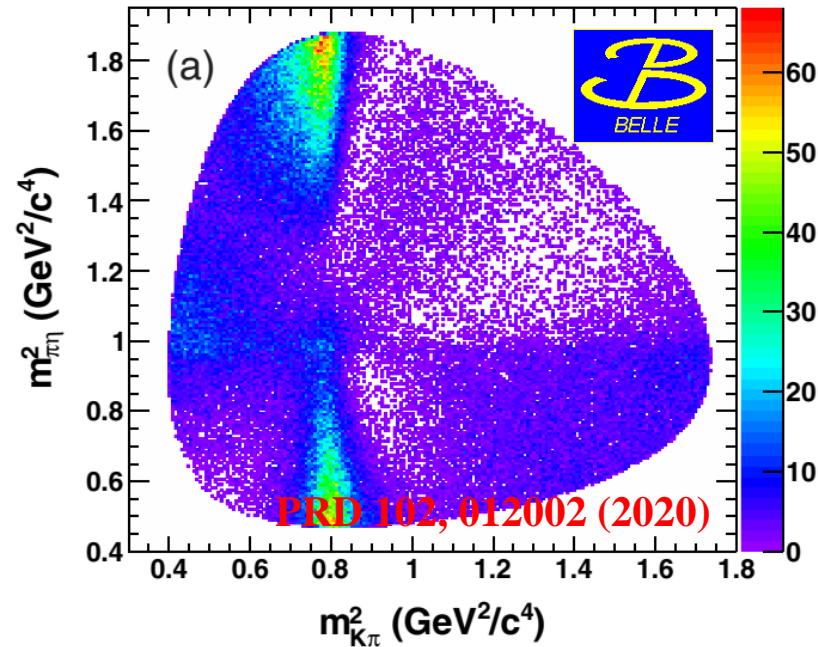


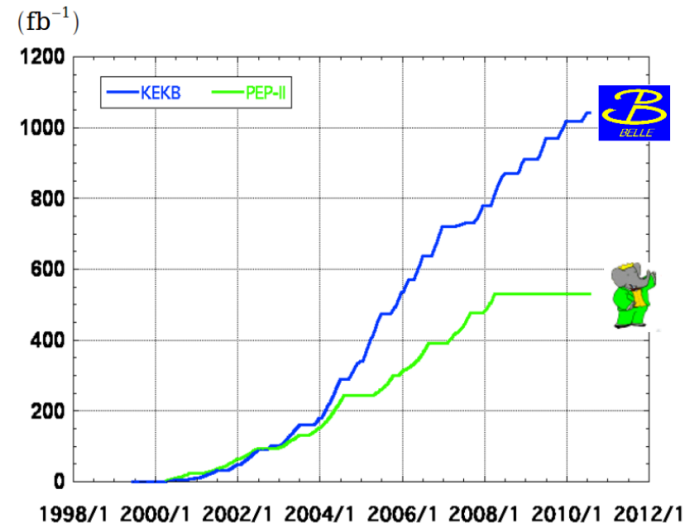
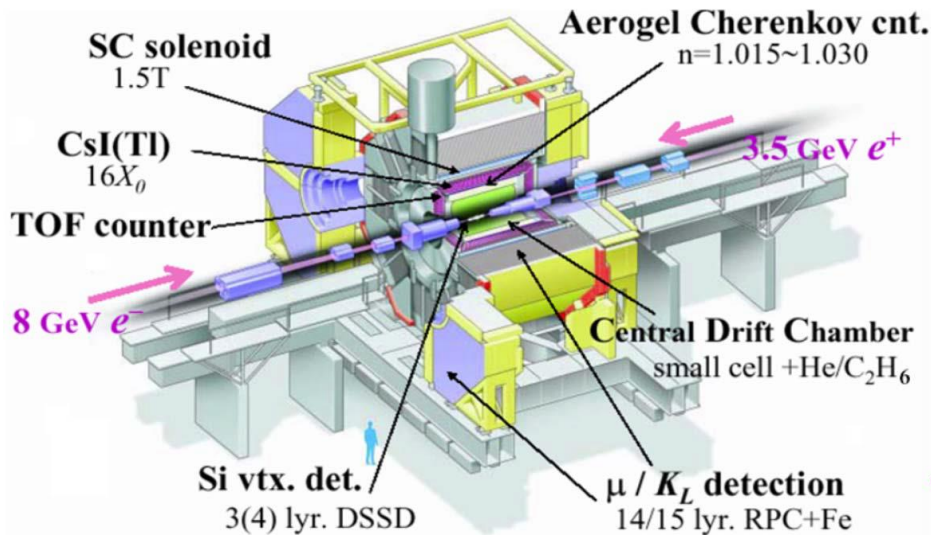
Dalitz plot analysis of $D^0 \rightarrow K^- \pi^+ \eta$

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第二届“强子物理和重味物理理论与实验联合研讨会”, 2021.03.28, 兰州

KEKB accelerator & Belle Detector



> 1 ab⁻¹
On resonance:
 $\Upsilon(5S)$: 121 fb⁻¹
 $\Upsilon(4S)$: 711 fb⁻¹
 $\Upsilon(3S)$: 3 fb⁻¹
 $\Upsilon(2S)$: 25 fb⁻¹
 $\Upsilon(1S)$: 6 fb⁻¹
Off reson./scan:
 ~ 100 fb⁻¹

~ 550 fb⁻¹
On resonance:
 $\Upsilon(4S)$: 433 fb⁻¹
 $\Upsilon(3S)$: 30 fb⁻¹
 $\Upsilon(2S)$: 14 fb⁻¹
Off resonance:
 ~ 54 fb⁻¹

- Asymmetric e⁺e⁻ collider
 - 8 GeV(e⁻); 3.5GeV(e⁺)
 - Around 10.58GeV ↔ Y(4S)
- B factory, also tau-charm factory

Process	σ (nb) @ Y(4S)
$b\bar{b}$	1.1
$c\bar{c}$	1.3
$q\bar{q}$ (q=u,d,s)	2.1
$\tau^+\tau^-$	0.93

$D^0 \rightarrow K^- \pi^+ \eta$

- $D^0 \rightarrow K^- \pi^+ \eta$ process

- Cabibbo favored via $c \rightarrow s u \bar{d}$

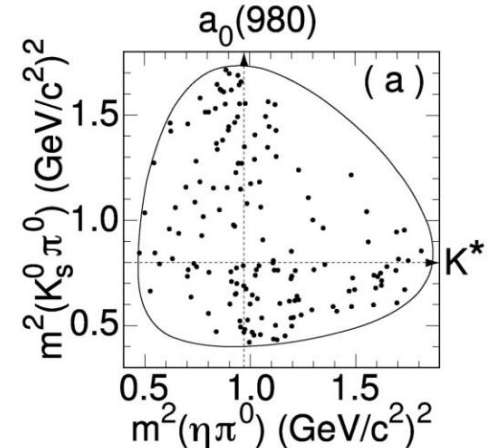
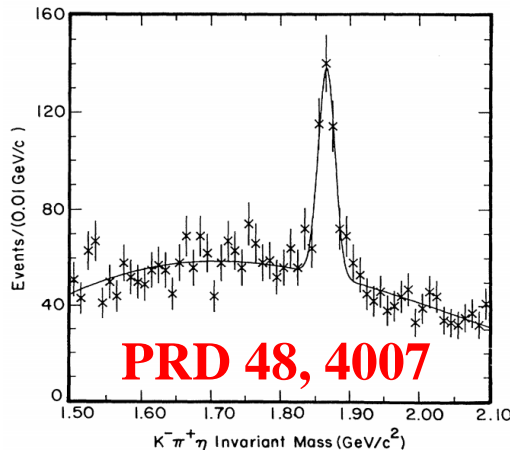
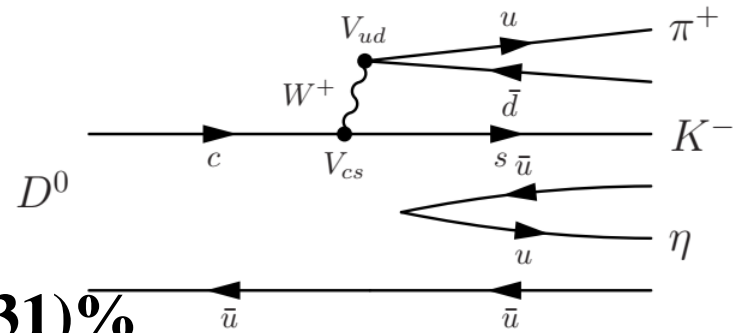
- Firstly observed by CLEO

- BESIII: $Br = (1.853 \pm 0.025 \pm 0.031)\%$

- $D^0 \rightarrow K^{*0} \eta$: W-exchange diagram

- $D^0 \rightarrow K_S^0 \pi^0 \eta$: $(1.02 \pm 0.30)\%$

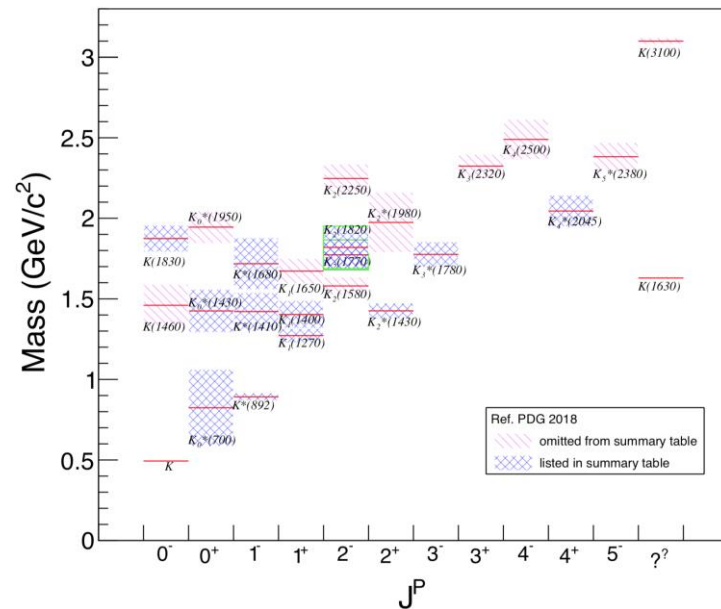
- Theory calculation: RR D81, 074021; D86, 036012; D89, 054006;



PRL 93, 111801

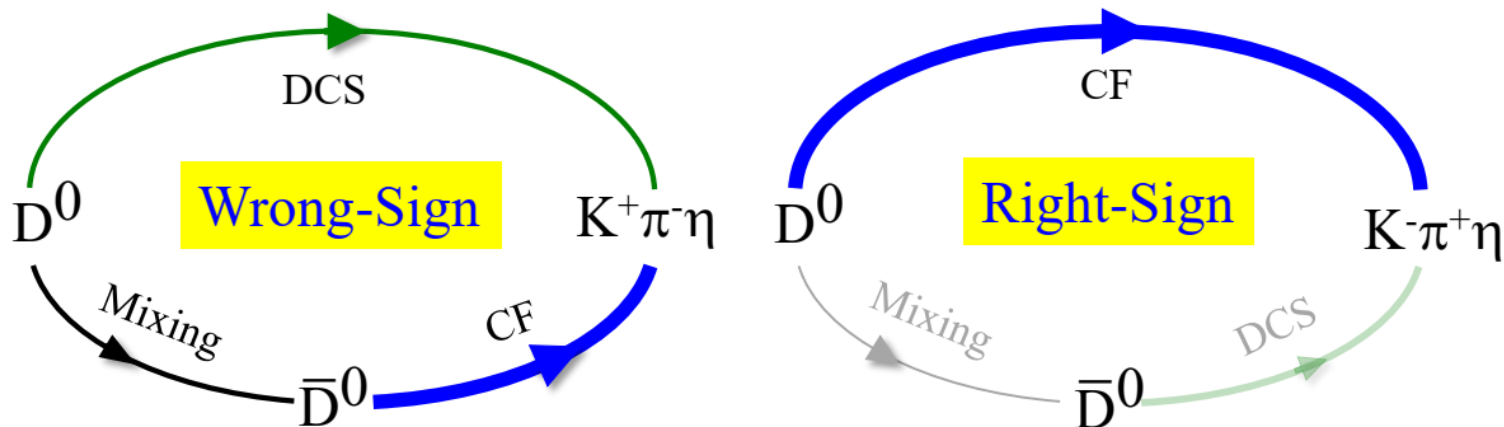
$D^0 \rightarrow K^- \pi^+ \eta$

- **D three-body hadronic decay**
 - study light hadron spectrum
- **Kaon spectrum**
 - PDG2020: 25 kaon states below 3.1 GeV/c²
 - Only 13 states well established, 12 states need confirmation
 - Little progress in past 30 years
- **K* @ Kπ & Kη**
 - Search K*(1410), K*(1680) and K₂*(1980) @ Kη
 - K*(1680) as pure 1³D₁ state
 - K*(1410) & K*(1680): mixture 2³S₁ and 1³D₁



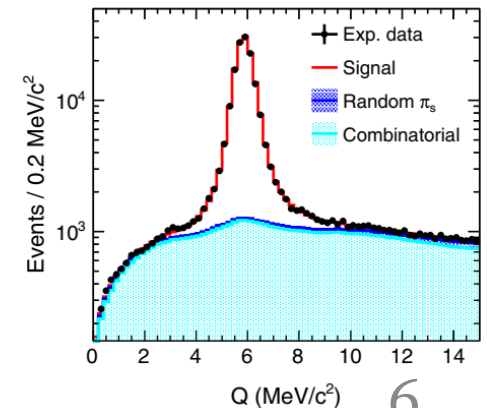
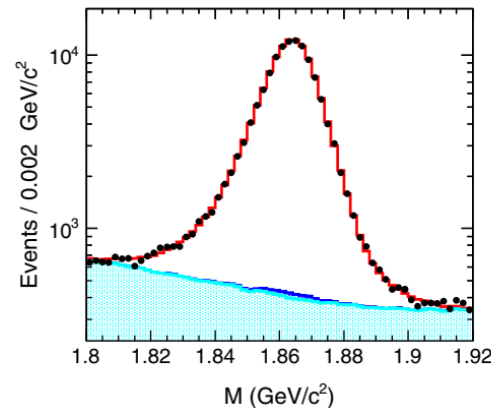
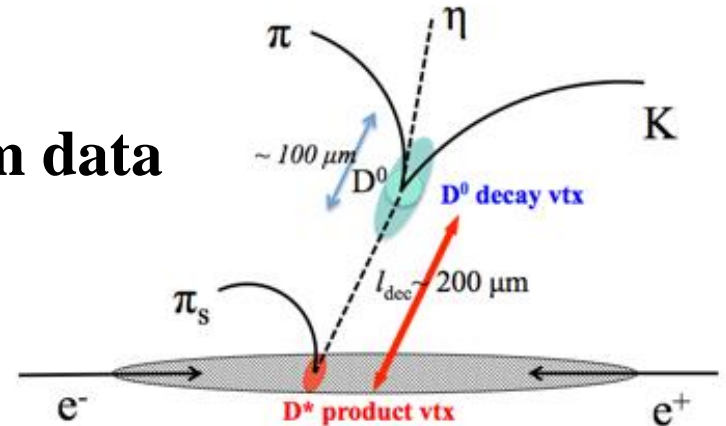
$D^0 \rightarrow K^- \pi^+ \eta$

- Wrong-sign decays have important role on D^0 - \bar{D}^0 mixing & CP violation
- Wrong-sign $D^0 \rightarrow K^+ \pi^- \eta$ for D^0 - \bar{D}^0 mixing: not yet
- With 50 ab^{-1} data @ Belle II, Time-dependent Dalitz analysis of $D^0 \rightarrow K^+ \pi^- \eta$: possible
- Right-sign $D^0 \rightarrow K^- \pi^+ \eta$: necessary input for $D^0 \rightarrow K^+ \pi^- \eta$



$D^0 \rightarrow K^- \pi^+ \eta$ @ Belle

- Belle data: total 953 fb^{-1}
 - $Y(nS)$ with $n=[1, 5]$ & continuum data
- $e^+e^- \rightarrow \gamma^* \rightarrow D^{*+} + X$
 - $D^{*+} \rightarrow D^0 \pi_s^+$ and $D^0 \rightarrow K^- \pi^+ \eta$
 - D^0/\bar{D}^0 tagged by π_s of D^*
 - Veto D^{*+} from B decay
- M-Q two-dimension fit:
 - $M \equiv M_{K\pi\eta}$ and $Q \equiv M_{K\pi\eta\pi_s} - M_{K\pi\eta} - m_{\pi_s}$
 - 105197 ± 60 events at signal region, with purity $(94.6 \pm 0.9)\%$

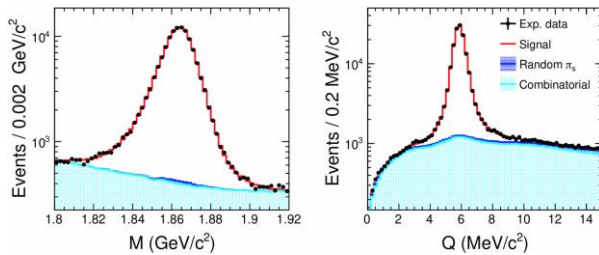


Generalization of Dalitz analysis

● Unbinned maximum likelihood method

$$-2\ln\mathcal{L}(m_{AB}^2, m_{BC}^2) = -2 \sum_{i=1}^n \ln[f_{sig}^i p_{sig}(m_{AB,i}^2, m_{BC,i}^2) + f_{bkg}^i p_{bkg}(m_{AB,i}^2, m_{BC,i}^2)]$$

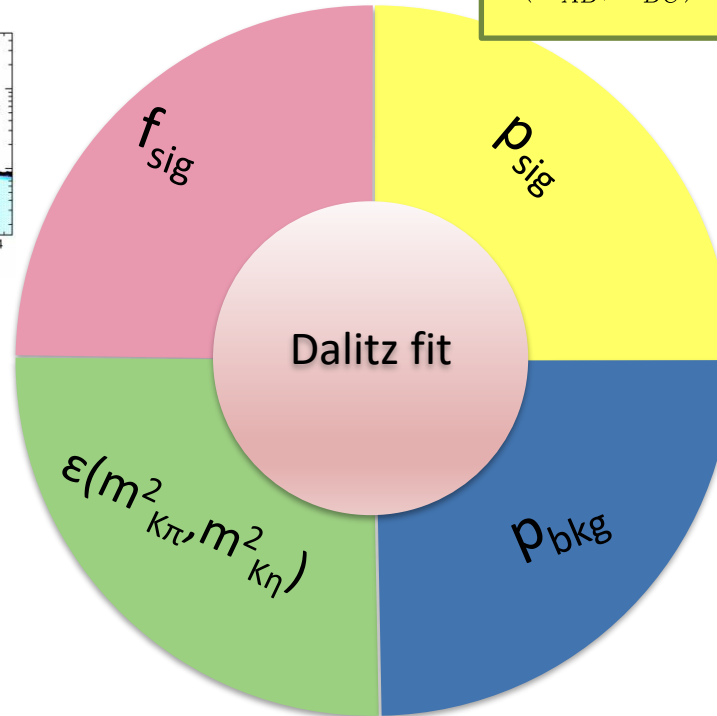
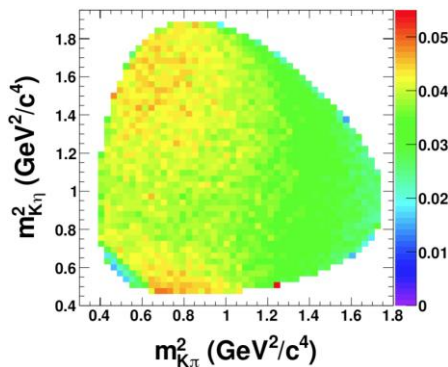
M-Q 2D fit



Isobar model

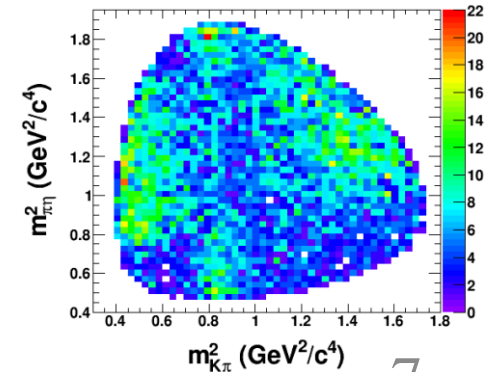
$$\mathcal{M}(m_{AB}^2, m_{BC}^2) = a_{NR} e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r} \mathcal{A}_r(m_{AB}^2, m_{BC}^2)$$

Efficiency plane



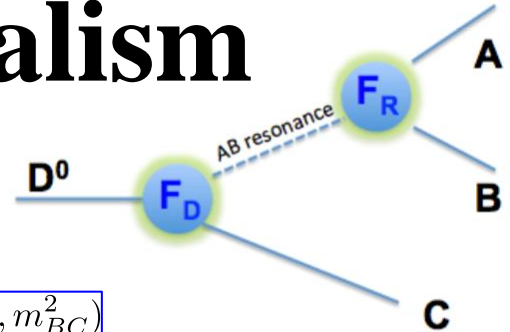
Background PDF

- $D^0 \sqrt{}, \pi_s \times$
- $D^0 \times, \pi_s \sqrt{}$
- $D^0 \times, \pi_s \times$
- M sideband @ Q signal



$$p_{sig} = \frac{\sum |\mathcal{M}(m_{12,i}^2, m_{23,i}^2)|^2 \epsilon_j(m_{12,i}^2, m_{23,i}^2)}{\sum \iint_{DP} dm_{12}^2 dm_{23}^2 |\mathcal{M}(m_{12}^2, m_{23}^2)|^2 \epsilon_j(m_{12}^2, m_{23}^2)}$$

Dalitz analysis formalism



- **Dalitz standard form** $d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |\mathcal{M}|^2 dm_{AB}^2 dm_{BC}^2$

- **Isobar model** $\mathcal{M}(m_{AB}^2, m_{BC}^2) = a_{NRE} e^{i\phi_{NR}} + \sum_r a_r e^{i\phi_r} \mathcal{A}_r(m_{AB}^2, m_{BC}^2)$

- **Matrix element \mathbf{A}_r** $\mathcal{A}(ABC|r) = F_D \times F_r \times T_r \times \Omega_J$

- **Blatt-Weisskopf centrifugal barrier factor: F_r, F_D**

- **Angular distribution function Ω_J by Zemach tensor**

- **Dynamical function T_r**

- ✓ **most of resonances by relativistic Breit-Wigner**

- ✓ **$a_0(980)$ by Flatte model**

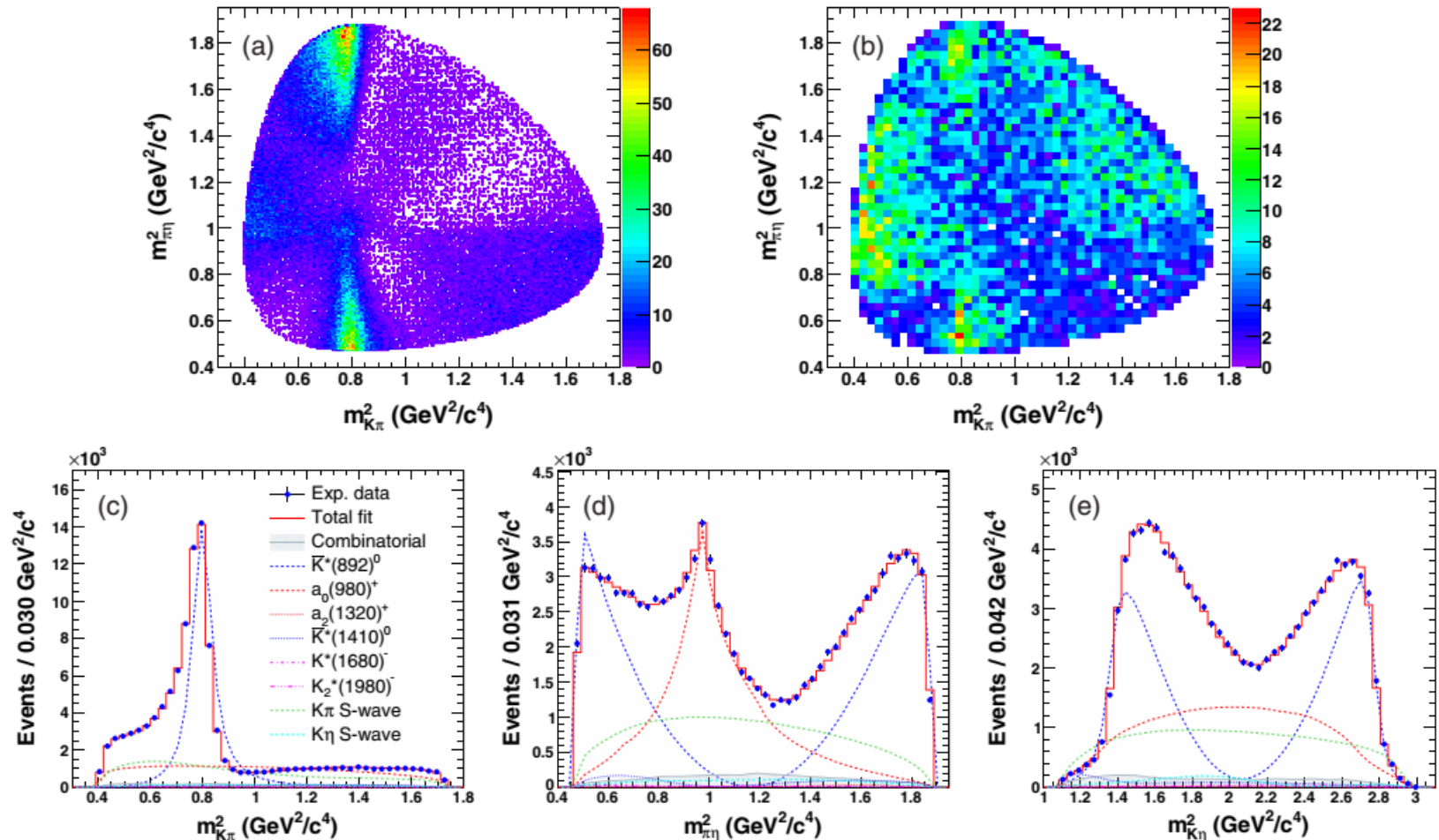
$$T_R(s) = \frac{1}{m_{a_0}^2 - s - i(g_{\pi\eta}^2 \rho_{\pi\eta} + g_{\bar{K}^0 K}^2 \rho_{\bar{K}^0 K} + g_{\pi\eta'}^2 \rho_{\pi\eta'})}$$

- ✓ **$K\pi$ & $K\eta$ S-wave contribution by LASS model**

$$A_{g\text{LASS}}(s) = \frac{\sqrt{s}}{2q} \cdot [B \sin(\delta_B + \phi_B) e^{i(\delta_B + \phi_B)} + \sin(\delta_R) e^{i(\delta_R + \phi_R)} e^{2i(\delta_B + \phi_B)}],$$

Non-resonant and $K_0^*(1430)$ components

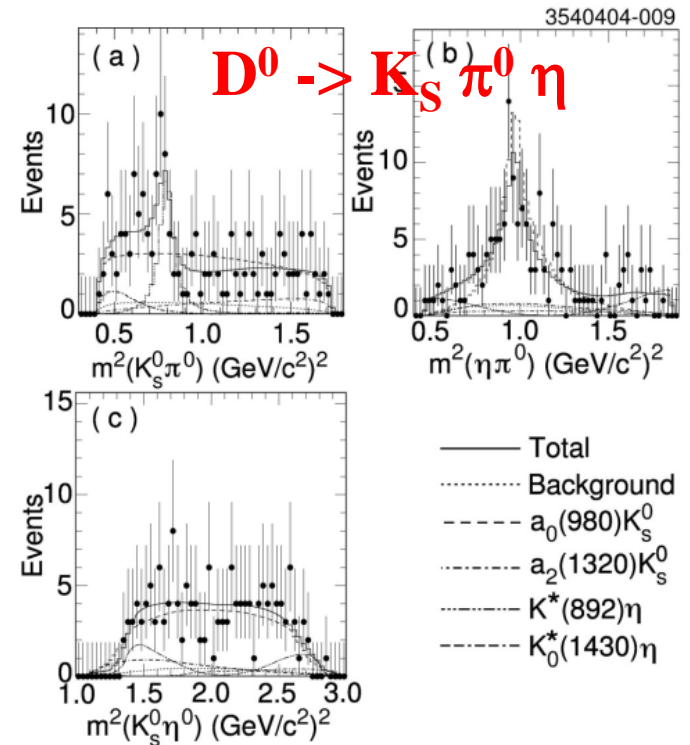
Dalitz plot fit results



- The fit quality $\chi^2/\text{d.o.f} = 1638/(1415-24) = 1.18$

Dalitz plot fit results

Component	Magnitude	Phase (°)	Fit fraction (%)
$\bar{K}^*(892)^0$	1	0	$47.61 \pm 1.32^{+0.24+3.64}_{-0.49-2.71}$
$a_0(980)^+$	2.779 ± 0.032	310.3 ± 1.1	$39.28 \pm 1.50^{+1.58+4.38}_{-0.51-3.30}$
$K\pi$ S-wave	10.82 ± 0.23	50.0 ± 5.7	$31.92 \pm 1.21^{+1.47+2.75}_{-0.53-2.87}$
$K\eta$ S-wave	1.70 ± 0.082	113.8 ± 13.6	$3.37 \pm 0.50^{+0.77+3.20}_{-0.27-1.21}$
$a_2(1320)^+$	1.27 ± 0.079	283.4 ± 4.7	$0.74 \pm 0.09^{+0.06+0.37}_{-0.04-0.17}$
$\bar{K}^*(1410)^0$	4.84 ± 0.36	352.7 ± 2.8	$6.94 \pm 0.85^{+0.55+2.37}_{-1.61-3.22}$
$K^*(1680)^-$	2.56 ± 0.18	232.2 ± 6.6	$1.07 \pm 0.16^{+0.11+0.58}_{-0.10-0.36}$
$K_2^*(1980)^-$	9.29 ± 0.69	207.7 ± 4.0	$1.13 \pm 0.15^{+0.05+0.88}_{-0.05-0.98}$
Sum	$D^0 \rightarrow K^+ \pi^- \eta$		$132.1 \pm 3.4^{+1.6+8.3}_{-0.7-4.5}$



- Dominant components: $\bar{K}^*(892)^0$ and $a_0(980)^+$
 - $K\eta$ S-wave with $K_0^*(1430)$: $> 30\sigma$
 - $K^*(1680)^-/K_2^*(1980)^- \rightarrow K-\eta$ are observed for the first time
- With $16\sigma/17\sigma$

$B(D^0 \rightarrow K^- \pi^+ \eta)$

- Normalized mode $D^0 \rightarrow K^- \pi^+$, fit M @ Q signal region

$$\frac{\mathcal{B}(D^0 \rightarrow K^- \pi^+ \eta) \mathcal{B}(\eta \rightarrow \gamma\gamma)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)} = \frac{N_1/\epsilon_1}{N_2/\epsilon_2}$$

$$\frac{B(D^0 \rightarrow K^- \pi^+ \eta)}{B(D^0 \rightarrow K^- \pi^+)} = 0.500 \pm 0.002(\text{stat}) \pm 0.020(\text{syst}) \pm 0.003(\text{PDG})$$

- PDG2018: $B(D^0 \rightarrow K^- \pi^+) = (3.950 \pm 0.031)\%$, then

$$B(D^0 \rightarrow K^- \pi^+ \eta) = 1.973 \pm 0.009(\text{stat}) \pm 0.079(\text{syst}) \pm 0.018(\text{PDG})$$

- BESIII: $B(D^0 \rightarrow K^- \pi^+ \eta) = (1.853 \pm 0.025(\text{stat}) \pm 0.031(\text{sys}))\%$

Branching fractions @ $D^0 \rightarrow K^- \pi^+ \eta$

- Combine Dalitz fit results and $B(D^0 \rightarrow K^- \pi^+ \eta)$, then have
- $B(D^0 \rightarrow K^*(892)^0 \eta \rightarrow K^- \pi^+ \eta) = (0.94 \pm 0.03 \pm 0.08 \pm 0.01)\%$
 - PDG: $B(K^*(892)^0 \rightarrow K^- \pi^+) = (66.503 \pm 0.014)\%$ @ PDG
 - $B(D^0 \rightarrow K^*(892)^0 \eta) = (1.41 \pm 0.04_{-0.11}^{+0.12} \pm 0.01)\%$
 - PDG: $B(D^0 \rightarrow \bar{K}^*(892)^0 \eta) = (1.02 \pm 0.30)\%$
 - Theory prediction: $(0.51-0.92)\%$
 - deviates theory prediction with significance of more than 3σ
- $B(D^0 \rightarrow K^*(1680)^- \pi^+ \rightarrow K^- \eta \pi^+) = (2.11 \pm 0.32_{-0.72}^{+1.16} \pm 0.02) \times 10^{-4}$
 - $B(D^0 \rightarrow K^*(1680)^- \pi^+ \rightarrow K^- \pi^0 \pi^+) = (0.19 \pm 0.07)\%$ @ PDG
 - $\frac{B(K^*(1680) \rightarrow K^- \eta)}{B(K^*(1680) \rightarrow K^- \pi)} = 0.11 \pm 0.02_{-0.04}^{+0.06} \pm 0.04$
 - $B(K^*(1680)^- \rightarrow K^- \pi^0) = (12.90 \pm 0.83)\%$ @ PDG
 - $B(K^*(1680)^- \rightarrow K^- \eta) = (1.44 \pm 0.21_{-0.49}^{+0.79} \pm 0.54)\%$
- $B(D^0 \rightarrow K_2^*(1980)^- \pi^+ \rightarrow K^- \pi^+ \eta) = (2.2 + 1.7 - 1.9) \times 10^{-4}$
 - Strongly suppressed do to phase-space and yet allowed due to large width of $K_2^*(1980)$

K*(1680)

- K*(1680) as pure 1³D₁ state
 - Kπ : Kη ≈ 1.0 @ theory
 - Belle: Kπ : Kη ≈ 0.11 ± 0.07
 - Kπ, Kρ, and K*(892)π
 - Any idea ?

Mode	EPJC 77, 861	PRD 68, 054014
Γ _{Kπ}	69.2 MeV	45 MeV
Γ _{Kη}	64.4 MeV	53 MeV

	EPJC 77, 861	PRD 68, 054014	Experiment
Γ _{Kπ} /Γ _{K*(892)π}	1.66	1.8	2.8 ± 1.1
Γ _{Kρ} /Γ _{Kπ}	0.65	0.58	1.2 ± 0.4
Γ _{Kρ} /Γ _{K*(892)π}	1.07	1.04	1.05+0.27-0.11







- K*(1410) and K*(1680): mixture 2³S₁ and 1³D₁
 - K*(1410)⁻ -> K⁻η: smaller than 3σ @ Belle

$$\begin{pmatrix} |K^*(1410)\rangle \\ |K^*(1680)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{sd} & \sin \theta_{sd} \\ -\sin \theta_{sd} & \cos \theta_{sd} \end{pmatrix} \begin{pmatrix} |1^3D_1\rangle \\ |2^3S_1\rangle \end{pmatrix}$$

Summary

- Based on 953 fb^{-1} Belle data, Dalitz analysis of $D^0 \rightarrow K^- \pi^+ \eta$ is performed for the first time.
- $B(D^0 \rightarrow K^- \pi^+ \eta)$ is consistent with BESIII's result.
- $B(D^0 \rightarrow \bar{K}^*(892)^0 \eta) = (1.41 +0.13 -0.12)\%$, deviates theory prediction with significance of more than 3σ
- $K^*(1680)^- / K_2^*(1980)^- \rightarrow K^- \eta$ are observed.
- $\frac{B(K^*(1680) \rightarrow K^- \eta)}{B(K^*(1680) \rightarrow K^- \pi)} = 0.11 \pm 0.02(\text{stat})^{+0.06}_{-0.04}(\text{sys}) \pm 0.04(\text{PDG})$, is not consistent with theory prediction (≈ 1) under assumption $K^*(1680)$ as pure 1^3D_1 state.

Charm meson data

Experiment	Machine	C.M. \sqrt{s}	Luminosity	charm sample	efficiency				
	CESR (e^+e^-)	3.77 GeV	0.8 fb ⁻¹	$2.9 \times 10^6 (D^0)$ $2.3 \times 10^6 (D^+)$	~10-30%				
		4.17 GeV	0.6 fb ⁻¹	$0.6 \times 10^6 (D_s^+)$					
	BEPC-II (e^+e^-)	3.77 GeV	2.9 fb ⁻¹	$10.5 \times 10^6 (D^0)$ $8.4 \times 10^6 (D^+)$					
		4.18 GeV	3.0 fb ⁻¹	$3 \times 10^6 (D_s^+)$					
		4.6 GeV	0.6 fb ⁻¹	$1 \times 10^5 (\Lambda_c^+)$					
	KEKB (e^+e^-)	10.58 GeV	1 ab ⁻¹	$1.3 \times 10^9 (D^0)$ $7.7 \times 10^8 (D^+)$ $2.5 \times 10^8 (D_s^+)$ $1.5 \times 10^8 (\Lambda_c^+)$	~5-10%				
						PEP-II (e^+e^-)	10.58 GeV	0.5 ab ⁻¹	$6.5 \times 10^8 (D^0)$ $3.8 \times 10^8 (D^+)$ $1.2 \times 10^8 (D_s^+)$ $0.7 \times 10^8 (\Lambda_c^+)$
	Tevatron (pp)	1.96 TeV	9.6 fb ⁻¹						1.3×10^{11}
			LHC (pp)						7 TeV
8 TeV	2.0 fb ⁻¹								

D^0 - \bar{D}^0 mixing

D^0 and \bar{D}^0 are flavor eigenstates, propagate and decays according to

$$i\frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

D^0 and \bar{D}^0 are combinations of mass eigenstates

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$$

$$|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$$

The mass eigenstates develop in time as

$$|D_{1,2}(t)\rangle = e_{1,2}(t)|D_{1,2}(0)\rangle$$

$$e_{1,2}(t) \equiv e^{[-i(M_{1,2} - \frac{i}{2}\Gamma_{1,2})t]}$$

Two parameters describe D^0 and \bar{D}^0 mixing

$$x \equiv \frac{\Delta M}{\Gamma} \quad \Delta M \equiv M_1 - M_2$$

$$y \equiv \frac{\Delta\Gamma}{2\Gamma} \quad \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$

If either x or y are not zero, mixing occurs

$$|\langle \bar{D}^0 | D^0(t) \rangle|^2 = \frac{1}{2} \left| \frac{q}{p} \right|^2 e^{-\Gamma t} [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

$$|\langle D^0 | \bar{D}^0(t) \rangle|^2 = \frac{1}{2} \left| \frac{p}{q} \right|^2 e^{-\Gamma t} [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

$D^0 \rightarrow K^- \pi^+ \eta$

