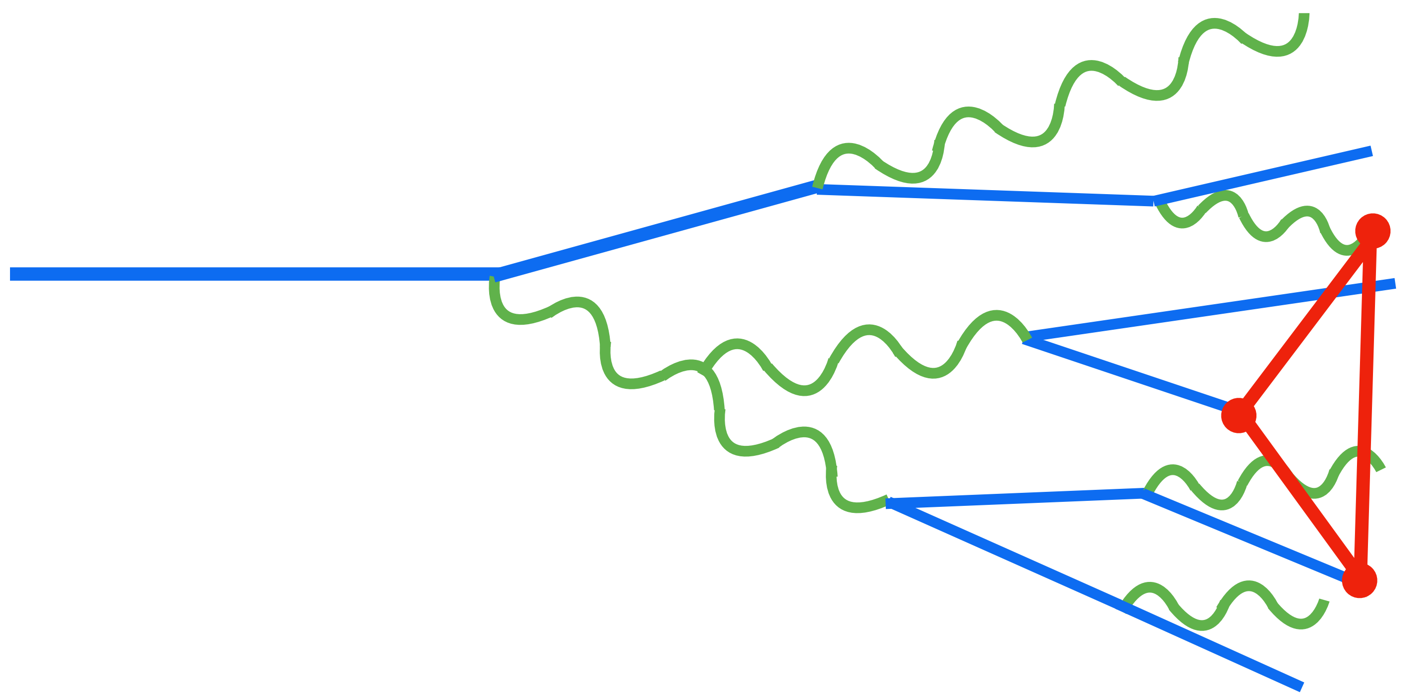
Exploring Jet Substructure with Energy Correlators



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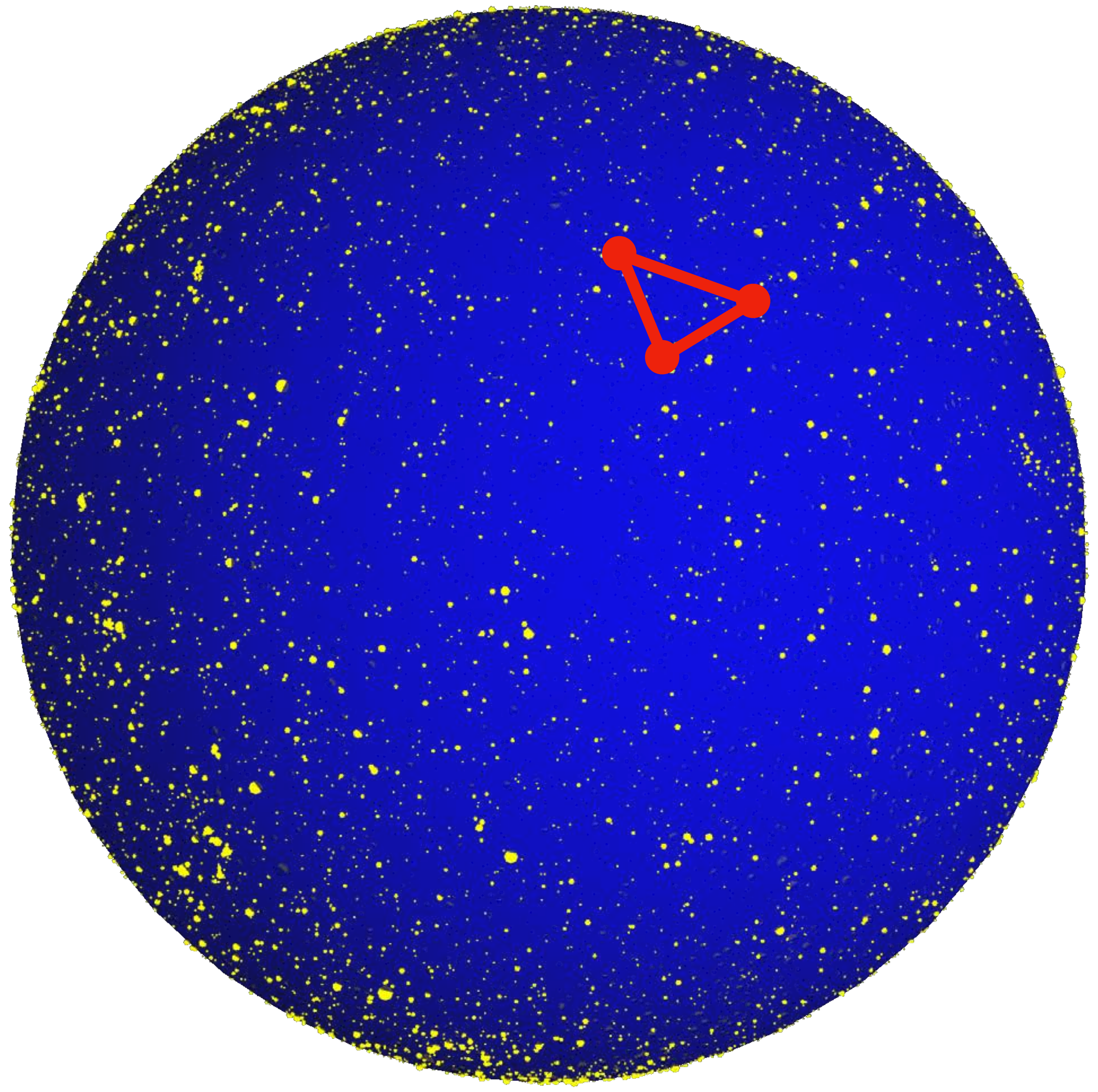

Based on:

HC, MX. Luo, I. Moult, TZ. Yang, XY. Zhang, HX. Zhu [1912.11050]

HC, I. Moult, HX. Zhu [2011.02492, 2104.00009]

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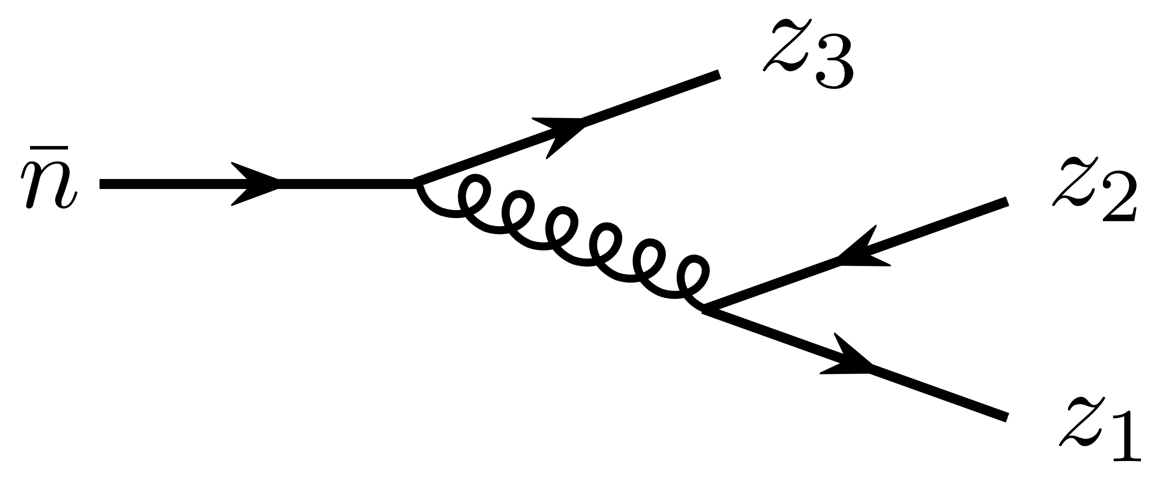
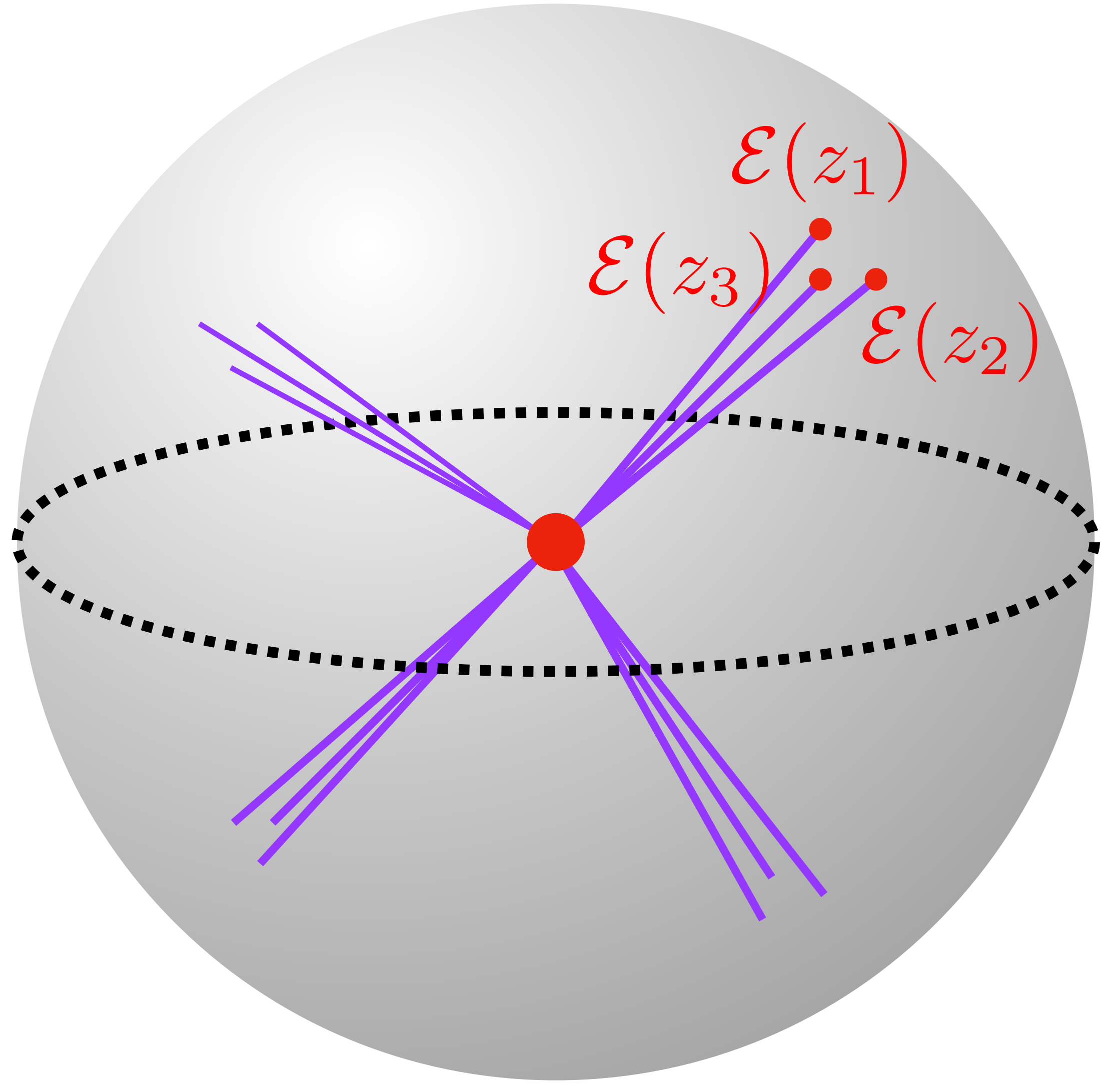
Outline

Collinear Triple Energy Correlation

- Amplitude Side
  - Definition and kinematics
  - Relation to Feynman integral



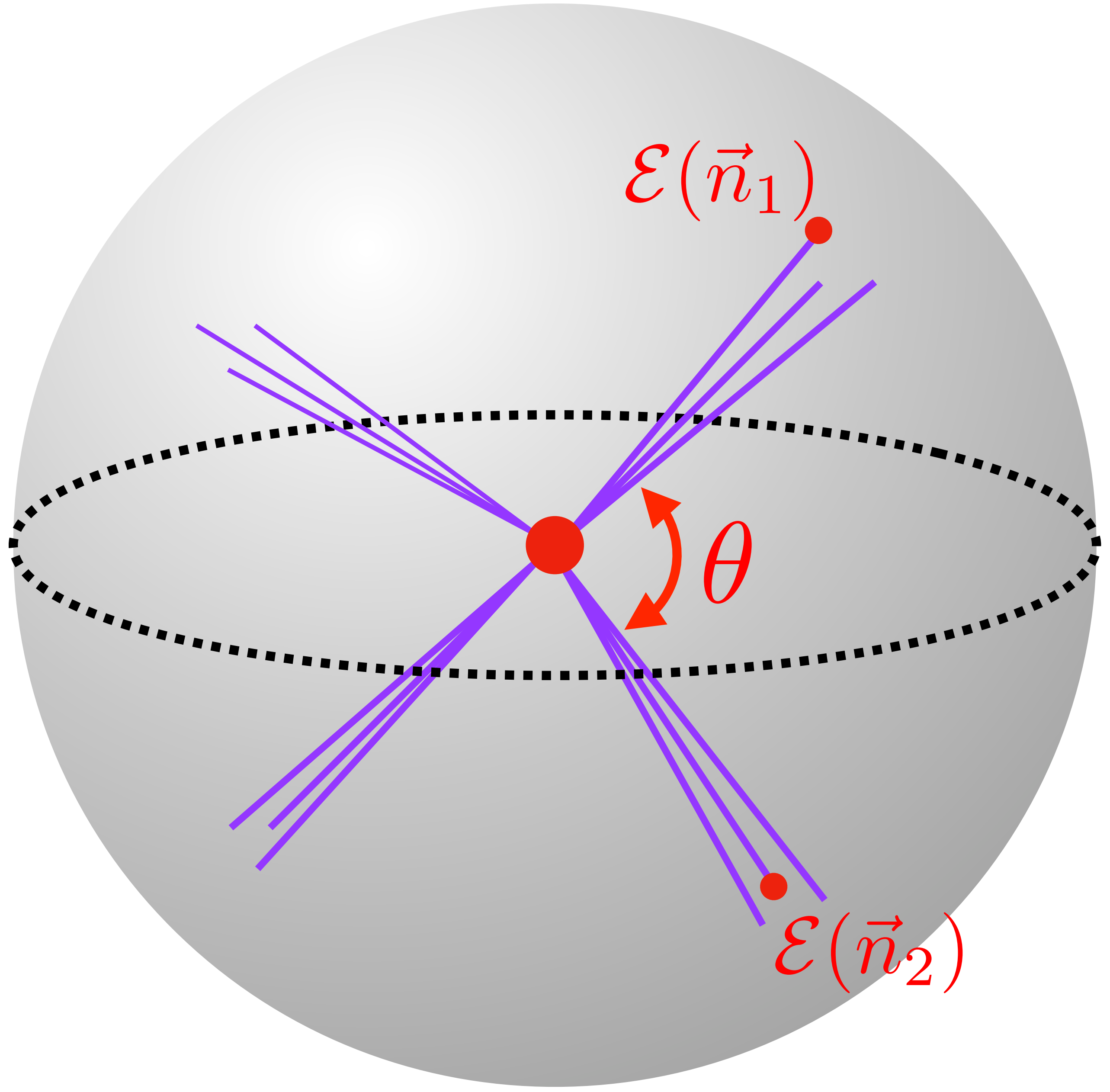
- Correlator Side
  - Celestial conformal symmetry
  - Conformal block decomposition on the celestial sphere



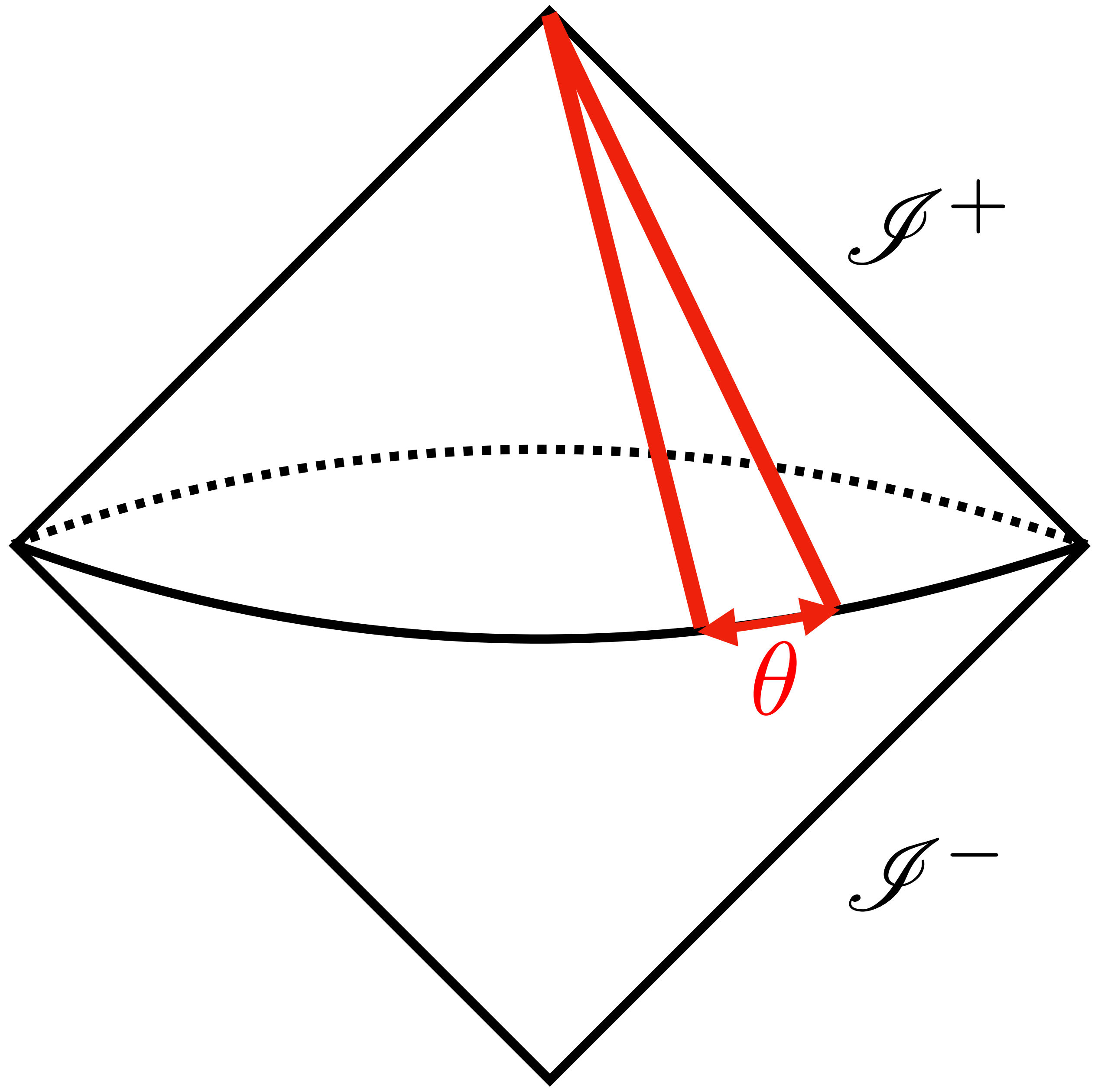
Energy Flow Operators and EEC

[Basham, Brown, Ellis and Love, 1978]
introduced energy-energy correlation

$$\frac{d\Sigma}{dz} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\theta_{ij}}{2}\right)$$



which characterizes the correlation of two energy detectors (calorimeters) at spatial infinity (celestial sphere).



The energy detector has a nice operator definition:

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\vec{n})$$

[Korchemsky, Sterman, 1999; Hofman, Maldacena, 2008; Bauer, Fleming, Lee, Sterman, 2008; ...]

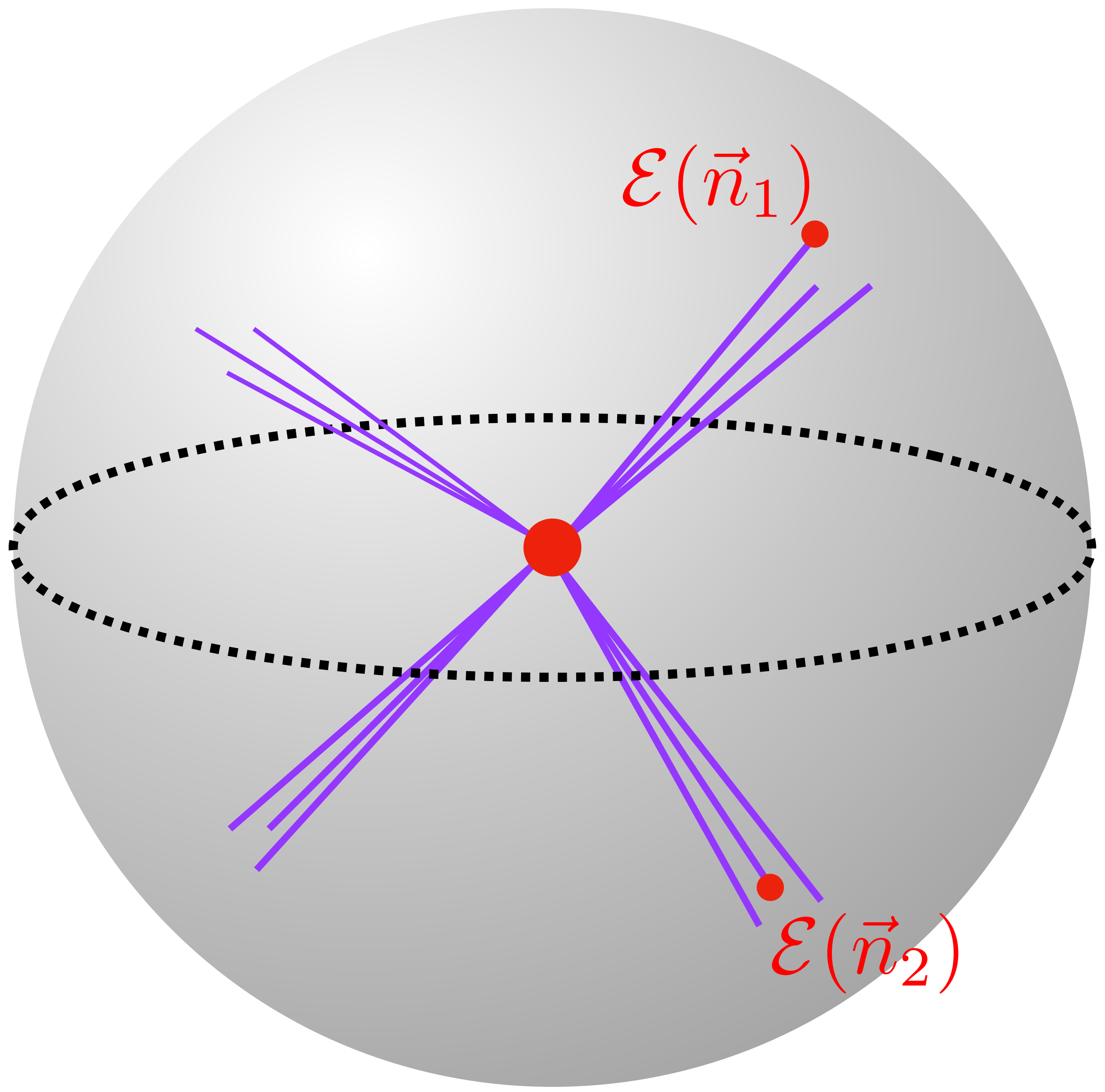
allowing alternative definition of EEC and its multi-point generalization as correlation function of multiple insertion of **energy flow** operators

$$\langle O'(-q)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots|O(q)\rangle$$

Source with total momentum $q = (Q,0,0,0)$

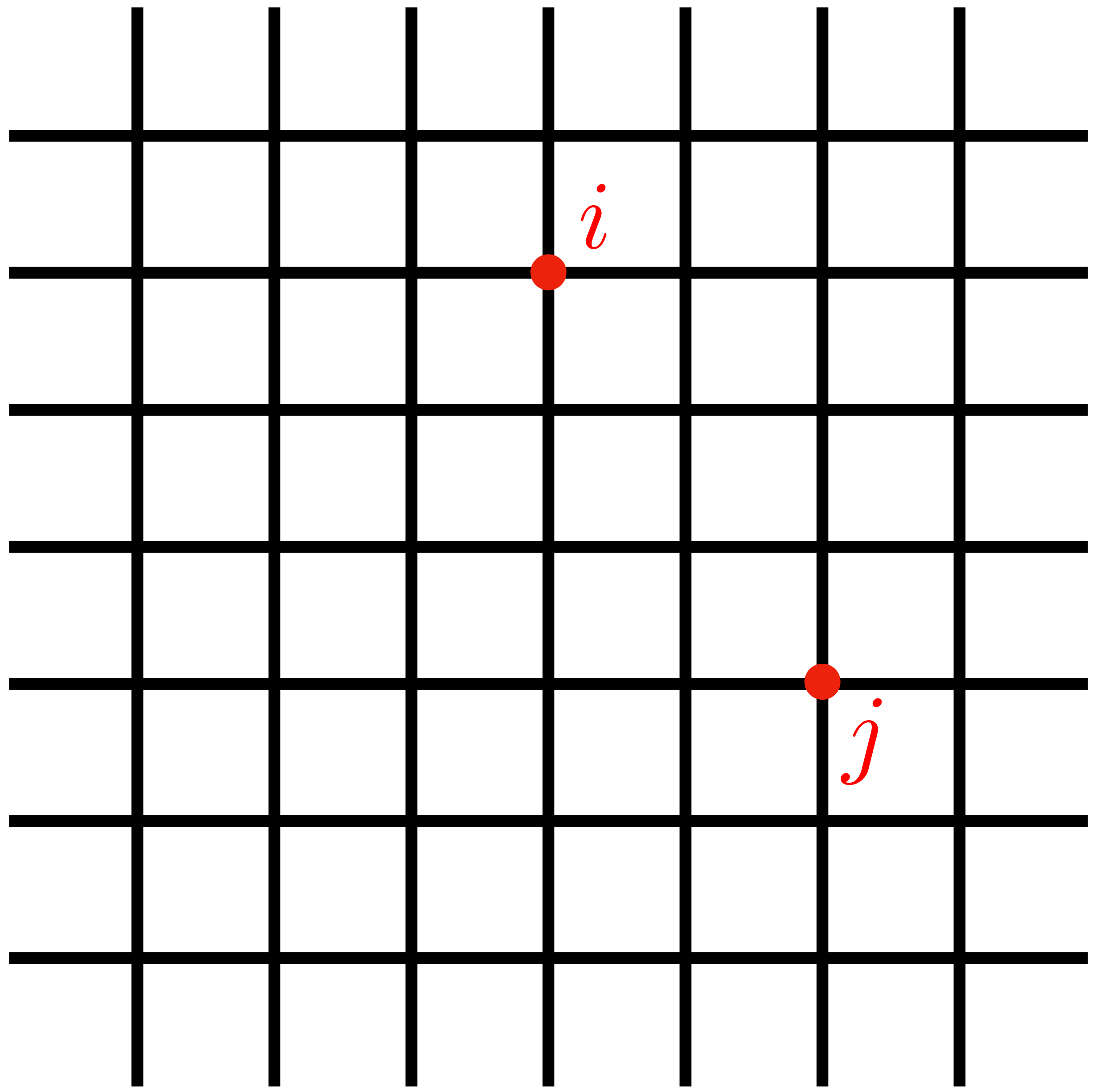
Energy Correlation on the celestial sphere

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \rangle$$



Spin Correlation on the plane (2D Ising)

$$\langle \sigma_i \sigma_j \rangle$$



Probability Distribution

differential cross section $d\sigma$ ⟷ Boltzmann factor $e^{-\beta H}$

Weighting Factor

eigenvalues of energy $\mathcal{E}$ ⟷ eigenvalues of spin σ

State of the art for Energy Correlators

EEC calculations (full angle):

QCD:	**LO**	**[Basham, Brown, Ellis, Love, 1978]**
	NLO	**[Dixon, Luo, Shtabovenko, Yang, Zhu, 2018]**
$\mathcal{N}=4$ SYM:	**NLO**	**[Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2014]**
	NNLO	**[Henn, Sokatchev, Yan, Zhiboedov, 2019]**

Collinear limit:

QCD:	**[Dixon, Moult, Zhu, 2019]**
$\mathcal{N}=4$ SYM:	**[Korchemsky, 2019; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]** **[Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020]**
3-point (LO):	**[HC, Luo, Moult, Yang, Zhang, Zhu, 2019]**

Back-to-back limit:

QCD:	**N^3LL**	**[Moult, Zhu, 2018] [Ebert, Mistlberger, Vita, 2020]**
	NLP	**[Moult, Vita, Yan, 2019]**
$\mathcal{N}=4$ SYM:		**[Korchemsky, 2019]**

Other colliders:

pp **[Gao, Li, Moult, Zhu, 2019]** ep **[Li, Vitev, Zhu, 2020; Li, Makris, Vitev, 2021]**

3-point Energy Correlator

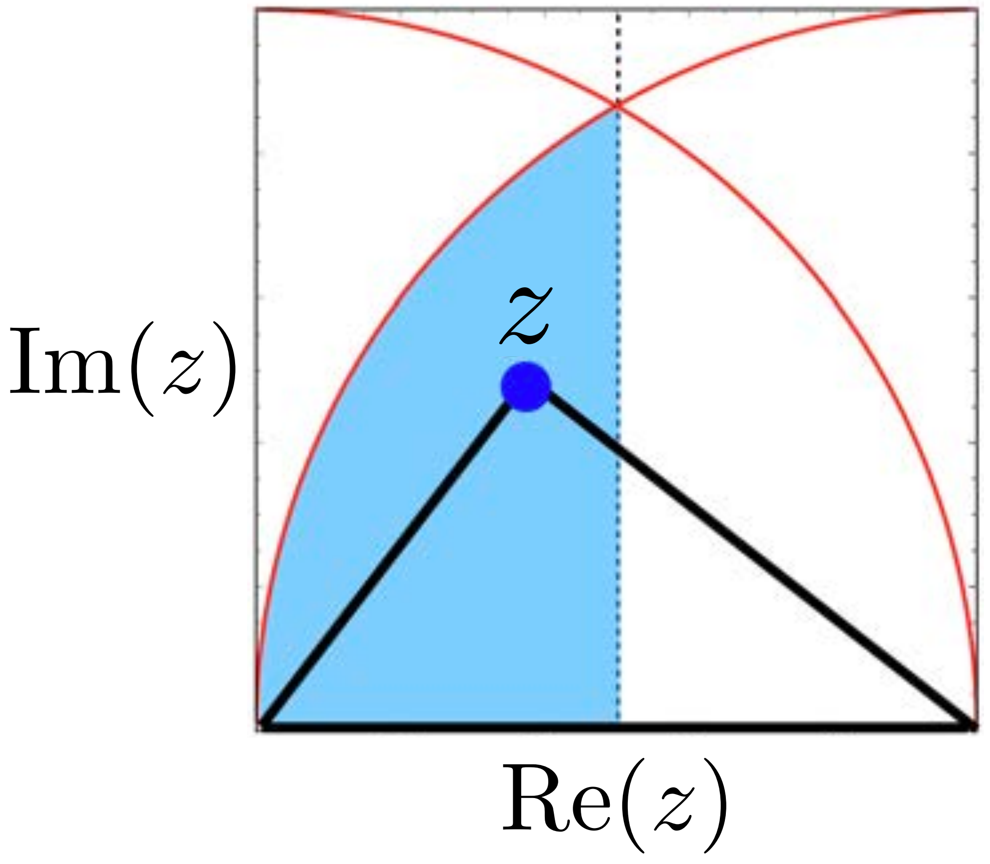
in the collinear limit

Kinematics

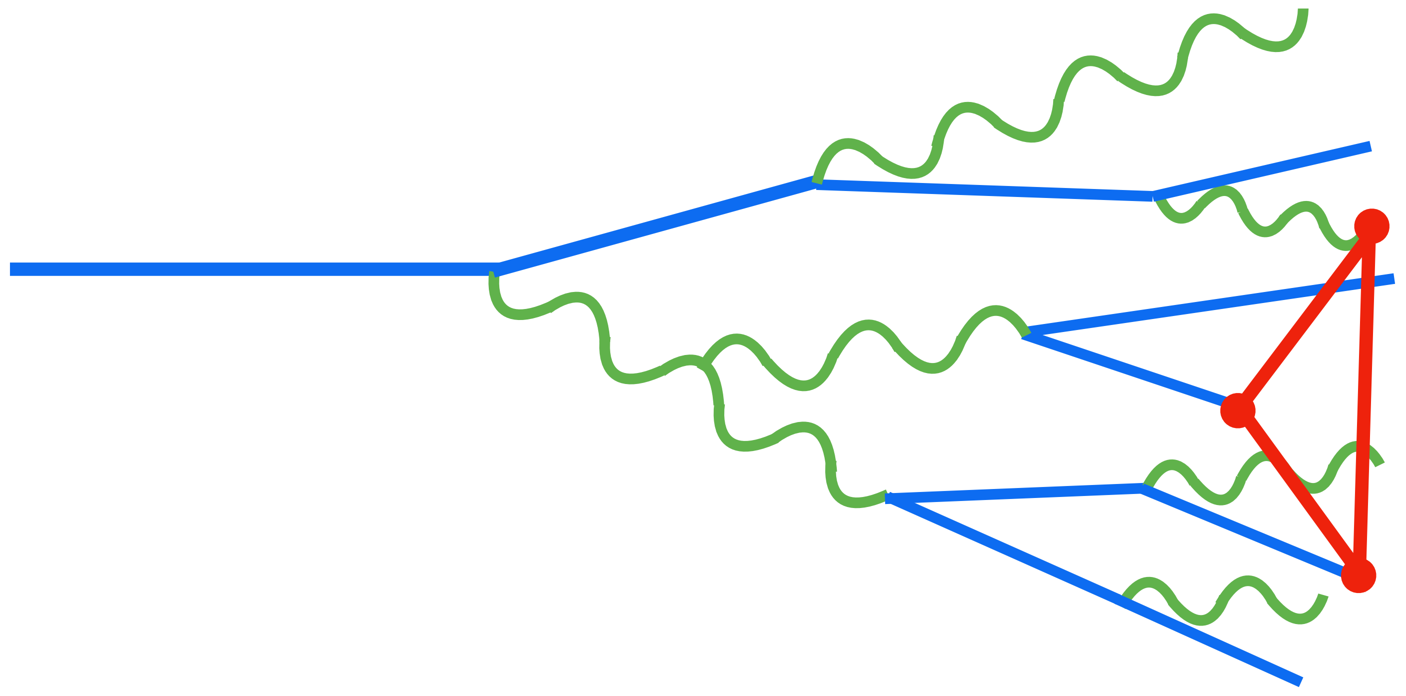
In the collinear limit, EEEC configuration can be approximated by a triangle.

[HC, Luo, Moult, Yang, Zhang, Zhu, 2019]

Moduli space of triangle shape



Parameterized in terms of
(1) the longest side^2 x_L **[Size]**
(2) a complex number z **[Shape]**



$$\frac{E_a E_b E_c}{(Q/2)^3}\,\delta\!\left(x_1 - \frac{s_{ab}}{4E_aE_b}\right)\delta\!\left(x_2 - \frac{s_{bc}}{4E_bE_c}\right)\delta\!\left(x_3 - \frac{s_{ac}}{4E_aE_c}\right)$$

Definition

collinear phase space **measurement**

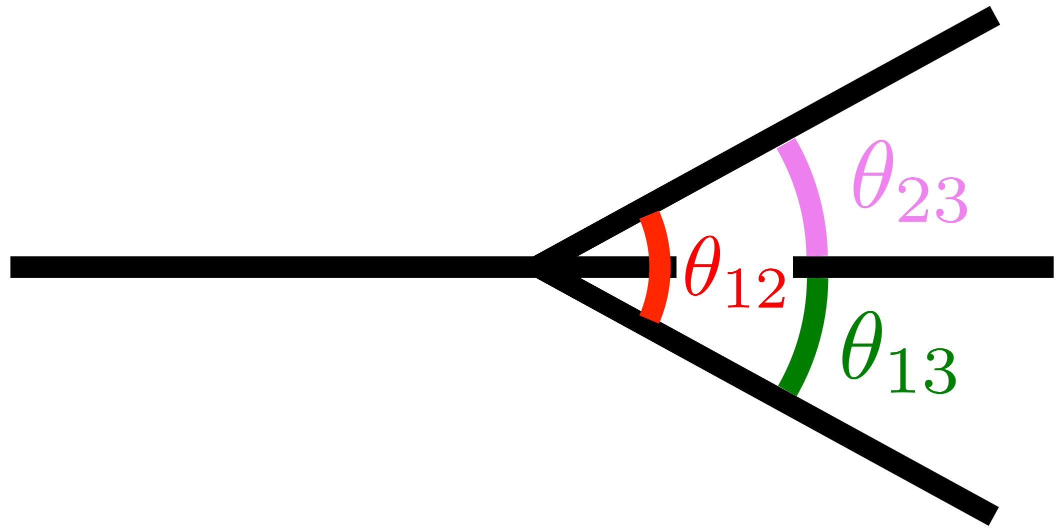
$$\frac{1}{\sigma_{\text{tot}}}\frac{d^3\Sigma_i}{dx_1dx_2dx_3} = \sum_{a,\,b,\,c}\int d\Phi_c^{(3)}\, P_{abc}^{(i)}\, \mathcal{M}_{\text{EEEC}}$$

$1 \to 3$ **splitting probability**

x_1, x_2, x_3 **length^2 of each side ~ angle^2**

Toy Example: Collinear Number Correlator

Though number correlators are not IR safe in QCD, they still make sense in ϕ^4 theory and capture some features of the energy correlators.



ϕ^4 theory $1 \to 3$ splitting probability

$$P = \frac{1}{s_{123}^2} \propto \frac{1}{(\omega_1\omega_2 x_3 + \omega_2\omega_3 x_1 + \omega_1\omega_3 x_2)^2}$$

$\omega_1, \omega_2, \omega_3$
momentum fractions

Measurement function fixes 3 angles θ_{12} θ_{23} θ_{13}

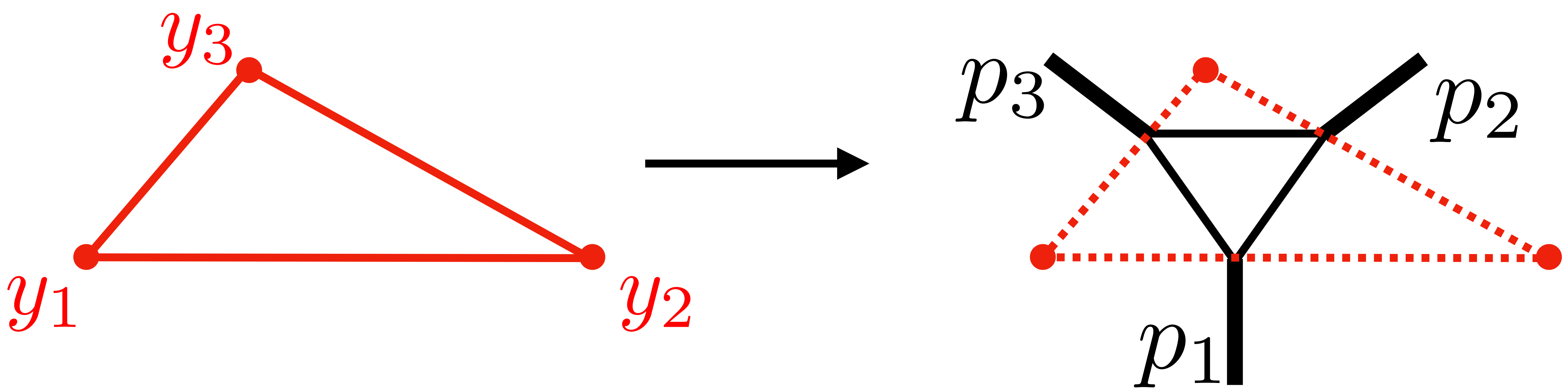
which reduces collinear PS to momentum fractions integration

$$\int d\Phi_c^{(3)} \longrightarrow N \int d\omega_1 d\omega_2 d\omega_3 \; \omega_1\omega_2\omega_3 \delta(1-\omega_1-\omega_2-\omega_3)$$

has the same measure as Feynman parametrization

Toy Example: Collinear Number Correlator

Interestingly, we can find dual Feynman diagram description:



dual coordinates

$$p_1 = y_2 - y_1$$
$$p_2 = y_3 - y_2$$
$$p_3 = y_1 - y_3$$

$$|y_{12}|^2 = x_3$$
$$|y_{23}|^2 = x_1$$
$$|y_{13}|^2 = x_2$$

Result

Leading singularity

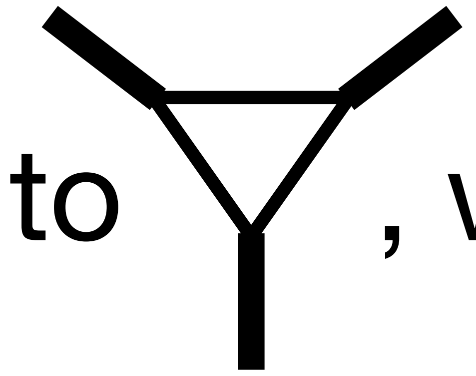
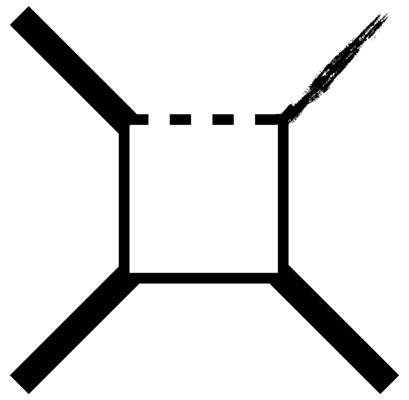
$$\frac{4i\left(u^3+v^3-u^2v-uv^2-u^2+6uv-v^2-u-v+1\right)}{(z-\bar{z})^5} D_2(z)$$
$$+\frac{2}{(z-\bar{z})^4}\left[\left(1+u-2u^2-2v+uv+v^2\right)\log u+(u\leftrightarrow v)\right]+\frac{2}{(z-\bar{z})^2}$$

Notations

cross-ratios $u = z\bar{z},\ v = (1-z)(1-\bar{z})$

Bloch-Wigner function $2iD_2(z) = \mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}(\log(1-z) - \log(1-\bar{z}))\log(z\bar{z})$

Collinear EEEC Results

In $\mathcal{N}=4$ SYM/QCD, in addition to  , we have another dual diagram 

1 massless leg
1 eikonal propagator

$\mathcal{N}=4$ SYM result:

$$G^{\mathcal{N}=4}_{\mathcal{E}\mathcal{E}\mathcal{E}}(z) = \frac{1+u+v}{2uv} - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v)$$
$$-(1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z)$$
$$+\frac{(u-1)(u+1)}{2uv^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right) + \frac{1+u+v}{2uv}\zeta_2$$

Two types of single-valued transcendental weight 2 functions:

$$\Phi(z) = \frac{1}{z-\bar{z}}\left(2\mathrm{Li}_2(z) - 2\mathrm{Li}_2(\bar{z}) + (\log(1-z) - \log(1-\bar{z}))\log(z\bar{z})\right),$$

$$D_2^+(z) = \mathrm{Li}_2\left(1-|z|^2\right) + \frac{1}{2}\log\left(|1-z|^2\right)\log\left(|z|^2\right)$$

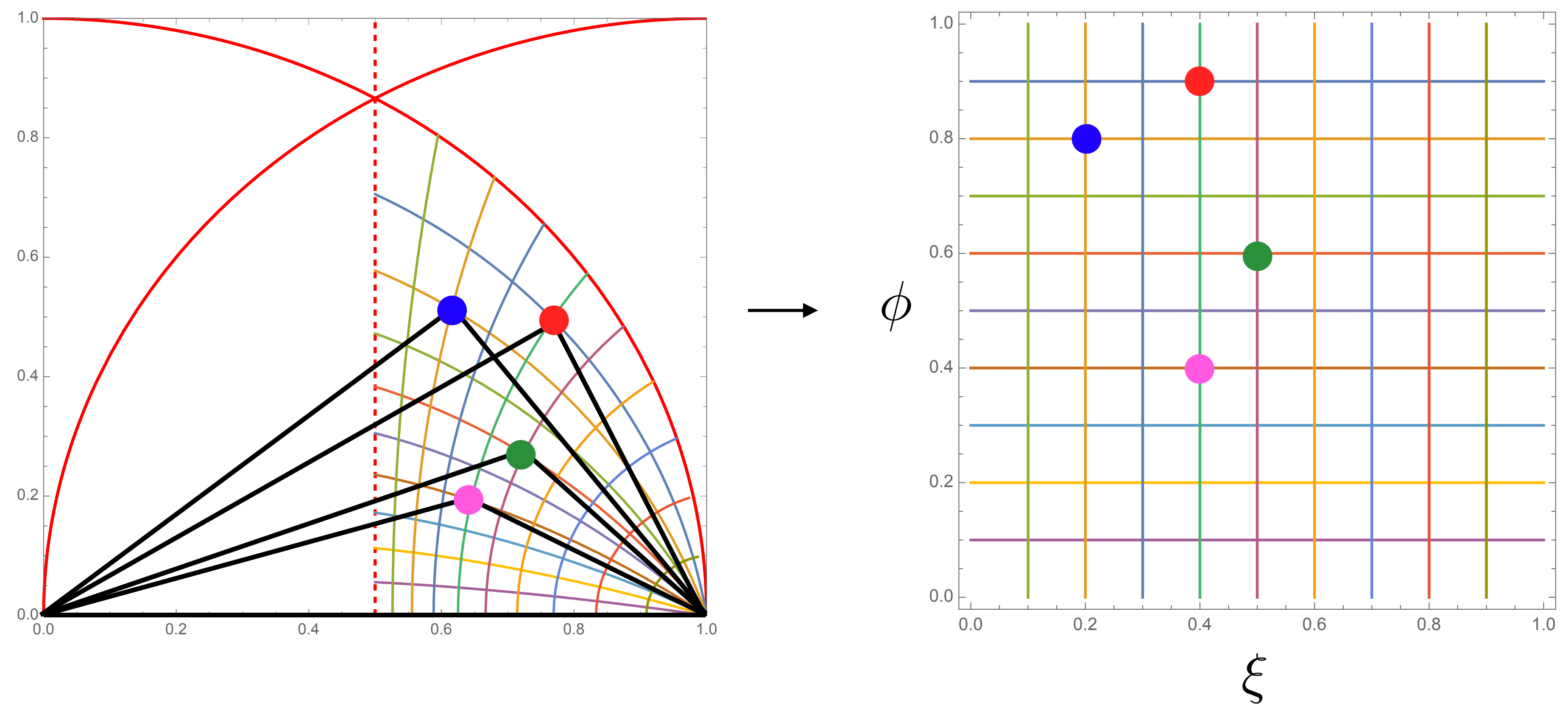
contribution from Wilson line, absent in scalar theory

QCD results have the same structure with more complicated rational functions.

Aside: Measuring Shape Dependence

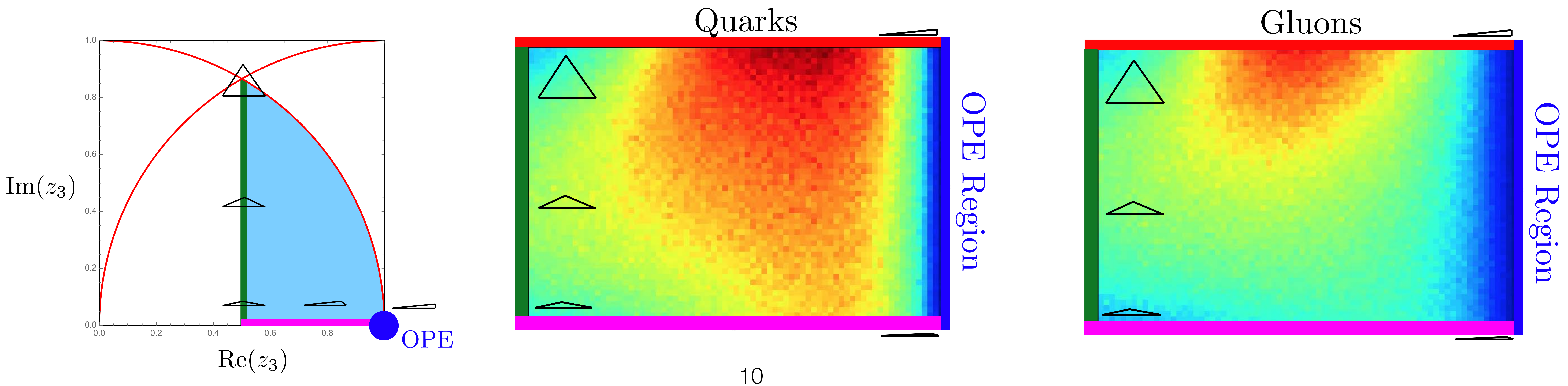
The shape dependence $g(z, \bar{z})$ in collinear EEEC can be directly measured at LHC!

[Komiske, Moult, Thaler, Zhu, Forthcoming]



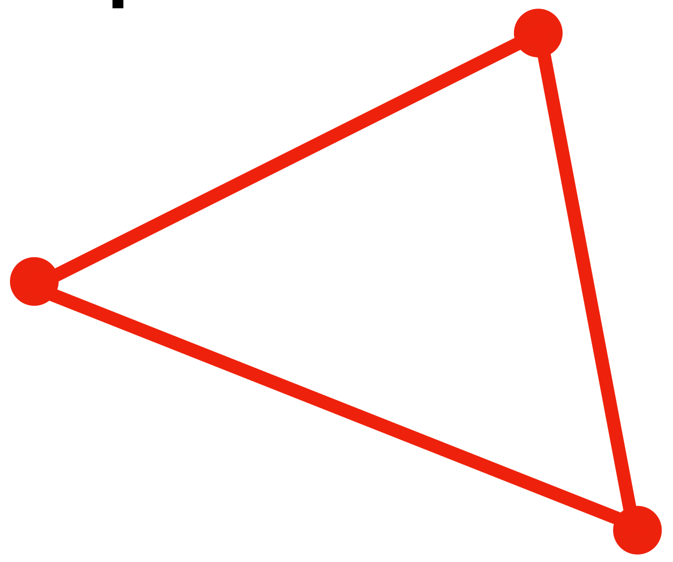
Moduli space and parametrization

Imaging of 3-point energy correlator $g(z, \bar{z})/(z\bar{z})^3$

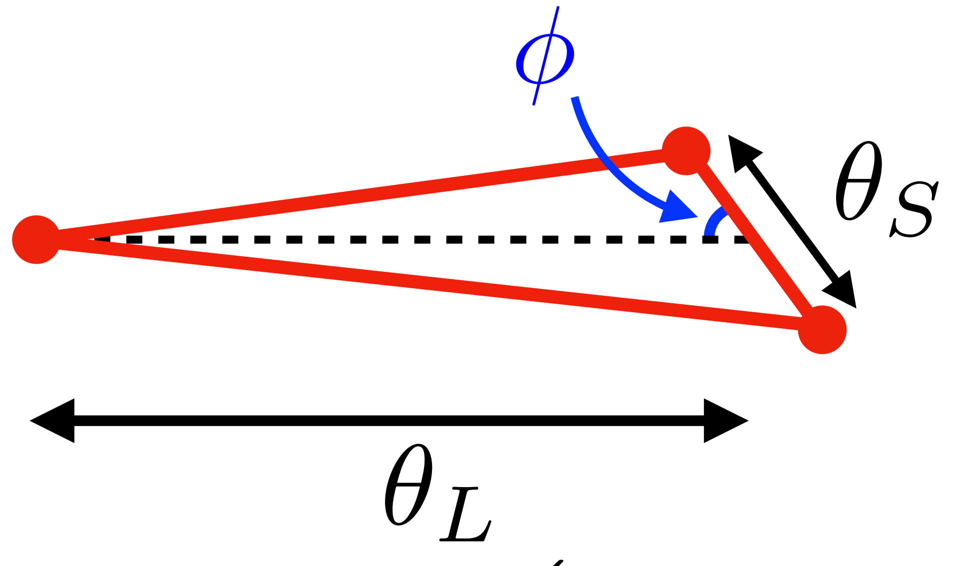


Squeezed Limit

Squeezed limit physically corresponds to bringing two detectors very close.



squeezed limit →



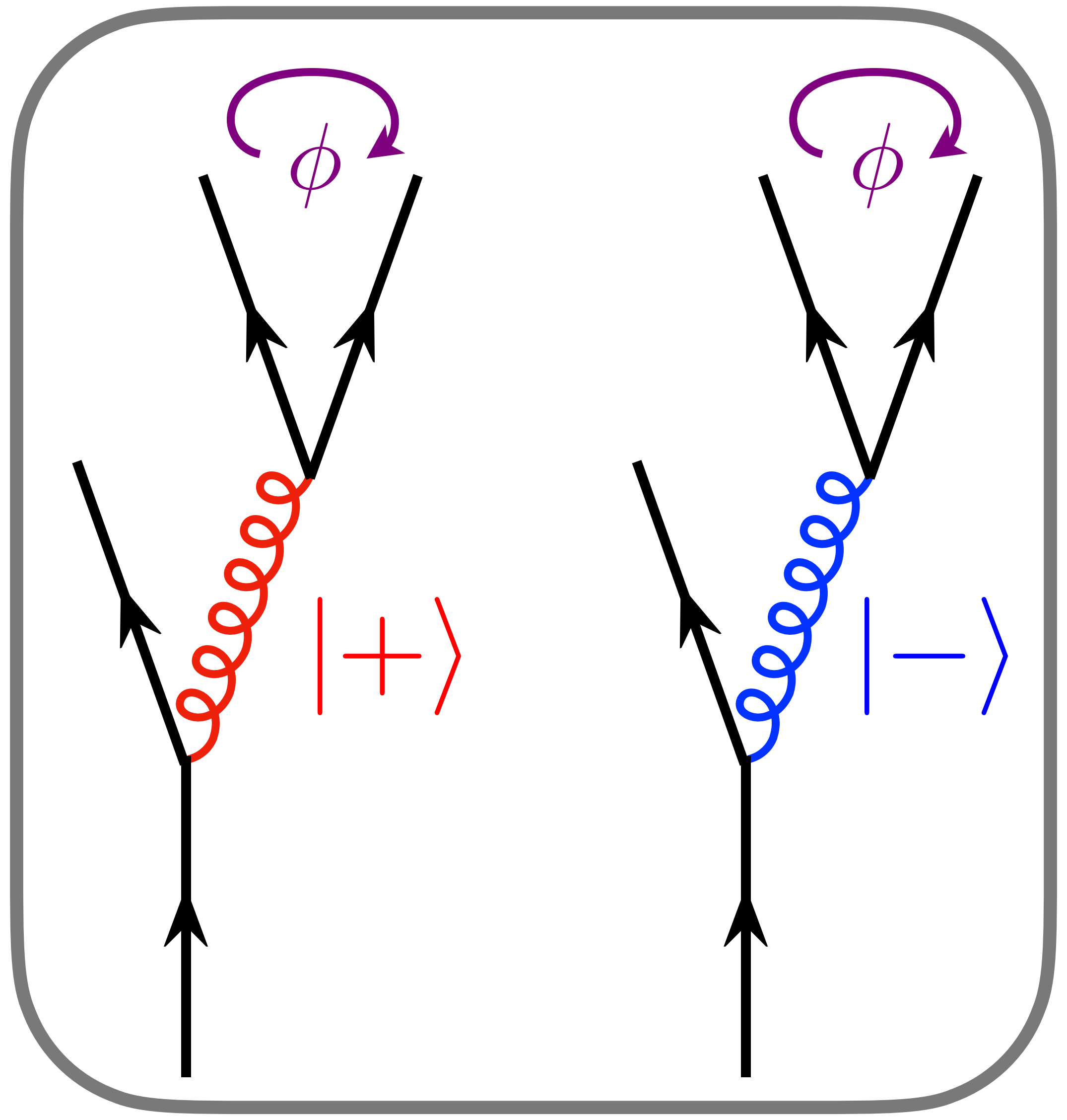
$$\frac{d^3\Sigma_i}{d\theta_L^2 d\theta_S^2 d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\mathrm{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$

$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2 \frac{16}{5}$$

$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F \frac{3}{5}$$

Squeezed limit encodes spin correlation information and the Leading Power resummation is done. **[HC, Moult, Zhu, 2020]**

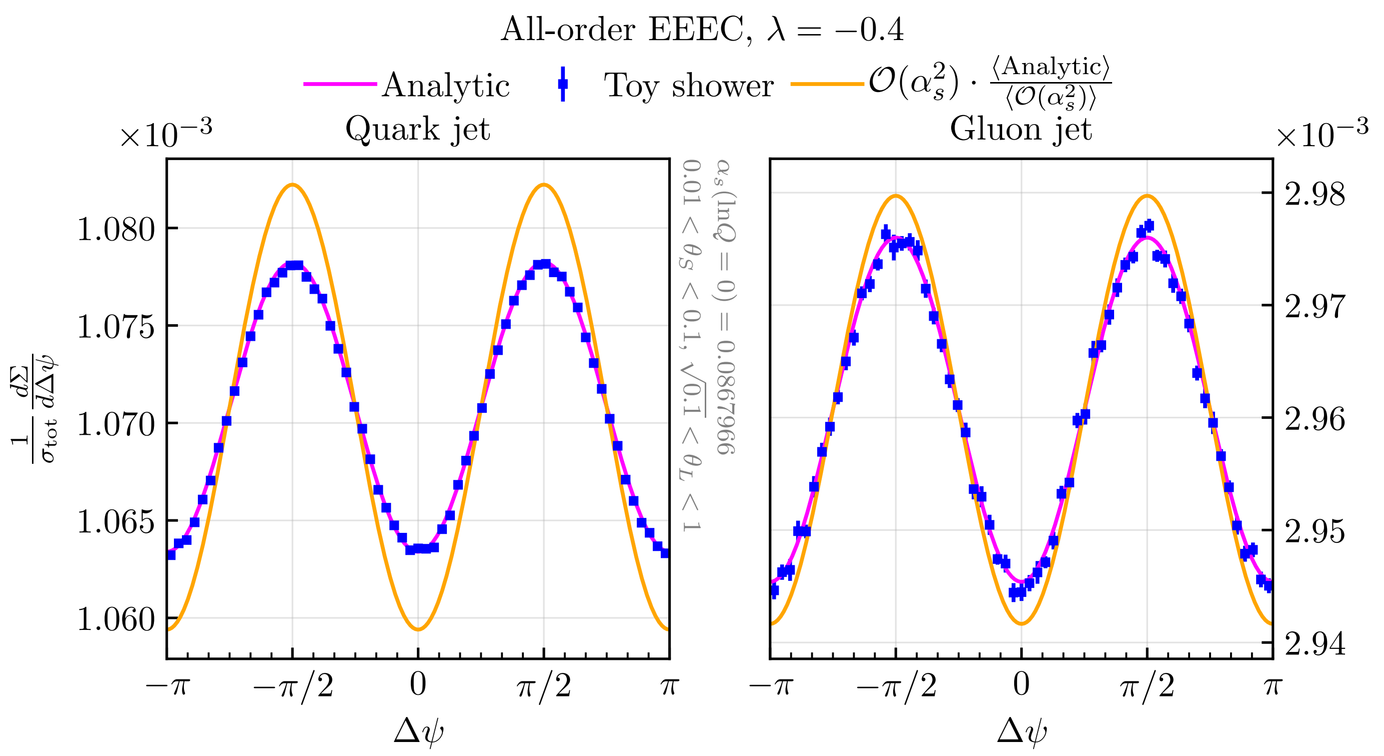
Interference Effect



Contributing Operator

$F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$

twist-2, transverse spin-2 gluonic operator



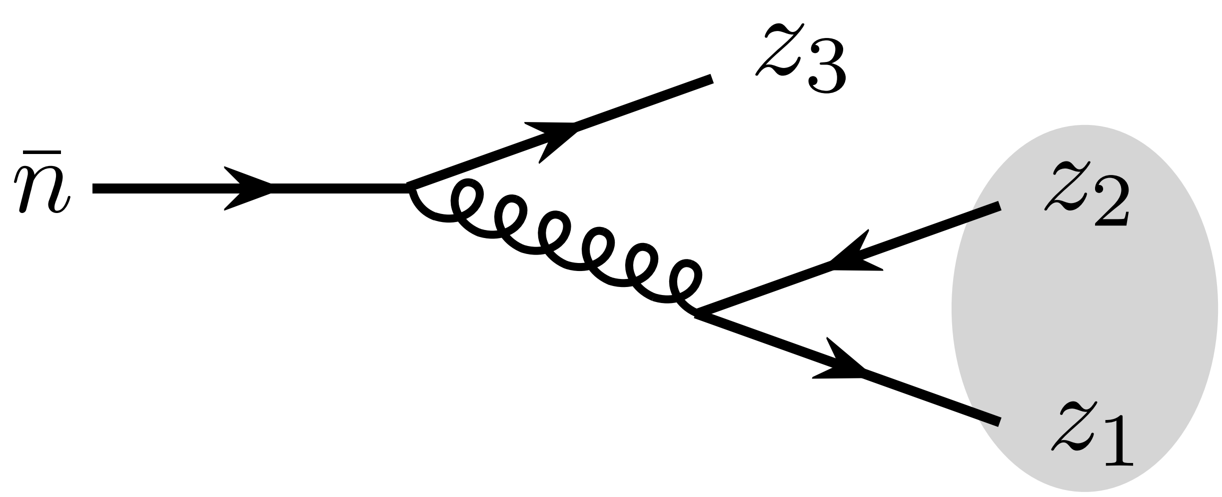
Recently, when collinear spin correlation is included in the **PanScales** family of parton showers, our resummed result provides validation of shower results.

[Karlberg, Salam, Scyboz, Verheyen, 2021]

Squeezed Limit

more power expansion

For simplicity, tagging final state quarks



Squeezed limit: $z_1 \cdot z_2 \to 0$

Expanding the full result:

highest transverse spin series

$$g(u,v) \equiv g(z,\bar{z}) \propto \quad -z^3\bar{z} \quad + \frac{39}{10}z^2\bar{z}^2 \quad - z\bar{z}^3 \qquad \textbf{LP}$$

$\cos 2\phi$

$$-z^4\bar{z} \quad + \frac{39}{20}z^3\bar{z}^2 \quad + \frac{39}{20}z^2\bar{z}^3 \quad - z\bar{z}^4 \qquad \textbf{NLP}$$

$\cos 3\phi$

$$-\frac{6}{7}z^5\bar{z} \quad + \frac{229}{140}z^4\bar{z}^2 \quad - \frac{211}{140}z^3\bar{z}^3 \quad + \frac{229}{140}z^2\bar{z}^4 \quad - \frac{6}{7}z\bar{z}^5 \qquad \textbf{NNLP}$$

$\cos 4\phi$

$$-\frac{5}{7}z^6\bar{z} \quad + \frac{207}{140}z^5\bar{z}^2 \quad - \frac{233}{140}z^4\bar{z}^3 \quad - \frac{233}{140}z^3\bar{z}^4 \quad + \frac{207}{140}z^2\bar{z}^5 \quad - \frac{5}{7}z\bar{z}^6 \qquad \textbf{NNNLP}$$

$\cos 5\phi$

$\cdots \qquad \cdots \qquad \cdots$

$$-z^3\bar{z}\ {}_2F_1(3,2,6,z)$$

How to understand?

Light-ray Operators and OPE

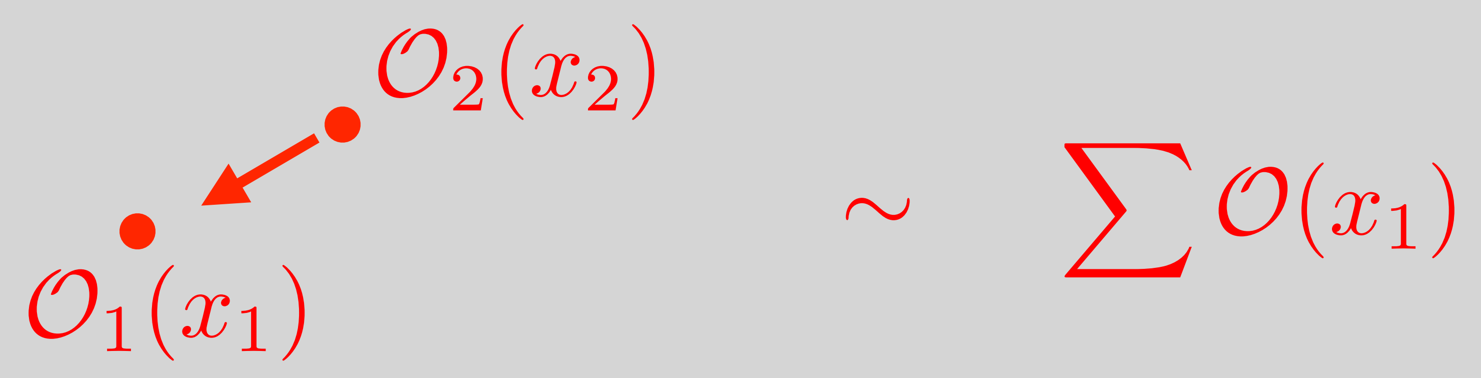
Generalization of $\mathcal{E}(\vec{n})$:

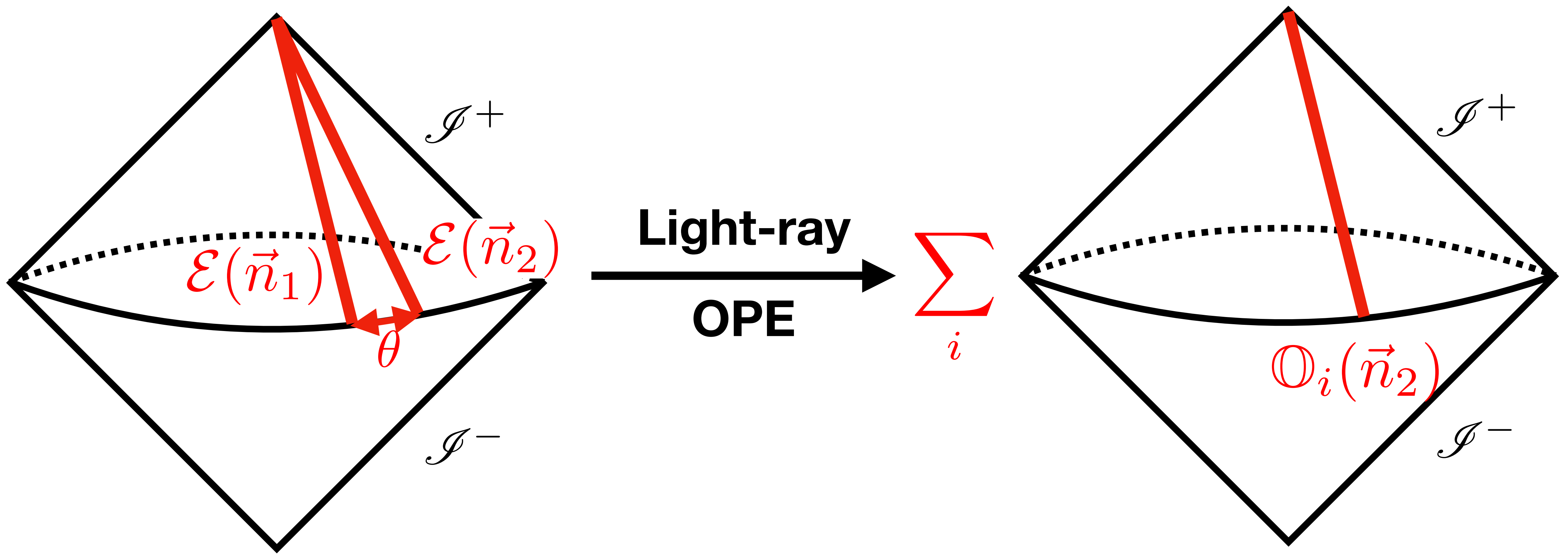
[Kravchuk, Simmons-Duffin, 2018]

twist

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta - J} \int_0^\infty dt\, O^{\mu_1\cdots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

Local Operator Analogy





Small angle behavior is controlled by the OPE of these light-ray operators.

Light-ray OPE $\quad \mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i - 4}\mathbb{O}_i(\vec{n}_2)$ [Hofman, Maldacena, 2008]

dominated by leading twist

Light-ray OPE in CFT is rigorous and convergent. [Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019] [Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2020]

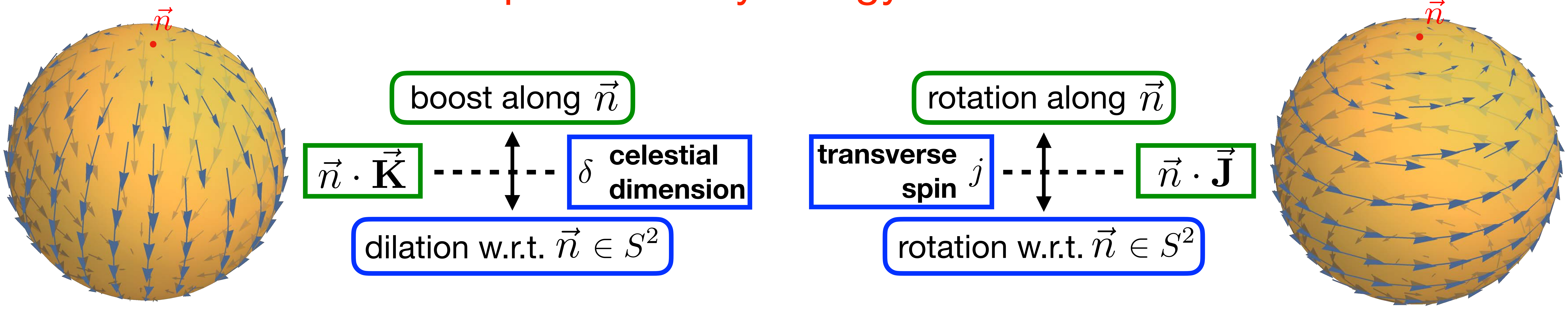
In QCD, things are less understood, but the leading power contribution is. [HC, Moult, Zhu, 2020]

Celestial Sphere and Celestial Block

Light-ray operators are local on the celestial sphere.

It has long been realized that the **Lorentz group** is equivalent to the **conformal group** on the **celestial sphere**.

Can we use CFT techniques to study energy correlators?



For 2-point EEC, **[Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]** had used this fact to give rigorous light-ray OPE and organize it into "**celestial blocks**".

sum all descendants

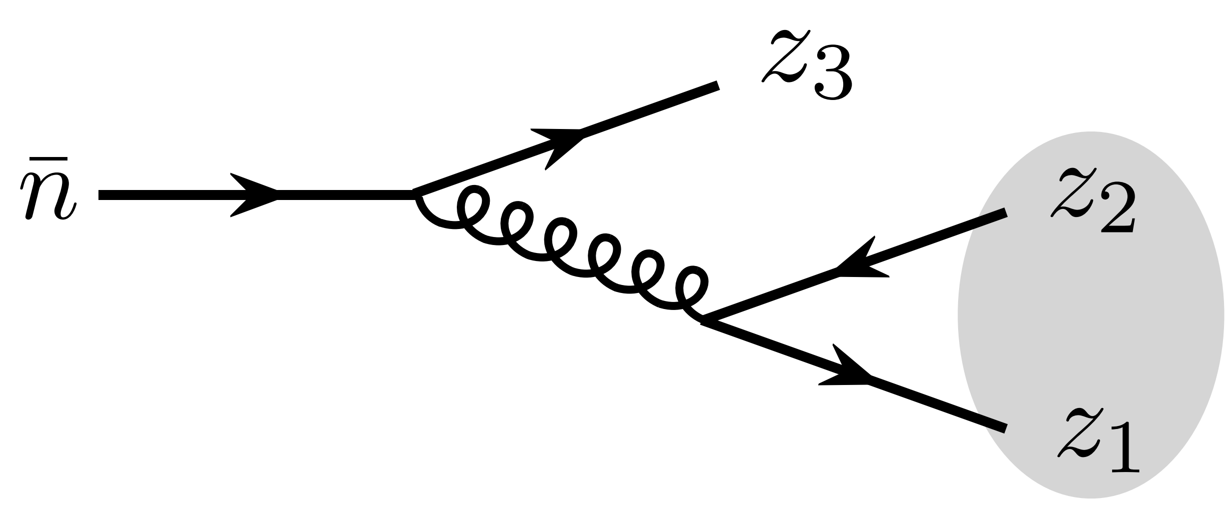
We generalize this idea to 3-point case. Interestingly, in the collinear limit, 3-point celestial blocks turn out to be **2D conformal blocks**. **[HC, Moult, Zhu, 2021]**

[HC, Moult, Sandor, Zhu, forthcoming]

Conformal Block Decomposition

on the celestial sphere

Example:



For simplicity, tagging final state quarks

Squeezed limit: $z_1 \cdot z_2 \to 0$

j

$$g(z,\bar{z}) = -\frac{1}{720} g_{4,2}(z,\bar{z}) + \frac{163}{252000} g_{6,2}(z,\bar{z}) - \frac{2057}{4233600} g_{8,2}(z,\bar{z}) - \frac{82667}{768398400} g_{10,2}(z,\bar{z})$$

$$+ \frac{13}{2400} g_{4,0}(z,\bar{z}) - \frac{139}{40320} g_{6,0}(z,\bar{z}) - \frac{10211}{5880000} g_{8,0}(z,\bar{z}) + \cdots$$

δ

$$- \frac{1}{168} \partial_\delta g_{8,0}(z,\bar{z}) - \frac{1}{1386} \partial_\delta g_{10,2}(z,\bar{z}) + \cdots$$

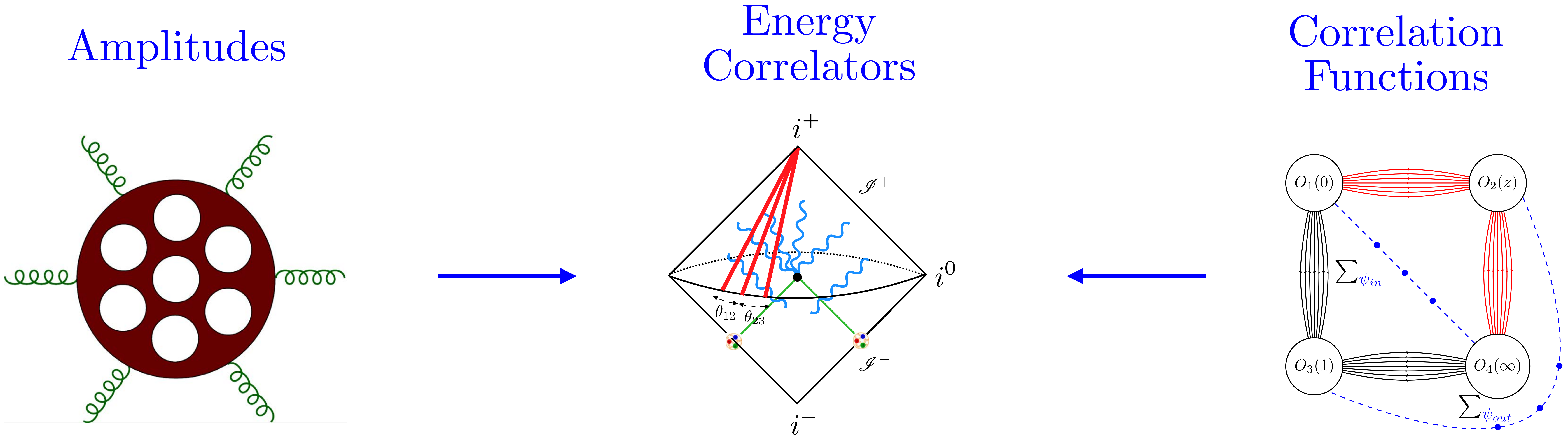
[derivative of blocks, contain $\log|z|$]

Conformal blocks nicely re-organize the power correction of this small angle expansion.

In particular, in this example, only $j = 0, 2$ blocks exist.

Summary

- Energy correlators are field-theoretically elegant observables, where we can apply sophisticated amplitude/correlation function techniques
- For collinear EEEC, we study
  - its relation to dual Feynman diagrams
  - its conformal block decomposition as a correlator on the celestial sphere




Thanks

Backup

3-Point Energy Correlator

with collinear quark source

Operator Definition $$\int dt\, e^{it\bar{n}\cdot P} \langle \Omega | \bar{\chi}(t\bar{n}) \not{\bar{n}} \mathcal{E}(z_1)\mathcal{E}(z_2)\mathcal{E}(z_3)\chi(0)|\Omega\rangle$$

Properties

- depends on scalar products $\bar{n}\cdot P,$ $\boxed{\bar{n}\cdot z_i, \quad z_i \cdot z_j}$ (dimensionless)
- dimension = 5
- homogeneous in $\bar{n},$ (RPI: 0) z_1, z_2, z_3 (-3) $\quad \mathcal{E}(\lambda z_i) = \lambda^{-3}\mathcal{E}(z_i)$ (celestial dimension: 3)

Functional Form $$(\bar{n}\cdot P)^5 \frac{1}{(z_1\cdot z_2)^3}\frac{1}{(z_3\cdot \bar{n})^4}\left(\frac{z_1\cdot z_3}{z_1\cdot \bar{n}}\right) g(u,v)$$

4 point conformal correlator on the celestial sphere

cross-ratios

$$u = \frac{(z_1\cdot z_2)(z_3\cdot \bar{n})}{(z_1\cdot z_3)(z_2\cdot \bar{n})}$$

$$v = \frac{(z_1\cdot \bar{n})(z_2\cdot z_3)}{(z_1\cdot z_3)(z_2\cdot \bar{n})}$$

Casimir Equation

on the celestial sphere

Finding a good **basis** that respects **symmetry**.

$$G(z_1, z_2, z_3, \bar{n}) = \frac{1}{(z_1\cdot z_2)^3}\frac{1}{(z_3\cdot \bar{n})^4}\left(\frac{z_1\cdot z_3}{z_1\cdot \bar{n}}\right) g(z,\bar{z}) \longrightarrow \boxed{???}$$

Symmetry: **Lorentz Group**

Representation labels:

- δ **celestial dimension** ------ $\vec{n}\cdot\vec{\mathbf{K}}$
- j **transverse spin** ------ $\vec{n}\cdot\vec{\mathbf{J}}$

Quadratic Casimir: $\frac{1}{2}M_{\mu\nu}M^{\mu\nu}$ $\xrightarrow{\text{eigenvalue}}$ $-\left(\delta(\delta-2)+j^2\right)$

Casimir Equation: acting Casimir operator on $z_1,\ z_2$

$$\mathcal{L}^{\mu\nu}(z_1,z_2)\mathcal{L}_{\mu\nu}(z_1,z_2)G_{\delta,j} = -(\delta(\delta-2)+j^2)G_{\delta,j}$$

[Dolan, Osborn, 2003]

$$\mathcal{L}^{\mu\nu}(z_1,z_2) \equiv \sum_{i=1,2}\left(z_i^\mu\frac{\partial}{\partial z_{i\nu}} - z_i^\nu\frac{\partial}{\partial z_{i\mu}}\right)$$

Rotation Group $\mathrm{SO}(3)$

$$f(\theta,\phi) = \sum_{\ell,m} f_{\ell,m}\, Y_{\ell,m}(\theta,\phi)$$

Origin of Spherical Harmonics

Cartan subalgebra basis: $\mathbf{L}_3$

Casimir operator: $\mathbf{L}_1^2+\mathbf{L}_2^2+\mathbf{L}_3^2$

Differential operator form:

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Solutions: $Y_{\ell,m}(\theta,\phi)$

Eigenvalue: $-\ell(\ell+1)$

Label ℓ **is the eigenvalue of** $\mathbf{L}_3$ **when the solution is annihilated by** $\mathbf{L}_1+i\mathbf{L}_2$

Conformal Blocks

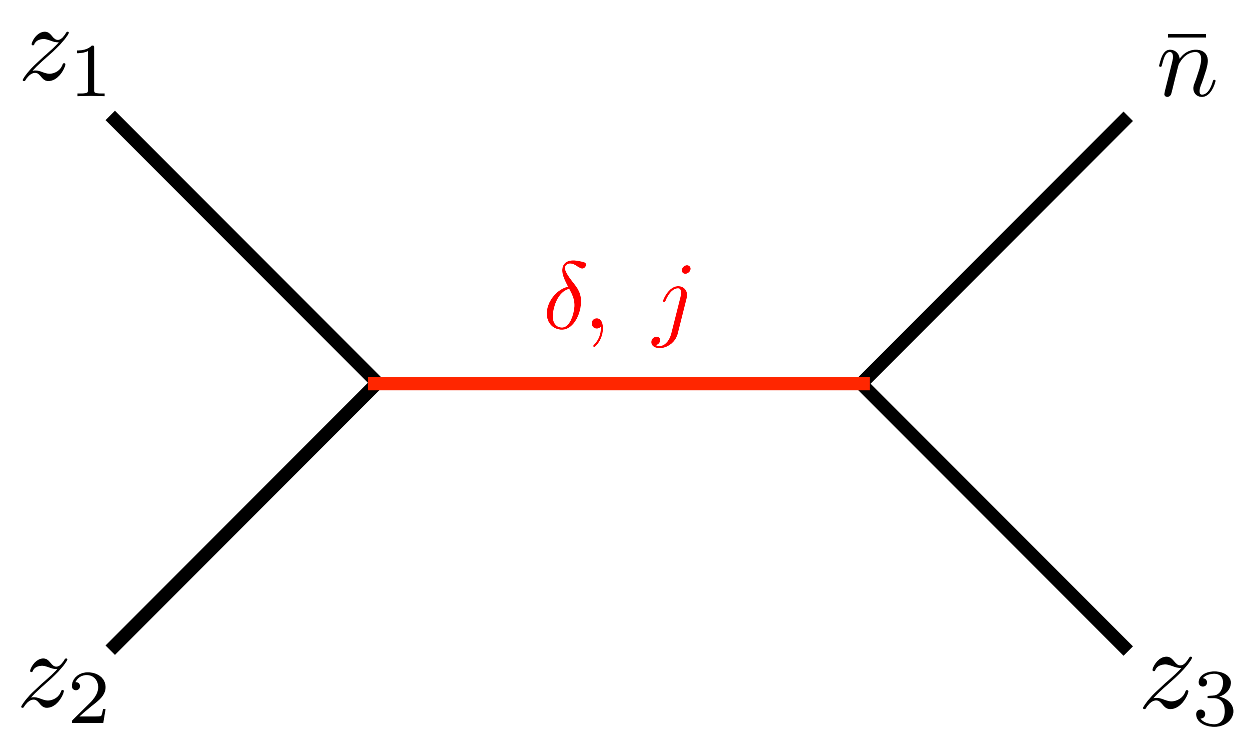
on the celestial sphere

Solutions:

$$g_{\delta,j}(z,\bar{z}) = \frac{1}{1+\delta_{j,0}}\left(k_{\delta-j}(z)k_{\delta+j}(\bar{z}) + k_{\delta+j}(z)k_{\delta-j}(\bar{z})\right)$$

[Notations] In our case, $a = 0,\ b = -1$

$$k_\beta(x) \equiv x^{\beta/2}\ {}_2F_1\left(\frac{\beta}{2}+a, \frac{\beta}{2}+b, \beta, x\right)$$



We find **celestial blocks** for **collinear EEEC** turn out to be **2D conformal blocks**.

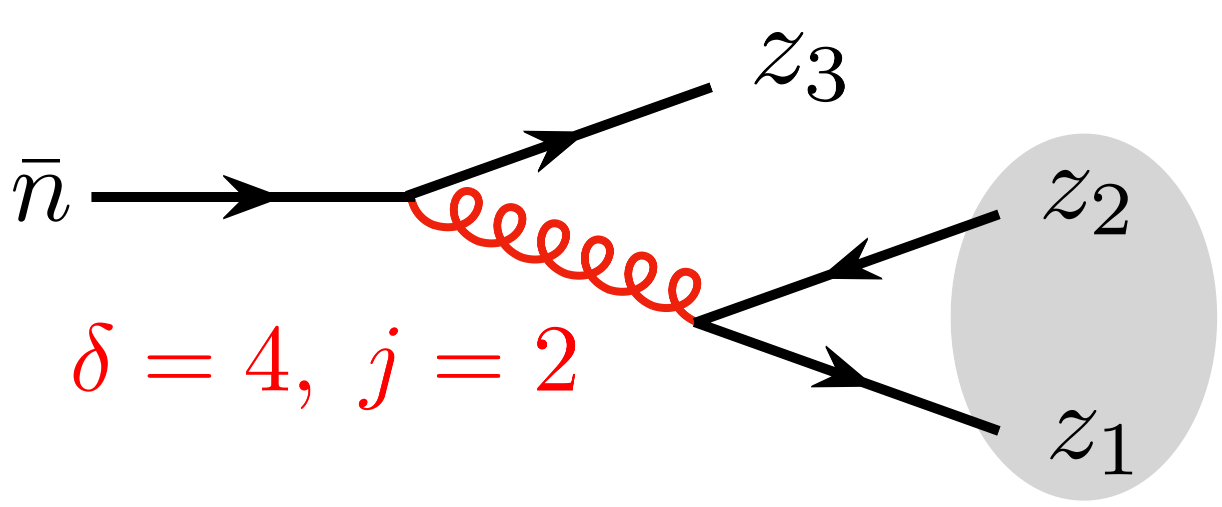
Decomposition:

$$g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$$

Previous example: $-z^3\bar{z}\ {}_2F_1(3,2,6,z)$

set $\delta = 4,\ j = 2$

$$k_6(z) = z^3\ {}_2F_1(3,2,6,z) \qquad k_2(\bar{z}) = \bar{z}$$



Contributing Operator

$$F_a^{\mu+}(iD^+)F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

twist-2, transverse spin-2 gluonic operator