# *A new method for amplitude with multi fermion line*

*——The development of FDC program*

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**微扰量子场论研讨会**

2021.05.14-16 上海

# Outline

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2. HOW(Method)
3. RESULTS(Speed)
4. SUMMARY

- In high energy physics, perturbation theory is widely used to do precise calculation.
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  (1) LEPII: $e^+e^- \to w^+w^- \to 4f$, nearly 100 processes
  (2) Future high energy collider (ILC, CLIC, and etc): $e^+e^- \to t\bar{t} \to 6f$
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- Helicity amplitude: some trouble in dealing with massive particle

# HOW(Improve the fermion line calculation)

*Step1: Simplification of spinor*

One term for a fermion line in Feynman amplitude can be generally expressed as:

$$M = \overline{u}(n_1, k_1, s_1) B_m u(n_2, k_2, s_2) = u^+(n_1, k_1, s_1)\gamma^0 B_m u(n_2, k_2, s_2), \tag{1}$$

$$u(n,k,s) = \begin{cases} \frac{\hat{k}+m}{\sqrt{k^0+m}}\mu(+,s) = \hat{k}'\mu(+,s), & n=+ \\ \frac{\hat{k}-m}{\sqrt{k^0+m}}\mu(-,s) = \hat{k}'\mu(-,s), & n=- \end{cases} \tag{2}$$

$$\mu(+,s) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}, \qquad \mu(-,s) = \begin{pmatrix} 0 \\ \chi_s \end{pmatrix}, \qquad \chi_s = \begin{cases} \begin{pmatrix} 1 & 0 \end{pmatrix}^T, & s=1 \\ \begin{pmatrix} 0 & 1 \end{pmatrix}^T, & s=2 \end{cases}$$

$$k'^{\mu} = (\sqrt{k^0+m}, \frac{\overrightarrow{k}}{\sqrt{k^0+m}}) \tag{3}$$

Substituting the new form, one can get

$$\begin{aligned} M &= \mu(n_1, s_1)^{+} \hat{k}_1'^{+} \gamma_0 B_m \hat{k}_2' \mu(n_2, s_2) \\ &= \mu(n_1, s_1)^{+} \gamma_0 A_n \mu(n_2, s_2) \end{aligned} \tag{4}$$

$$\mu(+, s) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}, \qquad \mu(-, s) = \begin{pmatrix} 0 \\ \chi_s \end{pmatrix}, \qquad \chi_s = \begin{cases} \begin{pmatrix} 1 & 0 \end{pmatrix}^T, & s = 1 \\ \begin{pmatrix} 0 & 1 \end{pmatrix}^T, & s = 2 \end{cases}$$

$$A_n \equiv \hat{k}_1' B_m \hat{k}_2', \quad n = m + 2. \tag{5}$$

Meanwhile, if there is a $\gamma_5$ in front of $B_m$,

$$M = \overline{u}(n_1, k_1, s_1) \gamma_5 B_m u(n_2, k_2, s_2) \tag{6}$$

the final result equals:

$$M = -\mu(n_1, s_1)^{+} \gamma_0 \gamma_5 A_n \mu(n_2, s_2). \tag{7}$$

In general, one can redefine

$$A_n \equiv \hat{p}_n \cdots \hat{p}_2 \hat{p}_1, \quad n = 2, 3, \cdots . \tag{8}$$

Form the calculation, the general result with n $p_i$, whose form has relationship with odevity, and equals:

$$A_n = \hat{p}_n \cdots \hat{p}_2 \hat{p}_1 = \begin{cases} \hat{p}_a \gamma^0 - i\gamma_5 \hat{p}_b \gamma^0, & \text{n is even} \\ \hat{p}_a - i\gamma_5 \hat{p}_b, & \text{n is odd} \end{cases} \tag{9}$$

If there is a $\gamma_5$ at the front, the general form is

$$\gamma_5 A_n = \gamma_5 \hat{p}_n \cdots \hat{p}_2 \hat{p}_1 = \begin{cases} -i(\hat{p}_b \gamma^0 + i\gamma_5 \hat{p}_a \gamma^0), & \text{n is even} \\ -i(\hat{p}_b + i\gamma_5 \hat{p}_a), & \text{n is odd} \end{cases} \tag{10}$$

The process introduced above should get and use final analytic results.

$$M = \mu(n_1, s_1)^{+}\gamma_0(\hat{p}_a\gamma^0 - i\gamma_5\hat{p}_b\gamma^0)\mu(n_2, s_2)$$

$$= \mu(n_1, s_1)^{+}\begin{pmatrix} p_a^0 - ip_b^3 & -ip_b^1 - p_b^2 & p_a^3 - ip_b^0 & p_a^1 - ip_a^2 \\ -ip_b^1 + p_b^2 & p_a^0 + ip_b^3 & p_a^1 + ip_a^2 & -p_a^3 - ip_b^0 \\ -p_a^3 + ip_b^0 & -p_a^1 + ip_a^2 & -p_a^0 + ip_b^3 & ip_b^1 + p_b^2 \\ -p_a^1 - ip_a^2 & p_a^3 + ip_b^0 & ip_b^1 - p_b^2 & -p_a^0 - ip_b^3 \end{pmatrix}\mu(n_2, s_2).$$

$$\mu(+, s) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}, \qquad \mu(-, s) = \begin{pmatrix} 0 \\ \chi_s \end{pmatrix}, \qquad \chi_s = \begin{cases} \begin{pmatrix} 1 & 0 \end{pmatrix}^T, & s = 1 \\ \begin{pmatrix} 0 & 1 \end{pmatrix}^T, & s = 2 \end{cases}$$

# *HOW(Improve the fermion line calculation)*

*Step3: Simplification of fermion line*

The process introduced above should get and use final analytic results.

$$M = \mu(n_1, s_1)^+ \gamma_0(\hat{p}_a\gamma^0 - i\gamma_5\hat{p}_b\gamma^0)\mu(n_2, s_2)$$
$$= \mu(n_1, s_1)^+ \begin{pmatrix} p_a^0 - ip_b^3 & -ip_b^1 - p_b^2 & p_a^3 - ip_b^0 & p_a^1 - ip_a^2 \\ -ip_b^1 + p_b^2 & p_a^0 + ip_b^3 & p_a^1 + ip_a^2 & -p_a^3 - ip_b^0 \\ -p_a^3 + ip_b^0 & -p_a^1 + ip_a^2 & -p_a^0 + ip_b^3 & ip_b^1 + p_b^2 \\ -p_a^1 - ip_a^2 & p_a^3 + ip_b^0 & ip_b^1 - p_b^2 & -p_a^0 - ip_b^3 \end{pmatrix} \mu(n_2, s_2).$$

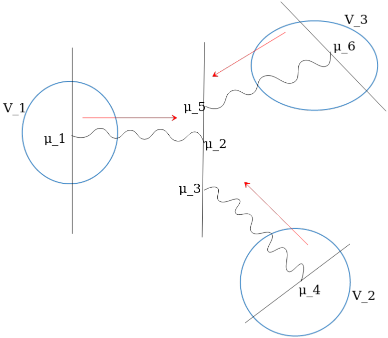
In fact, for a specific fermion line, both sides are particles:

$$M = \begin{cases} p_a^0 - ip_b^3, & s_1 = 1 \quad \text{and} \quad s_2 = 1 \\ -ip_b^1 - p_b^2, & s_1 = 1 \quad \text{and} \quad s_2 = 2 \\ -ip_b^1 + p_b^2, & s_1 = 2 \quad \text{and} \quad s_2 = 1 \\ p_a^0 + ip_b^3, & s_1 = 2 \quad \text{and} \quad s_2 = 2 \end{cases} \tag{11}$$

# HOW(Improve the fermion line calculation)

*Step4: Simplification of amplitude*

(1) contract to vector boson propagator;
(2) contract the vector obtained in to other fermion line;
(3) finally, all fermion lines can be contracted to one fermion line or scalar.



# HOW(Improve the fermion line calculation)

*Step4: Simplification of amplitude*

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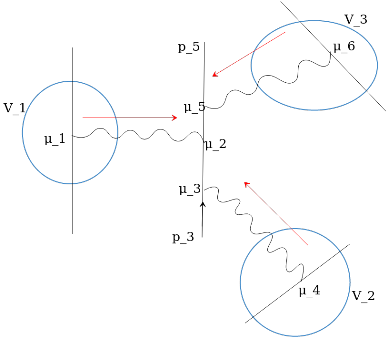
$$M = \overline{u}(p_5, s_5)\hat{V}_3(\hat{p}+m)\hat{V}_1(\hat{p}'+m)\hat{V}_2 u(p_3, s_3)$$



# RESULTS

*Compare the calculation speed with other programs*

| process | $e^+ + e^- \to c + \bar{c} + b + \bar{b}(8)$ | | $e^+ + e^- \to c + \bar{c} + c + \bar{c} + c + \bar{c}(576)$ | |
|---|---|---|---|---|
| $\sqrt{s}$ \ program | FDC | MadGraph | FDC | MadGraph |
| 20 | 0.6 | 2.26 | 103.4 | 4008 |
| 50 | 0.5 | 2.28 | 90.4 | 3936 |
| 100 | 0.5 | 2.23 | 111.4 | 3990 |
| 200 | 0.5 | 2.24 | 127.9 | 4044 |
| 500 | 0.5 | 2.24 | 154.2 | 4002 |
| 1000 | 0.5 | 2.23 | 172.8 | 4002 |

*Table:* The expected time for generating 10000 events for a certain process. ($\sqrt{s}$ in unit of GeV and expected time in unit of second.)

# RESULTS

*Compare the calculation speed with other programs*

| group | $1(e^+ + e^- \to c + \bar{c} + b + \bar{b})$ | | $2(e^+ + e^- \to c + \bar{c} + c + \bar{c} + c + \bar{c})$ | |
|---|---|---|---|---|
| program / $\sqrt{s}$ | FDC | WHIZARD | FDC | WHIZARD |
| 20 | 0.3 | 4 | 7.1 | 319 |
| 50 | 0.3 | 5 | 7.0 | 760 |
| 100 | 0.3 | 6 | 6.8 | 3935 |
| 200 | 0.2 | 8 | 6.8 | 6999 |
| 500 | 0.2 | 8 | 7.1 | 7905 |
| 1000 | 0.2 | 11 | 7.0 | 6277 |

*Table:* Group 1 represents the subprocess of $e^+ + e^- \to c + \bar{c} + b + \bar{b}$, which only contains the 4 diagrams of that. group 2 represents the subprocess of $e^+ + e^- \to c + \bar{c} + c + \bar{c} + c + \bar{c}$, which only contains 12 diagrams of that.

# *SUMMARY*

- The numerical calculation of the amplitude of multi fermion lines process is improved obviously, and the velocity is increased a lot.
- The improved algorithm has been programmed, and embedded into FDC. Any multi fermion lines process can be calculated.
- The article is in preparation and will be completed soon.

**The End**
**Thank you very much**

# Appendix

$$A_2 = \begin{pmatrix} p_2 \cdot p_1 - i\varepsilon_{ijk} p_2^i p_1^j \sigma^k & (p_1^0 \overrightarrow{p_2} - p_2^0 \overrightarrow{p_1}) \cdot \overrightarrow{\sigma} \\ (p_1^0 \overrightarrow{p_2} - p_2^0 \overrightarrow{p_1}) \cdot \overrightarrow{\sigma} & p_2 \cdot p_1 - i\varepsilon_{ijk} p_2^i p_1^j \sigma^k \end{pmatrix}. \tag{12}$$

one can define two new $p$ donated by $p_a$, $p_b$, which are

$$\begin{aligned} p_a &= (p_2 \cdot p_1, -p_2^0 \overrightarrow{p_1} + p_1^0 \overrightarrow{p_2}), \\ p_b &= (0, \quad \varepsilon_{ijk} p_2^i p_1^j). \end{aligned} \tag{13}$$

one can rewrite the form of Eq, which equals

$$A_2 = \hat{p}_a \gamma^0 - i\gamma_5 \hat{p}_b \gamma^0. \tag{14}$$

# Appendix

On the basis of the previous,

$$\begin{aligned} A_3 = \hat{p}_3\hat{p}_2\hat{p}_1 &= \hat{p}_3(\hat{p}_a\gamma^0 - i\gamma_5\hat{p}_b\gamma^0) \\ &= \hat{p}_3\hat{p}_a\gamma^0 + i\gamma_5\hat{p}_3\hat{p}_b\gamma^0 \end{aligned} \tag{15}$$

$$\begin{aligned} \hat{p}_3\hat{p}_a &= \hat{p}_{a'_1}\gamma^0 - i\gamma_5\hat{p}_{b'_1}\gamma^0, \\ \hat{p}_3\hat{p}_b &= \hat{p}_{a'_2}\gamma^0 - i\gamma_5\hat{p}_{b'_2}\gamma^0 \end{aligned} \tag{16}$$

Hence, one can submit them into formula (15) and continue to calculate:

$$\begin{aligned} A_3 &= (\hat{p}_{a'_1}\gamma^0 - i\gamma_5\hat{p}_{b'_1}\gamma^0)\gamma^0 + i\gamma_5(\hat{p}_{a'_2}\gamma^0 - i\gamma_5\hat{p}_{b'_2}\gamma^0)\gamma^0 \\ &= \hat{p}_{a'_1} - i\gamma_5\hat{p}_{b'_1} + i\gamma_5\hat{p}_{a'_2} + \hat{p}_{b'_2} \\ &= (\hat{p}_{a'_1} + \hat{p}_{b'_2}) - i\gamma_5(\hat{p}_{b'_1} - \hat{p}_{a'_2}). \end{aligned} \tag{17}$$

So one can also define two new $p$ donate by $p_{a_2}$, $p_{b_2}$, where

$$
\begin{aligned}
p_{a_2} &= p_{a'_1} + p_{b'_2}, \\
p_{b_2} &= p_{b'_1} - p_{a'_2},
\end{aligned}
\tag{18}
$$

and get the final result:

$$
A_3 = \hat{p}_{a_2} - i\gamma_5 \hat{p}_{b_2}. \tag{19}
$$