



Efficient computation of NLO corrections for HH production and HZ production via gluon fusion

G. Wang, Wang, Xu, Xu, Yang, arXiv:2010.15649, 2106.xxxxx

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微扰量子场论研讨会

05/15/2021

- Introduction to Higgs physics
- HH and HZ production @LHC.
- Calculation of the amplitudes.
- Numeric calculation and discussions.
- Summary and outlook.

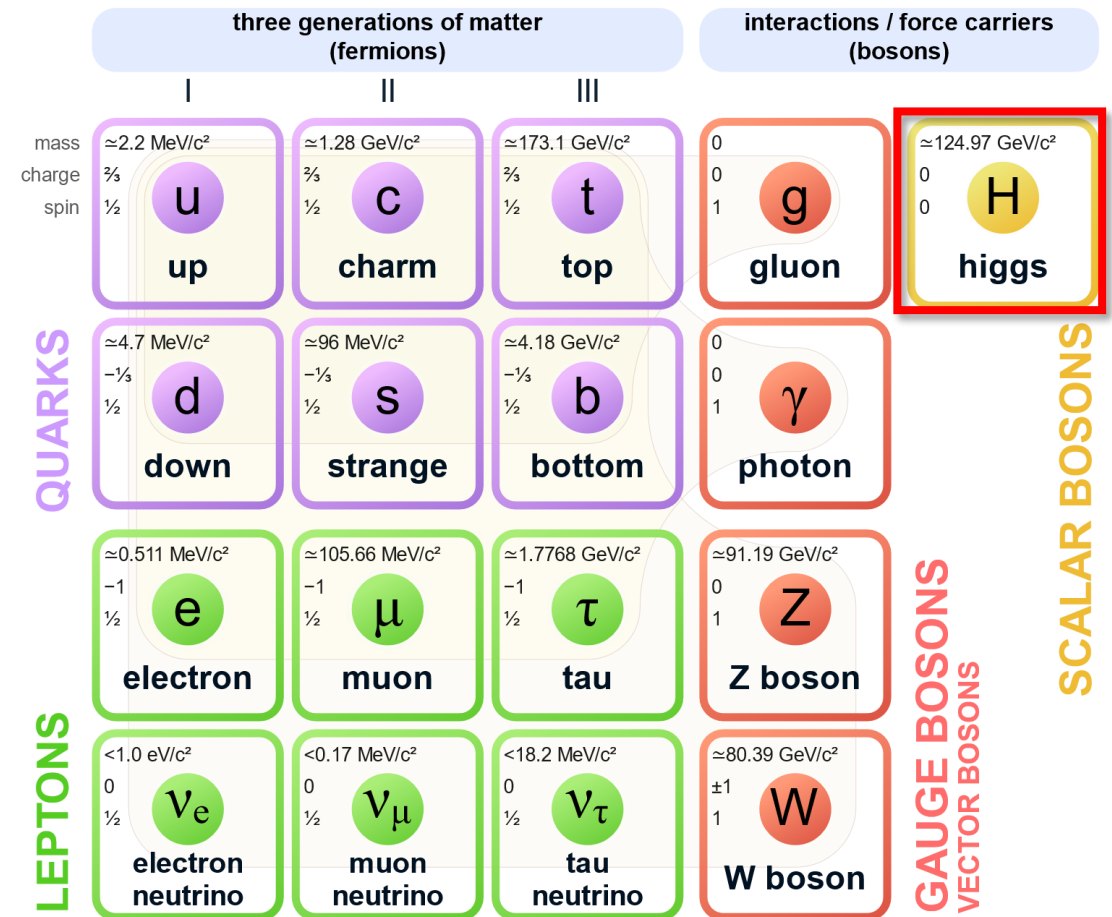
Basic facts about Higgs

- Large mass: $m_H \sim 125 \text{ GeV}$
- Small width: $\Gamma_H \sim 10 \text{ MeV}$
- EWSB: Fermion mass origin
Hierarchy problem
Vacuum stability
- Interactions:

$$g_{Hf\bar{f}}, g_{HVV}, g_{HHVV}$$

$$g_{HHH}, g_{HHHH}$$
- Production and decay :

Standard Model of Elementary Particles



Basic facts about Higgs

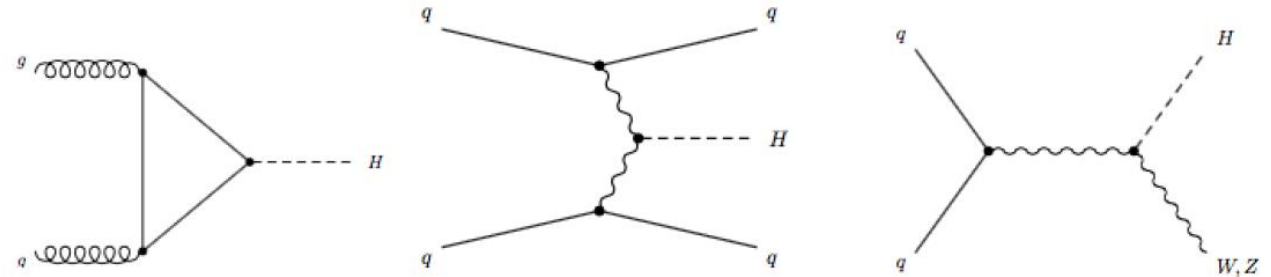
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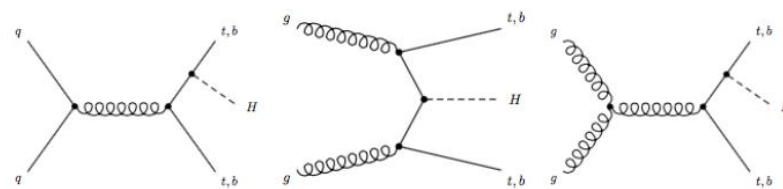
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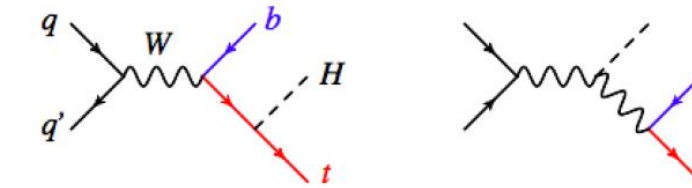
$gg \rightarrow H$

Vector boson fusion

Associated $V + H$ production



$\bar{t}t + H$ production

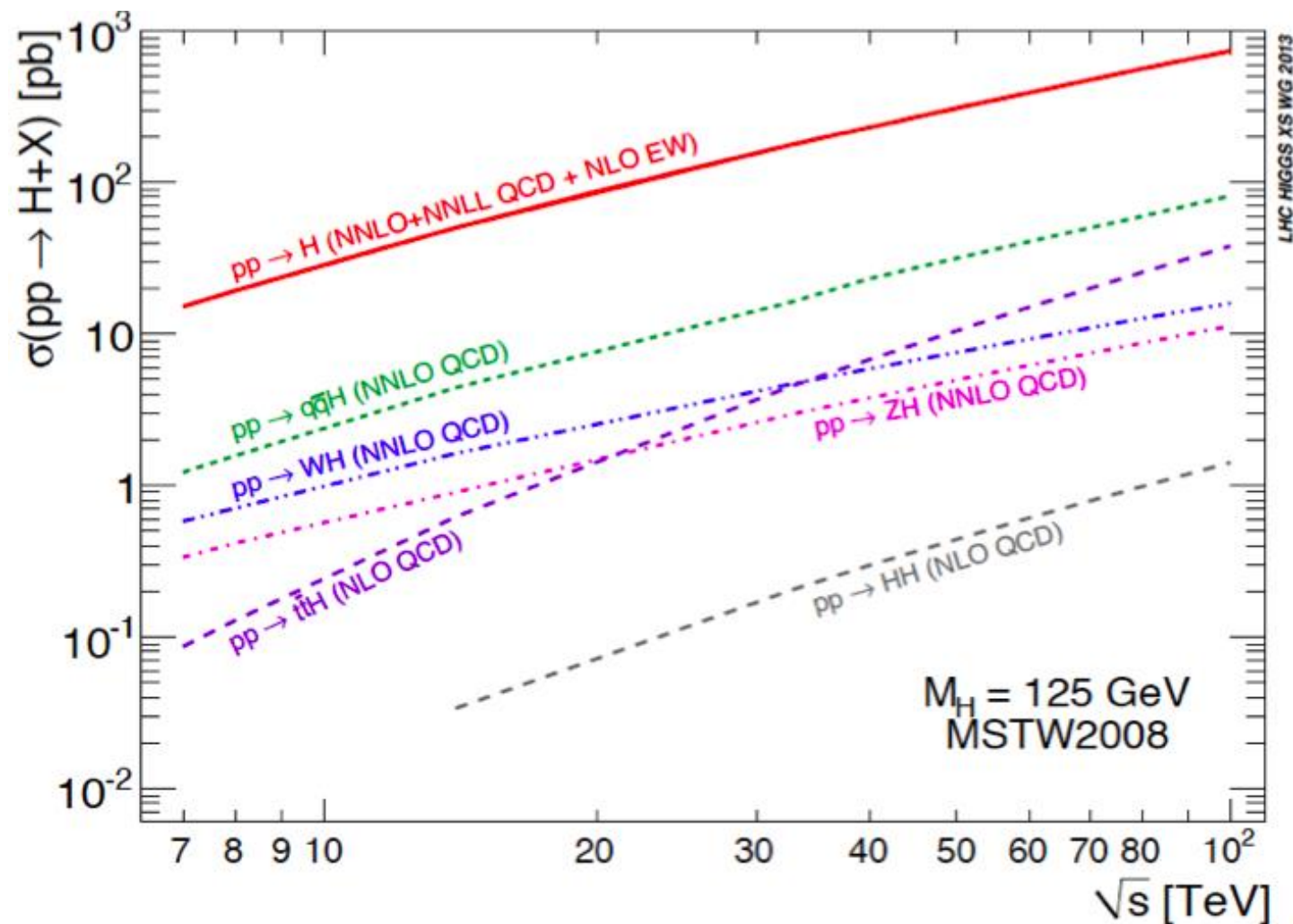


s -channel diagrams for $t + H$ production

J. Ellis: 1702.05436

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R. Contino et al: 1606.09408

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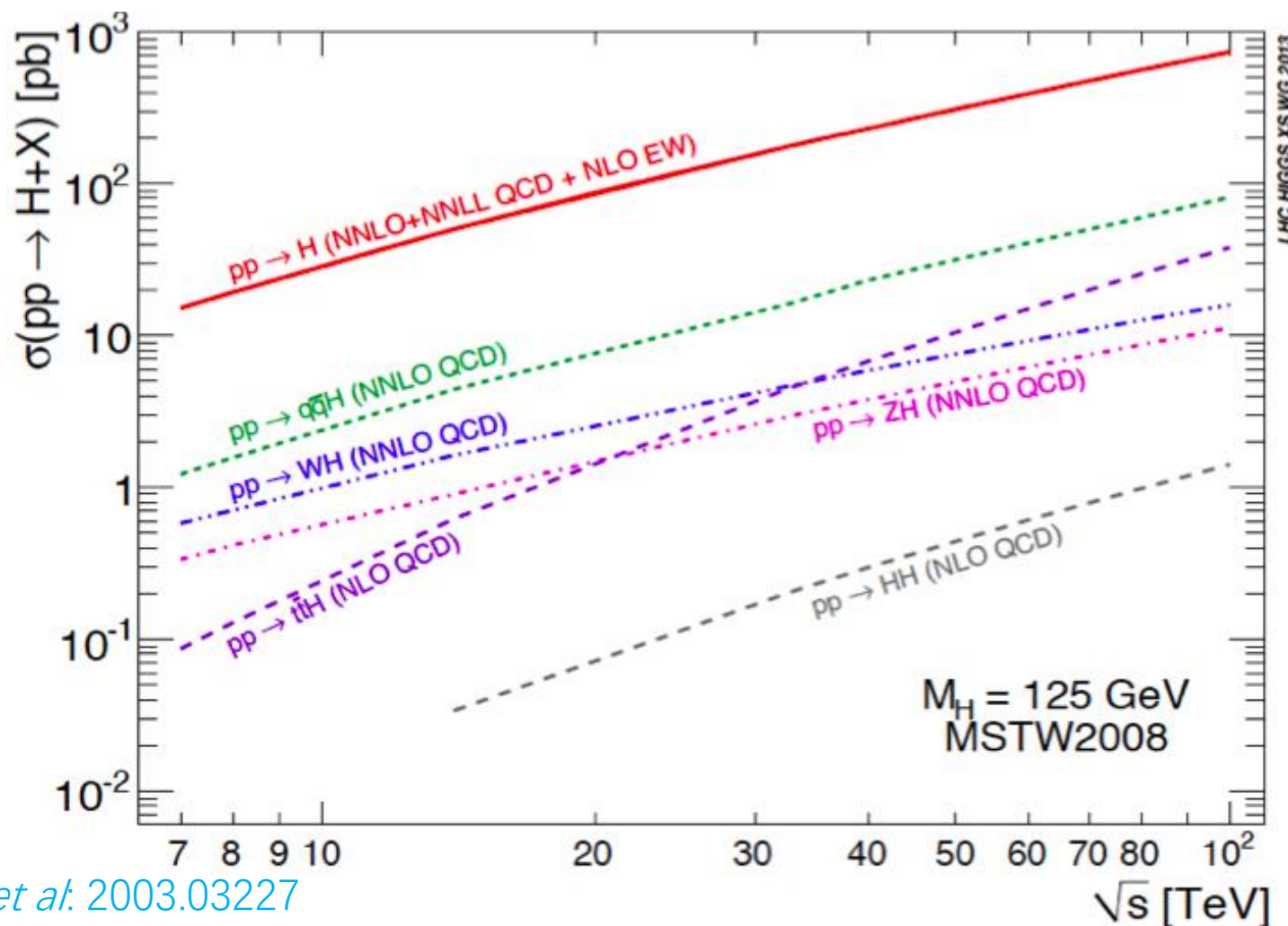
$$\sigma_{HH} \sim 30 \text{ fb} \quad @14\text{TeV LHC}$$

$$\sigma_{HZ} \sim 900 \text{ fb} \quad @14\text{TeV LHC}$$

[J. Baglio et al. 2003.03227](#)

[O. Breina, et al. 1111.0761](#)

[R. Contino et al. 1606.09408](#)



HH and HZ production

HH production

HZ production

- LO:

E. Glover, *et al.* Nucl. Phys. B 309, 282(1988)
T. Plehn, *et al.* Nucl. Phys. B 479, 46 (1996)

B. A. Kniehl, Phys. Rev. D 42 (1990) 2253

- NLO: absence of full analytic results

- Numeric results

e.g. S. Borowka, *et al.* 1604.06447
J. Baglio, *et al.* 2003.03227

L. Chen , *et al.* 2011.12325

- Approx. analytic results:

$1/m_t$, p_T , $1/\hat{s}$ expansion

e.g. S. Dawson, *et al.* hep-ph/9805244
R. Bonciani, *et al.* 1806.11564
J. Davies, *et al.* 1801.09696

e.g. L. Altenkamp,, *et al.* 1211.5015
J. Davies, *et al.* 2011.12314
L. Alasfar, *et al.* 2103.06225

HH and HZ production

HH production

- NLO: absence of full analytic results
 - Combination of former results:
 - high-energy expansion & sector decomposition via Padé approx.

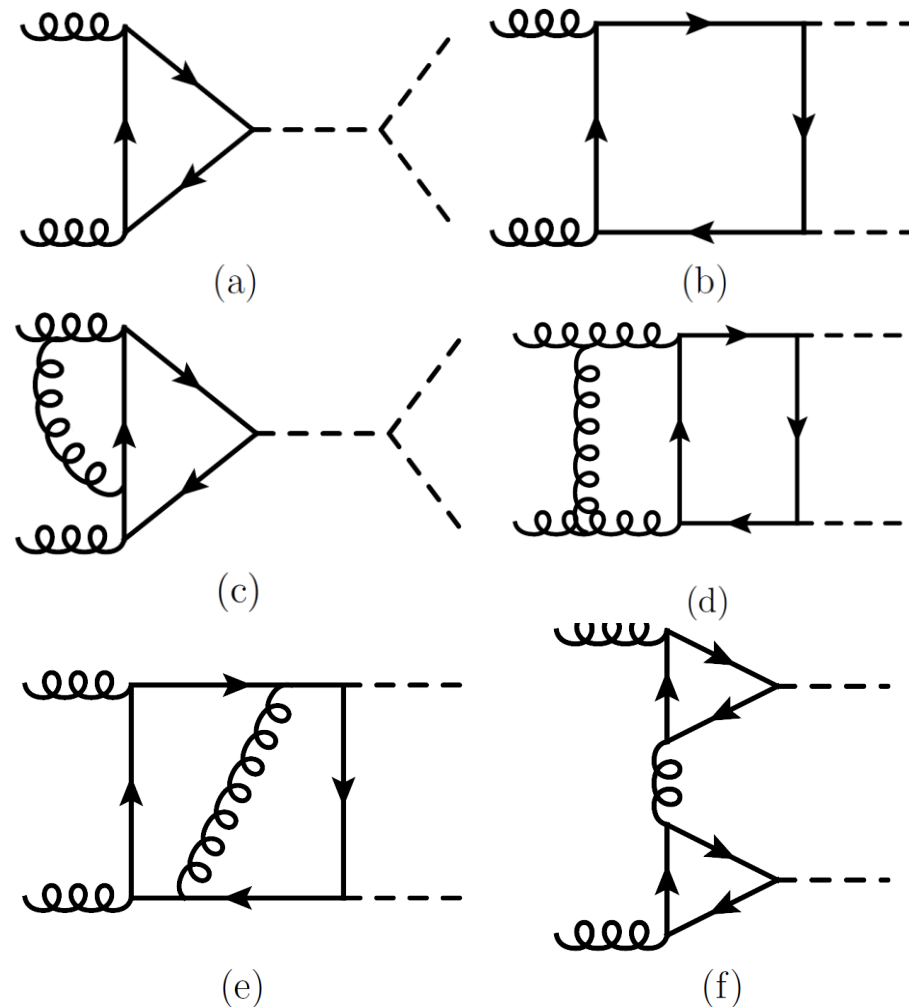
J.Davies , *et al.* 1907.06408

Small external mass expansion

$$m_H^2/m_t^2, m_H^2/\hat{s}, m_H^2/t_1$$

X. Xu, L. L. Yang. 1810.12002

G. Wang, Y. Wang, X. Xu, Y. Xu, L. L. Yang, 2010.15649



J. Davies, *et al.*, 1907.06408

Calculation of the amplitudes

Set up, HH as an example

• Partonic process: $g_a^\mu(p_1) + g_b^\nu(p_2) \rightarrow H(p_3) + H(p_4)$

• Mandelstam variables: $\hat{s} = (p_1 + p_2)^2$, $\hat{t}_1 = (p_1 - p_3)^2 - m_H^2$,
 $\hat{u}_1 = (p_2 - p_3)^2 - m_H^2 = -\hat{s} - \hat{t}_1$

• Amplitude: $\mathcal{M}_{ab}^{\mu\nu} = \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{2\pi} \hat{s} \delta_{ab} [A_1^{\mu\nu} F_1 + A_2^{\mu\nu} F_2]$,

$$A_1^{\mu\nu} = g^{\mu\nu} - \frac{2p_1^\nu p_2^\mu}{\hat{s}},$$

T. Plehn, *et al.* hep-ph/9603205

• Projection operators: $A_2^{\mu\nu} = g^{\mu\nu} + \frac{2m_H^2 p_1^\nu p_2^\mu + 2\hat{s} p_3^\mu p_3^\nu + 2\hat{u}_1 p_1^\nu p_3^\mu + 2\hat{t}_1 p_2^\mu p_3^\nu}{p_T^2 \hat{s}}$,

$$P_1^{\mu\nu} = \frac{A_1^{\mu\nu}(d-2) - A_2^{\mu\nu}(d-4)}{4(d-3)}, \quad P_2^{\mu\nu} = \frac{A_2^{\mu\nu}(d-2) - A_1^{\mu\nu}(d-4)}{4(d-3)}.$$

Calculation of the amplitudes

Set up

- Form factor: $F_i(\hat{s}, \hat{t}_1, m_t^2, m_H^2) = F_i^{(0)} + \frac{\alpha_s}{\pi} F_i^{(1)} + \mathcal{O}(\alpha_s^2)$.

UV & IR divergences

- Finite part:

$$\tilde{F}_i^{(1)} = F_i^{(1), \text{ren}} - K_g^{(1)} F_i^{(0)} - \beta_0 F_i^{(0)} \log\left(\frac{\mu^2}{-\hat{s}}\right),$$

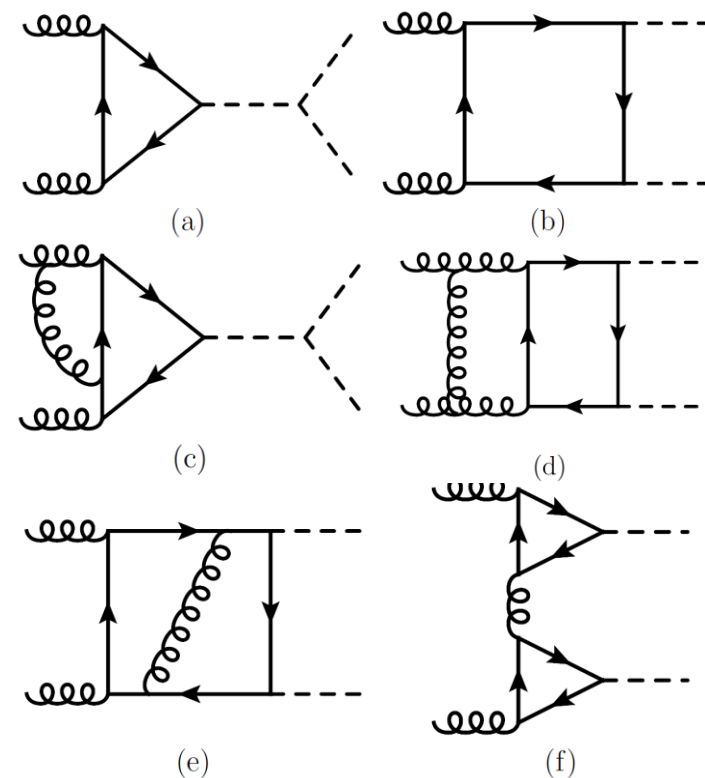
S. Catani, hep-ph/9802439

- Two-loop virtual corrections:

$$\mathcal{V}_{\text{fin}} = \frac{1}{16\pi^2} \frac{G_F^2 \hat{s}^2}{64} (C_0 + C_1)$$

$$C_0 = [|F_1^{(0)}|^2 + |F_2^{(0)}|^2] C_A \left(\pi^2 - \log^2 \frac{\mu^2}{\hat{s}} \right),$$

$$C_1 = 4 \Re \left[F_1^{(0)} \tilde{F}_1^{(1)*} + F_2^{(0)} \tilde{F}_2^{(1)*} \right].$$

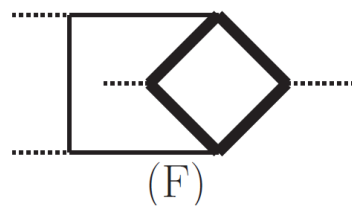
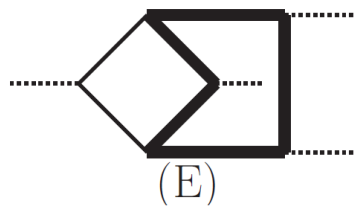
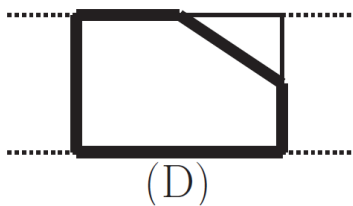
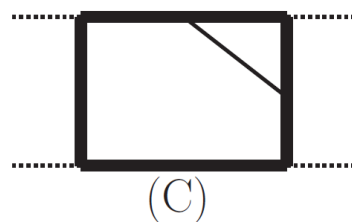
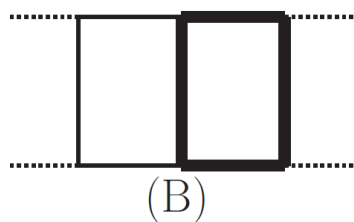
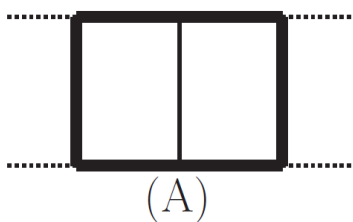


J. Davies, et al, 1907.06408

Calculation of the amplitudes

Expansion

- Form factor:
$$F_{i,\text{box}}^{(1)}(\hat{s}, \hat{t}_1, m_t^2, m_H^2) = \sum_{n=0}^{\infty} \frac{(m_H^2)^n}{n!} \left[\frac{\partial^n F_{i,\text{box}}^{(1)}}{\partial (m_H^2)^n} \right]_{m_H^2=0}.$$
- Integrand level:
$$\partial_{m_H^2} = \frac{\hat{u}_1 p_1^\mu + \hat{t}_1 p_2^\mu + \hat{s} p_3^\mu}{2m_H^2 \hat{s} - 2\hat{t}_1 \hat{u}_1} \partial_{p_3^\mu}.$$



S. Caron-Huot, *et al.* 1404.2922

M. Becchetti, *et al.* 1712.02537

X. Xu, L. L. Yang. 1810.12002

e.g. $\{k_1^2 - m_t^2, (k_1 + p_1)^2 - m_t^2, (k_1 + p_1 + p_2)^2 - m_t^2, (k_1 + k_2)^2, k_2^2 - m_t^2, (k_2 - p_3)^2 - m_t^2, (k_2 - p_1 - p_2)^2 - m_t^2\}$

Calculation of the amplitudes

HZ production

- Partonic process: $g_a^\alpha(p_1) + g_b^\beta(p_2) \rightarrow Z^\mu(p_3) + H(p_4)$

$$A_1^{\alpha\beta\mu}(p_1, p_2, p_3) = \frac{\hat{s}}{2} \epsilon^{\alpha\beta\mu\rho} p_{2\rho} - p_2^\alpha \epsilon^{\beta\mu\rho\sigma} p_{1\rho} p_{2\sigma},$$

$$A_2^{\alpha\beta\mu}(p_1, p_2, p_3) = -\frac{\hat{s}}{2} \epsilon^{\alpha\beta\mu\rho} p_{1\rho} + p_1^\beta \epsilon^{\alpha\mu\rho\sigma} p_{1\rho} p_{2\sigma},$$

$$A_3^{\alpha\beta\mu}(p_1, p_2, p_3) = -\left(p_3^\alpha + \frac{\hat{t}_1}{\hat{s}} p_2^\alpha\right) \epsilon^{\beta\mu\rho\sigma} p_{2\rho} p_{1\sigma},$$

$$A_4^{\alpha\beta\mu}(p_1, p_2, p_3) = -\left(p_3^\beta + \frac{m_H^2 - m_Z^2 - \hat{s} - \hat{t}_1}{\hat{s}} p_1^\beta\right) \epsilon^{\alpha\mu\rho\sigma} p_{1\rho} p_{2\sigma},$$

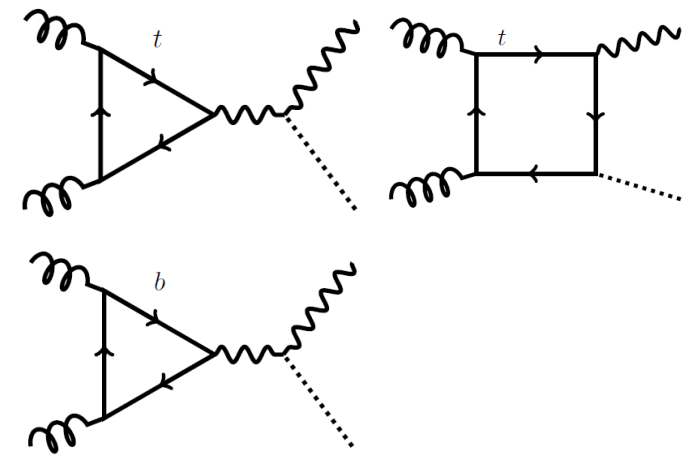
$$A_5^{\alpha\beta\mu}(p_1, p_2, p_3) = \left(p_3^\alpha + \frac{\hat{t}_1}{\hat{s}} p_2^\alpha\right) \epsilon^{\beta\mu\rho\sigma} p_{2\rho} p_{3\sigma},$$

$$A_6^{\alpha\beta\mu}(p_1, p_2, p_3) = \left(p_3^\beta + \frac{m_H^2 - m_Z^2 - \hat{s} - \hat{t}_1}{\hat{s}} p_1^\beta\right) \epsilon^{\alpha\mu\rho\sigma} p_{1\rho} p_{3\sigma},$$

$$A_7^{\alpha\beta\mu}(p_1, p_2, p_3) = -\frac{\hat{s}}{2} \epsilon^{\alpha\beta\mu\rho} p_{3\rho} + p_2^\alpha \epsilon^{\beta\mu\rho\sigma} p_{1\rho} p_{3\sigma} - p_1^\beta \epsilon^{\alpha\mu\rho\sigma} p_{2\rho} p_{3\sigma} - g^{\alpha\beta} \epsilon^{\mu\rho\sigma\tau} p_{1\rho} p_{2\sigma} p_{3\tau},$$

$$\bar{\psi} \gamma^\mu \gamma_5 \psi \rightarrow i \frac{\epsilon^{\mu\rho\sigma\tau}}{3!} \bar{\psi} \gamma_\rho \gamma_\sigma \gamma_\tau \psi$$

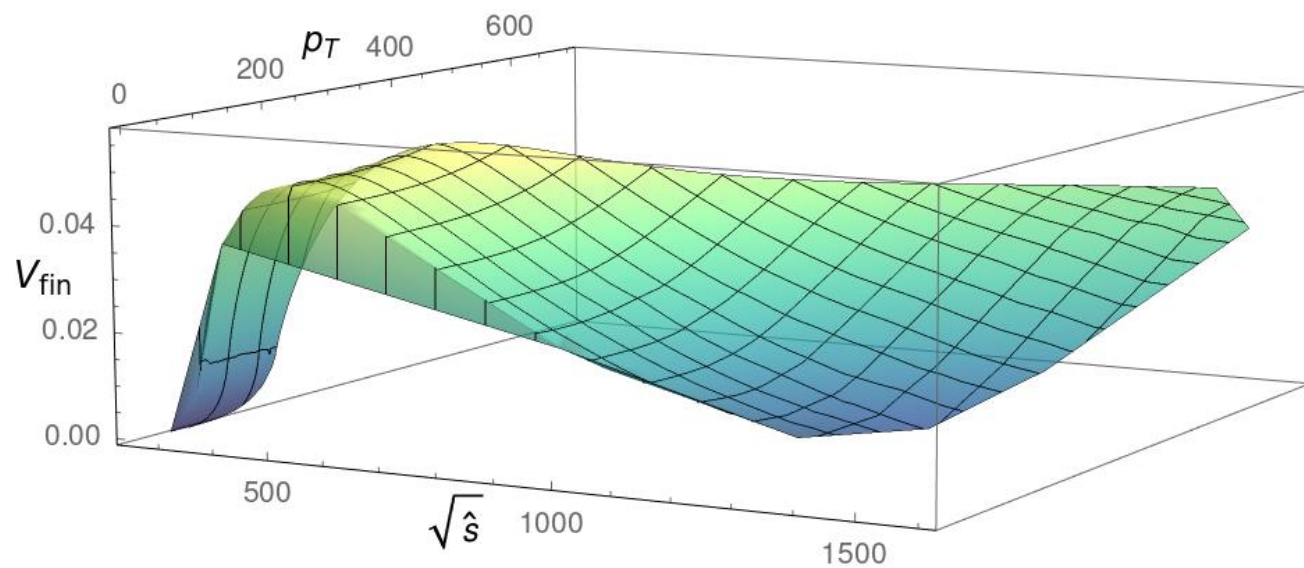
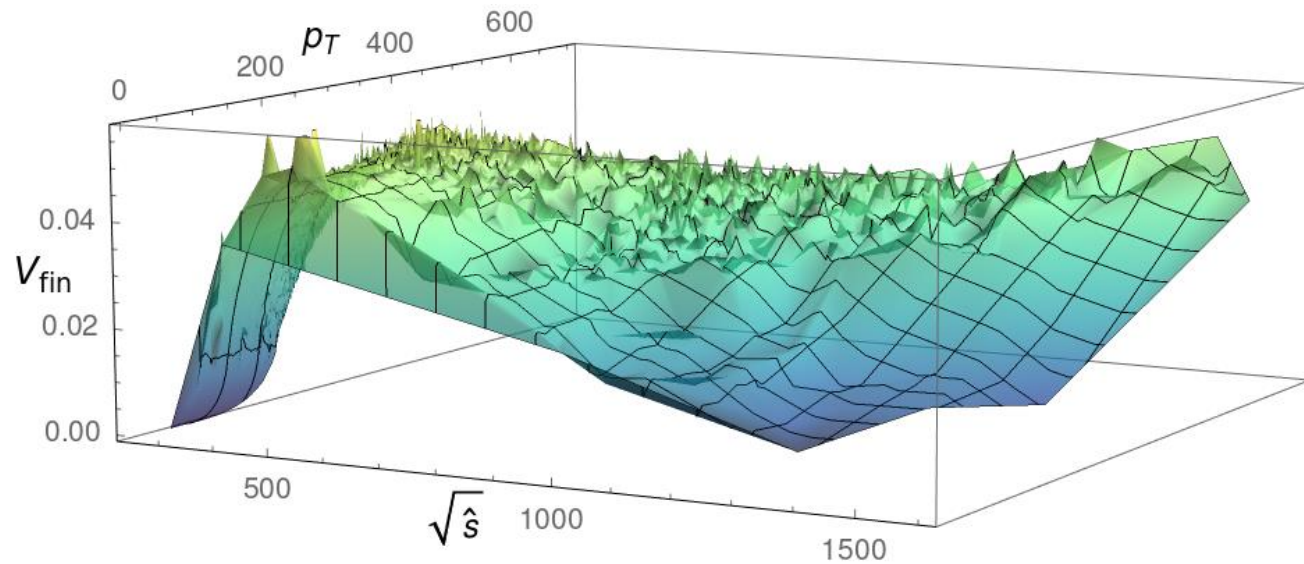
S. A. Larin, hep-ph/9302240



J. Davies, *et al.* 2011.12314

B. A. Kniehl, Phys. Rev. D 42 (1990) 2253

Numeric calculation and discussions



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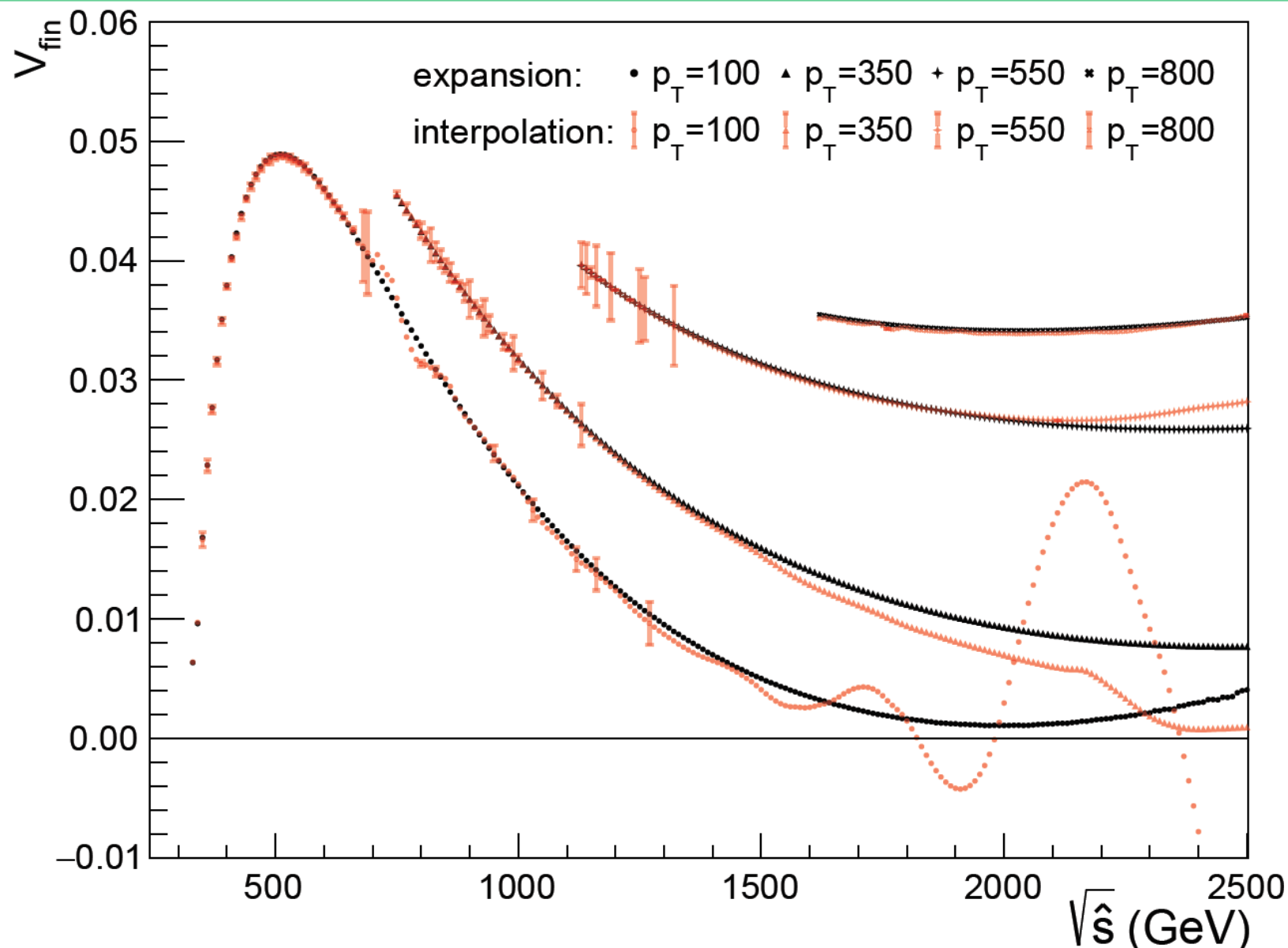
$$C_0 = [|F_1^{(0)}|^2 + |F_2^{(0)}|^2] C_A \left(\pi^2 - \log^2 \frac{\mu^2}{\hat{s}} \right),$$

$$C_1 = 4 \Re \left[F_1^{(0)} \tilde{F}_1^{(1)*} + F_2^{(0)} \tilde{F}_2^{(1)*} \right].$$

The upper plot uses the results from the grid file of [\[J. Davies, et al, 1907.06408\]](#)

while the lower one uses our results with the small-mass expansion up to $\mathcal{O}(m_H^4)$

Numeric calculation and discussions



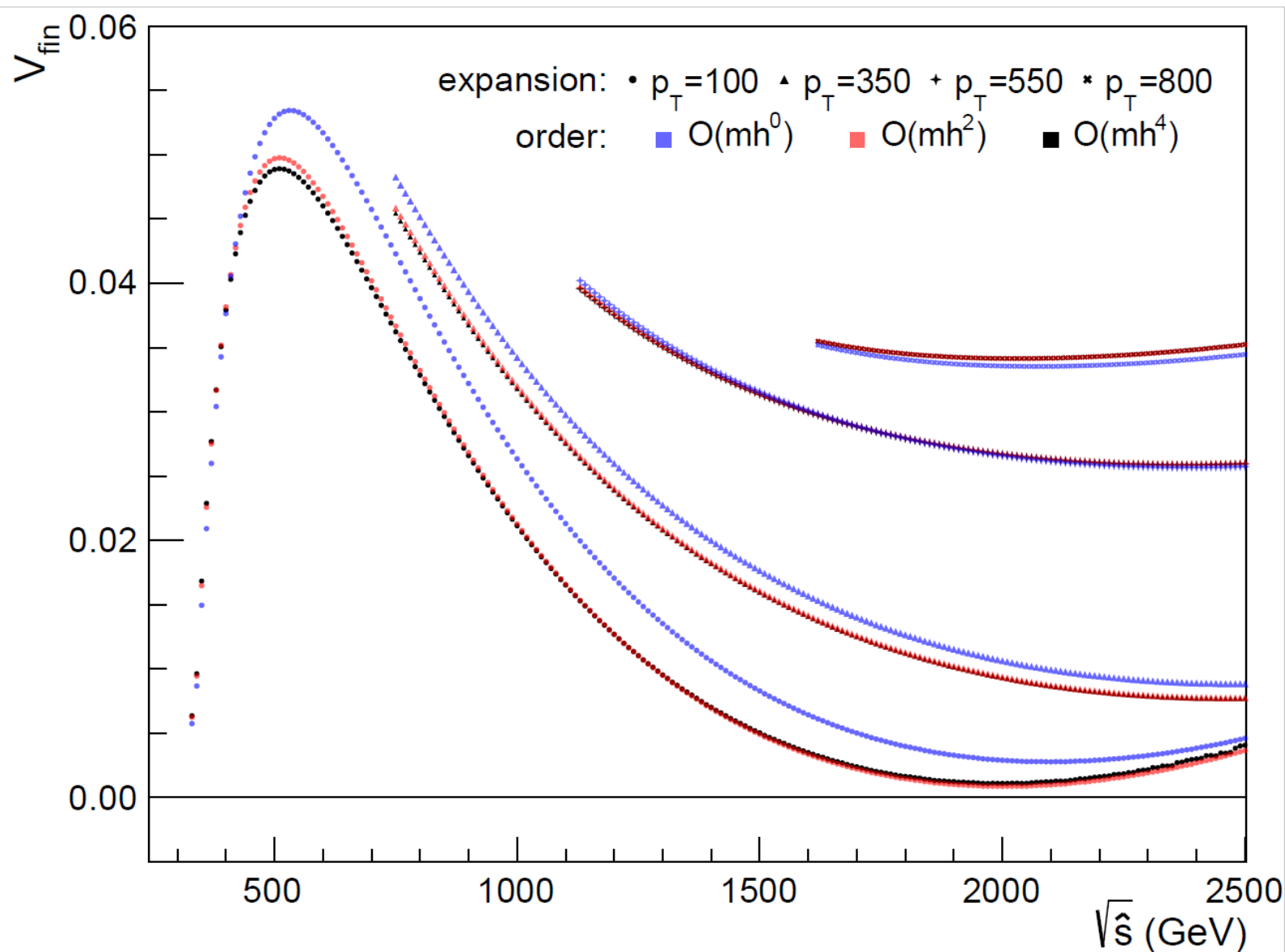
The black curves are obtained from our results with the small-mass expansion up to $\mathcal{O}(m_H^4)$, while the red ones are obtained using the interpolation code associated with

[\[J. Davies, et al, 1907.06408\]](#)

~7 GPGPU hours vs. ~10 seconds for one CPU core per phase-space point

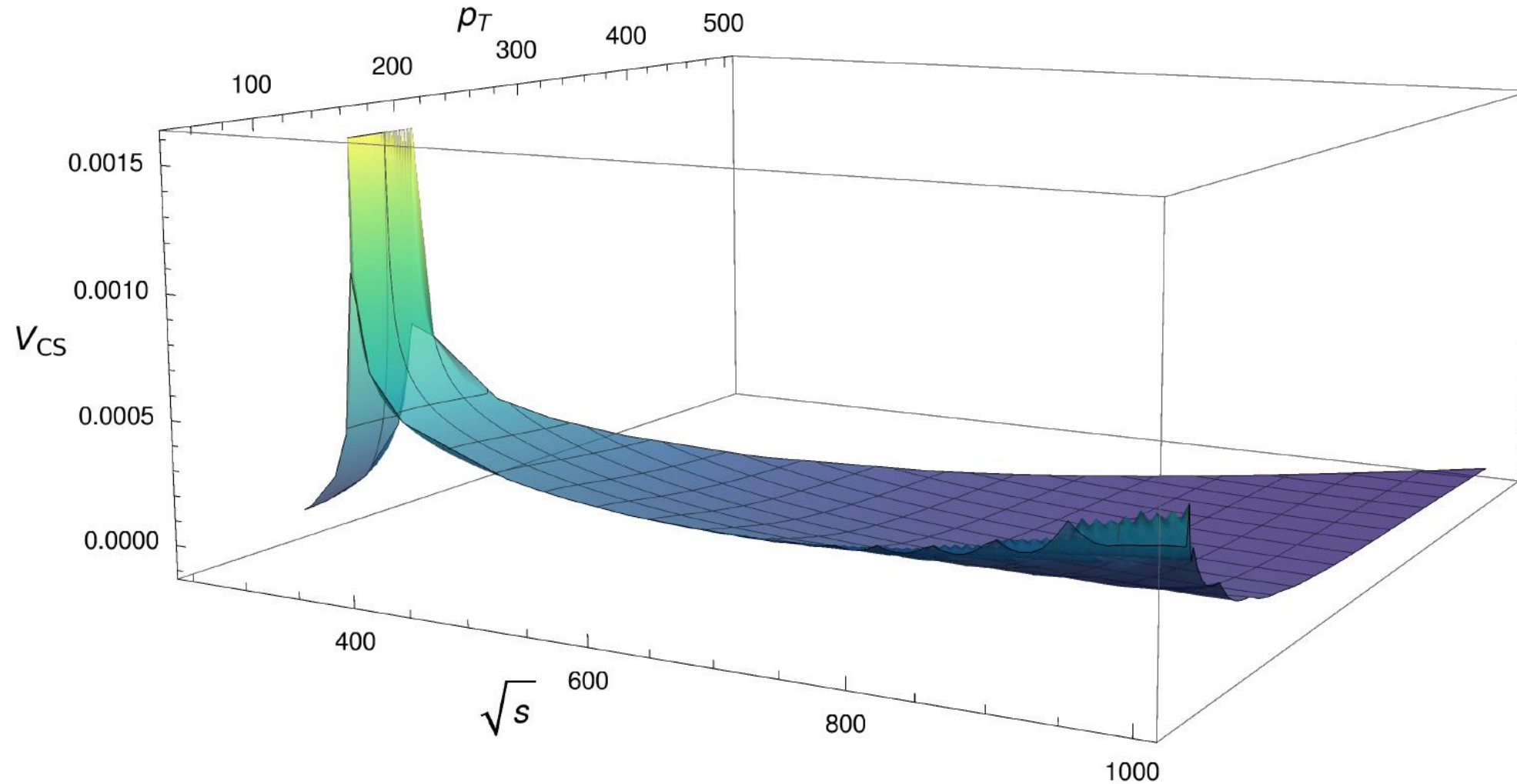
[S. Borowka. 1604.06447](#)

Numeric calculation and discussions



\mathcal{V}_{fin} as a function of $\sqrt{\hat{s}}$
 computed with the small-
 mass expansion up to $\mathcal{O}(m_H^0)$
 (blue), $\mathcal{O}(m_H^2)$ (red), and
 $\mathcal{O}(m_H^4)$ (black) for several
 values of p_T .

Preliminary results in HZ production



Virtual contribution
of HZ production:
squared amplitudes
at NLO(part of
phase space points)

Summary and outlook

- We present an efficient method to calculate the two-loop amplitudes for Higgs boson pair production via gluon fusion
- Our method is based on a series expansion in terms of the small mass m_H and m_Z : precision and efficiency
- Our method can also be implemented for other $2 \rightarrow 2$ processes with a heavy quark loop: $gg \rightarrow ZZ$, $gg \rightarrow Hj$

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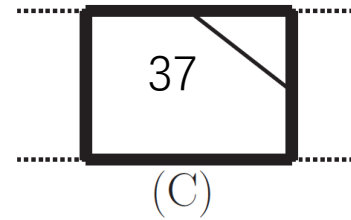
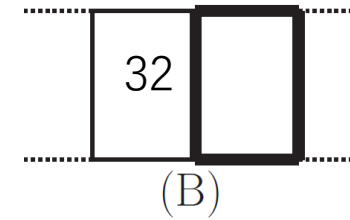
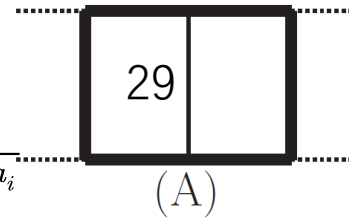
Thanks

Calculation of the amplitudes

- Topology, e.g. $\{k_1^2 - m_t^2, (k_1 + p_1)^2 - m_t^2, (k_1 + p_1 + p_2)^2 - m_t^2, (k_1 + k_2)^2, k_2^2 - m_t^2, (k_2 - p_3)^2 - m_t^2, (k_2 - p_1 - p_2)^2 - m_t^2, (k_2 - p_1)^2 - m_t^2, (k_1 + p_3)^2 - m_t^2\}$

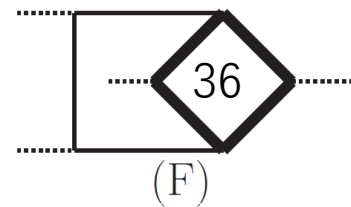
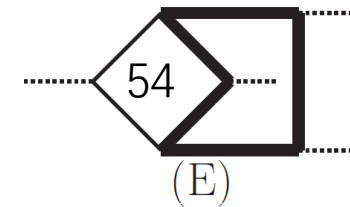
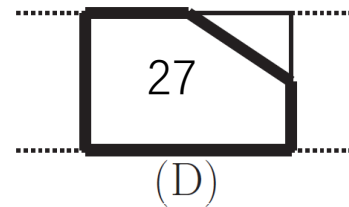
- Master integrals:

$$I_{\{a_i\}}(\hat{s}, \hat{t}_1, m_t^2, d) = C \int \frac{d^d k}{(2\pi)^d} \frac{d^d k}{(2\pi)^d} \prod_{i=1}^9 \frac{1}{D_i^{a_i}}$$



- Differential equations:

$$d\vec{f}(\vec{x}, \epsilon) = dA(\vec{x}, \epsilon) \vec{f}(\vec{x}, \epsilon)$$



- Canonical form: $d\vec{g}(\vec{x}, \epsilon) = \epsilon dB(\vec{x}) \vec{g}(\vec{x}, \epsilon)$ [J. Henn, 1304.1806](#) $\vec{g}(\vec{x}, \epsilon) = \hat{T}(\vec{x}, \epsilon) \vec{f}(\vec{x}, \epsilon)$

or
$$d\vec{h}(\vec{x}, \epsilon) = [\epsilon dB_1(\vec{x}) + dB_0(\vec{x})] \vec{h}(\vec{x}, \epsilon)$$

$$\vec{h}(\vec{x}, \epsilon) = \hat{T}'(\vec{x}, \epsilon) \vec{f}(\vec{x}, \epsilon)$$

Calculation of the amplitudes

- Canonical form: $d\vec{g}(\vec{x}, \epsilon) = \epsilon dB(\vec{x})\vec{g}(\vec{x}, \epsilon)$

- d-log form: $d\vec{g}(\vec{x}, \epsilon) = \epsilon \sum_k B_k d \log \alpha_k(\vec{x}) \vec{g}(\vec{x}, \epsilon)$ B_k is constant matrix,
 α_k is called a letter

- Solution: $\vec{g}(\vec{x}, \epsilon) = \mathcal{P} \exp \left[\epsilon \int_{\vec{x}_0}^{\vec{x}} \sum_k B_k d \log \alpha_k(\vec{x}) \right] \vec{g}_0(\vec{x}_0, \epsilon)$

- Square root: $\{\sqrt{1+a}, \sqrt{1+a+b}, \sqrt{a+b}, \sqrt{16a+(4+b)^2}\}$ $\{a, b\} \subset \{u, v, \frac{uv}{u+v}\}$

$$G(a_1, \dots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \dots, a_n; t)$$

$$u = -\frac{4m_t}{\hat{s}}, v = -\frac{4m_t}{\hat{t}_1}$$

M. Besier, *et al*, 1809.10983

A. Goncharov, 1105.2076

Calculation of the amplitudes

- $d\vec{h}(\vec{x}, \epsilon) = [dB_0(\vec{x}) + \epsilon dB_1(\vec{x})] \vec{h}(\vec{x}, \epsilon)$

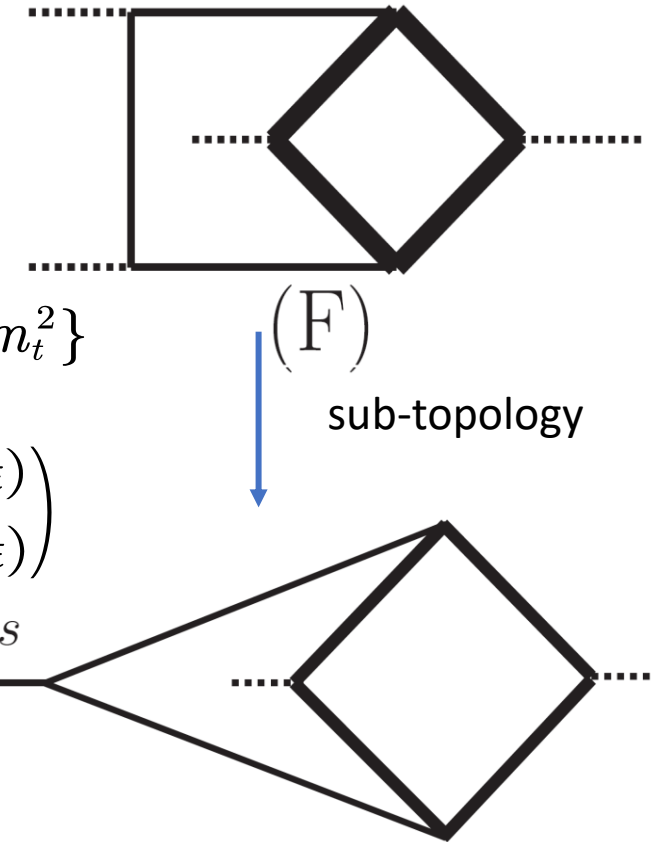
$$\{(k_1 - p_1)^2, k_1^2, (k_1 + p_2)^2, (k_1 + k_2 - p_1)^2 - m_t^2, k_2^2 - m_t^2, (k_2 - p_3)^2 - m_t^2, (k_1 + k_2 + p_2 - p_3)^2 - m_t^2, (k_1 - p_3)^2, (k_2 - p_1)^2 - m_t^2\}$$

- DEs: $\frac{d}{du} \begin{pmatrix} h_1(u, \epsilon) \\ h_2(u, \epsilon) \end{pmatrix} = [\tilde{B}_0(u) + \epsilon \tilde{B}_1(u)] \begin{pmatrix} h_1(u, \epsilon) \\ h_2(u, \epsilon) \end{pmatrix} + \tilde{C}(u, \epsilon) \begin{pmatrix} k_1(u, \epsilon) \\ k_2(u, \epsilon) \end{pmatrix}$

$$\tilde{B}_0(u) = \begin{pmatrix} 0 & \frac{4}{u} \\ \frac{1}{4(1-4u)} & \frac{4}{1-4u} \end{pmatrix}$$

- Homogeneous part:

$$\frac{d}{du} \begin{pmatrix} H_1^{(n)}(u) \\ H_2^{(n)}(u) \end{pmatrix} = \tilde{B}_0(u) \begin{pmatrix} H_1^{(n)}(u) \\ H_2^{(n)}(u) \end{pmatrix}$$

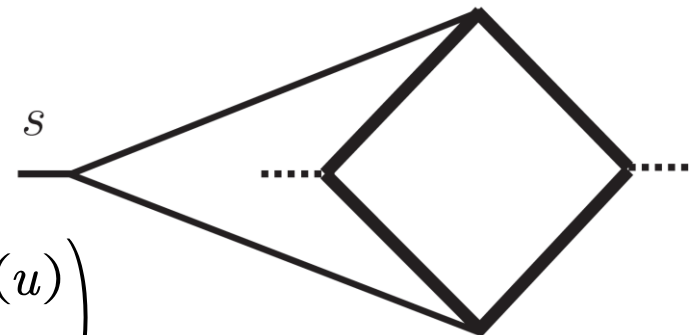


A. Manteuffel, *et al*, 1701.05905

Calculation of the amplitudes

- $$\frac{d}{du} \begin{pmatrix} H_1^{(n)}(u) \\ H_2^{(n)}(u) \end{pmatrix} = \tilde{B}_0(u) \begin{pmatrix} H_1^{(n)}(u) \\ H_2^{(n)}(u) \end{pmatrix}$$

$$G(u) = \begin{pmatrix} I_1(u) & J_1(u) \\ I_2(u) & J_2(u) \end{pmatrix} \xrightarrow{\text{transform}} \begin{pmatrix} H_1^{(n)}(u) \\ H_2^{(n)}(u) \end{pmatrix} = G(u) \begin{pmatrix} \tilde{H}_1^{(n)}(u) \\ \tilde{H}_2^{(n)}(u) \end{pmatrix}$$



- New DEs:**
$$\frac{d}{du} \begin{pmatrix} \tilde{H}_1^{(n)}(u) \\ \tilde{H}_2^{(n)}(u) \end{pmatrix} = G^{-1}(u) \tilde{B}_1(u) G(u) \begin{pmatrix} \tilde{H}_1^{(n-1)}(u) \\ \tilde{H}_2^{(n-1)}(u) \end{pmatrix} + G^{-1}(u) \begin{pmatrix} \tilde{k}_1^{(n)}(u) \\ \tilde{k}_2^{(n)}(u) \end{pmatrix}$$

Can be solved as usual !

- Top-topology:**
$$d\tilde{B}'_0 = \begin{pmatrix} 0 & \star & \star & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \star & \star \\ 0 & 0 & \star & 0 \end{pmatrix}$$

