The contribution of QCD trace anomaly to hadron mass

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Outline

① **Introduction to trace anomaly of QCD**

② **Hadron mass decomposition**

③ **Numerical results**

④ **Summary**

Scale Transformation

- **Scale transformation(dilatations):**

$$x \to xe^{-\sigma}$$
$$\phi(x) \to e^{-D\sigma}\phi(xe^{-\sigma})$$

D is the mass dimension of the field ϕ

- **Dilatational current** $D^{\mu} = T^{\mu\nu}x_{\nu}$ **T is the energy-momentum tensor**

- **Mass term will break down scale symmetry** $\partial_{\mu}D^{\mu} = T^{\mu}_{\mu} = m_q\bar{q}q$

Trace Anomaly of QCD EMT

- **Scale symmetry is broken when quantum corrections are included** Peskin and Schroeder, An Introduction to QFT, Chapter 19

$$(T^{\mu}_{\mu})^{a} = \frac{\beta_{QCD}}{2g} F^2 + \gamma_m m_q \bar{q} q$$

β_{QCD}: beta function of QCD

γ_m: Anomalous dimension of quark mass

J.Collins et,al. PRD16(1977) 438
N. K. Nielsen, NPB120 (1977) 212
X. Ji , PRL 74 (1995) 1071

- **Total trace term of QCD EMT**

$$(T^{\mu}_{\mu}) = (T^{\mu}_{\mu})^{a} + m_q \bar{q} q = \frac{\beta_{QCD}}{2g} F^2 + (1 + \gamma_m) m_q \bar{q} q$$

The Effect of Heavy quark

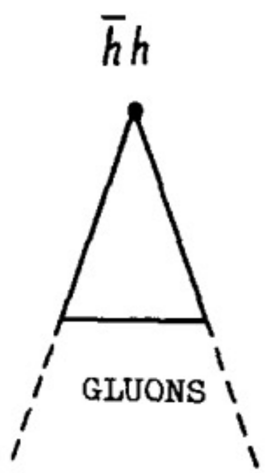
- **Trace term of ETM**

$$T^{\mu}_{\mu} = \frac{\beta_{QCD}}{2g} F^2 + \sum_l m_l (1 + \gamma_{m,l}) \bar{l} l + \sum_h m_h (1 + \gamma_{m,h}) \bar{h} h$$

- **The heavy quark terms can be changed into**

M.A. SHIFMAN et,al. PLB78(1978)

$$m_h \bar{h} h \to -\frac{2}{3} \frac{\alpha}{8\pi} n_h F^2 + O(1/m_h)$$



- **The final expression of trace anomaly is**

$$T^{\mu}_{\mu} = \frac{\tilde{\beta}}{2g} F^2 + \sum_l m_l (1 + \gamma_{ml}) \bar{l} l + O(1/m_h)$$

Outline

① Introduction to trace anomaly of QCD

② **Hadron mass decomposition**

③ Numerical results at large pion mass and physical point

④ Summary and outlook

Mass Decomposition

X. Ji, PRL 74,1071(1995)

- **Hadron energy can be decomposed as**

$$M = -\langle T_{44} \rangle = \langle H_m \rangle + \langle H_E \rangle(\mu) + \langle H_g \rangle(\mu) + \frac{1}{4}\langle H_a \rangle,$$

Quark mass

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

Parton Energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

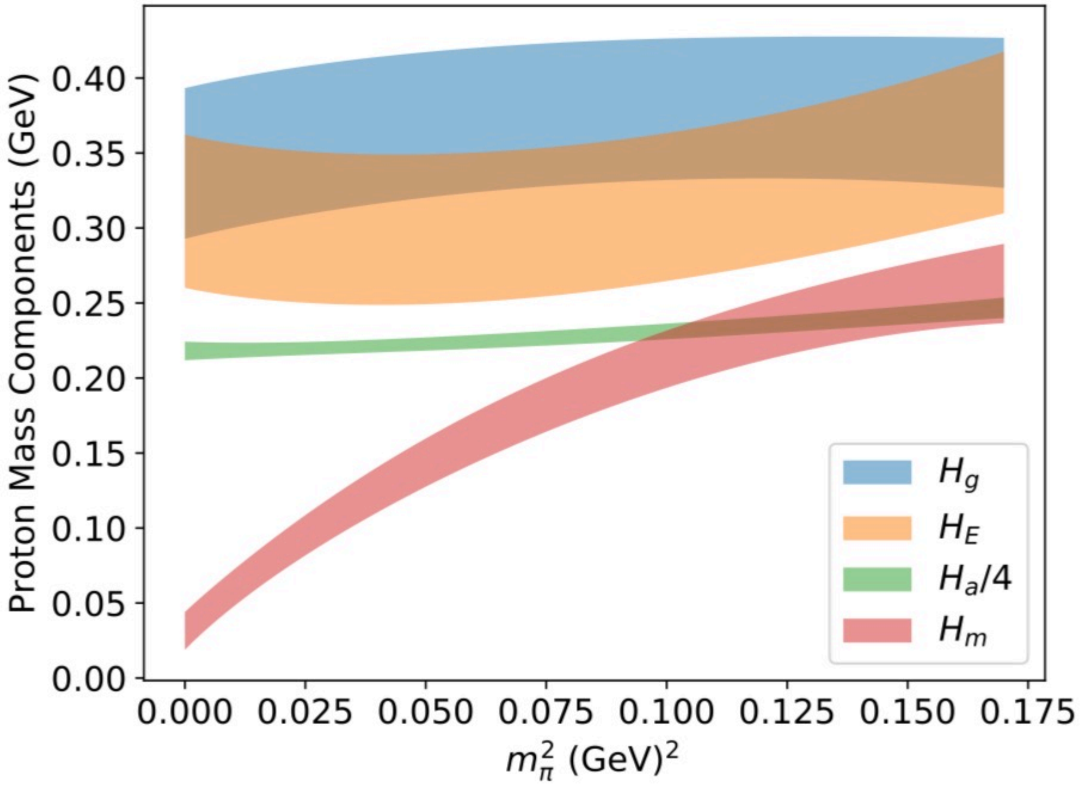
$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2).$$

Trace anomaly

$$H_a = H_g^a + H_m^\gamma,$$

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \gamma_m m\bar{\psi}\psi.$$



Y.B. Yang et,al. (χQCD Collaboration)PRL121(2018)

- **Hadron invariant mass can be decomposed as**

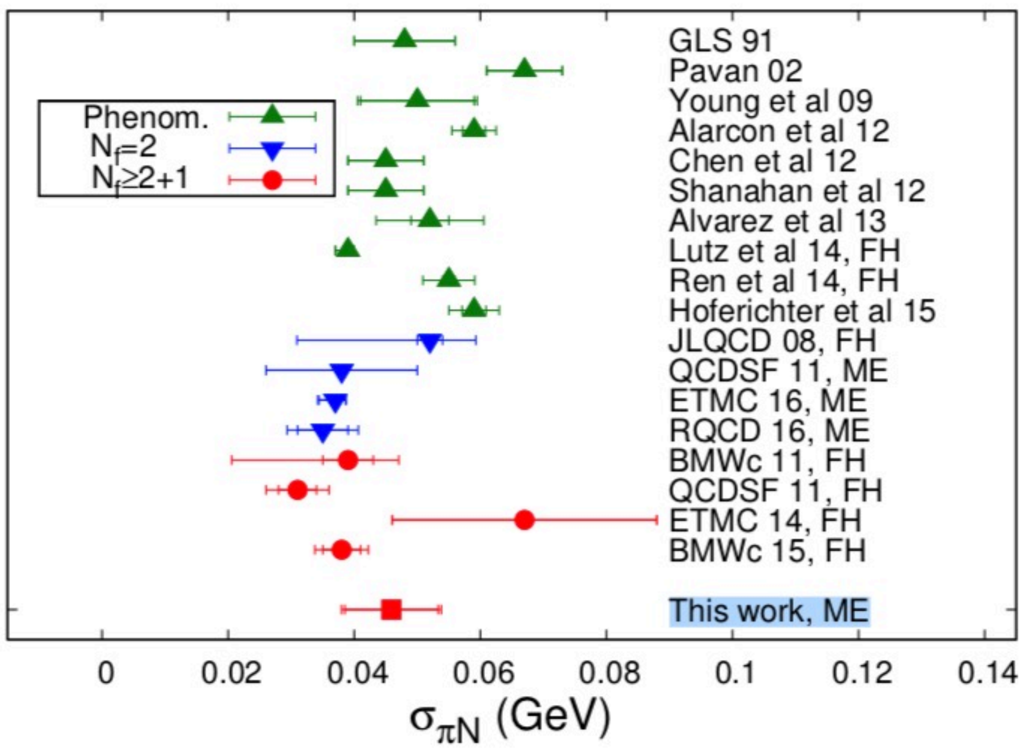
M.A. SHIFMAN et,al. PLB78(1978)

$$M = -\langle \hat{T}_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle.$$

Sigma Term

- **Quark mass contributes to proton mass (sigma terms)**

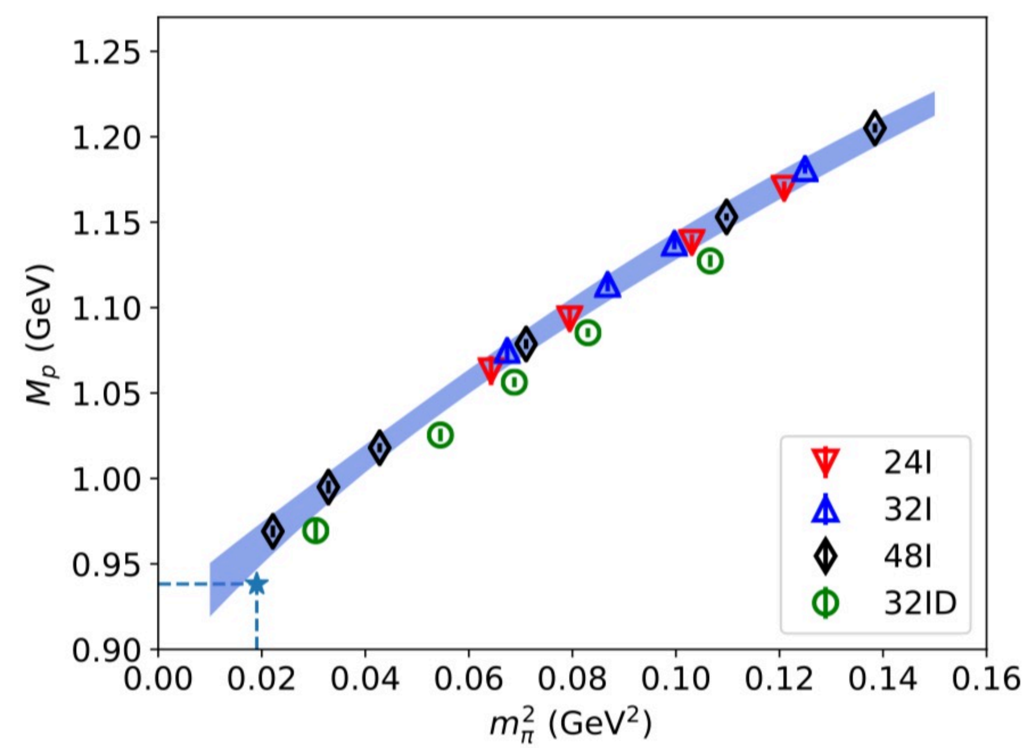
Sigma term can be calculated directly by Lattice qcd



$$H_{m,u+d} = 45.9(7.4)(2.8)MeV$$

Y.B. Yang et,al.(χQCD Collaboration)PRD(2016),054503

Sigma term can be estimated through proton mass derivative to quark mass

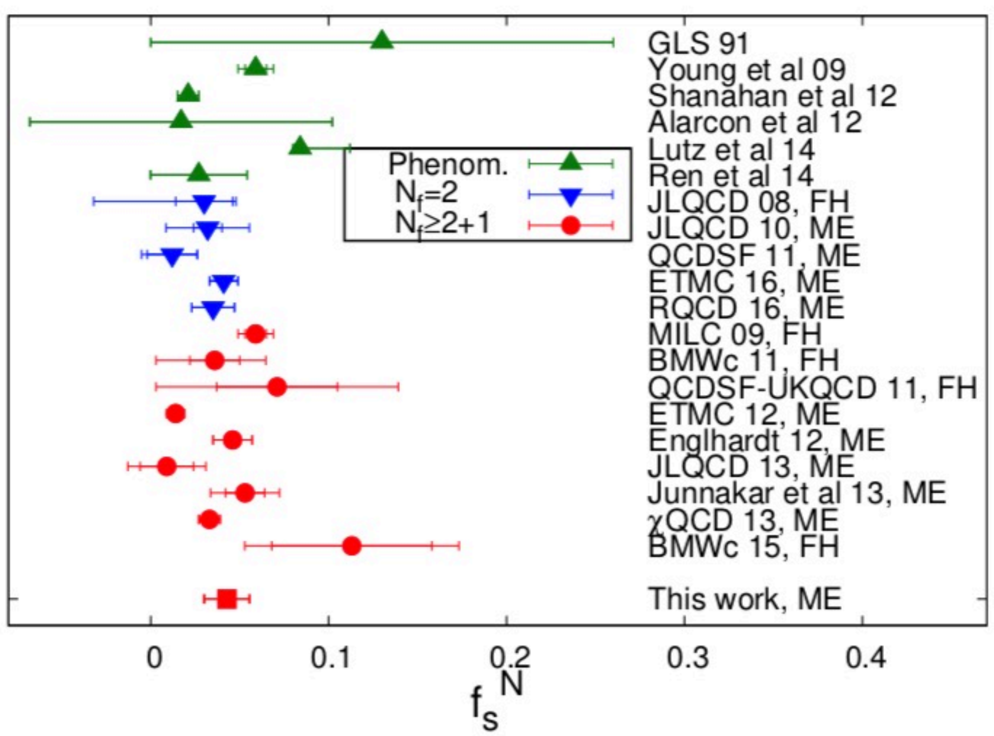


$$H_{m,u+d} \approx \frac{\partial M}{\partial m_\pi}\frac{m_\pi}{2} = 52(8)MeV$$

Y.B. Yang et,al. (χQCD Collaboration)PRL121(2018)

Sigma term of Strange Quark

Y.B. Yang et,al.PRD(2016),054503



$H_{m,s} = 40.2(11.7)(3.5)MeV$

- **The contribution of three light flavors quark condensate to proton mass is about 140-200MeV, According to sum rule:** $M_p = \sum_q < H_{m,q} > + H_a$

Most of proton mass is contribute by trace anomaly !

Outline

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③ Numerical results

Symbol	$L^3 \times T$	a (fm)	$6/g^2$	m_π	m_K	$N_{\rm cfg}$
24I	$24^3 \times 64$	0.1105(3)	2.13	340	593	203

④ **Summary and outlook**

Calculation Procedure

- **To check the mass sum rule** $M_H = < m_q \bar{q} q >_H + \gamma_m < m_q \bar{q} q >_H + \frac{\beta}{2g} < F^2 >_H$

our calculation are divided into the following steps:

① **Calculate the hadron mass and the matrix elements of quark mass, trace anomaly in different hadron, such as pion, rho and nucleon…**

② **Determine the value of γ_m and β. Since their value are unique in different hadron state, we can obtain them by solving the following equation**

$$M_\pi = (1 + \gamma_m) < m_q \bar{q} q >_\pi + \frac{\beta}{2g} < F^2 >_\pi$$
$$M_\rho = (1 + \gamma_m) < m_q \bar{q} q >_\rho + \frac{\beta}{2g} < F^2 >_\rho$$

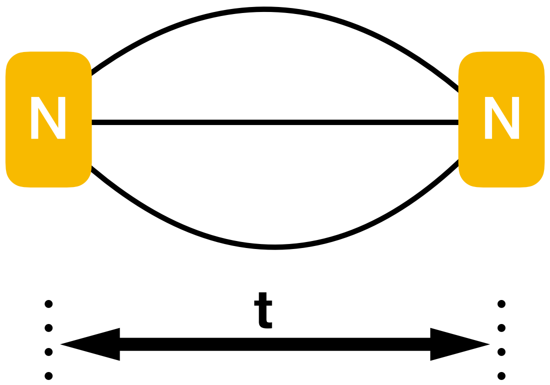
③ **Check the mass sum in different hadron state.**

Effective Mass

- **The effective mass can be extracted through 2pt correlation function**

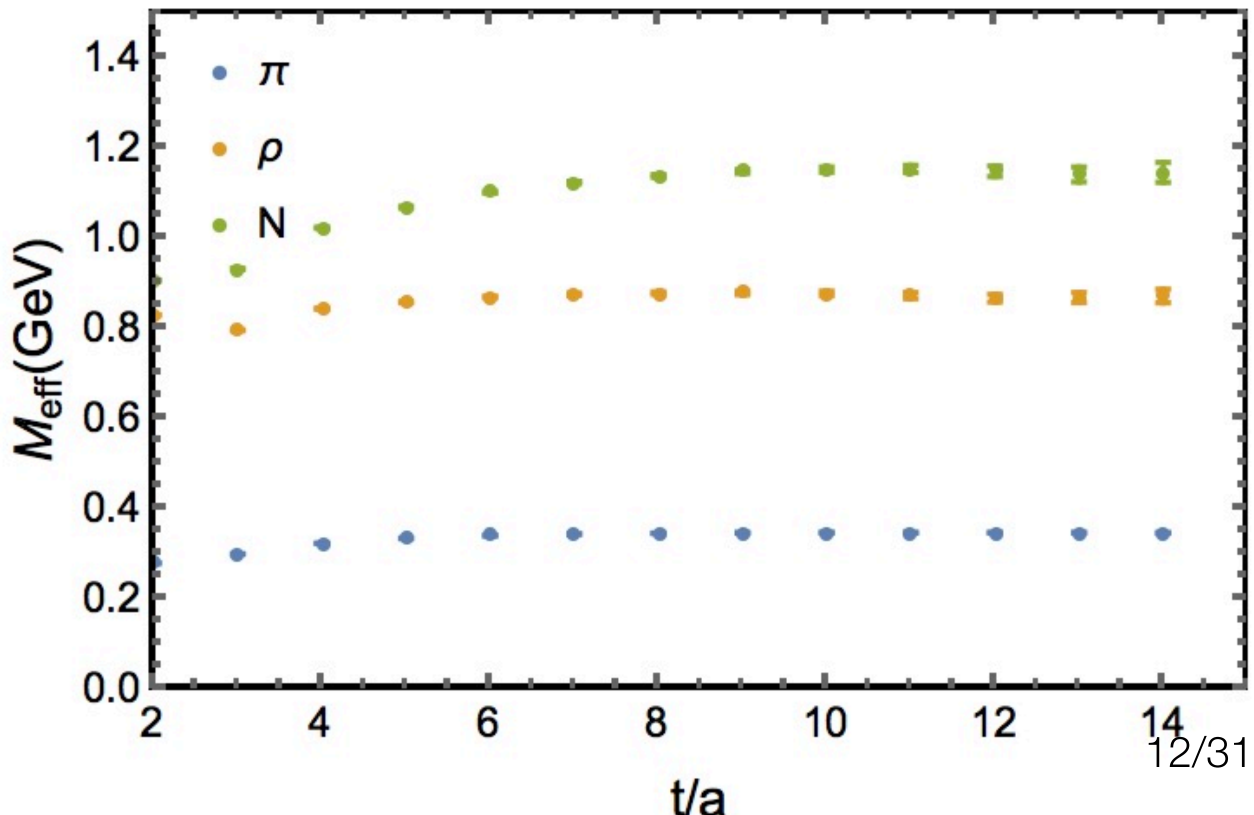
$$M_{eff} = -ln\left(\frac{C_2(t)}{C_2(t-1)}\right)$$

$C_2(t)$ =



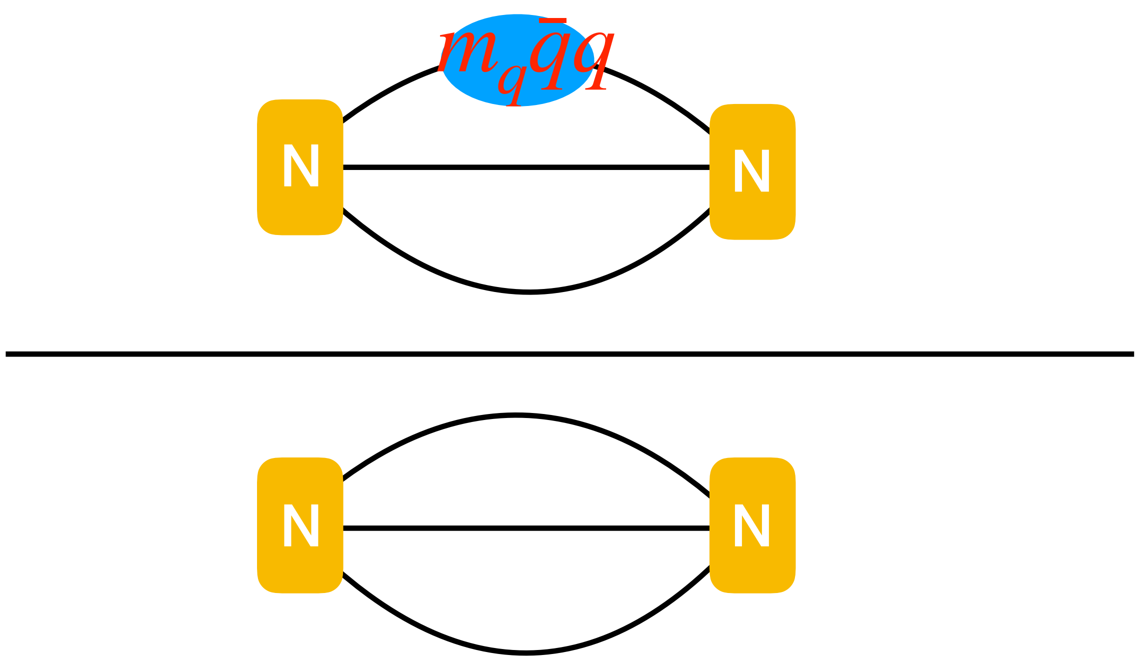
where $C_2(t)$ is two point correlation function

and t is time separation between source and sink



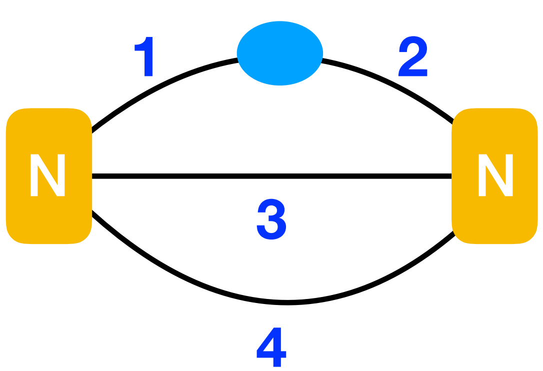
Quark mass term $\langle m_q \bar{q} q \rangle_H$

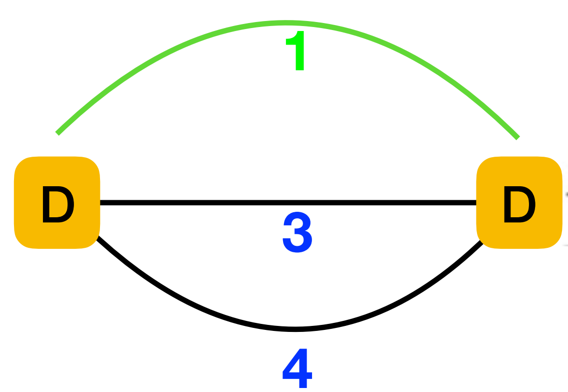
- **The quark condensate can be obtained by the ratio of 3pt to 2pt** (Large t limit)



= $\langle m_q \bar{q} q \rangle_H$

- **Curren sequential method** C.C. Chang et,al. Nature(2018),558

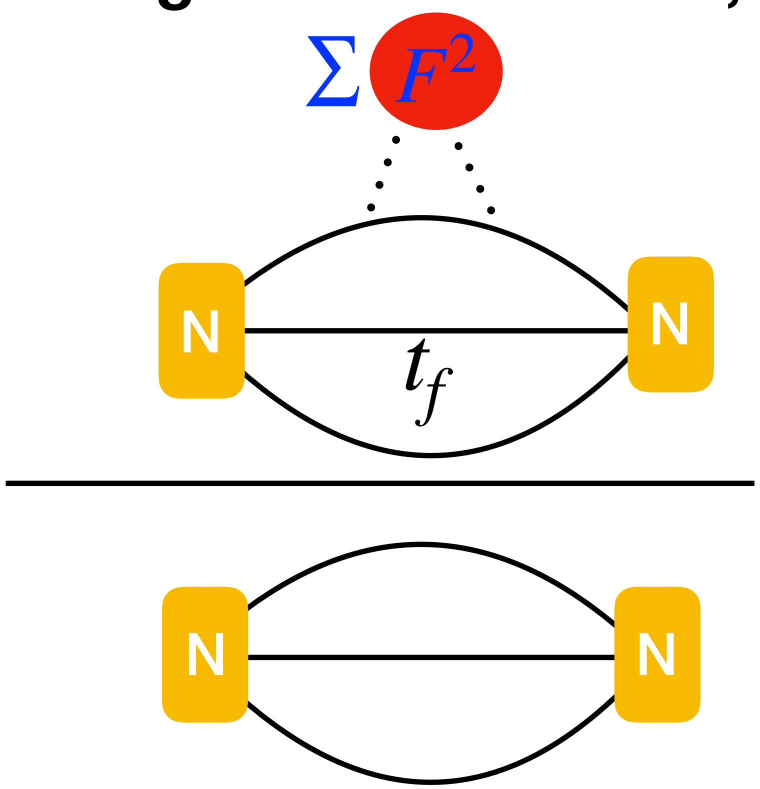




$$R(t_f, O) = \frac{\langle SC_3(t_f, O)\rangle}{\langle C_2(t_f)\rangle} - \frac{\langle SC_3(t_f-1, O)\rangle}{\langle C_2(t_f-1)\rangle} = \langle H|O|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Gluon Condensate $\langle F^2 \rangle_H$

- **For the gluon condensate, the ratio of 3pt to 2pt is defined as**



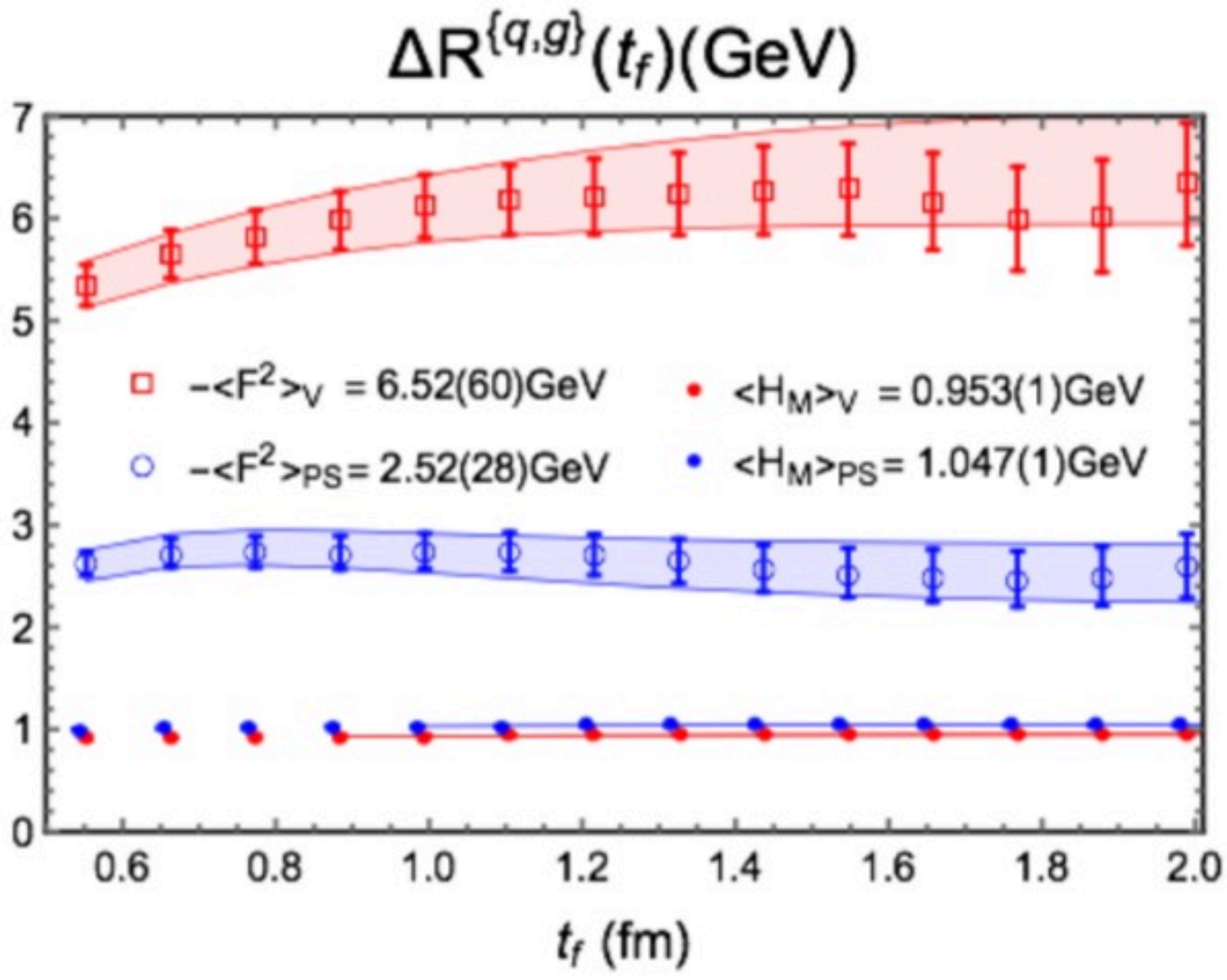
$$= \frac{\langle N(t_f) \sum_{t_f > t > 0} F^2(t) N(0) \rangle - \langle \sum_{t_f > t > 0} F^2(t) \rangle \langle N(t_f) N(0) \rangle}{\langle N(t_f) N(0) \rangle}$$

- **We can extract the matrix element of gluon condensate from 3pt over 2pt ratio**

$$\tilde{R}(t_f, \tilde{O}) = \frac{\sum_{t_f > t > 0} \langle C_3(t_f, t, \tilde{O}) \rangle}{\langle C_2(t_f) \rangle} - \frac{\sum_{t_f - 1 > t > 0} \langle C_3(t_f - 1, \tilde{O}) \rangle}{\langle C_2(t_f - 1) \rangle} = \langle H | \tilde{O} | H \rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Numerical Results

- **Numerical results for quark mass terms and gluon condensate in PS and V meson with** $m_q = 0.54 GeV$



The Values of γ_m and β

- **We can determinate the value of γ_m and β through**

$$M_\pi = (1+\gamma_m) < m_q \bar{q} q >_\pi + \frac{\beta}{2g^3} < g^2 F^2 >_\pi$$
$$M_V = (1+\gamma_m) < m_q \bar{q} q >_V + \frac{\beta}{2g^3} < g^2 F^2 >_V$$

Define $\beta' = \frac{\beta}{2g^3}$

Our result

$\gamma_m \approx 0.38(3)$

$\beta' \approx -0.028(3)$

4-loop result($\bar{MS}$ 1.785GeV)

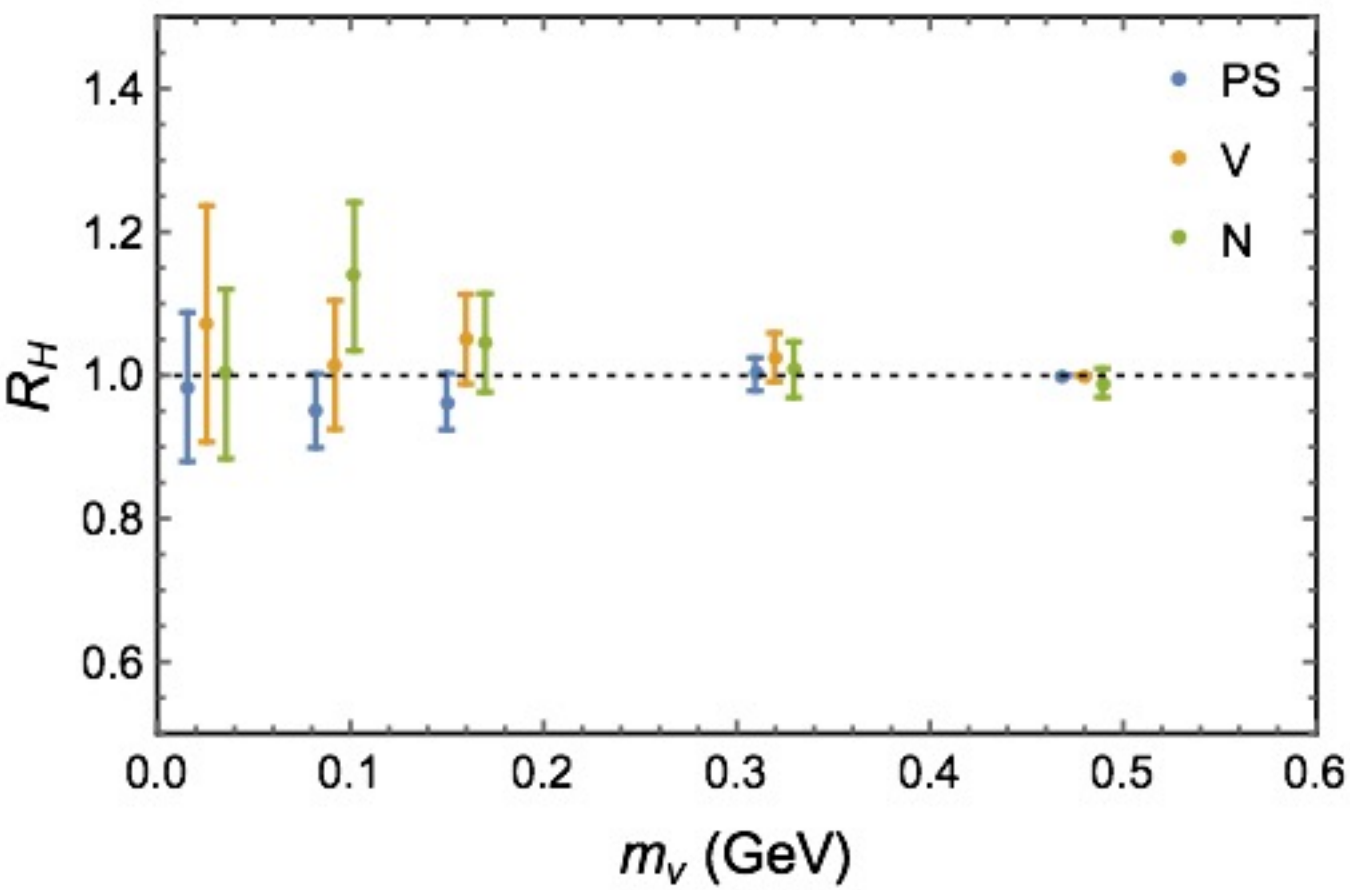
$\gamma_m(1/a) \approx 0.325(10)$

$\beta'(1/a) \approx -0.036(1)$

J.Vermaseren. PLB,405(1997) 327

Numerical Results

- **Verify Sum rules:** $M_H = < H_m >_H + \gamma_m < H_m >_H + \frac{\beta}{2g} < F^2 >_H$

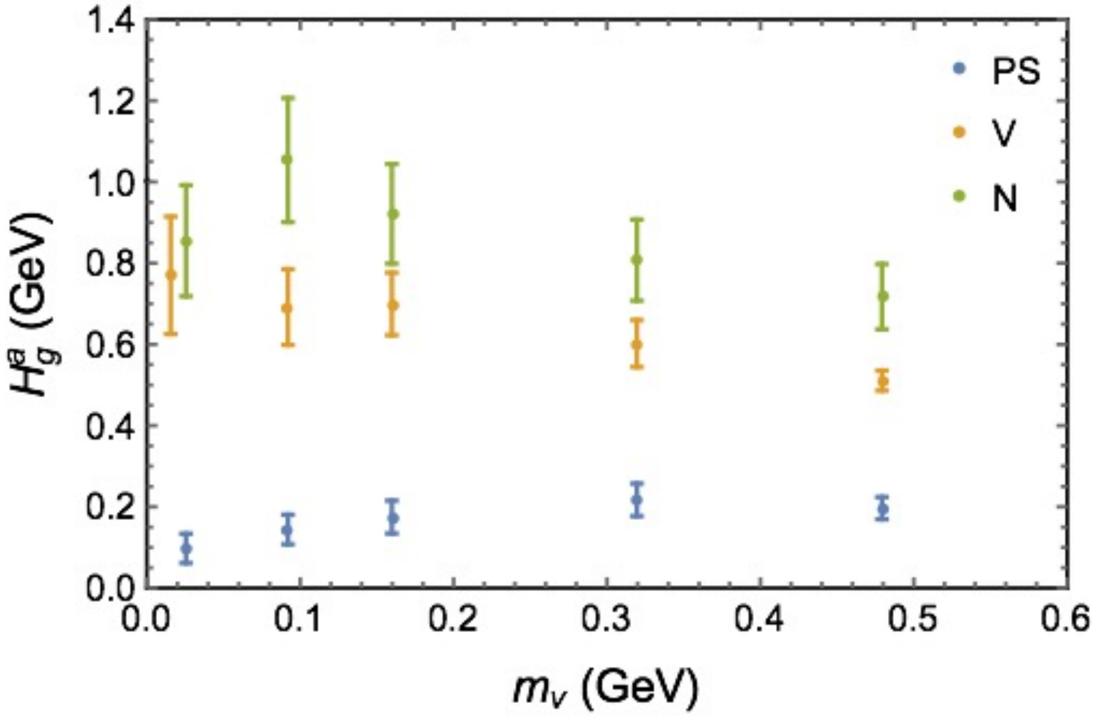


$$R_H = \frac{(1+\gamma_m) < H_m >_H + \frac{\beta}{2g} < F^2 >_H}{M_H}$$

We checked the trace anomaly sum rule. The ratio of sum rules to hadron mass is plotted, We can see that the R_H all the cases are consistent with one within the uncertainties.

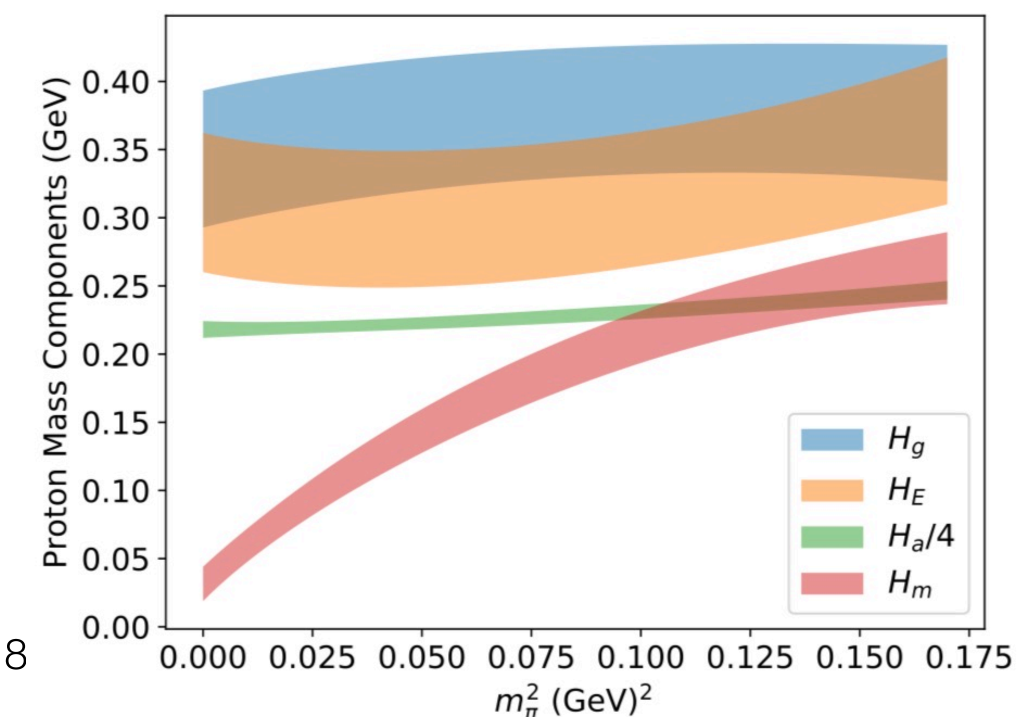
Numerical Results

The contribution from gluon($H_g^a = \frac{\beta}{2g} < F^2 >$)



$$H^a = \gamma_m \langle H_M \rangle_H + \langle H_g \rangle_H$$

The gluon trace anomaly contribution in the PS meson mass is always much smaller than that in the other hadrons, especially around the chiral limit.



Summary and Outlook

- **Summary**

① **We calculate the quark condensate and trace anomaly in different hadron, we also check the mass sum rule, the hadron mass obtained from sum rule is consistent with its ground state mass.**

② **We find the trace anomaly contribute most of the hadron masss, except the pion case.**

Thanks for your attention!

Backup

Energy Momentum Tensor of QCD

- **The energy momentum tensor of QCD:**

$$T^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} \psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}{}_{\alpha},$$

- $T^{\mu\nu}$ **can be decomposition into two parts:**

$$T^{\mu\nu} = \underbrace{\frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} \psi - \frac{1}{4} g^{\mu\nu} m \bar{\psi}\psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_{\alpha}}_{\text{Traceless}} + \underbrace{\frac{1}{4} g^{\mu\nu} m \bar{\psi}\psi}_{\text{Trace term}}$$

The relation of hardon mass and trace of EMT

$$M_H = < T^{\mu}_{\mu} >_H = \sum_l m_l < \bar{l} l >_H + \sum_h m_h < \bar{h} h >_H \quad ?$$

- **Puzzles:**
  1. **Does heavy quark dominate contribute to hadron mass?**
  2. **All hadron mass will become zero at chiral limit**

Overlap Fermion

- **Clover Fermion will introduce extra term to energy momentum tensor of QCD**

$$\left(D_w + c_{sw}\sigma^{\mu\nu}F_{\mu\nu} + (m_c + m_q)\right)\psi = 0$$

- **The definition of Overlap operator**

$$D_{ov} = \rho(1 + \gamma_5\epsilon_{ov}(\gamma_5 D_w))$$

Where $\epsilon_{ov}(\gamma_5 D_w)$ is the sign function of Wilson operator D_w

- **Overlap operator satisfies Ginsparg-Wilson Relation**

$$D_{ov}\gamma_5 + \gamma_5 D_{ov} = \frac{1}{\rho} D_{ov}\gamma_5 D_{ov}$$

Anomalous Breaking of Scale Invariance

- **The coupling constant and quark mass will vary with scale**

$$\frac{d\log(m)}{d\log(\mu)} = -\gamma_m$$

$$\mu\frac{dg}{d\mu} = \beta$$

$$\xrightarrow{\mu \to \mu + \sigma\mu}$$

$$g \to g + \sigma\beta(g)$$

$$m \to m - \sigma m\gamma_m$$

- **Corresponding change in the Lagrangian is**

$$\sigma\beta(g)\frac{\partial L}{\partial g} - \sigma\gamma_m m\frac{\partial L}{\partial m}$$

- **Trace anomaly can be obtained**

$$\sigma\partial_\mu D^\mu = \sigma T^\mu_\mu = \sigma\beta(g)\frac{\partial L}{\partial g} - \sigma\gamma_m m\frac{\partial L}{\partial m} \longrightarrow T^\mu_\mu = \frac{\beta}{2g}F^2 + m\gamma_m\bar{\psi}\psi$$

Trace anomaly in perturbation theory

- The trace of ETM in d dimension

$$T^{\alpha}_{\alpha} = -2\epsilon \frac{F^2}{4} + \bar{\psi} i \overleftrightarrow{\not{D}} \psi = -2\epsilon \frac{F^2}{4} + m\bar{\psi}\psi.$$

In d dimension

- Renormalization of FF.

$$F^2 = \left(1 - \frac{\beta}{g}\frac{1}{\epsilon}\right)(F^2)_R - 2\frac{\gamma_m}{\epsilon}(m\bar{\psi}\psi)_R$$

The bare operator FF is divergent

$$T^{\mu}_{\mu} = -2\epsilon \frac{F^2}{4} + m\bar{q}q$$
$$= \underbrace{\frac{\beta(g)}{2g}(F^2)_R + \gamma_m (m\bar{q}q)_R}_{\text{from } (T_g)^{\mu}_{\mu}} + \underbrace{(m\bar{q}q)_R}_{\text{from } (T_q)^{\mu}_{\mu}}$$

For the bare ETM, the anomaly entirely comes from the gluon part