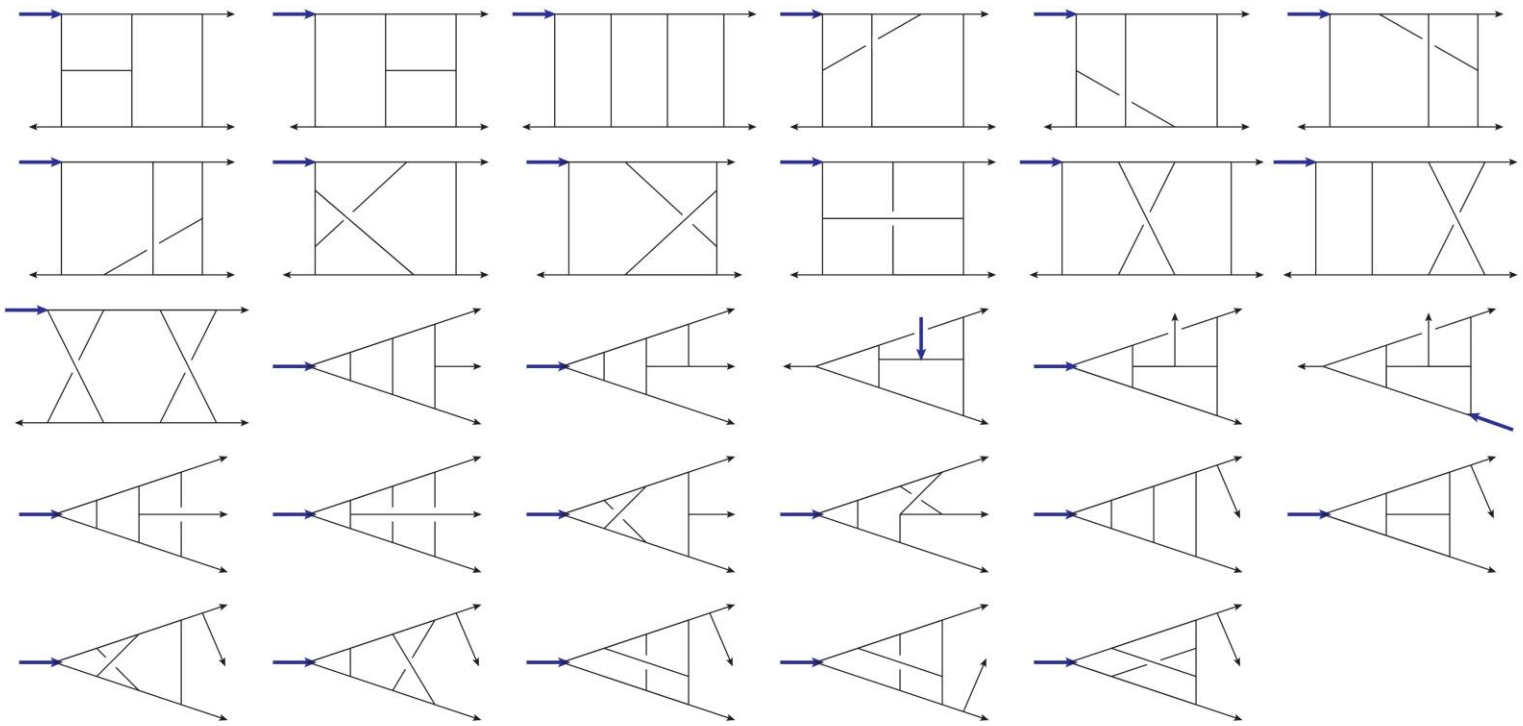
Procedure

Get integrand ansatz Color-kinematics duality $\mathcal{O} = \text{tr}\phi^2$



$$N_s + N_t + N_u = 0$$

Dual Jacobi Relation



New 3-loop result (3-loop 3-point case)

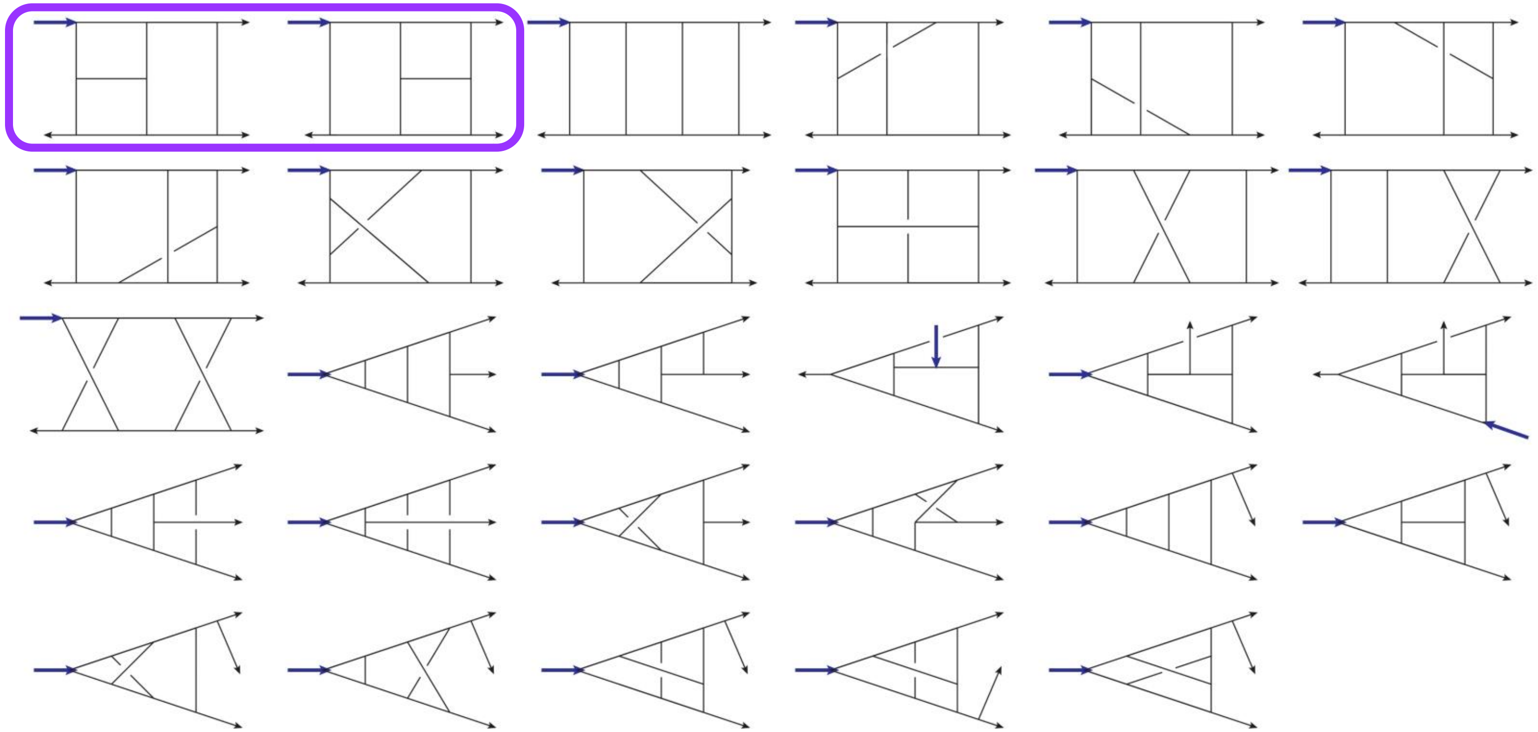
Procedure

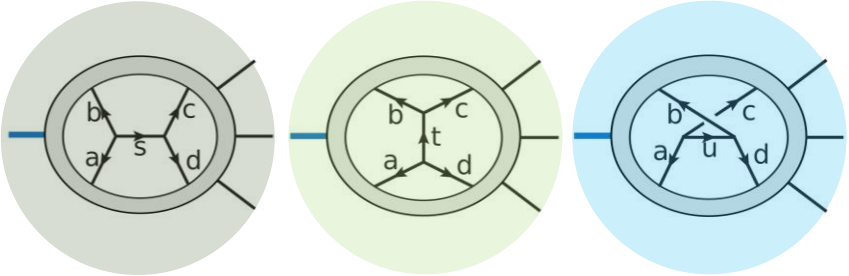
Get integrand ansatz

Color-kinematics duality

$\mathcal{O} = \mathrm{tr}\phi^2$

Master numerators





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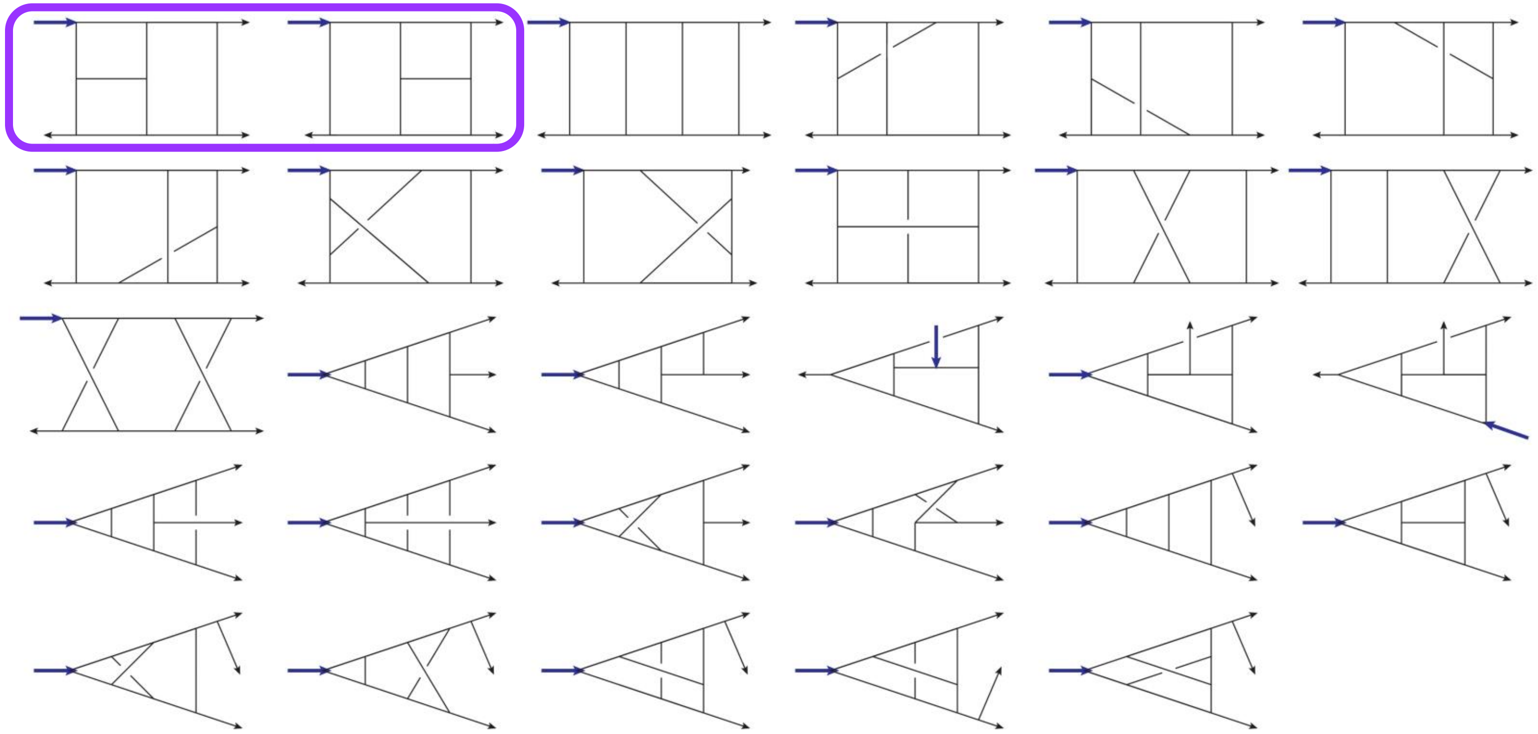
Dual Jacobi Relation

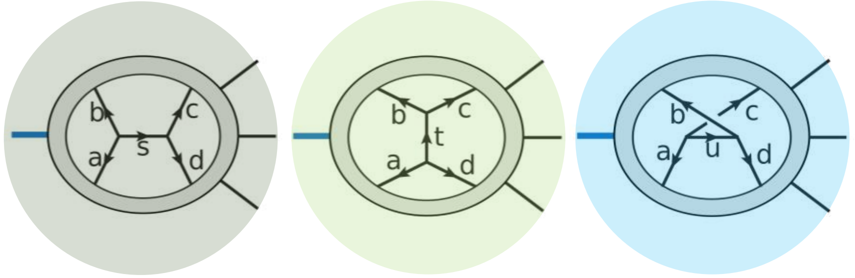
New 3-loop result (3-loop 3-point case)

Procedure

Get integrand ansatz   Color-kinematics duality $\mathcal{O} = \mathrm{tr}\phi^2$

Master numerators





$$N_s + N_t + N_u = 0$$

Dual Jacobi Relation

$$\boldsymbol{F}^{(3)}_{\mathrm{tr}(\phi^2),3} = \mathcal{F}^{(0)}_{\mathrm{tr}(\phi^2),3} \sum_{\sigma} \sum_{i=1}^{29} \int \prod_{j=1}^{3} \mathrm{d}^D \ell_j \frac{1}{S_i} \sigma \cdot \frac{C_i\, N_i}{\prod_{\alpha_i} d_{\alpha_i}}$$

Ansatz with 316 parameters

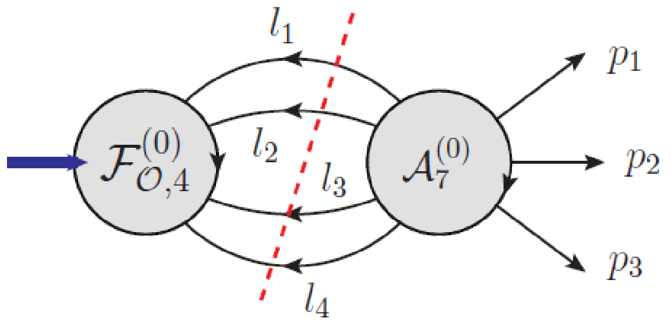
New 3-loop result (3-loop 3-point case)

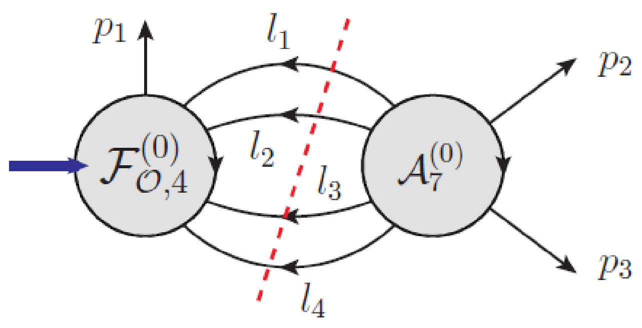
Procedure

Solve integrand ansatz

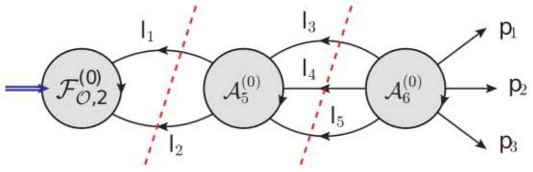
	Symmetry	Unitarity
$\mathcal{O} = \mathrm{tr}\phi^2$	$316 \to 105$	$105 \to 24$

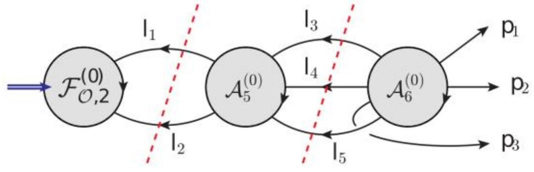
4-cuts to solve:

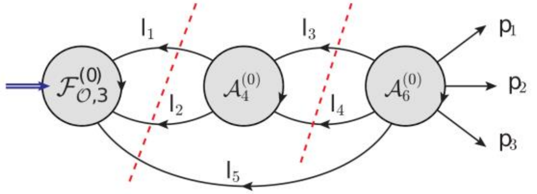




Other cuts to check:







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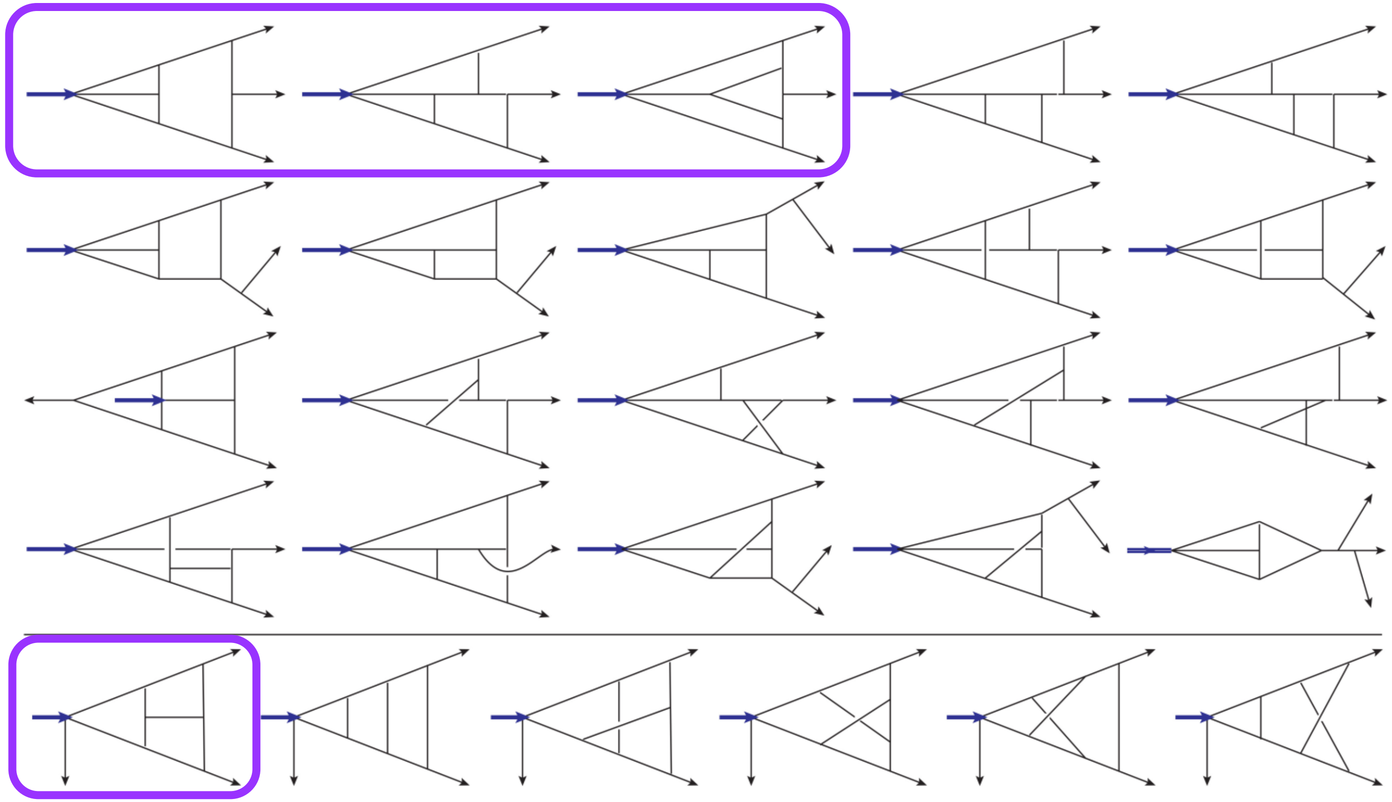
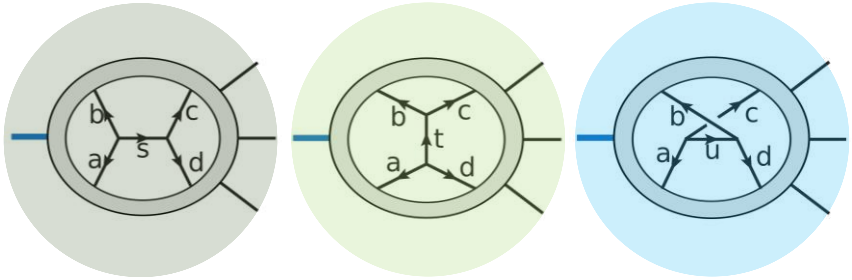
New 3-loop result (3-loop 3-point case)

Procedure

Get integrand ansatz Color-kinematics duality $\mathcal{O} = \mathrm{tr}\phi^3$

Master numerators





$$N_s + N_t + N_u = 0$$

Dual Jacobi Relation

$$\boldsymbol{F}^{(3)}_{\mathrm{tr}(\phi^3),3} = \sum_{\sigma}\sum_{i=1}^{26}\int\prod_{j=1}^{3}\mathrm{d}^D l_j \frac{1}{S_i}\sigma\cdot\frac{C_i N_i}{\prod_{\alpha_i} d_{\alpha_i}}$$

Ansatz with 273 parameters

Solve → 10 parameters

New 3-loop result (3-loop 3-point case)

Result

	Get integrand ansatz			Solve integrand ansatz		
	Topology	Master numerator	Ansatz	Symmetry	Unitarity	Free parameters
$\mathcal{O} = \mathrm{tr}\phi^2$	29	2	316	105	24	**24**
$\mathcal{O} = \mathrm{tr}\phi^3$	26	4	273	26	10	**10**

Integrand-level cancel

A short summary:

- New **3-loop** result:
  3-point form factor of $\mathrm{tr}\phi^2$ and $\mathrm{tr}\phi^3$ In $\mathcal{N} = 4$ Super Yang-Mills Theory

- Feature:
  - With non-planar contribution (**full-color** result)
  - With **color-kinematics-dual** structure

- Further discussion in Part 2:
  - Numerical integration
  - IR subtraction (planar and nonplanar)
  - Free parameters

Thanks!