Excited Spectra of Singly Heavy Baryon

单重味重子的激发谱

贾多杰

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With

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提要

- Singly Heavy hadron: Spectra
- P-wave masses of SH baryons: Mass scaling
- A mixing Rep. of P-wave SH Baryons
- A QM explanation of Mass splittings
- Summary

Singly Heavy Hadron :Spectra

单重味粲介子:D= $c\bar{q}$, q=u,d,s

J=½+½+L={0,1}+L

PDG有60多个已发现的粲及粲奇异介子候选粒子

H.X Chen, W.Chen, X. Liu, Y.R Liu, S-L Zhu, Rept.Prog. Phys. 80, 076201 (2017)

• P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$D^+ = c\,\bar{d}$, $D^0 = c\,\bar{u}$, $\overline{D}^0 = \bar{c}\,u$, $D^- = \bar{c}\,d$, similarly for D^*'s

See related reviews:

Review of Multibody Charm Analyses

$D^0 - \overline{D}^0$ Mixing

·已确立

•	$D^{\pm}$	$1/2(0^-)$
•	D^0	$1/2(0^-)$
•	$D^*(2007)^0$	$1/2(1^-)$
•	$D^*(2010)^{\pm}$	$1/2(1^-)$
•	$D_0^*(2300)^0$ was $D_0^*(2400)^0$	$1/2(0^+)$
	$D_0^*(2300)^{\pm}$ was $D_0^*(2400)^{\pm}$	$1/2(0^+)$
•	$D_1(2420)^0$	$1/2(1^+)$
	$D_1(2420)^{\pm}$	$1/2(?^?)$
	$D_1(2430)^0$	$1/2(1^+)$
•	$D_2^*(2460)^0$	$1/2(2^+)$
•	$D_2^*(2460)^{\pm}$	$1/2(2^+)$
	$D(2550)^0$	$1/2(?^?)$
	$D_J^*(2600)$ was $D(2600)$	$1/2(?^?)$
	$D^*(2640)^{\pm}$	$1/2(?^?)$
	$D(2740)^0$	$1/2(?^?)$
	$D_3^*(2750)$	$1/2(3^-)$

CHARMED, STRANGE MESONS ($C = S = \pm1$)

$D_s^+ = c\,\bar{s}$, $D_s^- = \bar{c}\,s$, similarly for D_s^*'s

See related reviews:

D_s^+ Branching Fractions

Leptonic Decays of Charged Pseudoscalar Mesons

•	$D_s^{\pm}$	$0(0^-)$
•	$D_s^{*\pm}$	$0(?^?)$
•	$D_{s0}^*(2317)^{\pm}$	$0(0^+)$
•	$D_{s1}(2460)^{\pm}$	$0(1^+)$
•	$D_{s1}(2536)^{\pm}$	$0(1^+)$
•	$D_{s2}^*(2573)$	$0(2^+)$
•	$D_{s1}^*(2700)^{\pm}$	$0(1^-)$
	$D_{s1}^*(2860)^{\pm}$	$0(1^-)$
	$D_{s3}^*(2860)^{\pm}$	$0(3^-)$
	$D_{sJ}(3040)^{\pm}$	$0(?^?)$

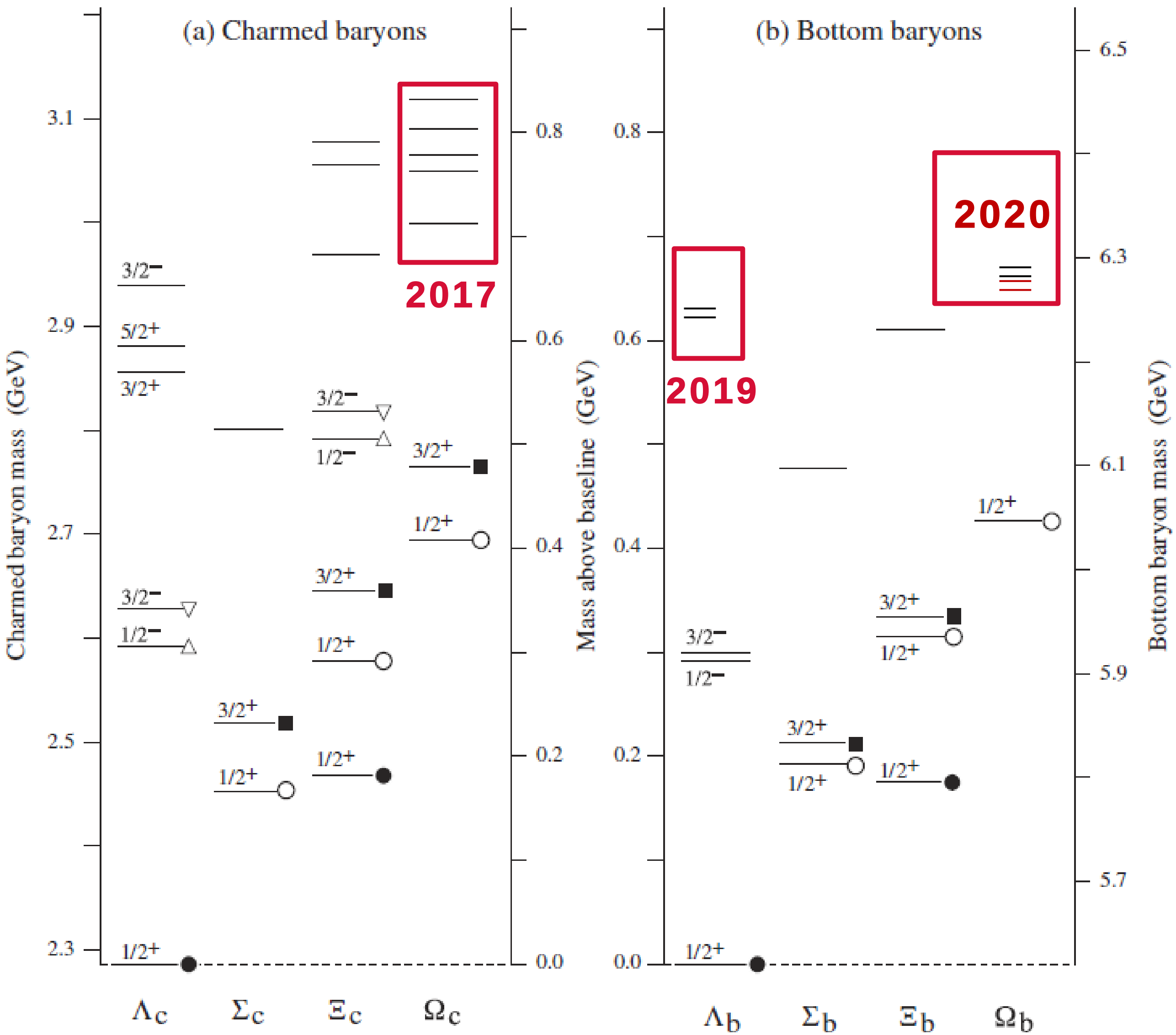
单重味底奇异介子:B= $b\overline{n}$

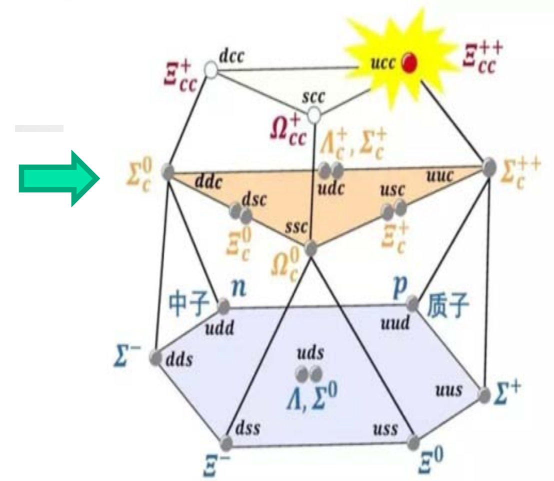
- P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

• $B^{\pm}$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^{\pm}/B^0$ ADMIXTURE	
• $B^{\pm}/B^0/B^0_s/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B^*_J(5732)$ aka B^{**}	$?(?^?)$
• $B^*_2(5747)^+$	$1/2(2^+)$
• $B^*_2(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

单重粲及底味重子: Baryon= c q q, bqq   q=u, d, s

- P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)





$J = ½ + ½ + ½ + L$

$= ½ + \{0,1\} + L$

$= ½ + 3/2 + 5/2 + \ldots$

单重粲味重子：cnn, cns n=u,d

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CHARMED BARYONS (C = +1)

$\Lambda_c^+ = udc$, $\Sigma_c^{++} = uuc$, $\Sigma_c^+ = udc$, $\Sigma_c^0 = ddc$,
$\Xi_c^+ = usc$, $\Xi_c^0 = dsc$, $\Omega_c^0 = ssc$

See related review:
Charmed Baryons

PDG有40多个已发现的单粲味重子候选者

**** Existence is certain, and properties are at least fairly explored.

*** Existence ranges from very likely to certain, but further confirmation is desirable and/or J^P, branching fractions, etc. are not well determined.

** Fair evidence

* Poor evidence

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$ or $\Sigma_c(2765)$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0		
$\Xi_c^{\prime+}$		
$\Xi_c^{\prime0}$		
$\Xi_c(2645)$		
$\Xi_c(2790)$		
$\Xi_c(2815)$		
$\Xi_c(2930)$		
$\Xi_c(2970)$		

Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***

单重底味重子：*bnn*, *bns*　n=u,d

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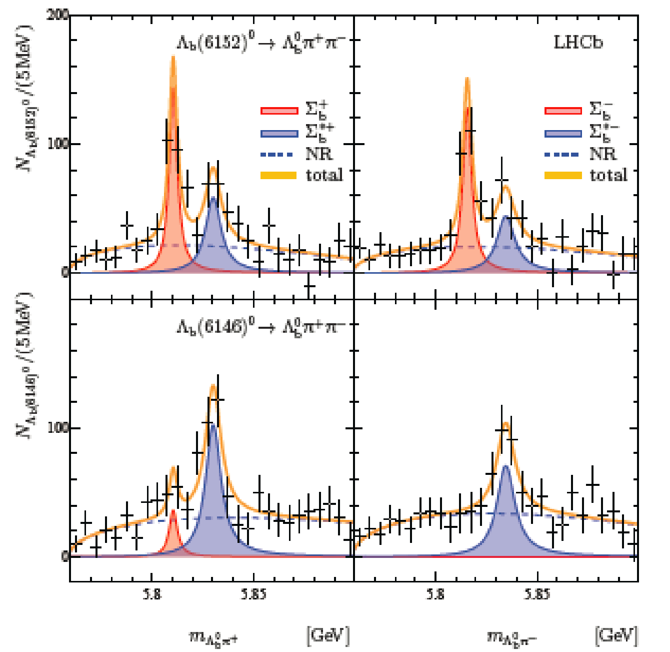
BOTTOM BARYONS ($B = -1$)

PDG有近20多个发现的底重子候选者

$\Lambda_b^0 = udb$, $\Xi_b^0 = usb$, $\Xi_b^- = dsb$, $\Omega_b^- = ssb$

Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6227)$		***
Ω_b^-	$1/2^+$	***



$\Lambda_b(6146,6152)$: 2019

b-baryon ADMIXTURE (Λ_b, Ξ_b, Ω_b)

*** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers fractions, etc. are not well determined.

P-Wave Spectra of SH Baryons: Mass scaling

P-wave SH 重子分类和自旋态劈裂

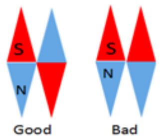
The spin configuration of diquark leads to two type of diquark: scalor(spin=0) or vector(spin=1),

- In the HQS limit, the spin states of the quark-diquark system can be classified by that of diquark

For 1P excitations, one has

$$(qq)_d = \begin{cases} [ud], & I = 0 = \text{spin,in } \Lambda_Q, \\ \{uu, ud, dd\}, & I = 1 = \text{spin,in } \Sigma_Q. \end{cases}$$

$$(qs)_d = \begin{cases} [us, ds], & I = 1/2, \text{spin} = 0, \text{ in } \Xi_Q, \\ \{us, ds\}, & I = 1/2, \text{spin} = 1, \text{ in } \Xi'_Q. \end{cases}$$

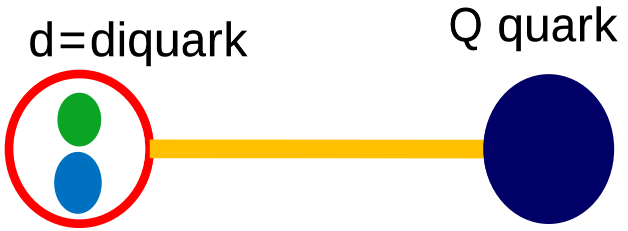


Good(scalor) diquark: spin=0
Bad(vector): spin=1

$$\left[0_d \otimes \left(\tfrac{1}{2}\right)_Q\right]_S \otimes 1_L = \frac{1}{2} \oplus \frac{3}{2}$$

$1^2P_{1/2}$	$1/2^-$
$1^2P_{3/2}$	$3/2^-$

Λ_Q, Ξ_Q (J=1/2,3/2)



$$\left[1_d \otimes \left(\tfrac{1}{2}\right)_Q\right]_S \otimes 1_L = \left[\frac{1}{2}' \oplus \frac{3}{2}\right]_S \otimes 1_L,$$

$$= \frac{1}{2} \oplus \frac{3}{2} \oplus \frac{1}{2}' \oplus \frac{3}{2}' \oplus \frac{5}{2},$$

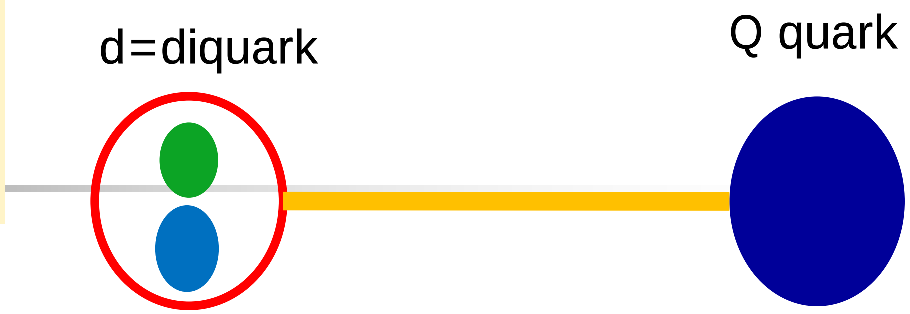
$(1P, 1/2)$
$(1P, 3/2)$
$(1P, 1/2')$
$(1P, 3/2')$
$(1P, 5/2)$

Σ_Q , Ξ'_Q(J): J=1/2, 3/2,1/2',3/2',5/2

Spin-dependent质量：P-wave splittings

Depending diquark(spin=0,1) and the strangeness, its mass m_d varys, due to different interactions.

In the quark-diquark picture



$$H = H^{Spin-indep.} + H^{SD}$$

$$(\bar{M} - M_Q)^2 = \pi a L + \left[m_d + M_Q\left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)\right]^2$$

$$H^{SD} = a_1 \mathbf{L}\cdot\mathbf{S}_d + a_2 \mathbf{L}\cdot\mathbf{S}_Q + b S_{12} + c\,\mathbf{S}_d\cdot\mathbf{S}_Q,$$

$$S_{12} = 3\mathbf{S}_d\cdot\hat{\mathbf{r}}\,\mathbf{S}_Q\cdot\hat{\mathbf{r}} - \mathbf{S}_d\cdot\mathbf{S}_Q,$$

(i) The spin-coupling parameter a_1 should be positive but smaller than $\Delta E(1P)$

(ii) a_2 is of same order with a_1 as a2/a1 scales as mss/Mc.

(iii) b should be smaller than a_1 and a_2 as b scales like $1/(m_d M_c)$.

(iv) c should be smallest, less than b as it scales as P-wave wave function near the origin.

D. Ebert, R. N. Faustov and V. O. Galkin Phys. Rev. D 84(2011)014025,arXiv:1105.0583
M. Karliner and J. L. Rosner, Phys. Rev. D 92, 074026 (2015):arXiv:1506.01702

激发态强子确定夸克有效质量

单重味重子---Regge Fit

Parameters	M_c	$m_d([nn])$	$a(\Lambda_c)$	$m_d([ns])$	$a(\Xi_c)$	$\bar{M}(\Lambda_c)$	$\bar{M}(\Xi_c)$
This work	1.44	0.535	0.212	0.718	0.255	2.618	2.804
EFG[17]	1.55	0.710	0.18	0.948	0.18	2.617	2.808

Parameters	M_c	$m_d(\{nn\})$	$a(\Sigma_c)$	$m_d(\{ns\})$	$a(\Xi'_c)$	$\bar{M}(\Sigma_c)$	$\bar{M}(\Xi'_c)$
This work	1.44 [input]	0.745	0.212	0.872	0.255	2.774	2.923
EFG[17]	1.55	0.99	0.18	1.069	0.18	2.780	2.919

TABLE VI. The masses (GeV) of the bottom quark and diquarks, and the tension (GeV^2) that match the measured spin-averaged masses of the Λ_b and the Ξ_b in Table I and the Σ_b and the Ξ'_b in Table II. Here, the RMS error $\chi_{\rm RMS} = 0.001$ GeV. The comparison with that by quark model is given.

Parameters	M_b	$m_d([nn])$	$a(\Lambda_b)$	$m_d([ns])$	$a(\Xi_b)$	$\bar{M}(\Lambda_b)$	$\bar{M}(\Xi_b)$
This work	4.48	0.534	0.246	0.718	0.307	5.917	6.125
EFG [17]	4.88	0.710	0.18	0.948	0.18	5.938	6.127

Parameters	M_b	$m_d(\{nn\})$	$a(\Sigma_b)$	$m_d(\{ns\})$	$a(\Xi'_b)$	$\bar{M}(\Sigma_b)$	$\bar{M}(\Xi'_b)$
This work	4.48 [input]	0.745	0.246	0.869	0.307	6.088	6.248
EFG [17]	4.88	0.909	0.18	1.069	0.18	6.090	6.228

D/Ds介子---Regge Fit

TABLE IX. The effective masses (in GeV) of quarks that match the observed spin-averaged masses in Table VII and VIII, with a in GeV and the RMS error $\chi_{\rm RMS} = 0.001$ GeV. The comparison with that by quark model is given.

Parameters	M_c	M_b	m_n	m_s	$a(c\bar{n})$	$a(c\bar{s})$	$a(b\bar{n})$	$a(b\bar{s})$
This work	1.44 [input]	4.48 [input]	0.23	0.328	0.223	0.249	0.275	0.313
EFG [34]	1.55	4.88	0.33	0.5	0.64/0.58	0.68/0.64	1.25/1.21	1.28/1.23

Spin-dependent Mass: Scaling relations

We use the **scaling relation** based on the **color-interaction similarity** between a singly heavy meson and a SH baryon, or partners between SH baryons.

Scaling relation(质量标度关系).

SH baryons. HL meson.

$$a_1(Qqq)m_d(qq) = a_1(Q\bar{s})m_s,$$

$$a_1(b)=a_1(c),\ a_2(b)=a_2(c)\left(\frac{M_c}{M_b}\right),\ b(b)=b(c)\left(\frac{M_c}{M_b}\right),$$

Σ_c^{++}	:	$a_1 = \left(\frac{m_s}{m_d([qq])}\right)a_1(c\bar{s}) = \left(\frac{328\text{MeV}}{745\text{MeV}}\right)89.4\text{MeV}^2 = 39.4\text{MeV}^2,$
$\Xi_c^{0\prime}$	:	$a_1 = \left(\frac{m_s}{m_d([qs])}\right)a_1(c\bar{s}) = \left(\frac{328\text{MeV}}{872\text{MeV}}\right)89.4\text{MeV}^2 = 33.6\text{MeV}^2,$

where m_s=328MeV is given by fitting the D_s/B_s mesons via the linear RT relation with the diquark mass replaced by m_s and $a_1(Ds)=89.4\text{MeV}^2$ is determined by matching the 1P-wave Ds mass spilitting with the diquark d replaced by the strange quark s

M. Karliner and J. L. Rosner, Phys. Rev. D 92, 074026 (2015):arXiv:1506.01702

Spin-dependent Mass: jj representation

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- When M_Q is very large, the interactions except for the $\mathbf{L}\cdot\mathbf{S}_d$ term suppressed by $1/M_Q$, and one can use $\mathbf{L}\cdot\mathbf{S}_d$ rep. (jj- rep.), with the other interactions treated perturbatively.

- Notice that $\langle \mathbf{L}\cdot\mathbf{S}_d \rangle$ =[j(j+1)-L(L+1)-Sd}(Sd+1)]/={-2, -1, 1} when j=0, 1,2, respectively. The mass splittings becomes

j-j coupling (transforming jj to LS coupling)

$$\Delta M(J,j) = \langle J,j|H^{SD}|J,j\rangle$$

(J,j)	$\langle \mathbf{L}\cdot\mathbf{S}_Q\rangle$	$\langle \mathbf{S}_{12}\rangle$	$\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle$
$(1/2, 0)$	0	-1	0
$(1/2, 1)$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$(3/2, 1)$	$\frac{1}{4}$	1	$\frac{1}{4}$
$(3/2, 2)$	$-\frac{3}{4}$	$\frac{1}{5}$	$-\frac{3}{4}$
$(5/2, 2)$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$

→

$$M^{SD}(1/2,0) = -2a_1 - b,$$
$$M^{SD}(1/2,1) = -a_1 - \frac{1}{2}(a_2 + c) - \frac{1}{2}b,$$
$$M^{SD}(3/2,1) = -a_1 + \frac{1}{4}(a_2 + c) + b,$$
$$M^{SD}(3/2,2) = a_1 - \frac{3}{4}(a_2 + c) + \frac{1}{5}b,$$
$$M^{SD}(5/2,2) = a_1 + \frac{1}{2}(a_2 + c) - \frac{3}{10}b.$$

M. Karliner and J. L. Rosner, Phys. Rev. D 92, 074026 (2015):arXiv:1506.01702
Duojie Jia, W-N Liu, A. Hosaka, PRD101, 034016 (2020);

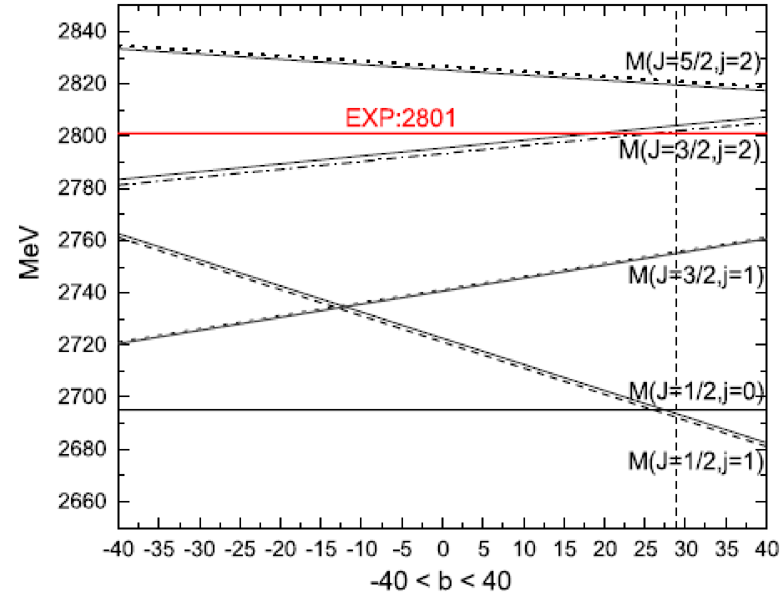
Mass splittings: 1P wave

Putting all togather and ignoring the spin-spin c-term in the P-wave, one obtains

$$M = \bar{M}(L = 1) + M^{SD}$$

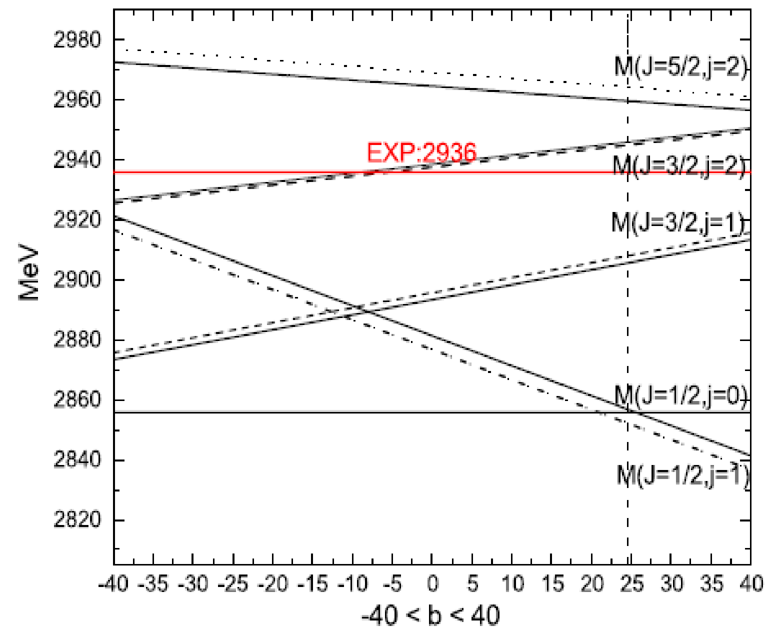
$$M(1/2,0) = 2694.74 - b,$$
$$M(1/2,1) = 2722.64 - \frac{1}{2}b,$$
$$M(3/2,1) = 2740.37 + b,$$
$$M(3/2,2) = 2795.63 + \frac{1}{5}b,$$
$$M(5/2,2) = 2825.63 - \frac{3}{10}b.$$

For Σ_c^{++}



$$M(1/2,0) = 2855.76 - b,$$
$$M(1/2,1) = 2881.38 - \frac{1}{2}b,$$
$$M(3/2,1) = 2893.38 + b,$$
$$M(3/2,2) = 2938.62 + \frac{1}{5}b,$$
$$M(5/2,2) = 2964.62 - \frac{3}{10}b,$$

For $\Xi_c'^0$



$|b| \le (3/5)|a_1| \simeq$ 24MeV(Σ_c), 16MeV(Ξ_c')

b = (QCD/M_Q) a_2= (1/5) · 39MeV ≃ 8MeV

P-wave Mass splitings: Bottom sector

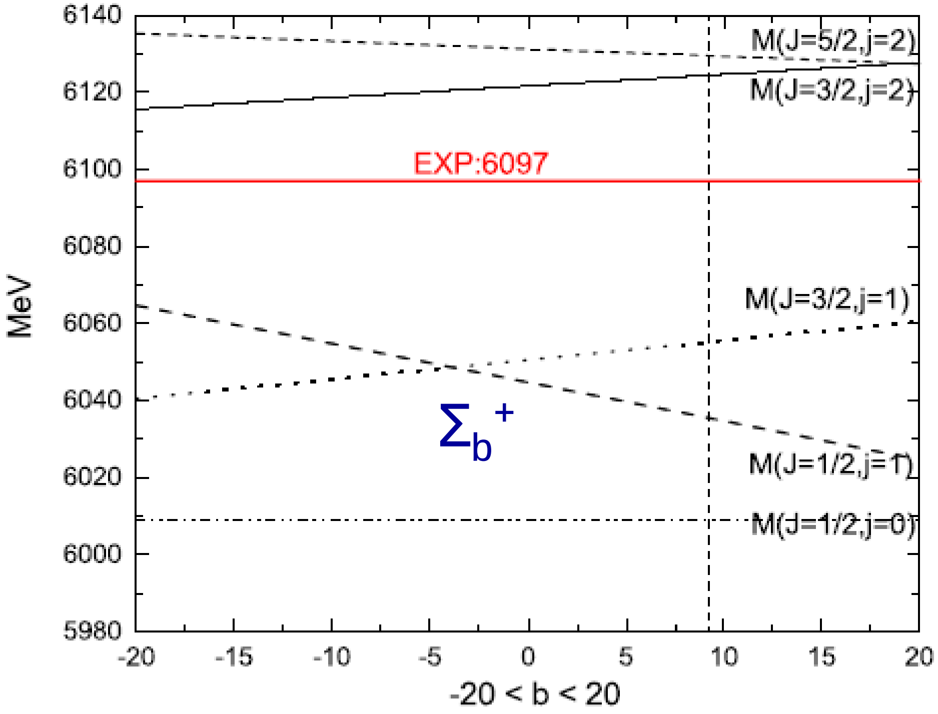


FIG. 3. The masses of the P-wave Σ_b against the parameter b.

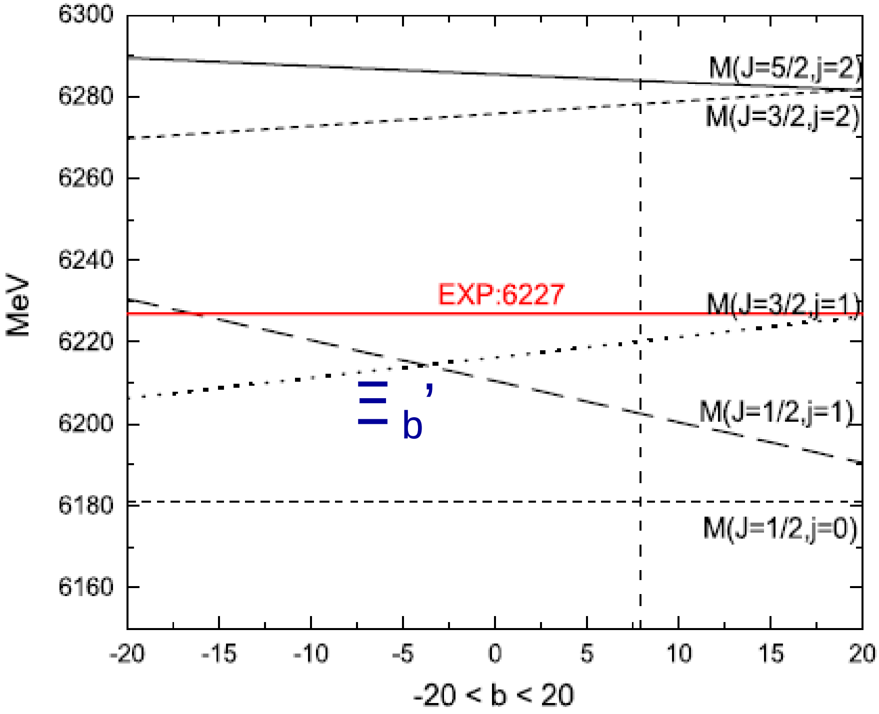


FIG. 4. The masses of the P-wave Ξ_b' against the parameter b.

Spin-coupling parameters: scaling

Employ same procedure to the singly bottom baryons, one can find

Σ_b :	$a_1 = \left(\frac{m_s}{m_d([qq])}\right) a_1(b\bar{s}) = \left(\frac{328\text{MeV}}{745\text{MeV}}\right) 89.4MeV = 39.4\text{MeV}^2,$
Ξ_b' :	$a_1 = \left(\frac{m_s}{m_d([qs])}\right) a_1(b\bar{s}) = \left(\frac{328\text{MeV}}{869\text{MeV}}\right) 89.4MeV = 33.6\text{MeV}^2.$

With $a_1(b\bar{s})=89.4\text{MeV}^2$ determined by matching the 1P-wave Bs mass spilitting given by H^{SD}, with m_d replaced by m_s.

$$a_2(\Sigma_b) = \frac{2}{3}[M(\Lambda_b^0, 3/2^-) - M(\Lambda_b^0, 1/2^-)] = 5.3\text{MeV}$$

$$a_2(\Xi_b') = \frac{2}{3}[M(\Xi_b^-, 3/2^-) - M(\Xi_b^-, 1/2^-)] = 6.7\text{MeV}.$$

Σ_b

$M(1/2, 0) = 6009 - b,$
$M(1/2, 1) = 6046 - \frac{1}{2} b,$
$M(3/2, 1) = 6050 + b,$
$M(3/2, 2) = 6124 + \frac{1}{5} b,$
$M(5/2, 2) = 6130 - \frac{3}{10} b.$

Ξ_b'

$M(1/2, 0) = 6181 - b$
$M(1/2, 1) = 6211 - \frac{1}{2} b,$
$M(3/2, 1) = 6216 + b,$
$M(3/2, 2) = 6277 + \frac{1}{5} b,$
$M(5/2, 2) = 6285 - \frac{3}{10} b.$

$$|b| \leq |a_1|/6 \simeq 6\text{MeV},$$

6130 - 6120 ←-RQM predctions

重味重子质量：低激发态预言

States, J^P	Baryon	Mass	This work	Baryon	Mass	This work
$1^2S_{1/2}, 1/2^+$	Λ_c^+	2286.46(14)	2286.0	Ξ_c^+	2467.87(30)	2469.1
$1^2P_{1/2}, 1/2^-$	$\Lambda_c(2595)^+$	2592.25(28)	2588.7	$\Xi_c(2790)^+$	2792.0(5)	2778.6
$1^2P_{3/2}, 3/2^-$	$\Lambda_c(2625)^+$	2628.11(19)	2628.9	$\Xi_c(2815)^+$	2816.67(31)	2816.5
$1^2D_{3/2}, 3/2^+$	$\Lambda_c(2860)^+$	$2856.1^{+2.3}_{-6.0}$	2857.3	$\Xi_c(3055)^+$	3055.9(4)	3058.7
$1^2D_{5/2}, 5/2^+$	$\Lambda_c(2880)^+$	2881.63(24)	2880.2	$\Xi_c(3080)^+$	3077.2(4)	3079.7
$1^2S_{1/2}, 1/2^+$	Λ_b^0	5619.60(17)	5615.5	Ξ_b	5791.9(5)	5792
$1^2P_{1/2}, 1/2^-$	$\Lambda_b(5912)^0$	5912.20(21)	5908.5	Ξ_b		6116.9
$1^2P_{3/2}, 3/2^-$	$\Lambda_b(5920)^0$	5919.92(19)	5921.4	Ξ_b		6129.1
$1^2D_{3/2}, 3/2^+$	$\Lambda_b(6146)^0$	6146.17	6144.8	Ξ_b		6376.9
$1^2D_{5/2}, 5/2^+$	$\Lambda_b(6152)^0$	6152.51	6152.2	Ξ_b		6383.6

TABLE II. The observed quantum numbers and masses (in MeV) [2] charmed and charm-strange baryons that contain vector-diquark. The J^P of some states indicated by the question marks are the quark-model predictions.

States, J^P	Baryon	Mass	This work	Baryon	Mass	This work
$1^2S_{1/2}, 1/2^+$	$\Sigma_c(2455)^{++}$	2453.97(14)	2452.7	Ξ_c'	2578.8(5)	2586.0
$1^4S_{3/2}, 3/2^+$	$\Sigma_c(2520)^{++}$	$2518.41^{+0.21}_{-0.19}$	2517.8	$\Xi_c'^0(2645)$	2646.32(31)	2641.6
$1^{2J+1}P_J, ?^?$	$\Sigma_c(2800)^{++}?$	2801^{+4}_{-6}	…	$\Xi_c'(2930)?$	2931(6)	…
$1^2S_{1/2}, 1/2^+$	Σ_b^+	5811.3(1.9)	5811.0	$\Xi_b'(5935)^-$	5935.02(05)	5937.1
$1^4S_{3/2}, 3/2^+$	Σ_b^{*+}	5832.1(1.9)	5832.0	$\Xi_b'(5955)$	5955.33(13)	5955.0
$1^{2J+1}P_J, ?^?$	$\Sigma_b(6097)^+?$	6095.8(2)	…	$\Xi_b'(6227)?$	6226.9(2.1)	…

Mass prediction: $\Sigma_b(6146,6152)$

西北師範大學 NORTHWEST NORMAL UNIVERSITY

2019 LHCb discovered two excited Σb(6146,6152) . Using the RT parameters fixed, one has

$$M(1D) = M_Q + \sqrt{\pi a L + \left[m_d + M_Q(1 - m_{bareQ}^2/M_Q^2)\right]^2}$$
$$= 4.48 + \sqrt{\pi * 0.246 * 2 + [0.534 + 4.48(1 - 4.18^2/4.48^2)]^2}$$
$$= 6.14927\text{GeV},$$

which agrees well (err=0.1MeV) with the observed spin-averaged mass of the $\Sigma_b(1D, J=3/2^+, 5/2^+)$:

$$\bar{M} = \frac{1}{10}(4 * 6146.17 + 6 * 6152.51) = 6149.97.$$

EXP

The 1D-mass splitting via the scaling relation with the charm partners $\Lambda_c(1D, 3/2^+, 5/2^+)$, in which $\mathbf{S}_d=0$, so that

$$H^{SD} = a_2 \mathbf{L} \cdot \mathbf{S}_Q = \{-3/2, +1\} a_2.$$

$$a_2[\Lambda_c(1D)] = \frac{2}{5}[\Lambda_c(5/2^+) - \Lambda_c(3/2^+)] = 10.212\text{MeV},$$

$$a_2[\Sigma_b(1D)] = a_2[\Lambda_c(1D)]\left(\frac{M_c}{M_b}\right) = 10.212\left(\frac{1.44}{4.48}\right) = 3.28\text{MeV},$$

which predicts as the Exp observed in LHCb (Err<1.8MeV):

$$M[\Sigma_b(1D)] = M(1D) + \{-3/2, +1\} a_2$$
$$= 6149.27 + \{-3/2, +1\} 3.28$$
$$= \{6144.35, 6152.55\}$$

A Mixing Rep. of SH baryons :odd Parity

Main idea

- 改进jj耦合为H近自身表象：Mixing 耦合(Jls) 表象
- 用Exp/LQCD的数据抽取自旋耦合参数
- 用基态和Regge质量关系定出P波平均质量

A Mixing Coupling Scheme

JIA Duojie, J-H. Pan, C-P, Pang,
arXiv:2007.01545v2 [hep-ph]

考虑到系数正比于$1/m_d$的 L · S的项有限大，需要计入重夸克的反冲效应：

$$a_1 \mathbf{L} \cdot [\mathbf{S}_{ss} + \epsilon \mathbf{S}_Q] + bS_{12} \simeq a_1 \mathbf{L} \cdot \mathbf{S}_{ss}, \text{ as } M_Q \to \infty,$$

•将前三项作为表象，接触项视为微扰/对角化H^{SD} --Mixing coupling scheme

Jj-耦合表象：第一项L · S对角的表象

State	Mass (MeV)	Width (MeV)	J^P Proposed
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1$	$4.5 \pm 0.6 \pm 0.3$	$1/2^-$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1$	$0.8 \pm 0.2 \pm 0.1^a$	$1/2^-$
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3$	$3.5 \pm 0.4 \pm 0.2$	$3/2^-$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5$	$8.7 \pm 1.0 \pm 0.8$	$3/2^-$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9$	$1.1 \pm 0.8 \pm 0.4^b$	$5/2^-$
$\Omega_b(6316)^-$	$6315.64 \pm 0.31 \pm 0.07^{\pm 0.5}$	$< 2.8,$	$1/2^-$
$\Omega_b(6330)^-$	$6330.30 \pm 0.28 \pm 0.07^{\pm 0.5}$	$< 3.1,$	$1/2^-$
$\Omega_b(6340)^-$	$6339.71 \pm 0.26 \pm 0.05^{\pm 0.5}$	< 1.5	$3/2^-$
$\Omega_b(6350)^-$	$6349.88 \pm 0.35 \pm 0.05^{\pm 0.5}$	$1.4^{+0.1}_{-0.8} \pm 1.0$	$3/2^-$
$\Sigma_c(2800)^{++}$	2801^{+4}_{-6}	75^{+22}_{-17}	$3/2^-$
$\Xi'_c(2930)$	2942 ± 5	36 ± 13	$3/2^-$
$\Sigma_b(6097)^-$	6098.0 ± 1.8	29 ± 4	$3/2^-$
$\Xi'_b(6227)^-$	6226.9 ± 2.0	18 ± 6	$1/2^-$

Spin couplings for LHCb measured css Baryons

JIA Duojie, J-H. Pan, C-P, Pang, arXiv:2007.01545v2 [hep-ph]

$$\Delta\mathcal{M}_{J=1/2} = \begin{bmatrix} \frac{1}{3}(a_2 - 4a_1) & \frac{\sqrt{2}}{3}(a_2 - a_1) + \frac{b}{\sqrt{2}} \\ \frac{\sqrt{2}}{3}(a_2 - a_1) + \frac{b}{\sqrt{2}} & -\frac{5}{3}(a_1 + \frac{1}{2}a_2) - b \end{bmatrix} + \begin{bmatrix} -c & 0 \\ 0 & \frac{1}{2}c \end{bmatrix},$$

$$\Delta\mathcal{M}_{J=3/2} = \begin{bmatrix} \frac{2}{3}a_1 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) - \frac{b}{2\sqrt{5}} \\ \frac{\sqrt{5}}{3}(a_2 - a_1) - \frac{b}{2\sqrt{5}} & -\frac{1}{3}(2a_1 + a_2) + \frac{4b}{5} \end{bmatrix} + \begin{bmatrix} -c & 0 \\ 0 & \frac{1}{2}c \end{bmatrix},$$

$$\Delta\mathcal{M}_{J=5/2} = a_1 + \frac{1}{2}a_2 - \frac{b}{5} + \frac{c}{2}.$$

$$\Delta M(J = 1/2, 0') = -\frac{a_1}{4}\left(6 + \sqrt{\Delta_1\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right)} + \frac{a_2}{a_1}\right) - \frac{b}{2} + c\Delta_3^+\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right),$$

$$\Delta M(J = 1/2, 1') = -\frac{a_1}{4}\left(6 - \sqrt{\Delta_1\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right)} + \frac{a_2}{a_1}\right) - \frac{b}{2} + c\Delta_3^-\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right),$$

$$\Delta M(J = 3/2, 1') = -a_1\left(\sqrt{\Delta_2\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right)} + \frac{a_2}{4a_1}\right) + \frac{2b}{5} + c\Delta_4^+\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right),$$

$$\Delta M(J = 3/2, 2') = a_1\left(\sqrt{\Delta_2\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right)} - \frac{a_2}{4a_1}\right) + \frac{2b}{5} + c\Delta_4^-\left(\frac{a_2}{a_1}, \frac{b}{a_1}\right),$$

$$\Delta M(J = 5/2, 2') = a_1 + \frac{a_2}{2} - \frac{b}{5} + \frac{c}{2},$$

$$\Delta_1(\epsilon, x) = 4 + 12x^2 + 4x(5\epsilon - 2) - 4\epsilon + 9\epsilon^2,$$

$$\Delta_2(\epsilon, x) = 1 + \frac{1}{5}x^2 - \frac{x}{5}(1 + 2\epsilon) - \epsilon + \frac{9}{16}\epsilon^2.$$

$$\Delta_3^+(\epsilon, x) = \frac{4-(2+6x+7\epsilon-3\sqrt{\Delta_1(\epsilon,x)})^2/(2\epsilon-2+3x)^2}{8+(2+6x+7\epsilon-3\sqrt{\Delta_1(\epsilon,x)})^2/(2\epsilon-2+3x)^2},$$

$$\Delta_3^-(\epsilon, x) = \Delta_3^+(\sqrt{\Delta_1} \to -\sqrt{\Delta_1}).$$

$$\Delta_4^+(\epsilon, x) = \frac{10-(40-24x+5\epsilon+60\sqrt{\Delta_2(\epsilon,x)})^2/(10-10\epsilon+3x)^2}{20+(40-24x+5\epsilon+60\sqrt{\Delta_2(\epsilon,x)})^2/(10-10\epsilon+3x)^2},$$

$$\Delta_4^-(\epsilon, x) = \Delta_4^+(\sqrt{\Delta_2} \to -\sqrt{\Delta_2}),$$

5! Permulations fit

$$\bar{M} = 3079.94\text{MeV},$$

$$\{a_1, a_2, b, c\} = \{26.96,\ 25.76,\ 13.51,\ 4.04\}(\text{MeV}),$$

$$a_1(css) = a_1(c\bar{s})\left(\frac{m_s}{m_{ss}}\right) = (89.4\ \text{MeV})\left(\frac{328}{991}\right) = 29.6\ \text{MeV},$$

$$a_2(css) = \frac{a_2(c\bar{s})}{1 + m_{ss}/M_c} = \frac{40.7\ \text{MeV}}{1 + 991/1440} = 24.1\ \text{MeV},$$

Inner-structure of Excited css baryons

计入重夸克的反冲效应后，Mixing 耦合表象更加接近LS耦合的预言

•Jj-耦合表象看

$$\left\{\begin{array}{llllll} \text{State } |J, j_{LS}\rangle: & |\frac{1}{2}, 0'\rangle & |\frac{1}{2}, 1'\rangle & |\frac{3}{2}, 1'\rangle & |\frac{3}{2}, 2'\rangle & |\frac{5}{2}, 2'\rangle \\ M(\Omega_c 1P): & 3000.4 & 3050.2 & 3065.6 & 3090.2 & 3119.1 \\ \text{Main comp.} & {}^4P_{1/2}(97\%) & {}^2P_{1/2}(97\%) & {}^4P_{3/2}(98\%) & {}^2P_{3/2}(98\%) & {}^4P_{5/2} \end{array}\right\}$$

$|J = 1/2, j' = 0'\rangle = -0.164|^2P_{1/2}\rangle + 0.986|^4P_{1/2}\rangle$, at 3000

$|J = 1/2, j' = 1'\rangle = 0.986|^2P_{1/2}\rangle + 0.164|^4P_{1/2}\rangle$, at 3050

$|J = 3/2, j' = 1'\rangle = 0.129|^2P_{3/2}\rangle + 0.992|^4P_{3/2}\rangle$, at 3066

$|J = 3/2, j' = 2'\rangle = -0.992|^2P_{3/2}\rangle + 0.129|^4P_{3/2}\rangle$, at 3090

$|J = 5/2, j' = 2'\rangle = |^4P_{5/2}\rangle$, at 3119.

$$\left|J = \frac{1}{2}, j' = 0'\right\rangle = 0.711\left|\frac{1}{2}, j = 0\right\rangle + 0.703\left|\frac{1}{2}, j = 1\right\rangle,$$

$$\left|J = \frac{1}{2}, j' = 1'\right\rangle = -0.703\left|\frac{1}{2}, j = 0\right\rangle + 0.711\left|\frac{1}{2}, j = 1\right\rangle,$$

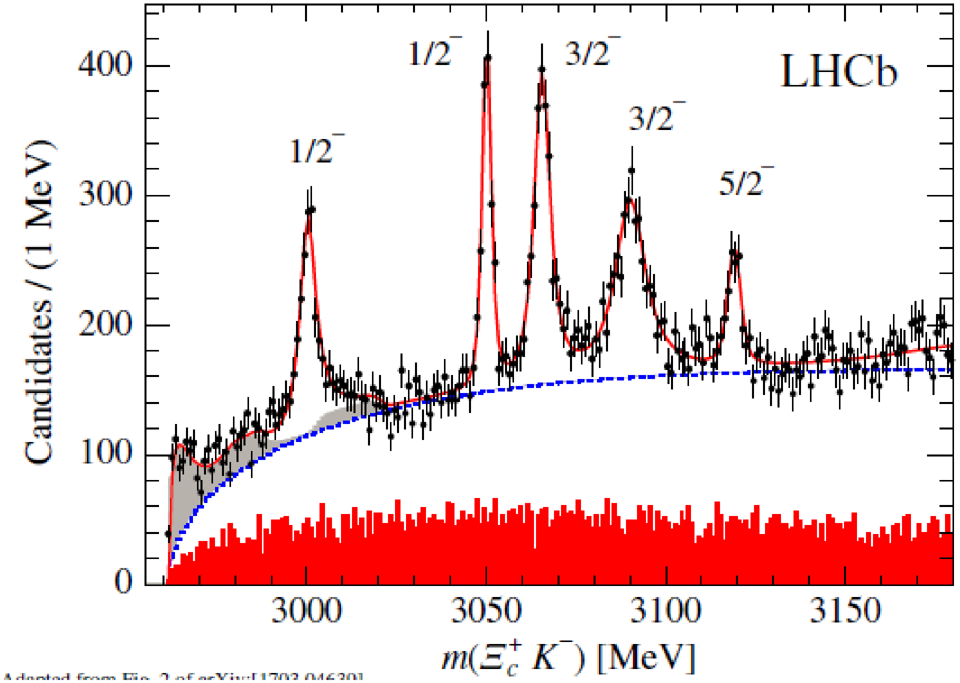
$$\left|J = \frac{3}{2}, j' = 1'\right\rangle = 0.958\left|\frac{3}{2}, j = 1\right\rangle + 0.286\left|\frac{3}{2}, j = 2\right\rangle,$$

$$\left|J = \frac{3}{2}, j' = 2'\right\rangle = -0.286\left|\frac{3}{2}, j = 1\right\rangle + 0.958\left|\frac{3}{2}, j = 2\right\rangle,$$

$\lvert\frac{1}{2}, 0'\rangle, \lvert\frac{1}{2}, 1'\rangle, \lvert\frac{3}{2}, 1'\rangle, \lvert\frac{3}{2}, 2'\rangle, \lvert\frac{5}{2}, 2'\rangle$	a_1	a_2	b	$\bar{M}(1P)$
[6314.5], 6332.0, 6337.8, 6350.0, 6351.5	9.02	4.44	7.92	6341.8
6315.4, [6332.1], 6337.8, 6350.0, 6351.5	8.91	4.27	7.53	6342.0
6315.4, 6332.0, [6337.7], 6350.0, 6351.5	8.95	4.25	7.48	6341.9
6315.4, 6332.0, 6337.8, [6350.5], 6351.5	8.99	4.01	7.12	6342.1

其他J^{P}安排的拟合

TABLE II: Mass and parameters for two J^P assignme
observed masses of the excited Ω_c systems. All 2 $*$ 5
measured masses are considered. The predicted m
bracket. All parameters except the Regge slope a in M

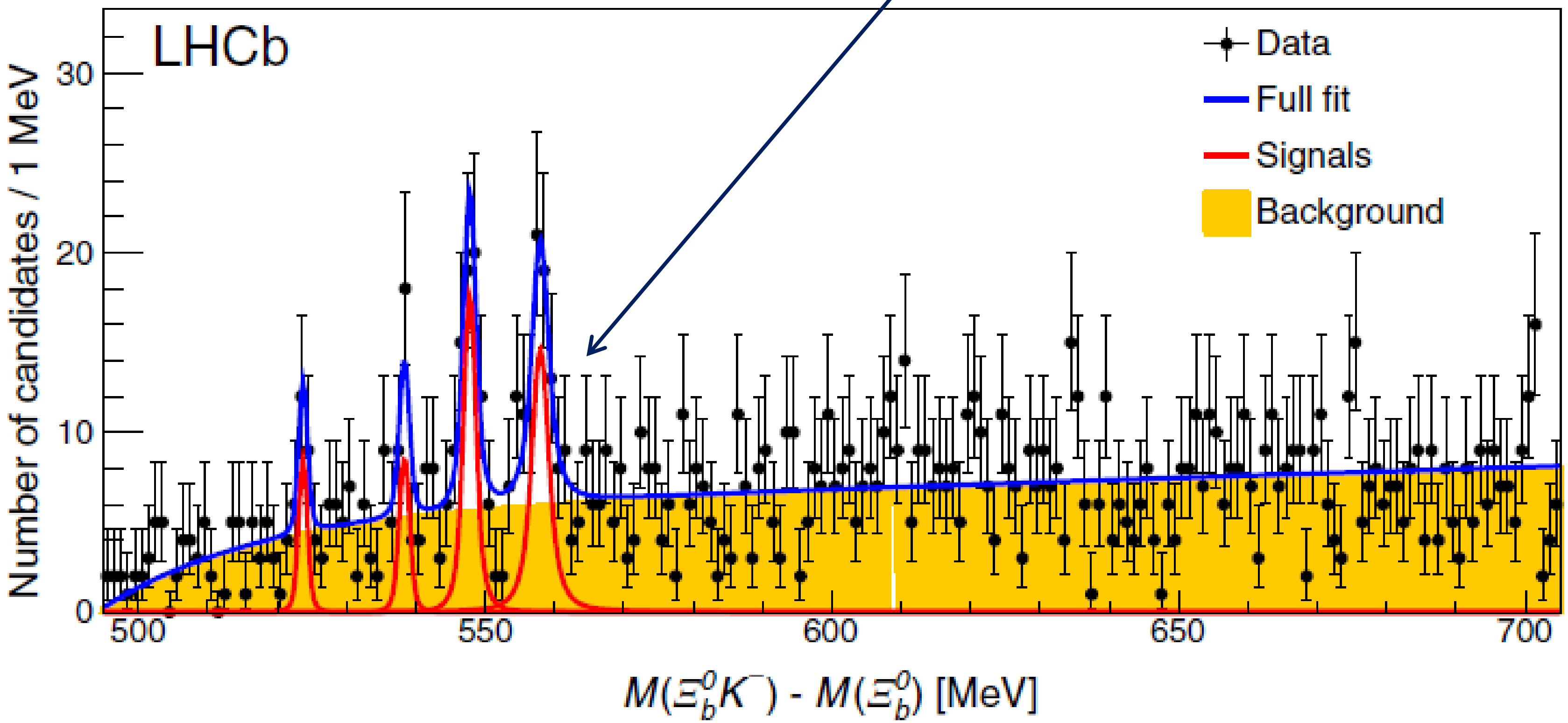


$\lvert\frac{1}{2},0'\rangle$	$\lvert\frac{1}{2},1'\rangle$	$\lvert\frac{3}{2},1'\rangle$	$\lvert\frac{3}{2},2'\rangle$	$\lvert\frac{5}{2},2'\rangle$	a_1	a_2	b	c	$a(GeV^2)$	$\bar{M}(1P)$	$\bar{M}(2S)$
[2995.0]	3050	3066	3090	3119	27.5	27.0	15.5	3.6	0.316	3079	3244
3000	[3049.0]	3066	3090	3119	27.2	25.2	13.7	4.4	0.316	3080	3244
3000	3050	[3068.2]	3090	3119	26.7	24.8	15.4	5.0	0.317	3081	3245
3000	3050	3066	[3095.4]	3119	28.2	23.1	14.4	2.3	0.317	3081	3246
3000	3050	3066	3090	[3115.6]	26.3	23.7	14.7	3.2	0.315	3079	3243
[3000.4]	3066	3050	3090	3119	21.4	40.8	5.7	0.44	0.314	3078	3242
3000	[3067.4]	3050	3090	3119	20.4	41.9	6.4	1.2	0.315	3078	3242
3000	3066	[3051.0]	3090	3119	21.4	40.4	6.1	0.52	0.315	3078	3242
3000	3066	3050	[3090.1]	3119	21.3	40.8	5.7	0.59	0.314	3078	3242
3000	3066	3050	3090	[3117.5]	21.4	39.7	5.7	-0.57	0.314	3078	3241

Mean and its splitting for Excited bss baryons

a_1 = 8.98 MeV, a_2 = 4.11 MeV, b = 7.61 MeV, $\bar{M}$ = 6342.0 Me

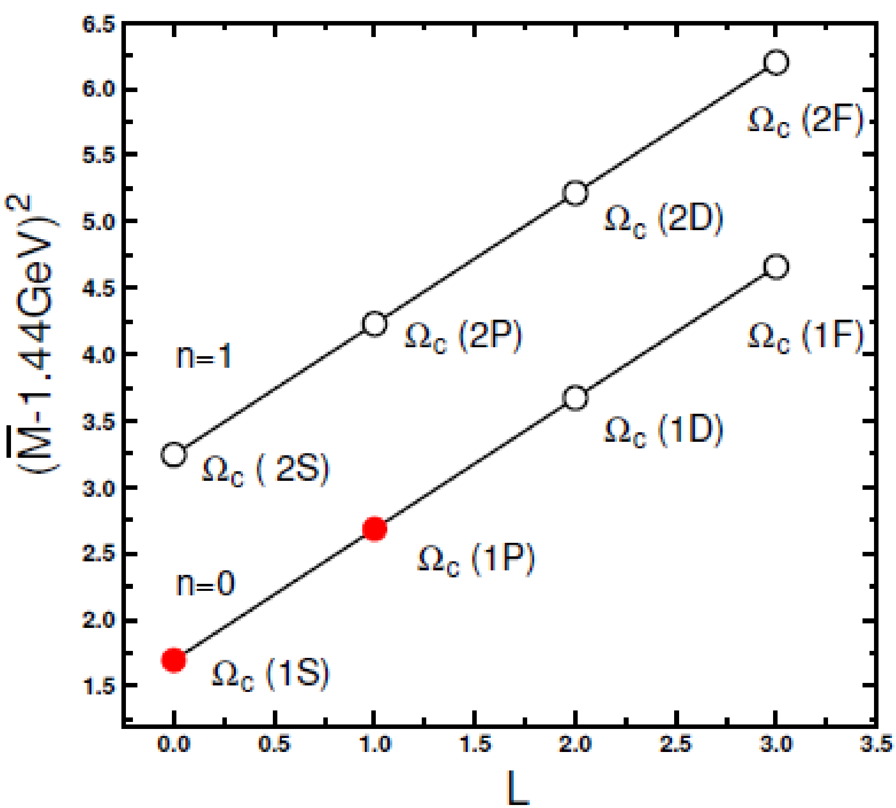
$$\begin{Bmatrix} \text{State}(J^P): & |1/2,0'\rangle & |1/2,1'\rangle & |3/2,1'\rangle & |3/2,2'\rangle & |5/2,2'\rangle \\ M(\Omega_b,1P): & 6315.4, & 6332.0 & 6337.8 & 6350.0 & 6351.5^{\text{Pd}} \end{Bmatrix}$$

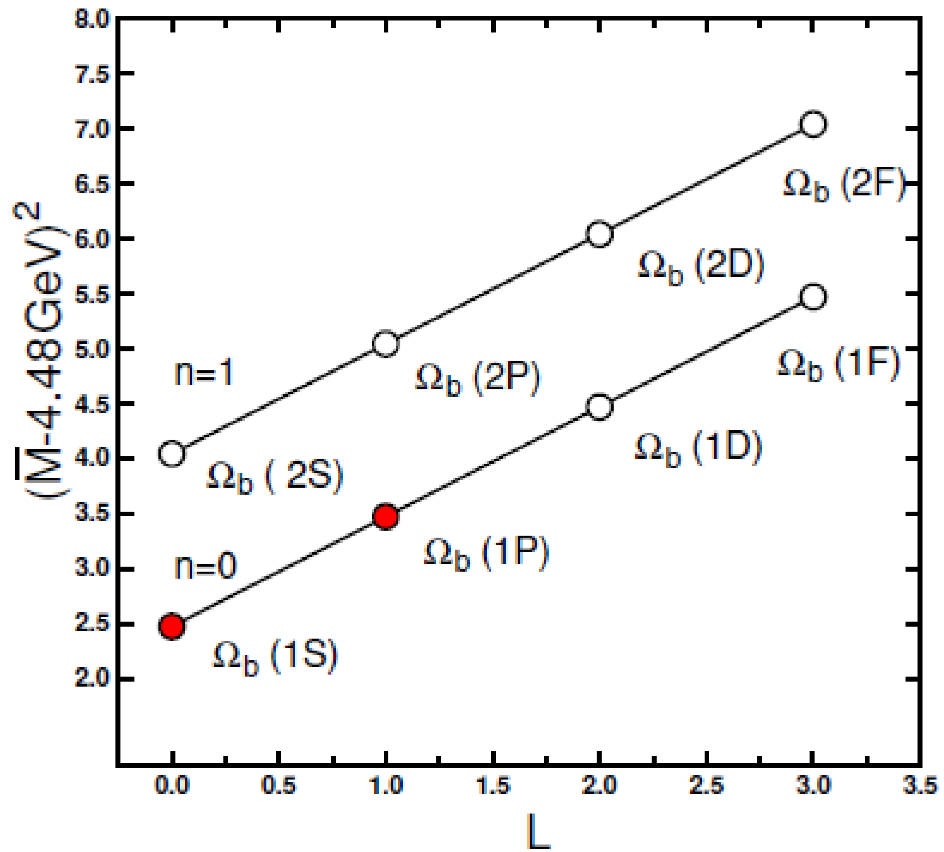


Excited spectra in Regge trajectories

$$(\bar{M} - M_c)^2 = \pi a L + \left[m_d + M_c \left(1 - \frac{m_{barec}^2}{M_c^2} \right) \right]^2;$$

Duojie Jia, W-N Liu, A. Hosaka, PRD101, 034016 (2020)





Excited Baryons Σ_{c} /Ξ_{c}′ and Σ_{b} /Ξ_{b}′

Initial: $c(cqq) \approx c(css)\left(\frac{m_{ss}}{m_{qq}}\right) = \begin{cases} 5.37\ \text{MeV}, & \Sigma_c, \\ 4.59\ \text{MeV}, & \Xi'_c. \end{cases}$

Initial: $c(bqq) \approx c(css)\left(\frac{M_c}{M_b}\right)\left(\frac{m_{ss}}{m_{qq}}\right) = \begin{cases} 1.73\ \text{MeV}, & \Sigma_b, \\ 1.48\ \text{MeV}, & \Xi'_b. \end{cases}$

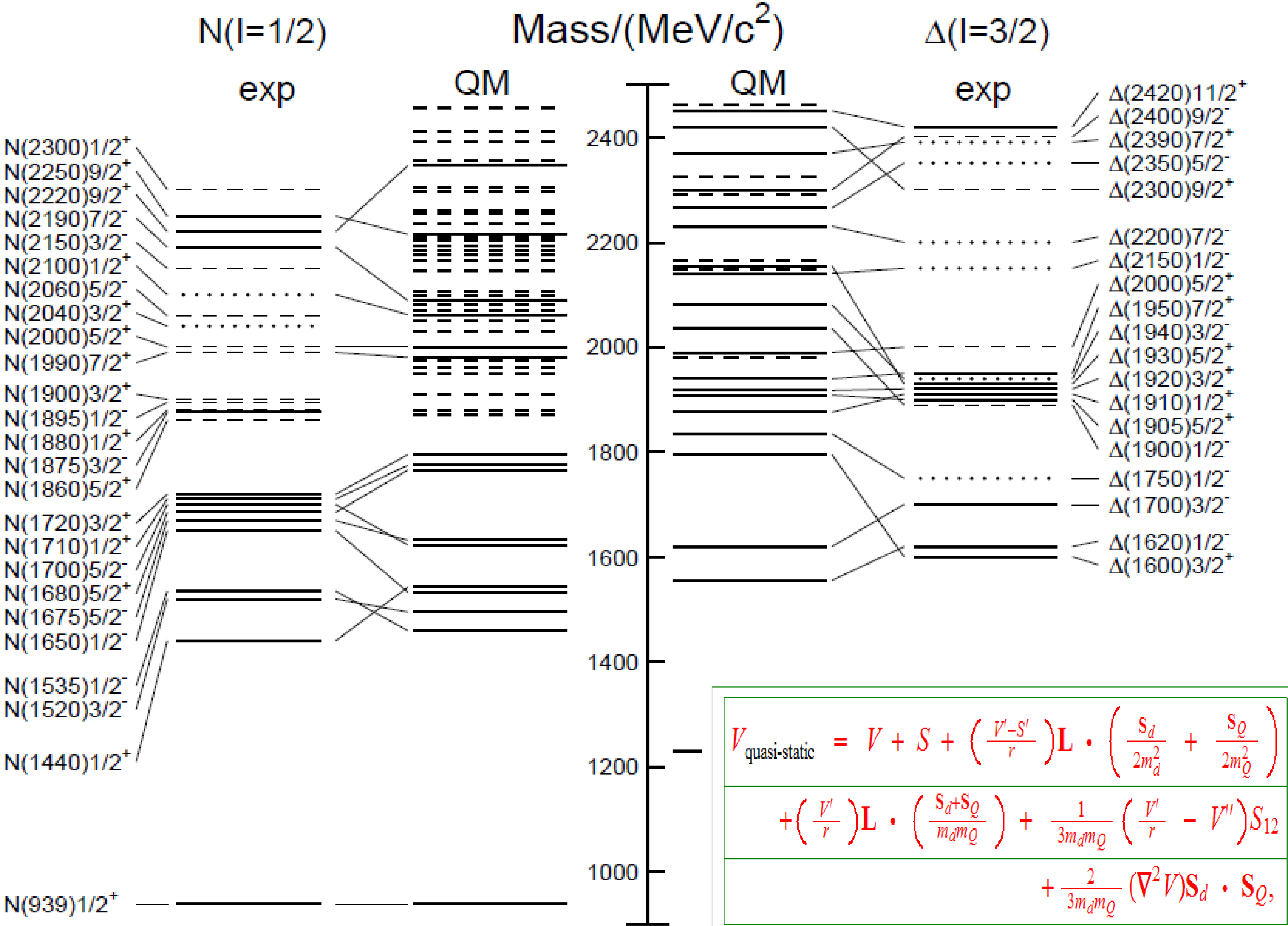
Initial input	a_1	a_2	b	c	$\bar{M}(1P)$
Σ_c(MeV)	39.4	26.8	20.1	5.37	2774.1
Ξ'_c(MeV)	33.6	25.3	17.9	4.59	2923.0
Σ_b(MeV)	12.7	8.61	6.45	1.73	6088.4
Ξ'_b(MeV)	10.8	8.10	5.76	1.48	6248.2

质量标度估计+用自洽迭代

State :	$\lvert\frac{1}{2},0'\rangle, \lvert\frac{1}{2},1'\rangle, \lvert\frac{3}{2},1'\rangle, \lvert\frac{3}{2},2'\rangle, \lvert\frac{5}{2},2'\rangle$	a_1	a_2	b	c	$\bar{M}(1P)$
Σ_c(MeV)	2668.4, 2723.1, 2757.3, 2801.0$^\diamond$, 2826.6	39.96	21.75	20.70	7.85	2776.4
Ξ'_c(MeV)	2840.6, 2881.6, 2908.9, 2942.3$^\diamond$, 2969.5	32.89	20.16	16.50	7.17	2925.9
Σ_b(MeV)	6053.9, 6071.8, 6082.8, 6098.0$^\diamond$, 6104.8	12.99	6.42	6.45	1.73	6089.1
Ξ'_b(MeV)	6226.9$^\diamond$, 6235.8, 6243.4, 6252.3, 6262.5	9.37	6.29	5.76	1.48	6249.1

A QM explanation for Mass splitings

Mass splittings of Light baryons in QM



$$V_{\text{quasi-static}} = V + S + \left(\frac{V'-S'}{r}\right)\mathbf{L}\cdot\left(\frac{\mathbf{S}_d}{2m_d^2}+\frac{\mathbf{S}_Q}{2m_Q^2}\right) + \left(\frac{V'}{r}\right)\mathbf{L}\cdot\left(\frac{\mathbf{S}_d+\mathbf{S}_Q}{m_d m_Q}\right) + \frac{1}{3m_d m_Q}\left(\frac{V'}{r}-V''\right)S_{12} + \frac{2}{3m_d m_Q}(\nabla^2 V)\mathbf{S}_d\cdot\mathbf{S}_Q,$$

Spin-couplings in Relativized QM

- QCD analogues of Breit-Fermi interaction in QED(in nonrelativistic version)
- IT can change due to the recoil of the charm quark (not heavy enough).

$$a_1 = \frac{1}{2m_d}\left[\frac{V'-S'}{m_d r} + \frac{2V'}{m_Q r}\right],$$

$$a_2 = \frac{1}{2m_Q}\left[\frac{V'-S'}{m_Q r} + \frac{2V'}{m_d r}\right],$$

$$b = \frac{1}{3m_d m_Q}(V'/r - V''),\ c = \frac{2\nabla^2 V}{3m_d m_Q},$$

$$V(r) \to \tilde{V}(r) = \left(\frac{m_d m_Q}{E_d E_Q}\right)^{1/2+\epsilon_V/2} V(r)\left(\frac{m_d m_Q}{E_d E_Q}\right)^{1/2+\epsilon_V/2},$$

$$S(r) \to \tilde{S}(r) = \left(\frac{m_d m_Q}{E_d E_Q}\right)^{1/2+\epsilon_S/2} S(r)\left(\frac{m_d m_Q}{E_d E_Q}\right)^{1/2+\epsilon_S/2},$$

相对论因子（尺缩效应）修正

$$m_i/E_i = m_i/\sqrt{m_i^2 + |\mathbf{p}|^2} = \sqrt{1 - v_i^2}\ (i = 1, 2)$$

$$F(r) \equiv 1 - e^{-\xi r - \zeta r^2}$$

$$V \to -\frac{k_s}{r}F(r).$$

$$a_1 = \frac{1}{2m_d^2}\left\langle\left(1 + \frac{2m_d}{m_Q}\right)\left(\frac{m_d m_Q}{E_d^H E_Q^H}\right)^{1+\epsilon_V} k_s\left(\frac{F}{r^3} - \frac{F'}{r^2}\right) - \left(\frac{m_d m_Q}{E_d^N E_Q^N}\right)^{1+\epsilon_S}\frac{a}{r}\right\rangle,$$

$$a_2 = \frac{1}{m_d m_Q}\left\langle\left(1 + \frac{m_d}{2m_Q}\right)\left(\frac{m_d m_Q}{E_d^H E_Q^H}\right)^{1+\epsilon_V} k_s\left(\frac{F}{r^3} - \frac{F'}{r^2}\right) - \left(\frac{m_d m_Q}{E_d^N E_Q^N}\right)^{1+\epsilon_S}\frac{m_d a}{2m_Q r}\right\rangle,$$

$$b = \frac{k_s}{3m_d m_Q}\left(\frac{m_d m_Q}{E_d^H E_Q^H}\right)^{1+\epsilon_V}\langle 3F/r^3 - 3F'/r^2 + F''/r\rangle,$$

$$c = \frac{2k_s}{3m_d m_Q}\left(\frac{m_d m_Q}{E_d^H E_Q^H}\right)^{1+\epsilon_V}\left[4\pi|R_{nL}^H(0)|^2 - \int dr r F''|R_{nL}^H(r)|^2\right],$$

Relativized QM

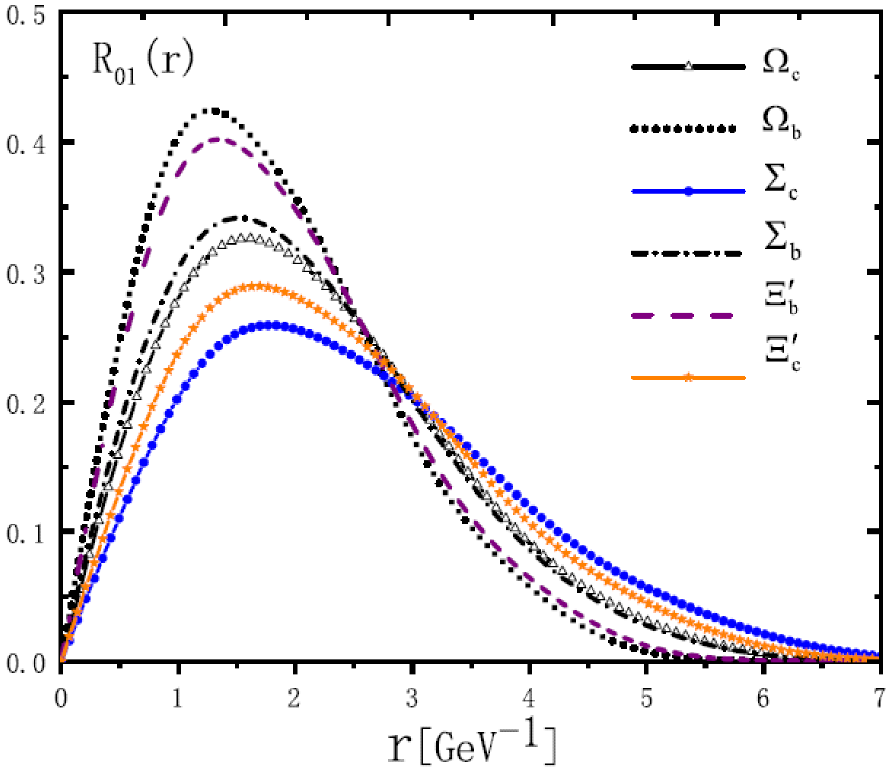
- Use RQM solve wavefunction and apply the Breit-Fermi formula to Compute the spin couplings
- Put form factor F(r) for short-range part

$$H = \sqrt{m_Q^2 + p^2} + \sqrt{m_d^2 + p^2} + V + S,$$

$$V = -k_s/r, S = ar + C_0, k_s = 4\alpha_s/3.$$

$$F(r) \equiv 1 - e^{-\xi r - \zeta r^2}$$

$$V \to -\frac{k_s}{r} F(r).$$



State	m_d	$a[GeV^2]$	α_s	$\xi(GeV^{-1})$	$\zeta(GeV^{-2})$	ϵ_V	ϵ_S
Ω_c	991	0.316	0.561	0.818	0.11	−0.10	2.65
Ω_b		0.318	0.543				
Ξ'_c	872	0.255	0.590	0.820	0.12	−0.06	2.20
Ξ'_b		0.307	0.545				
Σ_c	745	0.212	0.595	0.850	0.12	−0.05	1.80
Σ_b		0.246	0.549				

Spin-couplings in Relativized QM

- Spin couplings
- Scaling law tested

css: $\{a_1, a_2, b, c\} = \{26.96, 25.76, 13.51, 4.04\}$(MeV)

State	μ_d	μ	ν	μ_{dH}	μ_H	a_B[GeV^{-1}]	a_1	a_2	b	c
Ω_c	1291	681	909	1016	596	2.248	28.52	27.03	15.32	20.73
Ω_b	1379	1054	789	1034	840	1.701	10.30	10.25	5.61	9.26
Ξ'_c	1153	640	805	900	554	2.256	30.15	27.98	16.54	20.35
Ξ'_b	1273	992	787	912	757	1.882	11.42	11.06	6.14	9.82
Σ_c	1017	596	728	777	504	2.283	35.46	30.96	19.10	20.44
Σ_b	1117	894	703	781	665	2.130	13.93	11.15	6.61	8.93

bss: a_1=8.98 MeV, a_2=4.11 MeV, b=7.61 MeV

质量标度律验证

ratio:	r_1	r_2	r_b
Ω_b/Ω_c :	0. 361	0. 379*	0. 366*
Ξ'_b/Ξ'_c :	0. 379	0. 395	0. 370
Σ_b/Σ_c :	0. 393	0. 360	0. 346

$[r_1]_{Match}$	$[r_2]_{Match}$	$[r_b]_{Match}$
0. 33	0. 16	0. 56
0. 28	0. 31	0. 35
0. 33	0. 29	0. 31

Summary

- We re-examine the low-lying spectra of the SH baryons in Heavy quark-diquark picture, combining with Regge approach. We find that a linear Regge relation, derived from the rotating string model, is sufficient to describe the low-lying spectrum of the SH baryons. This is supported strongly by precise mass calculations of two Λ_b(6146,6152).
- We predict that quantum numbers of some parity-odd charmed/bottom baryons:
  Σc(2800/Ξc′(2930) : 1P-wave $J^P = 3/2^-$;
  Σb(6097/Ξb′(6227)$^-$: 1P-wave $J^P = 3/2^-$, $1/2^-$;
- We provide an QM explanation why the SH baryon mass splittings are normally smaller than QM predicted and thereby the mass patterns of the Excited Omega_c,b; The mass scaling law is valid roughly.

Thank you!

重轻介子RT之中的斜率比

We propose the slope ratio between the radial and angular-momentum Regge trajectories to be **π:2**,

and the mass **M** of mesons/baryons to be shifted by μ_{qQ}, the effective reduced mass of the heavy quark and light (anti)quark:

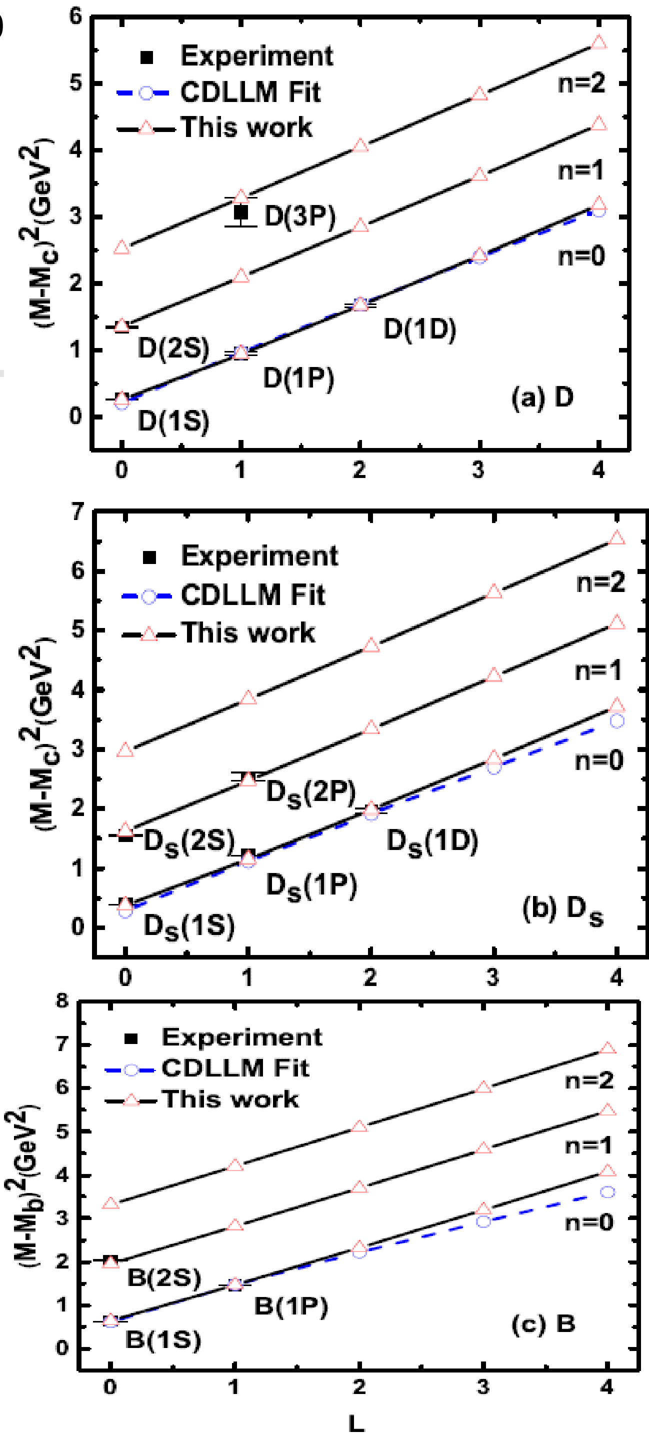
$$\beta : \alpha = \pi / 2$$

$$\left[M - (M - \mu_{qQ})\right]^2 = \pi a \left(L + \frac{\pi}{2} n \right) + a_I$$

$$\mu_{qQ} = \frac{k m_q}{1 + k m_q / M_Q}$$

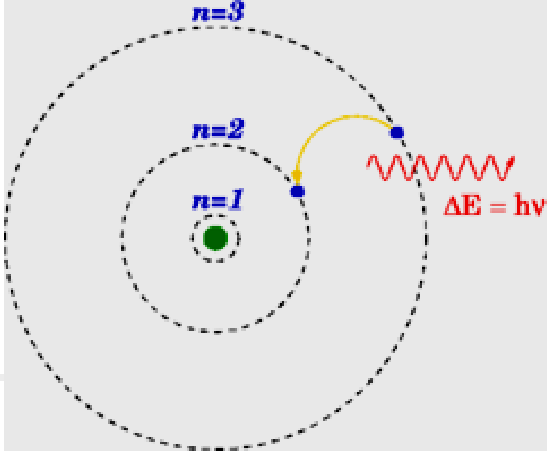
$$a_I = \left(m_q + M_Q v_Q{}^2\right)^2$$

Duojie. Jia*,Wen-Chao Dong, **Eur. Phys. J. Plus** (2019) 134: 123 arXiv:1811.04214

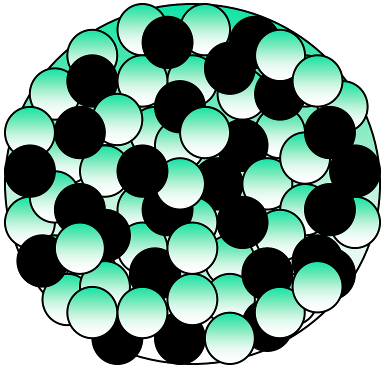


如何研究重味强子?

- 第一原理或类比QED
- 唯象学研究其性质(mass and lifetime)



原子



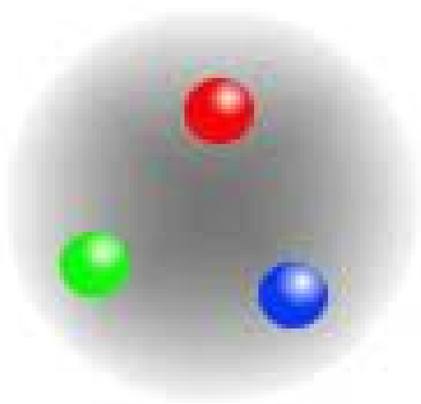
核

原子谱 ➡ 玻尔原子模型+量子理论

核谱 ➡ 壳模型+集体运动模型

强子谱 ➡ QCD ？

强子



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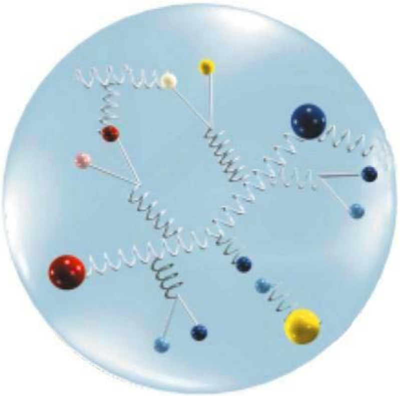


The body of hadrons are
INCREASING steadily !!!

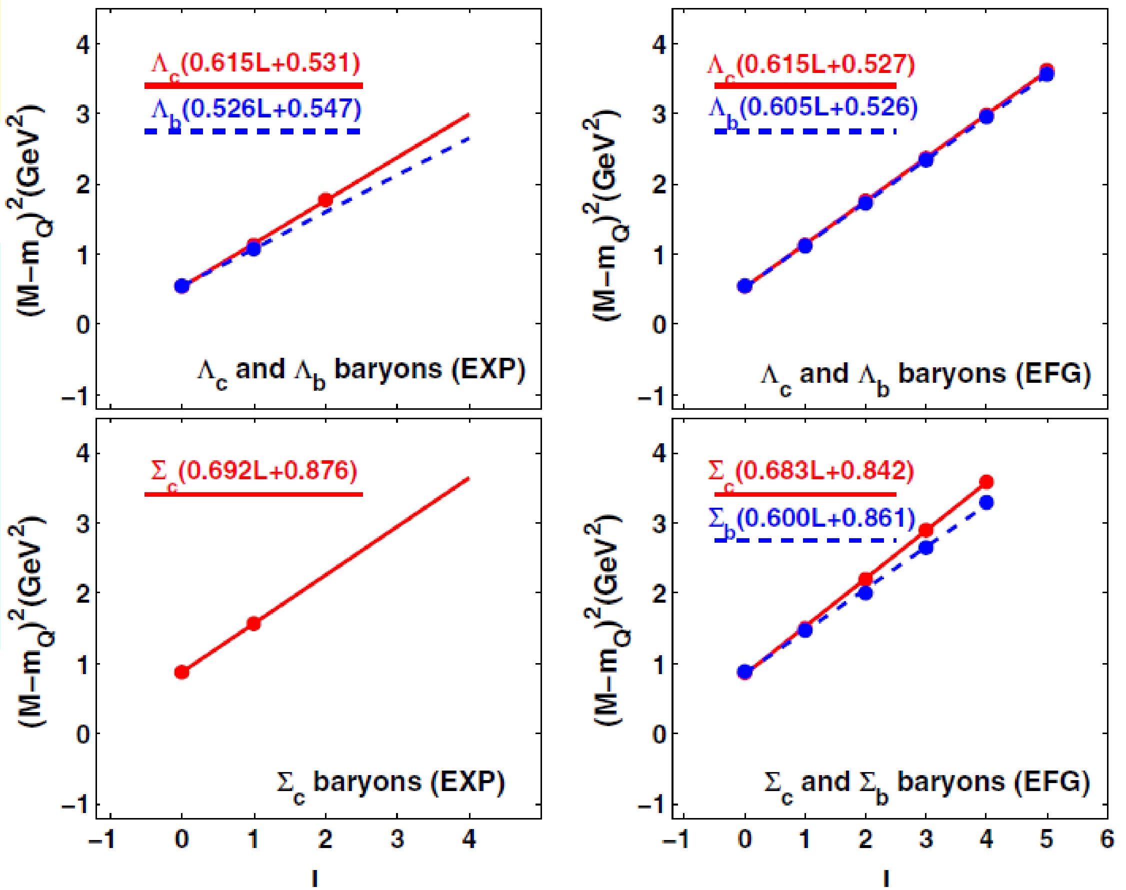
线性RT：重味重子

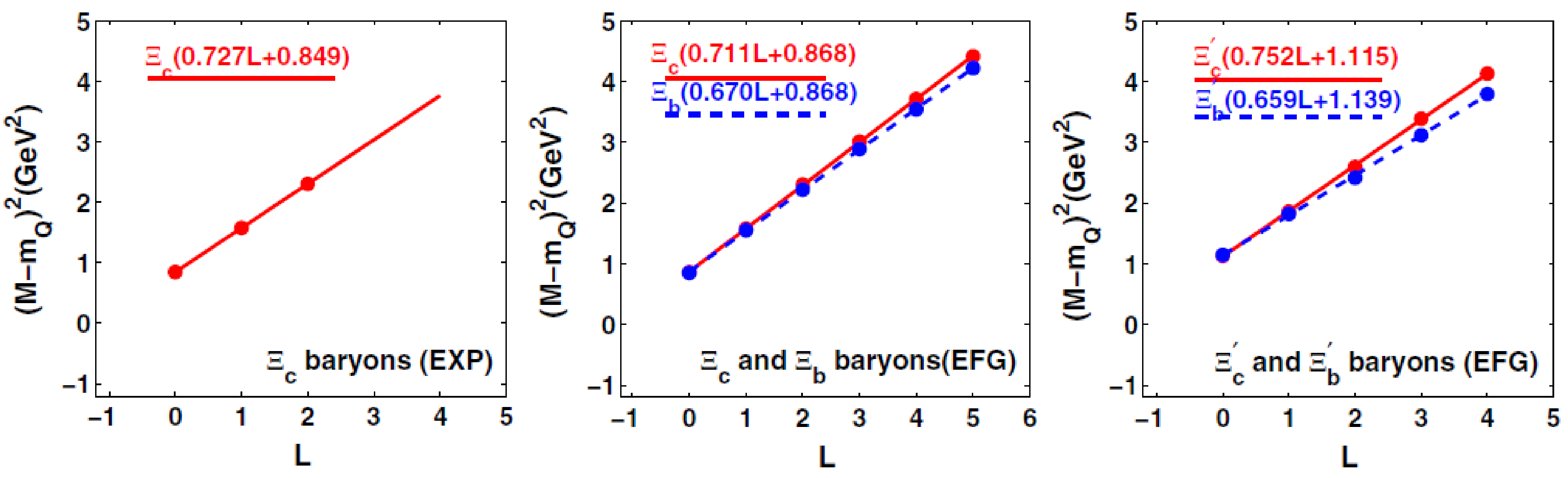
$(M-M_Q)^2 = \pi bL + c$

The linearity confirmed provided that

---the mass is shifted by M_Q
---the inverse slope is halved

K. Chen et al., Eur. Phys. J. C (2018) 78:20





轻介子几乎皆在线性RT上

A. V. Anisovich, V. V. Anisovich et al., Phys. Rev. D62(2000)051502
P. Masjuan, E. R. Arriola et al., Phys. Rev. D85(2012) 094006 .

Anisovich

$$M^2 = M_0^2 + n(1.25 \pm 0.3)GeV^2$$

$$M^2 = M_0^2 + J(1.25 \pm 0.15)GeV^2$$

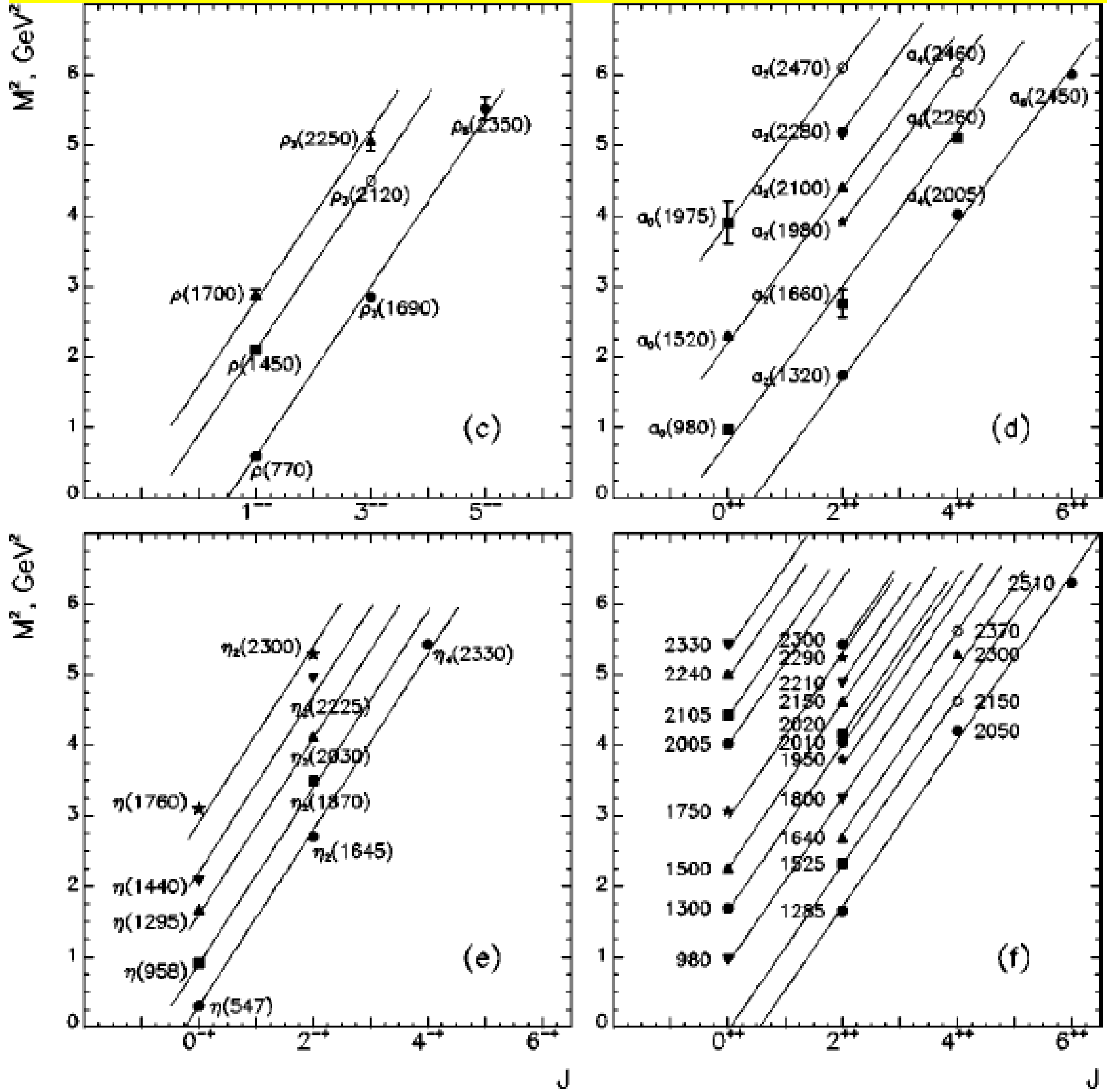
Masjuan,E. R. Arriola

$$M^2 = M_0^2 + n(1.38 \pm 0.3)GeV^2$$

$$M^2 = V_0 + J(1.16 \pm 0.04)GeV^2$$

S. S. Afonin, Mod. Phys. Lett. A 22, 1359 (2007)

$$M^2 = b + a(n + J)$$



Slope = 0.6 ±0.1 GeV²
= 1.25 之半

雷吉理论

P. D. B. Collins, An Introduction to Regge Theory and High-Energy Physics (Cambridge/1977).

Regge (1959,1960) 曾将量子散射振幅的高能行为和交叉(t)道中分波振幅在复数轨道角动量平面上的奇点相联系，提出了量子束缚态的Regge唯象理论。

要点：不用标准的量子场论和Feynman图技术，集中研究散射(S)矩阵的一般性质---幺正性、解析性和交叉对称性。

散射振幅最简单的奇点是极点(poles）

叫 Regge极点；

每条Regge轨迹看可以对应物理上分立（E<0)和准分立（E>0）的能级

The simplest singularities are poles (Regge pole)：

$$l = \alpha(M) = \alpha(0) + \alpha' M^2 + \alpha'' M^3$$

库仑势场之散射

$$S_l = \frac{\Gamma(l+1-i/k)}{\Gamma(l+1+i/k)}, k = i\sqrt{-2E}, if\ E<0$$

$$E = k^2/2 \cdot (m_e e^4/\hbar^2)$$

pole

$$\alpha(E) = -n_r - 1 + \frac{1}{\sqrt{-2E}}, E<0$$

$$E = -\frac{1}{2(n_r+1+l)^2} \cdot (m_e e^2/\hbar^2)$$

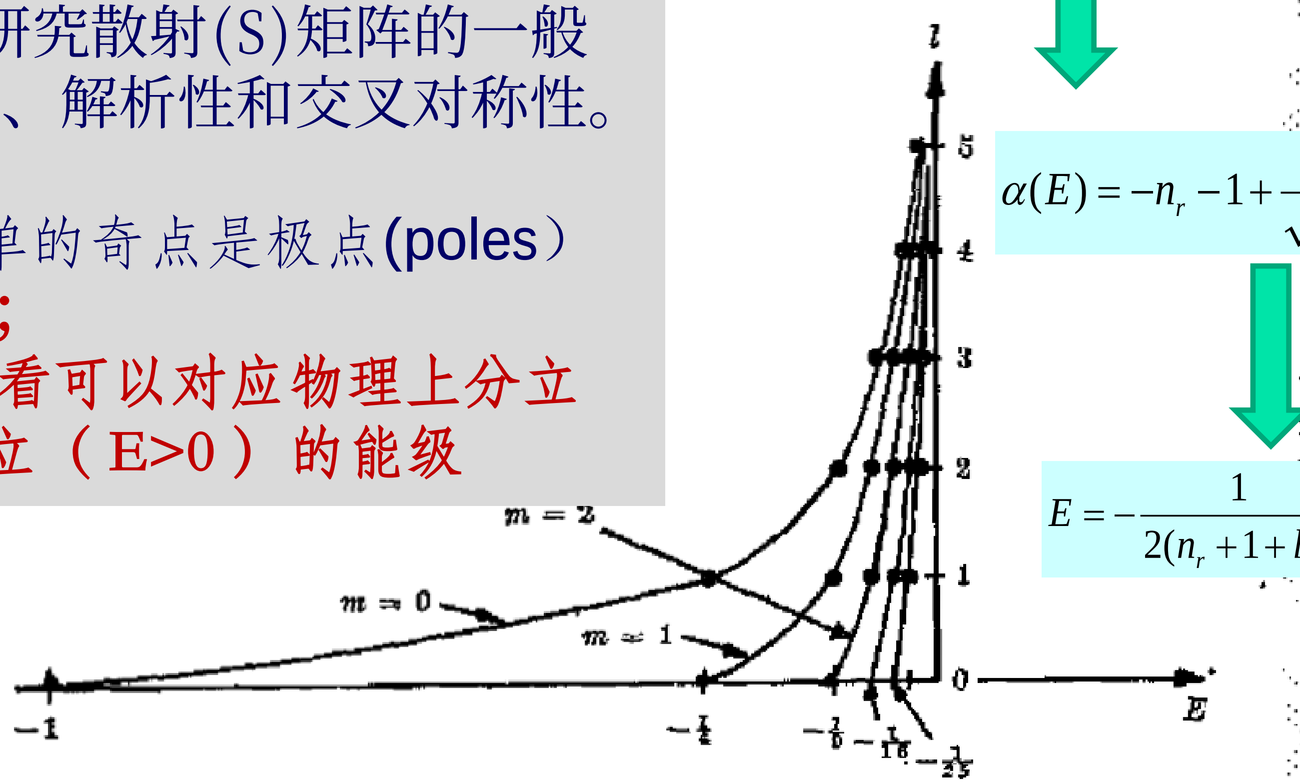
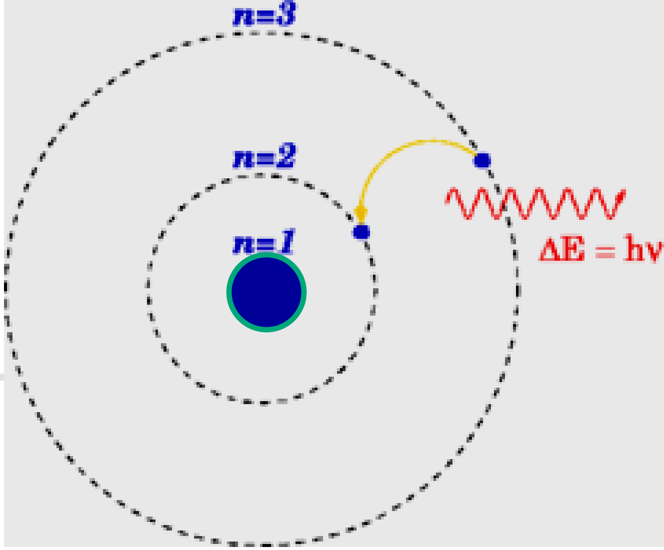


Fig. 3.1 Regge trajectories for the Coulomb potential from (3.2.29). For

与原子结构比较

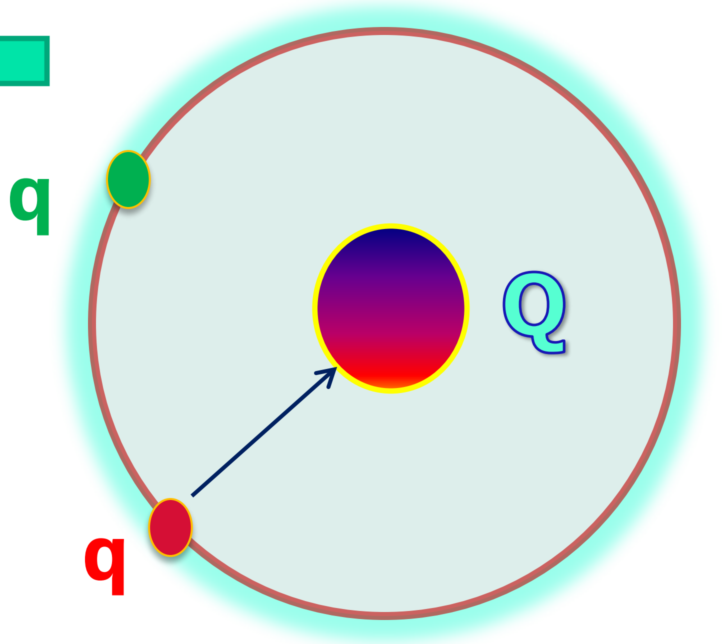
● **SIMPLER to understand** (QCD/Models): With one heavy quark, many **Simplifications /approximations** apply (**HQ** symmetry, diquark pitcture, Heavy-light 极限.),



当Q非常重时

类似氢或氦原子处理

More things concerved:
heavy quark spin
heavy quark velocity
Center-of-mass easier to be removed



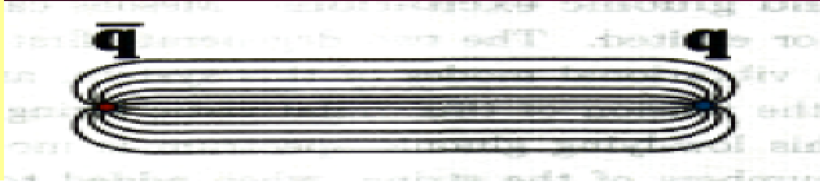
可以集中处理轻夸克动力学
如用手征模型/微扰论

观测激发谱小结：

- 重味重子满足线性的RT轨迹，若扣除重夸克质量之后；
- 线性RT暗示一种特殊的夸克间相互作用
- 重味重子的RT斜率大约是轻味RT的一半.
- RT的斜率具有微弱的味道依赖性，
  而截距严重地味道

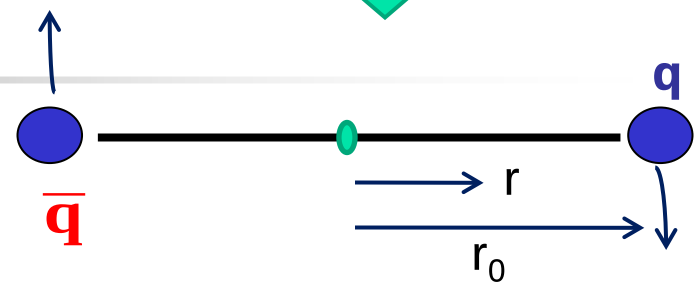
Low-Energy QCD: Effective Quark and String pictures

线性RT：一个简单的弦图像



- Assuming quark and antiquark in a light meson is connected by a flux-tube (string) and string rotates relativistically with the ends having speed of light (massless quark limit).

so that the small piece of string at r rotates with



$$v/c = r/r_0$$

$$\mathrm{E} = Mc^2 = 2\int_0^{r_0} \frac{kdr}{\sqrt{1-(v/c)^2}} = kr_0\pi$$

$$J = \frac{2}{\hbar c^2}\int_0^{r_0} \frac{krvdr}{\sqrt{1-(v/c)^2}} = \frac{kr_0{}^2\pi}{2\hbar c}$$

• Total energy and orbital angular momentum of system are

$$V = kr$$

$$J = \alpha' M^2 + const., \quad \alpha' = 1/(2\pi k\hbar c)$$

• If one starts with $V = k{*}r^n$

$$J \propto E^{(1+1/n)}$$

which is linear Regge iff n=1

$$For\, \alpha' = 0.93 GeV^{-2} (1/\alpha' = 1.075 GeV^2)$$

$$k = 0.87 GeV/fm = 0.17 GeV^2$$

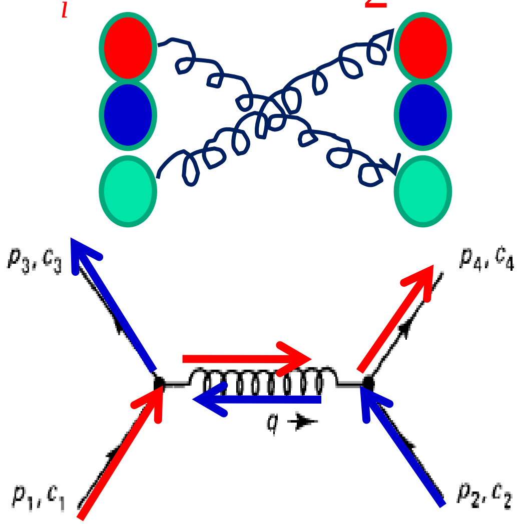
Y. Nambu, Phys. Rev. D10, 4262 (1974)
T. Goto, Prog. Theor. Phys. 46, 1560 (1971)

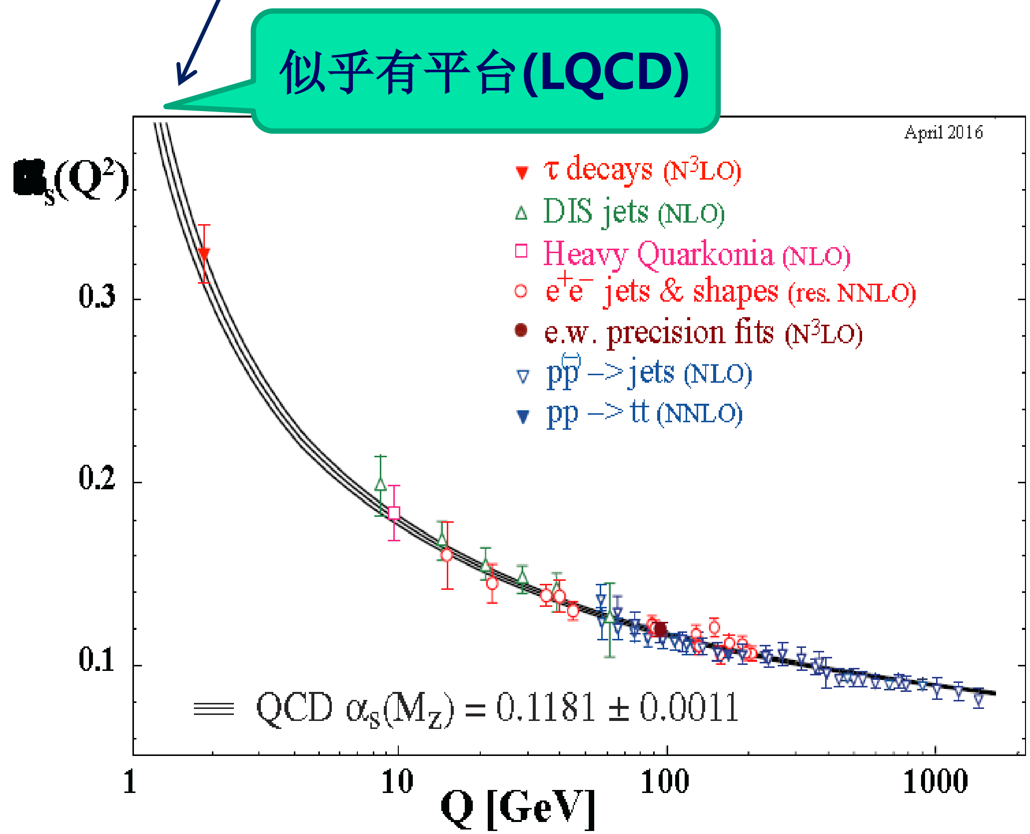
色动力学QCD：短程图像

从3MeV到300MeV
微扰论失效

胶子：光子的类似物

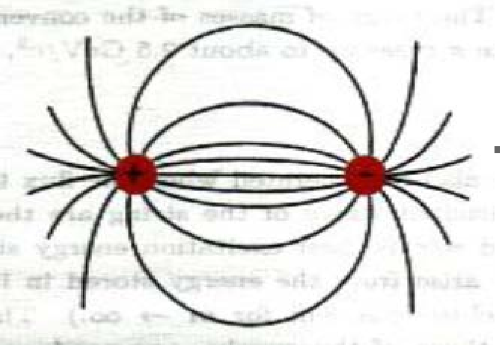
$$\mathcal{L} = \sum_i \bar{\mathbf{q}}_i [i\gamma\partial + G^a_\mu \frac{\lambda^a}{2}]\mathbf{q}_i - \frac{1}{4} trGG$$



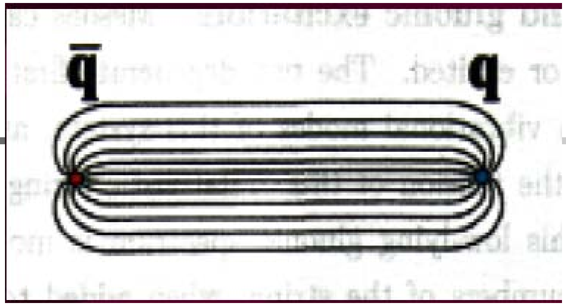


不同于QED，QCD是非阿贝尔规范理论：G^a胶子自作用/色荷反屏蔽(渐进自由)/夸克(色)禁闭 /手征对称SSB

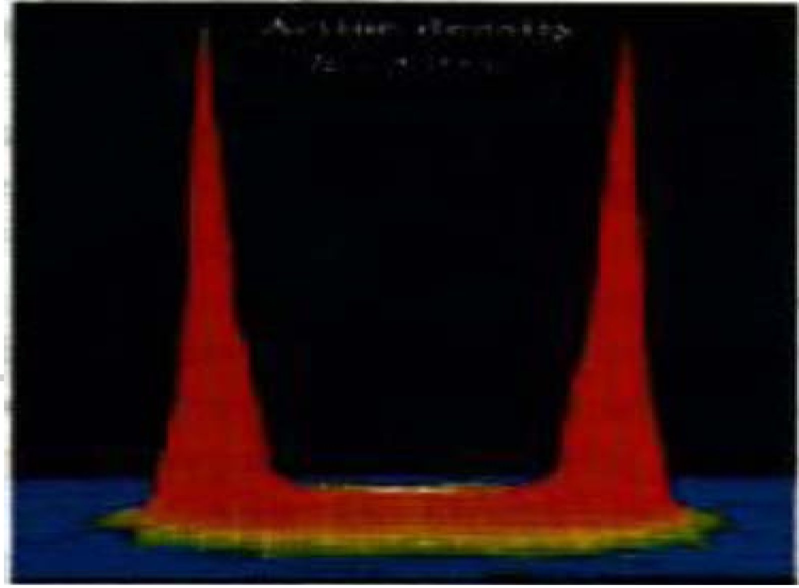
长程QCD： 色禁闭困难



QED的电力线



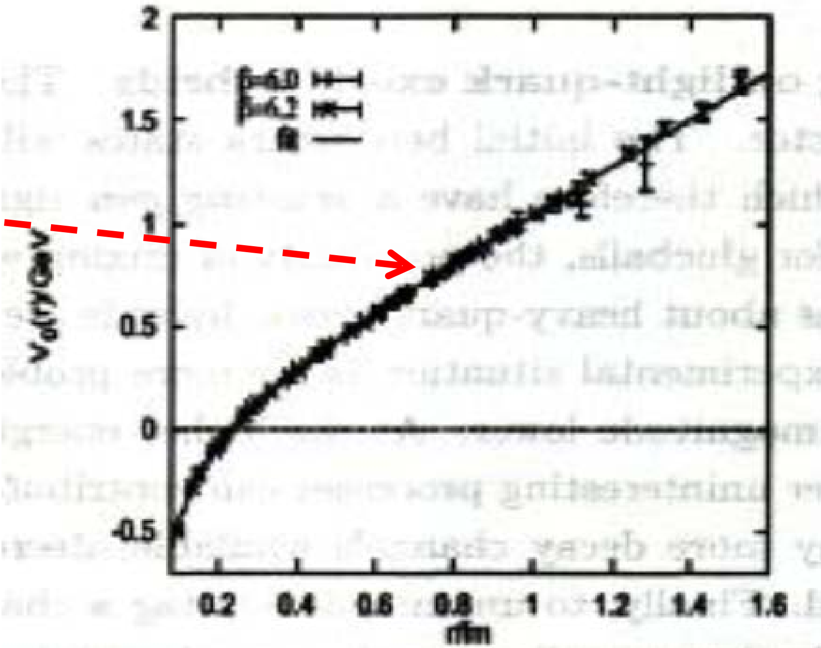
QCD的色力线



格点QCD计算:静源

$$V_{confining}(r) = -\frac{A}{r} + br + c \qquad \text{(L0)}$$

静源Q$\overline{Q}$ 间的势 ---短程库仑+长程线性位势



格点QCD研究的意义：

- 基本理解了粲（底）夸克偶素谱学
- 为QCD弦模型提供了依据。
- 成为多数Quark模型的基础（De Rujula-Georgi-Glashow, Godfrey-Isgur(Capstick)等
- 该势反映QCD夸克间作用多少？依然是个谜

G. S. Bali, Phys. Rept. 343, 1 (2001).
T. Kawanai and S. Sasaki, Prog. Part. Nucl. Phys. 67(2012) 130

Questions:

- 为何观测到单重味介子和重子也服从线性RT?
- 如何解释单重味介子和重子的RT质量公式的味道依赖性?
- 如何解释单重味重子的P波质量劈裂? 劈裂为何看起来较窄?

Questions:

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- 如何解释单重味重子的P波质量劈裂? 劈裂为何看起来较窄?

重轻介子和重子的普适RT

K. Chen, Y. Dong, X. Liu, Q. F. Lu and T. Matsuki, Eur. Phys.J. C 78, 20 (2018).

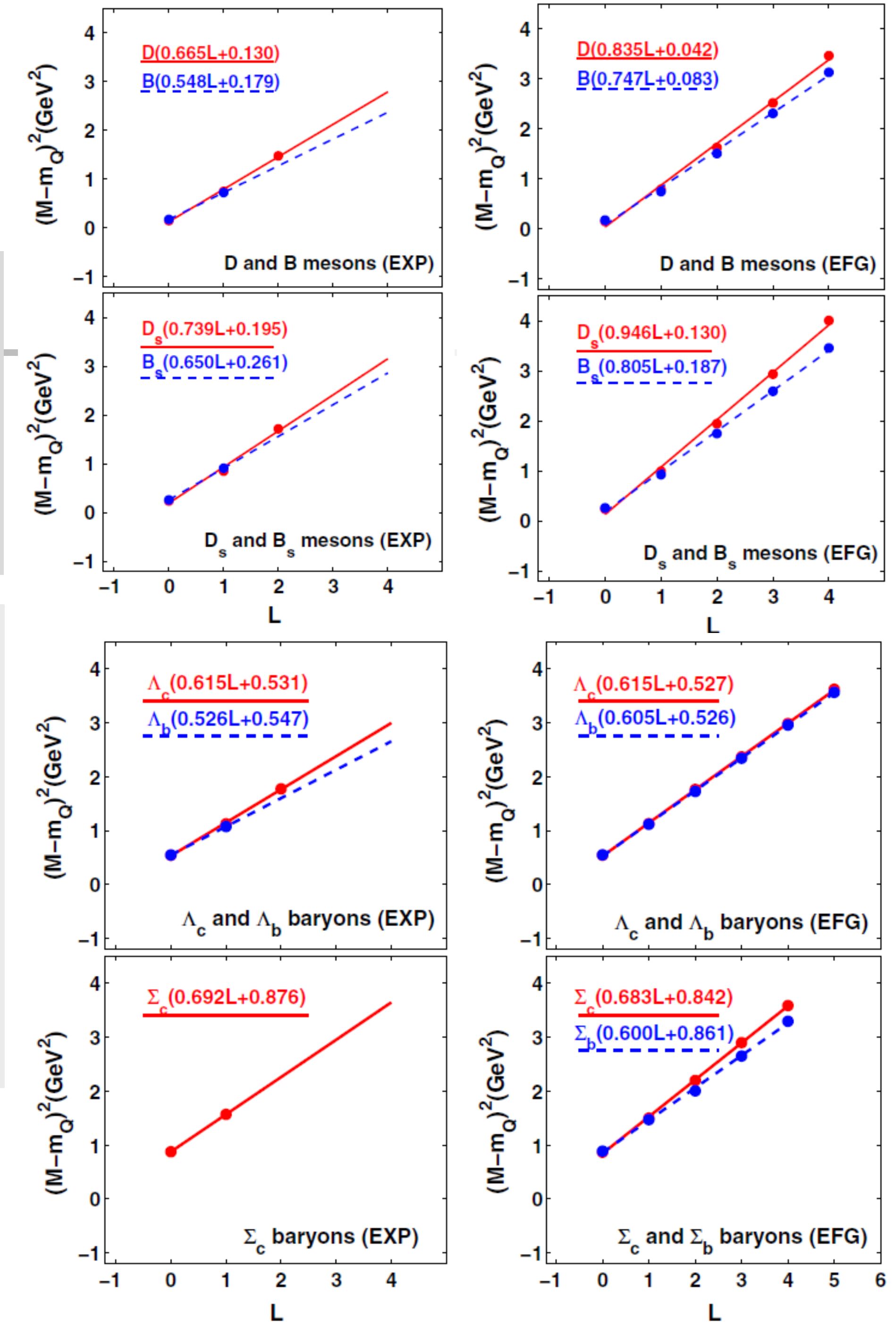
Analysis for the whole HL systems of mesons and baryons suggests that instead of mass itself, **gluon flux energy** is essential to obtain a linear trajectory

Finding： RT for HL systems are flavor independent IF the eff. Heavy quark masses m_Q subtracted.

Slope＝ 0.6 ±0.1 GeV²
= 1.25 之半

$$(M - M_Q)^2 = aL + a_0$$

shifted Regge relations



数值拟合和预言

Duojie. Jia*,Wen-Chao Dong, **Eur. Phys. J. Plus** (2019) 134: 123 arXiv:1811.04214

- 定出夸克有效质量和斜率之后，可以预言：
- The $D(3000)^0$: 3P state,
- $B_J(5840)/B_J(5970)$: 2S,

$D(2P) \approx$ 2908MeV

$D(2D) \approx$ 3148MeV.

waiting for forthcoming Belle II and LHCb experiments for test

TABLE V: The trajectory parameters (α', α_0) in (4) and predicted by Ref. [47]. The unit of the α' is in GeV^{-2}.

Traj. Parameters	$c\bar{n}$(natural J^P)	$c\bar{s}$(natural J^P)	$b\bar{n}$(natural J^P)	$b\bar{s}$(natural J^P)
This work(α', α_0)	(1.21, −0.52)	(1.01, −0.72)	(1.04, −0.97)	(0.86, −1.13)
EFG (α', α_0) [47]	(0.494, −1.00(4))	(0.469, −1.10(4))	(0.254, −6.30(36))	(0.249, −6.43(51))
	(0.548, −3.21(12))	(0.497, −3.16(12))	(0.263, −8.77(47))	(0.259, −8.87(58))

RT斜率比的一个论证

Why π/2 in mass relation?

利用约化的弦位势，借助**WKB**量子化条件

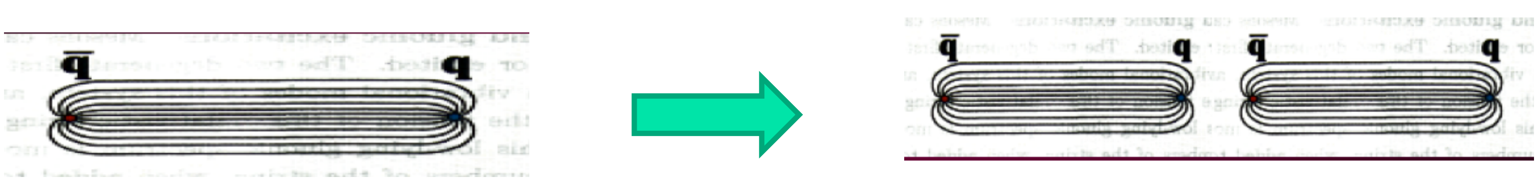
$$H = M_Q + \sqrt{\mathbf{p}^2 + m_q^2} + \frac{\pi}{2}Tr + Tr_Q\left[1 - \frac{\pi}{2}\right].$$

$$\cong M_Q + |\mathbf{p}| + \frac{\pi}{2}Tr, \quad M_Q >> 1$$

$$2\int_0^{(E-M_Q)/a} (E - M_Q - a|x|)dx = \pi(n+b),$$

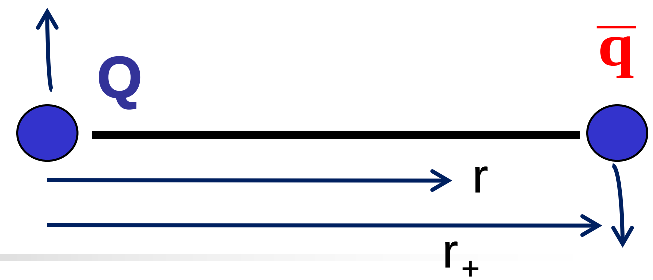
$a \equiv \pi T/2$, that is,

$$(E - M_Q)^2 = \pi a\,(n+b) = \pi T\left(\frac{\pi}{2}n + \frac{\pi}{2}b\right),$$



QCD弦模型： RT 推导

- Assuming linear confinement between quark and antiquark in a light meson as string indicated,
- the spinless Salpeter Hamiltonian



$$E=\frac{m_Q}{\sqrt{1-v_Q^2}}+\frac{m}{\sqrt{1-v_d^2}}+\frac{a}{\omega}\sum_{i=Q,d}\int_0^{v_i}\frac{du}{\sqrt{1-u^2}},\qquad (A1)$$

$$L=\frac{m_Q v_Q^2/\omega}{\sqrt{1-v_Q^2}}+\frac{m v_d^2/\omega}{\sqrt{1-v_d^2}}+\frac{a}{\omega^2}\sum_{i=Q,d}\int_0^{v_i}\frac{u^2du}{\sqrt{1-u^2}},\qquad (A2)$$

$$M_Q=\frac{m_Q}{\sqrt{1-v_Q^2}},\qquad m_d=\frac{m}{\sqrt{1-v_d^2}},\qquad (A3)$$

to rewrite (A1) and (A2) as

$$E=M_Q+m_d+\frac{a}{\omega}[\arcsin(v_d)+\arcsin(v_Q)],\qquad (A4)$$

$$L=\frac{1}{\omega}(M_Q v_Q^2+m_d v_d^2)+\frac{a}{2\omega^2}\sum_{i=Q,d}[\arcsin(v_i)-v_i\sqrt{1-v_i^2}],\qquad (A5)$$

$$\frac{a}{\omega}=\frac{m_Q v_Q}{1-v_Q^2}=\frac{M_Q v_Q}{\sqrt{1-v_Q^2}},$$

$$E=M_Q+m_d+\frac{\pi a}{2\omega}+\frac{a}{\omega}\left[v_Q-\frac{m}{m_d}+\frac{1}{6}v_Q^3\right]+\mathcal{O}[v_Q^5],\qquad (A8)$$

$$\omega L=m_d+M_Q v_Q^2+\frac{a}{\omega}\left[\frac{\pi}{4}-\frac{m}{m_d}\right]+\frac{a}{3\omega}v_Q^3+\mathcal{O}[v_Q^5].\qquad (A9)$$

Using Eq. (A7) and upon eliminating ω, Eqs. (A8)–(A9) combines to give, when ignoring the small term m/m_d,

$$(E-M_Q)^2=\pi a L+\left(m_d+\frac{P_Q^2}{M_Q}\right)^2-2mP_Q.\qquad (A10)$$

QM中的味道依赖性：短程色作用

Summary of the Cornell parameters and the quark mass determined from lattice QCD. For comparison, the corresponding values adopted in a non-relativistic potential (NRp) model [3] are also included.

	This work	Polyakov lines	NRp model
A	0.861 (17)	0.403 (24)	0.7281
$\sqrt{\sigma}$ [GeV]	0.394 (7)	0.462 (4)	0.3775
m_Q [GeV]	1.74 (3)	∞	1.4794

- 味道依赖性可以来自短程库仑相互作用 *A*.
- Also From the state-average of the 1/r interaction which depends on radius of meson considered
- The linear parameter b is roughly F-independent

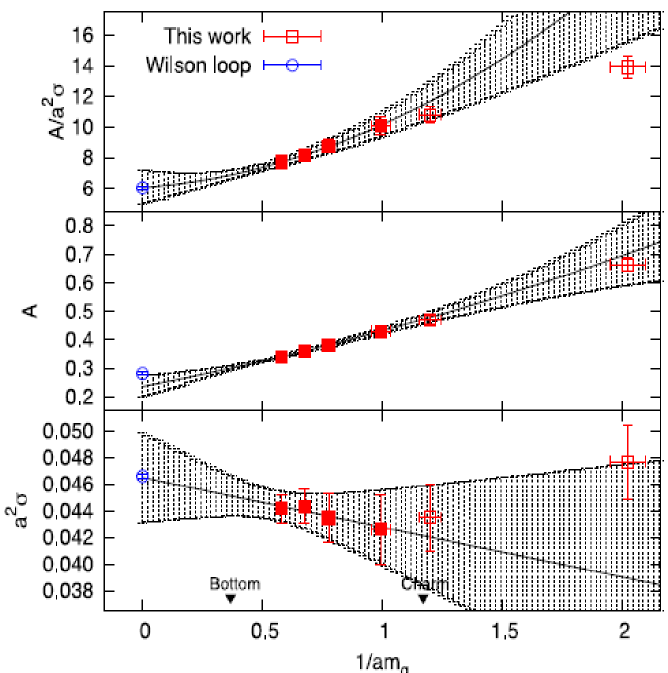
$$V_{\overline{Q}Q}(r) = br + c - \frac{A}{r}$$

$$\left\langle c - \frac{A}{r} \right\rangle = c - \left\langle \frac{A}{r} \right\rangle = c - \overline{A}\left\langle \frac{1}{r} \right\rangle$$

$$\overline{A}\left\langle \frac{1}{r} \right\rangle \propto \overline{A}\frac{1}{Radius} \propto \overline{A}\mu_{qQ}\sqrt{b}$$

$$by\ Radius \propto 1/(\mu_{qQ}\sqrt{b})$$

Binding among HF/strangeness



T. Kawanai and S. Sasaki, PRL 107, 091601 (2011)
T. Kawanai and S. Sasaki, Prog. Part. Nucl. Phys. 67(2012) 130

Mean–Mass and Splittings of Heavy Hadrons

重味重子：KR夸克模型

M.Karliner,J. Rosner, PRD 90, 094007 (2014)

$$M=\sum_i m_i + A\frac{\mathbf{S}_i\cdot\mathbf{S}_j}{m_i m_j}+B(Qs/QQ)$$

$A=4b\,(Mesons)$

$A=4a\,(Baryon)$

- 最低态原始夸克模型
—Karliner-Rosner 模型

1. Full balance： 重味和轻味重子谱整体平衡；夸克质量和耦合完全确定/NO freedom to adjust.
2. 无位势假定/预言能力较强
4. 问题：激发态需重新定参数

Quark	In a meson	In a baryon
u,d	$m^m_{u,d}=310$	$m^b_{u,d}=363$
s	$m^m_s=483$	$m^b_s=538$
c	$m^m_c=1663.3$	$m^b_c=1710.5$
b	$m^m_b=5003.8$	$m^b_b=5043.5$

TABLE II. Quark model description of ground-state mesons containing u, d, s. Here we take $m^m_u=m^m_d\equiv m^m_q=310$ MeV, $m^m_s=483$ MeV, $b/(m^m_q)^2=80$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$\pi(138)$	0	$2m^m_q-6b/(m^m_q)^2$	140
$\rho(775)$, $\omega(782)$	1	$2m^m_q+2b/(m^m_q)^2$	780
$K(496)$	0	$m^m_q+m^m_s-6b/(m^m_q m^m_s)$	485
$K^*(894)$	1	$m^m_q+m^m_s+2b/(m^m_q m^m_s)$	896
$\phi(1019)$	1	$2m^m_s+2b/(m^m_s)^2$	1032

TABLE I. Quark model description of ground-state baryons containing u, d, s. Here we take $m^b_u=m^b_d\equiv m^b_q=363$ MeV, $m^b_s=538$ MeV, and hyperfine interaction term $a/(m^b_q)^2=50$ MeV.

State (mass in MeV)	Spin	Expression for mass [24]	Predicted mass (MeV)
$N(939)$	1/2	$3m^b_q-3a/(m^b_q)^2$	939
$\Delta(1232)$	3/2	$3m^b_q+3a/(m^b_q)^2$	1239
$\Lambda(1116)$	1/2	$2m^b_q+m^b_s-3a/(m^b_q)^2$	1114
$\Sigma(1193)$	1/2	$2m^b_q+m^b_s+a/(m^b_q)^2-4a/m^b_q m^b_s$	1179
$\Sigma(1385)$	3/2	$2m^b_q+m^b_s+a/(m^b_q)^2+2a/m^b_q m^b_s$	1381
$\Xi(1318)$	1/2	$2m^b_s+m^b_q+a/(m^b_s)^2-4a/m^b_q m^b_s$	1327
$\Xi(1530)$	3/2	$2m^b_s+m^b_q+a/(m^b_s)^2+2a/m^b_q m^b_s$	1529
$\Omega(1672)$	3/2	$3m^b_s+3a/(m^b_s)^2$	1682

重味重子：双粲重子质量

$B(Qs/QQ)$

Pair q_1q_2	$B(q_1q_2)$	$B(q_1\bar{q}_2)$
cs	35.0	70.0
bs	41.8	83.6
cc	129	258
bc	170.8	341.5
bb	281.4	562.8

TABLE VII. Contributions to the mass of the lightest doubly charmed baryon Ξ_{cc}.

Contribution	Value (MeV)
$2m_c^b + m_q^b$	3783.9
cc binding	−129.0
$a_{cc}/(m_c^b)^2$	14.2
$-4a/m_q^b m_c^b$	−42.4
Total	3627 ± 12

M(Ξ_{cc}) = 3621.40 ± 0.78 MeV [LHCb]

M.Karliner,J. Rosner, PHYSICAL REVIEW D 90, 094007 (2014)

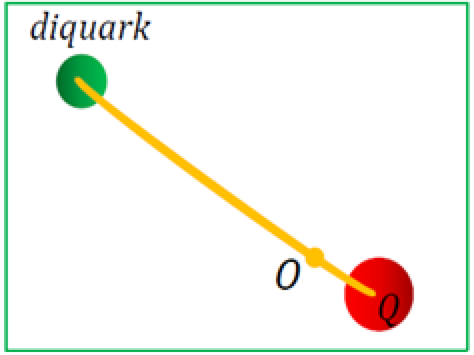
State (M in MeV)	Spin	Expression for mass	Exp	Predicted M (MeV)
$\Lambda_c(2286.5)$	1/2	$2m_q^b + m_c^b - 3a/(m_q^b)^2$	2286	Input
$\Sigma_c(2453.4)$	1/2	$2m_q^b + m_c^b + a/(m_q^b)^2 - 4a/(m_q^b m_c^b)$	2453	2444.0
$\Sigma_c^*(2518.1)$	3/2	$2m_q^b + m_c^b + a/(m_q^b)^2 + 2a/(m_q^b m_c^b)$	2518	2507.7
$\Xi_c(2469.3)$	1/2	$B(cs) + m_q^b + m_s^b + m_c^b - 3a/(m_q^b m_s^b)$	2468	2475.3
$\Xi'_c(2575.8)$	1/2	$B(cs) + m_q^b + m_s^b + m_c^b + a/(m_q^b m_s^b) - 2a/(m_q^b m_c^b) - 2a_{cs}/(m_s^b m_c^b)$	2578	2565.4
$\Xi_c^*(2645.9)$	3/2	$B(cs) + m_q^b + m_s^b + m_c^b + a/(m_q^b m_s^b) + a/(m_q^b m_c^b) + a_{cs}/(m_s^b m_c^b)$	2445	2632.6
$\Omega_c(2695.2)$	1/2	$2B(cs) + 2m_s^b + m_c^b + a/(m_s^b)^2 - 4a_{cs}/(m_s^b m_c^b)$	2695.2	2692.1[a]
$\Omega_c^*(2765.9)$	3/2	$2B(cs) + 2m_s^b + m_c^b + a/(m_s^b)^2 + 2a_{cs}/(m_s^b m_c^b)$	2765.9	2762.8[a]

[a]Difference between experimental values used to determine $6a_{cs}/(m_s^b m_c^b) = 70.7$ MeV.

- 基态预言能力较强

激发态重味重子：RT

Duojie Jia, W-N Liu, A. Hosaka, PRD101, 034016 (2020)



• Using heavy quark-diquark picture, One can do the same fit for the HL baryons, to propose a mass relation :

1. Heavy quark masses, fixed by HL mesons, can be used to determine the massed of scalor qq and qs diquark fitting the data in 4MeV error
2. Without any potential assumption
4. Once Traj. given, it is of predictive

Proposal for HL baryons

$$M_L = M_Q + \sqrt{\pi a\left(L + \frac{\pi}{2} n\right) + a_M}$$

$$a_M = \left(m_d + M_Q\left(1 - \frac{m_{bareQ}^2}{M_Q^2}\right)\right)^2$$

Numerical results:

TABLE IX. The effective masses (in GeV) of quarks that match the observed spin-averaged masses in Table VII and VIII, with a in GeV and the RMS error $\chi_{\mathrm{RMS}} = 0.001$ GeV. The comparison with that by quark model is given.

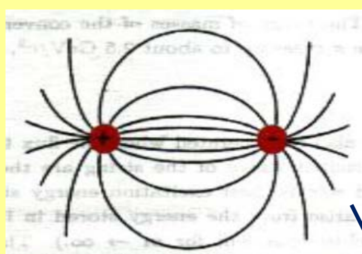
Parameters	M_c	M_b	m_n	m_s	$a\,(c\bar{n})$	$a\,(c\bar{s})$	$a(b\bar{n})$	$a(b\bar{s})$
This work	1.44 [input]	4.48 [input]	0.23	0.328	0.223	0.249	0.275	0.313
EFG [34]	1.55	4.88	0.33	0.5	0.64/0.58	0.68/0.64	1.25/1.21	1.28/1.23

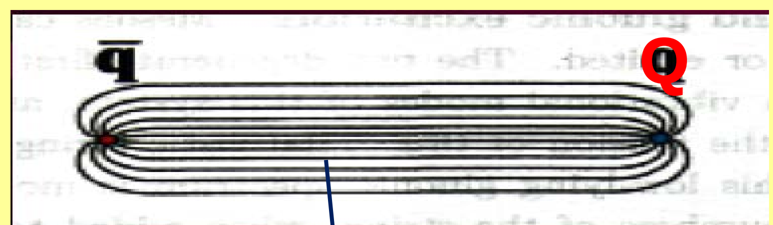
Traj.	$Q = c$	$Q = b$
Λ_Q	$1.37059 + \sqrt{0.827665 + 0.714879(L + n\pi/2)}$	$4.40982 + \sqrt{1.48262 + 0.790019(L + n\pi/2)}$
Ξ_Q	$1.34907 + \sqrt{1.24725 + 0.890221(L + n\pi/2)}$	$4.37522 + \sqrt{2.02969 + 1.038(L + n\pi/2)}$

EFG: D. Ebert, et al., Phys. Rev. D 84, 014025 (2011)

Regge-like 质量关系的解释

- Both the heavy quark mass M_Q and short-distance Binding B need to be removed from hadron mass M in order to restore the linear Regge trajectory;
- The square root in M is due to relativistic motion of flux-tube (long-distance glue-dynamics) in chiral limit, as Nambu and others illustrated.
- The factor *k* remains to be explored, universal/flavor-dependent?





$$M_{nL} = M_Q - k\frac{m_d}{1 + m_d/M_Q} + \sqrt{\pi b\left(L + \frac{\pi}{2}n\right) + a_M}$$

Duojie. Jia*,Wen-Chao Dong, **Eur. Phys. J. Plus** (2019) 134: 123 arXiv:1811.04214

Duojie Jia, W-N Liu, A. Hosaka, PRD101, 034016 (2020)；arXiv:1907.04958：

轻介子的RT质量关系

With dynamical computation by spinless BS Hamiltonian we show that (IJMPA,2017) NO shift by quark mass m happen but mass corrects the slope and squre-root flux-tube part onlinearly:

$$M = \left(\frac{3}{2}+\frac{1}{c_N}\right)\sqrt{2a(L+3/2-2\overline{\alpha}_s)-2\overline{m}^2}+\frac{(4/3)a\overline{\alpha}_s}{\sqrt{2a(L+3/2-2\overline{\alpha}_s)-2\overline{m}^2}}+V_0$$

$$c_N = \left[1-\frac{2\overline{\alpha}_s}{3(L+3/2)}-\frac{\overline{m}^2}{a(L+3/2)}\right]^2$$

Flavor-dependent due to m in Regge traj.

$$(M-V_0)^2 = 2a\left(\frac{3}{2}+\frac{1}{2c_N}\right)^2\left[L+\frac{3}{2}-\frac{5\overline{\alpha}_s}{3}\right]\propto L$$

$$\alpha' = \frac{1}{2a}\left(\frac{3}{2}+\frac{1}{2}\left(1-\frac{2\overline{\alpha}_s}{3(L+3/2)}-\frac{\overline{m}^2}{a(L+3/2)}\right)^{-2}\right)^{-2}$$

Jia Duojie, C.Q Pang,Atsushi Hosaka, **Int. J. Mod. Phys. A** 32 (2017) 1750153; arXiv:1706.02788v1

重味重子的RT质量关系

西北师范大学 NORTHWEST NORMAL UNIVERSITY

给出重味重子激发谱的雷吉质量公式，统一计算和预言了的负宇称$\Sigma_{c,b}/\Xi'_{c,b}$重子的量子数（$3/2^-$）， 预言LHCb2019年新发现的中性$\Lambda_b(6146,6152)$ 乃Λ_b之D波激发态。此项研究对于Gell-mann夸克模型和组分质量概念提供了新的解释，推动了重味强子激发态进展。

预言Ξ_b / Λ_b 之高激发态D波质量

Duojie Jia, W-N Liu, A. Hosaka, PRD101, 034016 (2020)；arXiv:1907.04958；

ırkably well with the observed spin-averaged mass 6149.97MeV of orted very recently by LHCbcite: Lamdab19:

EXP:

$$M[\Lambda_b(6146)^0] = 6146.17 \pm 0.33 \pm 0.22 \pm 0.16\text{MeV},$$
$$M[\Lambda_b(6152)^0] = 6152.51 \pm 0.26 \pm 0.22 \pm 0.16\text{MeV}.$$

ce 5/2 in $\mathbf{L}\cdot\mathbf{S}_Q$ between the $5/2^+$ and $3/2^+$ states, one can extra)] $= \frac{2}{5}[\Lambda_c(5/2^+) - \Lambda_c(3/2^+)] = 10.21$MeV. The scaling rel

$$a_2[\Lambda_b(1D)] = a_2[\Lambda_c(1D)]\left(\frac{1.44}{4.48}\right) = 3.28\text{MeV},$$

MeV), by Eq. (ref: HDw),

Theory:

$$M[\Lambda_b(1D)] = 6149.3 + 3.28\begin{bmatrix} -3/2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6144.4 & 0 \\ 0 & 6152.6 \end{bmatrix}.$$

$J = 3/2^+, 5/2^+$, respectively. This is in good agreement with the o

Baryon	Mass (Exp:)	This work (预言)
Ξ_c^+	2467.87(30)	2469.1
$\Xi_c(2790)^+$	2792.0(5)	2778.6
$\Xi_c(2815)^+$	2816.67(31)	2816.5
$\Xi_c(3055)^+$	3055.9(4)	3058.7
$\Xi_c(3080)^+$	3077.2(4)	3079.7
Ξ_b	5791.9(5)	5792
Ξ_b	??	6116.9
Ξ_b		6129.1
Ξ_b		6376.9
Ξ_b		6383.6

MeV