

Ecal Digitization Check

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Digitization plan: smallest time in all steps:

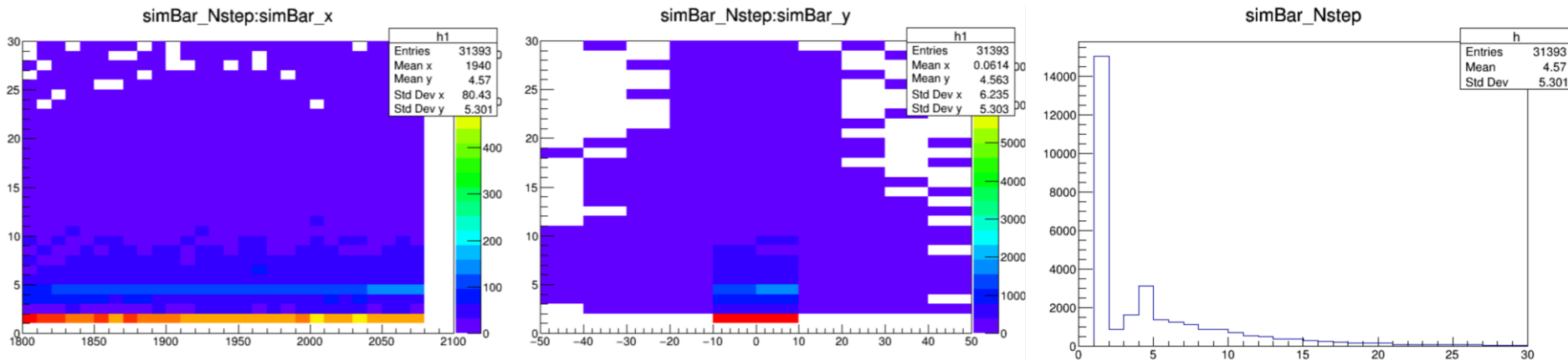
$$T_{\pm}^{step_i} = \text{Gaus}(z_{\pm}^i/v, \sigma_T), \quad T_{\pm}^{bar} = \min(T_{\pm}^{step_i})$$

Mathematical model:

- Crystal bar: $z_- = -20\text{cm}, z_+ = 20\text{cm}$.
- Use a Gaussian to describe step hits: $z_{step} \sim \text{Gaus}(z_0, \sigma_z)$.
- Step time $T_{\pm}^{step} \sim \text{Gaus}\left(\frac{L/2 \pm z_{step}}{v}, \sigma_T\right) \sim \text{Gaus}\left(\frac{L/2 \pm z_0}{v}, \sigma_T \oplus \frac{\sigma_z}{v}\right)$.
- Bar time: Order statistics of steps:
$$f(T_{\pm}) \sim n[1 - F_{\pm}(x)]^{n-1} f_{\pm}(x)$$
 - n : step hits number. $F(x)$: CDF of Gaussian. $f(x)$: PDF of Gaussian.

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Check with muon sample: influencing pars: n_{step} , σ_Z , σ_T .

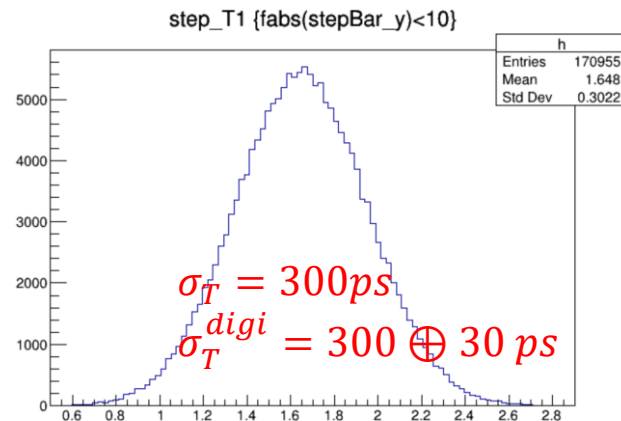
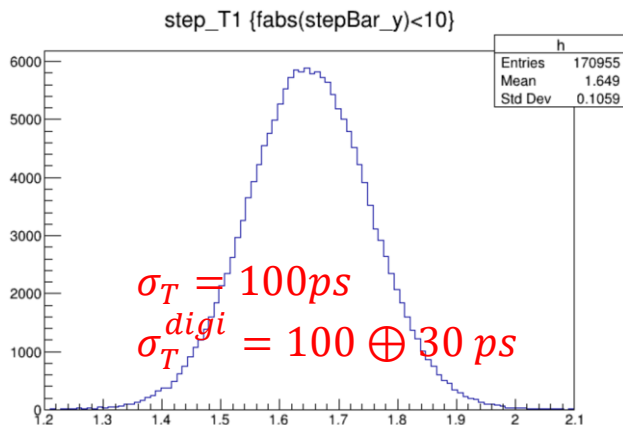
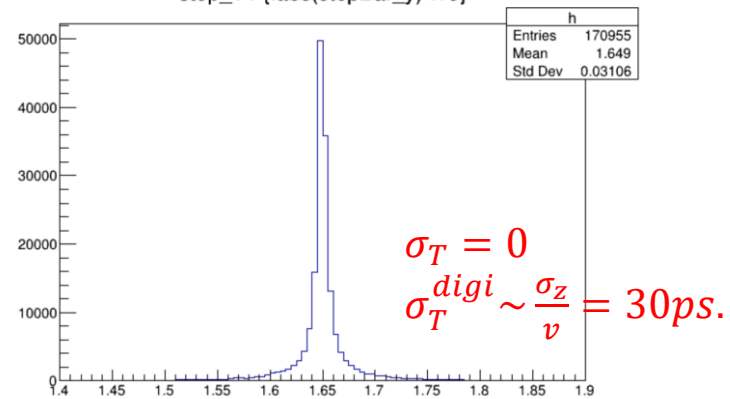
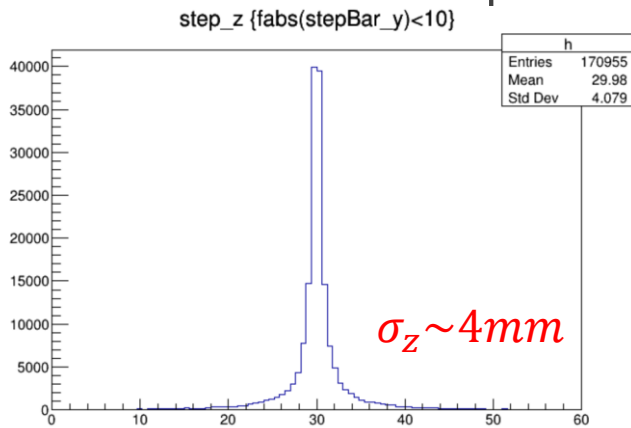


n_{step} doesn't depends on longitude depth, and most are 1 and 4.

If $n_{step} = 1$, $f(T_{\pm}) = \text{Gaus}(\frac{L/2 \pm z_0}{v}, \sigma_T \oplus \frac{\sigma_Z}{v})$.

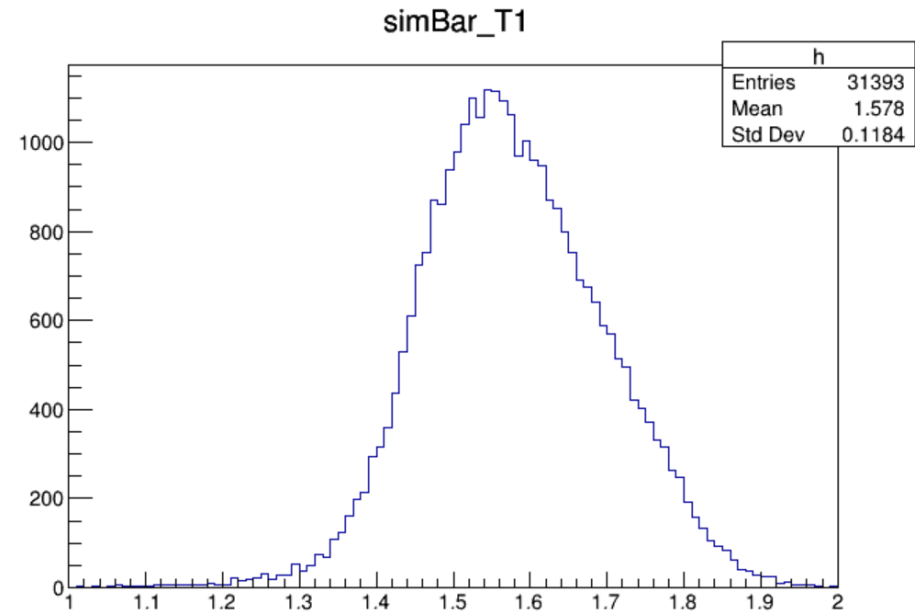
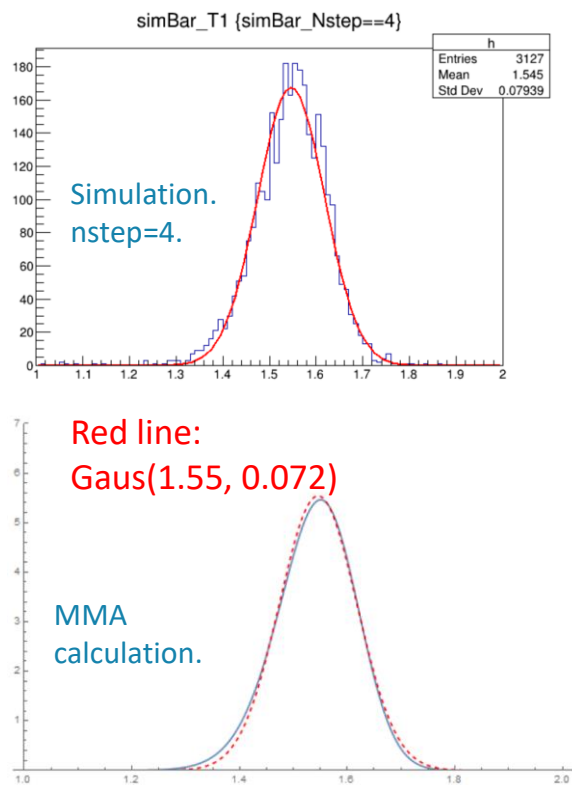
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Check with muon sample: influencing pars: $\sigma_z, \sigma_T, n_{step}$.



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Digitalized bar time: only see in $\sigma_T = 100ps$.



Should be sum of weighted order function: $f(T_-) = \sum_{nstep} w_i f(T_- | n_i)$.
 $w_i = \frac{N(nstep=n_i)}{N_{tot}}$, $n_i = 1$ is dominant.

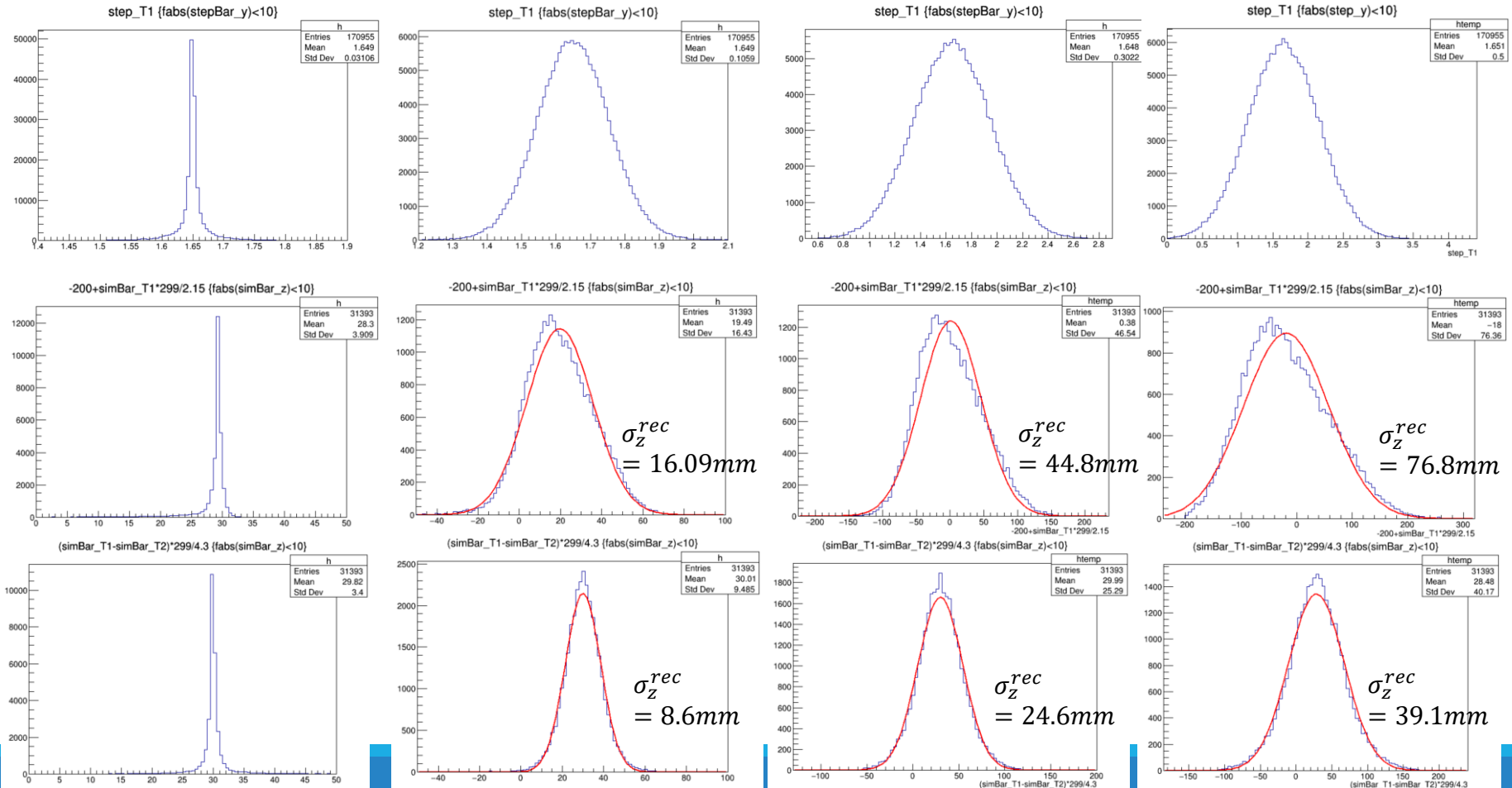
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$\sigma_T = 0$

$\sigma_T = 100ps$

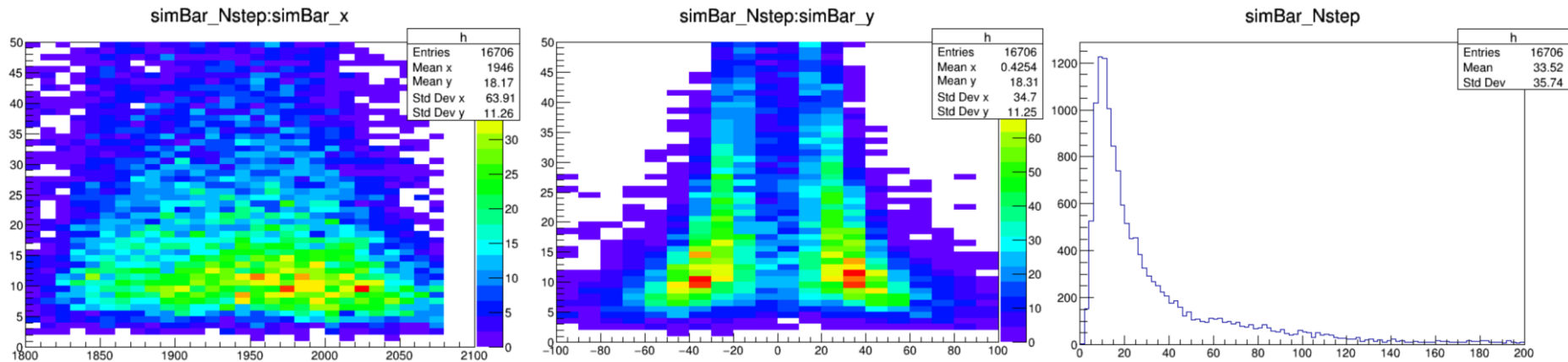
$\sigma_T = 300ps$

$\sigma_T = 500ps$



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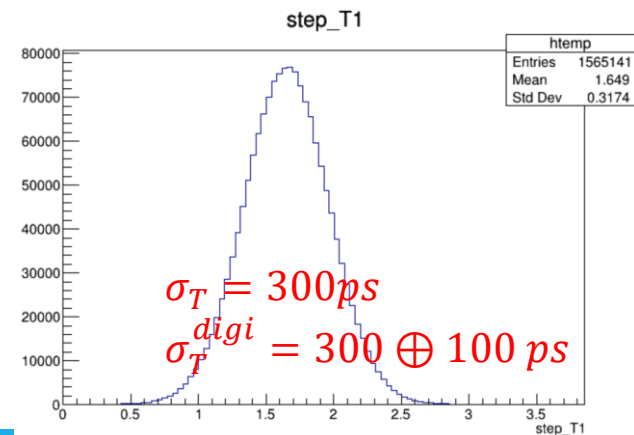
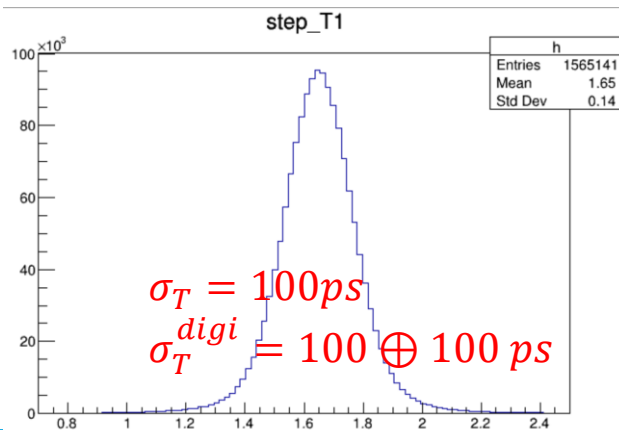
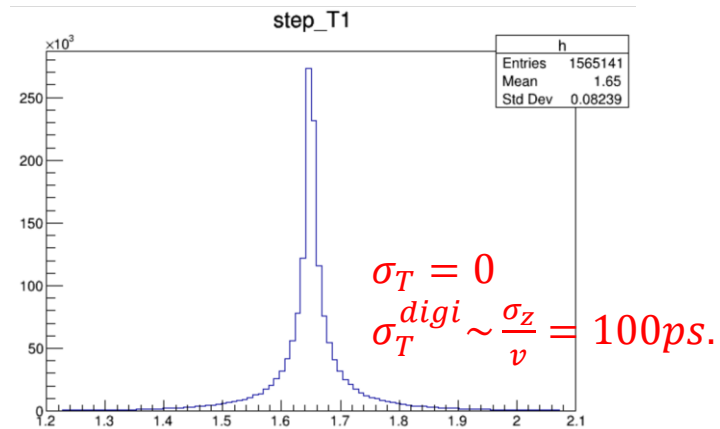
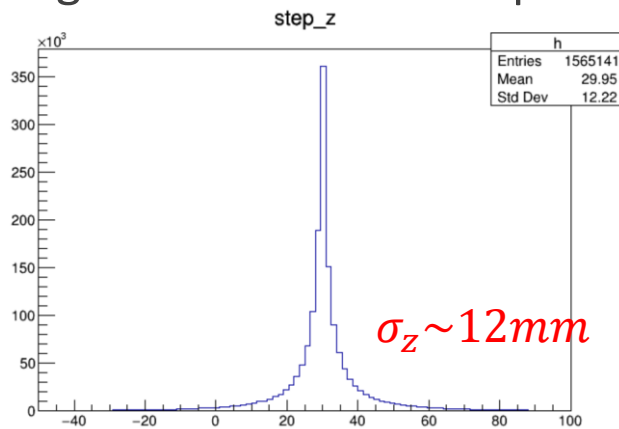
Digitization check with photon:



More complex than muon. Ordered distribution only works for specific condition.

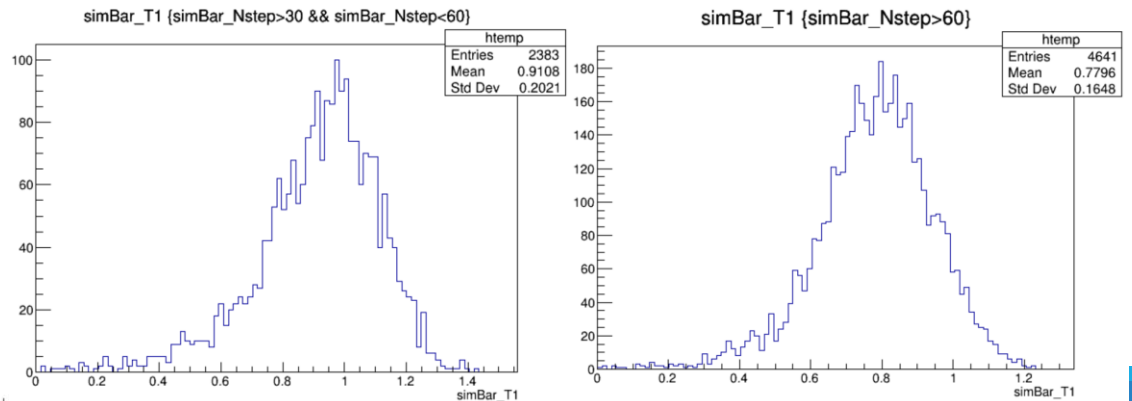
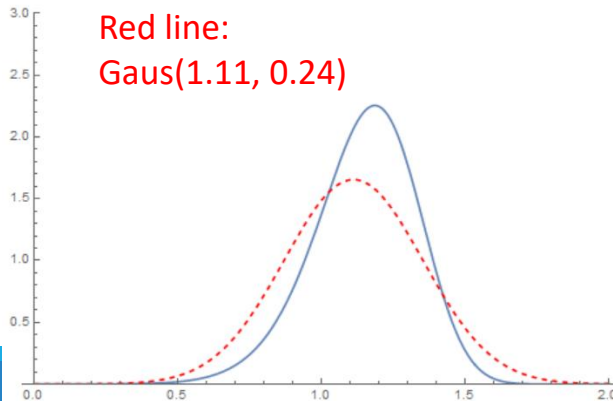
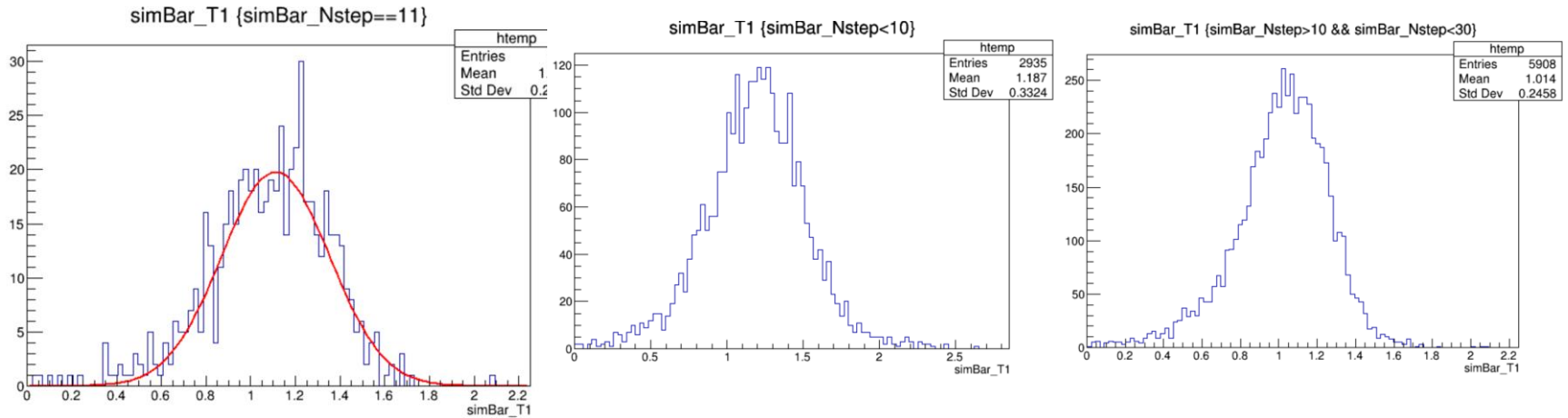
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Digitization check with photon:



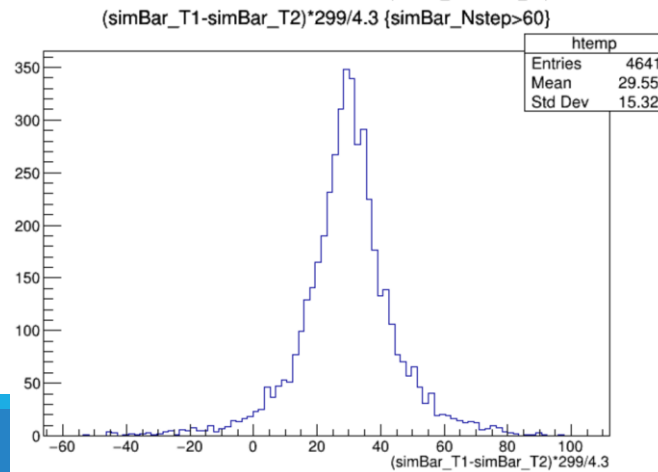
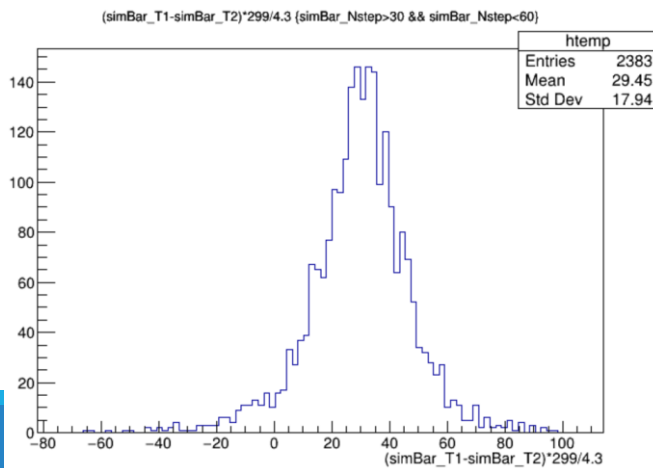
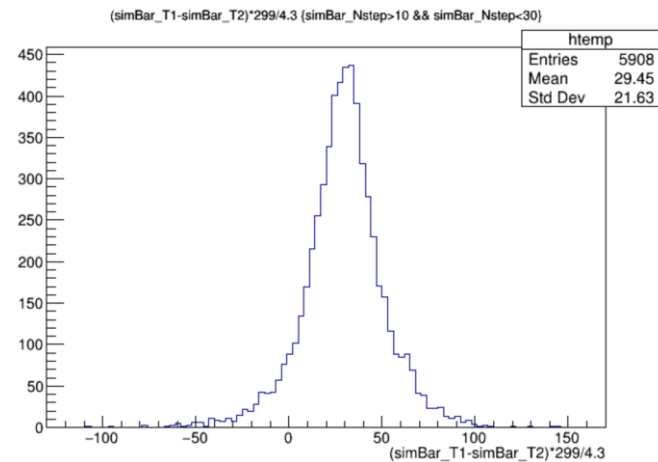
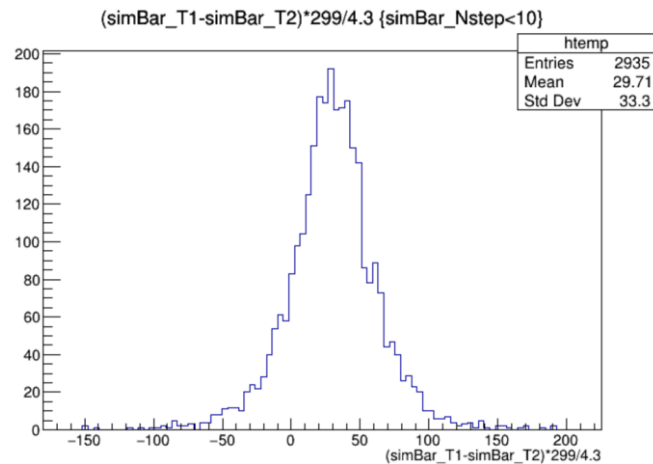
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Check with $\sigma_T = 300ps$



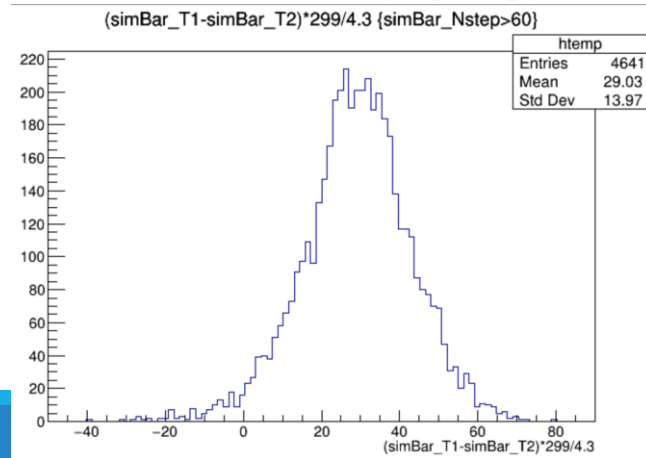
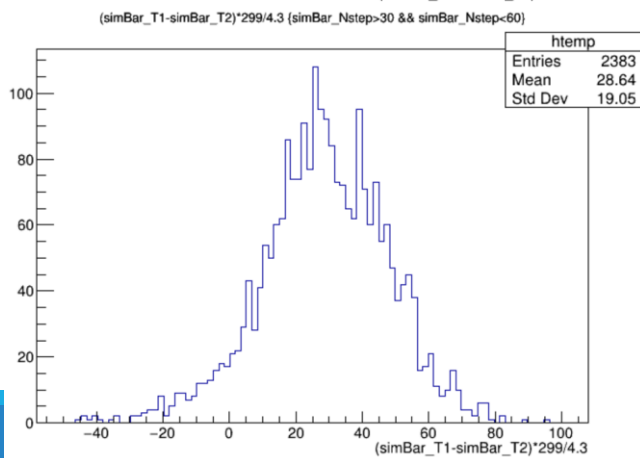
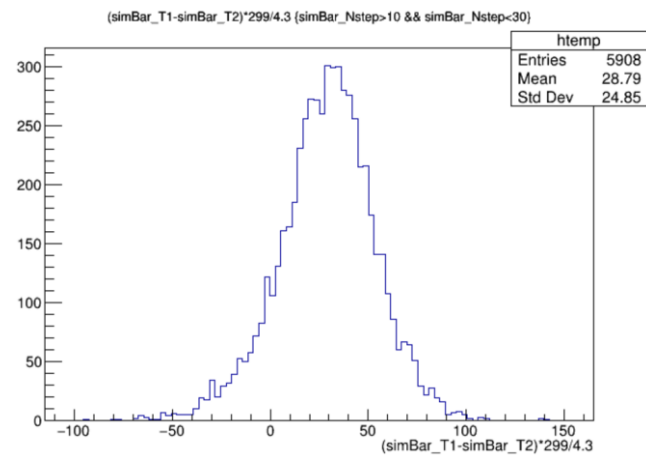
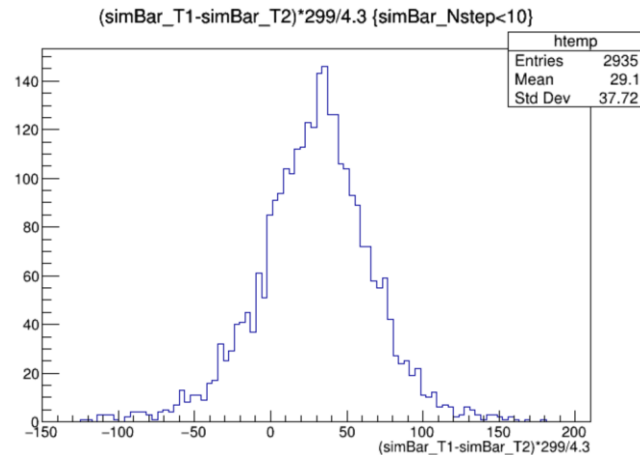
Ecal Digitization Check

$$\sigma_T = 100ps$$



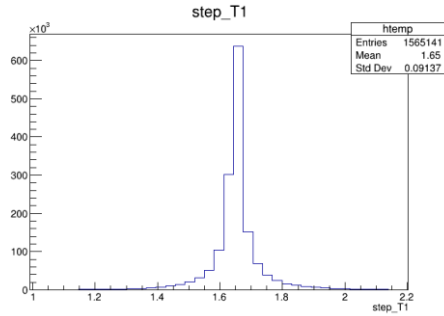
Ecal Digitization Check

$$\sigma_T = 300ps$$

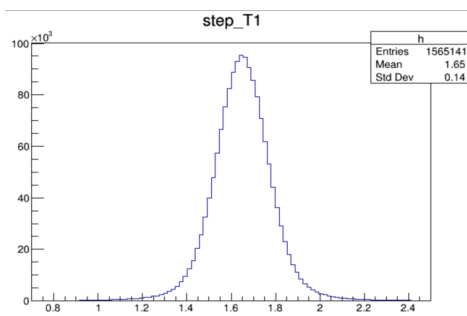


Ecal Digitization Check

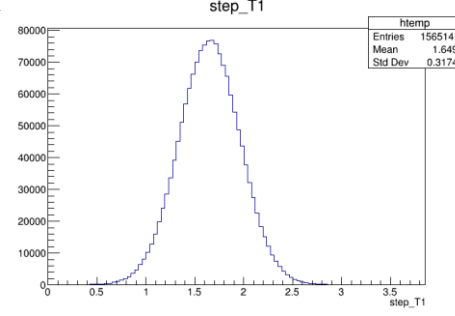
$\sigma_T = 0$



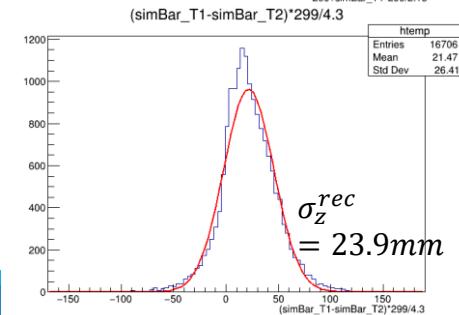
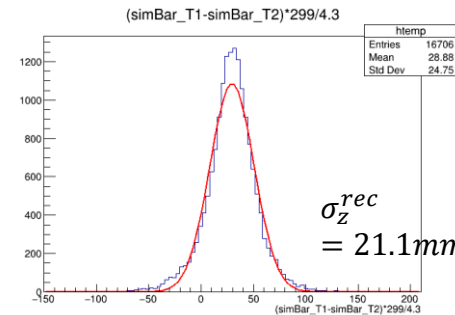
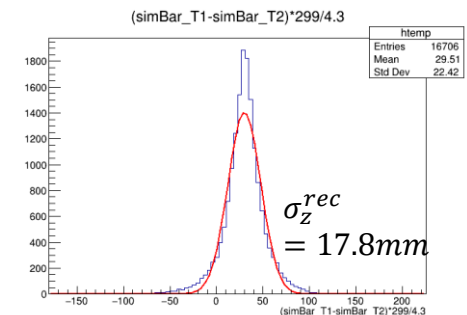
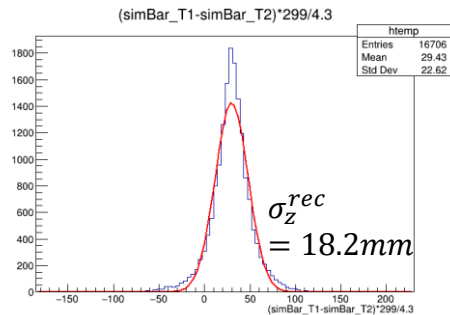
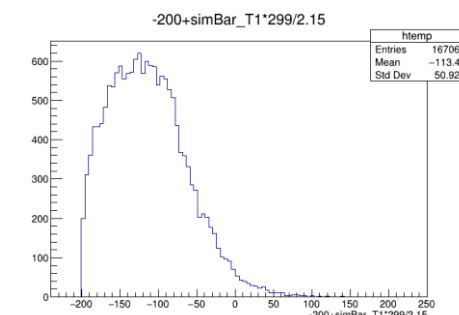
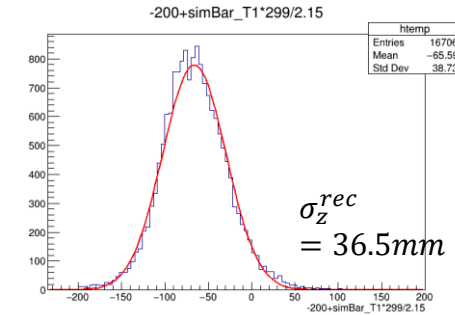
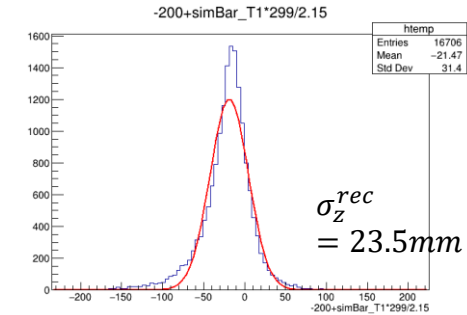
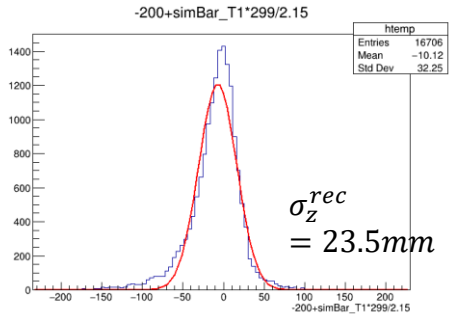
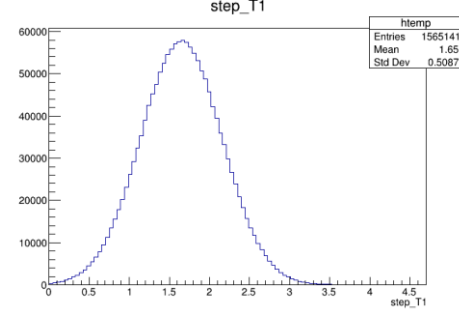
$\sigma_T = 100ps$



$\sigma_T = 300ps$



$\sigma_T = 500ps$



Conclusion

For MIP like muon:

- Only few steps in each crystal bar.
- Bar time $T_{\pm} \sim$ Ordered statistical distribution
- Can have precise reconstructed position, $\delta z_{rec} < 4cm$.
- δz_{rec} changes with σ_T .

For EM particle like photon:

- $n_{step} \sim O(10)$, and varies with longitude/transvers development.
- For a specific bar(with certain n_{step}), δz_{rec} depends on σ_T .
- Averagely $\delta z_{rec} \sim f(\delta z_{shower}, \sigma_T, n_{step}, \dots)$, while n_{step} is dominant.