Recent applications of PMC

贵州民族大学

王声权

微扰量子场论研讨会

2021.5.16 上海

Outline

一. Introduction

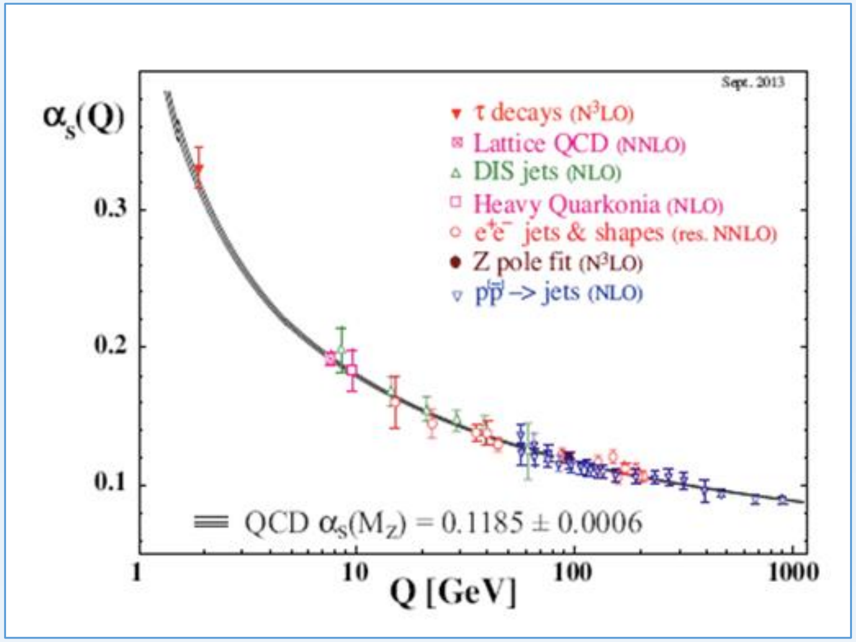
二. Principle of Maximum Conformality (PMC)

三. Recent applications of PMC

1. A novel method for the determination of α_s
2. Basic process $\gamma\gamma*->\eta_c$ puzzle

四. Summary

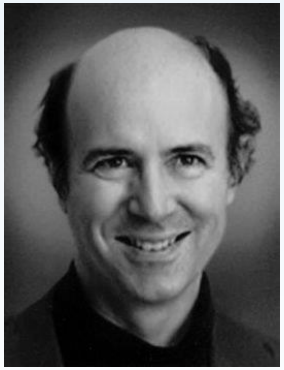
一. Introduction

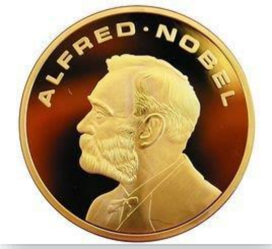


强相互作用

- 夸克禁闭（不存在自由夸克）
- 渐进自由（微扰可算）

强耦合常数的大小决定研究方法 < 非微扰 / **微扰**





$$\alpha_s\left(Q \gg \Lambda_{QCD}\right) < 1$$

一. Introduction

$$\rho(\mu_R) = r_0\alpha_s(\mu_R)[1+\sum_{k=1}^{\infty} r_k(\frac{Q}{\mu_R})\frac{\alpha_s^k(\mu_R)}{\pi^k}]$$

为消除红外发散或紫外发散
引入重整化理论

$$g_0 = Z_g \mu^{\varepsilon/2} g \quad (\varepsilon\text{=4-d})$$

正规化、重整化、能标设定

准确预言具同等重要性

计算到无穷阶的微扰论预言需与人为引入的参数无关
- - 重整化群不变性

物理量

$$\frac{\partial\rho}{\partial\mu_R} \equiv 0; \frac{\partial\rho}{\partial \mathrm{R}} \equiv 0$$

一. Introduction

传统观点：重整化能标依赖性不可避免，
因此，怎么选择重整化能标才是关键

Conventional Scale Setting Method

$$\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \quad n-\text{微扰阶数}$$

- fixed-order, the prediction, scheme- and scale-dependence
- Guessing a renormalization scale Q “typical momentum transfer”, or to eliminate large logs or to improve convergence
- Varying the scale in a certain range, e.g. [Q/2, 2Q] to discuss its uncertainty

成为当前理论中重要系统误差之一，
极大地影响微扰论计算精度及预言能力

一. Introduction

如何解决能标问题

Brodsky–Lepage–Mackenzie method (BLM)

引用1214次

PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

On the elimination of scale ambiguities in perturbative quantum chromodynamics

Stanley J. Brodsky
Institute for Advanced Study, Princeton, New Jersey 08540
*and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**

G. Peter Lepage
Institute for Advanced Study, Princeton, New Jersey 08540
*and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Paul B. Mackenzie
Fermilab, Batavia, Illinois 60510
(Received 23 November 1982)

源自QED观察
引入轻子圈
问题来自高阶如何处理

Principle of Minimum Sensitivity (PMS)

引用1200次

PHYSICAL REVIEW D VOLUME 23, NUMBER 12 15 JUNE 1981

Optimized perturbation theory

P. M. Stevenson
Physics Department, University of Wisconsin-Madison, Madison, Wisconsin 53706
(Received 21 July 1980; revised manuscript received 17 February 1981)

源自数学处理
引入驻点
问题来自物理

RG-improved effective coupling method (FAC)

引用553次

Volume 95B, number 1 PHYSICS LETTERS 8 September 1980

RENORMALIZATION GROUP IMPROVED PERTURBATIVE QCD

G. GRUNBERG [1]
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA

源自实验 - 理论一致性
引入有效耦合常数
问题来自与微扰论理念冲突

一. Introduction

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

The renormalization scale-setting problem in QCD

Xing-Gang Wu[a,*], Stanley J. Brodsky[b], Matin Mojaza[b,c]

[a] Department of Physics, Chongqing University, Chongqing 401331, PR China
[b] SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA
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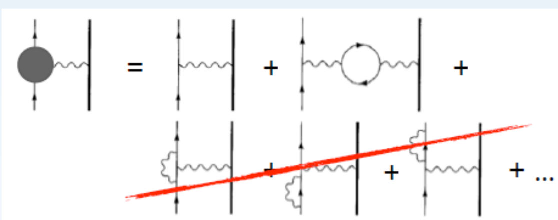
BLM/FAC/PMS-分析



BLM=> nf-term
QED极限＝GM-L方案
CSR＝>解决方案不确定性

所有真空极化图贡献相加确定能标

GM-L方案
QED不存在重整化能标设定问题
"1/137-本身数值小"

Quantum Electrodynamics at Small Distances

M. Gell-Mann and F. E. Low
Phys. Rev. **95**, 1300 – Published 1 September 1954

二. principle of maximum conformality

PMC首篇正式论文

最初想法是将BLM推到无穷阶

后期发现两者在低阶等价，但PMC理念更基础

PHYSICAL REVIEW D 85, 034038 (2012)

Scale setting using the extended renormalization group and the principle of maximum conformality: The QCD coupling constant at four loops

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, China*

(Received 30 November 2011; published 22 February 2012)

PRL 109, 042002 (2012) PHYSICAL REVIEW LETTERS week ending 27 JULY 2012

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

Stanley J. Brodsky[1,*] and Xing-Gang Wu[1,2,†]

[1]*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*
[2]*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China*

(Received 29 March 2012; published 23 July 2012)

PRL 110, 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

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(Received 13 January 2013; published 10 May 2013)

二. principle of maximum conformality

PMC基本思想

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2}\left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right) = -\sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi}\right)^{i+2}$$

基于重整化群方程，利用微扰序列中的非共形β项确定高能物理过程的有效强耦合常数数值，获得与重整化能标选择无关的理论预言。通过最大程度的逼近共形微扰序列，可同时获得与重整化方案无关的理论预言，符合重整化群不变性要求。

附产品：由于消除具有发散性质的重整化子项，PMC序列将自然地具有更好的微扰收敛性。该收敛性与重整化能标选择无关，因此可以将之认为是高能物理过程的内禀属性。在阿贝尔极限下，将回归QED理论中的GM-L方案。

$n!\beta_i^n\alpha_s^n$

二. principle of maximum conformality

最大共形原理（PMC）

1981年，Brodsky-Lepage-Mackenzie提出BLM机制
1992年，Gruberg-Kataev提出延拓方案认为BLM机制存在问题
1995年，Brodsky-Lu提出自洽能标对应关系CSR，拓展BLM机制到两圈
1997年-2011年，Brodsky提出采用β-函数替换BLM中nf-项想法，未明确如何做
2007年，Mikhailov提出seBLM，基于大β0近似提高微扰收敛性

萌 芽

2011年，Brodsky-Giustino明确β-函数想法，命名PMC（实际是单圈层次重新表述BLM）
2011年，Brodsky-Wu提出PMC－BLM对应原理，实现PMC
实现BLM到任意圈（突破进展，四圈为例，PMC首篇正式论文，PRD）
2012年，Brodsky-Wu将PMC用于Top对截面及正反不对称性分析，取得成功（PRL）
2013年，Wu-Brodsky-Mojaza完成PPNP邀请综述（国际首篇重整化能标系统综述）
2014年，Brodsky-Mojaza-Wu（BMW）给出对应原理解释，完善PMC公式体系
PMC获赞意义"接近重整化理论"（PRL）
2015年，Bi-Wu等证明简并关系普适性、PMC两种方案的微扰等价性（PLB）
Ma-Wu等基于重整化群不变性完成PMC与PMS深入对比（PRD）
Wu-Ma-Wang-Brodsky等完成RPP邀请综述、完成国内FOP综述

发 展

2016年，Shen-Wu等提出PMC单能标实现方案，证明不同方案的微扰等价性（PRD）
2017年，Shen-Wu等将自洽能标对应关系CSR推到任意高阶（PLB）
Deur-Shen-Wu等初步考虑将PMC应用于低能区的可能性（PLB）
2018年，Shen-Wu等证明PMC对于任意重整化方案均有方案无关性（PRD）
Du-Wu等提出基于PMC共形序列以及Pade估算未知高阶项方法（PRD）
2019年，Wu-Shen-Wang-Brodsky等完成PPNP邀请综述
2020年，Giustino-Brodsky-Wu-Wang提出有效方案-基于Intrinsic Conformality （PRD）

负重前行

二. principle of maximum conformality

Scale Setting Using the Extended Renormalization Group and the Principle of Maximum Conformality: the QCD Coupling Constant at Four Loops.
Phys.Rev. D85 (2012) 034038.

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality
Phys.Rev.Lett. 109 (2012) 042002.

Self-Consistency Requirements of the Renormalization Group for Setting the Renormalization Scale
Phys.Rev. D86 (2012) 054018.

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD
Phys.Rev.Lett. 110 (2013) 192001.

基础概论 ←

The Renormalization Scale-Setting Problem in QCD
Prog.Part.Nucl.Phys. 72 (2013) 44-98.

Reanalysis of the BFKL Pomeron at the next-to-leading logarithmic accuracy
JHEP 1310 (2013) 117

Systematic Scale-Setting to All Orders: The Principle of Maximum Conformality and Commensurate Scale Relations
Phys.Rev. D89 (2014) 014027.

重整化群不变性 ←

Renormalization Group Invariance and Optimal QCD Renormalization Scale-Setting
Rept.Prog.Phys. 78 (2015) 126201.

二. principle of maximum conformality

General Properties on Applying the Principle of Minimum Sensitivity to High-order Perturbative QCD Predictions
Phys.Rev. D91 (2015) , 034006.

Setting the renormalization scale in perturbative QCD: Comparisons of the principle of maximum conformality with the sequential extended Brodsky-Lepage-Mackenzie approach.
Phys.Rev. D91 (2015), 094028.

Degeneracy Relations in QCD and the Equivalence of Two Systematic All-Orders Methods for Setting the Renormalization Scale
Phys.Lett. B748 (2015) 13-18.

The Generalized Scheme-Independent Crewther Relation in QCD
Phys.Lett. B770 (2017) 494-499

Novel All-Orders Single-Scale Approach to QCD Renormalization Scale-Setting
Phys.Rev. D95 (2017) , 094006.

Renormalization scheme dependence of high-order perturbative QCD predictions
Phys.Rev. D97 (2018), 036024.

Novel demonstration of the renormalization group invariance of the fixed-order predictions using the principle of maximum conformality and the C -scheme coupling
Phys.Rev. D97 (2018), 094030.

最新进展 ←

The QCD Renormalization Group Equation and the Elimination of Fixed-Order Scheme-and-Scale Ambiguities Using the Principle of Maximum Conformality
Prog.Part.Nucl.Phys. 108 (2019) 103706

二. principle of maximum conformality

$$\rho(Q)=\sum_{i=1}^{n}\left(\sum_{j=0}^{i-1}c_{i,j}n_f^j\right)a_s^{p+i-1}(\mu_r)+\mathcal{O}(a_s^{n+1}),$$

利用重整化群方程，可将传统微扰序列转换为β-函数序列

$$\begin{aligned}\rho(Q)=\ &r_{1,0}a_s(\mu_r)+(r_{2,0}+\beta_0 r_{2,1})a_s^2(\mu_r)\\&+(r_{3,0}+\beta_1 r_{2,1}+2\beta_0 r_{3,1}+\beta_0^2 r_{3,2})a_s^3(\mu_r)\\&+(r_{4,0}+\beta_2 r_{2,1}+2\beta_1 r_{3,1}+\frac{5}{2}\beta_1\beta_0 r_{3,2}\\&+3\beta_0 r_{4,1}+3\beta_0^2 r_{4,2}+\beta_0^3 r_{4,3})a_s^4(\mu_r)+\mathcal{O}(a_s^5)\end{aligned}$$

$$\beta(a_s)=\frac{da_s(\mu_r)}{d\ln\mu_r^2}=-a_s^2(\mu_r)\sum_{i=0}^{\infty}\beta_i a_s^i(\mu_r),$$

逐阶消去β-函数，微扰级数中只有共形系数

$$\begin{aligned}\rho(Q)=\ &\hat{r}_{1,0}a_s(Q_1)+\hat{r}_{2,0}a_s^2(Q_2)+\hat{r}_{3,0}a_s^3(Q_3)\\&+\hat{r}_{4,0}a_s^4(Q_4)+\mathcal{O}(a_s^5),\end{aligned}$$

Phys. Rev. Lett. 110, 192001 (2013).
Phys. Rev. D 89, 014027 (2014).

三. Recent applications of PMC

1. Novel method for the determination of α_s

PHYSICAL REVIEW D **99,** 114020 (2019)

Thrust distribution in electron-positron annihilation using the principle of maximum conformality

Sheng-Quan Wang,[1,2,*] Stanley J. Brodsky,[2,†] Xing-Gang Wu,[3,‡] and Leonardo Di Giustino[2,4,§]

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[4]*Department of Science and High Technology, University of Insubria, via valleggio 11, I-22100, Como, Italy*

(Received 11 February 2019; published 24 June 2019)

PHYSICAL REVIEW D **100,** 094010 (2019)

Novel method for the precise determination of the QCD running coupling from event shape distributions in electron-positron annihilation

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[2]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*
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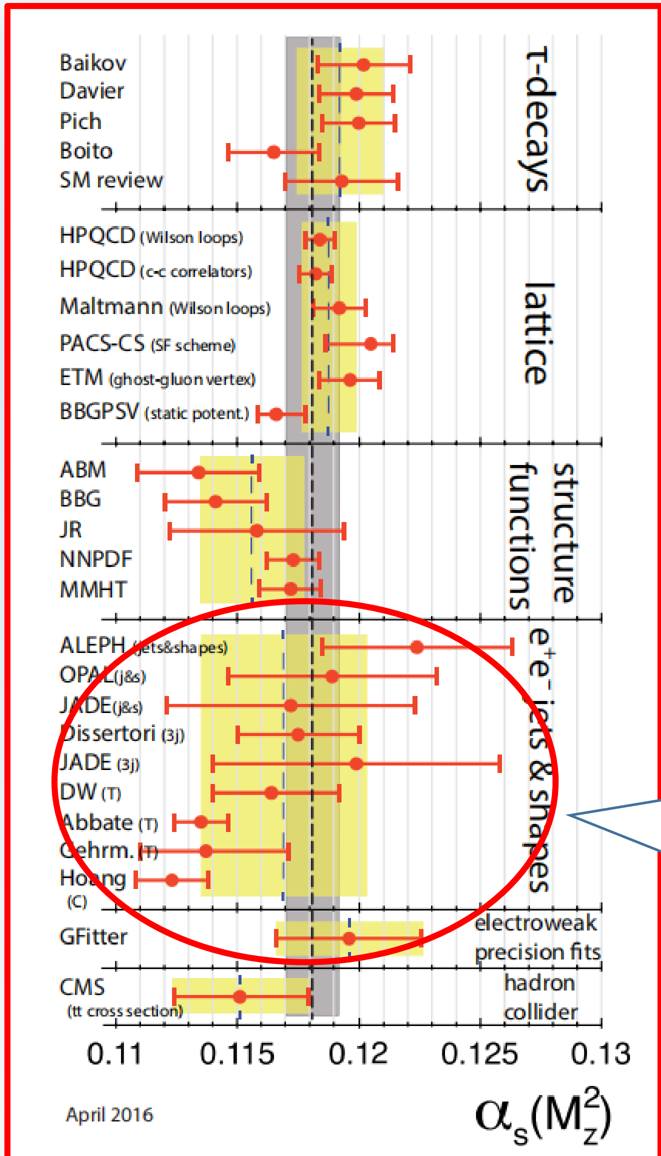
(Received 5 August 2019; published 11 November 2019)

0.9%

$$\alpha_s(M_Z^2) = 0.1181 \pm 0.0011 \ ,$$

[Particle Data Group],
Phys. Rev. D98, 030001 (2018)

三. Recent applications of PMC



419. G. Dissertori *et al.*, JHEP **0908**, 036 (2009).
420. G. Abbiendi *et al.*, Eur. Phys. J. **C71**, 1733 (2011).
421. S. Bethke *et al.*, [JADE Collab.], Eur. Phys. J. **C64**, 351 (2009).
422. G. Dissertori *et al.*, Phys. Rev. Lett. **104**, 072002 (2010).
423. J. Schieck *et al.*, Eur. Phys. J. **C73**, 2332 (2013).
424. R.A. Davison and B.R. Webber, Eur. Phys. J. **C59**, 13 (2009).
425. R. Abbate *et al.*, Phys. Rev. **D83**, 074021 (2011).
426. T. Gehrmann *et al.*, Eur. Phys. J. **C73**, 2265 (2013).
427. A.H. Hoang *et al.*, Phys. Rev. **D91**, 094018 (2015).
428. R. Frederix *et al.*, JHEP **1011**, 050 (2010).

- The $a_s(M_Z)$ are plagued by significant scale uncertainty
- Some extracted $a_s(M_Z)$ are deviated from the world average
- non-self-consistent

三. Recent applications of PMC

The method for extracting $a_s(M_Z)$ in e^+e^- collider:

- predictions matched Monte Carlo models to correct for hadronization effects
- based on analytic calculations of non-perturbative and hadronization effects, using methods like power corrections, factorization of soft-collinear effective field theory, dispersive models and low scale QCD effective couplings

We note that there is criticism on both classes of α_s extractions described above: those based on corrections of non-perturbative hadronization effects using QCD-inspired Monte Carlo generators (since the parton level of a Monte Carlo simulation is not defined in a manner equivalent to that of a fixed-order calculation), as well as studies based on non-perturbative analytic calculations, as their systematics have not yet been fully verified. In particular, quoting rather small overall experimental, hadronization and theoretical uncertainties of only 2, 5 and 9 per-mille, respectively [425,427], seems unrealistic and has neither been met nor supported by other authors or groups.

?

[Particle Data Group], Phys. Rev. D98, 030001 (2018)

三. Recent applications of PMC

The two classic event shapes: the thrust (T) and C-parameter e+e- collider

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2},$$

Phys. Lett. 12, 57 (1964).
Phys. Rev. Lett. 39, 1587 (1977).

Phys. Lett. B 74, 65 (1978).
Phys. Rev. D20, 2759 (1979).

Currently, the main obstacle for achieving a precise determination of $a_s(M_Z)$ is not the lack of precise experimental data, especially at Z^0 peak, but the ambiguity of theoretical predictions.

Rep. Prog. Phys. 69, 1771 (2006).

三. Recent applications of PMC

The differential distribution for T or C:

$$\frac{1}{\sigma_h}\frac{d\sigma}{d\tau} = \bar{A}(\tau)\, a_s(Q) + \bar{B}(\tau)\, a_s^2(Q) + \mathcal{O}(a_s^3).$$

$Q=\sqrt{s}$ using conventional method

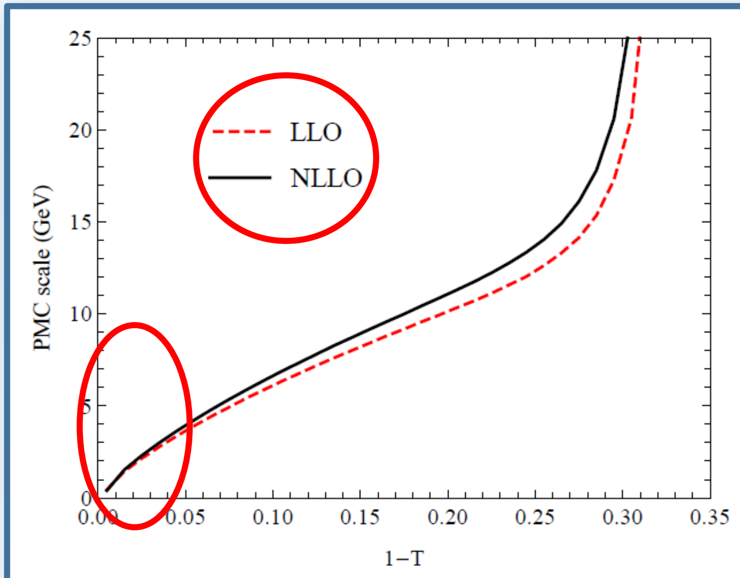
$$\frac{1}{\sigma_h}\frac{d\sigma}{d\tau} = \bar{A}(\tau) a_s(\mu_r^{\rm pmc}) + \bar{B}(\tau,\mu_r)_{\rm con} a_s^2(\mu_r^{\rm pmc}) + \mathcal{O}(a_s^3)$$

$$\bar{B}(\tau,\mu_r)_{\rm con} = \frac{11 C_A}{4 T_R}\bar{B}(\tau,\mu_r)_{n_f} + \bar{B}(\tau,\mu_r)_{\rm in},$$

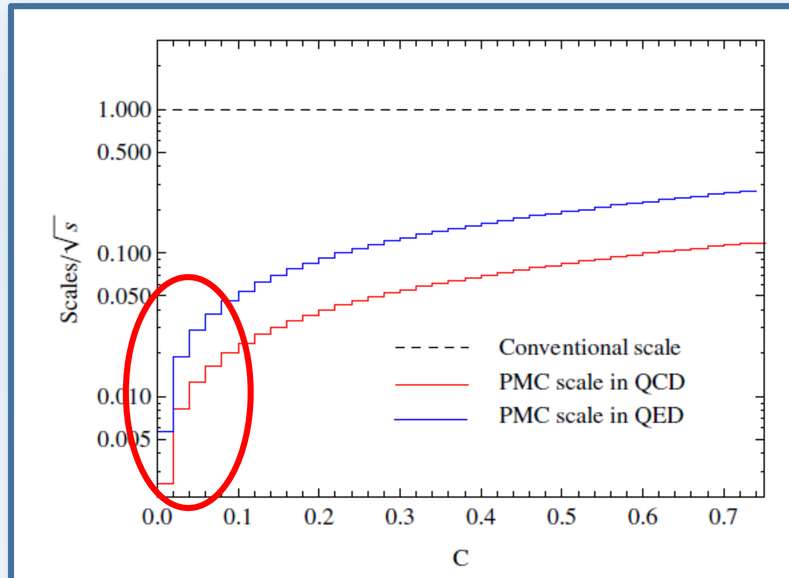
$$\mu_r^{\rm pmc} = \mu_r \exp\left[\frac{3\bar{B}(\tau,\mu_r)_{n_f}}{4 T_R \bar{A}(\tau)} + \mathcal{O}(a_s)\right].$$

三. Recent applications of PMC

T



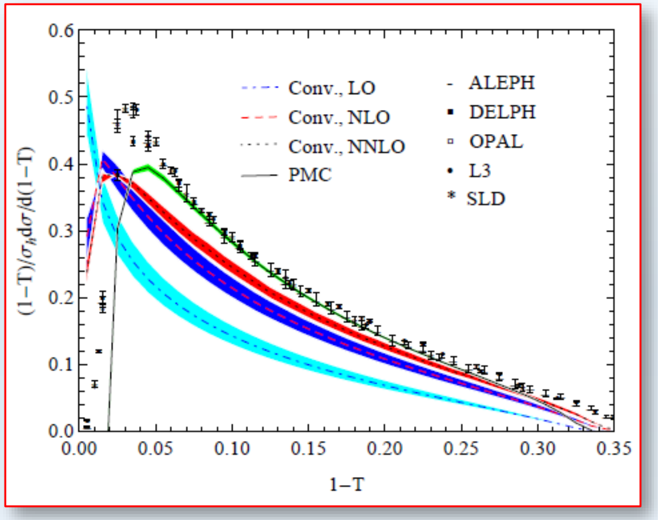
C

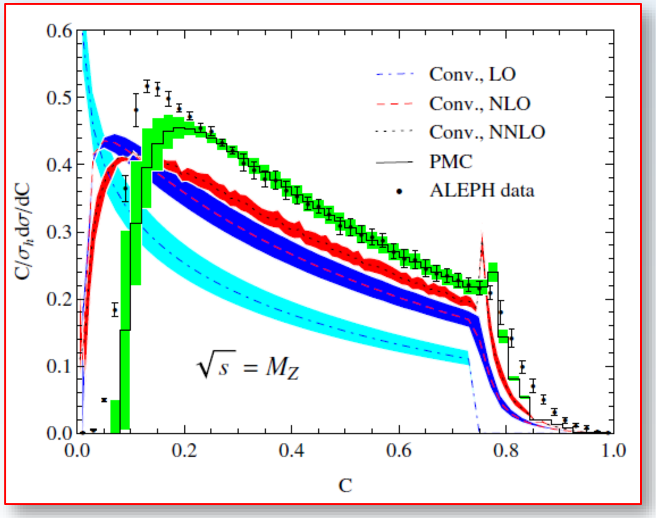


Q = M_Z using conventional method

➢ Scale monotonously increases with T (C). ➢ In the two-jet region, has a bound and low scale. ➢ active flavors nf changes with T.	yields the correct physical behavior, and similar behavior are obtained in the SCET theory and other literatures (ZPA 339, 189 (1991); EPJC 74, 2896 (2014)).

三. Recent applications of PMC





- The NLO and NNLO are large and the pQCD series shows a slow convergence.

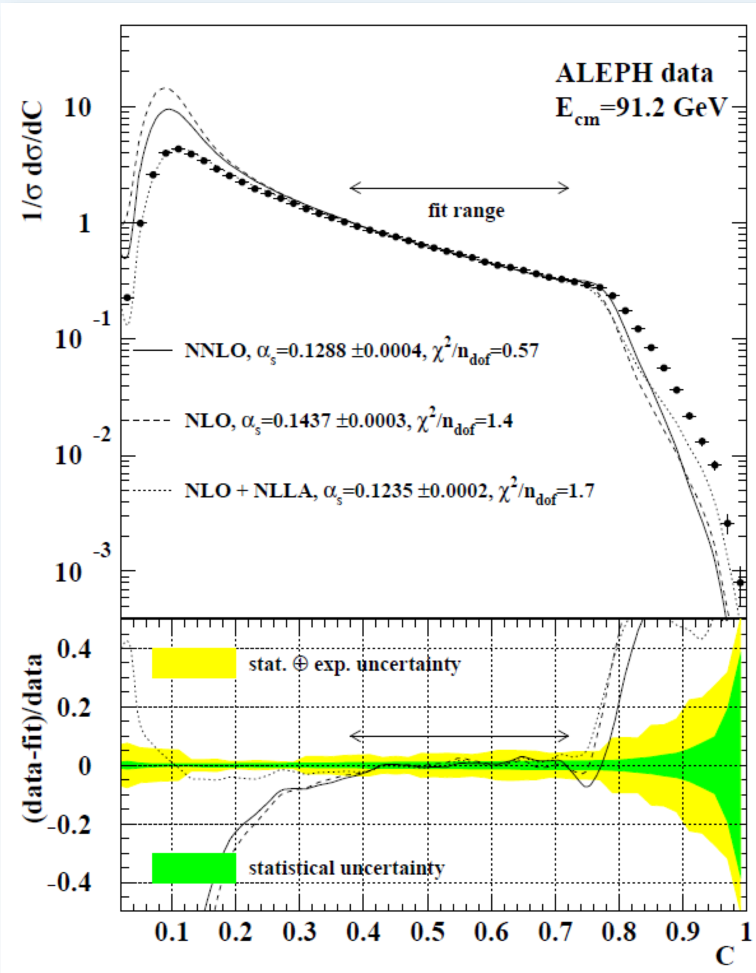
Conv.

- Estimating the unknown higher order QCD by varying the scale [1/2Q, 2Q] is unreliable.
- The predictions are plagued by scale uncertainty, and even up to NNLO, the predictions do not match the data.
- The extracted coupling constants are deviated from the world average, and are also plagued by scale uncertainty.

PMC

- The scale uncertainty is eliminated, predictions are in agreement in data.

三. Recent applications of PMC



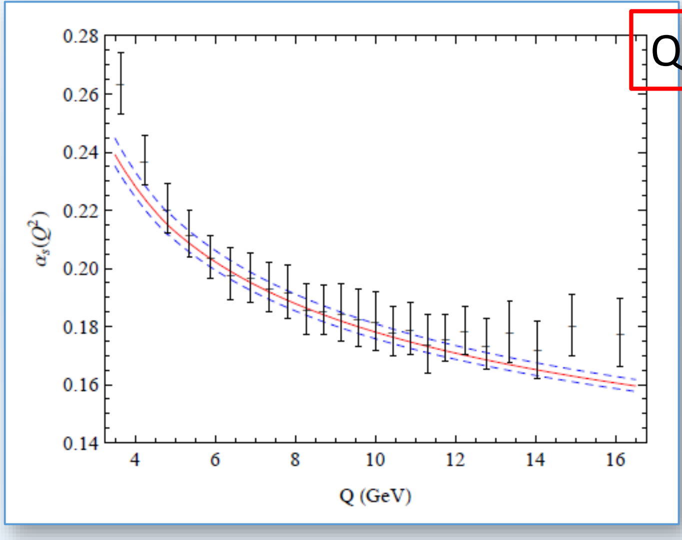
JHEP 0802 (2008) 040

$Q = \sqrt{s} = M_Z$

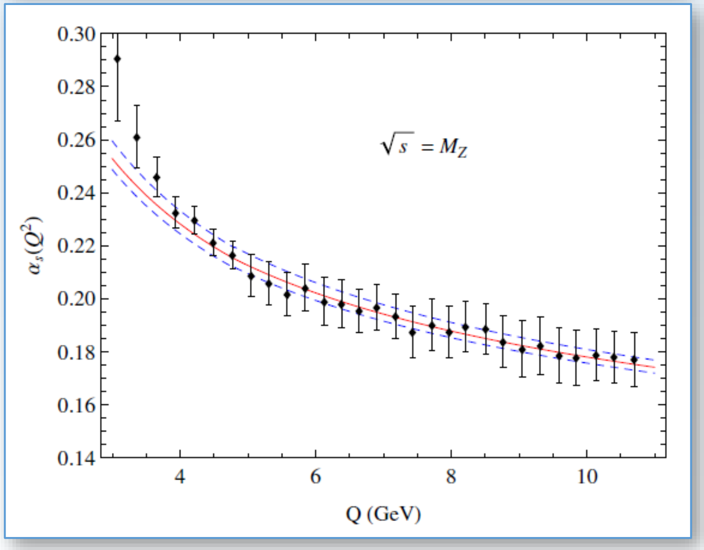
Conv.

- ✓ One value α_s at scale M_Z is extracted ($\alpha_s(M_Z)$).
- ✓ the fit range of T (C) distribution is narrow.
- ✓ the fit range is arbitrary, different fit range leads to different α_s.

三. Recent applications of PMC



$Q=\sqrt{s}=M_Z$



PMC

- ✓ The extracted α_s are in agreement with the world average.
- ✓ The extracted α_s are not plagued by scale uncertainty.
- ✓ Since PMC scale varies with T (C), we can extract the strong coupling at a wide scale range using the experimental data at single center-ofmass-energy.

In QED, the running of the QED coupling at a wide scale range can be determined from events at a single energy $\sqrt{s}$

e.g., (OPAL Collaboration), EPJC 45, 1 (2006)

三. Recent applications of PMC

the mean value of event shapes,

$$\langle y \rangle = \int_0^{y_0} \frac{y}{\sigma_h} \frac{d\sigma}{dy} dy,$$

- ✓ it involves an integration over the full phase space.
- ✓ it provides an important complement to the differential distributions and to determinate α_s

$$\mu_r^{\rm pmc}|_{\langle 1-T \rangle} = 0.0695\sqrt{s}, \text{ and } \mu_{\rm r}^{\rm pmc}|_{\langle {\rm C} \rangle} = 0.0656\sqrt{\rm s},$$

- ✓ PMC scales of differential distribution are also very small.
- ✓ the average of the PMC scale for differential distribution is close to the scale of mean value. self-consistent.

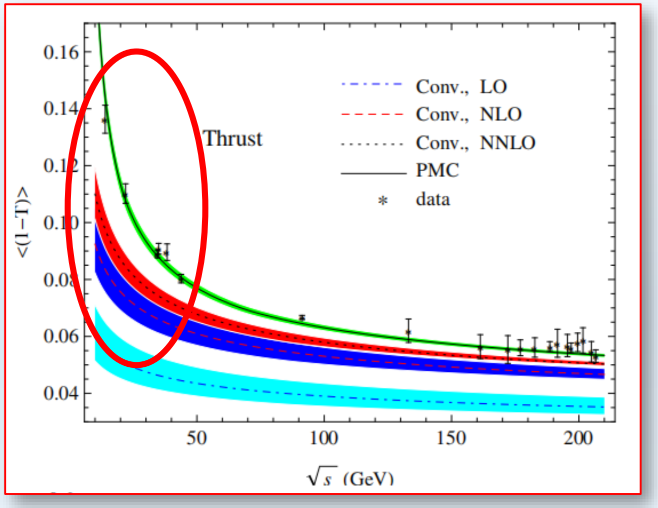
$\mu_r^{\rm pmc} \ll \sqrt{s}$ is also suggested by

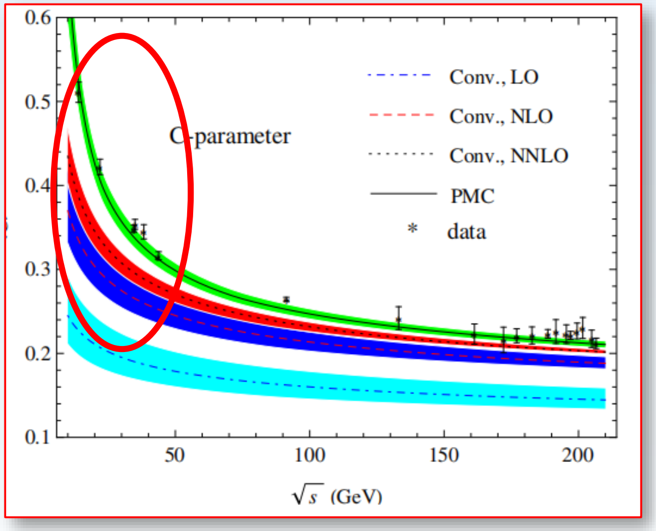
Studies of QCD at e^+e^- centre-of-mass energies between 91 and 209 GeV

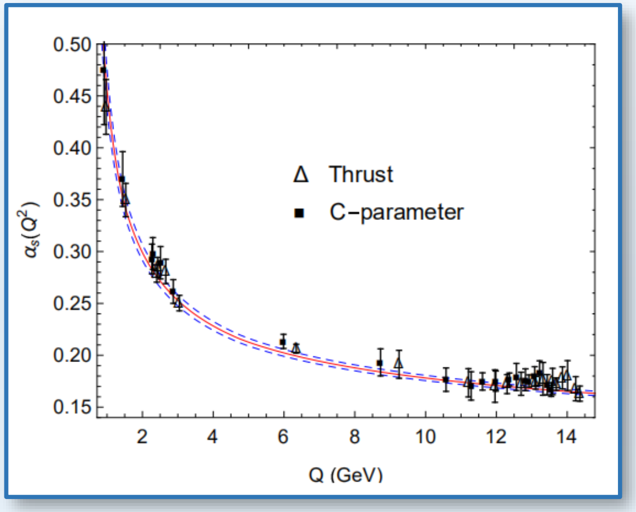
The ALEPH Collaboration

Eur. Phys. J. C 35, 457 – 486 (2004)

三. Recent applications of PMC







$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0011(\text{Exp.}) \pm 0.0005(\text{Theo.}) = 0.1185 \pm 0.0012, \quad (3)$$

$$\alpha_s(M_Z^2) = 0.1193^{+0.0009}_{-0.0010}(\text{Exp.})^{+0.0019}_{-0.0016}(\text{Theo.}) = 0.1193^{+0.0021}_{-0.0019}, \quad (4)$$

The Large Hadron-Electron Collider at the HL-LHC
LHeC Collaboration and FCC-he Study Group (P. Agostini (Santiago
CERN-ACC-Note-2020-0002, JLAB-ACP-20-3180
e-Print: arXiv:2007.14491 [hep-ex] | PDF

other event shapes ,ρ,B_W,B_T…

CEPC

三. Recent applications of PMC

2. Basic process $\gamma\gamma*->\eta_c$ puzzle

The simplest charmonium production process $\gamma^*\gamma \to \eta_c$

PRL **115**, 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

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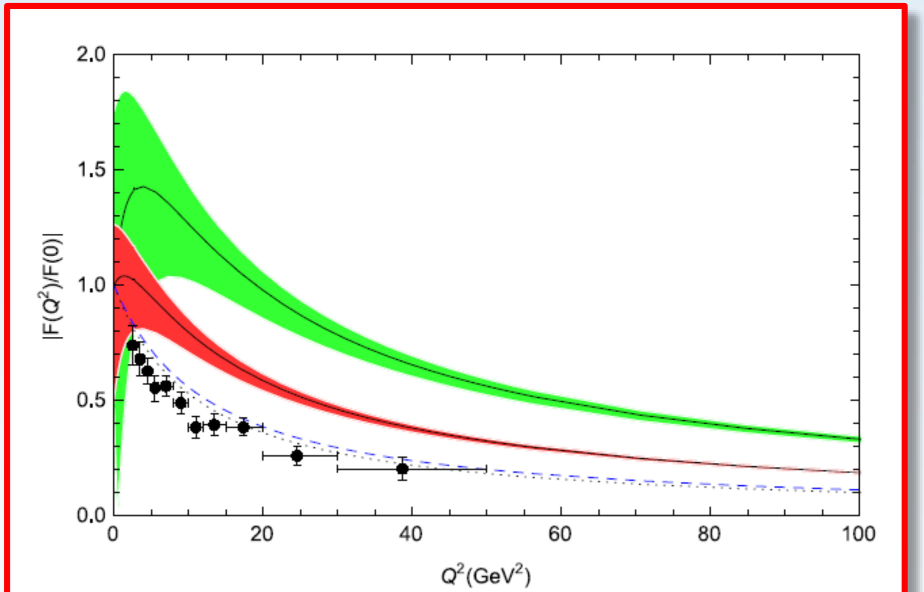
(Received 12 May 2015; published 25 November 2015)



The NNLO NRQCD prediction fails to explain the BaBar measurements

applicability of NRQCD ?

New physics ?

三. Recent applications of PMC

$$F(Q^2) = c^{(0)}\left[1 + \delta^{(1)} a_s(\mu_r) + \delta^{(2)}(\mu_r) a_s^2(\mu_r)\right],$$

$$c^{(0)} = \frac{4e_c^2 \langle \eta_c | \psi^\dagger \chi(\mu_\Lambda)|0\rangle}{(Q^2 + 4m_c^2)\sqrt{m_c}},$$

$$\delta^{(1)} = C_F f^{(1)}(\tau)$$

$$\delta^{(2)}(\mu_r) = B_{\rm con}^{(2)}(\mu_r) + C_{n_f}^{(2)}(\mu_r) n_f,$$

$$\begin{aligned} B_{\rm con}^{(2)}(\mu_r) &= f_{\rm lbl}^{(2)}(\tau) + f_{\rm regcon}^{(2)}(\tau) + \frac{11}{4}\ln\left[\frac{\mu_r^2}{Q^2 + m_c^2}\right] \\ &\quad C_F f^{(1)}(\tau) - \frac{\pi^2}{2} C_F (C_A + 2C_F)\ln\left[\frac{\mu_\Lambda}{m_c}\right] (5) \\ C_{n_f}^{(2)}(\mu_r) &= f_{{\rm reg}n_f}^{(2)}(\tau) - \frac{1}{6} C_F f^{(1)}(\tau)\ln\left[\frac{\mu_r^2}{Q^2 + m_c^2}\right]. (6) \end{aligned}$$

$$F(Q^2) = c^{(0)}\left[1 + \delta^{(1)} a_s(\mu_r^{\rm PMC}) + \delta_{\rm con}^{(2)}(\mu_r) a_s^2(\mu_r^{\rm PMC})\right]$$

$$\mu_r^{\rm PMC} = \mu_r \exp\left[\frac{3C_{n_f}^{(2)}(\mu_r)}{2T_F \delta^{(1)}}\right],$$

$$\delta_{\rm con}^{(2)}(\mu_r) = \frac{11 C_A}{2} C_{n_f}^{(2)}(\mu_r) + B_{\rm con}^{(2)}(\mu_r).$$

三. Recent applications of PMC

Conventional scale setting:

$$F^{\rm Conv}(0)|_{\mu_r=1{\rm GeV}} = c^{(0)}(1-0.25-1.11).$$
$$F^{\rm Conv}(0)|_{\mu_r=m_c} = c^{(0)}(1-0.18-0.60).$$
$$F^{\rm Conv}(0)|_{\mu_r=2m_c} = c^{(0)}(1-0.13-0.36).$$

NLO is moderate

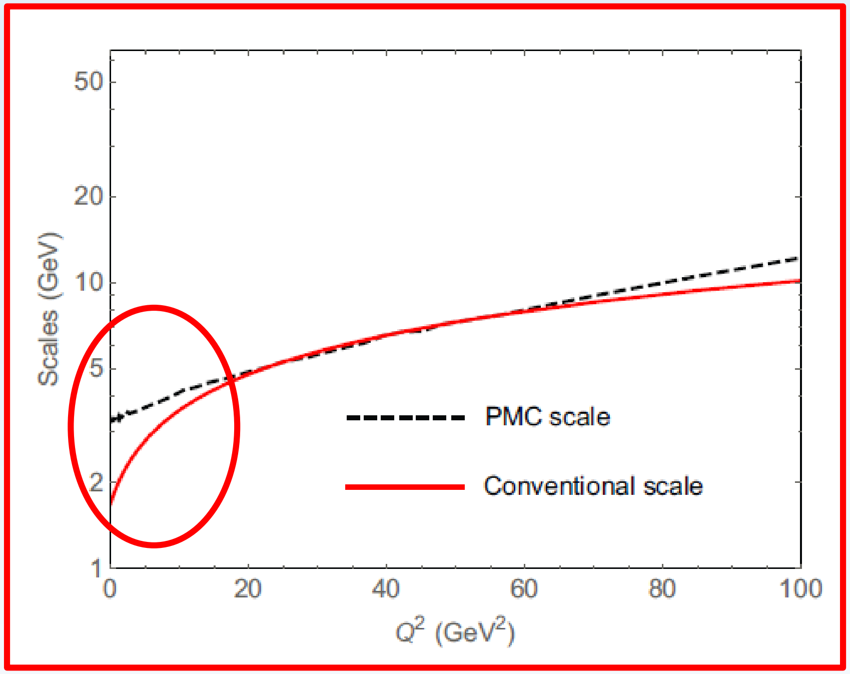
NNLO gives large negative contributions, encounters large scale uncertainty

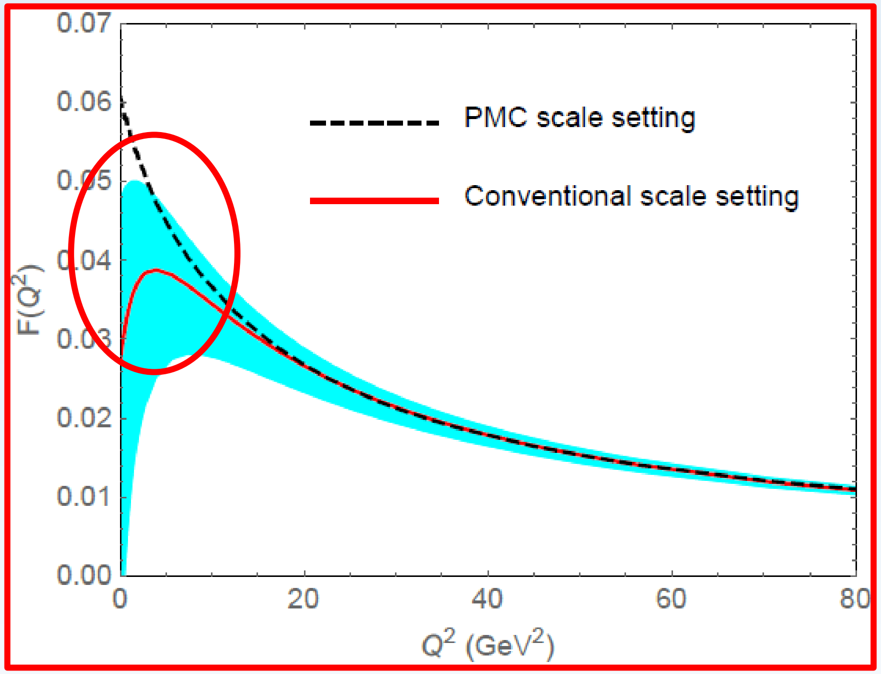
In fact, when μr < 1.3 GeV, the F(0) becomes negative

PMC scale setting:

$$F^{\rm PMC}(0) \equiv c^{(0)}(1-0.13-0.37)$$

三. Recent applications of PMC





三. Recent applications of PMC

Factorization scale uncertainty:

Conv.:

$$F^{\rm Conv}(0)|_{\mu_r=m_c} = 0.43c^{(0)},\ \ 0.22c^{(0)},\ \ -0.06c^{(0)}$$

PMC:

$$F^{\rm PMC}(0) = 0.62c^{(0)},\ \ 0.50c^{(0)},\ \ 0.34c^{(0)}.$$

for μf= 1 GeV, mc and 2mc

Mass mc uncertainty:

Conv.:

$$F(0)\ =\ 0.22c^{(0)},\ 0.14c^{(0)},\ 0.07c^{(0)}$$

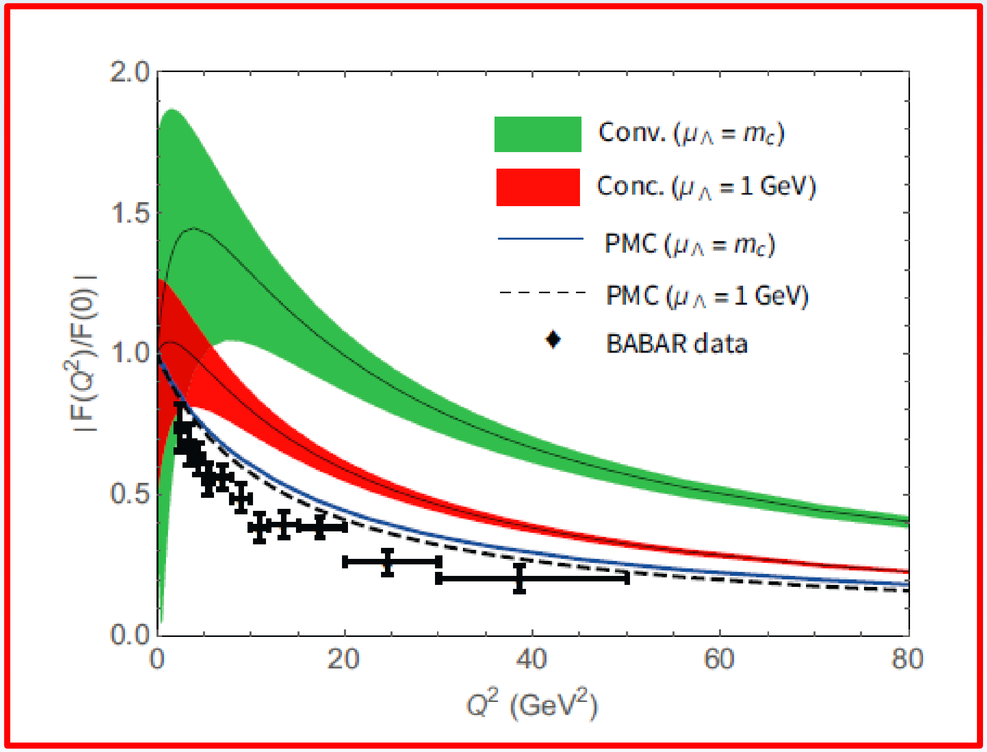
PMC:

$$F(0)\ =\ 0.50c^{(0)},\ 0.46c^{(0)},\ 0.43c^{(0)}$$

for mc= 1.68, 1.5 and 1.4 GeV

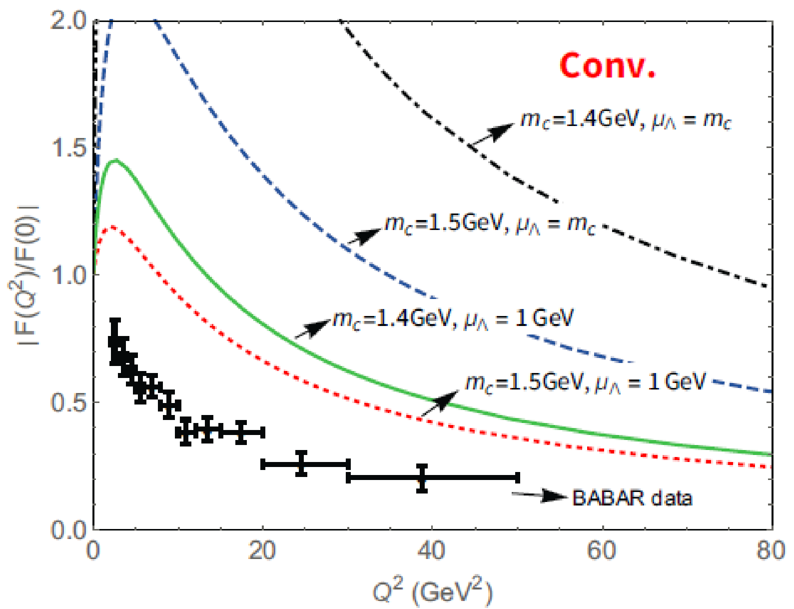
三. Recent applications of PMC

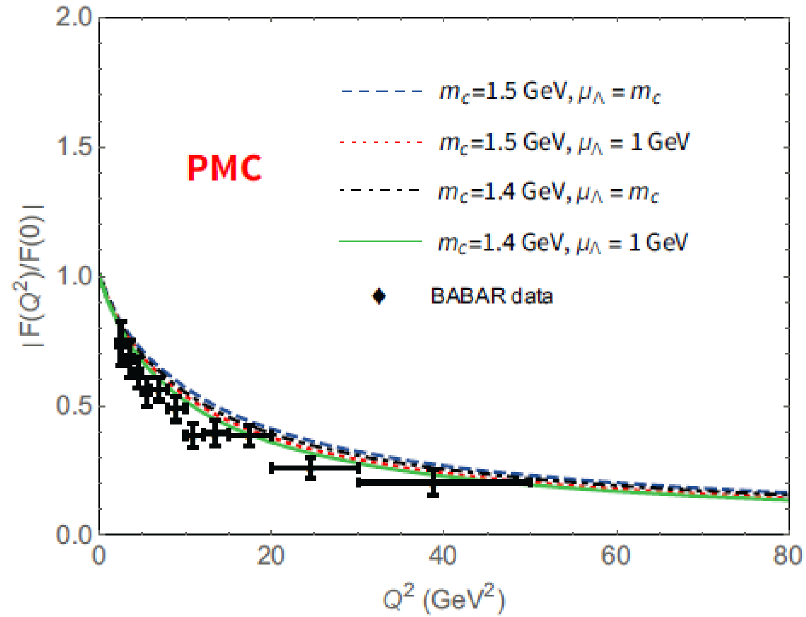
Renormalization and factorization scale uncertainties



三. Recent applications of PMC

Mass mc and factorization scale uncertainties





All the uncertainties are included

S-Q Wang, X-G Wu, W-L Sang, S J. Brodsky, Phys.Rev. D97 (2018), 094034

四. Summary

基于-重整化群方程以及基本重整化群不变性--提供具可系统设定高能物理过程“正确动量流动” 的方案--从而解决传统方案下的重整化能标和重整化方案依赖问题

PMC能标设定方案

1）可自然改变微扰收敛性
2）可更快地逼近物理量的真实值
3）微扰低阶下就可与重整化能标选择无关，获得每一阶准确值
4）采用与方案无关的共形序列，得到微扰展开收敛性的固有属性，可用于估算未知高阶项贡献

粲夸克偶素?

传统能标设定方案

1）收敛慢（重整化子项发散）
2）计算到任意高阶也无法获得每一阶准确值
3）足够高阶时才能获得与重整化能标无关的物理量的真实值
4）因每阶的数值都不准确，不能很好判断微扰展开的收敛性，无法给出令人信服的未知高阶项估算值

更多验证，更多实例应用

thanks