Classification of High Dimensional Gluon Operators

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Based on work in preparation with Ke Ren, Gang Yang and Rui Yu

Gluon Operators

- **High dimensional operators**
- Operator diagram and primitive operators
- Classify operators by Sn Symmetry
- Summary and outlook

High Dimensional Operators

- The Physics beyond the Standard Model: Supersymmetry? Composite Higgs? Extra dimension?
- Extend SM by adding new particles with masses much larger than the Electroweak Scale. $M \gg v$
- The complete Lagrangian of the extended theory is unknown, but we can construct the low energy effective field theory.

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{n\geq 1} \sum_i \frac{C_i^{(n)}}{M^n} \mathcal{O}_i^{(n)} \,.$$

High Dimensional Operators

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{n\geq 1} \sum_{i} \frac{C_i^{(n)}}{M^n} \mathcal{O}_i^{(n)} .$$

Wilson coefficients

Infinite tower of high dimensional operators

- It can be very hard to generate and classify the high dimensional operators. The number of operators increase very fast as dimension increases.
- We start by construct a simpler set of operators: gluon operators.
- Gluon operators also play major role in Higgs-EFT, obtained by integrating out top quarks.

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Kinetic Operators

- Gluon operators only contain field strength tensor $F^a_{\mu\nu}$ and covariant derivative D_μ . For example,

$$O = \mathrm{Tr}(F^{\mu\nu} D_\mu F_{\rho\sigma} D_\nu F^{\rho\sigma})$$

- It can be split into a kinetic part K and a color part C:

$$K = F^{\mu\nu a_1} D_\mu F_{\rho\sigma}{}^{a_2} D_\nu F^{\rho\sigma a_3}$$

$$C = \mathrm{Tr}(T^{a_1} T^{a_2} T^{a_3})$$

- K will be called a kinetic operator.

Kinetic Operators: notation

- A kinetic operators can be split into several DF blocks:

$$K = F^{\mu\nu a_1} D_\mu F_{\rho\sigma}{}^{a_2} D_\nu F^{\rho\sigma a_3}$$

- For compactness we drop the color indices:

$$K = F^{\mu\nu} D_\mu F_{\rho\sigma} D_\nu F^{\rho\sigma}$$

- The color indices can be retrieved by adding a_i to the i-th DF block.
- Simplify K further by replacing Lorentz indices by numbers:

$$K \rightarrow F_{12} D_1 F_{34} D_2 F_{34}$$

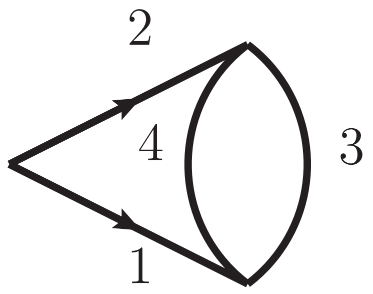
Other Representations

- Besides the DF-form, we propose two other representations of kinetic operators, which can be useful in different problems.
- 1, The operator diagram.
- 2, The minimal form factor.

Operator Diagrams

- A diagrammatical representation of operators.
- Each DF-block corresponds to a vertex.
- Each pair of Lorentz index μ corresponds to an edge.
- Suppose μ is a D-index in the vertex v, then add an arrow to the edge μ pointing to v.

$$F_{12}D_1F_{34}D_2F_{34} \rightarrow$$



- The diagrammatical representation makes the relations among operators more transparent.

Minimal Form Factors

$$\mathcal{F}(K) = K\big|_{D_{1\cdots n}F_{ab}\to p_{i1}\cdots p_{in}(p_{ia}\epsilon_{ib}-p_{ib}\epsilon_{ia})}$$

$$\mathcal{F}(F_{12}D_1F_{34}D_2F_{34}) = (p_1\cdot p_2\epsilon_1\cdot p_3 - p_1\cdot p_3\epsilon_1\cdot p_2)$$
$$(p_2\cdot p_3\epsilon_2\cdot\epsilon_3 - p_2\cdot\epsilon_3 p_3\cdot\epsilon_2)$$

- Physical constraints like equation of motion, Bianchi identities are automatically satisfied by minimal form factors.
- There is a one-to-one correspondence between minimal form factors and inequivalent operators. DF-form and operator diagrams have redundancy.
- Drawback: may have lengthy expressions.

Physical constraints

$$[D_1, D_2] = \mathcal{O}(F) \Rightarrow [p_1, p_2] = 0$$

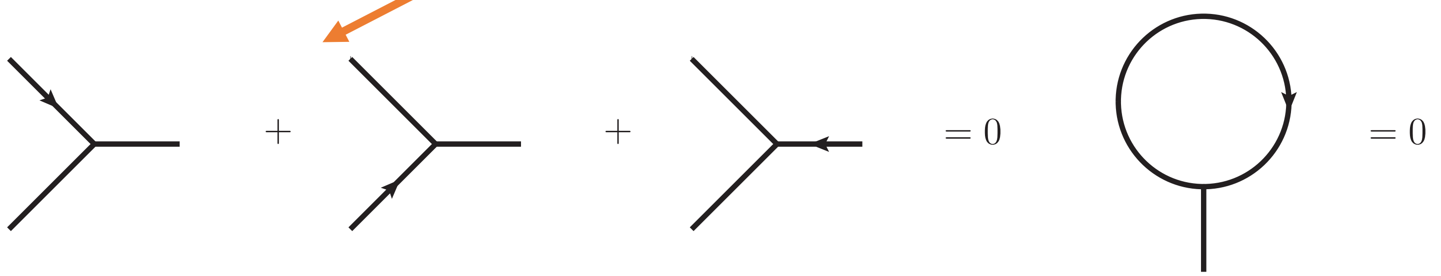
From inequivalent condition

$$D_1 F_{12} = 0 \Rightarrow p_1(p_1\epsilon_2 - p_2\epsilon_1) = p^2\epsilon_2 - (p\cdot\epsilon)p_2 = 0$$

$$D_{[1}F_{23]} = 0 \Rightarrow p_{[1}p_2\epsilon_{3]} = 0\ .$$

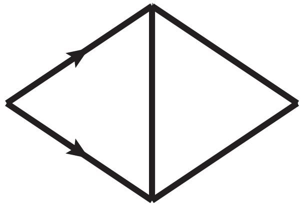
Equation of motion

Bianchi identity

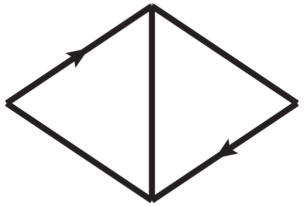


Arrow Configurations

- Each v-vertex has v-2 arrows coming in.
- A topology may have multiple arrow configurations:



$F_{12}D_1F_{34}D_2F_{45}F_{35}$



$F_{12}D_2F_{34}D_5F_{14}F_{35}$

- There are $\prod_{i=1}^{V} C_{v_i}^2$ arrow configurations for a topology. Very large number for complicated diagrams. How to reduce the number?

Reduce the number: Bianchi identity

- For $D_{1\cdots n}F_{ab}$, each D_i is associated with a Bianchi identity.
- Reduce the configurations from C_{n+2}^2 to n+1.
- We are left with $\prod_{i=1}^{V}(v_i - 1)$ configurations.
- Still can be large number for complicated diagrams.

Primary Operators

- If $K_1 = \partial^2 K_2$, then K_1 is called a descendant of K_2 .
- If an operator K is not a descendant, it is called a primary operator.
- The physical properties of descendants can be completely determined by the primary operators.
- Their minimal form factors are related by:

$$\mathcal{F}(K_1) = -(p_1 + \cdots + p_n)^2 \mathcal{F}(K_2)$$

- However, the number of primary operators still increase very fast as the dimension increases.

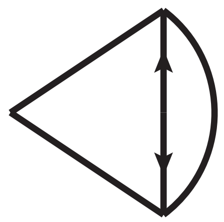
Primitive Operators

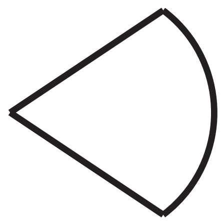
- More operators can be related to lower dimension operators if we use less stringent relations:

$$\mathcal{F}(K_1) = -(p_1 \cdot p_j)\mathcal{F}(K_2)$$

$$K_1 = F_{12}D_4F_{23}D_4F_{31}, \;\; K_2 = F_{12}F_{23}F_{31}$$

$$\mathcal{F}(K_1) = -p_2 \cdot p_3\mathcal{F}(K_2)$$

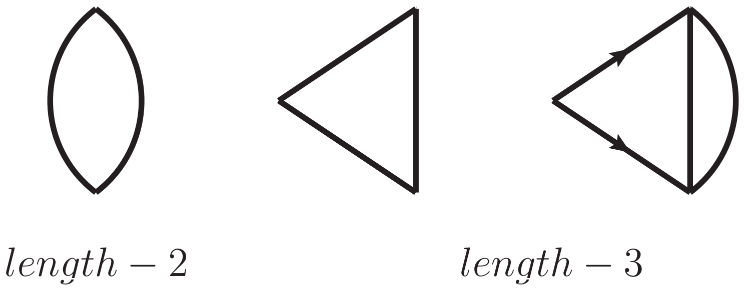




- K_1 will be called a non-primitive operator, which contains a D-pair(a pair of D with the same index). Operators without D-pair will be called primitive operators.

Primitive Operators

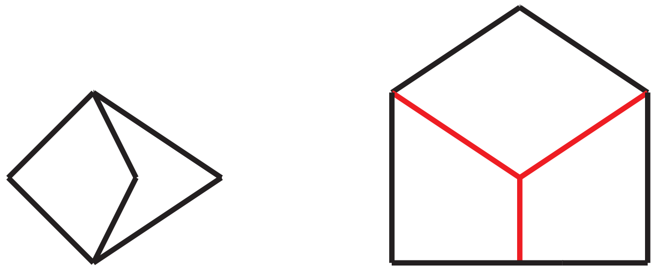
- There are only a finite number of primitive operators for a given length.



- Non-primitive operators can be generated by adding D-pairs to primitive operators.

Non-primitive Subdiagram

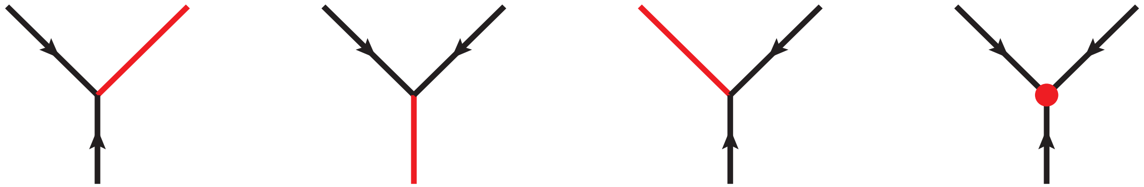
- A diagram has non-primitive configurations only if it contain edges without 2-vertex.



- These edges are called non-primitive edges. The sub-diagram formed by non-primitive edges is called the non-primitive sub-diagram.

Converge Regions

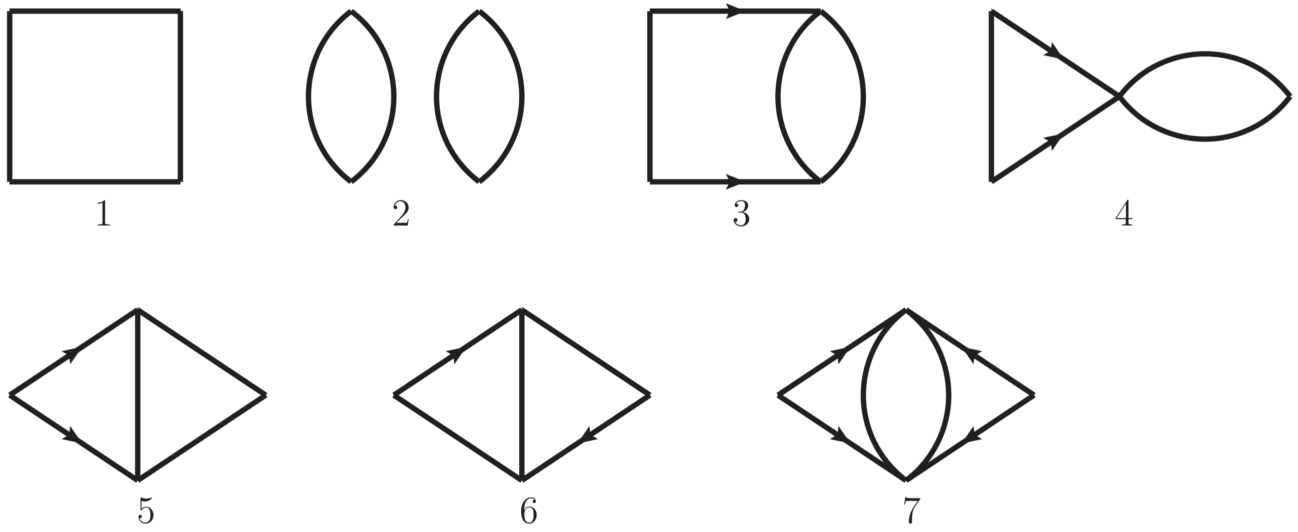
- The primitive configurations are characterized by converge regions of non-primitive sub-diagrams.



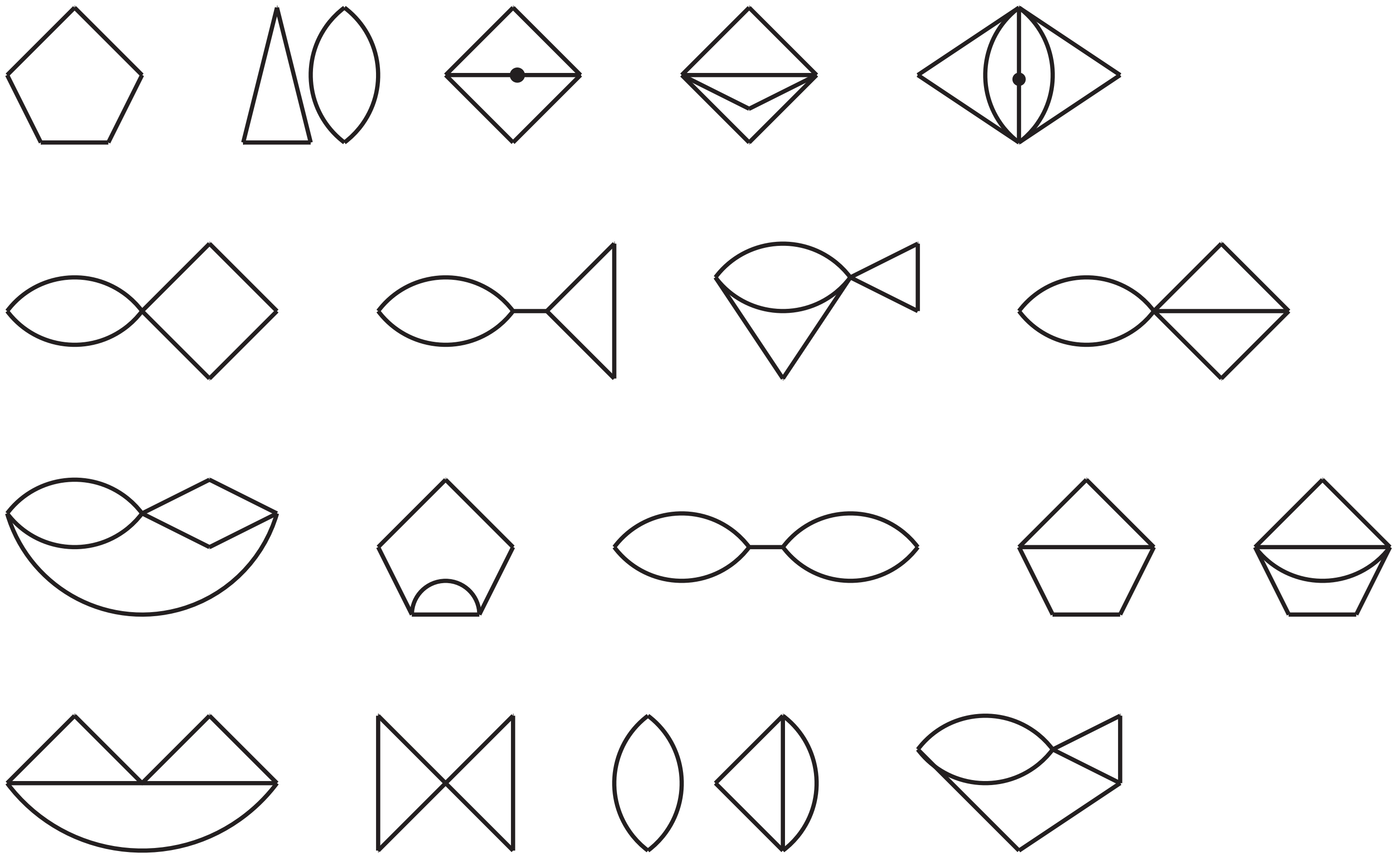
- Each vertex and edge in a tree sub-diagram can be a converge region. All arrows in the sub-diagram flow to the converge region.
- In a 1 loop sub-diagram, the converge region is the loop. Multi-loop sub-diagram has no primitive configurations.

Length-4 Primitive Operators

- 6 topologies, 7 configurations:



Length-5 Topologies



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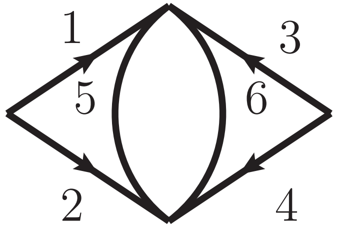
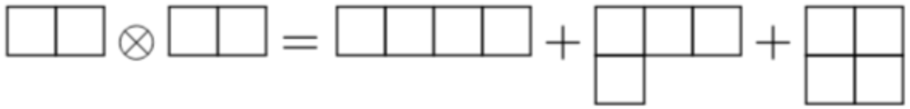
Outer Production

$$P = F_{12}F_{34}D_{13}F_{56}D_{24}F_{56}$$



- The topology has the symmetry $S_2 \times S_2$.
- The configuration is a  representation of $S_2 \times S_2$.
- The S_4 representation can be found by the outer production:



CSCO Method

- The CSCO method is probably the most convenient and “physical” way to find the explicit expressions of irreducible representations.
- CSCO: $\{O_1\} = \{(12) + (13) + (14) + (23) + (24) + (34)\}$
- The basis of $\{\sigma P | \sigma \in S_4\}$ can be chosen as:

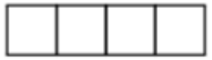
$$e_i = \{P,\ (23)P,\ (24)P,\ (13)P,\ (14)P,\ (13)(24)P\}$$

- The matrix of CSCO in this basis is

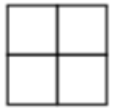
$$O_1 = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 \end{pmatrix}$$

CSCO Method

- The eigenspaces of CSCO gives the irreducible representations:

 $: \ \lambda = 6, \ L_1 = \mathrm{span}\{e_1 + e_2 + e_3 + e_4 + e_5 + e_6\},$

 $: \ \lambda = 2, \ L_2 = \mathrm{span}\{e_1 - e_6, \ e_2 - e_5, \ e_3 - e_4\},$

 $: \ \lambda = 0, \ L_3 = \mathrm{span}\{e_1 - e_3 - e_4 + e_6, \ e_2 - e_3 - e_4 + e_5\},$

D-pairs

- Non-primitive operators are generated from P by adding D-pairs, or s_{ij} factors in the language of form factors. These factors should also be classified by Sn symmetry.
- For example, for dim=14 operators generated by P, the factors are

$$s_{12}^2 : \yng(2) \otimes \yng(2) = \yng(4) + \yng(3,1) + \yng(2,2)$$

$$s_{12}s_{13} : \yng(1) \otimes \yng(2) \otimes \yng(1) = \yng(4) + 2\,\yng(3,1) + \yng(2,2) + \yng(2,1,1)$$

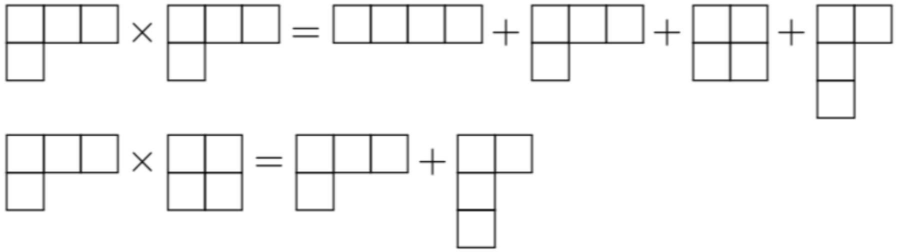
$$s_{12}s_{34} : \yng(4) : \ \lambda = 6,\ L_1 = \mathrm{span}\{e_1 + e_2 + e_3\},$$

$$\yng(2,2) : \ \lambda = 0,\ L_3 = \mathrm{span}\{e_1 - e_2,\ e_1 - e_3\},$$

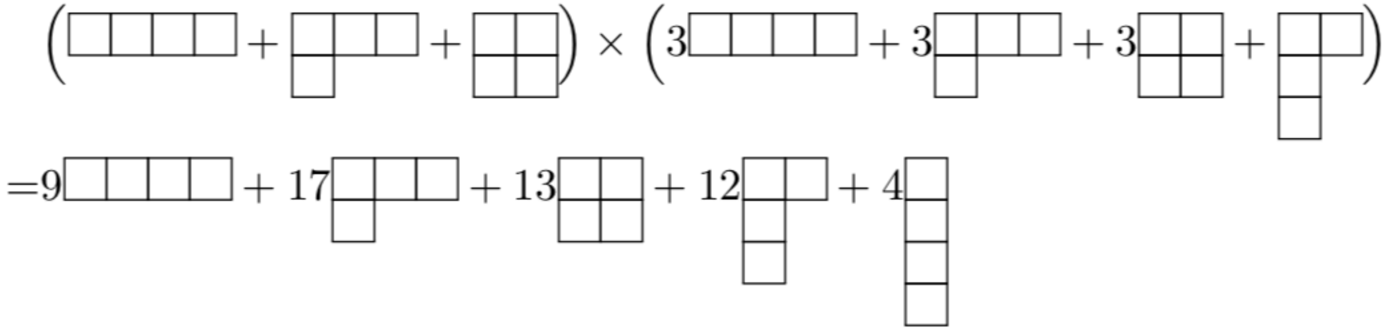
$$e_i = \{s_{12}s_{34},\ s_{13}s_{24},\ s_{14}s_{23}\}$$

Tensor products

- The representation of $s_{ij}s_{kl}P$ can be obtained by the tensor products of S_4 representations. For example:



- The result is



Color Factors

- The representations of color factors can be decomposed similarly:

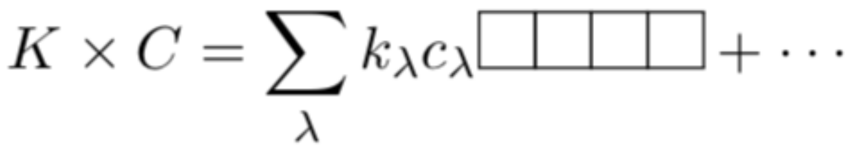
$$\mathrm{Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}) = \yng(4) + \yng(2,2) + \yng(2,1,1)$$

$$\mathrm{Tr}(T^{a_1}T^{a_2})\,\mathrm{Tr}(T^{a_3}T^{a_4}) = \yng(4) + \yng(2,2)$$

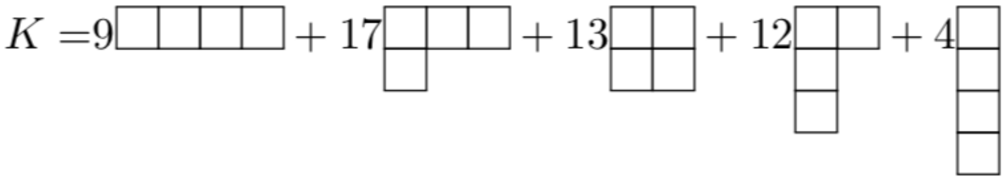
- Due to Bose symmetry, the color dressed operator must be invariant under S_4, which means it is a $\yng(4)$ representation.
- So physical form factors corresponding to $\yng(4)$ in the tensor product of kinetic form factors and color factor.

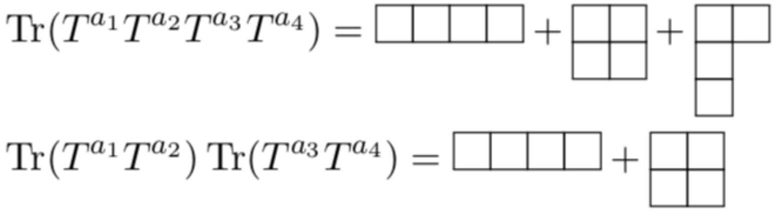
Color Factors

- Two irreducible representations contains a $[4]$ if and only if they are equivalent.
- Suppose the kinetic form factors $K = \sum_\lambda k_\lambda [\lambda]$, and color factors $C = \sum_\lambda c_\lambda [\lambda]$ their tensor product



- So P generates 9+13+12=34 single trace operators, and 9+13=22 double trace operators at dim=14.





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Summary and Outlook

- A diagrammatic representation of gluon operators is proposed to manifest their symmetries and relations.
- The infinite set of high dimensional operators are generated from a finite set of "primitive operators" by adding pairs of covariant derivatives.
- Can we extend the method to operators with fermions, like those appear in SMEFT?

Thank you!