Pion form factor in pQCD at the intermediate and large momentum transfers

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Overview

1. Motivation

2. Formulism

3. Numerics

4. Outlook

"Hitting one constituent of a bound state with a photon initiates a complicated process if the bound state is not to fall apart. The momentum gained must be redistributed between all the constituents so that the whole convey can slew round into the new direction. A price is paid in terms of a reduced interaction strength between the photon and the bound state and this is known as the electromagnetic form factor - a function of the square of the (spacelike) 4-momentum, q^2, transferred from initial to final state."

[HPQCD collaboration, Phys.Rev.D96(2017)054501]

Motivation

† Open question: Hadron matrix element
Factorization approaches: LCDAs $\otimes$ HK
Exclusive B,D decays: CPV and NP

† From factor (FF): The most simple HME
Pion e.m. FF: The most direct and easy way to study QCD
Both for pert. and nonpert. information

† Direct and indirect measurements: [0.25, 3] GeV^2,
[CLEO 2005; Jefferson Lab 2001,2006,2008; et.al.,]

† Lattice QCD (LQCD): a few points of small Q^2
[HPQCD, UKQCD, MILC and Fermilab Lattice Collaborations 2004]
[QCDSF/UKQCD 2007; ETM 2009; JLQCD and TWQCD 2009; JLQCD 2016, et.al.,]
up to 1 GeV^2, [χQCD 2020]

† QCD-based approaches
DSE: 0-6 GeV^2 Lei Chang's talk;
QCDSR: 1-2 GeV^2; LCSRs: 1-15 GeV^2; pQCD: $\gtrsim$ 10 GeV^2;
[B.L Ioffe 1982] [P. Ball 1991, V. Braun 1994,2000, **SC** 2020] [Li 2001, Li 2012, **SC** 2014]

Motivation

† External reason (large Q^2) of the small relative distance $\bar{z}_i \bar{p}_i \sim 1$
$\rightarrow$ OPE in twist(power);
~~Internal reason (M_Z, M_b): $\bar{z}_i \ll 1/\mu_t$~~

† the controversy between LCSRs and pQCD in small-x region,

$\ln(Q^2/(p-q)^2)$	*VS*	$\ln(Q^2/\Lambda^2_{QCD})$;		
$\ln(Q^2/M^2)$ and $\ln(Q^2/S_0)$	*VS*	$\ln(Q^2/K_T^2)$;		
not like to occur	*VS*	Sudakov suppresssion ;		
"soft" + "hard"	*VS*	Hard scattering	$\leftarrow$	Leading power;

† "k_T" + multiparton $\leftarrow$ High powers;

† Accurancy:

'$q\bar{q}$' + '$qg\bar{q}$', twist four DAs [V. Braun 1994]	*VS*	'$q\bar{q}$','$qg\bar{q}$', twist three [Li 2001]	$\leftarrow$	LO
'$q\bar{q}$' + '$qg\bar{q}$', twist six DAs [V. Braun 2000, J. Bijnens 2002]	*VS*	'$q\bar{q}$','$qg\bar{q}$', twist three [Li 2012, **SC** 2014]	$\leftarrow$	NLO-2p

Motivation

Contribution from twist four DAs

† LCSRs: a visible enhancement at $Q^2 > 10$ GeV^2, $\sim 40\%$,
the same asymptotic behaviour as twist two $\mathcal{O}(1/Q^4)$ [J. Bijnens 2002]
LCSRs holds in the intermediate energy region.

† Check the twist expansion in pQCD at large Q^2

† Improve the precision of pQCD prediction

† In turn, with measurements, to determine the nonpert. parameters

† Pion, Kaon, $SU(3)_f$ breaking

Formulism

- Factorization formula

$$
\begin{aligned}
& \langle \pi^-(p_2) | J_\mu^{e.m.} | \pi^-(p_1) \rangle \\
= \quad & \oint dz_1 dz_2 \left\langle \pi^-(p_2) \right| \left\{ \overline{d}_\gamma(0) \exp\left(i g_s \int_{z_2}^{0} d\sigma_{\nu'} A_{\nu'}(\sigma) \right) u_\beta(z_2) \right\}_{kj} \left| 0 \right\rangle_{\mu_t} \\
& \cdot H^{ijkl}_{\gamma\beta\alpha\delta,\mu}(z_2, z_1) \cdot \left\langle 0 \right| \left\{ \overline{u}_\alpha(z_1) \exp\left(i g_s \int_{0}^{z_1} d\sigma_\nu A_\nu(\sigma) \right) d_\delta(0) \right\}_{il} \left| \pi^-(p_1) \right\rangle_{\mu_t}, \quad (1)
\end{aligned}
$$

- Seperation: SD (propagator) and LD (Heisenberg operator).
- Hard kernel and nonlocal MEs $\Leftarrow$ spin structures (Fierz trans.).
- nonlocal MEs, defined in terms of LCDAs by twist.
- Truncated (factorizable) scale μ_t.

Formulism

- hard kernel associated with the lowest Fock state

$$H^{ijkl}_{\gamma\beta\alpha\delta}(z_1, z_2) = (-1)\,[ig_s\gamma_m]_{\alpha\beta}\, T^{ij}\left[(ie_q\gamma_\mu)S_0(0-z_1)(ig_s\gamma_n)\right]_{\gamma\delta} T^{kl}\left[-iD^0_{mn}(z_1-z_2)\right], \tag{2}$$

- the free propagators in the coordinate space

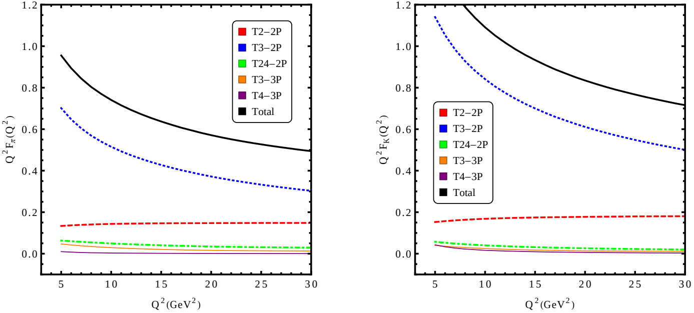
$$S_0(z) = \frac{i}{2\pi}\frac{\not{z}}{z^4}, \quad D^0_{mn}(z) = \frac{1}{4\pi}\frac{g_{mn}}{z^2}. \tag{3}$$

- nonlocal matrix elements in Eq. (1), the mesons breaking-up into a pair of soft quarks, different spin structures

$$\begin{aligned}
&\langle 0\Big|\left\{\bar{u}_\alpha(z_1)\exp\left(ig_s\int_0^{z_1} d\sigma_\nu A_\nu(\sigma)\right)d_\delta(0)\right\}_{il}\Big|\pi^-(p_1)\rangle_{\mu_t}\\
&= \frac{\delta_{il}}{3}\left\{\frac{1}{4}(\gamma_5\gamma^\rho)_{\delta\alpha}\langle 0|\bar{u}(z_1)\exp\left(ig_s\int_0^{z_1} d\sigma_\nu A_\nu(\sigma)\right)(\gamma_\rho\gamma_5)\,d(0)|\pi^-(p_1)\rangle_{\mu_t}\right.\\
&\quad+\frac{1}{4}(i\gamma_5)_{\delta\alpha}\langle 0|\bar{u}(z_1)\exp\left(ig_s\int_0^{z_1} d\sigma_\nu A_\nu(\sigma)\right)(i\gamma_5)\,d(0)|\pi^-(p_1)\rangle_{\mu_t}\\
&\quad+\frac{1}{8}\left(\sigma^{\tau\tau'}\gamma_5\right)_{\delta\alpha}\langle 0|\bar{u}(z_1)\exp\left(ig_s\int_0^{z_1} d\sigma_\nu A_\nu(\sigma)\right)(i\sigma_{\tau\tau'}\gamma_5)\,d(0)|\pi^-(p_1)\rangle_{\mu_t}\\
&\quad\left.+\cdots\right\}.
\end{aligned} \tag{4}$$

Formulism

- 2p-2p scattering: T2⊗T2, T3⊗T3, T2⊗T4
  3p-3p scattering: T3⊗T3, T4⊗T4
  ~~2p-3p, 3p-2p scattering~~ $\leftarrow$ color transparency mechanism
- Gauge invariance: $\propto k_T$ in 2p-2p cancels with 3p-3p scattering
  four categories: $N_g = 0, 1, 2$ and the 3g vertex configuration [Li 2001]
- 3p-3p scattering:
  dominated by the gauge invariant configuration with 4g vertex
- Power expansion: $\mathcal{O}(1) : \mathcal{O}(\frac{m_0^{\pi 2}}{Q^2}) : \mathcal{O}(\frac{\delta_{\mathcal{P}}^2}{Q^2}) : \mathcal{O}(\frac{f_{3\mathcal{P}}^2}{f_{\mathcal{P}}^2 Q^2}) : \mathcal{O}(\frac{\delta_{\mathcal{P}}^2}{Q^4})$



- 2p-2p scattering: T2⊗T2, T3⊗T3 ← LO+NLO [Li 2012, **SC** 2014].
- Inputs: LQCD, QCDSR and χPT relation for m_0^π.
- The chiral enhancement at T3, more stronger in Kaon case.
- $SU(3)_f$ breaking $\sim 30\%$, $R_P(Q^2) \equiv F_P^{T2}/F_P^{T3-2p}$, $A(Q^2) \equiv 1 - R_\pi/R_K$.

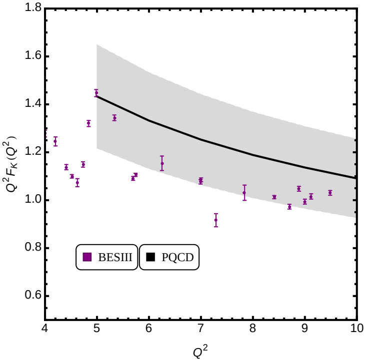
$Q^2(\mathrm{GeV}^2)$	$Q^2 F_\pi^{\mathrm{PQCD}}(Q^2)$	$Q^2 F_\pi^{\mathrm{LCSRs}}(Q^2)$	$Q^2 F_K^{\mathrm{PQCD}}(Q^2)$	$Q^2 F_K^{\mathrm{LCSRs}}(Q^2)$
10	0.75(10)	0.51(15)	1.08(15)	0.76(22)

Numerics

- Fruitful discussions with BES-III collaboration,

  in particular with W.B. Yan and G.X Huang [BES-III 2019]
- $e^+e^- \to K^+K^-$ at $\sqrt{s} = 2 - 3.08$ GeV
- Analytical continuum, $\ln(-Q^2 - i\epsilon) = \ln(Q^2) - i\pi$

$$\ln(k_T^2 - xQ^2 + i\epsilon) = \ln(k_T^2 - xQ^2) + i\pi\Theta(k_T^2 - xQ^2)$$



Outlook

- Jefferson Lab 12 GeV upgrade program $(9\,\mathrm{GeV}^2)$
  to extract the nonpert. paras (besides the LQCD contribution).

  *"provide new insights into the structure of the nucleon,
  the transition between the hadronic and quark/gluon description of matter,
  and the nature of quark confinement."* [JLAB CDR]

- Dispersion relation: spacelike $\leftrightarrow$ timelike regions
  resonances (measurements) + high energy tail (pQCD)

- BEPCII, 4.7-4.95 GeV in 2021, STCF (2-7 GeV)

  "Physics highlights: QCD and hadronic physics, Flavor physics and CP violation, New Physics search."

- nonpert. paras, fact. scale, Nucleons $\cdots$

The End, Thanks.