Compton Scattering Total Cross Section at NLO

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with Roman Lee and Matthew Schwartz

Compton Scattering:

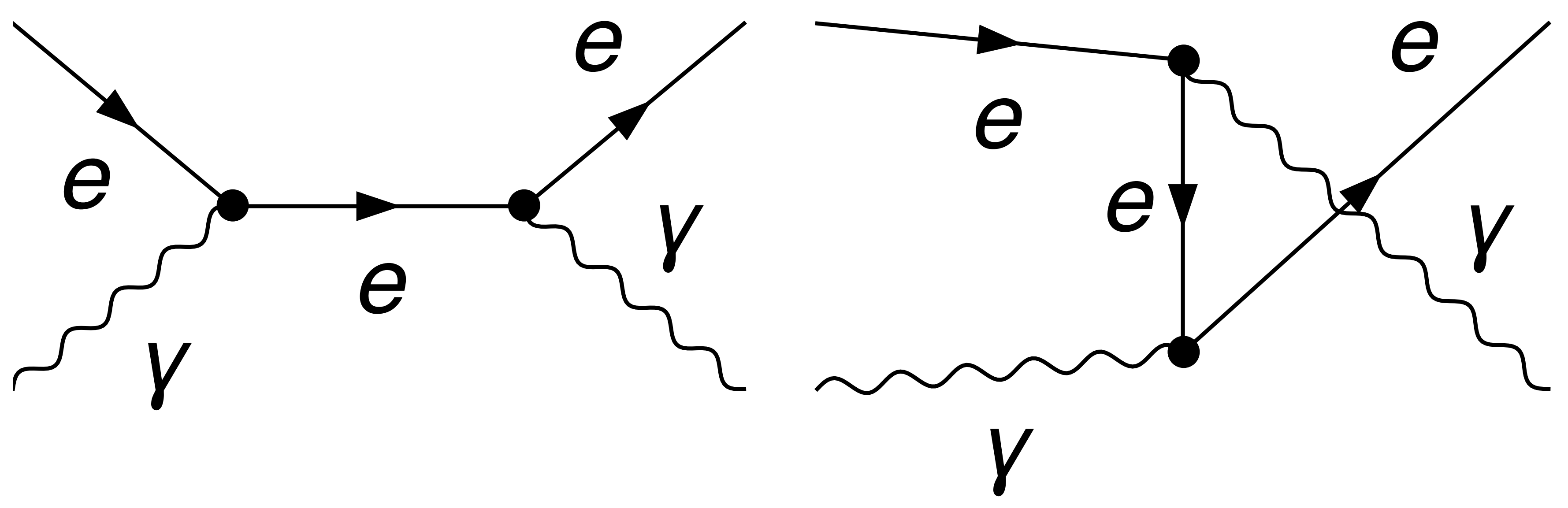
witnesses the development of QFT

- One of the fundamental processes in Quantum Field Theory
- Compton effect (1923): quantum effect

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

- Klein-Nishina formula (1929):

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right)$$



$$\sigma = \pi\alpha^2\left(\frac{2\left(3m^4 + 6m^2 s - s^2\right)\log\left(\frac{s}{m^2}\right)}{\left(m^2 - s\right)^3} + \frac{m^6 - m^4 s + 15m^2 s^2 + s^3}{s^2\left(m^2 - s\right)^2}\right) + \mathcal{O}(\alpha^3)$$

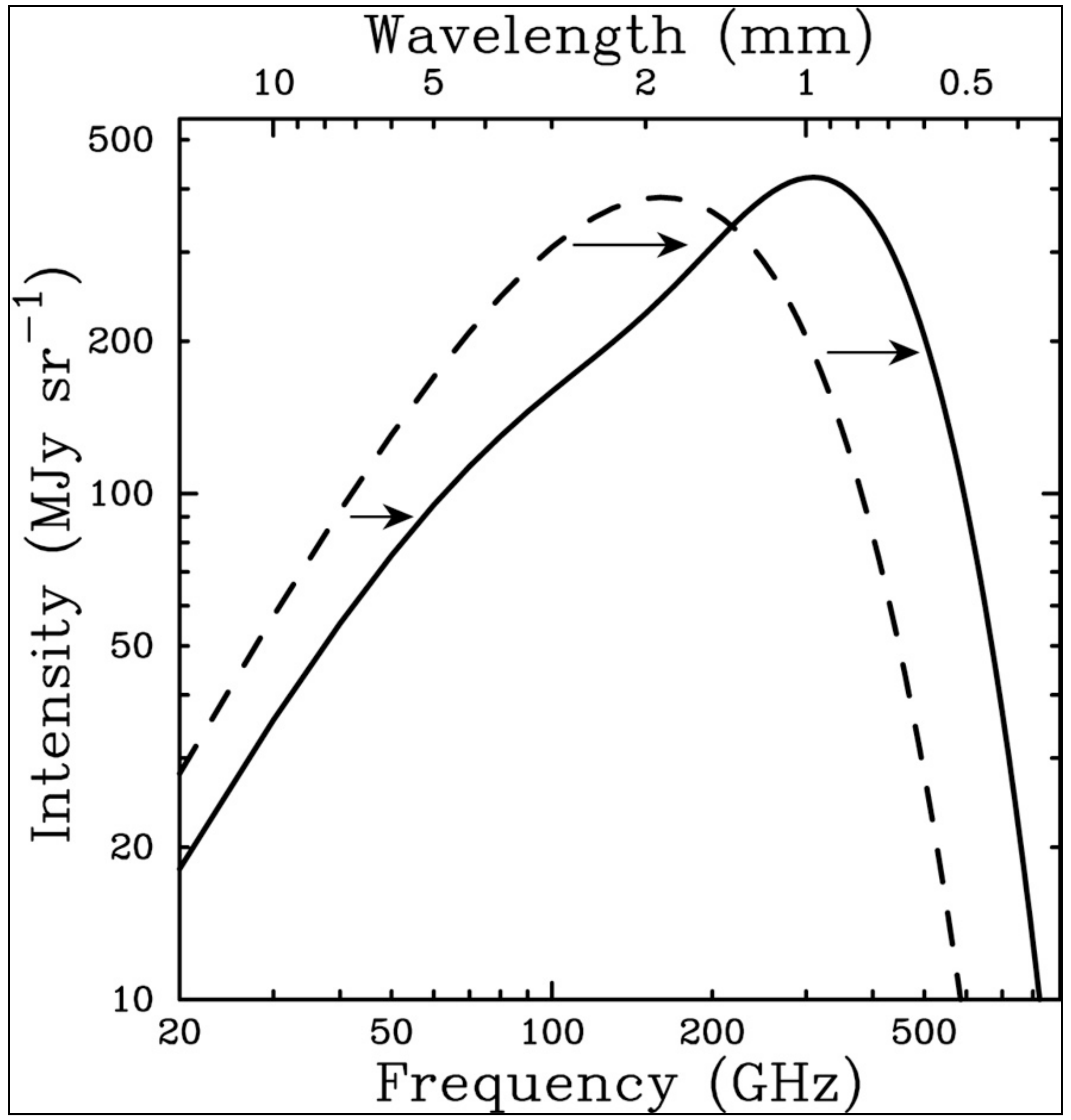
One of the first results in QED!

Motivation for studying Compton scattering

- Important in many aspects of physics: from X-ray crystallography to astrophysics
  - A luminosity monitor for the electron-photon collider
  - A clean process: to measure the coupling constant
  - In astrophysics: inverse Compton scattering

- Theoretical side:
  - the fundamental question: what is an electron?
  - Total cross sections involve forward-scattering region, where off-shell Glauber modes are essential.

e.g. Sunyaev–Zeldovich effect



[Sunyaev, Zeldovich, 1980]

Motivation for studying Compton scattering:

Forward scattering

[1810.10022, Frye, Hannesdottir, Paul, Schwartz, Yan]

- There is already a single log at the tree-level.

$$\sigma \sim \frac{\pi\alpha^2}{s}\left[2\log\left(\frac{s}{m^2}\right)+1\right], \quad s \gg m^2$$

- Usually we expect a log to show up at 1-loop, and can be resummed with RG equations. For example, we introduce the running coupling to improve the efficiency in QED.

- Use dim reg and set $m = 0$, we see explicit divergence in the t-channel:

$$\sigma_t = \frac{16\pi\alpha^2}{Q^2}\Gamma_d\left(-\frac{1}{2\epsilon}+1\right), \quad \text{with} \quad \Gamma_d = \left(\frac{4\pi e^{-\gamma_E}\mu^4}{Q^2}\right)^{\epsilon} \qquad d = 4-2\epsilon$$

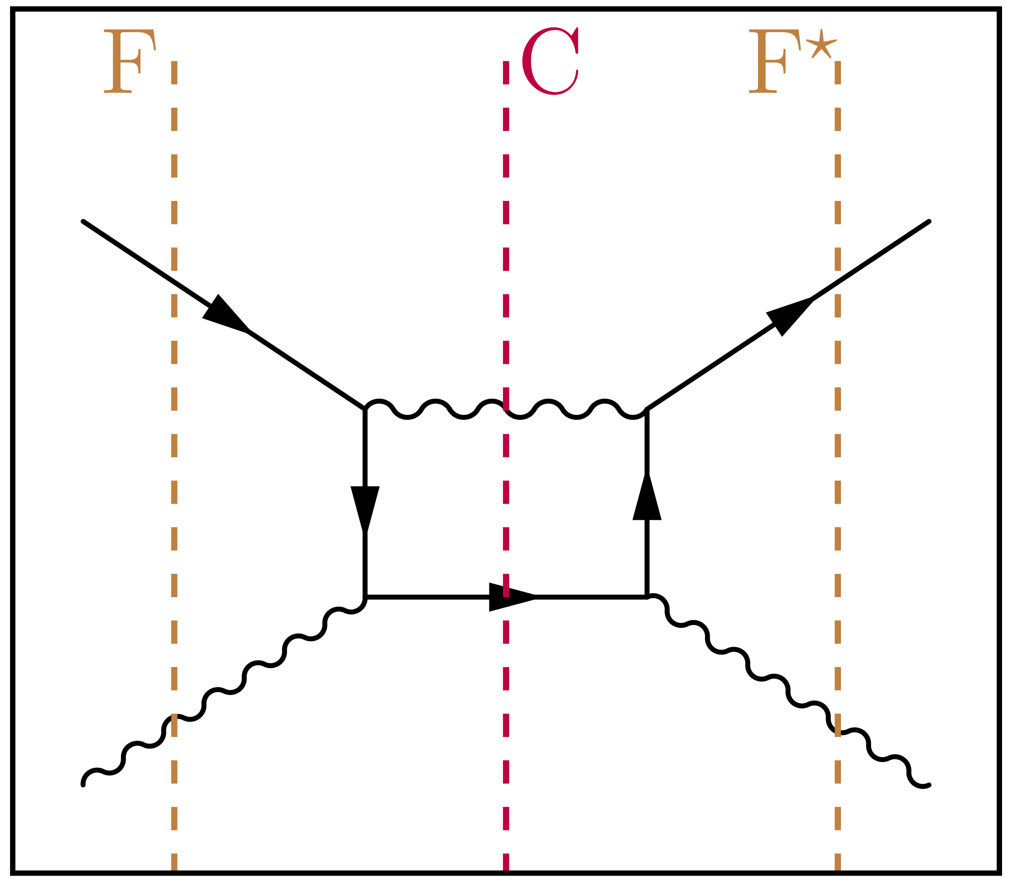
IR divergence comes from the outgoing γ collinear to the incoming e^- **Collinear Logarithms**

Motivation for studying Compton scattering:

Forward scattering

[1810.10022, Frye, Hannesdottir, Paul, Schwartz, Yan]

- IR finiteness requires the forward scattering included, where outgoing γ collinear to the incoming γ



$$\sigma_t = \frac{16\pi\alpha^2}{Q^2}\Gamma_d\left(-\frac{1}{2\epsilon}+1\right)$$

$$\sigma_F = \frac{16\pi\alpha^2}{Q^2}\Gamma_d\left(\frac{1}{2\epsilon}-1\right)$$

cancel!

A hard photon and electron become effectively indistinguishable at high energies

- Kinoshita-Lee-Nauenberg (KLN) theorem: Unitarity guarantees the cancellation of infrared divergences when all final states and initial states are summed over.
- However, one only need to sum over initial or final states once the forward scattering is included.
- If we want to resum the logarithms, we need NLO as a check

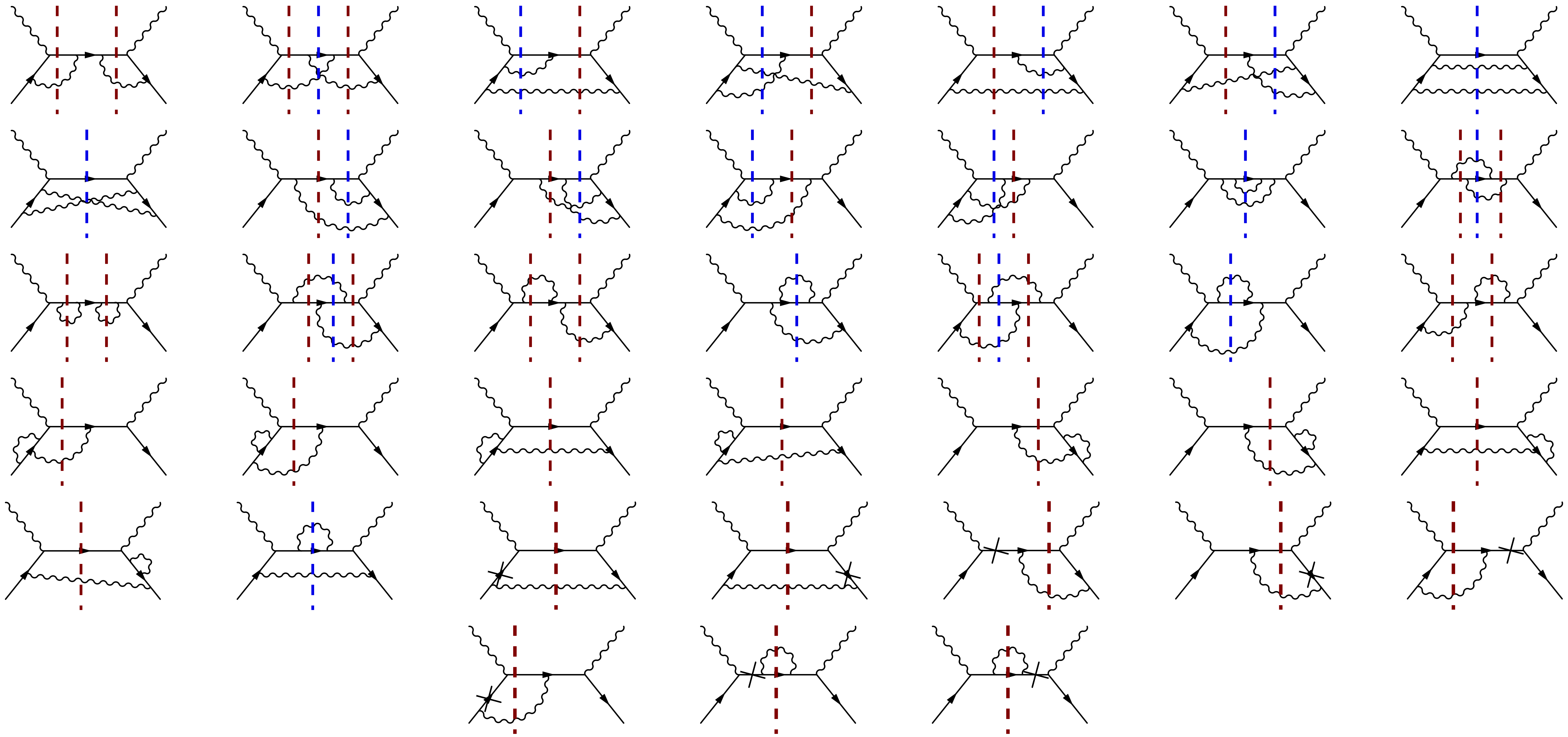
Compton scattering beyond leading order

- There are very few analytic results of the total cross section beyond LO in QED. For Compton,
  - real emissions: $e^-(p_1) + \gamma(k_1) \rightarrow e^-(p_2) + \gamma(k_2) + \gamma(k_3)$
  - virtual corrections: $e^-(p_1) + \gamma(k_1) \overset{1-loop}{\rightarrow} e^-(p_2) + \gamma(k_2)$
  - pair productions:$e^-(p_1) + \gamma(k_1) \rightarrow e^-(p_2) + e^+(k_2) + e^-(k_3), e^-(p_2) + \mu^+(k_2) + \mu^-(k_3)\ldots$
- Several developments:
  - Brown and Feynman (1951): the virtual differential cross section regulated by the photon mass
  - Mandl and Skyrme (1952): double Compton scattering: the amplitudes for the hard-photonic bremsstrahlung
  - Milton, Tsai, De Raad (1972), Gongora-T. and Stuart (1989), Veltman (1989), Swartz, hep-ph/9711447
  - Denner and Dittmaier, hep-ph/9805443 polarized scattering; numerical total cross section
  - Lee, Lyubyakin, Stotsky, 2010.15430 first analytic result for real emissions and pair productions!

Compton scattering beyond leading order

Failure: discontinuities of forward amplitude

- The main difficulties:
  - two-loop massive diagrams are hard to evaluate, even after region expansions
  - cannot separate different processes (difficult to separate elliptical sectors)



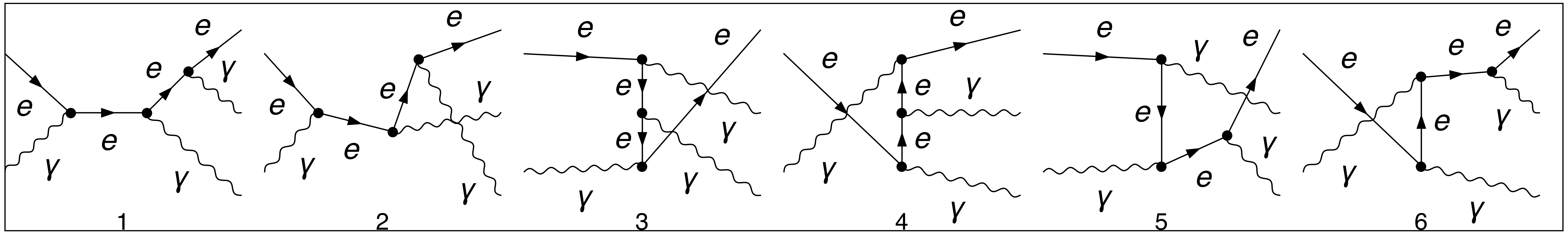
Instead we use direct phase space integrations and evaluate the forward diagrams in Fiesta as numerical checks

NLO total cross section

- Real emissions: $e^{-}(p_1) + \gamma(k_1) \rightarrow e^{-}(p_2) + \gamma(k_2) + \gamma(k_3)$

$$\sigma^{(R)} \sim \int \mathrm{dLIPS}_3 \sum |M^{(R)}|^2$$

IBP reduction: LiteRed2;
Canonical form: Libra

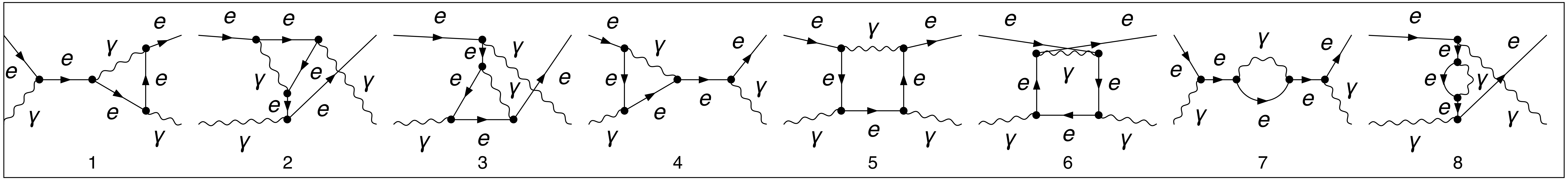


2 sectors:
14 master
integrals

- Virtual corrections: $e^{-}(p_1) + \gamma(k_1) \xrightarrow{1-loop} e^{-}(p_2) + \gamma(k_2)$

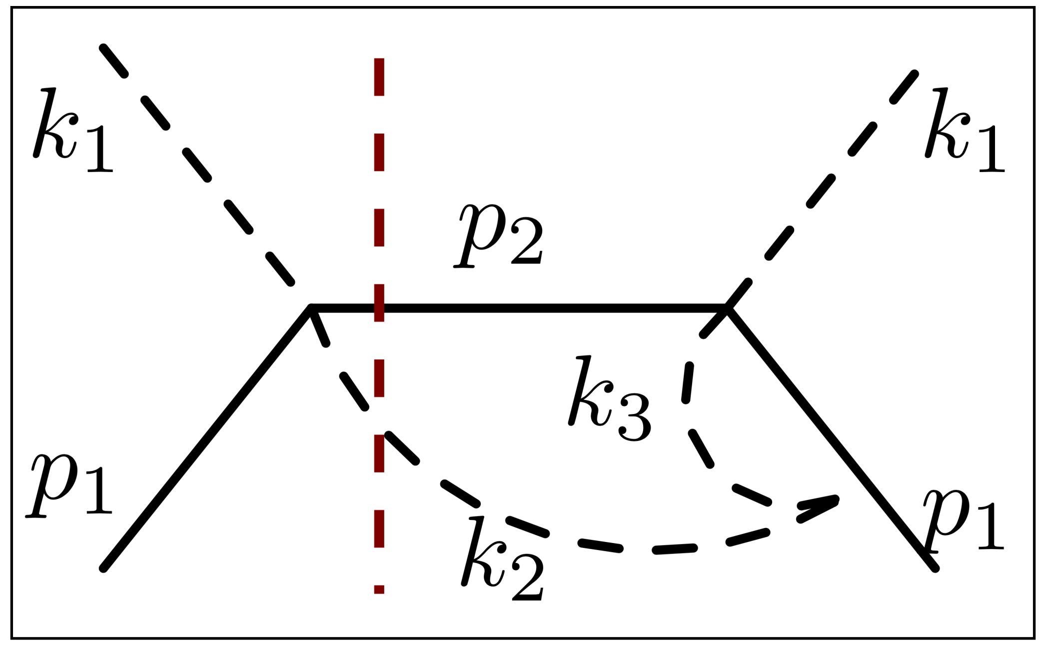
$$\sigma^{(V)} \sim \int \mathrm{dLIPS}_2 \int \frac{d^d k_3}{i\pi^{d/2}} \sum 2\mathrm{Re}\left[M^{(V)} \times M^{(T)*}\right]$$

2 sectors:
24 master
integrals



Boundary conditions for DE

- Real emissions: only 1 nonzero boundary; similar to tree-level
- Virtual corrections: 4 nonzero boundaries
  - Remarkably, the loop integral and phase space integral factorizes at threshold $s \to m^2 = 1$ or $y = \sqrt{(s-m^2)/(s+3m^2)} \to 0$



$$\mathcal{I} = \frac{(2\pi)^2}{2i\pi^d} \int d^d k_3 d^d p_2 d^d k_2 \delta(p_2^2 - m^2)\theta(p_2^0)\delta(k_2^2)\theta(k_2^0)$$

$$\times \delta^d(p_1 + k_1 - p_2 - k_2) \frac{1}{k_3^2[(p_1 - k_2 - k_3)^2 - m^2]}$$

$$\int \frac{d^d k_3}{i\pi^{d/2}} \frac{1}{k_3^2[(p_1 - k_2 - k_3)^2 - m^2]} \overset{p_1 \cdot k_2 \to 2y^2}{\approx} \int dx_1 dx_2 \delta(1 - x_1 - x_2)\Gamma(\epsilon)(x_1 + x_2)^{2\epsilon - 2} x_2^{-\epsilon} \left(4x_1 y^2 + x_2\right)^{-\epsilon}$$

$\mathcal{I} \overset{\text{factorizes}}{\approx} \left(\int \text{dLIPS}_2^{(soft)} \right) \times (\text{feynman integral})$

no long depends on any outgoing momentum!

NLO total cross section

- on-shell renormalization scheme

$$\sigma^{NLO} = \sigma^{NLO}_{bare} + (Z_\psi^2 Z_A^2 Z_\alpha^2 - 1)\sigma^{born} + \delta\sigma_m$$

$$\sigma^{NLO} = \frac{\alpha^3}{m^2x^3}\Bigg\{-\frac{x\left(273x^3-982x^2-2960x-1744\right)}{24(x+1)^2}$$

$$+\frac{37x^4-54x^3-339x^2-428x-184}{4(x+1)^2}\ln(x+1)+\frac{x^2\left(14x^4+17x^3-17x^2-22x-8\right)}{2(1-x)(1+x)^3}\ln x$$

$$-\frac{4x^6+35x^5-31x^4-755x^3-1765x^2-1506x-440}{2(x+1)^2(x+4)}\ln^2(x+1)+\left(x^2-x+2\right)\left[\mathrm{Li}_2(1-x)-\frac{\pi^2}{6}\right]$$

$$-\frac{x^6+7x^5-28x^4-239x^3-449x^2-338x-88}{(x+1)^2(x+4)}\mathrm{Li}_2(-x)+\frac{x^4+7x^3+x^2-3x-2}{(x+1)^2}\ln(x+1)\ln x$$

$$-\frac{4\left(x^5+26x^4+146x^3+316x^2+288x+96\right)}{(x+1)^2(x+4)}G(-2,-1;x)+\frac{3x^4+18x^3+44x^2-8x-64}{x}yG(y,-1;x)+T_3(x)\Bigg\}+\mathcal{O}(\epsilon)$$

transcendental weight 3

where

$$x = \frac{s-m^2}{m^2} \qquad y = \sqrt{\frac{s-m^2}{s+3m^2}}$$

$$G(a, a_1, \ldots, a_n; x) = \int_0^x dw_a(x')G(a_1, \ldots, a_n; x'),$$

$$dw_y(x) = \frac{ydx}{x}, \quad dw_a(x) = \frac{dx}{x-a} \quad (a = -4, -2, -1, 0)$$

NLO total cross section

$$T_3(x) = \left(x^2+2x-6\right)g_1 - \frac{1}{3}\left(x^2-16x-23\right)g_2 + 8\left(x^2-4x-6\right)g_3 + 4(2x^2-x-6)g_4 + 2\left(2x^2-7x-12\right)g_5$$
$$-(5x^2+32x-8)g_6 - 3(x-2)(x+4)yg_7 + 3\left(3x^2-8\right)g_{10} - \frac{8y\left(x^4+3x^3-18x^2-68x-24\right)}{(x+4)x}g_8$$
$$+\frac{3y\left(5x^4+14x^3-96x^2-352x-128\right)}{(x+4)x}g_9 - \frac{16y\left(x^4+2x^3-24x^2-80x-48\right)}{(x+4)x}g_{11} - \frac{6y\left(x^3-12x-8\right)}{x}g_{12}$$

transcendental weight 3 bases

$$g_1 = \left[\mathrm{Li}_3(x^2) - \mathrm{Li}_2(x^2)\ln x\right], g_2 = \ln^3(x+1), g_3 = G(-1,-2,-1;x), g_4 = G(-1,-1,0;x), g_5 = G(-1,0,-1;x),$$
$$g_6 = G(0,-1,-1;x), g_7 = \left[G(0,y,-1;x) + 2G(y,-1,0;x)\right], g_8 = G(y,0,-1;x), g_9 = G(y,-1,-1;x),$$
$$g_{10} = G(y,y,-1;x), g_{11} = G(y,-2,-1;x), g_{12} = G(-4,y,-1;x)$$

where

$$x = \frac{s-m^2}{m^2} \qquad y = \sqrt{\frac{s-m^2}{s+3m^2}}$$

$$G(a,a_1,\ldots,a_n;x) = \int_0^x dw_a(x')G(a_1,\ldots,a_n;x'),$$
$$dw_y(x) = \frac{ydx}{x}, \quad dw_a(x) = \frac{dx}{x-a} \quad (a = -4,-2,-1,0)$$

Discussions

- Bloch-Nordsieck theorem: IR divergences cancel when both real and virtual contributions are summed over

[9704368, Dittmaier]

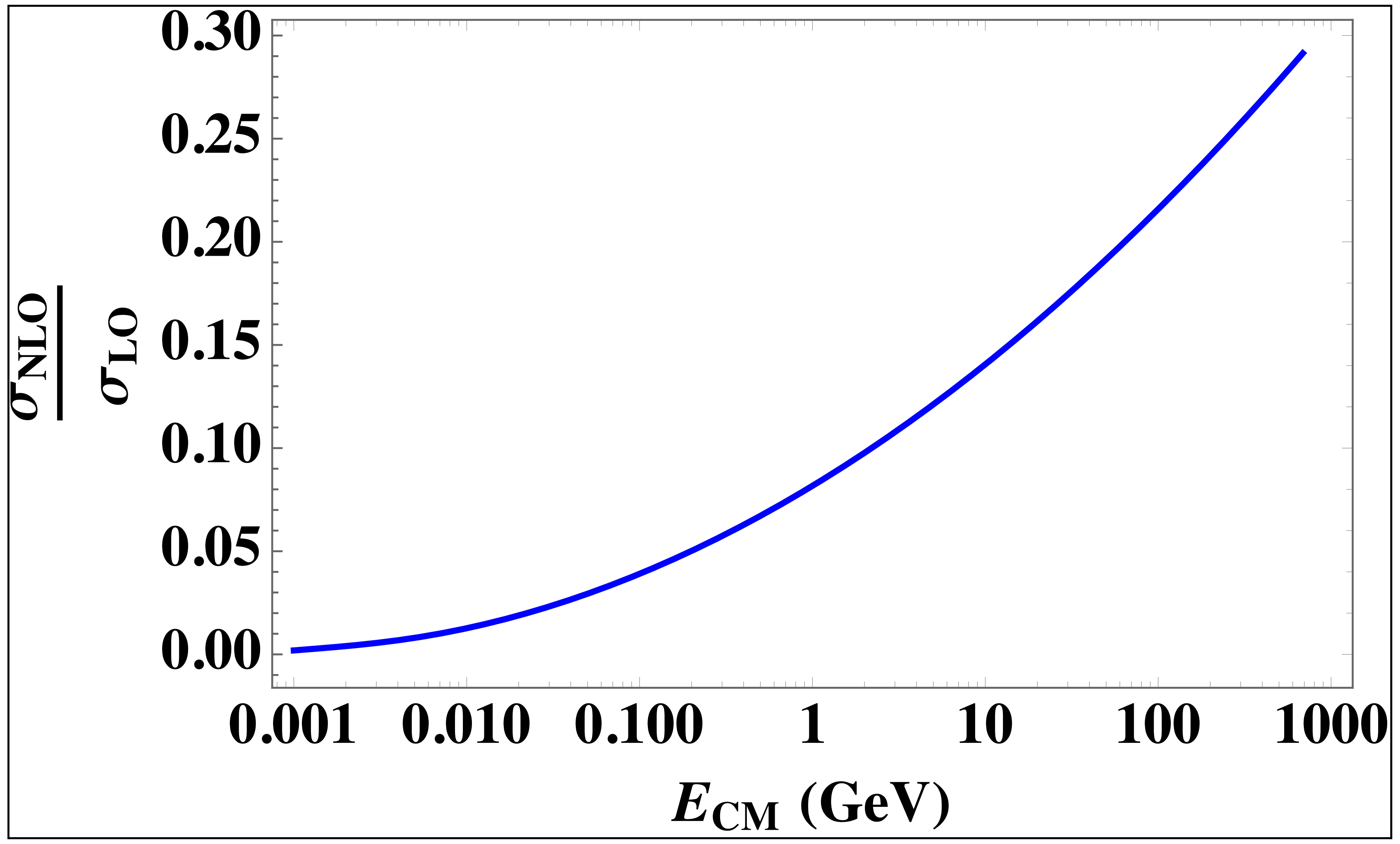
- Thirring's theorem: near the threshold $s \to m^2$, NLO cross section vanishes

Threshold:

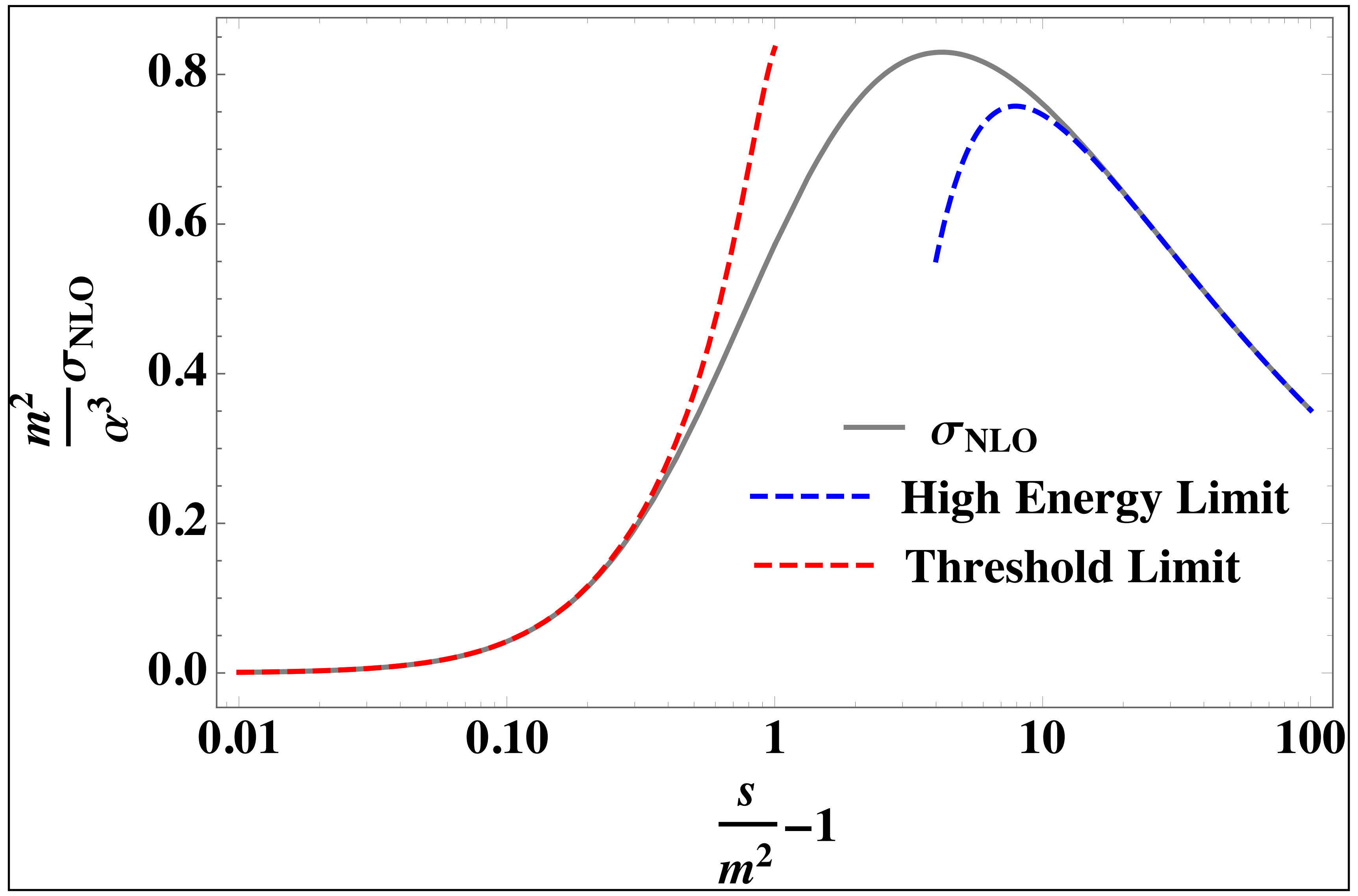
$$\sigma = \frac{\pi\alpha^2}{m^2}\left[\frac{8}{3} - \frac{8}{3}x + \cdots\right] + \frac{\alpha^3}{m^2}x^2\left[-\frac{16}{9}\ln x + \frac{7}{15} + \cdots\right], \quad x = \frac{s-m^2}{m^2}$$

High energy:

$$\sigma = \frac{\pi\alpha^2}{s}\left[2\ln\frac{s}{m^2} + 1 + \cdots\right] + \frac{\alpha^3}{s}\left[\frac{1}{3}\ln^3\frac{s}{m^2} - \frac{1}{2}\ln^2\frac{s}{m^2} + \frac{17}{4}\ln\frac{s}{m^2} - \frac{75}{8} - \frac{\pi^2}{2} + 4\zeta_3 + \cdots\right]$$



NLO corrections to total cross section



Asymptotics

Total Cross Sections in QED

- Compton scattering: $e^-\gamma \to e^-\gamma$ $$\sigma = \frac{2\pi\alpha^2}{s}\ln\frac{s}{m^2}\left(1+\frac{\alpha}{6\pi}\ln^2\frac{s}{m^2}+\cdots\right)$$
- Pair production: $\gamma\gamma \to e^+e^-$ $$\sigma = \frac{4\pi\alpha^2}{s}\ln\frac{s}{m^2}\left(1+\frac{\alpha}{12\pi}\ln^2\frac{s}{m^2}+\cdots\right)$$
- Annihilation: $e^+e^- \to \gamma\gamma$ $$\sigma = \frac{2\pi\alpha^2}{s}\ln\frac{s}{m^2}\left(1+\frac{\alpha}{6\pi}\ln^2\frac{s}{m^2}+\cdots\right)$$

- DGLAP equations cannot reproduce all logarithms:

  PDFs predict $\frac{\alpha^3}{s}\ln^2\frac{s}{m^2}$ at NLO (collinear logarithms)

- It is important to understand these logarithms (Glauber modes) and resum them in the future.

[1703.08572, Schwartz, Yan, Zhu]
[Bhattacharya, Schwartz, in progress]

- It is also essential to construct IR finite cross section beyond LO.