forward hadron production

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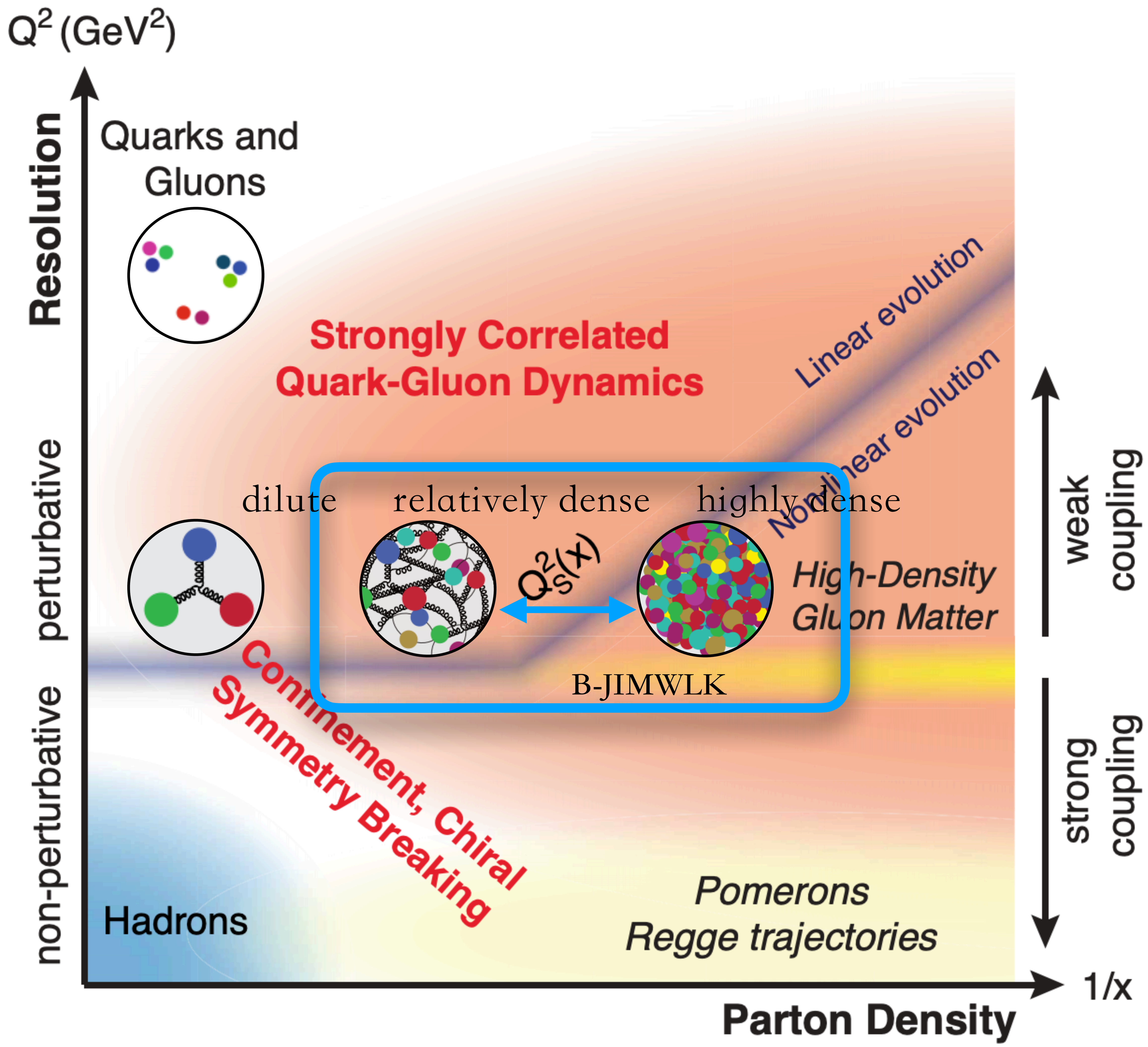
Kang, Liu, 1910.10166

Liu, Kang, XL, 2004.11990

A Landscape of the strong interaction

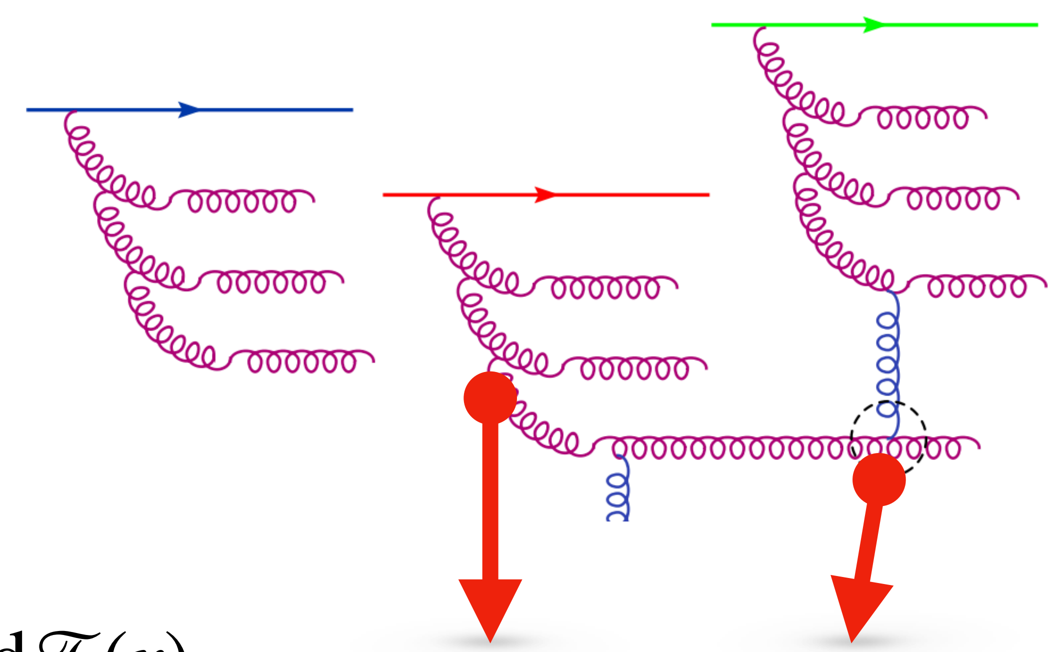


- Highly dense region: gluon splittings and recombinations.
  - non-linear evolution, gluon saturation

At high density, gluon wave functions start to overlap and recombinations are likely to happen, which leads to the gluon saturation

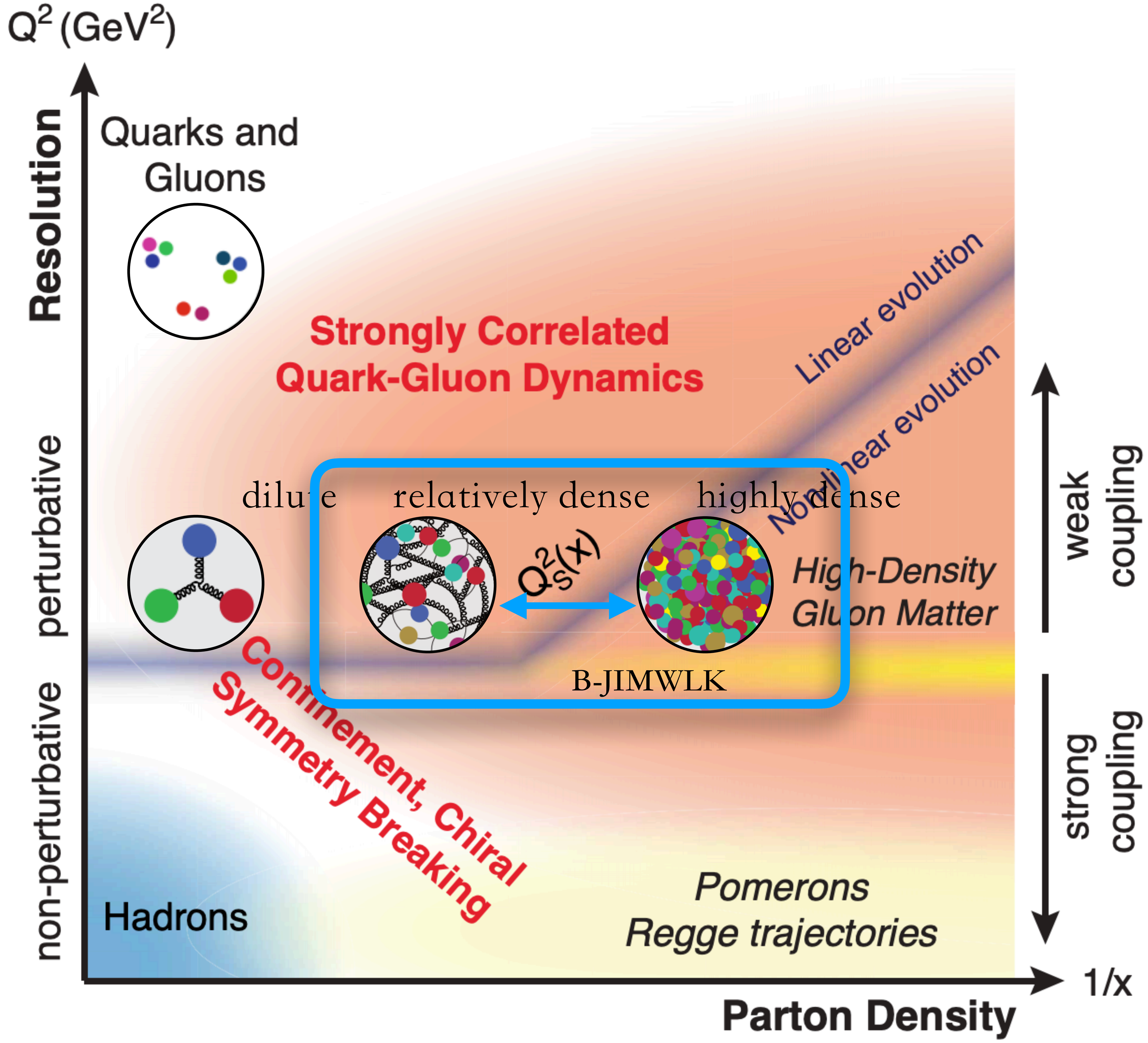
$$\rho\sigma \sim 1 \sim \frac{x\,G(x)}{\pi R_A^2}\frac{\alpha_s}{Q_s^2}$$

$$\rightarrow Q_s^2(x) \sim \frac{\alpha_s}{\pi R_A^2} x\,G(x) \sim x^{-\lambda}$$

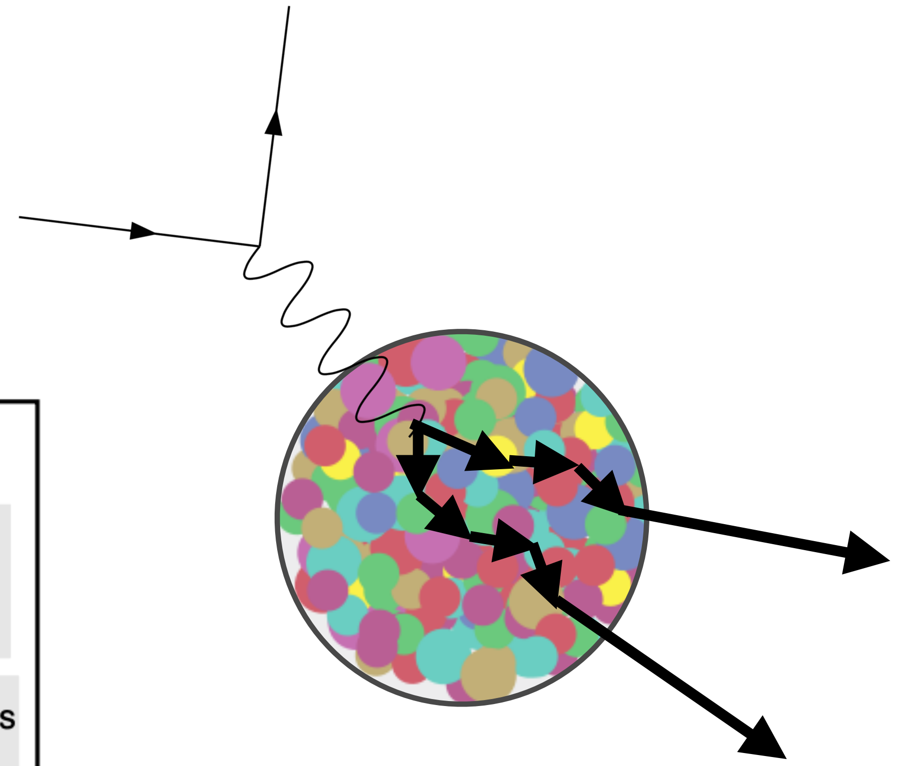


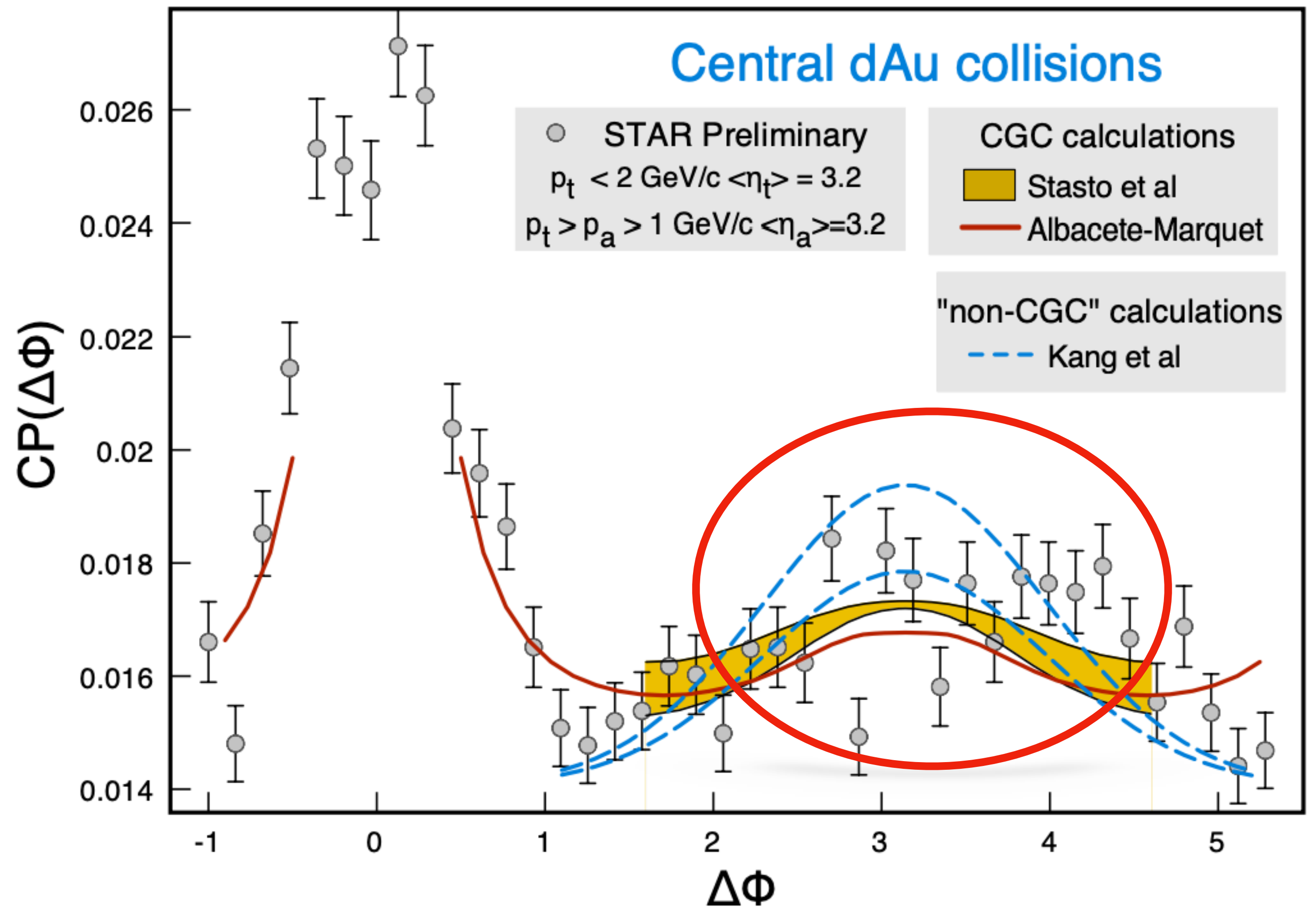
$$\frac{\mathrm{d}\mathscr{F}(x)}{\mathrm{d}\ln(1/x)} \propto \kappa \otimes \mathscr{F}(x) - \alpha_s[\mathscr{F}(x)]^2$$

Precision matters

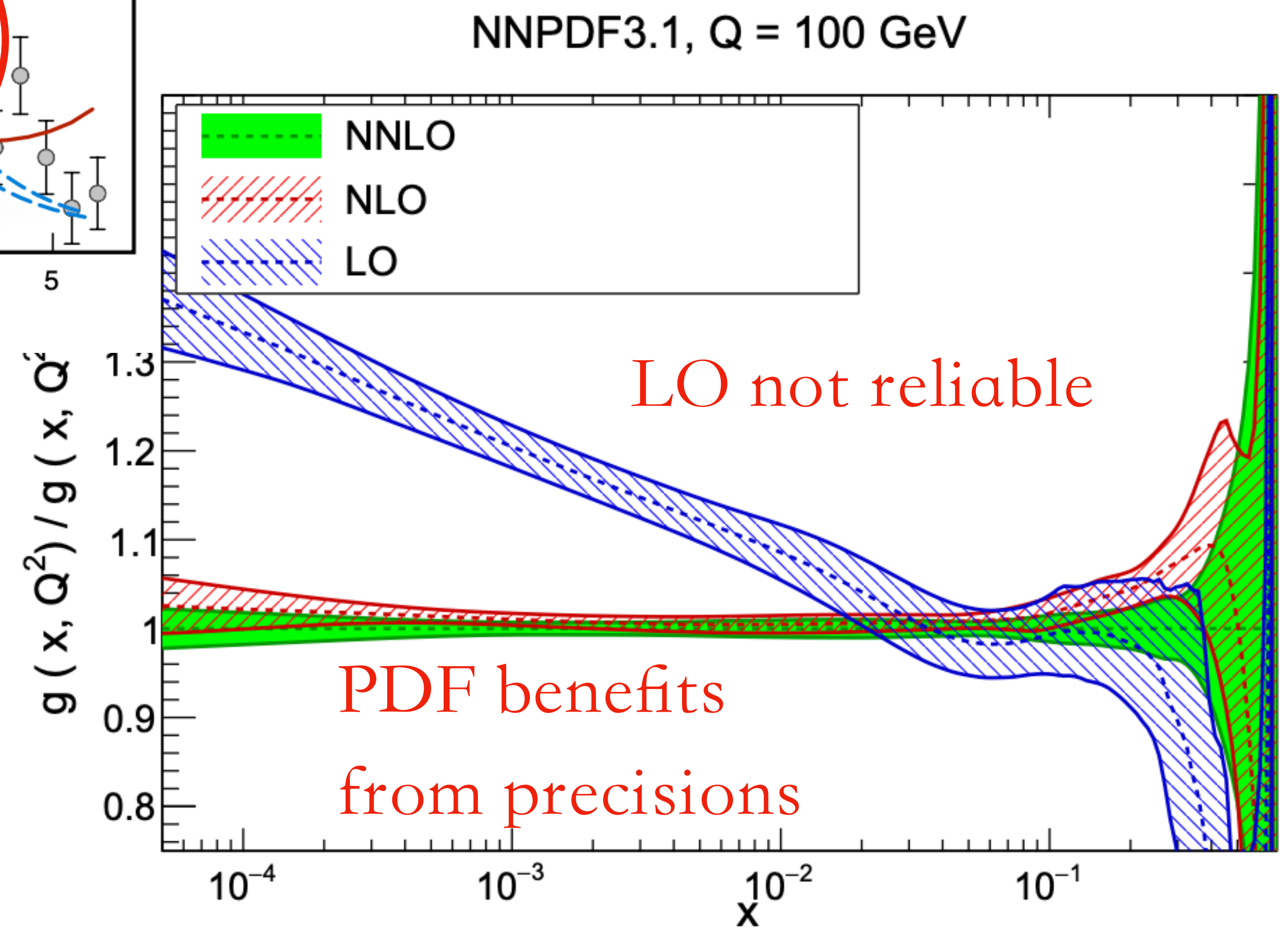


- Highly dense region: gluon splittings and recombinations.
  - non-linear evolution, gluon saturation

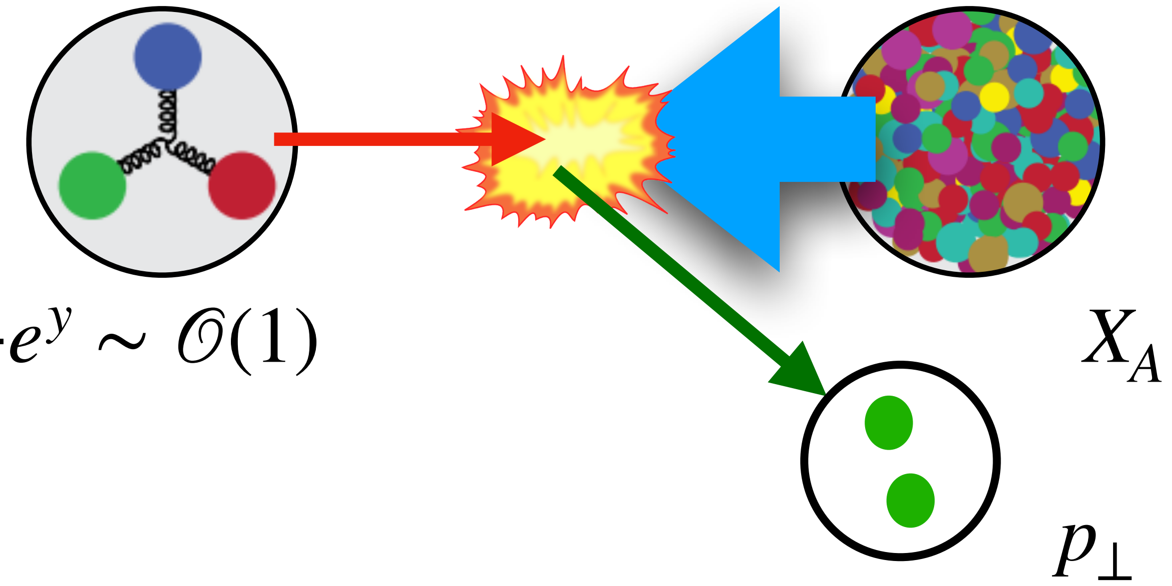




Compatible with both CGC and collinear twist predictions



Forward hadron production in CGC



$x \sim \frac{p_\perp}{\sqrt{s}} e^{y} \sim \mathcal{O}(1)$

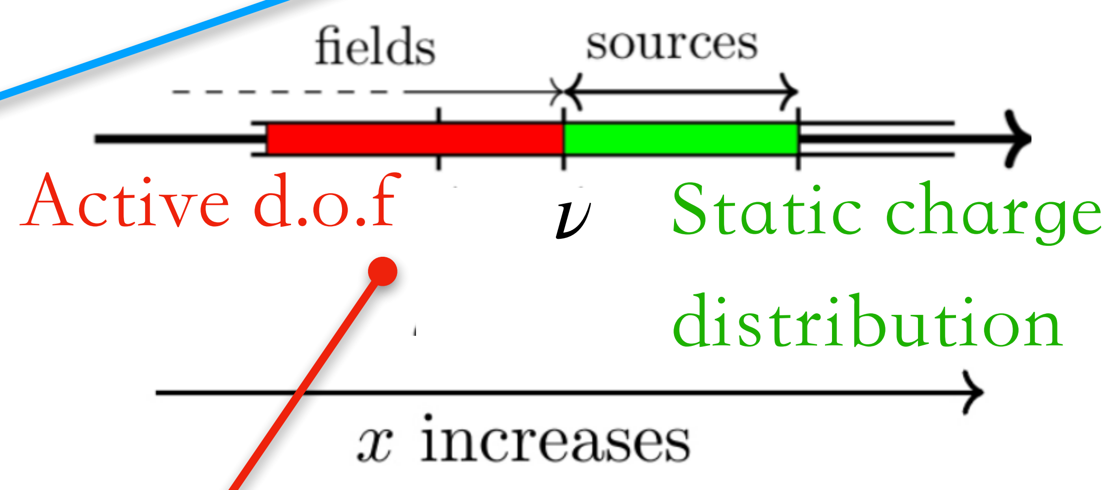
$X_A \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$

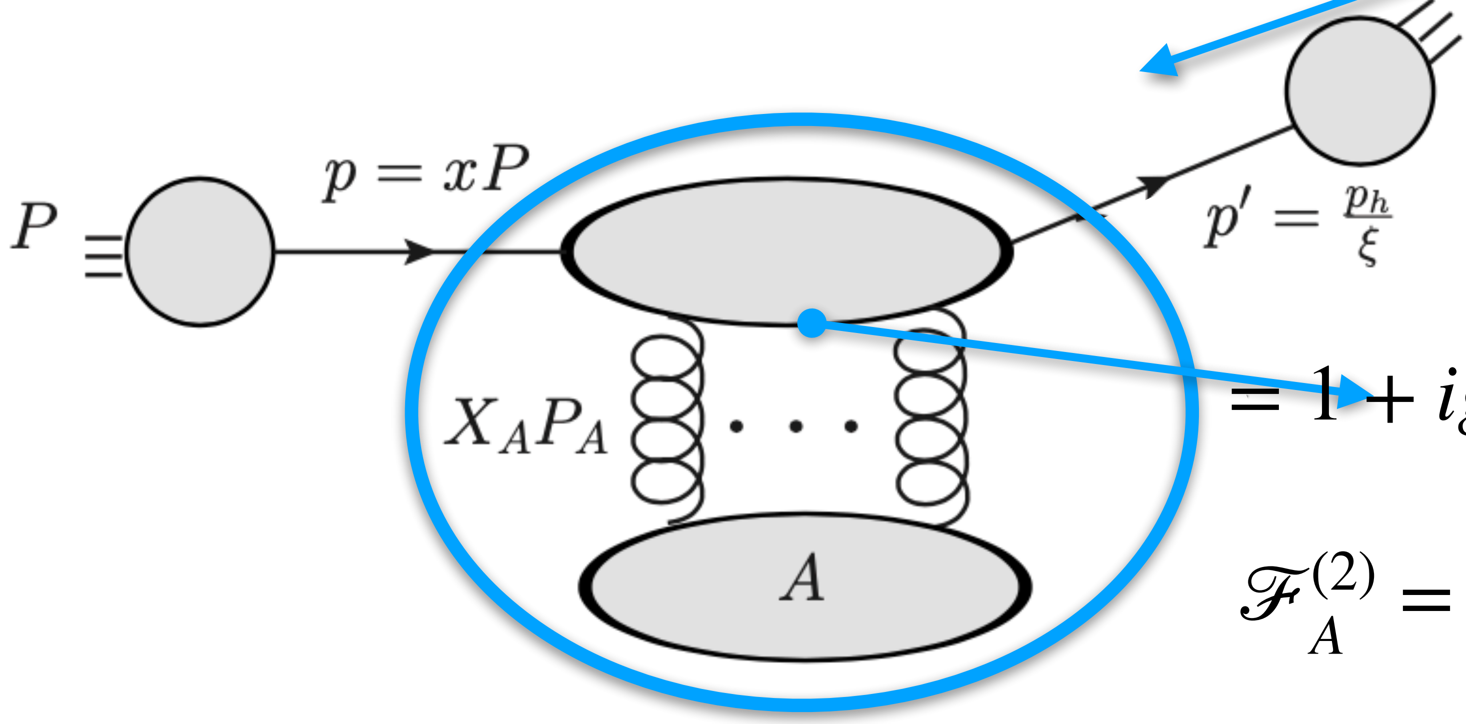
$\frac{p_\perp}{\sqrt{s}} \ll 1$

$$\frac{d\sigma}{dp_{h,\perp}^2 dy_h} = \int_\tau^1 \frac{d\xi}{\xi^2} \left(\hat{\sigma}^0 + \mathcal{O}(\alpha_s)\right) D_{h/j}(\xi,\mu) x f_{i/P}(x,\mu)\, \mathcal{F}_A^{(2)}(k_\perp,\nu)$$

Focus of the talk

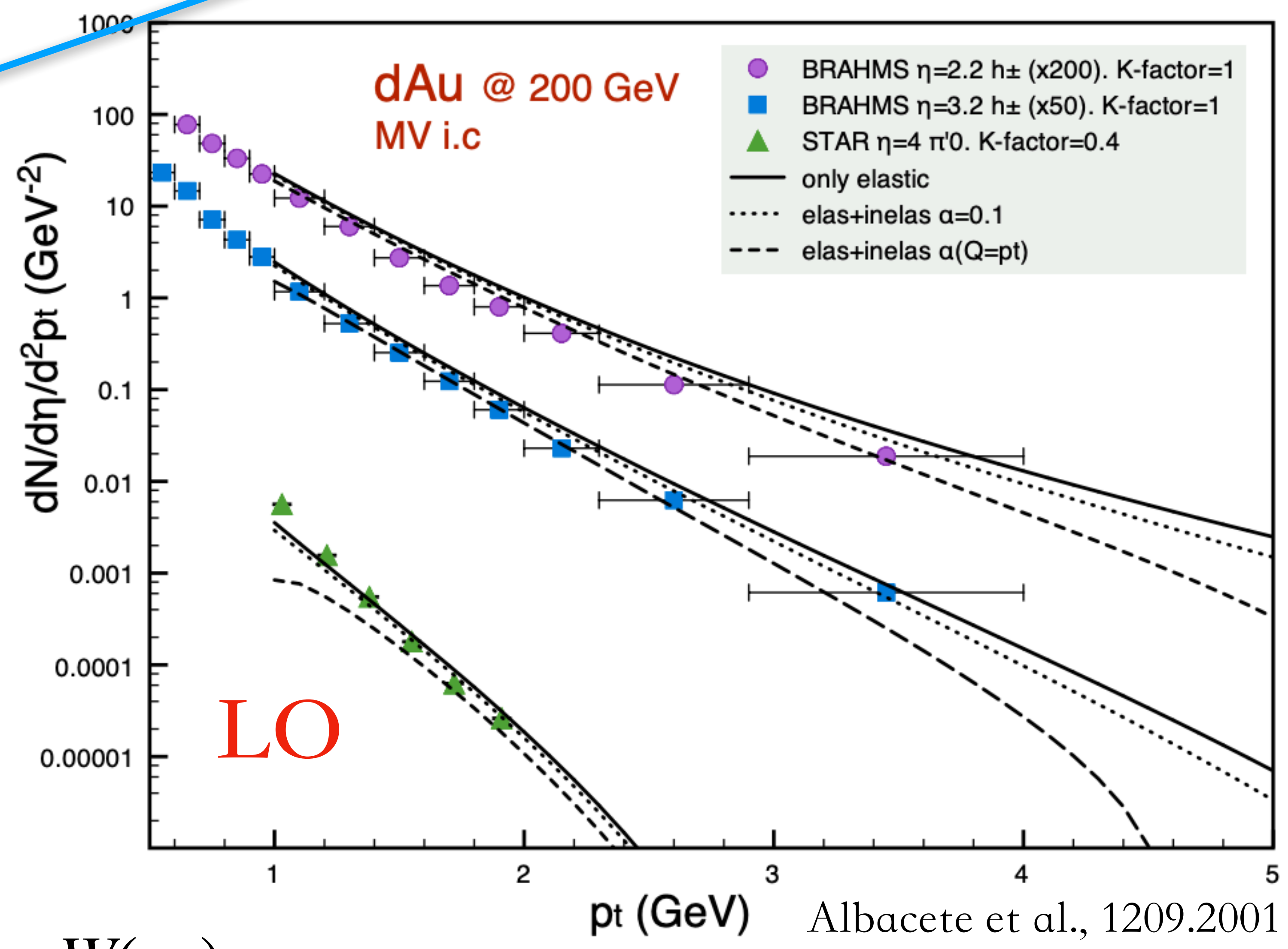
$\nu \sim X_A P_A$





$$= 1 + ig_s \int dz^+ A^-(x_\perp, z^+) + \ldots = \mathcal{P} \exp\left[ig_s \int dz^+ A^-(x_\perp, z^+)\right] = W(x_\perp)$$

$$\mathcal{F}_A^{(2)} = \mathrm{F.T.} \frac{1}{N_c} \langle W(x_\perp) W^\dagger(x'_\perp)\rangle_\nu$$

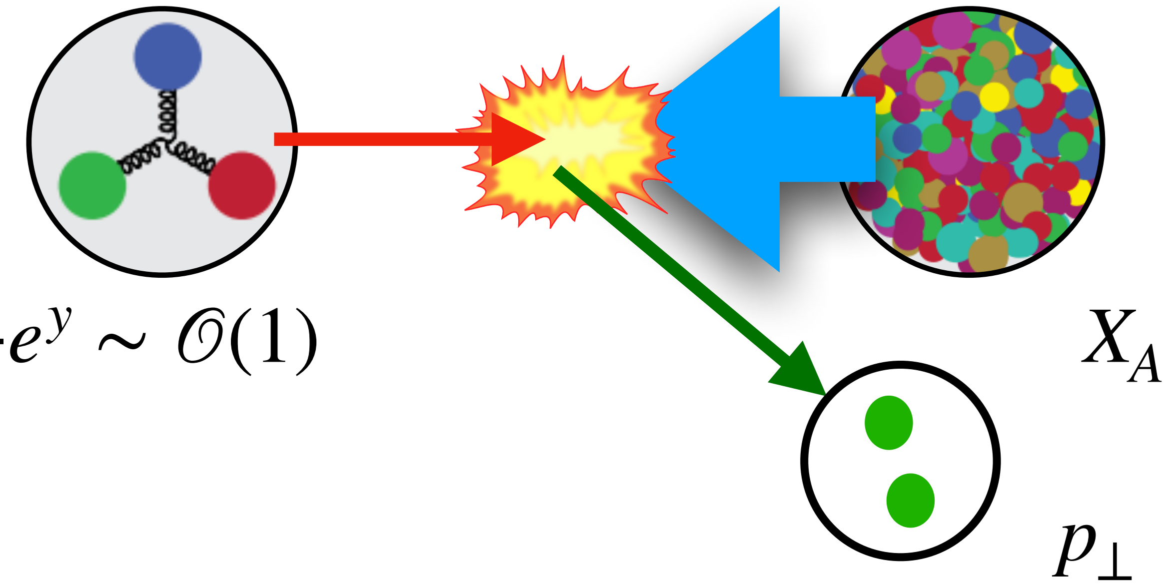


Albacete et al., 1209.2001

LO works pretty well but remember

$\alpha_s(Q_s)$ correction could be large

Forward hadron production in CGC



$x \sim \frac{p_\perp}{\sqrt{s}} e^{y} \sim \mathcal{O}(1)$

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$\frac{p_\perp}{\sqrt{s}} \ll 1$

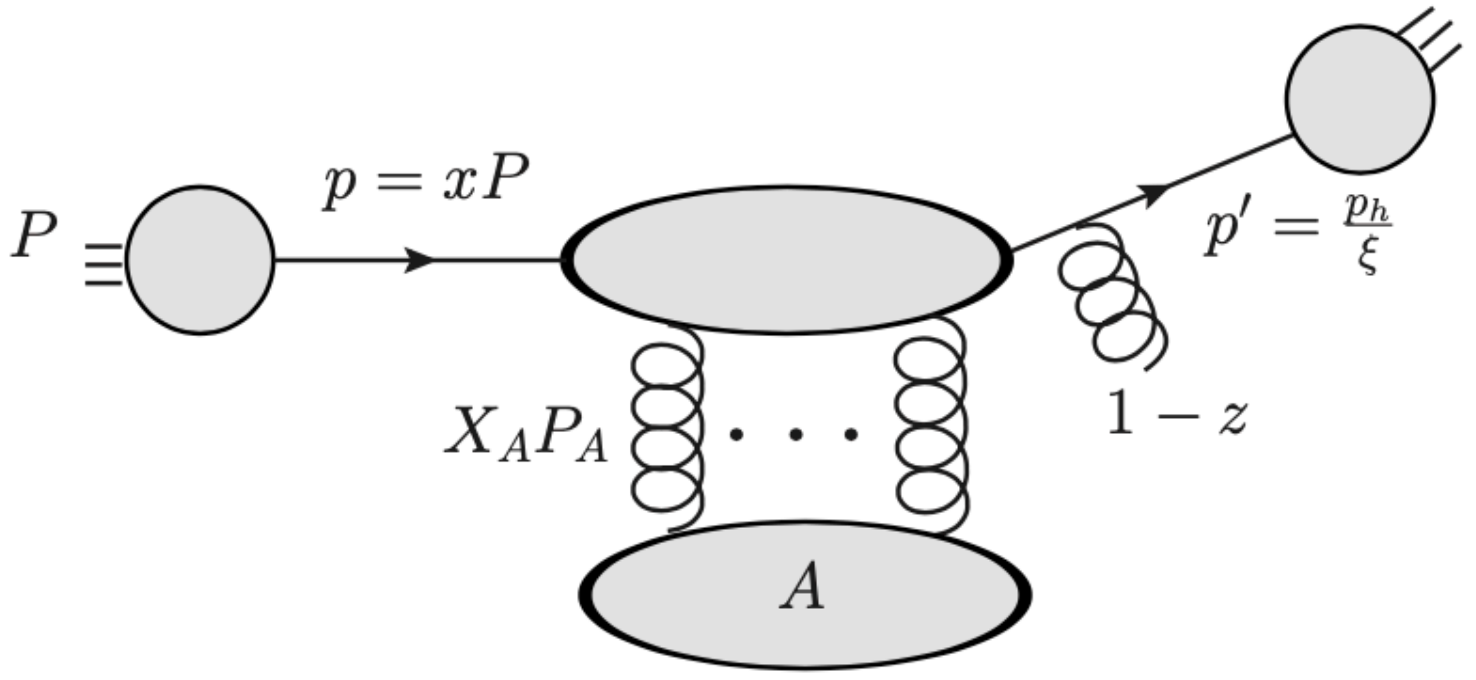
NLO:

- Demonstrate the CGC factorization at NLO
- Rapidity divergence leads to the BK evolution

Chirilli, Xiao, Feng, 2012

NLO:

$$d\sigma^{(1)} \propto \int^1 dz \left(-\frac{1}{\epsilon} P^{(1)}_{i\to j}(z) - \frac{1}{\epsilon}\frac{1}{z^2} P^{(1)}_{j\to k}(z) - P^{(1)}_{i\to j}(z) \ln \frac{\mu^2}{p_\perp^2} + \ldots \right) + \frac{1}{1-z}\left(\kappa^{(1)} \otimes \mathscr{F}_A - \alpha_s \left[\mathscr{F}_A\right]^2 \right) + \ldots$$



Rapidity divergence

BK evolution kernel

$z \to 1, y \to \infty$

Forward hadron production in CGC

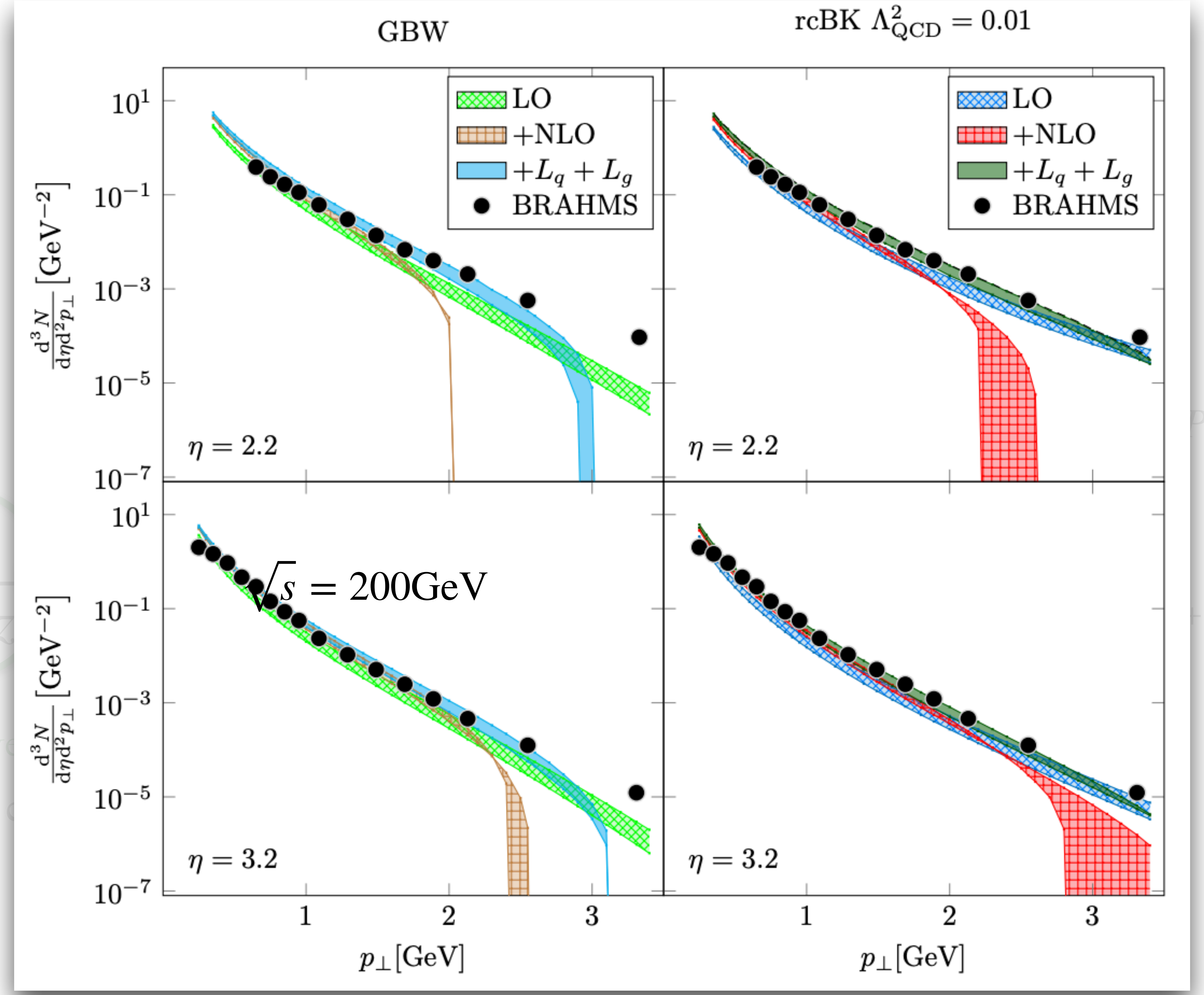
Negative cross section problem

Breakdown of CGC?

$$\alpha_s(p_T)\frac{p_T}{\sqrt{s}}\log\left[\frac{p_T}{\sqrt{s}}\right] \sim \mathcal{O}\left(10^{-2}\right)$$

Or breakdown of the perturbation series?

Large source ?



Watanabe, Xiao, Yuan and Zaslavsky 1505.05183

Forward hadron production in CGC

Large source ? Logs to worry about

$$\frac{d^2\hat\sigma^{(1)}}{dz\,d^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{dx_\perp}{\pi}\left\{\frac{1}{z}\tilde P_{i\to i}(z)\, e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\right.$$

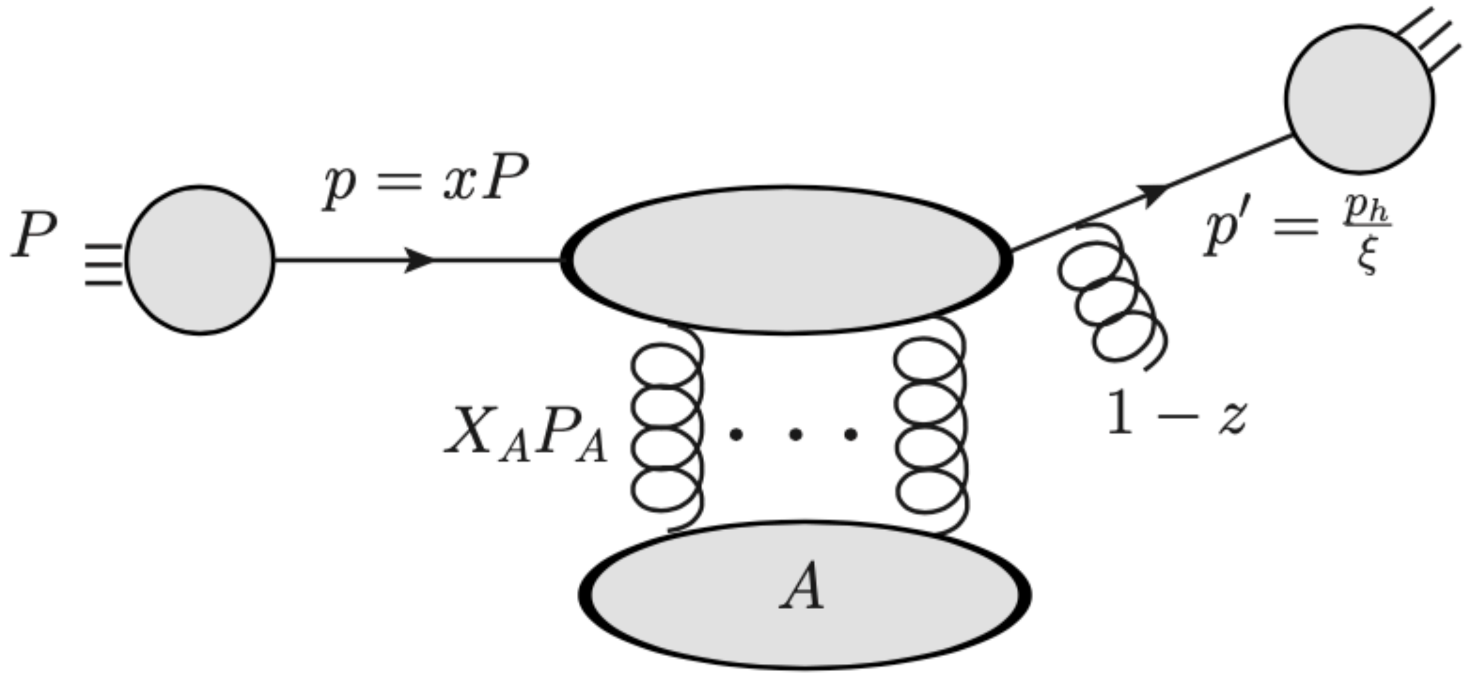
$$\left.+\,\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+\right\}W_{aa'}(x_\perp)+\dots$$

resummed, threshold PDFs/FFs

Kang, XL, 2019

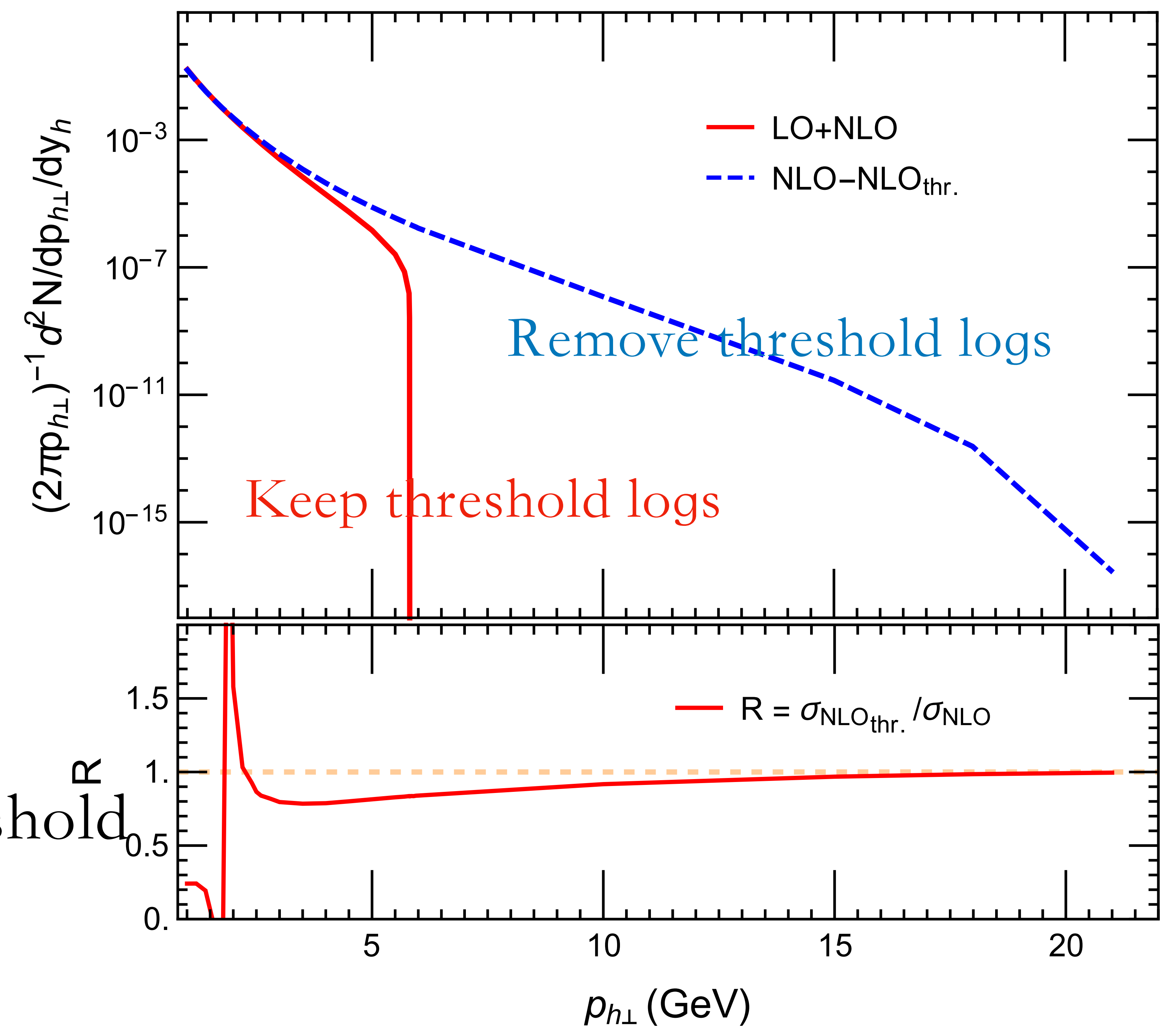
$X_f \equiv \nu/P_A$ resummed by BK

$$\tilde P \sim \frac{2}{(1-z)_+} \qquad \frac{p_{h,\perp}}{\sqrt{s}}e^y < z < 1$$



When $1-z$ is effectively small, the threshold contributions could be large and breaks down the fixed order results.

(Xiao, Yuan, 19, Kang, XL, 19)



Kang, Liu, XL, 2020

Forward hadron production in CGC

Large source ? Logs to worry about

$$\frac{\mathrm{d}^2\hat{\sigma}^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

resummed, threshold PDFs/FFs

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\right.$$

$$\left.+\,\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+\right\}W_{aa'}(x_\perp)+\dots$$

Kang, XL, 2019

$X_f \equiv \nu/P_A$

resummed by BK

$$\tilde{P}\sim\frac{2}{(1-z)_+}\qquad \frac{p_{h,\perp}}{\sqrt{s}}e^y < z < 1$$

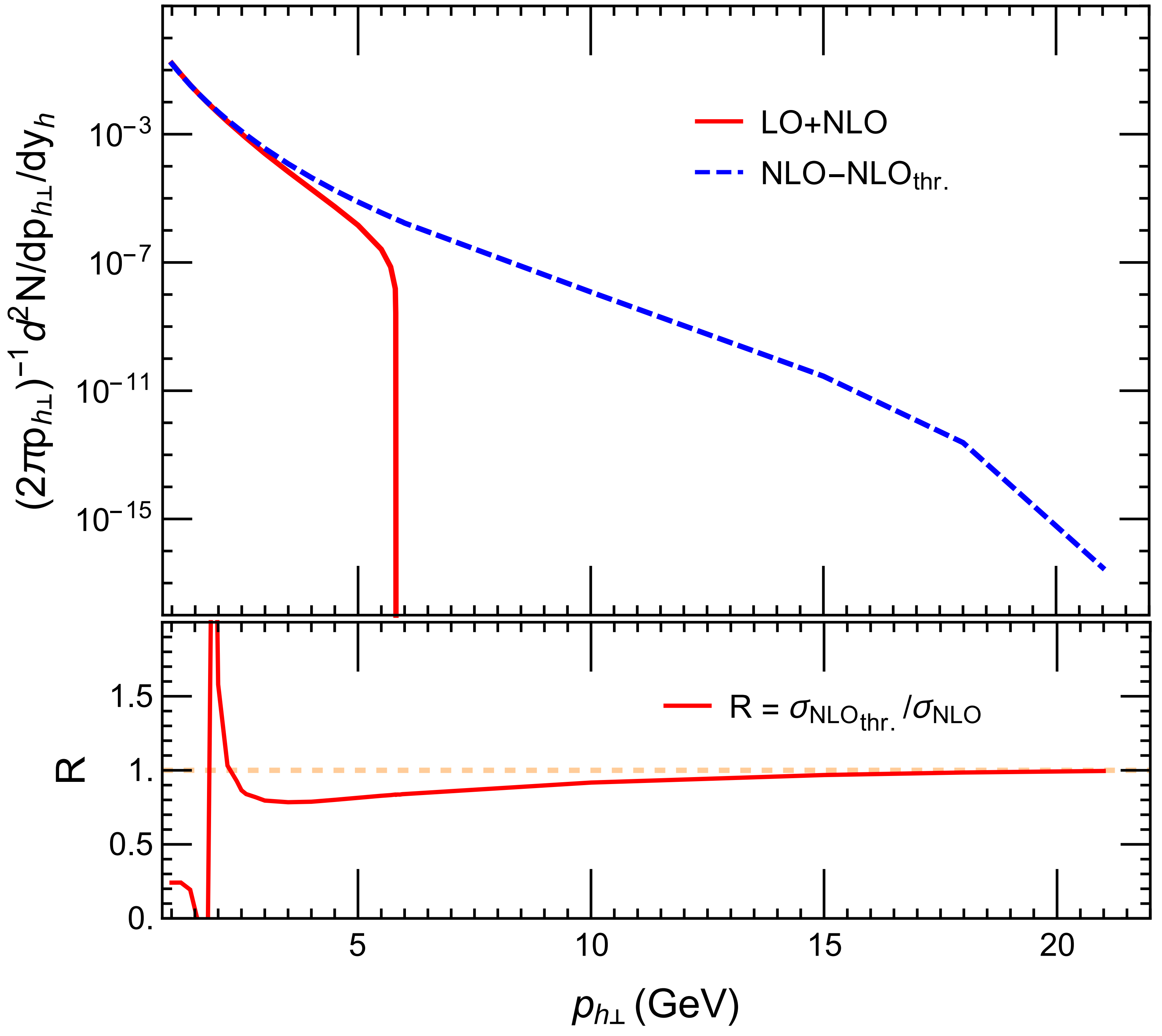
SEVERAL SOLUTIONS IN THE MARKET

Kang, Vitev, Xing 2014

Iancu, Mueller, Triantafyllopoulos, 2016

Duclou, Lappi, Zhu, 2017

Liu, Ma, Chao, 2019, Kang, Liu 2019, Kang, Liu, Liu, 2020 + ⋯



Kang, Liu, XL, 2020

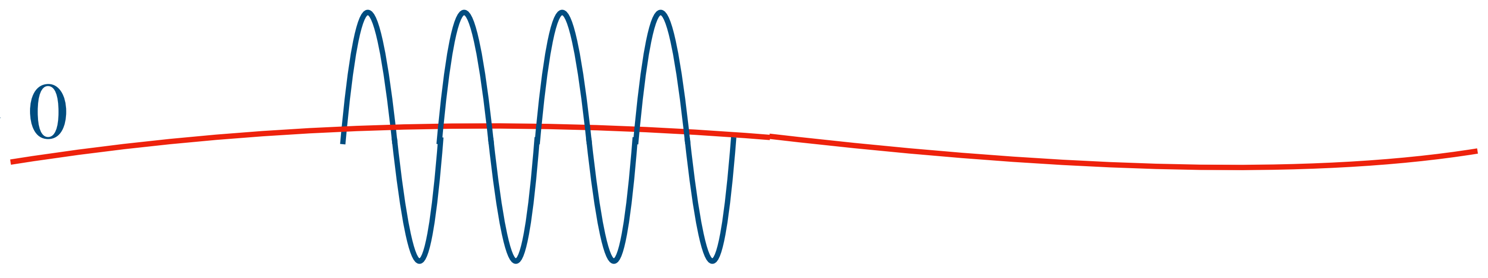
Forward hadron production in CGC

What we should expect from a perturbative calculation

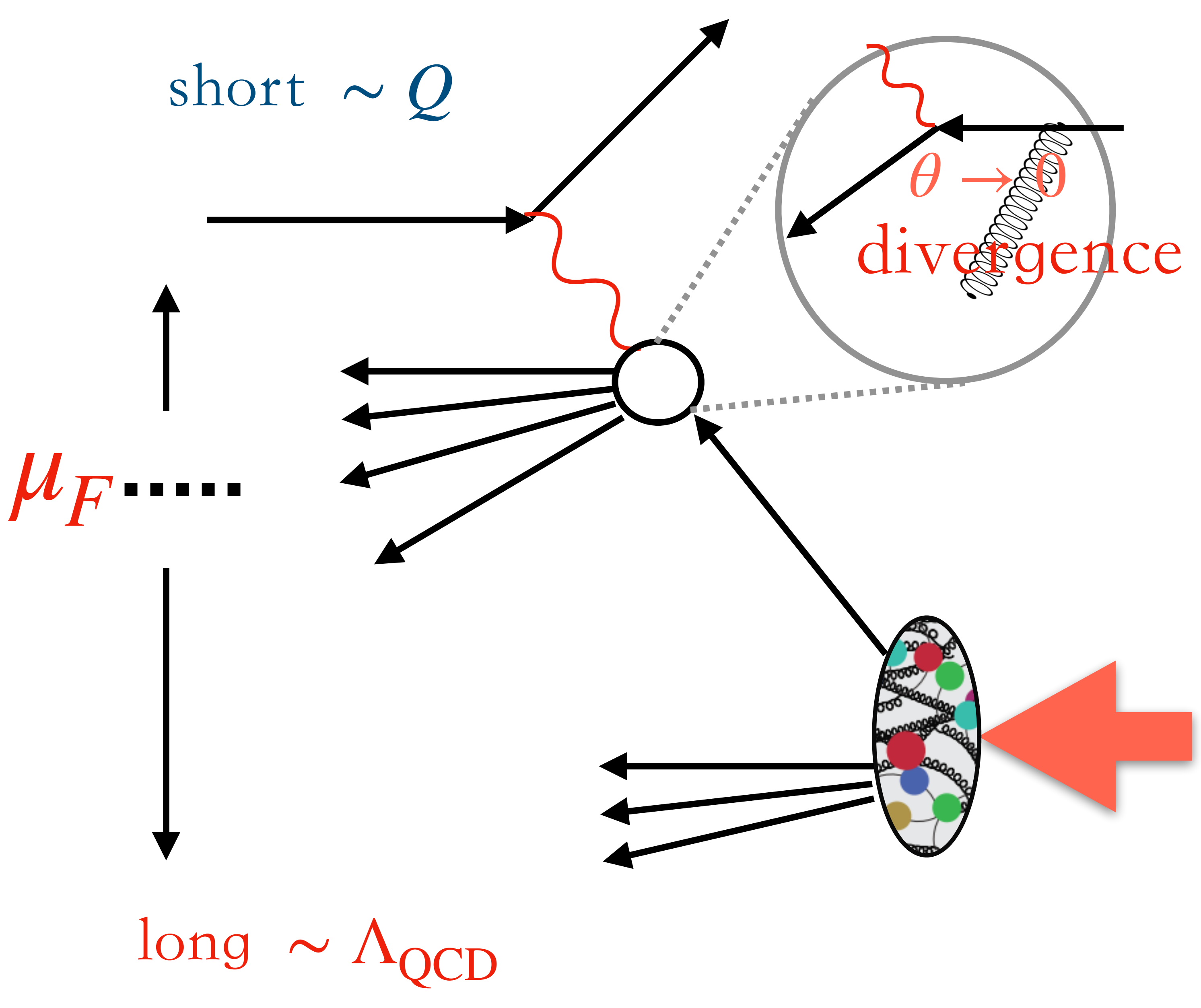
$$\sigma = \sum_i \int \mathrm{d}x\, \hat{\sigma}_i(x) f_{i/P}(x) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

$\lambda_{long} \sim \frac{1}{\Lambda_{\rm QCD}} \to \infty$ Crucial!!

$\lambda_{short} \sim \frac{1}{Q} \to 0$



EFT point of view:



Dim Reg

Preserve power counting

$$\sigma \propto \left[\hat{\sigma}_i^{(0)} + \alpha_s \left(-\frac{1}{\epsilon} P(x) - P(x) \log\frac{\mu_F}{Q} + \ldots\right)\right] f(x, \mu_F)$$

$\mu_F \, \mathrm{d}\hat{\sigma}_i / \mathrm{d}\mu_F = -P(x) \otimes \hat{\sigma}_i$

$\mu_F \, \mathrm{d}f_i / \mathrm{d}\mu_F = P(x) \otimes f_i$

$\mu_F = Q$ the scale to evaluate distribution w/o running $\hat{\sigma}_i$

$\theta > \Lambda_{\rm QCD}/Q$

Violate power counting

$$\sigma \propto \left(\hat{\sigma}_i^{(0)} + \alpha_s \int^{1-\Lambda_{\rm QCD}/Q} \frac{\mathrm{d}\theta}{1-\theta} \ldots\right) f_{i/P}(x, \Lambda_{\rm QCD})$$

Wrong scale to evaluate the distribution

Large logs, mixing with non-perturbative objects

Forward hadron production in CGC

What we should expect from a perturbative calculation

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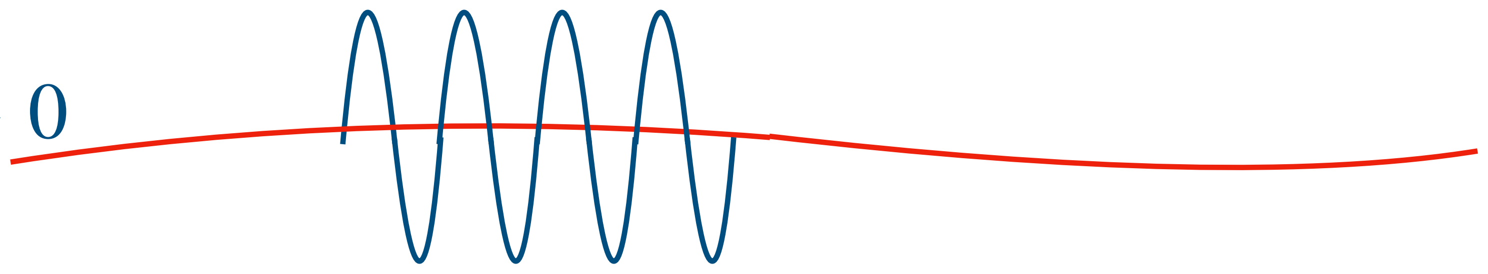
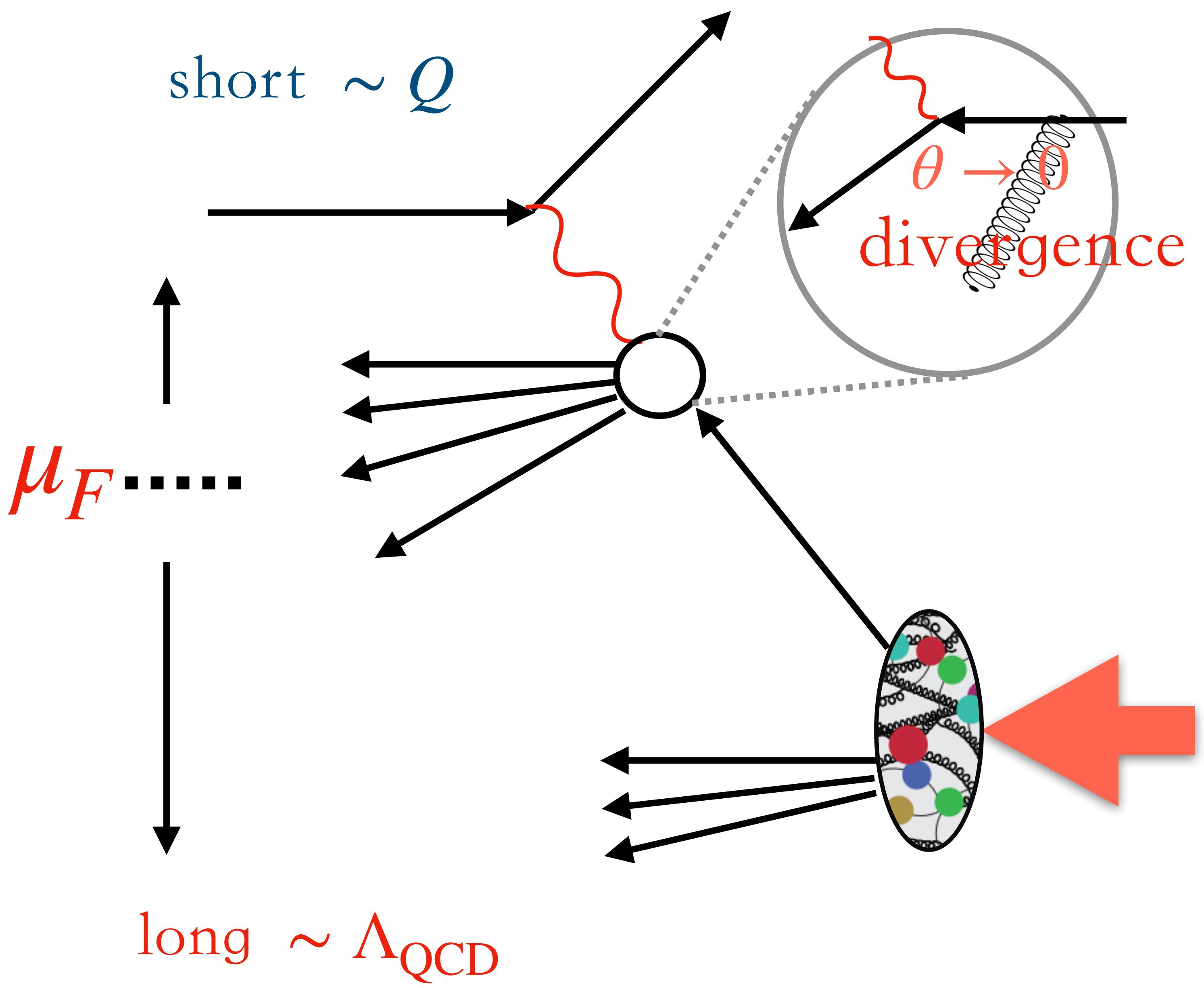


EFT point of view:

$$\sigma \propto \left[\hat{\sigma}_i^{(0)} + \alpha_s\left(-\frac{1}{\epsilon}P(x) - P(x)\log\frac{\mu_F}{Q} + \dots\right)\right] f(x,\mu_F)$$

$$\left[\frac{\ln^n(1-z)}{1-z}\right]_+ \cdots$$



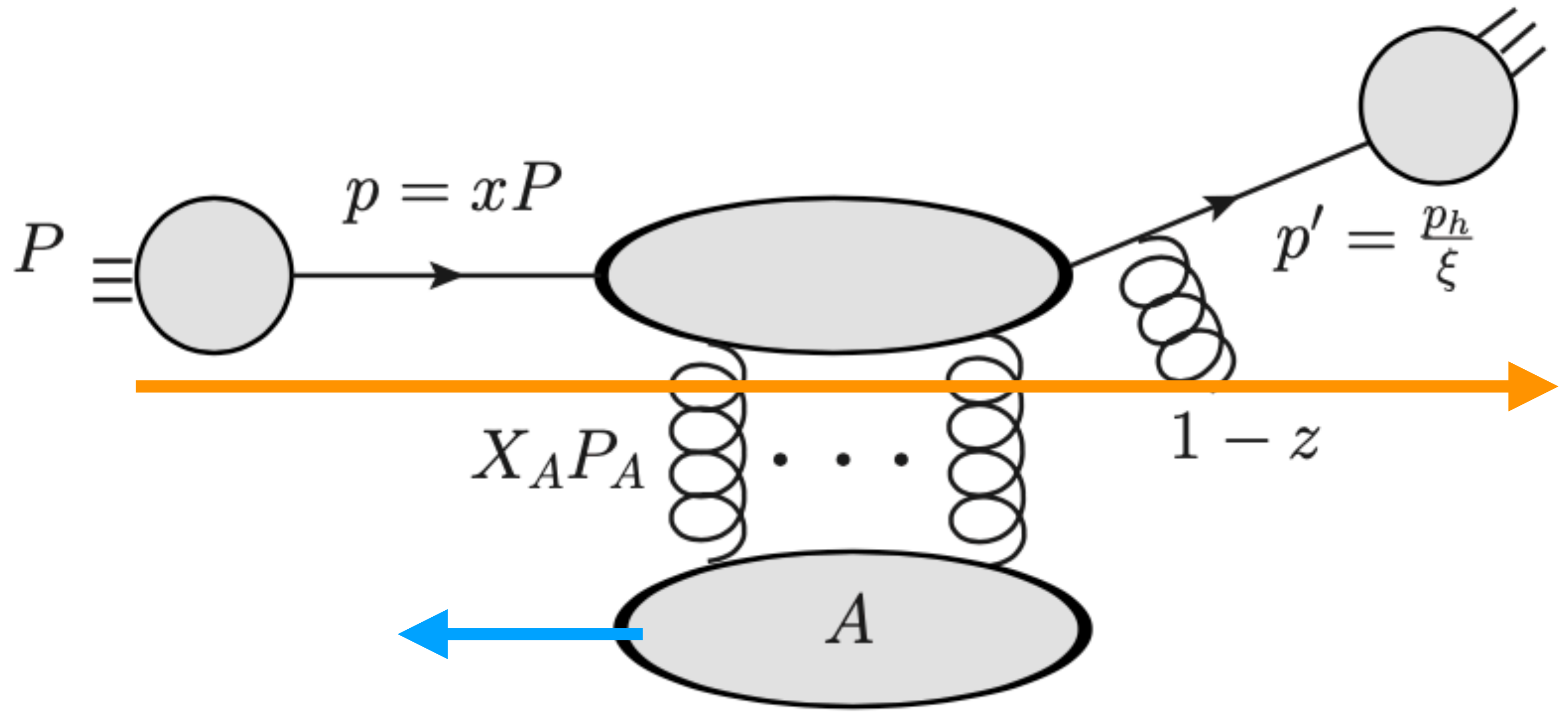
Dim Reg

Preserve power counting

$$\frac{\mathrm{d}^2\hat{\sigma}^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right) - \frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp} + \delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+\right\}W_{aa'}(x_\perp)+\dots$$

Additional perturbative re-factorization of the short distance coefficient is required to resum those logs, for instance SCET, conventional QCD approach ⋯

Power counting



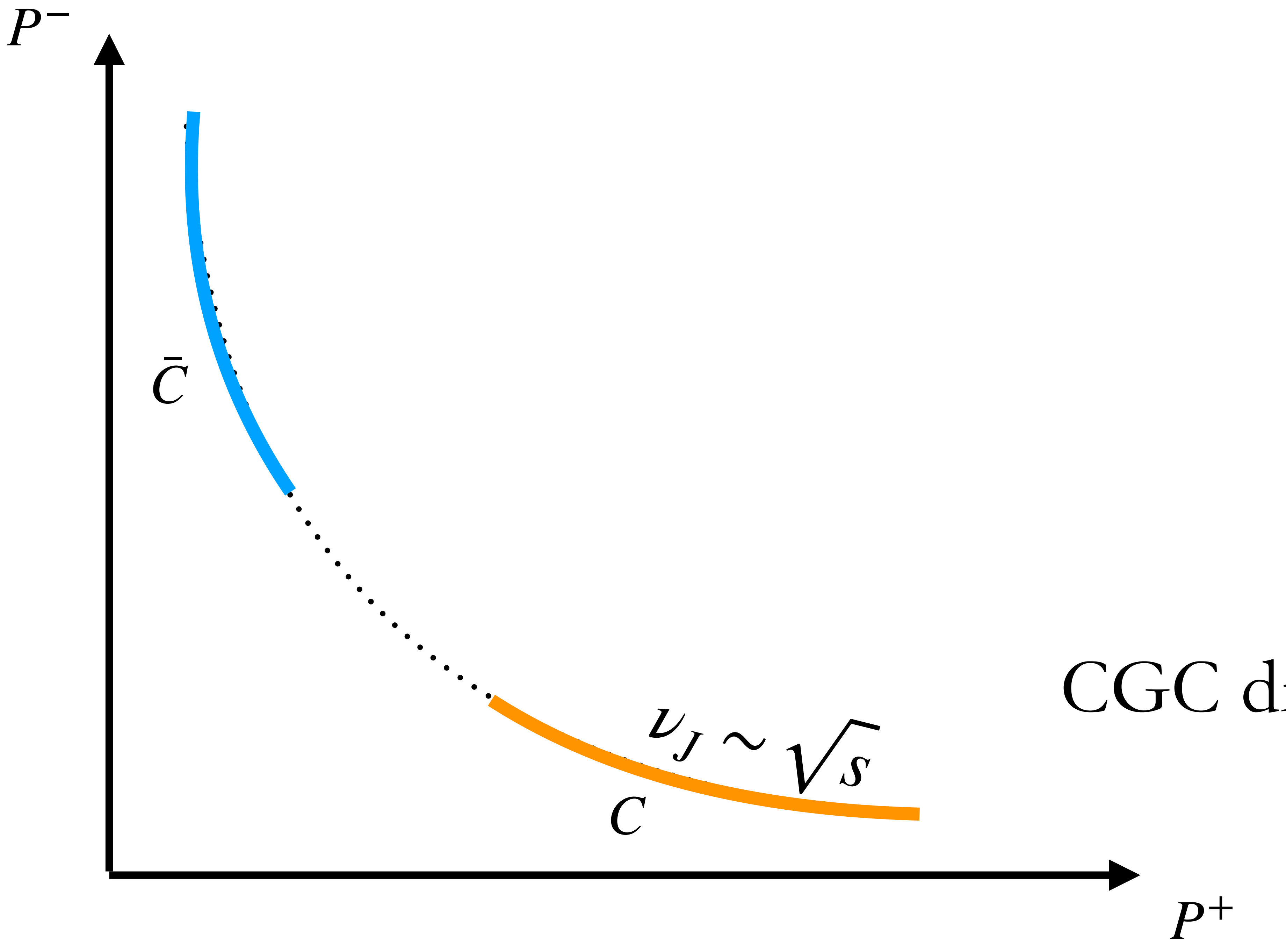
$$\lambda \sim \mathcal{O}\left(\frac{p_\perp}{\sqrt{s}}\right) \ll 1 \qquad z \sim 1-z \sim \mathcal{O}(1)$$

Away from the threshold

Modes contribute to $p_\perp$

$(+, \perp, -)$

$$P_c = \frac{\sqrt{s}}{2}(1, \lambda, \lambda^2)$$ forward rapidity



CGC dipole

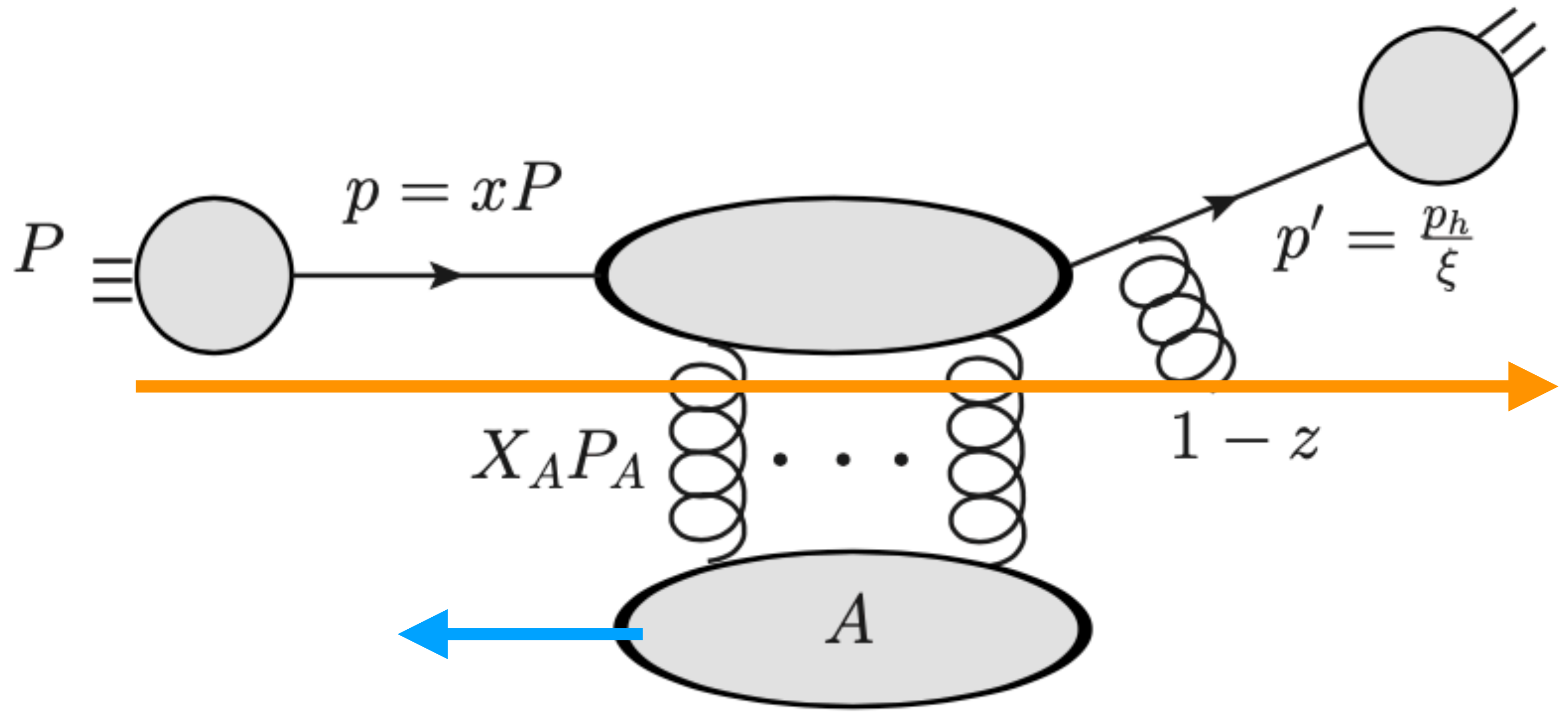
$$P_{\bar{c}} = \frac{\sqrt{s}}{2}(\lambda^2, \lambda, 1)$$

$$q \sim \Delta P_c \sim \Delta P_{\bar{c}} \sim \frac{\sqrt{s}}{2}(\lambda^2, \lambda, \lambda^2)$$

- Small x
- Off-shell Glauber mode

Kang, XL, 1910.10166

Power counting



$$\lambda \sim \mathcal{O}\left(\frac{p_\perp}{\sqrt{s}}\right) \ll 1 \qquad z \sim 1 - z \sim \mathcal{O}(1)$$

Away from the threshold

Modes contribute to $p_\perp$

$(+, \perp, -)$

SCET

$$P_c = \frac{\sqrt{s}}{2}(1, \lambda, \lambda^2)$$ forward rapidity

$$k_s = \frac{\sqrt{s}}{2}(\lambda, \lambda, \lambda)$$ central rapidity

CGC dipole

$$P_{\bar{c}} = \frac{\sqrt{s}}{2}(\lambda^2, \lambda, 1)$$

$$q \sim \Delta P_c \sim \Delta P_{\bar{c}} \sim \frac{\sqrt{s}}{2}(\lambda^2, \lambda, \lambda^2)$$

- Small x
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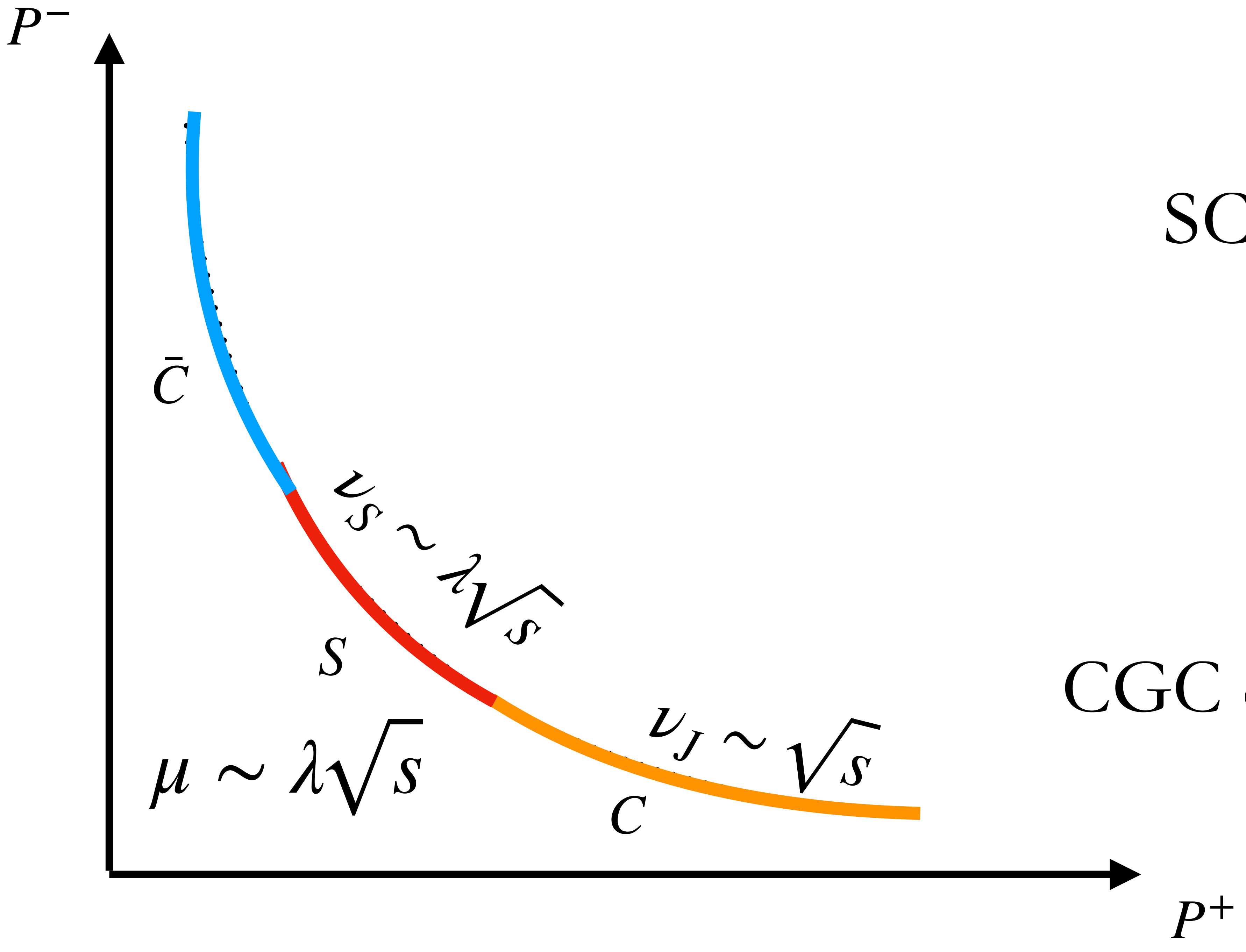
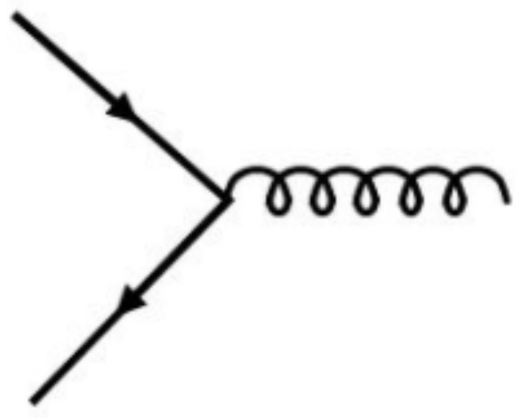


Kang, XL, 1910.10166

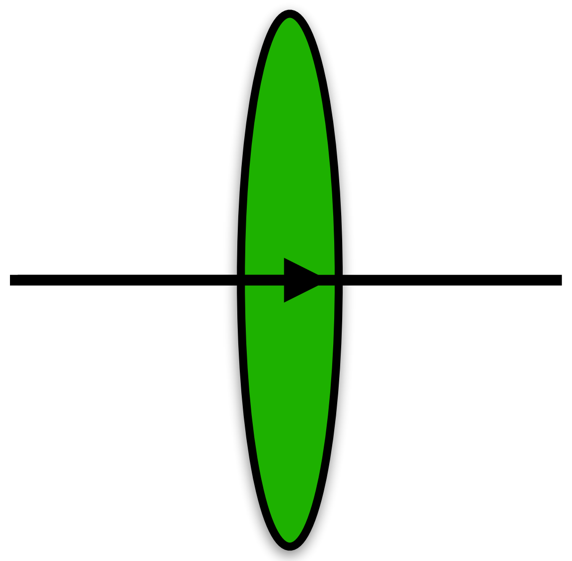
Power counting

Collinear Interactions $\bar{n}\cdot A = 0$



$$= i g_s t^a \left(n^\alpha + \frac{\not{q}_\perp^\alpha \not{\gamma}_\perp}{\bar{n}\cdot q} + \frac{\not{\gamma}_\perp \not{q}_\perp^{\prime\,\alpha}}{\bar{n}\cdot q'} \right) \frac{\not{\bar{n}}}{2}$$

Bad components in terms of the good components

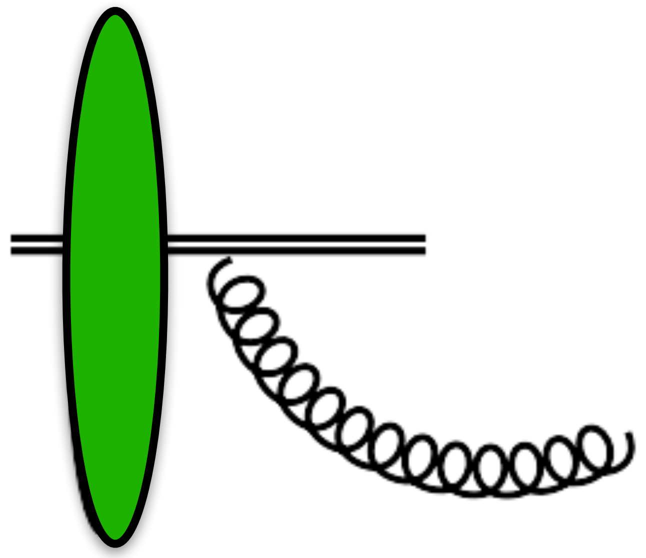


$$= \frac{\not{\bar{n}}}{2} \int \mathrm{d}b_\perp \, W_{ij}(b_\perp) \;\; 2(2\pi)\delta(p^+ - p'^+)$$

$+ \ldots$

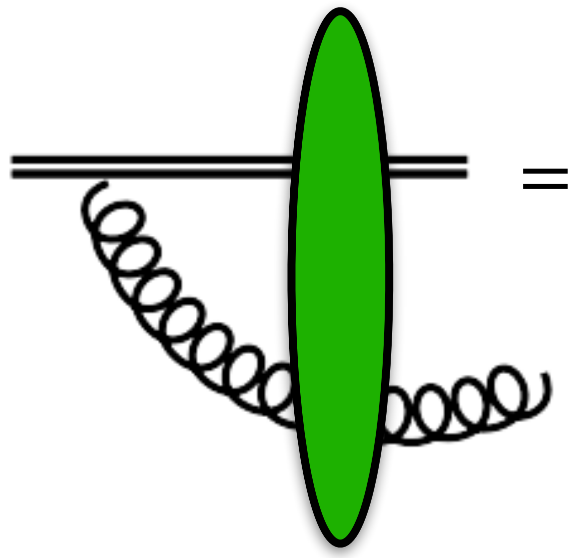
Soft Current

rapidity regulator Chiu, et al. 2011



$$= -\, g_s \mathbf{T}^a \frac{n^\alpha}{n\cdot k} \left(\frac{\nu}{k_\perp}\right)^{\frac{\eta}{2}} e^{-\frac{\eta}{2}|y|}$$

Nothing but eikonal approximation

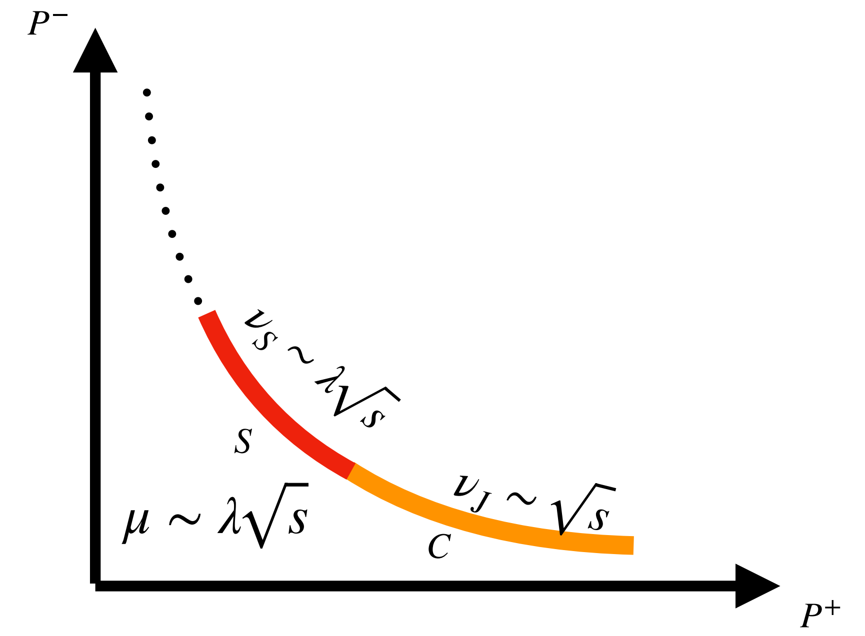


$$= i \frac{g_s \mathbf{T}^b}{(4\pi)^{1-\epsilon}} \mu^{2\epsilon} \nu^{\frac{\eta}{2}} e^{-\frac{\eta}{2}|y|} \frac{\Gamma[1-\epsilon-\eta/4]}{\Gamma[1+\eta/4]} \int \mathrm{d}x_\perp \, r_\perp^{\prime\,\alpha} \left[\frac{r_\perp^{\prime\,2}}{4}\right]^{-1+\epsilon+\eta/4} W_{ab}(x_\perp) e^{i k_\perp \cdot x_\perp}$$

Can be viewed as additional sources of soft radiations

Kang, XL, 1910.10166

Power counting



Collinear contribution

$$-\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)+\ldots$$

- Reproduce exactly the NLO using LFPT
- Regulator to turn rapidity divergence to pole (forward)
- Generate the rapidity scale for the collinear sector, $\nu_J \sim p^+$

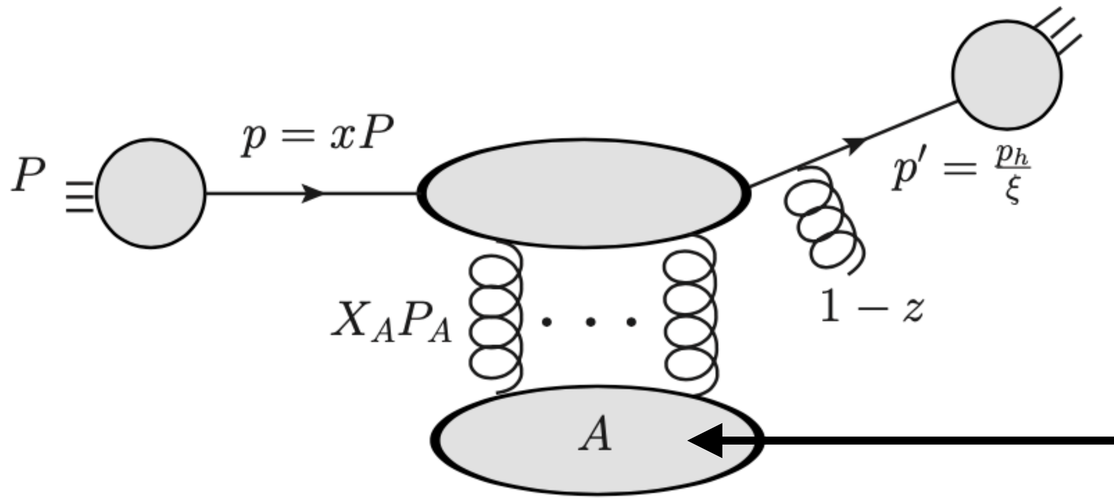
$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_h \mathrm{d}^2 p_{h\perp}} = \sum_{i,j=g,q}\frac{1}{4\pi^2}\int\frac{\mathrm{d}\xi}{\xi^2}\frac{\mathrm{d}x}{x}\, z x f_{i/P}(x,\mu) D_{h/j}(\xi,\mu)$$
$$\times\int \mathrm{d}^2 b_\perp \mathrm{d}^2 b'_\perp\, e^{ip'_\perp\cdot r_\perp}$$
$$\times\left\langle \langle \mathcal{M}_0(b'_\perp)|\,\boldsymbol{\mathcal{J}}(z,\mu,\nu,b_\perp,b'_\perp)\boldsymbol{\mathcal{S}}(\mu,\nu,b_\perp,b'_\perp)|\mathcal{M}_0(b_\perp)\rangle\right\rangle_\nu .$$

Soft contribution

$$\delta(1-z)\left(\frac{2}{\eta}+\ln\frac{\nu^2}{p_\perp^2}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)$$

$$+\delta(1-z)\frac{\alpha_s}{\pi}\frac{N_C}{2}\frac{1}{\pi}\left(\frac{\ln(r'^2_\perp p_\perp^2/c_0^2)}{r'^2_\perp}+\frac{\ln(r''^2_\perp p_\perp^2/c_0^2)}{r''^2_\perp}+\frac{2r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\ln\frac{r''_\perp r'_\perp p_\perp^2}{c_0^2}\right)_+\mathscr{F}$$

- Poles (forward + backward)
- Reproduce the kinematic constraints, automatically arises
- Rapidity scale for the soft sector, $\nu_S \sim p_\perp$



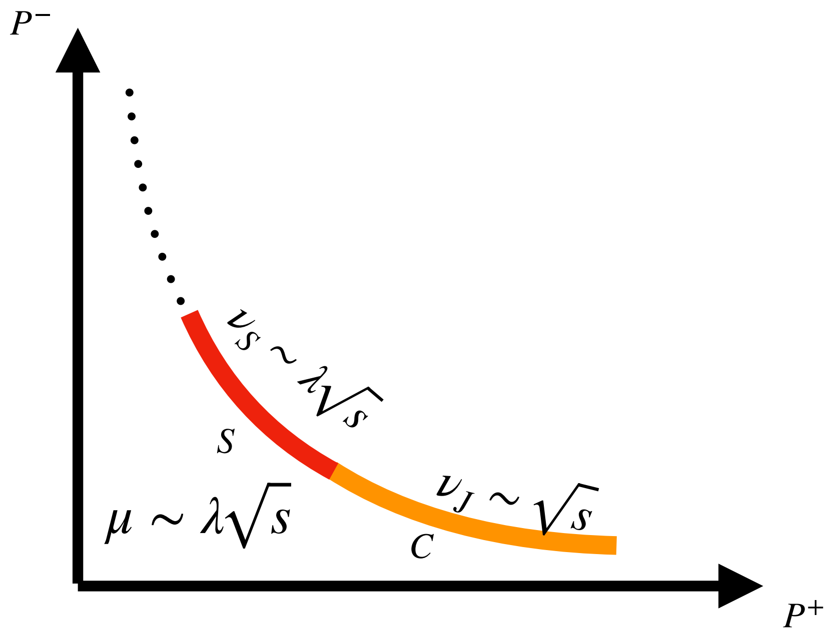
$$\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p_\perp^2/p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)+\ldots$$

- Remaining rap. pole to be absorbed (cancelled) by the small-x distribution
- Rap. scale arises naturally, $\nu \sim p_\perp^2/p^+ \sim x_A P_A \sim e^{-Y_A}$
- Reproduce the BK equation

See also, Liu, Ma, Chao 1909.02370

Kang, XL, 1910.10166

Power counting



Collinear contribution

$$-\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)+\ldots$$

- Reproduce exactly the NLO using LFPT
- Regulator to turn rapidity divergence to pole (forward)
- Generate the rapidity scale for the collinear sector, $\nu_J \sim p^+$

Soft contribution

$$\delta(1-z)\left(\frac{2}{\eta}+\ln\frac{\nu^2}{p_\perp^2}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)$$

$$+\delta(1-z)\frac{\alpha_s}{\pi}\frac{N_C}{2}\frac{1}{\pi}\left(\frac{\ln(r_\perp'^2\,p_\perp^2/c_0^2)}{r_\perp'^2}+\frac{\ln(r_\perp''^2\,p_\perp^2/c_0^2)}{r_\perp''^2}+\frac{2r_\perp'\cdot r_\perp''}{r_\perp'^2 r_\perp''^2}\ln\frac{r_\perp'' r_\perp' p_\perp^2}{c_0^2}\right)_+\mathscr{F}$$

- Poles (forward + backward)
- Reproduce the kinematic constraints, automatically arises
- Rapidity scale for the soft sector, $\nu_S \sim p_\perp$

$$\tilde{\sigma}\sim\delta(1-z)\left(\frac{1}{\eta}+\ln\frac{\nu}{p_\perp^2/p^+}\right)\left(\kappa^{(1)}\otimes\mathscr{F}-\alpha_s[\mathscr{F}]^2\right)+\ldots$$

Same log

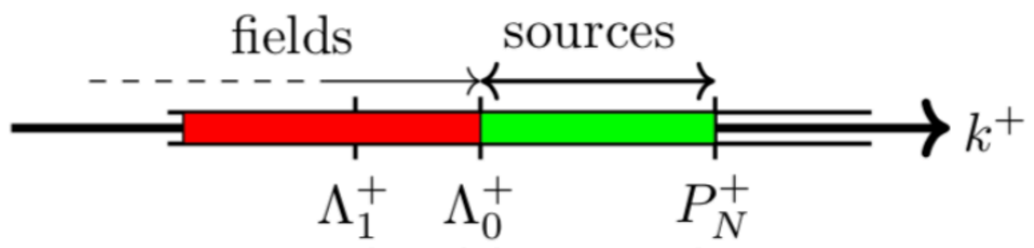
$$\frac{d^2\hat{\sigma}^{(1)}}{dz d^2p_\perp'}\propto-\frac{\alpha_s}{2\pi}\mathbf{T}_i^2P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p_\perp'\cdot r_\perp}\right)$$
$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{dx_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p_\perp'\cdot r_\perp'}\frac{r_\perp'\cdot r_\perp''}{r_\perp'^2 r_\perp''^2}\right.$$
$$\left.+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r_\perp'^2 r_\perp''^2}\right]_+\right\}W_{aa'}(x_\perp)+\ldots$$

In principle, we need to worry about the hierarchy and turn on resummation.

$$\boldsymbol{U}_J\boldsymbol{U}_S=\exp\left[\gamma_\nu\ln\frac{\nu\,\nu_J}{\nu_S^2}\right]=\exp\left[\gamma_\nu\ln\frac{X_f}{X_A}\right]$$

$$\gamma_\nu=-\frac{\alpha_s}{\pi}\int\frac{dx_\perp}{\pi}\left[\frac{r_\perp^2}{r_\perp'^2 r_\perp''^2}\right]_+\mathbf{T}_i^a\mathbf{T}_j^{a'}\,W_{aa'}(x_\perp)$$

$\nu \sim X_A P_A$



- Complicated evolution
- Is 1, once the CGC rap. scale is made. All logs are then absorbed into the small-x distribution

Liu, Kang, XL, 2004.11990

Threshold $1-z \sim \mathcal{O}(\lambda)$ Liu, Kang, XL, 2004.11990

$$\frac{d^2\hat{\sigma}^{(1)}}{dz\,d^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

resummed

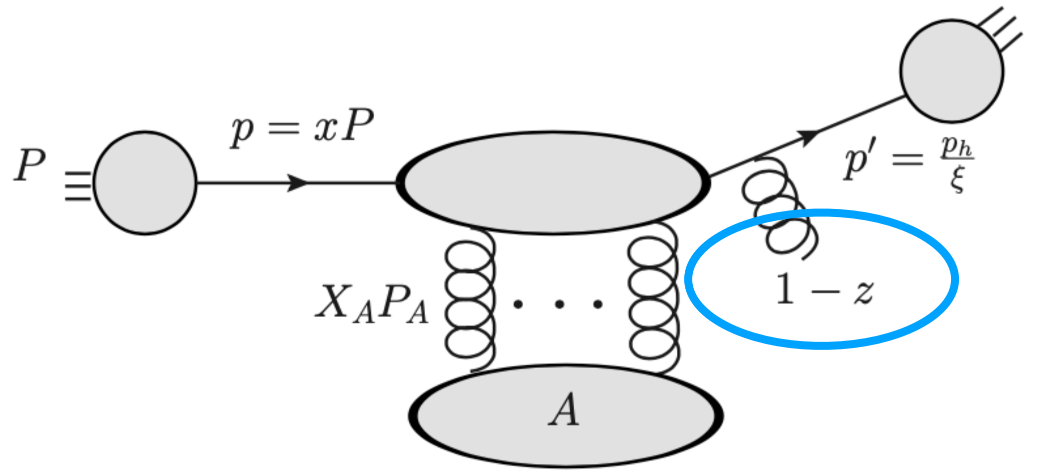
$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{dx_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{r'^2_\perp r''^2_\perp}\right.$$

$$\left.+\,\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{r'^2_\perp r''^2_\perp}\right]_+\right\}W_{aa'}(x_\perp)+\dots$$

resummed

$$\tilde{P} \sim \frac{2}{(1-z)_+}$$

$$\frac{p_{h,\perp}}{\sqrt{s}}e^y < z < 1$$



No real energetic collinear radiations allowed. Collinear momentum occurs only in the virtual loops

$$\frac{d\sigma}{dy_h d^2p_{h\perp}} = \sum_{i,j=g,q}\frac{1}{4\pi^2}\int\frac{d\xi}{\xi^2}\frac{dx}{x}\,zx f_{i/P}(x,\mu)D_{h/j}(\xi,\mu)$$

$$\times\int d^2b_\perp d^2b'_\perp\, e^{ip'_\perp\cdot r_\perp}$$

$$\times\left\langle\langle\mathcal{M}_0(b'_\perp)|\,\boldsymbol{\mathcal{J}}(z,\mu,\nu,b_\perp,b'_\perp)\boldsymbol{\mathcal{S}}(\mu,\nu,b_\perp,b'_\perp)|\mathcal{M}_0(b_\perp)\rangle\right\rangle_\nu .$$

$$\boldsymbol{\mathcal{J}}(z) \to \boldsymbol{\mathcal{J}}_{thr.} \qquad \boldsymbol{\mathcal{S}} \to \boldsymbol{\mathcal{S}}_{thr.}(z)$$

Contains only loops

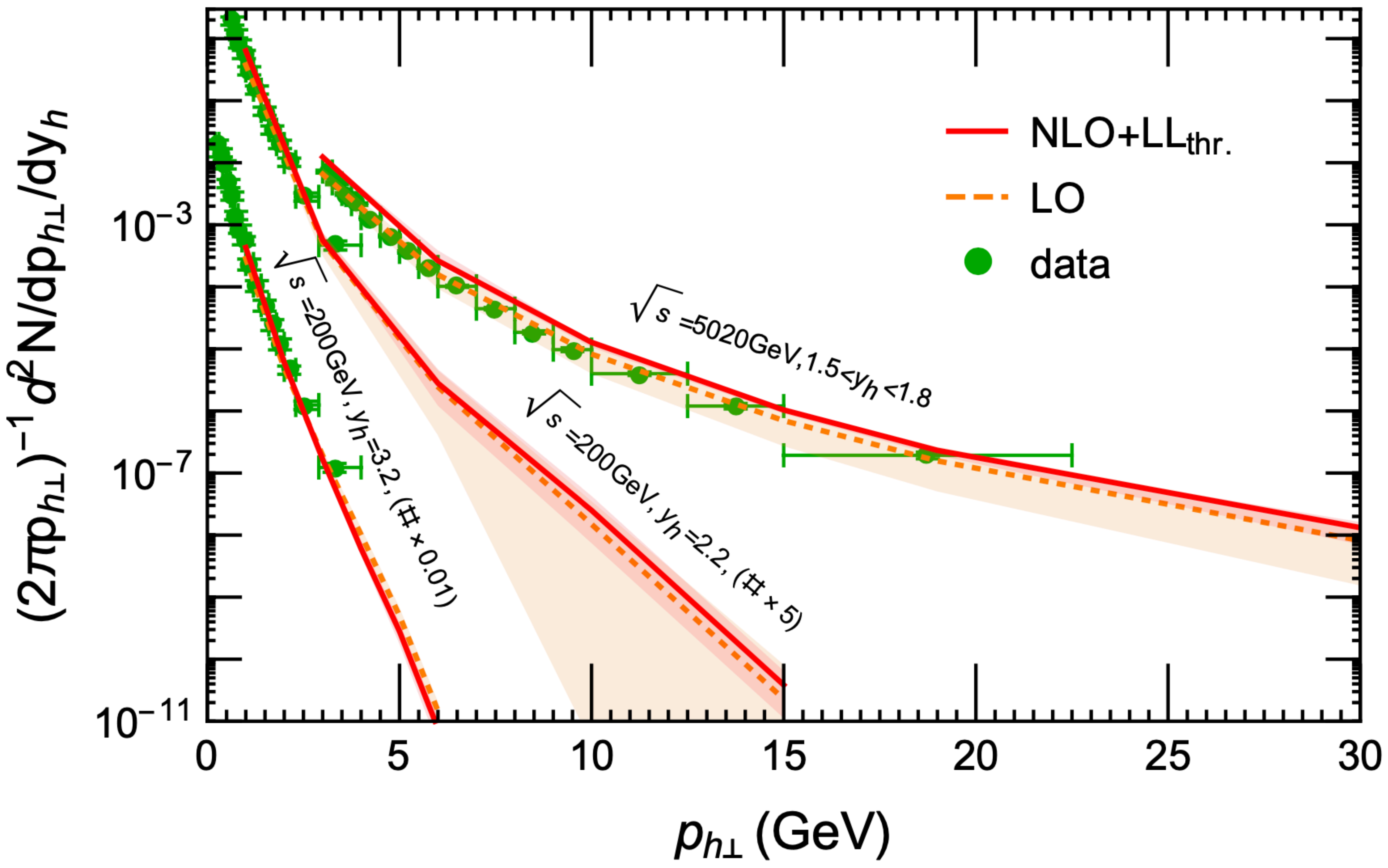
$$\boldsymbol{U}_{J_{thr.}}\boldsymbol{U}_{S_{thr.}} = \exp\left[-\frac{\alpha_s}{\pi}\int\frac{dx_\perp}{\pi}\left(\ln\frac{\nu_S}{\nu_J}I_{BK,r}\right.\right.$$

$$\left.\left.+\ln\frac{X_f}{X_A}I_{BK}\right)\mathbf{T}_i^a\mathbf{T}_j^{a'}\,W_{aa'}(x_\perp)\right]$$

- Novel threshold resummation structure.
- CGC rap. scale choice can not resum threshold logs
- Dynamical scale X_f can be determined numerically to minimize the evolution.

Threshold

Numerics



- Dynamical scale X_f determined numerically to minimize the evolution.
- Stay positive
- good agreements with data and ready for phenomenological applications

Liu, Kang, XL, 2004.11990

Conclusions

- CGC physics to a precision physics playground
- Power counting plays the role with additional soft corrections and threshold resummation resolves the negative issue for the forward hadron inclusive production
- More to study in the future:
  - Jet, DIS, spin $\cdots$
  - Push precisions with SCET, Multi-loop techniques