Three-loop color-kinematics duality and Higgs amplitudes: Part 2

Guanda Lin, Peking University

based on work with Gang Yang and Siyuan Zhang(ITP,CAS), to appear

May 15, 2021

Review of the previous talk

- **Integrands in color-kinematics duality representation**

$$\boldsymbol{F}^{(3)}_{\mathcal{O}_L,3} = \mathcal{F}^{(0)}_{\mathcal{O}_L,3} \sum_{\sigma_3} \sum_i \int \prod_{j=1}^{3} d^D \ell_j \frac{1}{S_i} \frac{C_i N_i}{\prod_{\alpha_i} P^2_{\alpha_i}}$$

$\mathrm{tr}(\phi^L)$, with $L = 2, 3$

$$C_s = C_t + C_u \quad \Rightarrow \quad N_s = N_t + N_u$$

$\mathrm{tr}(\phi^2)$	$\mathrm{tr}(\phi^3)$
29 diagrams	26 diagrams
2 planar masters	4 planar masters
24 free parameters	10 free parameters

- **Free parameters cancel at the** integrand **level**

Simplified full-color result

- $\mathrm{tr}(\phi^2)$ form factor
  Both leading and subleading N_c contributions

$$\boldsymbol{F}^{(3)}_{\mathrm{tr}(\phi^2),3} = \mathcal{F}^{(0)}_{\mathrm{tr}(\phi^2),3}\tilde{f}^{123}\left(N_c^3\mathcal{I}^{(3)}_{\mathrm{tr}(\phi^2)} + 12N_c\mathcal{I}^{(3)}_{\mathrm{tr}(\phi^2),\mathrm{NP}}\right)$$

$\tilde{f}^{123} = \mathrm{tr}(T^{a_1}T^{a_2}T^{a_3}) - \mathrm{tr}(T^{a_1}T^{a_3}T^{a_2})$

- $\mathrm{tr}(\phi^3)$ form factor
  Only leading N_c contribution

$$\boldsymbol{F}^{(3)}_{\mathrm{tr}(\phi^3),3} = \mathcal{F}^{(0)}_{\mathrm{tr}(\phi^3),3}\tilde{d}^{123}N_c^3\mathcal{I}^{(3)}_{\mathrm{tr}(\phi^3)}$$

$\tilde{d}^{123} = \mathrm{tr}(T^{a_1}T^{a_2}T^{a_3}) + \mathrm{tr}(T^{a_1}T^{a_3}T^{a_2})$

Each of the $\mathcal{I}$ contains only about 60 integrals.

IR structure

IR divergences @ 3 loops — { important checks / research frontiers }

- **Planar IR structure**

  BDS ansatz [Bern,Dixon,Smirnov,2005]:

  @ 3 loops

$$\mathcal{I}^{(3)}(\epsilon) = -\frac{1}{3}\left(\mathcal{I}^{(1)}(\epsilon)\right)^3 + \mathcal{I}^{(2)}(\epsilon)\mathcal{I}^{(1)}(\epsilon) + f^{(3)}(\epsilon)\mathcal{I}^{(1)}(3\epsilon) + \mathcal{R}^{(3)} + C^{(3)} + O(\epsilon)$$

  Properties:

  IR cancellation

  $\mathcal{R}$ finite remainders and $\mathcal{R}_n \to \mathcal{R}_{n-1}$ in the collinear limit

IR structure

- **Full Color IR structure**

$$\boldsymbol{F}(p_i, a_i, \epsilon) = \boldsymbol{Z}(p_i, \epsilon)\boldsymbol{F}^{\text{fin}}(p_i, a_i, \epsilon)$$

$$\boldsymbol{Z} = \mathcal{P}\exp\left[-\sum_{\ell=1}^{\infty} g^{2\ell}\left(\text{dipole terms} + \boxed{\frac{1}{\ell\epsilon}\boldsymbol{\Delta}^{(\ell)}}\right)\right]$$

planar non-planar

non-dipole start from **3-loop**

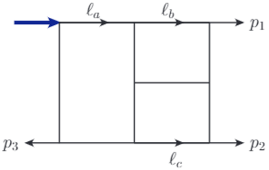
non-dipole terms $\boldsymbol{\Delta}^{(3)} = \boldsymbol{\Delta}_3^{(3)} + \boldsymbol{\Delta}_4^{(3)}$ [Gardi,Almelid,Duhr,2016]

- 3-pt form factors
  - dipole terms $\leftrightarrow$ N_c-leading (planar)
  - non-dipole terms $\leftrightarrow$ N_c-subleading (non-planar) $\&$ no cross-ratio
- non-planar IR divergence
  - pole: only ϵ^{-1}
  - residue: constant $\zeta_5 + 2\zeta_2\zeta_3$

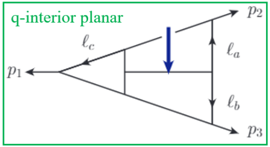
Numerics and checks

- **The integrals**

  N_c-leading: $\mathcal{I}^{(3)}$ (e.g. $\mathrm{tr}(\phi^2)$ form factors)

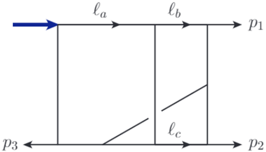


$$\left((\ell_a - p_1)^2\right)^2$$



$$(\ell_a - p_1 - p_2)^2(\ell_b - p_1 - p_3)^2$$

N_c-subleading: $\mathcal{I}^{(3)}_{\mathrm{NP}}$



$$\left((\ell_a - p_1)^2\right)^2$$

$$(\ell_a - p_3)^2(\ell_c - p_1 - p_2)^2$$

Numerics and checks

- **and the evaluations**
  Analytical:
  IBP reduction (hard) Master integrals (non-planar unknown)
  Numerical:
  Method: sector decomposition
  Tools: pySecDec [Borowka,et al.2017]
  FIESTA [Simirnov,2015]

Numerics and Checks

The planar result and check

Form factor	$\mathcal{I}^{(3)}_{\mathrm{tr}(\phi^2)}$						
(s_{12}, s_{23}, s_{13})	ϵ^{-6}	ϵ^{-5}	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
(-2,-2,-2)	-4.5	9.3575	-22.613	55.893	-77.25	92.8	-338.19
est. error	8×10^{-10}	2×10^{-4}	0.001	0.006	0.03	0.2	1.7

universal IR structure: (determined by lower loops) — ϵ^{-6} to ϵ^{-1}

match OPE result — ϵ^{0}

1. $O(1 \times 10^4)$ cpu core hours (3.7GHz) in total, mainly pySecDec
2. IR: correctly captured by BDS
3. finite order: match the OPE result (planar only) [Dixon,McLoed,Wilhelm,2020]

Numerics and Checks

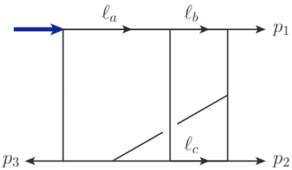
About the non-planar

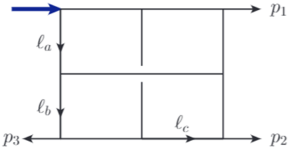
- **The result**
  1. $\epsilon^{-6} - \epsilon^{-2}$: vanish ✓ ($\epsilon^{-2}$ -0.055±0.27)
  2. ϵ^{-1}: high precision calculation in progress
- **and the difficulties**

  "highly non-planar integrals"

  a single integral: $\epsilon^{-6} - \epsilon^{-2}$ (ϵ^{-2} 378.9±0.13)

  $O(3 \times 10^4)$ CPU core hours





FIESTA √

pySecDec ×
less efficient

Discussions

Cancellation of free parameters

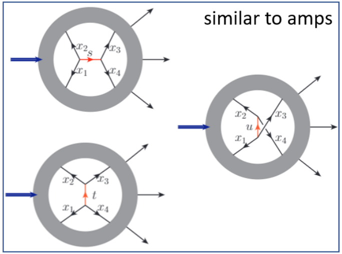
$$\sum_{\sigma_3}\sum_{\Gamma_i}\frac{1}{S_i}\frac{C_i\Delta N_i}{D_i}=0$$

deformation by free parameters

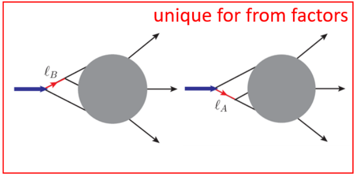
Generalized gauge transformation

Jacobi-like: $N_s \to N_s + s\Delta, \quad N_t \to N_t + t\Delta, \quad N_u \to N_u + u\Delta$

Beyond Jacobi: $N_A \to N_A + l_A^2\Delta, \quad N_B \to N_B + l_B^2\Delta$



Jacobi-like



Beyond-Jacobi

Discussions

- Non-triviality

  CK-preserving generalized Gauge Transformation(GT)

  Jacobi-like GT usually breaks CK-duality: $s + t + u \neq 0$

  Beyond Jacobi GT can help to restore CK-duality
- Significance

  Form factors are **nice arena** for applying color-kinematics duality

  Possible to construct CK-representation for **higher loops**

  May also have ramifications on "double copy"

Summary and Outlook

We study the three-loop three-point form factors of $\mathrm{tr}(\phi^L), L = 2, 3$

- construct integrands via **unitarity** and **color-kinematics duality**
- get solutions with **free parameters** and understand their cancellation
- implement the **numerical integration** and analyse the properties

Outlook

- ☐ Improved numerical results, *e.g.* non-planar remainders
- ☐ Analytic calculation:
  - integral reduction and master integrals
  - maximally transcendental part of $H \to ggg$
- ☐ Higher loops or more general operators

Supplements

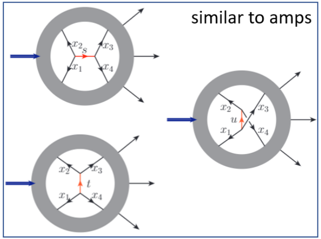
Generalized gauge transformation

Generalized gauge transformation → cancellation of free parameters

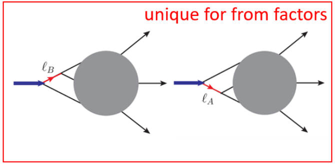
$$\sum_{\sigma_3}\sum_{\Gamma_i}\frac{1}{S_i}\frac{C_i\Delta N_i}{D_i}$$

↓ collect independent $\mathcal{K}$

$$\sum \mathcal{K}_{\rm I}\underbrace{(C_s+C_t+C_u)}_{\text{Jacobi-like: irrelevant to } C_O} + \sum \mathcal{K}_{\rm II}\underbrace{(C_A+C_B)}_{\text{Beyond Jacobi: relevant to } C_O}$$



Jacobi-like



Beyond-Jacobi

Implication on double copy

Double copy

$$\boldsymbol{A}^{\mathrm{YM}} = \sum_{\sigma}\sum_{\Gamma_i}\frac{1}{S_i}\frac{C_i N_i}{D_i} \to \boldsymbol{M}^{\mathrm{GRA}} = \sum_{\sigma}\sum_{\Gamma_i}\frac{1}{S_i}\frac{N_i^{\mathrm{CK}} N_i}{D_i}$$

Implications on the double copy of form factors?

we do **NOT** impose beyond Jacobi relations like $N_A + N_B = 0$

the double copy may not be well-defined

Type I Only

Amplitudes: $\sum \frac{1}{S_i}\frac{C_i \Delta N_i}{D_i} = 0 \Rightarrow \sum \frac{1}{S_i}\frac{N_i^{\mathrm{CK}} \Delta N_i}{D_i} = 0$

Form factors: $\sum \frac{1}{S_i}\frac{C_i \Delta N_i}{D_i} = 0 \not\Rightarrow \sum \frac{1}{S_i}\frac{N_i^{\mathrm{CK}} \Delta N_i}{D_i} = 0$

Both Type I and Type II

$N_A + N_B \neq 0$